Dynamic Stress Test Diffusion Model Considering the Credit Score Performance

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Abstract

After the crisis of 2008, and the important losses and shortfall in capital that it revealed, regulators conducted massive stress testing exercises in order to test the resilience of financial institutions in times of stress conditions. In this context, and considering the impact of these exercises on the banks' capital, organization and image, this white paper proposes a methodology that diffuses dynamically the stress on the credit rating scale while considering the performance of the credit score. Consequently, the aim is to more accurately reflect the impact of the stress on the portfolio by taking into account the purity of the score and its ability to precisely rank the individuals of the portfolio.

Key words: Basel III, Dodd Frank, Stress testing, CCAR, Gini, Rating scale

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1. Introduction

After the 2008 crisis, regulators identified many shortcomings in both regulatory requirements as well as internal risk management practices. As a consequence, regulators conducted massive stress testing exercises in Europe (with the Asset Quality Review and the stress testing exercises for which the results were published late October 2014) and in North America (with the CCAR exercise). The purposes of these exercises is to make sure that in times of stress (similar to those of 2008), banks hold sufficient capital and are well provisioned in order to absorb losses that could occur.

Moreover, if these exercises have a significant impact on banks' capital and therefore on the Return on Equity, there is much certitude that they will be performed on a regular basis. Knowing this, financial institutions will have to enhance current methodologies to more accurately measure the risks embedded in their portfolios, to show regulators their willingness to pursue constant and regular efforts in enhancing their stress testing exercises, and optimize the impact on capital.

The purpose of this paper is to provide a methodology that diffuses the stress applied on a credit portfolio (Retail or SME counterparties mainly) while taking into account the level of risk embedded in each internal rating category as well as the performance of the score that allowed to build the rating scale. In fact, the riskier the rating class is (generally the rating scale contains 10 classes, class #1 being the less risky and class #10 being the riskier class), the higher the level of potentially applied stress. The level of stress applied to each class obviously depends on the performance of the score, as well as its ability to isolate risky individuals and discriminate them.

First, the importance of stress testing and the impacts these exercises have on the reputation of a financial institution will be analyzed. In addition, considering the importance of credit risk in a vast majority of financial institutions, the white paper will be placed from this perspective.

Second, the approach and the methodology will be discussed in detail. More specifically, from stressing the average default rate of the portfolio, to diffusing the stress by rating scale (on the default rate of each class) and then to analyzing the relationship between the stress applied and the performance of the score, the step-by-step approach will be detailed.

Third, a study will be conducted on a real SME portfolio comprised of investment loans. A score has been built on this portfolio and therefore a rating scale with good performances (based on the Gini index and other relevant performance and discrimination power measures). The diffusion model will be applied on this portfolio and the relationship between the level of stress by rating class and the Gini index will be established. Finally, the ability of the model to diffuse the stress dynamically will be considered as well as the way to cover the model errors and therefore cover model risks.

2. Stress testing, an essential program for regulators and financial institutions

In times where banks and regulators are heavily investing in stress testing, the purpose here is to provide some elements around the importance of stress testing and show why existing regulations (Basel II, Basel 2.5, etc…) showed certain limits in times of stress conditions.

2.1. What is new in a stress testing exercise?

Over the past few years, stress testing is rapidly moving to become one of the most powerful tools for assessing the resilience of the banking industry. Regulators around the world conducted these exercises and relied on the results to determine the shortfalls in capital buffers. More than just a regulatory requirement, these exercises took a whole new dimension. In fact, banks considered stress testing very seriously (by investing a lot on both internal resources as well as external resources) because of the impact that the results could have on their image and reputation.

More precisely, stress testing revealed the important gaps to be filled by banks in order to succeed this exercise. If having adequate resources as well as getting a strong involvement from the Board is key, other aspects are as equally important. As part of the top priorities, Data quality and modeling will require extensive efforts to meet the standards required by the regulators as well as to create a sound base for accurate measurements and projections. In fact, data quality is a prerequisite for building any stress test model. In fact, even though the model built is as perfectly as possible, if the data inputs are not accurate, the outputs will be inaccurate. As part of the recent concerns issued by regulators towards banks' data quality, the EBA launched in Q2-2014 the Asset Quality Review exercise. Similarly, the FED required from banks to put in place an organization dedicated to Data governance and quality. Finally, the BCBS 239 rule is dedicated to insuring data quality across 11 principles. The same concerns are raised around the quality of the underlying models used for stress tests (as well as other purposes). Consequently, within the CCAR exercise, banks are required to provide a clear explanation of the methodologies used, the assumptions chosen and the results provided. Independent validation teams must be constituted to validate these models and test their reliability with high standards. Similarly, in Europe, it is very certain that a Model Quality Review (similar to the AQR exercise but for models) will be conducted.

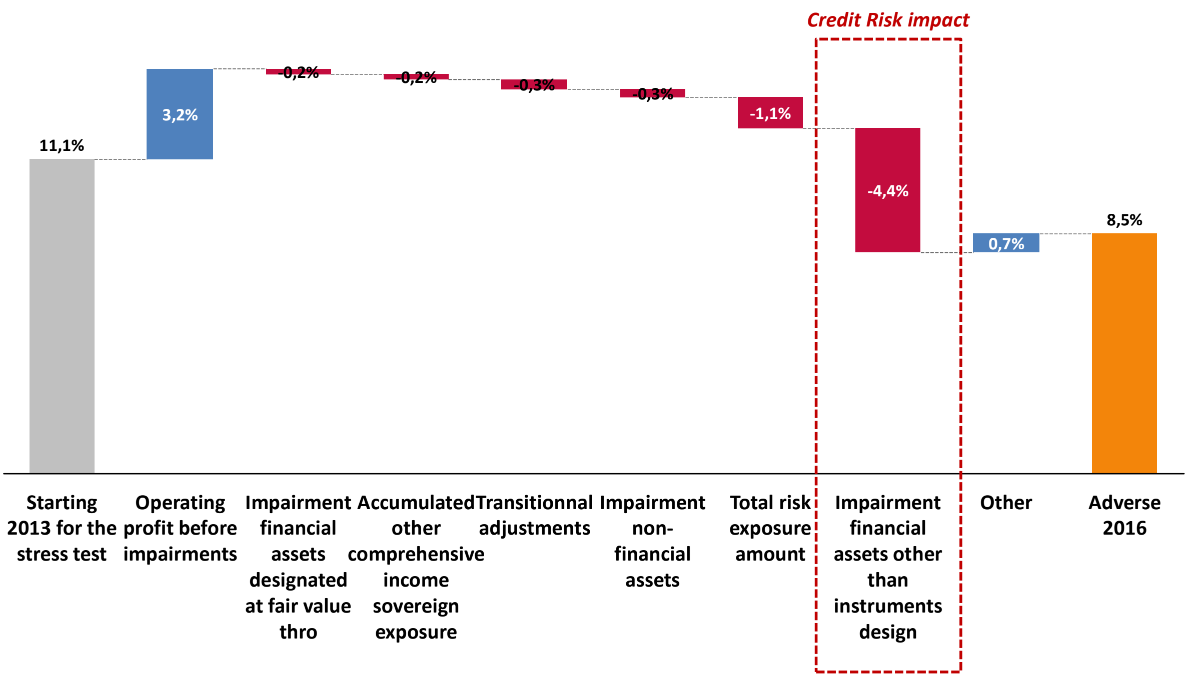
In addition, the stress testing exercise is complementary to current regulations. In fact, Basel regulations are backwards-looking. It reckons that the assets that were risky in the past are the same as the assets that will be risky in the future. Moreover, these regulations failed to identify tail events. Consequently, the Basel committee recognized these shortcomings and tried to bring answers (for instance, the Fundamental Review of the Trading Book recognized that the VaR models were unable to capture extreme events and suggested to replace it by the expected shortfall). Stress testing provides "forward-looking assessments of risk" as described in the Principles For Sound Stress Testing Practices and Supervision (BCBS 147). In fact, the forward-looking scenarios used (whether central, adverse or extremely adverse scenarios) allow for measuring the impacts of future economic turmoil on the banks' financial stability. Moreover, the adverse and extremely adverse scenarios could contribute in measuring extreme events.

2.2. Credit risk remains a top issue in today's banking industry

The credit risk component remains an important subject for banks and its impact on capital is still a major topic. Even though banks have been conducting some de-risking and deleveraging actions in Europe, these actions mainly concerned wholesale assets, and more particularly in the FICC activity (Fixed Income, Currencies, Commodities) and not really in the banking book area. Moreover, "The quality of banks' loan portfolios continued to decline in 2013 and in the first months of 2014 and remains a concern, harmfully contributing to the continuation of elevated risk premiums levels on European banks," as described in the *Risk Assessment Of The European Banking System* of June 2014. This means that, on the one hand, the banking book is not subject to significant deleveraging actions, and on the other hand, it is getting riskier in times of zero growth. Besides, "Provisioning has not increased in conformity with rising credit risks."

All these factors tend to show how important the credit risk component is and its impact in terms of capital requirements. Moreover, the last stress testing results led by the ECB (and published on the 26th of October 2014) revealed the impacts of credit risks on the solvency ratio, and were categorized as "the most important driver by far" in Europe.

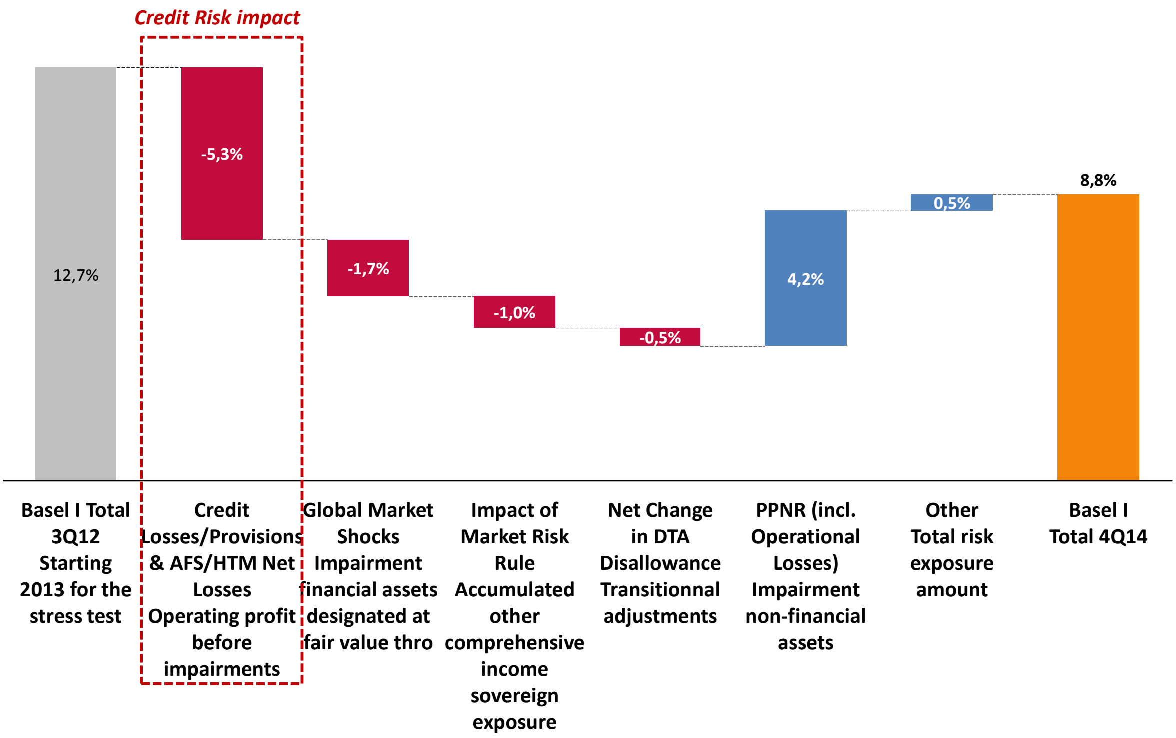
Contribution of different drivers to the change in Common Equity Tier 1 Capital ratio from 2013 to 2016 in the adverse scenario



Source: EBA

In the US, the results of the CCAR exercise for CitiBank (published in March 2013) reveal the same kind of tendency:

Contribution of different drivers to the change in Common Equity Tier 1 Capital ratio – Supervisory Severely Adverse Scenario



Source: CitiBank

2.3. Context and purpose of the white paper

In times where regulators rely more and more on stress testing to assess capital needs and shortfall, where credit risk remains a top issue for financial institutions (in both Europe and North America), the purpose of the white paper is to provide a new approach on how to diffuse the stress test impacts on the rating scale of a credit portfolio in order to more accurately reflect the real behavior of the portfolio in times of stress.

2.3.1. Context

Generally, when performing a stress testing exercise on a credit portfolio (Mainly Retail or SMEs), the average default rate of the portfolio is projected over the time horizon, based upon the macro-economic drivers that most impact the default rate (these drivers depend on the loan types, counterparty types, geographical zone, etc.). For banks that fall under the IRBA approach, the average stressed default rate must be diffused on the rating scale. In other words, the way the average stressed default rate impacts the individual default rate of each rating class must be considered.

Moreover, the performance of the score could have an impact on the distortion of the curve (default rate by rating class) post-stress. In fact, it is possible to assume that the higher the performance of the score, the more often the impact of the stress is observed on the "bad" rating classes. This assumption relies indeed on two main facts.

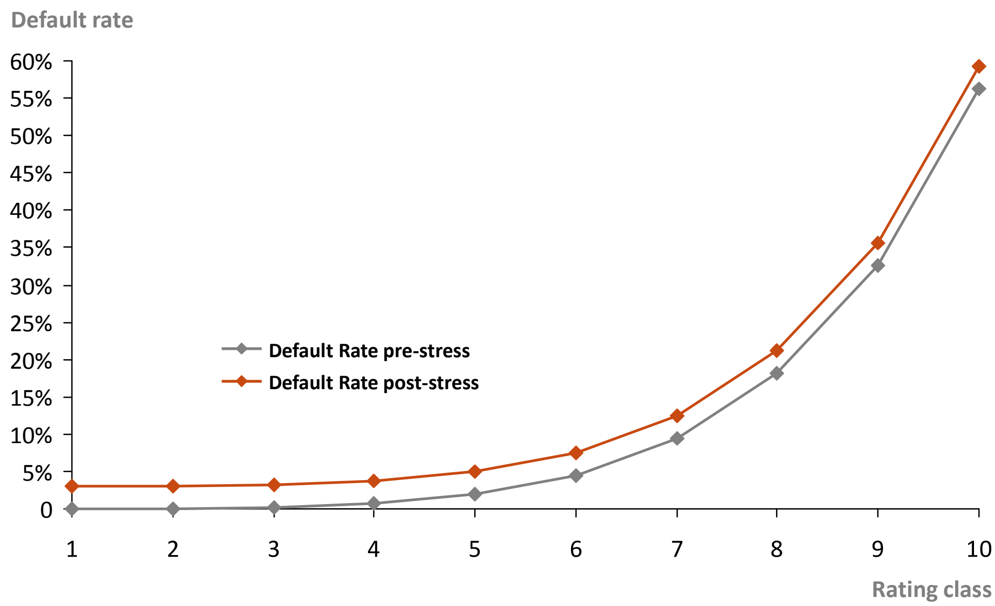
The first one is related to the behavior of individuals in times of stress. In other words, when a crisis occurs, all potentially risky individuals will be downgraded and will migrate to the "bad" rating classes. Plus, the risk level embedded in the bad rating classes will increase because of the macro-economic conditions that weaken the financial health of the counterparties knowing that they are already considered as fragile individuals. This means that the distance to default of these "fragile" individuals is shortened.

The second fact is related to the ability of the score to correctly order the individuals of the portfolio. Knowing the performance and the discriminatory power of the score, the behavior of the individuals will be captured accordingly. In other words, the more precise the score, (and therefore its performances are good), the more exactly it will capture the behavior of the counterparties and therefore, order them in an explicitly correct manner.

2.3.2. Current market practices

Even though some big players put advanced methodologies in place to diffuse the stress by rating class, a major portion of financial institutions used simple methods due to lack of time or lack of resources. These methods are considered proxies since they do not capture the real behavior of the portfolio in times of stress, but instead were used because of their simplicity and because they are not time consuming.

One of the current market practices applies the stress uniformly on the entire curve (default rate by rating class) or in other words to uniformly increase the individual default rate of each rating class:



If the advantages of this approach are obvious (simplicity, timeliness, etc.), it does present some loopholes:

- First, it overestimates the risk on the "good" classes and underestimates the risk on the "bad" classes;
- Second, it does not capture the real behavior of the portfolio and could be considered as a static approach for stress diffusion;
- Third, in many cases it is not optimized and therefore overestimates capital needs.

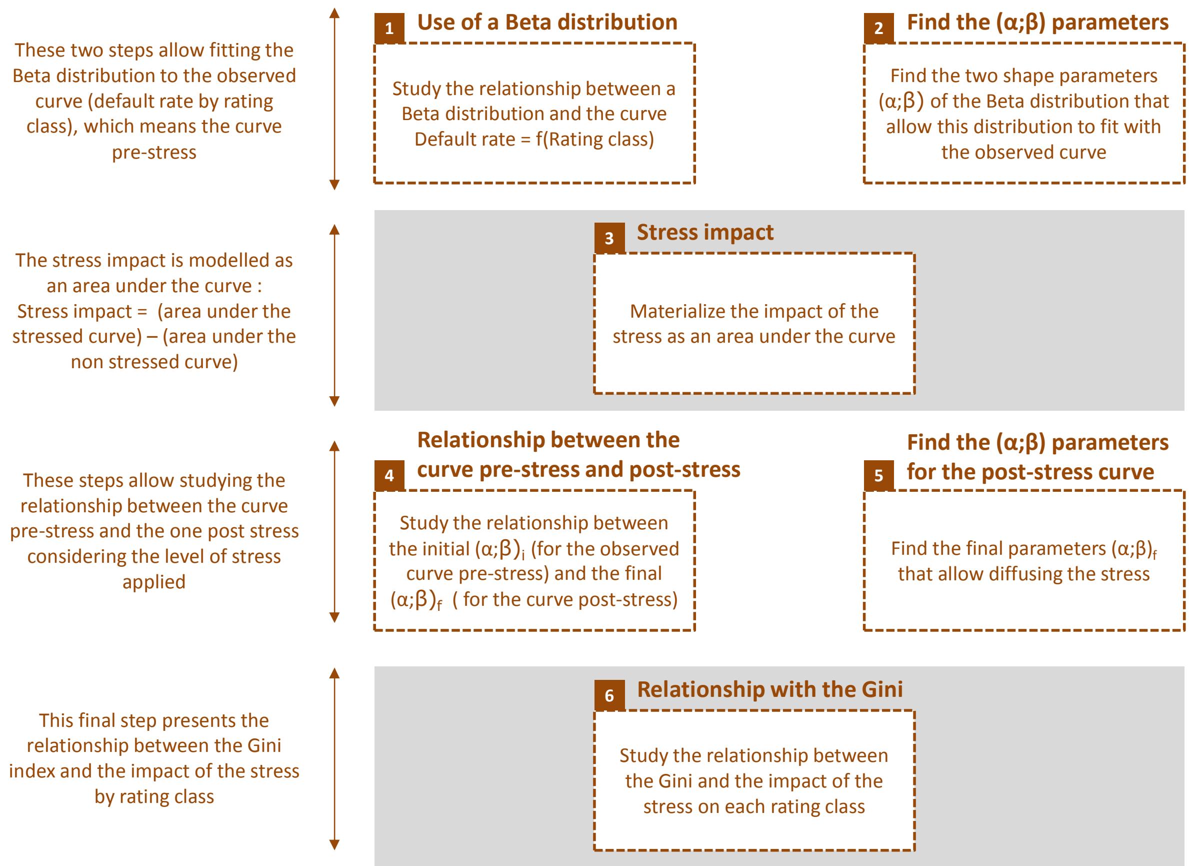
2.3.3. Purpose and objective of the study

The purpose of the study is to propose an approach that allows diffusing the stressed average default rate on the rating scale considering the behavior of the portfolio in times of stress while studying the relationship between the performances of the score (measured essentially by the Gini index and the ROC curve) and the way the curve (default rate by rating class) is impacted post-stress.

3. The approach for a dynamic stress diffusion model

3.1. The overall approach

The approach proposed in this white paper is constituted of 6 steps:



These steps will be further detailed below.

3.2. Step #1 – Use of a Beta distribution

The purpose of this step is to analyze the possibility of using a beta distribution that can be fitted to the observed curve default rate = f(rating class), the logic behind this approach being to model the observed curve under a known form or distribution, and deduce from it the form of the curve post-stress.

The beta distribution has been chosen because of its ability to model the behavior of different random variables as well as its ability to take many different shapes, giving the Beta distribution a high level of adaptability and a high degree of customization. Moreover, it takes values between 0 and 1 which is adequate to model the default rate. Plus, this distribution presents some remarkable properties that could be interesting for the purpose of this study.

In order to assess the possibility of using the Beta distribution, a Kolmogorov-Smirnov will be performed. This nonparametric test allows for comparing the probability distribution of a sample with a reference probability distribution. In other words, the default rate curve (by rating class) will be compared to a Beta distribution in order to make sure that it is possible to use this distribution while modeling the default rate curve.

3.3. Step #2 – Find the (α,β) parameters for the initial curve

The Beta distribution is characterized by two shape parameters (α;β). These parameters allow defining the shape of the distribution and therefore fitting it with the observed default rate curve (by rating class). In other words, finding the right couple (α;β) will allow for modeling the observed default rate curve.

Assume that random variable F follows a Beta distribution:

$$F \sim B(\alpha;\beta)$$

With (α;β) the two shape parameters of a Beta distribution.

The distribution function of F is:

$$B(x,\alpha,\beta) = \int_0^x t^{\alpha-1}(1-t)^{\beta-1}dt$$

$$B(\alpha,\beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt$$

$$F(x) = \frac{B(x,\alpha,\beta)}{B(\alpha,\beta)} = \frac{\int_0^x t^{\alpha-1}(1-t)^{\beta-1}dt}{\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt}$$

Moreover the default rate for the class i is denoted DR_i.

To find the couple (α;β), a commonly used method is the Maximum Likelihood Estimation (MLE) and the Generalized method of moments estimation. This technique is possible when the sample is constituted of n independent points, identically distributed. The system of equations to be solved is:

$$DR_i - F\left(\frac{i}{10}\right) = \epsilon_i \quad \forall\, i \in [\![1;10]\!]$$

With $\epsilon_i \sim N(0,\varepsilon)$, is the error term.

$$DR_i - \frac{\int_0^{\frac{i}{10}} t^{\alpha-1}(1-t)^{\beta-1}dt}{\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt} = \epsilon_i \qquad \forall\, i \in [\![1;10]\!]$$

To solve this optimization problem and therefore find the adequate (α;β), a Generalized Reduced Gradient Algorithm has been utilized, based on the least squares method:

$$(\alpha,\beta) = \arg min_{\alpha,\beta} \left\{ \sum_{i=1}^{10} \left(DR_i - \frac{\int_0^{\frac{i}{10}} t^{\alpha-1}(1-t)^{\beta-1}dt}{\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}dt} \right)^2 \right\}$$

3.4. Step #3 – Model the stress impact

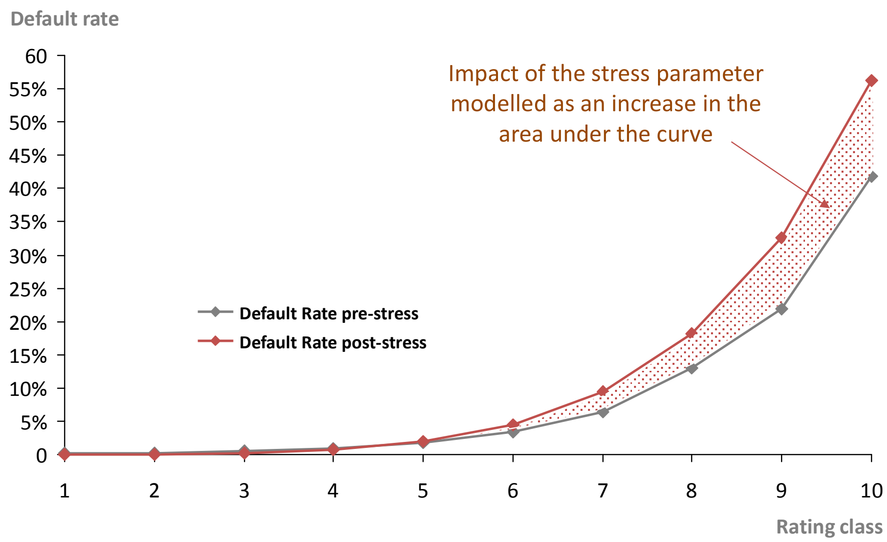
In a credit stress testing exercise, the average default rate of the portfolio is projected under an adverse scenario. The stressed average default rate at time t+1 is:

$$DR(t+1) = DR(t) + s$$

s is the stress applied to the average default rate.

This stress must be diffused on the rating scale. In other words, considering the distribution of default rates by rating class before stress, how the stress will impact this distribution is the primary focus of the paragraph.

The approach used here is to model the impact of the stress as an area under the curve. In fact, generally, the stress will contribute in increasing the individual default rates of the different rating classes (DR_i), shifting the default rate distribution upwards. Consequently, the area under the shifted curve is expected to increase in comparison with the non-stressed distribution. The increase in the area could be linked with the level of stress applied (the "s" parameter):



To calculate the area under the curve, the trapezoidal rule is used. This method is an approximation of the definite integral, or in other words, the exact area under the curve. Assume A1 is the area under the pre-stress curve and A2 the area under the post-stress curve. Assume as well that the rating scale is constituted by n classes, class n having a 100% default rate and class 0 having a 0% default rate (in the graph above, n equals 11).

$$A_1 = \sum_{i=0}^{n} \frac{DR_{i+1}(t) + DR_i(t)}{2}(i+1-i)$$
$$= \sum_{i=0}^{n} \frac{DR_{i+1}(t) + DR_i(t)}{2} = \sum_{i=1}^{n-1} DR_i(t) + \frac{DR_0(t) + DR_n(t)}{2}$$

$$A_2 = \sum_{i=1}^{n} \frac{DR_{i+1}(t+1) + DR_i(t+1)}{2}(i+1-i)$$
$$= \sum_{i=1}^{n-1} DR_i(t+1) + \frac{DR_0(t+1) + DR_n(t+1)}{2}$$

$$A_2 - A_1 = \sum_{i=1}^{n-1} \big(DR_i(t+1) - DR_i(t)\big)$$

Because $DR_0(t+1) = DR_0(t) = 0\%$ and $DR_n(t+1) = DR_0(t+1) = 100\%$

By dividing A_2-A_1 by (n-1):

$$\frac{A_2 - A_1}{n-1} = \frac{\sum_{i=1}^{n-1} DR_i(t+1)}{n-1} - \frac{\sum_{i=1}^{n-1} DR_i(t)}{n-1} = DR(t+1) - DR(t) = s$$

Finally, the increase in area between the pre-stress and post-stress curve depends on both the level of stress applied and the number of classes:

$$A_2 - A_1 = s.(n-1)$$

3.5. Step #4 – Establish a relationship between the curve pre-stress and post-stress

The purpose of this paragraph is to establish a relationship between the curve pre-stress and the curve post-stress. To do so, the Beta distribution used to model the pre-stress curve will be used and the curve post-stress is assumed to follow a Beta distribution as well. In other words, the link between the two curves corresponds to establishing a link between the shape parameters of the curve pre-stress (denoted α_1 and β_1) and the shape parameters post-stress (denoted α_2 and β_2).

One of the advantages of the Beta function is its remarkable properties. The Beta function is linked with the Gamma function as follows:

$$B(\alpha,\beta) = \frac{\Gamma(\alpha).\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

The gamma function can be written as follows:

$$\Gamma(z) = (z-1)! = (z-2)!\,(z-1) = \Gamma(z-1).(z-1)$$

This means that:

$$B(\alpha,\beta+1) = \frac{\Gamma(\alpha).[\Gamma(\beta).\beta]}{\Gamma(\alpha+\beta).(\alpha+\beta)}$$

Finally,

$$B(\alpha,\beta+1) = B(\alpha,\beta)\frac{\beta}{\alpha+\beta}$$

As mentioned above, the distribution functions of the two curves are:

$$F_1 \sim B(\alpha_1,\beta_1)$$

$$F_2 \sim B(\alpha_2,\beta_2)$$

The relationship between the two curves can be explicated as follows, in relationship with the stress parameter s:

$$\int_0^1 F_2(x) - F_2(x)dx = s$$

Plus,

$$B(\alpha,\beta)\int_0^1 F(x)dx = \int_0^1\int_0^x t^{\alpha-1}(1-t)^{\beta-1}dt.dx$$

$$= \int_0^1\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}.\mathbb{1}_{t\leq x}.dt.dx$$

By applying Fubini's theorem, the equation is:

$$B(\alpha,\beta)\int_0^1 F(x)dx = \int_0^1\int_t^1 t^{\alpha-1}(1-t)^{\beta-1}.dx.dt$$

$$= \int_0^1\left(\int_t^1 dx\right)t^{\alpha-1}(1-t)^{\beta-1}.dt = \int_0^1 (1-t)t^{\alpha-1}(1-t)^{\beta-1}.dt = \int_0^1 t^{\alpha-1}(1-t)^{\beta}.dt$$

This means that,

$$B(\alpha,\beta)\int_0^1 F(x)dx = \; B(\alpha,\beta+1)$$

Using the remarkable propriety of the B function, the equation is:

$$\int_0^1 F(x)\,dx = \frac{\beta}{\alpha+\beta}$$

Therefore, we can write:

$$\int_0^1 F_2(x) - F_2(x)dx = \frac{\beta_2}{\alpha_2+\beta_2} - \frac{\beta_1}{\alpha_1+\beta_1} = s$$

Finally, the relationship between the shape parameters pre and post stress is:

$$\beta_2 = \frac{\alpha_2\left(s + \frac{\beta_1}{\alpha_1 + \beta_1}\right)}{1 - s - \frac{\beta_1}{\alpha_1 + \beta_1}}$$

This equation shows that there is one degree of freedom between the post-stress parameters. In other words, if α2 is correctly estimated, the couple (α_2, β_2) will be estimated and therefore the post-stress curve.

3.6. Step #5 – Find the (α,β) parameters for the post-stress curve

In step 4, a relationship has been established between the pre-stress and post-stress curves. In this step, the purpose is to define the system of equations and the problem to be solved in order to determine the shape parameters for the post-stress curve.

Considering the different steps above, the optimization problem to be solved can be written as follows:

$$\alpha_2 = \arg\min_{\alpha_2} \mid \int_0^1 F_2(x) - F_1(x)dx - s \mid$$

With

$$\beta_2 = \frac{\alpha_2\left(s + \frac{\beta_1}{\alpha_1 + \beta_1}\right)}{1 - s - \frac{\beta_1}{\alpha_1 + \beta_1}}$$

By using the trapezoidal rule results the equation becomes:

$$\sum_{i=1}^{n-1} F_2(i) - F_1(i) - (n-1).s \sim 0$$

With the following constraints:

$$\forall\, k \in [\![1; n-1]\!] : F_1(k) \leq F_2(k)$$

$$\forall\, k, k' \in [\![1; n-1]\!] \mid k \leq k' : F_2(k) - F_1(k) \leq F_2(k') - F_1(k')$$

The first constraint refers to the fact that default rates post-stress must be higher than those pre-stress. The second constraint materializes the fact that the impact of the stress increases with the rating class.

To solve this optimization problem and therefore find the adequate α_2, a Generalized Reduced Gradient Algorithm has been utilized, based on the least squares method. This algorithm works on multiple iterations and finds the most optimized solution.

3.7. Step #6 – Analyze the relationship with the Gini index

As part of the study, and as mentioned earlier, one can assume that based on the portfolio behavior in times of stress, the performance of the score has an impact on how the stress is diffused on the rating scale. Consequently, it is possible to assume that the higher the Gini score, the higher the impact of the stress on the “bad” classes, showing the ability of the score to accurately rank risky individuals and their behavior in times of tumultuous conditions.

The purpose of this step is to confirm that the assumption is correct and make sure that the diffusion model proposed for this study allows for the consideration of the Gini index.

4. Building a diffusion model on a real SME portfolio

4.1. Framework of the study

4.1.1. Purpose of the analysis

The analysis presented in this section regards building an efficient stress diffusion model based on the methodology enclosed in this article. A real portfolio of SME counterparties will be considered on which a score has been built with a rating scale that ranks counterparties in 10 classes (class 1 being the less risky).

The model will be calibrated for this portfolio and the way the stress is diffused will be challenged.

4.1.2. What are the dimensions involved in the study?

There are three dimensions that are taken into account for the study:

The first dimension concerns the optimization problem to be solved. In fact, during the study, the ability to find accurate, stable and precise solutions for the problem will be considered. If there is no solution, or there are inaccurate solutions, then the reliability of the model and its applicability in real conditions will be questioned.
The second dimension concerns the results obtained and their reliability from a risk management point of view. In fact, the model risk will be considered for two reasons. The first one is related to the importance of stress testing and its impacts on regulatory capital.

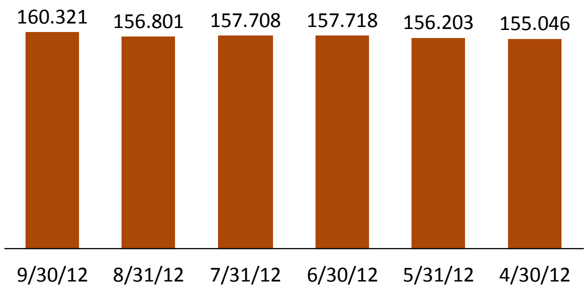
Consequently, knowing the sensitivity of capital to risk parameters, it is crucial to make sure that the results obtained are reliable and to cover the uncertainty of the model (or in other words, the error term). The second reason is related to regulatory pressure from a documentation point of view. In fact, regulators ask for a clear documentation around stress test models in order to justify assumptions and projection results.

The third dimension is timeliness of the approach. As mentioned above, the stress testing exercise is most likely to become a permanent exercise. Consequently, any approach built around this topic must take into account time constraints, computational time, resources workload, etc. Consequently, while implementing the model on real portfolio, the timeliness of the approach will be measured.

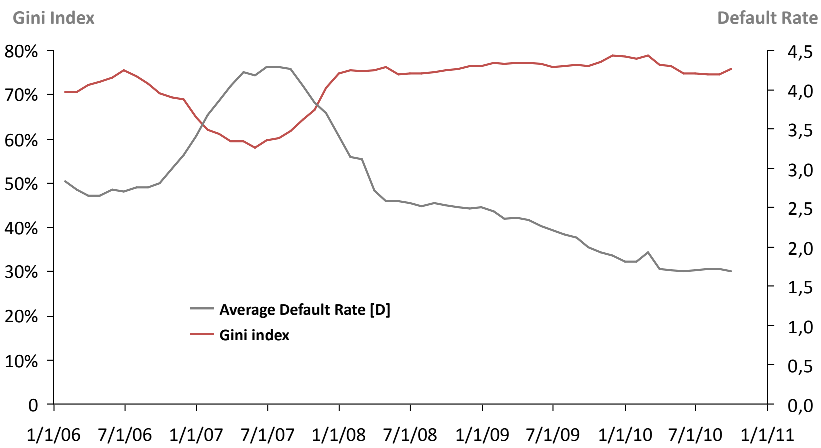
4.1.3.Description of the portfolio on which the study was conducted

As mentioned above, the portfolio on which the study was conducted is an SME portfolio. The relevant data is available from January 2006 until today. The score has been built on the modeling window of September 2012. The default rate series on which the stress testing model has been built is identical to the score modeling sample.

The portfolio is constituted by a high number of counterparties with an average default rate of 2.73%. Moreover, the evolution of the individuals over 6 months' time horizon is:



For the purpose of the study, the score has already been constructed, based on the best practices in the industry. The score shows good performances with a Gini index 78%. The methodology used to build the score is not the core subject of the study. Yet, one interesting result is the relationship between the default rate and the Gini index as shown above:



The graph shows that the default rate and the Gini index are negatively correlated (-89% over the time horizon). In fact, the default rate increases significantly from August 2006 (2.76%) to reach its peak on June 2007 (4.31%). In the meantime, the Gini index decreases from 73% to 61.8% over the same time horizon. This is explained by the fact that the score is calibrated on a modeling window with specific conditions. Once these conditions change, the performances of the score are impacted showing the limitation of the score to capture accurately the behavior of the portfolio in times of stress conditions. Consequently, the purpose of the stress diffusion model as proposed within this approach is to understand the relationship between the performance of the score and the way the stress is diffused on the rating scale.

4.2. Model calibration and results

4.2.1. Step #1 – Confirming the use of the Beta distribution

The first step of the study is to confirm the use of the Beta distribution. For this matter, a Kolmogorov-Smirnov test is performed on the existing default rate curve by rating class as provided by the score on this portfolio. A distribution fitting algorithm is used and a KS test is performed.

The non-parametric test performed is:

H_0 : The distribution follows a Beta distribution
H_1 : The distribution does not follow a Beta distribution

The results are shown above:

Statistics	Values
Distance	0,167
p-value	0,918

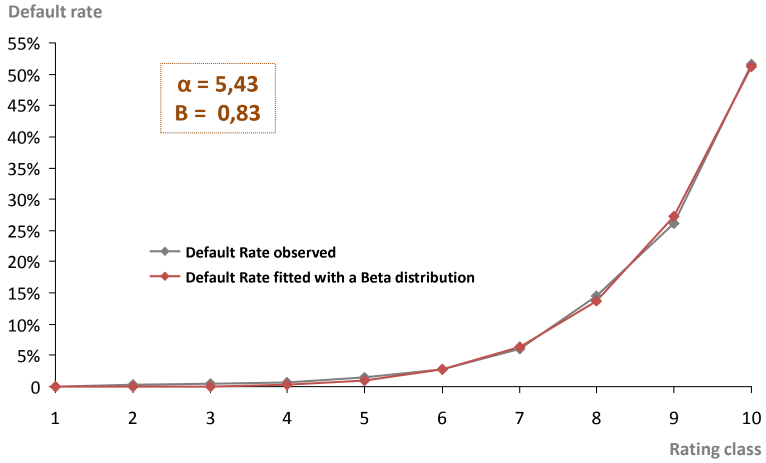
The p-value with a 5% confidence level shows that the null hypothesis cannot be rejected. In other words, the assumption for which the default rate curve follows a Beta distribution can be applicable. Therefore, this curve will be modeled using this distribution.

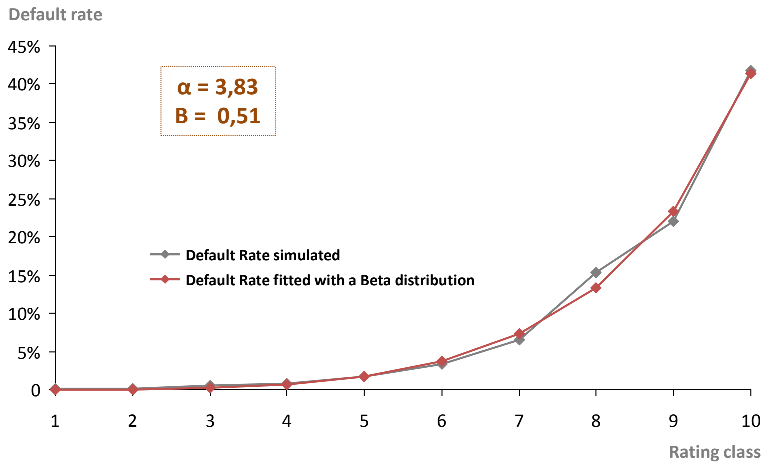
4.2.2. Step #2 – Modeling the default rate curve with a Beta distribution

Once the hypothesis of using a Beta distribution is validated, the purpose here is to model the default rate curve using this distribution. The shape parameters will be calculated in order to fit the distribution with the observed curve as much as possible.

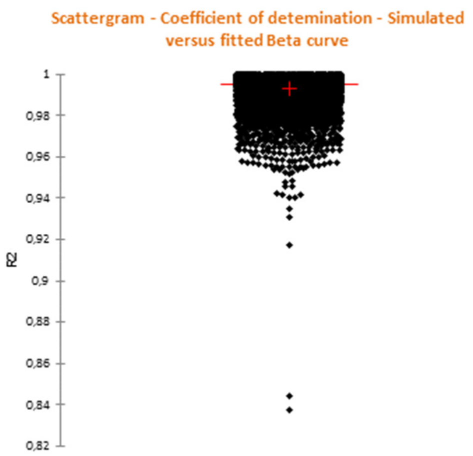
In order to generalize the problem and verify its applicability in various conditions, the distribution of default rate by rating class has been simulated using a Monte Carlo simulation. 10,000 classifications have been simulated using a normal distribution for each individual rating class.

The algorithm for computing the shape parameters provided positive and accurate results for 9813 out of 10000, which means a positive response of 98.2%. Examples of the results are presented below:





The performances of the fitting between the simulated curve and the fitted Beta distributions have been measured. The distributions of the coefficient of determination for the 9813 classifications are shown below:



Lower limit	*Higher limit*	*Number*	*%*
80,0%	82,1%	0	0,0%
82,1%	84,2%	1	0,0%
84,2%	86,3%	1	0,0%
86,3%	88,4%	0	0,0%
88,4%	90,5%	0	0,0%
90,5%	92,6%	1	0,0%
92,6%	94,7%	9	0,1%
94,7%	96,8%	112	1,1%
96,8%	98,9%	1608	16,4%
98,9%	100,0%	8081	82,3%

The Root Mean Squared Error (RMSE) has been computed as well:

RMSE	Number	%
<0.5%	9 513	96,94%
[0.5%;1%[	287	2,92%
[1%;1.5%[	10	0,10%
>=1.5%	3	0,03%

The conclusions are that the performances of the fitted curves are high with more than 98.7% of the classifications that have a coefficient of determination of more than 98.9%. The RMSE is low as well with more than 97% of the classifications that have a RMSE lower than 0.5%. The results provided are then considered as compliant with the expectations.

4.2.3. Step #3 and Step #4 – Providing inputs to the optimization problem of Step #5

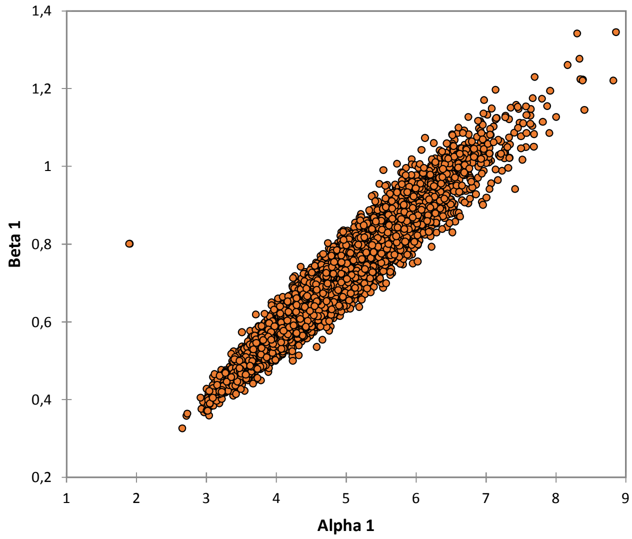
Step #3 and Step #4 provide inputs to the system of equations to be solved in Step #5. Basically, there is no computation at this stage of the approach. As part of the Step #3, and for the purpose of this portfolio, the number of rating classes is 10 for the performing loans and 1 class (class#11) for the non-performing loans. Moreover, for the purpose of this study, it is assumed that the stress to be applicable to the average default rate is an add-on of 2%. In reality, the level of stress will be determined using a time series model and by projecting the default rate based on macro-economic covariate.

Consequently, by applying the formula in step 3.4, the increase in the area under the curve due to the stress applied is of 20%.

4.2.4. Step #5 – Finding the (α,β) for the post-stress curve

The purpose of this paragraph is to solve the system of equations provided in part 3.6. This will allow the shape parameters for the stressed default rate curve to be determined. On one hand, the positive response of the model is key to validating its applicability in real conditions. On the other hand, the quality and the reliability of the results will show the ability of the model to diffuse the stress on the rating scale accordingly to the expected behavior of the portfolio in tumultuous conditions.

First, one interesting finding that must be mentioned is related to the relationship between the shape parameters α_1 and β_1 which means the relationship between the shape parameters of the fitted pre-stress curve. By drawing the point cloud between these two parameters, a linear relationship is identified between the two:



By performing a linear regression between these two parameters, the results are as follows:

Model parameters

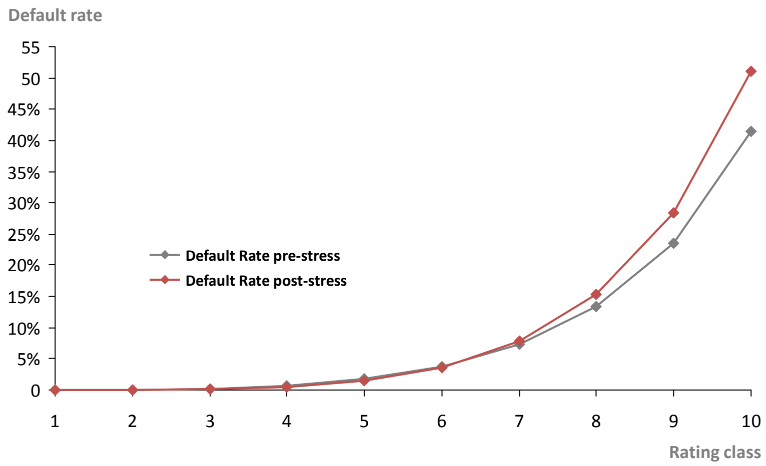
Source	Value	Standard error	t	Pr > |t|	Lower bound (95%)	Upper bound (95%)
Intercept	0,843	0,012	71,364	< 0,0001	0,819	0,866
Beta	5,686	0,017	334,629	< 0,0001	5,652	5,719

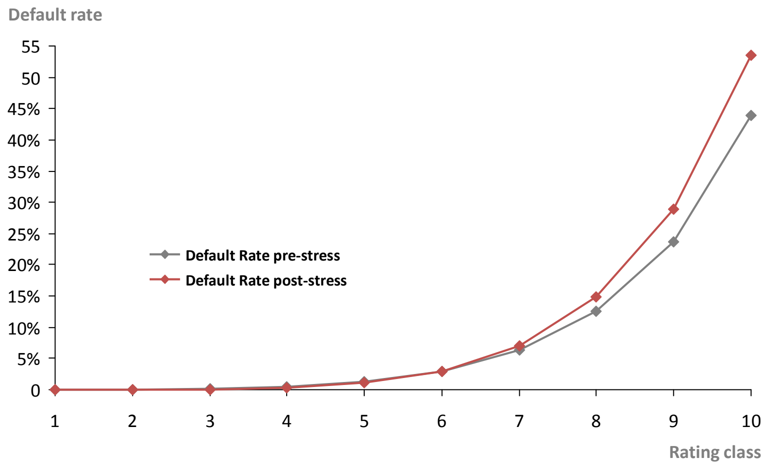
Goodness of fit statistics

Observations	9813
Sum of weights	9813
DF	9811
R^2	**0,919**
Adjusted R^2	**0,919**
MSE	0,050
AIC	-29397,818
SBC	-29383,435
PC	0,081

Even though this result won't be used in the system of equations, and therefore won't be used for the purpose of this study, it remains important to notice that finding such a relationship can eventually have applications for other studies in the same field.

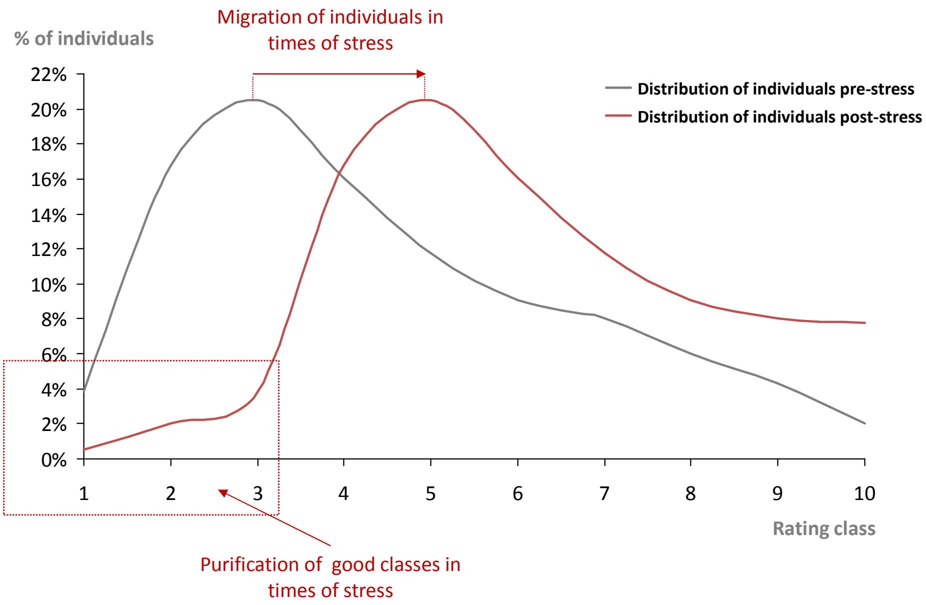
Second, to solve the problem, a Generalized Reduced Gradient Algorithm has been applied on the 9813 classifications for which the default rate curve has been modeled using the Beta distribution. From a computational point of view, the time span for finding the solutions was about a dozen of hours for all of the classifications. This time span could be considered as acceptable knowing that it could have been optimized using a computer with better performances and by optimizing the algorithm used. Out of 9813 classifications, 9803 positive response were determined while 10 errors has been identified with negative shape parameters and stressed curves that don't meet the requirements of this study. Consequently, the total positive responses for the entire study (including shape parameters for the pre-stress curve and those for the post-stress curve) is about 98% which basically indicates the reliability of the model and its ability to be applicable and adaptable to various portfolios and default rate distributions. Some examples of the results are shown above in terms of the stress diffusion on the rating scale in comparison with the non-stressed curve:





Third, the model's errors and accuracy have been measured and analyzed with the purpose of understanding these errors and cover them with relevant margin of prudence. Two main points have been identified as weaknesses in the model, which will require some adjustments and on which a margin will calibrated.

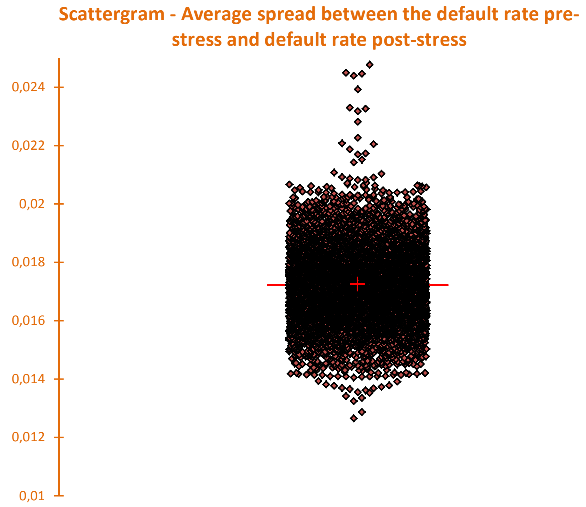
The first point is related to the levels of pre-stress default rates for each rating class in comparison to the levels of post-stress default rates. In fact, for some classifications, and mainly for the "good classes" (class #1 to class #4 mainly), the post-stress default rates could be lower that the pre-stress default rates. This phenomenon is not observed for the other classes but is nonetheless interesting because it can be interpreted in various ways. One way to interpret that is to consider that in times of stress, the migration of potential "risky" individuals from good classes to bad classes purifies the "good" classes, meaning that in times of stress, the remaining counterparties classified within the "good" rating classes are considered risk-free (even in crisis times). Therefore, this purification of the good classes contributes in lowering the default rate for these classes:



For the purpose of this study, this phenomenon will be considered as a model error. Yet, the interpretation from an economic point of view could be relevant and this feature is in line with findings identified during the AQR exercise in Europe. In fact, the Portfolio Improvement Effect (PIE) as described in the ECB's *Comprehensive Assessment Stress Test Manual* is related to this phenomenon which means that from a regulatory perspective, it could be acceptable to present such results.

The second point is related to estimating the impact of the stress. In fact, as mentioned above, the target for the model is to diffuse the impact of the stress (parameter "s" as detailed above) on the rating scale. The purpose here is to make sure that the average default rate post-stress meets the targeted "s" parameter. For the 9803 classifications, the difference between the average pre- and post-stress default rates has been computed. The distribution is shown below:

Statistics	Values
Obervations	9803
Minimum	1,27%
Maximum	2,48%
1st Quartile	1,64%
Mediane	1,72%
3rd Quartile	1,80%
Mean	1,72%
Variance (n-1)	0,00%
Deviation (n-1)	0,12%



As shown above, the average spread between the pre- and post-stress default rates is 1.72%. Basically, we would expect to have a 2% spread as the stress parameter "s" has been set at 2%

for the purpose of this study. This shows that the model tends to underestimate the diffusion of the stress, and this error must be covered by a margin of prudence.

The methodology proposed to determine the margin of prudence has been chosen to cover both points listed above. In fact, one simple method has been proposed to cover both points at the same time: The margin of prudence is calculated as the average spread between the curve pre-stress and post-stress and the "s" parameter. This margin of prudence is added to the individual default rates (post stress) of each rating class, which will contribute to translating the curve upwards while introducing two benefits:

- First, the stressed default rates for the "good" classes are translated upwards and therefore contribute to a more prudential and conservative approach;
- Second, the level of stress finally applied will be equivalent to the "s" parameter.

To determine the margin of prudence MP:

$$MP = s - \left(DR(t+1) - DR(t)\right) = s - s'$$

4.2.5. Step #6 – Analyzing the relationship between the Gini index and the diffusion of the stress

The purpose of this paragraph is to analyze the relationship between the Gini index and the way the stress is diffused on the rating. It is possible to imagine that the more the Gini index is high the more the stress impacts the "bad" classes. If such a relationship is plausible then the purpose here is two-fold:

- First, it is important to verify the ability of the model to capture that information and to reflect this relationship in real conditions;
- Second, it will allow for measuring the bias introduced by scores with low or medium performances by properly ranking the individuals in times of stress. Therefore, knowing the level of the bias, the diffusion of stress can be covered or corrected, consequently enhancing the ability of financial institutions to more precisely measure the risks that arise in times of stress.

To do so, scores with different performances have been considered. The stress diffusion model detailed within this paper has been implemented and the impact of the stress has been measured for each rating class according to the performances of the score (and more precisely, the Gini index). The results above show the difference between the default rate pre- and post-stress for each rating class:

	Rating Class									
Gini	1	2	3	4	5	6	7	8	9	10
0,75	0,27%	0,26%	0,20%	0,12%	0,08%	0,24%	0,91%	2,53%	5,57%	9,83%
0,76	0,27%	0,26%	0,23%	0,19%	0,21%	0,43%	1,15%	2,73%	5,55%	9,38%
0,77	0,27%	0,26%	0,21%	0,14%	0,12%	0,31%	1,01%	2,62%	5,58%	9,67%
0,78	0,27%	0,26%	0,22%	0,17%	0,17%	0,40%	1,13%	2,76%	5,64%	9,46%
0,79	0,27%	0,26%	0,23%	0,18%	0,21%	0,46%	1,22%	2,86%	5,69%	9,31%
0,80	0,27%	0,26%	0,24%	0,21%	0,26%	0,56%	1,36%	3,01%	5,75%	9,07%
0,81	0,27%	0,26%	0,24%	0,22%	0,29%	0,61%	1,44%	3,10%	5,79%	8,93%
0,82	0,27%	0,27%	0,26%	0,27%	0,40%	0,80%	1,71%	3,37%	5,87%	8,44%
0,83	0,27%	0,27%	0,24%	0,23%	0,32%	0,68%	1,57%	3,29%	5,94%	8,73%
0,84	0,28%	0,29%	0,35%	0,55%	1,01%	1,85%	3,12%	4,66%	5,94%	5,71%

The results above show two main features of the study:

- The first point is related to the relationship between the Gini and the stress impact across the rating scale (horizontal axis of the matrix). As the results show, the lower the Gini index, the less the evolution of the stress impact on the "good" classes is monotonously increasing:
  - For instance, if we consider the score with a 75% Gini index, the impact of the stress decreases from class 1 to class 5
  - On the contrary, if we consider the score with 85% Gini index, the impact of the stress is monotonously increasing on these classes

- The second point is related to the relationship between the Gini index and the stress impact for one rating class (vertical axis of the matrix). For basically all classes (except class 10), the stress impact for a rating class increases with the Gini index. In other words, the higher the Gini, the higher the stress impact for a rating class. Yet, for class 10 the relationship between the Gini and the stress impact is inverted. One explanation for this phenomenon is that this class is at the boundary with the default class (i.e. class 11). In fact, the counterparties of this class are highly risky and their behavior presents risky patterns in normal conditions. The distance to default is normally short. In times of stress these counterparties are most likely to default. Consequently, the better the performances of the score, the easier it will be able to identify these counterparties and classify them as highly risky. The impact on the distance to default post-stress in comparison to the one pre-stress is relatively short because these counterparties are already highly risky in normal conditions. This depends obviously on the precision of the score.

These results prove that a relationship exists between the Gini index and the stress diffusion. They prove as well that the model designed for the purpose of this study captures that relationship and transcribes it in real conditions.

Conclusion

As stress testing exercises become more and more important in today's financial and regulatory world, the accuracy and the reliability of stress models on which the risk components are estimated must be subject to a special focus. In fact, the areas of optimization are numerous and the regulators are more and more concerned about the quality of internal models and the discrepancies identified from one bank to another. In this context, this white paper tries to provide an approach that allows for diffusing the stress on the rating scale more accurately and more friendly with real behaviors of the portfolios. Therefore, the stress impact depends on the severity of risks in the rating scale. Moreover, this study shows the impact of the Gini index on the stress diffusion and the bias introduced considering the level of performance of a credit score. This model, coupled with a migration model in stress conditions, will create some areas for RWA optimization and capital shortfall estimations, which might be interesting to look at.

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