Value-at-Risk in turbulence time

Genest, Benoit and Cao, Zhili

27 January 2014

Value-at-Risk in turbulence time

27/01/2014

By Zhili Cao (Paris office) & Benoit Genest (London office)

Global Research & Analytics Dpt.

Value-at-Risk in turbulence time

Abstract

Value-at-Risk (VaR) has been adopted as the cornerstone and common language of risk management by virtually all major financial institutions and regulators. However, this risk measure has failed to warn the market participants during the financial crisis. In this paper, we show this failure may come from the methodology that we use to calculate VaR and not necessarily for VaR measure itself. we compare two different methods for VaR calculation, 1. by assuming the normal distribution of portfolio return, 2. by using a bootstrap method in a nonparametric framework. The Empirical exercise is implemented on CAC40 index, and the results show us that the first method will underestimate the market risk - the failure of VaR measure occurs. Yet, the second method overcomes the shortcomings of the first method and provides results that pass the tests of VaR evaluation.

Keywords: Keywords: Value-at-risk, GARCH model, Bootstrap, hit function, VaR evaluation.

*ChappuisHalder - Global Research&Analytic. E-mail: zcao@chappuishalder.com

1 Introduction

Many risk management tools and their calculations implemented by most financial institutions and regulators have been designed for "normal times". They typically assume that financial markets behave smoothly and follow certain distributions to depict the financial environment during this period. However, the problem discovered recently is that we are living in more and more "turbulence times": large risks happened without any warnings and come along with huge impacts in both time and cross-sectional dimensions.

The late 2000's financial crisis, which started in August 2007 (Subprime Crisis) and followed in May 2010 (European Sovereign Crisis), is a good explanation for why we should be involved in this point. It must be viewed as an historical event of major importance, with worldwide and durable consequences on the economic behavior and on the organization of financial and banking activities in many countries over the world. It has completely shattered public confidence into the Risk Management methods that were used by banks and financial intermediaries. These Risk Management methods must be adapted to "turbulent times". The Value at Risk (VaR) is a very important concept in Risk Management and it is one of the most widely used statistics that measures the potential of economic loss. It has been adopted as the cornerstone and common language of risk management by virtually all major financial institutions and regulators. The main objective of this paper is to dissect VaR methods, to explain why it has failed, and propose amendments for the future. Of course there are some other useful risk measures in the literature, however, these are not in the scope the research in this paper.

Conceptually speaking, we are not simply saying that VaR is bad and we should ban this measure for any use. However, we will take the more reasonable view that VaR is potentially useful but have to be used with a lot of caution. we focus on two VaR calculation methods:[1] 1) GARCH with normal distribution assumption method, 2) Nonparametric bootstrap method. Here, VaR is implemented on stressing the riskiness of portfolio. Portfolios are exposed to many classes of risk. These five categories represent an overview of the risk factors which may affect a portfolio.

- Market risk - the risk of losses in positions arising from movements in market prices.
- Liquidity risk - in addition to market risk by measuring the extra loss involved if a position must be rapidly changed.
- Credit risk - also known as default risk, covers cases where counterparties are unable to pay back previously agreed terms.
- Model risk - A type of risk that occurs when a financial model used to measure a firm's market risks or value transactions does not perform the tasks or capture the risks it was designed to.

[1] we won't focus on the three standard methods - 1.historical VaR, 2. Parametric VaR, 3. Monte Carlo method - in this paper.

- Estimation risk - captures an aspect of risk that is present whenever econometric models are used to manage risk since all model contain estimated parameters. Moreover, estimation risk is distincted from model risk since it is present even if a model is correctly specified.

In this paper, we focus on the three categories of risk, 1) market risk, 2) Model risk and 3) Estimation risk. The market risk is illustrated by implementing our analysis on CAC40 index, which can be viewed as virtual portfolio. The time horizon is from January 2002 to October 2013 that captures at least one business cycle and allows us to differentiate "normal period" and "turbulence period". we will also provide in sample and out of sample check to justify if the GARCH model is well specified, this procedure is in the purpose of dealing with model risk. Finally, on top of the advantage of nonparametric estimation, bootstrap methodology also cope with estimation risk of the model, since it takes the average of estimations as the final result to establish the VaR calculation.

The rest of the paper proceeds as follows. Section 2 describes the different methodology of VaR calculating in the literature. Section 3 describes two different test for VaR evaluation. This section is the core of the paper, since it shows us why in some circumstance VaR won't correctly measure the market risk. Section 4 provides the empirical results on CAC40 index. Section 5 concludes.

2 Value-at-Risk

The most common reported measure of risk is Value-at-Risk (VaR). The VaR of a portfolio is the amount risked over some period of time with a fixed probability. VaR provides a more sensible measure of the risk of the portfolio than variance since it focuses on losses, although VaR is not without its own issues. One of the most critical points for VaR is that this measure is not a coherent risk measure, since, in some circumstances, VaR is not endowed with the so called subadditivity property. There exists a rather large literature interested in these suitable properties of a risk measure, however, this interesting point is not in the scope of research in this paper.

The VaR of a portfolio measures the value (in \$, £, €, etc) which an investor would lose with some small probability, usually between 1 and 10%, over a selected period of time. Because the VaR represents a hypothetical loss. Recall that VaR is defined as the solution to

$$P(r_t < VaR_t^q) = q, \tag{1}$$

VaR_t^q is the q-quantile of the return r_t. Note that with this definition, VaR_t^q is typically a negative number.

2.1 VaR calculation

There are three common methodologies for calculating VaR: Historical simulation, Parametric Modeling and Monte Carlo Simulation. In addition, we will present some more advanced methodologies to calculate VaR.

2.1.1 Historical simulation

The historical method simply re-organizes actual historical returns, putting them in order from worst to best. Then the VaR is obtained according to the confidence level q set by the investors. This method assumes that history will repeat itself, from a risk perspective. An advantage of this method is that we don't make any distribution assumption on portfolio returns, meanwhile, it is simple to compute. However, when computing Historical VaR, sampling history requires care in selection. Also, high confidence levels (e.g. 99%) are coarse for a short historical horizon.

2.1.2 Parametric method

The Parametric Model estimates VaR directly from the Standard Deviation of portfolio return. It assumes that returns are normally distributed. In other words, it requires that we estimate only two factors - an expected (or average) return and a standard deviation - which allow us to plot a normal distribution curve.

The idea behind the parametric method is similar to the ideas behind the historical method - except that we use the familiar curve instead of actual data. The advantage of the normal curve is that we automatically know where the confidence level q (worst 5% and 1%) lies on the curve.

2.1.3 Monte Carlo simulation

A Monte Carlo simulation refers to any method that randomly generates trials, but by itself does not tell us anything about the underlying methodology. This method involves developing a model for future returns and running multiple hypothetical trials through the model. For example, we can consider the stock prices movements follow some diffusion processes and then run different trials for the processes to obtain the results.

2.1.4 Quantile regression method

In this method, we first consider that the portfolio returns have the following relationship with a set of state variable M

$$r_t = \alpha + \beta M_{t-1} + E_t,$$

we then denote the cumulative distribution function (cdf) of r_t by $F_{r_t}(r_t)$, and its inverse cdf by $F^{-1}_{r_t}(q)$ for percentile q. It follows that the inverse cdf of r_t is

$$F^{-1}_{r_t}(q|M_{t-1}) = \hat{\alpha}_q + \hat{\beta}_q M_{t-1}, \tag{2}$$

where $\hat{\alpha}_q$ and $\hat{\beta}_q$ are the coefficients for quantile regression with $q \in (0,1)$.

$F^{-1}_{r_t}(q|M_{t-1})$ is called the conditional quantile function. From the definition of VaR, we obtain

$$VaR_q = \inf_{VaR_q} \{P(r_t \leq VaR_t|M_{t-1}) \geq q\} = F^{-1}_{r_t}(q|M_{t-1}).$$

The conditional quantile function $F^{-1}_{r_t}(q|M_{t\_1})$ is the VaR_q conditional on M_{t-1}.

A possible set of state variable is mentioned in Adrian and Brunnermeier (2011). In AB11, they have proposed 7 state variables to estimate time-varying VaR_t. Those are:

i) VIX, which captures the implied volatility in the stock market.

ii) A short term liquidity spread, defined as the difference between the three-month repo rate and the three-month bill rate, which measures short- term liquidity risk. The three-month general collateral repo rate is available on Bloomberg, and The three-month Treasury rate is obtained from the Federal Reserve Bank of New York.

iii) The change in the three-month Treasury bill rate from the Federal Reserve Boards H.15. By using the change in the three-month Treasury bill rate, not the level, since the change is most significant in explaining the tails of financial sector market-valued asset returns.

iv) The change in the slope of the yield curve, measured by the yield spread between the ten-year Treasury rate and the three-month bill rate obtained from the Federal Reserve Boards H.15 release.

v) The change in the credit spread between BAA-rated bonds and the Treasury rate (with the same maturity of ten years) from the Federal Reserve Boards H.15 release.

Note that iv) and v) are two fixed-income factors that capture the time variation in the tails of asset returns.

Then introduce two variables to control for the following equity market returns:

vi) The weekly equity market return.

vii) The one-year cumulative real estate sector return is the value weighted average of real estate companies (SIC code 65-66) from CRSP.

These state variables are well specified to capture time variation in conditional moments of asset returns, and are liquid and easily traded. However, a big shortcoming by using these variables is that it makes the results less robust. Imagine, if we add or delete one variable that will affect the final results of quantile regression, therefore the results of measure. Meanwhile, in financial data, the state variables at time $t-1$ have little predictive power. If we use M_t as explanatory variable, one may introduce the endogeneity problem in the regression, since $E(M_t E_t) \neq 0$, hence the estimated coefficients will be biased.

2.1.5 Conditional Value-at-Risk with normal assumption

This method is similar to the parametric method, the crucial point is to estimate the standard deviation of returns. In parametric model, the standard deviation is calculated from the whole sample; however, in the conditional VaR model, the standard deviation is computed from a GARCH process which provides us the conditional volatility of portfolio return.

We need to compute portfolio returns' conditional volatility σ_t at first place. To do so, in wide world of GARCH specifications, TGARCH is picked to model volatility to capture so called "leverage effect", which has the fact that a negative return increases variance by more than a positive return of the same magnitude. We also do a diagnosis of this GARCH model to show that the model is well specified in the Appendix. The evolution of the conditional variance dynamics in this model is given by:

$$\begin{aligned} r_t &= E_t\sigma_t, \\ \sigma_t^2 &= \omega_G + \alpha_G r_{t-1}^2 + \gamma_G r_{t-1}^2 I_{t-1}^- + \beta_G \sigma_{t-1}^2, \end{aligned}$$

with $I_{t-1}^- = r_t \leq 0$. The model is estimated by Quasi-MLE which guarantees the consistency of the estimator. We have:

$$\sqrt{T}(\hat{\theta}^{MLE} - \theta^o) \sim N(0, J^{-1} I_0 J^{-1}),$$

with $\hat{I} = \frac{1}{T} \sum_{t=1}^{T} S_t(\hat{\theta}) S_t(\hat{\theta})'$ and $\hat{J} = \frac{1}{T} \sum_{t=1}^{T} \frac{\partial^2 logL}{\partial \theta \partial \theta'}$ is the Hessian of total log-likelihood function. Where $\theta = (\alpha_G, \gamma_G, \beta_G)$. Therefore the VaR is derived from the q-quantile of a normal distribution,

$$VaR_t = \sigma_t \Phi^{-1}(q),$$

where $\Phi^{-1}(\cdot)$ is the inverse normal CDF.

As we will see later on, the normal distribution assumption is appropriate only in the calm period of financial market. In turbulence time, this assumption will be violated and if the VaR is calculated based on normal distribution, the portfolio risk is probably going to be underestimated.

2.1.6 Bootstrap method

Instead of using a distribution based approach to calculate VaR, here, we loosen the distribution assumption and use non-parametric bootstrap methodology to determine portfolio VaR. The nonparametric bootstrap allows us to estimate the sampling distribution of a statistic empirically without making assumptions about the form of population, and without deriving the sampling distribution explicitly. The key bootstrap concept is that the population is to the sample as the sample is to the bootstrap sample. Then, we proceed the bootstrap technique in the following way.

For a given series of returns $\{r_1, ..., r_T\}$, consider a TGARCH model as in the previous case, whose parameters have been estimated by Quasi-MLE. Then

we can obtain the standardized residuals, $\hat{E}_t = \frac{r_t}{\hat{\sigma}_t}$, where $\hat{\sigma}^2_t = \hat{\omega}_G + \hat{\alpha}_G r^2_{t-1} + \hat{\gamma}_G r^2_{t-1} I^-_{t-1} + \hat{\beta}_G \sigma^2_{t-1}$, and $\hat{\sigma}^2_1$ is long-run variance of the sample.

To implement the bootstrap methodology, it is necessary to obtain bootstrap replicates $R^*_T = \{r^*_1, ..., r^*_T\}$ that mimic the structure of original series of size T. R^*_T are obtained from following recursion (Pascual, Nieto, and Ruiz (2006))

$$\begin{aligned} \hat{\sigma}^{*2}_t &\equiv \hat{\omega}_G + \hat{\alpha}_G r^{*2}_{t-1} + \hat{\gamma}_G r^{*2}_{t-1} I^*_{t-1} + \hat{\beta}_G \hat{\sigma}^{*2}_{t-1}, \\ r^*_t &\equiv E^*_t \hat{\sigma}^*_t, \text{ for } t = 1, ..., T, \end{aligned}$$

where $\hat{\sigma}^{*2}_1 = \hat{\sigma}^2_1$ and E^*_t are random draws with replacement from the empirical distribution of standardized residuals $\hat{E}_t$.[2] This bootstrap method incorporate uncertainty in the dynamics of conditional variance in order to make useful to estimate VaR. Given the bootstrap series R^*_T, we can obtain estimated bootstrap parameters, $\{\hat{\omega}^b_G, \hat{\alpha}^b_G, \hat{\gamma}^b_G, \hat{\beta}^b_G\}$, The bootstrap of historical values are obtained from following recursions

$$\begin{aligned} \hat{\sigma}^{b*2}_t &= \hat{\omega}^b_G + \hat{\alpha}^b_G r^{b*2}_{t-1} + \hat{\gamma}^b_G r^{b*2}_{t-1} I^-_{t-1} + \hat{\beta}^b_G \hat{\sigma}^{b*2}_{t-1}, \\ \hat{r}^{b*}_t &= E_t \hat{\sigma}_t \text{ for } t = 1, ..., T, \end{aligned}$$

where $\hat{\sigma}^{b*}_1$ is the long-run variance of the bootstrap sample R^{b*}_T, note that the historical values is based the original series of return and on the bootstrap parameters. we repeat the above procedure B times, and estimated $VaR^*_t(q)$ is k^{th}-order of series $\hat{r}^{b*}_t$, for $b = 1, ..., B$, where $k = B \times q$.

3 Evaluation of VaR

Over the past two decades, banks have increased their quantitative models to manage their market risk exposure for a significant increase in trading activity. As the fast growth of trading activity, financial regulators have also begun to focus their attention on the use of such models by regulated institutions. Therefore, the Value-at-Risk has been used to determine banks' regulatory capital requirements for financial market exposure in Basel Accord since 1996. Given the importance of VaR estimates to banks and to the regulators, evaluating the accuracy of the model underlying them is a necessary exercise.

To formulate the evaluation of accuracy of VaR, suppose that we observe a time series of past ex-ante VaR forecasts and past ex-post returns, we can define the "HIT function (sequence)" of VaR violations as

$$H_t = \begin{cases} 1, & \text{if } r_t < VaR^q_t \\ 0 & \text{if } r_t \geq VaR^q_t \end{cases}.$$

If the model is correctly specified, H_t should be a Bernoulli(q) and independent identical distributed (i.i.d.). In this subsection, we focus on two tests: 1)

[2] It is necessary to sample with replacement, because one would otherwise simply reproduce the original sample.

the unconditional distribution of H_t is Bernoulli(q); 2) the H_t are i.i.d. and Bernoulli(q).

First test: the unconditional of H_t is Bernoulli(q). VaR forecast evaluation can be implemented by that HIT at time t, H_t, is a Bernoulli random variable, and a powerful test can be constructed using a likelihood ratio test. Under the null that model is correctly specified, the likelihood of hit sequence is

$$\begin{aligned} L(H_1, \ldots, H_T) &= \Pi_{t=1}^{T}(1-\pi)^{1-H_t}\pi^{H_t} \\ &= (1-\pi)^{T-\sum H_t}\pi^{\sum H_t} \\ &= (1-\pi)^{T_0}\pi^{T_1}, \end{aligned}$$

where T_0 and T_1 are the numbers of zeros and ones in the sample. The unconditional MLE estimator of π is $\hat{\pi} = \frac{T_1}{T}$. The log-likelihood ratio test is given by

$$LR_{uc} = 2[\log(L(\hat{\pi})) - \log(L(q))] \sim \chi^2(1),$$

under the null ($H_0 : \pi = q$).

Second test: H_t are i.i.d. and Bernoulli(q). The likelihood based test for unconditionally correct VaR can be extended to conditionally correct VaR by examining the sequential dependence of HITs. We need to define what type of serial correlation we want to test against. A simple alternative is a homogenous Markov chain. A simple first order binary valued Markov chain produces Bernoulli random variables which are not necessarily independent. It is characterized by a transition matrix which contains the probability that the state stays the same. The transition matrix is given by

$$\Omega = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} = \begin{bmatrix} \pi_{00} & 1-\pi_{00} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix},$$

where $P[H_t = 1|H_{t-1} = 1] = \pi_{11}$, $P[H_t = 1|H_{t-1} = 0] = \pi_{01}$. In a correct specified model, the probability of a HIT in the current period should not depend on whether the previous period was a HIT or not. In other words, the HIT sequence, $\{H_t\}$ is i.i.d., and so that $\pi_{00} = 1 - q$ and $\pi_{11} = q$ when the model is conditionally correct. The likelihood of Markov chain (ignoring the unconditional distribution of the first observation) is

$$L(\Omega) = (1-\pi_{01})^{T_{01}}\pi_{01}^{T_{01}}(1-\pi_{11})^{T_{10}}\pi_{11}^{T_{11}},$$

where T_{ij} is the number of observations with a j following an i. The MLE estimator of π_{01} and π_{11} are

$$\hat{\pi}_{01} = \frac{T_{01}}{T_{00}+T_{01}}, \hat{\pi}_{11} = \frac{T_{11}}{T_{11}+T_{10}}.$$

Under independence, one has

$$\pi_{01} = \pi_{11} = \pi,$$

and

$$\Omega_0 = \begin{bmatrix} 1-\pi & \pi \\ 1-\pi & \pi \end{bmatrix},$$

while the MLE of π is again $\hat{\pi} = T_1/T$. Hence, the Likelihood ratio test of the independence assumption is given by

$$LR_{ind} = 2[\log(L(\hat{\Omega})) - \log(L(\hat{\Omega}_0))] \sim \chi^2(1),$$

Under the null, where

$$\hat{\Omega} = \begin{bmatrix} 1-\hat{\pi}_{01} & \hat{\pi}_{01} \\ 1-\hat{\pi}_{11} & \hat{\pi}_{11} \end{bmatrix} \text{ and } \hat{\Omega}_0 = \begin{bmatrix} 1-\hat{\pi} & \hat{\pi} \\ 1-\hat{\pi} & \hat{\pi} \end{bmatrix}.$$

However, one may want to test independence and Bernoulli(q) (conditional coverage test), i.e. $\pi_{01} = \pi_{11} = q$. The test is

$$LR_{cc} = 2[\log(L(\hat{\Omega})) - \log(L(q))] = LR_{uc} + LR_{ind} \sim \chi^2(2).$$

4 Empirical analysis with CAC 40

In this section we will implement two different methods to calculate VaR on CAC 40, they are conditional VaR with normal assumption and Bootstrap VaR. The purpose of using these two methods is to give a direct and clear example to explain why we should consider VaR calculation in a “turbulent framework” to incorporate tail events on the financial market. The reason to chose CAC 40 index is that it is considered as the most diversified portfolio in French stock market, therefore to make useful for our VaR calculation and evaluation. The time horizon is from 1 January 2002 to 10 October 2013 with daily data that fully captures a business cycle as shown in the first part of Figure 1. The subprime crisis started from mid-2007, since then, CAC40 decreased. The crisis has been amplified by the bankruptcy of Lehman Brother in September 2008, then CAC40 tumbled to its lowest level in early 2009. After the publication of Supervisory Capital Assessment Program (SCAP) in May 2009, CAC40 restored until the first phase of Sovereign crisis. It was followed by the second phase of Sovereign crisis in the Summer of 2011. The second part of Figure 1 is the Log return of CAC40, it is derived as $r_t = \log P_t - \log P_{t-1}$. The graph clearly shows that a “Normal Period” from early 2004 to the end 2005. However, the whole sample period depicts several turbulence periods, especially after the subprime crisis. Therefore, by simply assuming the returns follow a normal distribution would be inappropriate for further modeling process. As showed in Figure 2, the QQ-plot of the whole sample does depict a fat tail distribution rather than normal distribution. Both One-sample Kolmogorov-Smirnov test and Shapiro-Wilk normality test show that the normality assumption will be violated in whole sample period.

	Normal Period		Whole Sample	
	D Statistics	P-Value	W Statistics	P-Value
KS test	0.0344	0.6847	0.0805	< 2.2e-16
SW test	0.995	0.1768	0.9336	< 2.2e-16

Tab. 1: Normality Test

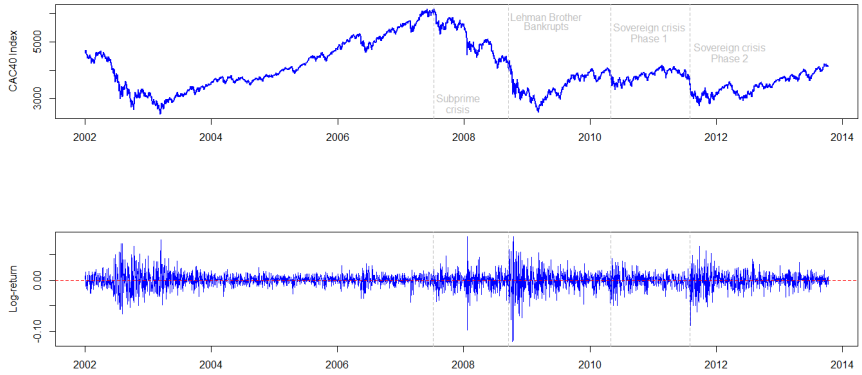

Fig. 1: CAC 40 Index

The second part of Figure 1 is the Log return of CAC40, log return is derived as $r_t = \log P_t - \log P_{t-1}$. The graph clearly shows that a "Normal Period" from early 2004 to the end 2005. However, the whole sample period depicts several turbulence periods, especially after the subprime crisis. Therefore, by simply assuming the returns follow a normal distribution would be inappropriate for further modeling process. As showed in Figure 2, the QQ-plot of the whole sample does depict a fat tail distribution rather than normal distribution. Both One-sample Kolmogorov-Smirnov test and Shapiro-Wilk normality test show that the normality assumption will be violated in whole sample period as showed in Table 1.

4.1 Evaluation of VaR for CAC40

4.1.1 Results for TGARCH model

In this section, we will provide the empirical results of VaR for CAC40. Before showing the results of VaR, we present the results of conditional volatility. No matter for calculating conditional VaR with normal assumption or Bootstrap VaR, we need to run the TGARCH process at first place to determine the con- ditional volatility of CAC40 return. In this TGARCH model, α_G measures the extent to which a volatility shock today feeds through into next period's volatil-

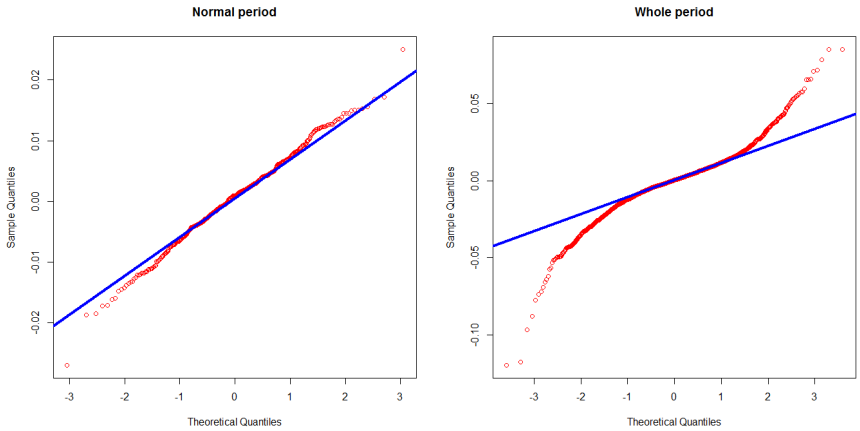

Fig. 2: QQ-plot

	α_G	γ_G	β_G
Estimator	0.103	0.091	0.801
Std. Error	0.224	0.191	0.111
t stat	0.460	0.476	7.215
P value	0.645	0.633	0.000

Tab. 2: TGARCH estimation results

ity, γ_G measures the extent to which an volatility shock today feeds through into next period's additional volatility if today's return is negative (leverage effect) and β_G measures the persistency of volatility. The estimation results are presented in Table 2. We provide the dynamics of volatility in figure 3. During the calm period, volatility stayed at a low level, however,it has sharply increased since the beginning of financial crisis.

We also provide a diagnosis of the TGARCH model to take into account the model risk to see whether this model is well specified for both in sample and out of sample check. The objective of variance modeling is to construct a variance measure, which has the property that standardized squared returns, $r_t^2/\hat{\sigma}_t^2$ have no systematic autocorrelation. We can see from figure 4 that the standardized squared returns have no autocorrelation as the sticks of different lags are all in their standard error banks, however the squared returns have strong autocorrelation. Model is well specified in terms of in sample check.

Out of sample check is done by using a simple regression where squared returns in the forecast period, $t+1$, is regressed on the forecast from the variance model, as in

$$r_{t+1}^2 = b_0 + b_1 \hat{\sigma}_{t+1|t}^2 + e_{t+1}.$$

In this regression, the squared return is used as a proxy for the true but unob-

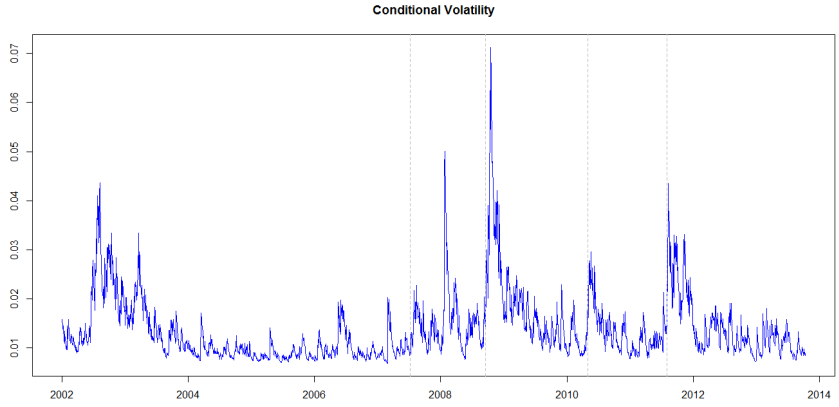

Fig. 3: Conditional Volatility

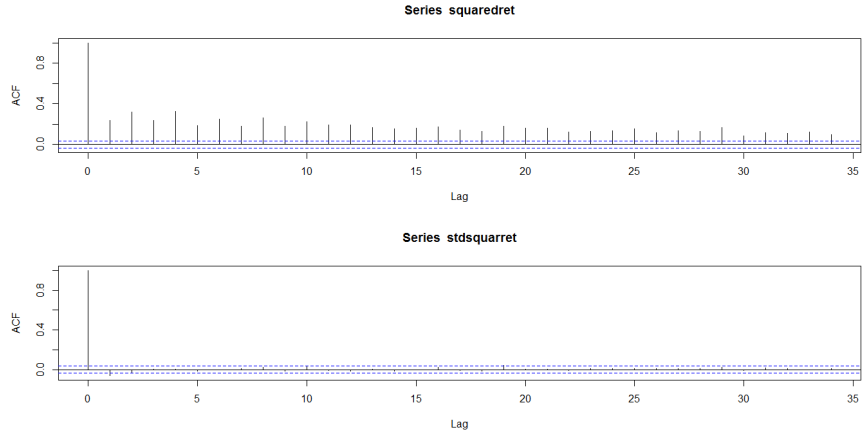

Fig. 4: Diagnostic of TGARCH model (In sample check)

Coefficients	Estimate	Std.Error	t-Stat	P-value
b_0	2.495e-05	1.397e-05	1.786	0.0741 .
b_1	9.584e-01	3.531e-02	27.145	<2e-16 ***
			F-Stat	736.8
			Adj.R^2	0.1962

Tab. 3: Diagnostic of TGARCH model (Out of sample check)

	Min	1Q	Median	Mean	3Q	Max
Conditional VaR	-0.166	-0.037	-0.027	-0.032	-0.021	-0.016
Boot VaR (1000)	-0.166	-0.039	-0.030	-0.035	-0.025	-0.017

Tab. 4: Result of VaR

served variance in period $t + 1$. What needs to be done is to test $b_0 = 0$ and $b_1 = 1$. The results show usb_0 is not statistically significant and b_1 is very close to 1, with an acceptable adjusted R^2 equals to 0.19. Therefore, we can conclude that our variance model is well specified.

4.1.2 Results for VaR

Table 5 shows us the results for two different methods of VaR calculation. The mean of Conditional VaR is -3.2%, which is lower (in absolute value) than the mean of Boot VaR, -3.5%. Moreover, for each quartile, the VaR of bootstrap method is lower than the VaR of conditional GARCH method as well as for minimum value and maximum value. This is because the bootstrap methodology doesn't assume any distribution assumption on CAC 40 return, therefore it does consider fatter tail events than conditional GARCH method. The dynamics of VaR is presented in Figure 5. The green line represents the VaR of GARCH with normal assumption method and red line represents the VaR of Bootstrap method. In general, green line exhibits a lower market risk than blue line, which is in line in Table 5. Therefore, it will be useful to check the hit function in order to evaluate the VaR results to determine a robust method for VaR computation.

In line with section 3, Figure5 shows the results of evaluation of VaR for CAC40. For TGARCH VaR method, both first test and second test passed the critical value 6.635 and 9.21 respectively,[3] which means that the hit function generated by TGARCH VaR method does not follow the Bernoulli (99%). This is because, by assuming the normal distribution of CAC40 return, one would underestimate the tail risk during the crisis (turbulence) period, therefore the hit function will be violated more frequently implicitly. However, the Bootstrap VaR method does have first and second tests below the critical values. As mentioned above this method is a nonparametric method, therefore it does not assume any distribution to depict CAC40's return and can fully capture the return movement in both calm and turbulence period. The inconvenience

[3]The confidence level is 99%.

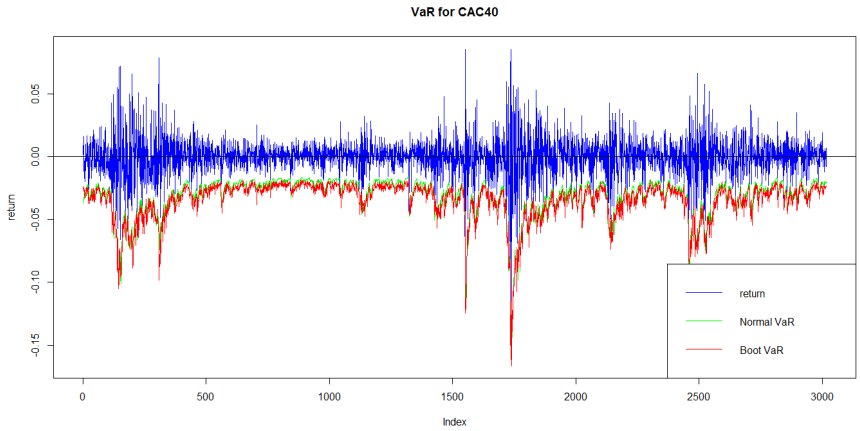

Fig. 5: Time varying VaR

	TGARCH VaR	Bootstrap VaR
First Test	26.017	5.619
Second Test	28.642	5.805

Tab. 5: Evaluation results of VaR for CAC40

Notes: The numbers in this table refer to the results of Likelihood Ratio Test and the confidence level in this paper is 99%. The critical level for the first test is 6.635 and for the second test 9.21. We did the bootstrap for 1000 times.

for this method is the computational burden, usually it takes more time than previous method. Fortunately, we have shown in this paper, bootstrap for 1000 times is enough for CAC40 VaR calculation.

5 Conclusion

Different methods used to calculate VaR will imply significant different results as showed in the main body of the paper. This gives the insights to the risk managers and other market participants, we should be prudential in implementing the methods to calculate VaR. In addition, it would be interesting to see others risk measures in a “turbulence time”, such as stressed VaR and expected shortfall, and it opens the door for the future research.

References

Acharya, V., and M. Richardson (2009): "Restoring Financial System: How to repair a Failed System," *John Wiley and sons*.

Adrian, T., and M. Brunnermeirer (2011): "CoVaR," *Fedral reserve of NewYork Working paper*.

Adrian, T., and H. Shin (2011): "Liquidity and Leverage," *Journal of Financial Intermediation*, 19, 418–437.

Artzner, P., F. Delbaen, J. Eber, and D. Heath (1997): "Thinking coherently," *Risk*, 10, 68–71.

Brunnermeier, M., and L. Pedersen (2009): "Market liquidity and funding liquidity," *Review of Financial Studies*, 22.

Christofferson, P., and S. GonCalves (2005): "Estimation risk in financial risk management," *Journal of Risk*, 7, 1–28.

Cont, R., R. Deguest, and G. Scandolo (2010): "Robustness and sensitivity analysis of risk measurement procedures," *Quantitative finance*, 10, 593–606.

Engle, R. (2002): "Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models," *Journal of Buisiness and Economic Statistics*, 20, 236–249.

Gourieroux, C., and A. Monfort (2011): "Allocating Systematic and Unsystematic Risks in a Regulatory Perspective," *CREST Working paper*.

Jorion, P. (2006): "Value at Risk," *McGraw-Hill, 3rd edn*.

Kashyap, A., and J. Stein (2004): "Cyclical Implications of the Basel II Capital Standards," *Federal Reserve Bank of Chicago Economic Perspectives*, 20.

Lehar, A. (2005): "Measuring systemic risk: A risk management approach," *Journal of Banking and Finance*, 29, 2577–2603.

Nieto, M., and E. Ruiz (2009): "Measuring financial risk: Comparison of alternative procedures to estimates VaR and ES," *Universidad Carlos III de Madirid Working paper*.

——— (2010): "Bootstrap prediction intervals for VAR and ES in the context of GARCH models," *Universidad Carlos III de Madirid Working paper*.

Pascual, L., M. Nieto, and E. Ruiz (2006): "Bootstrap prediction for returns and volatilities in GARCH models," *Computational Statistics and Data Analysis*, 50, 2293–2312.

Ruiz, E., and L. Pascual (2002): "Bootstrap financial time series," *Journal of Ecomonic survey*, 16, 271–300.