An algorithm for estimating the volatility of the velocity of money

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Real or nominal shock – which one does more to destabilize developing economies? A case study of money velocity in Kazakhstan

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Abstract

This note develops a gauge of the volatility of money velocity, based on the quantity equation of exchange. In contrast to ad hoc regression, the gauge measures the impacts of the three determinants of velocity – money supply, output, and the price level. An application to a fast-growing transition economy, Kazakhstan, finds that at the margin, price shocks affect the volatility of velocity more than do monetary or real shocks, by several orders of magnitude.

Keywords: real shocks, monetary shocks, monetary policy, simulations, forecasting in transitional economies, mathematical statistics in economics

JEL Classifications: E47; E52
I. Introduction

To steady an economy, policymakers need to know the relative impacts of real and nominal shocks on spending. In theory, only supply shocks can affect output in the long run, but the impacts of various shocks in shorter periods are ambiguous. Vector autoregression (VAR) can compute the effects, but one would like to supplement these atheoretic estimates with structural ones. This paper treats the turnover rate of a unit of money (velocity) as a function of three random variables (output, the price level, and money supply), as in a stochastic version of the quantity theory of exchange. Since the equation of exchange is a tautology, rooting a model of velocity in it may have some advantages over an ad hoc function of such independent variables as real income, the real interest rate and expected inflation; and, similarly, over a structural VAR (SVAR) model with a priori restrictions.

In the equation-of-exchange model, one can compute the relative contributions of the variances of output, prices and money supply to the variance of velocity. The note applies the algorithm to Kazakhstan since we know less about shocks to developing economies than about those to developed ones; indeed,
a dynamic transition economy may be more prone to fluctuations in velocity than are more established economies.

As a side benefit, the equation-of-exchange approach may improve central bank forecasts of the effects of monetary policy. When data are scarce and institutions are changing – for example, early in the transition of post-Soviet economies to markets – velocity can be hard to estimate (Citrin 1995). For better forecasts, we would like to gauge possible errors in estimates of velocity. While a Monte Carlo simulation may accomplish this, it would be informative to relate the variations of velocity to those of its determinants.

Here’s a road map to the paper. Section II surveys the literature. Modern studies concede that velocity is not constant, but they disagree over its determinants. Section III discusses velocity in Kazakhstan. It varies in the short run as well as in the long, but in general it has been falling since 2001, possibly because of monetization. Section IV models the variance of velocity as a function of the variances and covariances of the three other variables in the equation of exchange. Taking derivatives and estimating their values suggests that, at the margin, the variance of velocity in Kazakhstan has been most influenced by the
variance of the price level. Section V concludes and reflects. As an oil exporter,
Kazakhstan is vulnerable to the volatility of the global prices of crude.

II. Literature review

We begin with the familiar equation $MV = PQ$, where $M$ is money supply,
$V$ velocity, $P$ the price level, and $Q$ output. Given $V$ and $Q$, a change in $M$ induces
a proportional change in $P$, which simplifies forecasting. When the economy
produces at full capacity, $Q$ may be constant, but a constant $V$ is harder to justify.
Marshall (1923) suggested that it may be slow to change because habit determines
the share of income that people spend. In contrast, modern theories of velocity
tend to explain why it changes. The institutional approach focuses on
monetization, innovation and stability (Bordo and Jonung, 1981). At first,
monetization increases the ratio of money supply to spending, so velocity declines;
over time, financial innovations accumulate and the economy stabilizes, increasing
the efficiency of spending and consequently velocity. One computational
approach attributes the volatility of velocity to the household’s smoothing of
consumption over time (Cao-Alvira, 2012).
Velocity is often variable in the short run as well as in the long. For the velocity of the currency in Kazakhstan, the tenge, the ratio of the standard deviation to the mean ranged from .044 in 2005 to .213 in 2009 (Table 1). The ratio was more than twice as high in the period 2009-2011 (.197) -- which began with an economic slowdown and a 25% devaluation of the tenge -- as in the period 2000-2008 (.076). From 2001 to 2011, the velocity of M1 money (comprised mainly of currency and checkable deposits) halved.

Modeling velocity is hard partly because its link to lagged money volatility is unclear. Friedman (1984) argued, in effect, that velocity would fall when economic uncertainty increased, since people would hold money as a precaution. Uncertainty may cause, or result from, money volatility. Hall and Noble (1987) tested for Granger causality in United States data and concluded that the log of \( M1 \) velocity was “caused” partly by its own lags and by lags of the volatility of money growth. Other studies indicated that these results might vary with the period studied, since the monetary environment evolves due to such factors as regulation.

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3 Research into the tenge supply often uses \( M2 \) or \( M3 \). But \( M1 \) is more typical than these measures are of research into the variance of velocity.
4 For broad perspectives, see Friedman (1970, pp. 227-9) -- and Mascaro and Meltzer (1983), which “develops a general equilibrium model in which variability or risk affects the choice of portfolios” (p. 488). From this point of view, the increased volatility of the tenge in the period of 2009-11 may have reflected uncertainty about the optimal amount of money to hold following the global financial crisis of 2008.
and inflation (Brocato and Smith 1989; Mehra 1987 and 1989). The results in Mehra’s 1989 article were also sensitive to specification of the equation – e.g., in levels or in first differences. In addition, Granger-causality estimates often depend on the lengths of the lags specified, concluded Thornton and Batten (1985). Thornton (1995) turned up evidence supporting Friedman’s hypothesis for three of nine industrial countries studied, but only in certain time periods. “The Friedman hypothesis would appear to have little general applicability” (p. 290). For additional rigor and insight, one would like to derive estimates directly from the quantity theory of exchange.

### III. Velocity in Kazakhstan

An overview of velocity in Kazakhstan will help motivate the analysis below of the marginal impacts of determinants on its volatility. From 2001 through 2011 in Kazakhstan, M1 velocity fell roughly from 16 to 8, despite output growth for most of this period (the average annual rate for the period was 15.6%) and a fairly steady rate of inflation of 7% or 8% (7.3%), according to data from the central bank, the National Bank of Kazakhstan. Curiously, the rate of decline in velocity in the recession of 2008-9 was no higher than in previous years of recovery. The evident reason for the long-run fall in velocity was the sustained
rise in M1 (as the Bordo and Jonung model might have suggested). The average of the annual rates of M1 growth over the 11-year period was 33.1%, outstripping the corresponding rate for nominal income (24.0%). In contrast, the average growth rate of M0 money (currency) was 26.6%, although it tended to follow the same patterns as did M1; the simple correlation coefficient for the two measures of money, over quarters, was .98 (Figure 2).

The sustained rise in M1 may have stemmed from accommodative monetary policy. The most remarkable peak of M1 growth occurred in 2003, when it rose from 12.6% in 2002 to 54.4%; the M0 growth rate doubled, from 21.0% to 42.0%. The National Bank may have been reacting to an apparent slowdown in 2002, when the output growth rate fell to 9.0%, from 15.3% in 2001. This rate did recover in 2003, to 12.9%.

The National Bank may have reacted to inflation as drastically as to output slowdown. In 2008, when output rose 15.3%, the M1 growth rate fell to 13.3%, from 44.4% in 2007; the M0 rate fell from 42.4% to 5.9%. In 2009, when output fell by .3 of a percent, the M1 rate was 35.9%. (The M0 rate fell to 3.4%.) These trends raise the possibility that the National Bank over-reacted to inflation in 2007-08, when consumer prices were rising nearly 9% per year, then swung
almost as drastically in the other direction when facing stagnation. Might such oscillations affect the volatility of velocity? We’ll see.

IV. Analysis

*Deriving the variance of velocity.* From the equation of exchange,

\[ V = \frac{PQ}{M}. \]

(IV.1)

\(P, Q\) and \(M\) may be random variables. A Taylor series and a well-known property of variance give a first-order approximation of the variance of \(V\) (Appendix):\(^6\)

\(^5\) The velocity of money increases in nominal income \((PQ)\), since people will spend a given money supply faster when they can afford more purchases; and velocity falls with an increase in money, since there are more tenge now to finance aggregate purchases of a given size.

\(^6\) Statisticians refer to a linear Taylor approximation as the “delta method” (Greene 2008, pages 1055-6). In a useful illustration, Larsen and Marx (2006, p. 238) apply the method in order to interpret dental X-rays.
\[
\text{\text{var}(V) \approx} \frac{Q^2}{M^2} \text{var}(P) + \frac{P^2}{M^2} \text{var}(Q) + \frac{P^2Q^2}{M^4} \text{var}(M) + 2 \frac{QP}{M^2} \text{cov}(P,Q) - 2 \frac{PQ^2}{M^3} \text{cov}(P,M) - 2 \frac{P^2Q}{M^3} \text{cov}(M,Q).
\] 

(IV.2)

In short-run cases, \(P\), \(Q\) and \(M\) may be independent of one another -- each subject to random factors, such as a specific measurement error, which need not affect the other two variables. The zero covariances would eliminate the last three terms in Equation IV.2. This paper, however, considers longer time periods and so will account for covariances. Given the long-run variances and covariances of \(P\), \(Q\) and \(M\), the equation can forecast the variance of velocity in a scenario specifying the levels of the former three variables.

*Marginal impacts of the variances of the determinants.* Since output, money supply and the price level increase over time in Kazakhstan, their marginal impacts on velocity may indicate future trends. Table 3 reports the marginal impacts of the variances of \(P\), \(Q\), and \(M1\) on the variance of velocity, taking
covariances into account.\textsuperscript{7} The estimates use the average annual levels for these four variables for the period from 2000 through 2011. The price level creates the largest impact, by several orders of magnitude. Since these estimates are at the margin, they suggest that nominal causes of volatility are becoming increasingly important relative to real causes.

Volatility in spending may destabilize output in the short run. In the context of the Blanchard and Quah (1989) model, nominal causes are demand disturbances. In their SVAR studies of United States quarterly data for 1948 through 1987, Blanchard and Quah conclude that “demand disturbances make a substantial contribution to output fluctuations at short- and medium-term horizons…” (p. 668).

\textit{Forecasting}. At times, we may have ambiguous estimates for the three right-hand variables in Equation IV.1. For example, we may lack reliable monthly data. Or the analyst may base her prediction of velocity on assumed values of the independent variables that turn out to be unrealistic. In either case, Equation IV.1 may mis-estimate $V$. To get an idea of how large the mis-estimates might be, let’s measure $V$’s volatility.

\textsuperscript{7} Section VII.3 of the appendix derives the marginal estimators.
To illustrate the algorithm in forecasting, suppose that the National Bank of Kazakhstan considers an increase in the money supply equal to the forecasted annual rate of growth in $Q$, 7.5%. The Bank assumes that $P$ will not change. In addition to the levels of $P$, $Q$, and $M1$, assume for these variables their average annual variances for the period 2000-2011 (Table 2). By Equation IV.1, the predicted value of M1 velocity is 7.7. By Equation IV.2, the predicted standard deviation of velocity is 1.39, or .18 of the mean. This ratio is 70% higher than the average for 2000-2011, so the Bank may wish to act on its scenario forecast with caution. If velocity follows a normal distribution, then its 95% confidence interval is (4.9, 10.5).

Covariances among $M$, $P$ and $Q$ may affect forecasts. Keynesian policy assumes that the short-run correlation between prices and money is low enough to permit an infusion of money to affect real GDP rather than the price level. But in Kazakhstan, using annual data for 2000 through 2011, the simple correlations of the Consumer Price Index, the money supply ($M0$ or $M1$), and output in Kazakhstan all exceed .97. For monthly data, the correlation between the CPI and $M1$ also exceeds .97. The estimates here take covariances into account.
V. Conclusions and reflections

Keynesian theory emphasizes shifts in aggregate demand as causes of the business cycle; real business cycle theory emphasizes shifts in aggregate supply. This paper finds that at the margin, price shocks dominate the volatility of spending in Kazakhstan, which is consistent with Keynesian theory. Conceivably, with an adjustment for correlation among prices, output and money supply, the Keynesian approach is apt for developing economies because their agents know less about them than real business cycle theory may assume.

The dominance of price shocks at the margin has a practical implication for Kazakhstan, where oil exports account for roughly a fourth of gross domestic product. Since oil prices are unusually volatile, they may destabilize spending, and thus real GDP, more than other variables do.

As a function of random variables, velocity is subject to uncertainty that clouds forecasts. This paper’s algorithm measures the randomness and finds that it may be considerable in Kazakhstan. It may be most valuable when applied to short-run monetary relationships. True, recent work challenges the conventional view that the short-run demand for money is hard to explain. In the United States,
the return to savings accounts and money market mutual funds may help explain
$M1$ demand in a volatile period, 1959 through 1993 (Ball, 2012). But money
markets may be less developed in transition economies. In Kazakhstan, a model
of $M2$ demand based on an output proxy, the interest rate, and on foreign exchange
rates, has the expected coefficient signs in the long run but not in the short
(Yilmaz, Oskenbayev and Kanat, 2010).
VI. References


VII. Appendix

VII.1 The Case of Independent Random Variables

In a Taylor series, a first-order expansion approximates a function $g$ around some point $(\mu_1, \mu_2, \ldots, \mu_n)$:

$$
g(W_1, W_2, \ldots, W_n) \approx g(\mu_1, \mu_2, \ldots, \mu_n) + \sum_{i=1}^{n} \frac{\partial g}{\partial W_i}(W_i - \mu_i),
$$

where the derivatives are evaluated at the point $(\mu_1, \mu_2, \ldots, \mu_n)$.

For velocity, such a Taylor series would be

$$
V(M, P, Q) \approx V(\mu_m, \mu_p, \mu_q) + \frac{\partial V}{\partial M}(M - \mu_m) + \frac{\partial V}{\partial P}(P - \mu_p) + \frac{\partial V}{\partial Q}(Q - \mu_q),
$$

(VII.1.1)

where $\mu_m$, $\mu_p$ and $\mu_q$ are arbitrary constants. $V(\mu_m, \mu_p, \mu_q)$ is also a constant.
A basic result concerning the variance of a linear sum of independent random variables $W_i$ with finite means is that

\[
\text{Var} \left( \sum_{i=1}^{n} a_i W_i \right) = \sum_{i=1}^{n} a_i^2 \text{Var}(W_i)
\]

(VII.1.2)

where $a_i$ is a constant. Applying Equation VII.1.2 to Equation VII.1.1 gives us

\[
\text{Var}(V) \approx \text{Var} \left( \frac{\partial V}{\partial M} (M - \mu_m) + \frac{\partial V}{\partial P} (P - \mu_p) + \frac{\partial V}{\partial Q} (Q - \mu_q) \right)
\]

\[
= \left( \frac{\partial V}{\partial M} \right)^2 \text{Var}(M - \mu_m) + \left( \frac{\partial V}{\partial P} \right)^2 \text{Var}(P - \mu_p) + \left( \frac{\partial V}{\partial Q} \right)^2 \text{Var}(Q - \mu_q)
\]

\[
= \left( \frac{\partial V}{\partial M} \right)^2 \text{Var}(M) + \left( \frac{\partial V}{\partial P} \right)^2 \text{Var}(P) + \left( \frac{\partial V}{\partial Q} \right)^2 \text{Var}(Q),
\]

(VII.1.3)

where the last line uses Equation VII.1.2 again:
\[ \text{Var}(X - \mu) = \text{Var}(X) + \text{Var}(\mu) = \text{Var}(X), \]

since \( \mu \) is a constant.

**VII.2 The General Case**

When covariances are not zero, the general version of Equation VII.1.2 is

\[ \text{Var}\left(\sum_{i=1}^{n} a_i W_i\right) = \sum_{i=1}^{n} a_i^2 \text{Var}(W_i) + 2 \sum_{j<k} a_j a_k \text{Cov}(W_j, W_k), \]

\[ \text{(VII.2.1)} \]

where we have used the result

\[ \text{Cov}(a_i W_i, a_j W_j) = E(a_i W_i a_j W_j) - E(a_i W_i)E(a_j W_j) = a_i a_j E(W_i W_j) - a_i E(W_i) a_j E(W_j) \]
\[ = a_i a_j \left[ E(W_i W_j) - E(W_i) E(W_j) \right] = a_i a_j \text{Cov}(W_i, W_j). \]

Equation IV.2 specifies Equation VII.2.1.
VII.3. Deriving estimators of the marginal impacts of real and monetary shocks on the volatility of velocity

From Equation IV.2,

\[
\frac{\partial \text{Var}(V)}{\partial \text{Var}(P)} = \frac{Q^2}{M^2} + 2\frac{QP}{M^2} \frac{\partial \text{Cov}(P,Q)}{\partial \text{Var}(P)} - 2\frac{PQ^2}{M^3} \frac{\partial \text{Cov}(P,M)}{\partial \text{Var}(P)},
\]

\[
\frac{\partial \text{Var}(V)}{\partial \text{Var}(Q)} = \frac{P^2}{M^2} + 2\frac{PQ}{M^2} \frac{\partial \text{Cov}(P,Q)}{\partial \text{Var}(Q)} - 2\frac{P^2Q}{M^3} \frac{\partial \text{Cov}(M,Q)}{\partial \text{Var}(Q)},
\]

\[
\frac{\partial \text{Var}(V)}{\partial \text{Var}(M)} = \frac{P^2Q^2}{M^4} - 2\frac{PQ^2}{M^3} \frac{\partial \text{Cov}(P,M)}{\partial \text{Var}(M)} - 2\frac{P^2Q}{M^3} \frac{\partial \text{Cov}(M,Q)}{\partial \text{Var}(M)}.
\]

(VII.3.1)

One can rewrite Equation VII.2.1 as

\[
\text{Cov}(W_1, W_2) = \frac{\text{Var}(aW_1 + bW_2) - a^2\text{Var}(W_1) - b^2\text{Var}(W_2)}{2ab}.
\]

This formula yields derivatives of the covariance with respect to a variance.

Applying these derivatives to Equation VII.3.1 and simplifying yields
\[
\frac{\partial \text{Var}(V)}{\partial \text{Var}(P)} = \frac{Q^2}{M^2},
\]
\[
\frac{\partial \text{Var}(V)}{\partial \text{Var}(Q)} = \frac{P^2}{M^2},
\]
\[
\frac{\partial \text{Var}(V)}{\partial \text{Var}(M)} = \frac{3P^2Q^2}{M^4}.
\]
### Table 1

<table>
<thead>
<tr>
<th>Year</th>
<th>Standard Deviation</th>
<th>Velocity mean</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>0.267</td>
<td>3.985</td>
<td>0.067</td>
</tr>
<tr>
<td>2001</td>
<td>0.408</td>
<td>3.993</td>
<td>0.102</td>
</tr>
<tr>
<td>2002</td>
<td>0.359</td>
<td>4.134</td>
<td>0.087</td>
</tr>
<tr>
<td>2003</td>
<td>0.227</td>
<td>3.290</td>
<td>0.069</td>
</tr>
<tr>
<td>2004</td>
<td>0.212</td>
<td>2.822</td>
<td>0.075</td>
</tr>
<tr>
<td>2005</td>
<td>0.113</td>
<td>2.585</td>
<td>0.044</td>
</tr>
<tr>
<td>2006</td>
<td>0.139</td>
<td>2.422</td>
<td>0.057</td>
</tr>
<tr>
<td>2007</td>
<td>0.270</td>
<td>2.112</td>
<td>0.128</td>
</tr>
<tr>
<td>2008</td>
<td>0.129</td>
<td>2.321</td>
<td>0.056</td>
</tr>
<tr>
<td>2009</td>
<td>0.385</td>
<td>1.805</td>
<td>0.213</td>
</tr>
<tr>
<td>2010</td>
<td>0.362</td>
<td>1.890</td>
<td>0.192</td>
</tr>
<tr>
<td>2011</td>
<td>0.356</td>
<td>1.911</td>
<td>0.186</td>
</tr>
</tbody>
</table>

*Notes: Column 2 gives the standard deviation of velocity, calculated for each year from the quarterly estimates of velocity; Column 3, the mean of velocity, calculated as the annual average of quarterly estimates; and Column 4, the ratio of the standard deviation to the mean. Source of raw data: The National Bank of Kazakhstan*
Table 2

<table>
<thead>
<tr>
<th>Variables</th>
<th>Level</th>
<th>Variance</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price level</td>
<td>228.7</td>
<td>1,647.0</td>
<td>40.6</td>
</tr>
<tr>
<td>Output</td>
<td>128,036.7</td>
<td>979,119,830.8</td>
<td>31,290.9</td>
</tr>
<tr>
<td>M1 money</td>
<td>3,819,483.9</td>
<td>1,301,845,692,972.7</td>
<td>1,140,985.0</td>
</tr>
<tr>
<td>Velocity</td>
<td>7.7</td>
<td>1.9</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Table 3

| Marginal effects on variance of velocity |
|-----------------|-----------------|-----------------|
| Variable        | Formula         | Estimate        |
| Price level     | Q^2/M^2         | 0.001123724     |
| Output          | P^2/M^2         | 3.58487E-09     |
| Money           | 3P^2Q^2/M^4     | 1.20852E-11     |
Figure 1

Velocity falls in Kazakhstan

Data source: National Bank of Kazakhstan
Figure 2

M0 and M1 money move together

Annual growth rates

<table>
<thead>
<tr>
<th>Time</th>
<th>P</th>
<th>Q</th>
<th>M0</th>
<th>M1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>105.1</td>
<td>24,697.9</td>
<td>95,844.1</td>
<td>162,832.9</td>
</tr>
<tr>
<td>2001</td>
<td>114.0</td>
<td>28,477.7</td>
<td>112,141.3</td>
<td>202,735.2</td>
</tr>
<tr>
<td>2002</td>
<td>121.4</td>
<td>31,042.1</td>
<td>135,651.4</td>
<td>228,326.6</td>
</tr>
<tr>
<td>2003</td>
<td>131.3</td>
<td>35,045.2</td>
<td>192,612.3</td>
<td>352,616.1</td>
</tr>
<tr>
<td>2004</td>
<td>140.8</td>
<td>41,639.6</td>
<td>290,824.4</td>
<td>524,762.3</td>
</tr>
<tr>
<td>2005</td>
<td>150.0</td>
<td>50,504.2</td>
<td>388,599.3</td>
<td>732,620.9</td>
</tr>
<tr>
<td>2006</td>
<td>159.8</td>
<td>63,707.8</td>
<td>506,327.0</td>
<td>1,052,436.7</td>
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<tr>
<td>2007</td>
<td>174.0</td>
<td>73,659.0</td>
<td>720,892.9</td>
<td>1,520,003.7</td>
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<tr>
<td>2008</td>
<td>188.8</td>
<td>84,930.4</td>
<td>763,243.5</td>
<td>1,722,722.4</td>
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<tr>
<td>2009</td>
<td>200.2</td>
<td>84,678.0</td>
<td>789,508.7</td>
<td>2,340,956.5</td>
</tr>
<tr>
<td>2010</td>
<td>213.5</td>
<td>101,818.4</td>
<td>1,015,448.1</td>
<td>2,854,529.7</td>
</tr>
<tr>
<td>2011</td>
<td>228.7</td>
<td>119,103.9</td>
<td>1,196,024.8</td>
<td>3,553,008.3</td>
</tr>
</tbody>
</table>

Notes: P, the Consumer Price Index, is averaged from monthly data. Q, real output, equals nominal gross domestic product divided by the CPI. Each annual estimate of Q sums the four quarterly estimates. The money supplies M0 and M1 are measured in millions of tenge. They are averaged from monthly data. Source of raw data: The National Bank of Kazakhstan.