Volatility says less about the future than accounting rules suggest

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Volatility Indexes seem to point to the Past

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„…when you're dating someone and you know the person will marry you, you have an option that you can exercise at any time by agreeing to marry.“

Robert J. Shiller[1] teaching Options Markets.

Abstract:

In theory the market participants use the Black-Scholes-formula to asses the fair value of options as a benchmark. However, the formula requires the future volatility that has to be assessed itself. On the other hand once the prices are known the so called implied volatility matching the prices can be determined.

The volatility of the S&P 500 index, the VIX, is such an implied index published by the CBOE. Backtesting suggests that to a reasonable degree rather historical market data are used to determine future values.

Further analysis shows that neither the VIX nor historical volatility are sufficient predictors. In reality both values differ from future values significantly. The results are compared with prominent option market lectures.

Testing the prediction quality by option strategies suggests that the observed level pricing of mispricing cannot be explained by wrong volatility prediction only. The pricing formulas in use need to be revised.

Yet, this questionable practice is covered by international accounting standards (IAS/IFRS) allowing "historical data and implied volatility" for "reasonable estimations". Regression analysis shows a kind of circular reasoning when historical volatility are used for options trading and returned as implied volatility.

* Diplom-Kaufmann, University of Goettingen, March 2015 revised version of a 2008 paper

Introduction

The volatility is a measure of the "current market heat" but is assumed to be a "predictor" for the future statistical standard deviation or the risk of estimates (value at risk).

Implied volatility is computed for leading indexes like S&P 500, DJIA, NASDAQ, FTSE and DAX. Volatility is one of the most important factors for most of the financial market models including retail. This is a reason to review this property on the basis of historical data. The VIX an indicator for the volatility of the Standard & Poor's index 500 with quotations of each trading day over fifteen years is even sometimes referred to as the „fear index“ or the fear gauge, representing one measure of the market's expectation of stock market volatility over the next period.

The result was checked for. A strong correlation the VIX and the forward volatility found by backtesting. The correlation between the recent standard deviation (or actual volatility) and the VIX is even stronger. The best results are achieved when a time span of about 29 trading days or approximately 6 weeks (42 calender days) is considered. The NASDAQ Volatility, VXN, achieves even stronger correlation intensities. Methodology - Lag Correlation

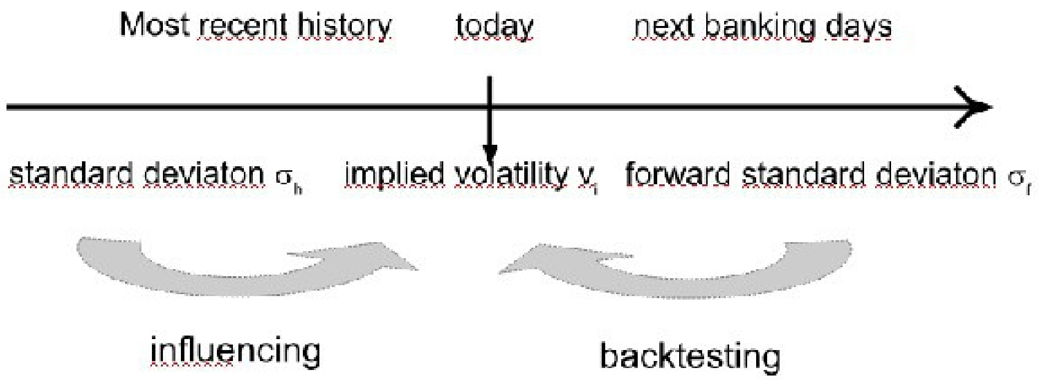
Correlation means similar behavior of two variables. The intensity of the linearity is measured by the correlation coefficient (σ, sigma) which can reach from -1 to +1. The coefficient is always measured over a span of time variables. One would speak of strong correlation if a coefficient is greater than 0.8 (very strong, "stramm" if greater than 0.9). Correlation as such suggest a hypothesis of a functional, linear dependency. More research is required to find out which variable would be the independent and which the dependent one.

Lag correlation does not compare synchronous values, but the presumably independent variable to values - later at a time lag - of the presumably dependent variable. Varying the time span, a margin with which the correlation precipitates at most can be determined if necessary. This implies the hypothesis that original information requires some time to impact the dependent, derivative variable. More general: processes require some time to perform. A classical population statistics example, the correlation of births and marriages, may illustrate the point. Marriages in the past were considered as independent events and birth as the dependent ones. This is not always unequivocal in social "processes" in contrast to natural scientific ones: It could be as well that only the knowledge about a pregnancy stimulates the decision to marry. Lag correlation can conclude what is valid or dominant in a given sample.

The „actual historical“ volatility is known „today“ but reflecting the past while the „actual future“ volatility“, to be predicted refers to the remaining time to maturity. The row of historical data starts. In this sample is about 29 (banking) days or 6 weeks respectively. There is a saying that this might be the short-term „memory“ of financial analysts.

The naming may be misleading: The standard deviation σh based on most recent historical and thus actual data is a variable which is available or can be computed easily. It can be used as a best guess for the actual volatility or for the future volatility despite the mathematical definitions are different.

The Black and Scholes option pricing formula is the most frequently used one. It expresses the call price as a function of the share price, the interest rate, time to maturity, the exercise price, and the volatility. The differential equation cannot be solved for the volatility.



Thus the implied volatility v_i is computed reversely by Monte Carlo techniques finding out the volatility that would fit the current set of option prices assumed by marketing professionals. V_i can be interpreted as a measure of the current „market heat“.

If the implied volatility is computed at day t, the data used to compute the historical standard deviation may range from t-m to t–1 with m being the number of days of one or more months. The future volatility e.g. predicted by the VIX is meant for option pricing originally for the next month (30 calendar days).

Volatility is annualized for comparison purposes. Due to the scaling formula daily volatility has to be multiplied by √252 (=average banking days) or √52 for weekly or √12 for monthly figures. However, to use the scaling formula to extend the prediction horizon is high risk[2].

To be specific v_i is a kind of „historical“ also. Normally the volatility doesn’t change too fast. However, today the implied volatility is computed, can change any other minute and remains a prediction for the next minute and can – of course – jump in case of breaking news that way.

The standard deviation σ_{future} is assumed to be the key variable for option pricing. It has to be predicted. In case there isn’t any new information available $\sigma_{history}$ is a best guess for σ_f. In case of news or even breaking news v_i will differ to incorporate the new information. σhistory is given and independent from any news - v_i. based on current market prices is not.

The prediction quality can be tested only ex post – after the fact. There are different way to test the quality:

One can test whether the the implied volatility v_i is a good predictor for the standard deviation over the future lead time which - of course - can be done ex post only. One can base any individual option strategy on either v_i or on σ_h (or any other) and find out what would have been the best the strategy. Even when v_i would predict σ_f exactly it is still uncertain whether the pricing model used is appropriate to find the right fair value matching the return of the option when executed. A way to test the prediction quality is to introduce fictitious options.

The method was introduced 2005[*]. The postulation that the total of a long series of fair vair values should match or exceed the total amount of payoffs wasn’t used by other authors yet.

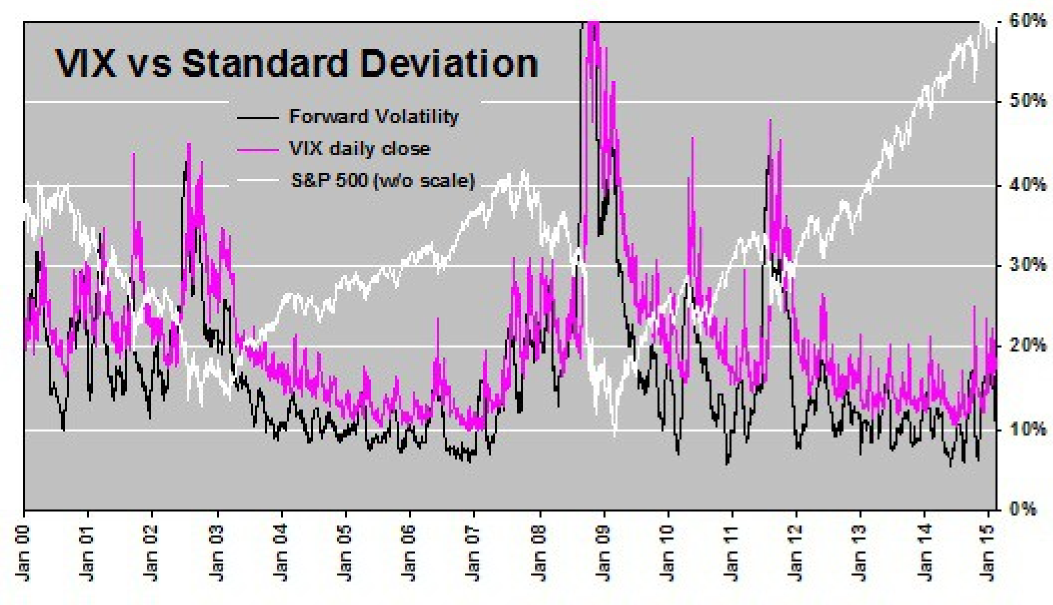
Analyzing the VIX

The intention for publishing is „The CBOE Volatility Index® (VIX® Index) is a leading measure of market expectations of near-term volatility conveyed by S&P 500 Index (SPX) option prices[3].“

Comparing the curves of the S&P stock standard deviation and the VIX a significant affinity is obvious. The first impression suggests – of course – „they line up fairly well[**]“. However, VIX and

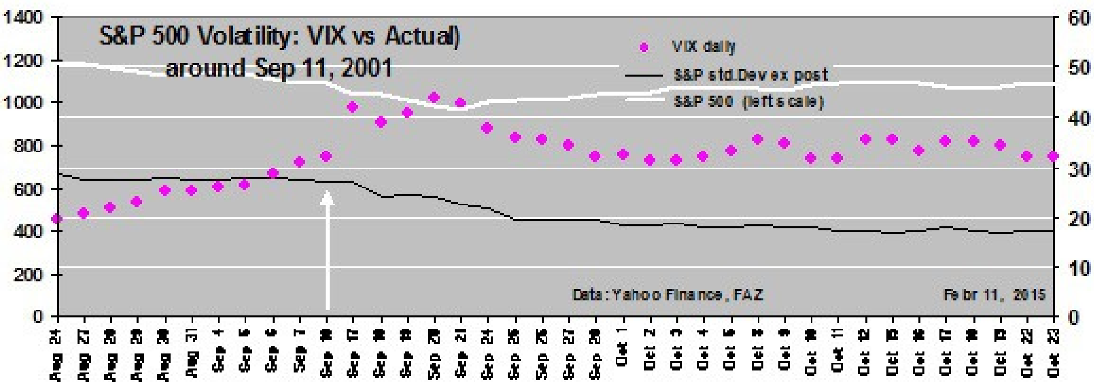
[*] Empirical Contributions to Optionpricing analyzing Black and Scholes and other Models http://128.118.178.162/eps/if/papers/0510/0510024.pdf
[**] See section: Section: “The VIX in Yale Lessons”

σhistory limb some days behind the factual figures found by backtesting. In addition the VIX is in average more than four percent higher than the deviation measured later.



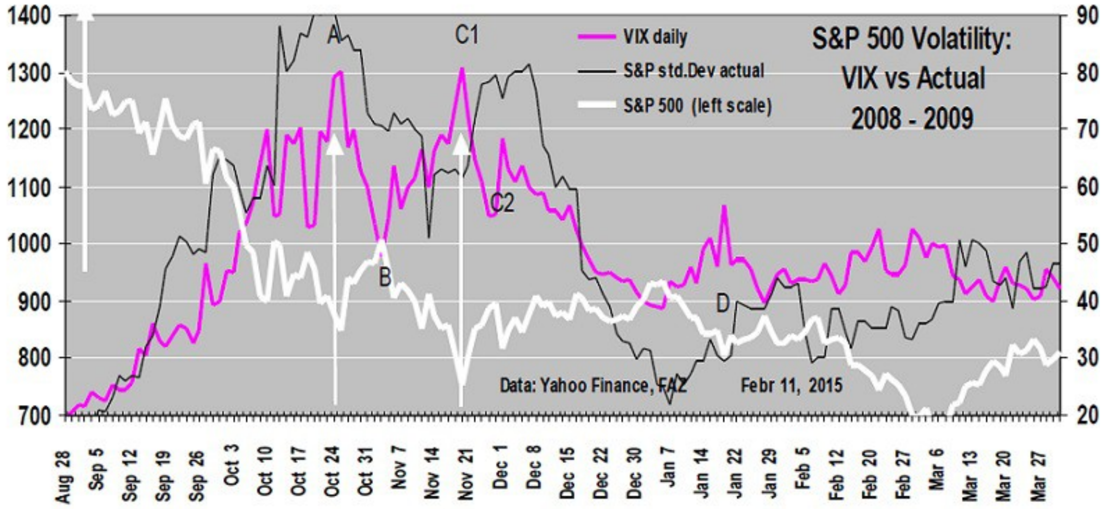
This analysis uses daily „adjusted close" figures[4] starting 2000 (instead of weekly or monthly figures. The red line represents the VIX, the green line the actual standard deviation. The black line represents the future volatility compute as the standard deviation for this period ex post.

The VIX around Sept. 11



Is the VIX is a fear index indicating significant jumps in advance? The dots are to indicate that only one VIX quotation, the close, was taken per day. There was a significant volatility jump between September 10 and 17. until end of September the level stayed obove 30 Percent. The real volatility turned out to be less than 20 percent - back to normal.

The VIX following the Lehman Collapse



Following the Lehman Brothers collapse the VIX increased from 20 to 80 percent within two months indicating turbulence until ye 2008. A (Sept. 9, 2008) and C1 are absolute VIX maxima but synchronous with the S&P lows. Volatility 80 would mean that within one year the total value of the 500 S&P-stock could vary from 1,8 times or 0,55 times the current value. Or starting at B with the S&P around 1000 the VIX could end up in the range of 1800 and 555. The S&P low Feb r. 2009 was 743 starting from 1300 Aug. 2008, The relative maximum at C1 is also synchronous with a relative VIX spike.

A VIX level of 60 percent or more indicates enormous market heat and crashing stocks very probable. However the actual standard deviation does that as well but rather synchronous than in advance of a precise due time.

VIX „explaining“* the Future?

The correlation coefficient r is a measure of the potential linearity between two variables. The square of r is the so called coefficient of determination indicating how much one variable might be „explained“ by another one. Since the actual standard deviation doesn’t „know“ the VIX to come one could say the actual volatility „explains“ 81 percent of the future volatility. 19 percent are market intuition so to speak. However that is not a total dependency. The VIX influences the option pricing which somehow influences the stock market and thus influences the next computation of the actual volatility.

* „Explaining“ as a terminus technicus represented by the square of the correlation coefficient

Table 1 Coefficients of Correlation and Determination

2000-2014	r	r^2
VIX by actual	90%	81%
Forward by VIX	79%	62%
Forward by actual	73%	53%

The computation is based on moving recent (for the prediction next) series of banking days starting 2000 until 2014. By testing different numbers of past banking days to be included the strongest correlation was achieved when including 29 banking days or approximately six weeks of recent trading. 81 percent of the VIX are determined by the recent standard deviation. 19 percent are market intuition processing relevant information as far as the option prices is concerned. However, the VIX is not the future as the standard deviation of the term until expiration.

The influence of the - moving - actual (historical) volatility or the recent average standard deviation (last line) respectively on the future volatility is still strong but lowest relatively because of three steps that have to be considered:

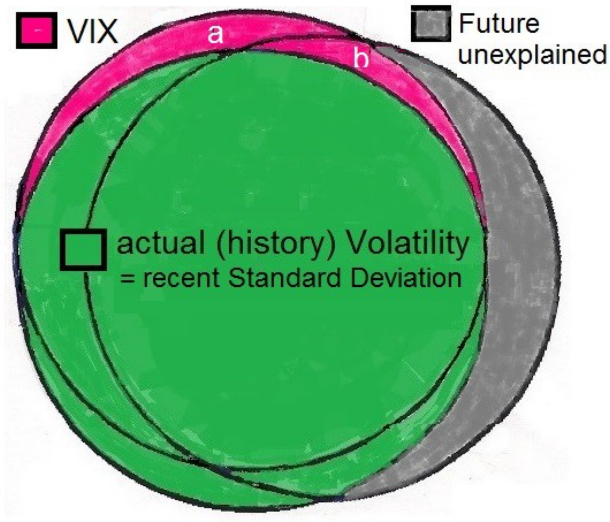
1. the determination of the VIX,
2. the impact of the VIX as an important variable of the B&S formula for options pricing and
3. the influence of the option prices on the S&P stocks and vice versa.

The influence of each of the possible pairs of data have a smoothing effect to „return“ to the standard deviation.

Visualizing the Effect of historical Data on the Future

The illustration shows the influence of the recent standard deviation on the VIX-Index and the influence of both data - actual volatility (green) and VIX (magenta) - on the future volatility of options during their run time. No prediction starts from scratch. The recent market data are obviously determining the VIX and explaine approximately 80 percent. Segments a and b of the VIX are closely related to the options prices by definition: The VIX is representing the options market.

Only a part of the VIX (segment b) adds further explanation – approximately 6 percent – to the pricing of the underlying of the 500 S&P stocks and the daily return of the next S&P course.

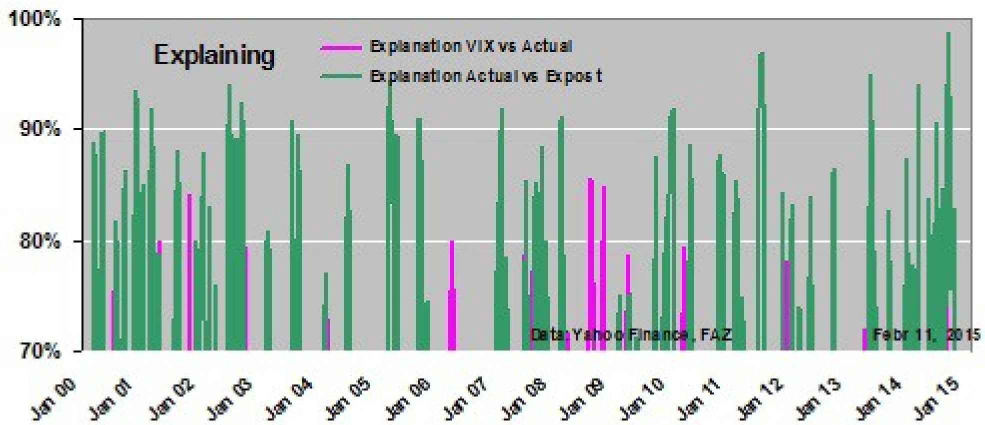


The rest (gray) - approximately 38 percent - is unpredicted influence on or reflection of the S&P 500 market that in turn is basis for the next determination of the actual volatility. The size of the partial areas are supposed to represent the degree of determination (r square).

Disclaimer: The illustration reflects the data of the recent 15 years. The analysis is done and only possible by backtesting. Tomorrow might be different.

Using moving Correlation Coefficients

Another approach is to determine the **explained** sum of squares (SQE) of the total sum of squares of the deviations of the predicted values total of a variable using a small moving sample. It can be applied again in two ways:



1. How much does the current (historic) volatility explain the implied volatility
2. How much does the implied volatility explain the volatility really happening – which can be computed only after the fact – ex post.

The explanatory power of the actual volatility (green, recent sigma) is dominant compared with the VIX.

Testing the quality of predictions by S&P 500 related option strategies

Mispricing of Fictitious Options

Std. Deviation S&P

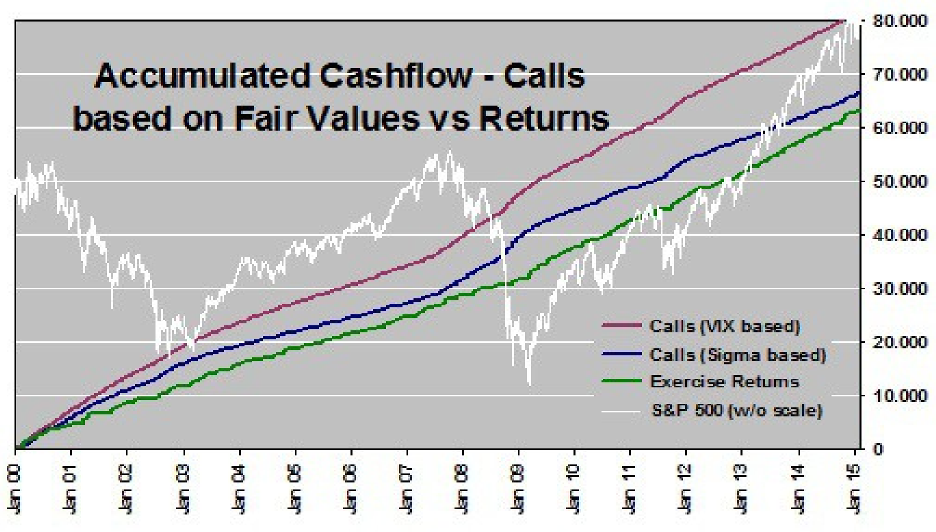
	Calls	Puts
FV(B&S):	**17,57**	**17,02**
Hits	**57%**	**43%**
	2144	1602
Mean Payoff	**16,68**	**14,83**
Mispricing:	**5%**	**15%**

VIX

	Calls	Puts
FV(B&S):	**21,52**	**21,01**
Hits	**57%**	**43%**
	2144	1602
Mean Payoff:	**16,68**	**14,83**
Mispricing:	**29%**	**42%**

The rational: Whether a single option is successful is arbitrary. However, the 2144 successful calls (57 percent) should amount in settlements equal to the payment for all 3746 calls - transaction costs not considered – if the pricing formula is appropiate. The costs however exceed the settlements by 5 percent and when using the VIX for the B&S formula even by 29 percent.

Accumulated Cash flow



Another way of demonstration is to accumulate costs and returns.:

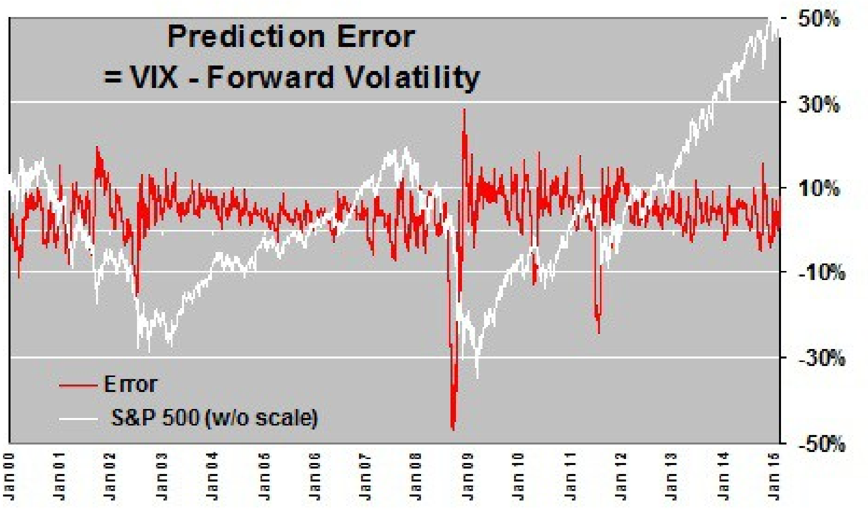
The strategy insinuated means buying VIX calls every banking day at fair value level either computed with the recent standard deviation (sigma) or with the VIX – 3746 times. Risk less interest rates were set to 1 percent. When buying S&P 500 stock at fair value level according this strategy it would cost accumulated 81 thousand dollar (VIX case) or 66 thousand dollar (sigma case). However, the total payoff of exercised options cumulate in 63 thousand dollar only.

Resume: Buying at VIX based fair values every day would have been definitely too expensive for the last 15 years. Buying at sigma based fair values would have caused a smaller loss. Nobody would follow either strategy. The intention is testing the prediction. Of course, the 57 percent „winning“ calls are profitable. Buying calls (or puts) means assuming an increase (decrease).

The differences between the sigma case and the returns must be related to properties of the B-S formula and the violated assumptions. The difference between VIX and Sigma dased strategies is due to the fact that the VIX is exceeding the standard deviation by about for percent in average.

Error Considerations

Prediction Error (percent)



The diagram suggests, that the prediction VIX is the more uncertain the more it is of interest in times of turbulence.

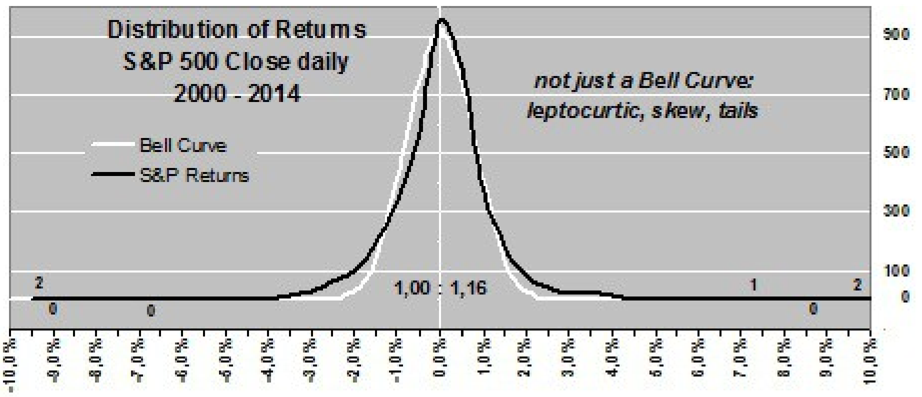
Besides the four percent the B&S formula requires properties of the S&P returns that are not existent:

They are not distributed normally and the volatility not stationary i.e. not at a constant level.

The B&S formula has some disadvantages: As shown by the micpricing table the formula privileges calls: Fair values of calls exceed fair value of puts slightly.

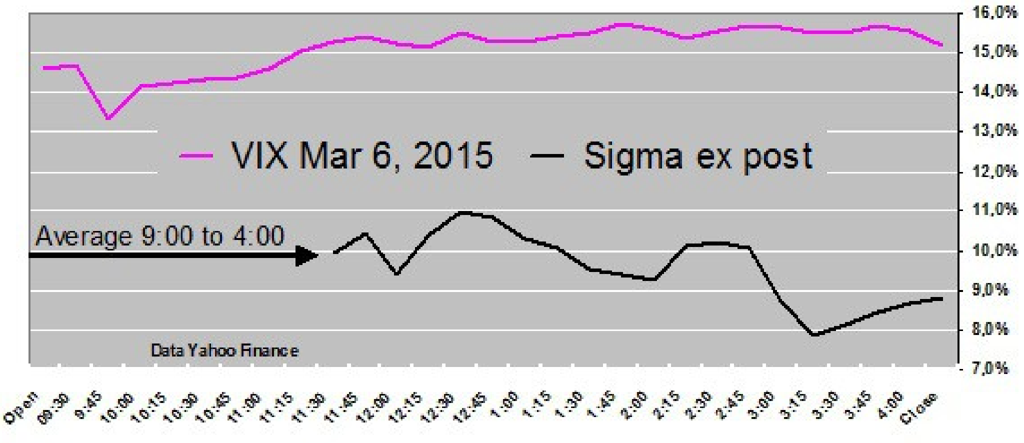
The disadvantages make the put-call parity questionable since it is using the formula for puts and calls – assuming the no-arbitrage condition. Corrections would not mean just subtracting four percent from the VIX to determine the most probable volatility levels:

Returns of historic data are not distributed normally:



The abnormal properties are not too obvious: the positive space under the curve is a little larger than the negative space. The decrease on the right side between 0 and 1 percent is less abrupt than on the left side. “Fat tails” happened in 2008: Sept 29 and Oct 15 ≈ -10 percent and ≈ +10 percent Oct 29, 2008. This is also an indication for scale invariance - intraday returns which would be remarkable for a year.

VIX Intraday



Instead of using the VIX quotations during the day one could gather the continuous intra day S&P 500 quotations – for this analysis every quarter of an hour – and start computing the standard deviation. This results in returns per quarter of an hour. Assigning Open to 9:15[*] and Close to 4:15[*] a banking day

[*] Times for Open and Close may differ.

amounts to 7 hours or 28 quarters.. Thus for scaling the factor should be 84 (root of '28 times 252').
This is acceptable for comparison. However extrapolating from a quarter of an hour is even more questionable when used as a prediction for a year or more.

Since 10 quotations have been taken for the moving sigma continuous quotations could start late am. The total intraday value was 9,95 percent (am 10,38%, pm 9,20%). The VIX was exceeding 4 percent (12:30h) or more!

VIX influencing the Korean Index VKOSPI as an Example

Heejoon Han (2015[5]) finds „ that some Korea's macroeconomic variables significantly explain the VKOSPI. In addition, we find that the stock market return and implied volatility index of the US market (i.e., the S&P 500 spot return and the VIX from S&P 500 options) play a key role in predicting the level of VKOSPI and explaining its dynamics, and their explanatory power dominates that of Korea's macro-finance variables. Further, while Korea's stock market return **does not** predict the VKOSPI, US stock market return well predicts the future VKOSPI level. When both US stock market return and US implied volatility index are incorporated into the HAR framework, the model's both **in-sample fitting** and out-of-sample forecasting ability exhibits the best performance."

The resume is plausible as other leading indexes in Japan and Europe show the influence of S&P, 500, DJIA and NASDAQ as well. However, there seem to be a misunderstanding of scientific modeling: In-sample fitting is used to calibrate a model. The quality of a model depends on out-of-sample forecasting only.

Excursion: The VIX in Yale Lessons

Discussing the VIX in the Yale Open Course: „Options Markets" 2008[6], Chapter 5 and compares the VIX with the „actual volatility" meaning the recent S&P standard deviation over the last month Robert J. Shiller said *„that they line up fairly well, ... but this shows the strength of the Black-Scholes formula."*

When 80 percent of the VIX is „explained" by the actual volatility an the VIX is computed by using the B-S-formula reversely this can't be taken as a proof of the formula. It is rather a kind of circular reasoning. The actual (historic) volatility is the independent variable - the VIX, also called "Fear Index", must be considered as the dependent one.

The lecture uses monthly data since for the very historic data only monthly data are available.. Fore comparison these values have to be multiplied by $\sqrt{12}$=3,464 while daily figures require multiplying by $\sqrt{252}$=15,474. The only difference is that monthly – and yearly even more - figures have a smoothing effect. One can use volatily to review the impact of global events, however looking at the stock or at its returns in particular is less unbiased.

Actual volatility or actual past volatility is „the standard deviation of actual stock prices over the preceding year, of monthly changes, annualized..." ... „multiplied by the square root of 12" p. 12 [$\sqrt{12} \approx 3{,}464$]. The longer the period the stronger the smoothing effect for any volatility variable. In chapter 1 (2011) Shiller quotes Avinash Dixit: „*when you're dating someone and you know the person will marry you, you have an option that you can exercise at any time by agreeing to marry." Obviously a call option is meant: „... you usually don't want to exercise a call option early".*

Why not thinking of a put option? If you love someone you don't want to loose her or him – or if it happens get at least some cash instead.

Chapter 8: „*Black and Scholes invented their equation in response to the founding of the CBOE*".

This statement is misleading: „*What the VIX is, is [essentially, not exactly] the sigma in the Black-Scholes equation. But it is, in effect, the market's expected standard deviation of stock prices. And to get it more precise, it's the standard deviation of the S&P 500 Stock Price Index for one month, multiplied by the square root of 12, because they want to annualize it (2011 lesson)."*

Yes, the VIX is essentially and as exactly as wanted, the sigma in the Black-Scholes equation when used by the reverse approximation. It is a prediction to be used for the next pricing when using the B&S formula.

The standard deviation, however, is the root of the variance of a random variable which is the expected value of the squared deviation from the mean. The symbol is also sigma (σ). One can take σ_b&s as a predictor for σ_stdvar and vice versa. However the formulas are different and in case of VIX the σ_b&s is about four percent higher than σ_stdvar. For correction the four percent cannot be simply deducted to return to the factual level measured ex post. As shown in section „Mispricing..." the option prices are in the long term significantly higher than the exercised returns. One could say the VIX adds an important tendency to the standard actual deviation.

In chapter 8 is stated: „*Black-Scholes is not a black swan theory. It assumes normality of distributions, and so, it's not always reliable. ... The Black-Scholes theory is very elegant and a very useful tool, especially useful when things behave normally. But, I think, one always has to keep in the back of one's mind, the risk of sudden major changes like we've seen here"*.

It's worse: The isn't any stock known showing returns - daily or weekly or monthly - which is distributed normally. And there is no stationary volatility. It is volatile itself – violating the other requirement for the B&S formula.

Accounting standards IAS and IFRS referring to Volatility and Black&Scholes:

The International Accounting Standard IAS 39, AG 82 (f) states: "... *Measures of the volatility of actively traded items can normally be reasonably estimated on the basis of historical market data or by using volatility implied in current market prices.*"

The use of the B&S formula is already involved by IAS 39 as it is required to compute the implied volatility. The International Financial Reporting Standard IFRS[7] 13 Appendix A, B11 mentions valuation techniques explicitly: *„... for example, the following: present value techniques (...); option pricing models, such as the Black-Scholes-Merton formula or a binomial model (ie a lattice model), that incorporate present value techniques and reflect both the time value and the intrinsic value of an option; and the multi-period excess earnings method, which is used to measure the fair value of some intangible assets.*

Thus the problematic practices using historical and implied volatility are rubber-stamped by the standards IAS 39 and IFRS 13 (referring to IFRS 9). The market participants use historical volatility when they consider to trade options and - no wonder - the approximation of the volatility by using the Black Scholes formula reversibly produces - apart from formula caused distortion (smile effect) - again the historical values.

Independently the prediction power can be tested in a different way: The VIX volatility over the last 29 trading days differs from the one over the 29 following days on average by about 4 percent points (absolute percentage values). This is too high for an predictor, particularly since the volatility affects disproportionately model prices. The accounting standard that allows historical volatility as an adequate predictor for the future ones, does not comply with empiric observations of VIX, VDAX, VXN, VFTSE and others.

The use of the latest historical volatility would be model-theoretically allowed with a constant volatility only - as it is assumed by the Black Scholes formula. However, the volatility is itself "volatile".

The statement, the "volatility" being the " *magnitude of future changes in price of the financial instrument or other item*" is in a statistical view with low explanatory power: The standard deviation (sigma) says that - normal distribution assumed - about 68 percent only of the cases would fall in the predicted range being the last exchange rate plus / minus sigma. This forecast is not of much use.

Anyway, it is not plausible economically that a future value should depend on the performance today or on a extrapolated volatility for the future. The application guideline 82 (legal part of IAS 39) is misleading.

Excursion: Are GARCH-Techniques contributing these questions?

Robert F. Engle (1982)[8] and Tim Bollerslev (1986) count as the founder of forecasting techniques based on autogression phenomena. Their approaches are used often to forecast volatility. Two hypotheses:

- They didn't prevent important indexes of implied volatility seem to be dependent on nine week old historical data. The GARCH approach is the reason that implied volatility correlate to two month old data. They use historical data also!
- May be they prevented implied volatility predictions from being less future oriented.

The variable lag correlation approach should be used to examine GARCH based predictions - room for further research. R. Cumby 1993[9]

Another paper[10], Jorion, P. (1995), analyzes a mean reversion strategy using the VIX as a **contrarian** indicator of hope and fear to time decisions at extreme levels that have been determined through statistical analysis. This thesis found through back testing that market timing is possible at extreme levels of fear but is less reliable during extreme levels of hope and complacency.

Jinji Hao (2012[11]) comes to the following conclusion: „When jointly estimated with both returns and VIX, the parameters are distorted unreasonably, and the GARCH implied VIX still **cannot fit** the CBOE VIX from various statistical aspects. ... We conclude that the GARCH option pricing under the LRNVR fails to incorporate the price of volatility or variance risk premium."

Autoregression suggests that a variable, often volatility, depends from historic data of the same variable. The probable explanation is rather, that stock indexes react to exogenous information – global events - in a similar way. The problem is to anticipate the point in time when such events happen to be.

Data

Daily/weekly exchange data, „adjusted close", of major indexes are used to compute standard deviation and to compare them with quoted implied volatility of Standard & Poor's 500, DAX, FTSE, and NASDAQ. The paper focuses on the VIX only. A few quotations had to be eliminated for precise matching. This is due to different bank holidays in Chicago, New York, London and Frankfurt.

Data Source: courtesy Yahoo Finance and FAZ.

There are no quoted implied volatility data for the NIKKEI 225 available. The DJIA volatility seem to require a fee. Intentionally only data are used that are free available.

Summary

There seem to be a fundamental difference between Anglo-American and Continental European statisticians. The former assume that the future will follow the Gaussian bell curve or historical behavior – the latter find that the curve describes the distribution of deviations („Errors") from a value, a mean, an intended size in a process if no other influence is known. As soon as other experience is available, observations, recent and „historic" data, they should be preferred instead of the idealistic „bell curve".

Can the „predictive analytics" approach, to use as much data as available, be sure to find enough substantial enough to base significant decision on it?

The Yale lessons are probably considered as text book standard. They skip theoretical and practical deficits in pricing of options and derivatives. Despite it seems to work as long as all market participants follow the same rules – even if the are wrong.

There are studies to overcome the B&S requirement of stationary volatility. The idea is to model volatility by GARCH or by stochastic processes like the Heston[12] model. It's still leaving the open question whether the specific volatility of a term ending at expiration date will be close to the prediction.

Engle[13], (1993) said „Volatility forecasting is a little like predicting whether it will rain: You can be in correct in predicting the probability if rain, but still have no rain."

Duffie[14], in an eMail Jan 6, 1999: *„..the Black-Scholes formula, it* ***is certainly a useful formula****, and* ***under standard technical conditions,*** *is correct. (There is an immense literature regarding this, and absolutely no uncertainty regarding the validity of the formula* ***provided you accept the axioms****.) I agree, of course****, that the axioms, in particular the "clean" log-normal model"*** *as you put it, are unrealistic, but there are now 25 years of hard research* ***relaxing*** *those axioms, and getting more general and applicable option pricing rules."*

Neither the axioms nor the model itself nor the volatility prediction is proven once the tst by fictitious options is accepted. Thus far option pricing ist just a risky industrie standard.

The findings suggest to follow Jean-Philippe Bouchaud (2008, École Polytechnique), „that „financial engineers have put too much faith in untested axioms and faulty models...To prevent economic havoc, that needs to change." There is no major change known since 2008.

Table of Content

References

[1] Shiller, Robert J. referring to Avinash Dixit http://oyc.yale.edu/economics/econ-252-11 (2011), Chapter 1

[2] Diebold, Francis X. et al, July 1997, „Converting 1-Day Volatility to h-Day Volatility: Scaling by Root-h is Worse Than You Think“ http://fic.wharton.upenn.edu/fic/papers/97/9734.pdf

[3] Official CBOE VIX related publications: http://www.cboe.com/micro/vix-options-and-futures.aspx and http://www.cboe.com/framed/pdfframed.aspx?content=/micro/vix/vixwhite.pdf&section=SECT_MINI_SITE&title=White+Paper

[4] Up to 400 daily quotations are published – minute by minute. There are plans for extensions.

[5] Heejoon Han et al.: „Modeling and Predicting the Market Volatility Index: The Case of VKOSPI, 2015

[6] Shiller, Robert J. „http://oyc.yale.edu/economics/econ-252-08“ 2008, Chapter 5. Accounting for Volatility in the B&S-Formula [51:35 min] http://oyc.yale.edu/sites/default/files/Lecture23.pdf

[7] All standards material is published by the IASB in London. Because of copyright restrictions please refer to http://shop.ifrs.org/ProductCatalog/Default.aspx. The IFRS are supposed to replace all IAS as a final set of accounting standards. However, currently some IAS are still valid with numerous cross references.

[8] Engle. Robert F., David M. Lilien. and Russell P. Robins (1987) Estimating Time Varying Risk in the Term Structure. Economics, 55, pp. 391-407.

[9] Cumby, R.; Figlewski, S.; Hasbrouck, J. (1993). "Forecasting Volatility and Correlations with EGARCH models". Journal of Derivatives 1 (2): 51–63. doi:10.3905/jod.1993.407877.

[10] Jorion, P. (1995). "Predicting Volatility in Foreign Exchange Market". Journal of Finance 50 (2): 507–528. doi:10.1111/j.1540-6261.1995.tb04793.x. JSTOR 2329417

[11] Jinji Hao, „GARCH Option Pricing Models, the CBOE VIX, and Variance Risk Premium“, 2012, p. 1

[12] Heston, Steven L. (1993). "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options". The Review of Financial Studies 6 (2): 327–343.

[13] Engle, R. F., 1993 "Statistical Models for Financial Volatility," Financial Analysts Journal (Jan/Feb 1993): p. 72

[14] Duffie, Darrel, SBS Stanford, by eMail Jan 6, 1999