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17 March 2015

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ABSTRACT

This paper provides an assessment of the comparative effectiveness of four econometric methods in estimating the optimal hedge ratio in an emerging equity market, particularly the South African equity and futures markets. The paper bases the effectiveness of hedging on volatility reduction and minimisation of the coefficient of variation of hedged returns as well as risk-aversion based utility maximisation. The empirical analysis shows that the single equation method estimated by ordinary least squares is the most effective over daily hedging periods. However, the vector error-correction method and multivariate GARCH methods are most effective over weekly and monthly hedging periods.

Key words: emerging markets, optimal hedge ratio, South Africa, index futures hedging, Vector autoregression, Vector error-correction, GARCH

JEL Classification: G11, G15, C51, C52

1. INTRODUCTION

This paper compares the effectiveness of different econometric methods for the estimation of the hedge ratio in an emerging equity market, the South African equity and futures market, during periods of financial crises. The paper focuses on the 2008-2010 global financial crisis.

Contrary to tranquil periods, turmoil periods triggered by financial crises create the possibility of forward discount and the prospect of high volatility of prices in the equity and futures market. Moreover, a number of studies have shown that emerging equity markets are susceptible to contagion during periods of global financial crises, leading investors to actively consider mechanisms that mitigate the increasing risk (Bonga-Bonga and Hoveni, 2013; Bekaert et al., 2005; Bouaziz et al., 2012)). These realities are likely to influence the relationship between spot and futures prices and thus, the choice of the appropriate econometric methods necessary for the estimation of the hedge ratio in an emerging market.

Emerging markets are set to make up an increasing share of global equity portfolios in the decades to come. An economic report from Goldman Sachs (Moe et al., 2010) estimates that emerging markets will contribute up to 55% of global equity market capitalisation by 2030. Following South Africa's inclusion in the BRICs economic bloc and its status as the gateway to Africa, South Africa's equity market is a key emerging financial market for investors. Its market capitalisation was estimated at US$ 612 in 2012, the largest in Africa (World Bank, 2013). A number of asset managers rely on the South African equity market as an important investment opportunity for risk diversification among the emerging markets. Given this reality, the risks of investing in the South African equity market and its mitigation thereof need to be assessed. Hearn et al. (2010) highlighted the high risk premiums faced by emerging market equity investors relative to developed market equity investors. Thus, the mechanism and procedure for hedging equity market risk should be a priority for investors interested in emerging market equity, especially for investor choosing the South African equity market as an investment prospect for risk diversification and hedging.

Equity market offers a large possibility for hedging, especially by combining positions in the spot and derivatives markets. In the context of the combination of spot and futures products for hedging, making the right hedging decision involves finding an optimal hedge ratio, i.e. the ratio of the size of the position taken in futures contracts to the size of the exposures (Wen, et. al, 2011). However, there is no consensus in financial econometrics as to which econometric methodology provides an efficient and consistent measure of an optimal hedge ratio. Hedging of equity exposure by determining the optimal hedge ratio can be approached in various ways. Studies by Ederington (1979), Franckle (1980), Figlewski (1985), Vishwanath (1993), Ghosh (1993), Chou et al. (1996), Myers (1991), Park and Switzer (1995), Holmes (1996), Laws and Thompson (2005) as well as Yang and Allen (2005) have applied different econometric hedging methods in order to estimate the optimal hedge ratio. These methods range from the Single Equation Method estimated by Ordinary Least Squares (Ederington (1979), Franckle (1980) and Figlewski (1985)); the Vector Autoregression Method (Vishwanath (1993), Ghosh (1993) and Chou et al. (1996)); Vector Error-Correction Method (Vishwanath (1993), Ghosh (1993) and Chou et al. (1996)); to a class of Multivariate GARCH methods (Myers (1991), Park and Switzer (1995) Wahab (1995)). Consequently, it is important to determine the suitable econometric methodology for the appraisal of optimal hedge ratio.

While hedge effectiveness measure is often used to identify the correct econometric methodology for the estimation of optimal hedge ratio and the construction of hedged portfolio, the myriad of hedge effectiveness measures raises a question as to which of these measures is appropriate in identifying the optimal hedge ratio estimate. A good hedge effectiveness measure should assist investors to construct an effective hedged portfolio. A highly effective hedged portfolio should be able to offset the changes in the fair value of the hedged item with the value of the hedged derivative. However, different methods or measures of testing hedge effectiveness of an estimated optimal hedge ratio aim at different outcomes. For example, the Variability-Reduction method for testing hedge effectiveness compares the variance of the fair value of the hedged portfolio, constituted of a derivative and the underlying item or asset, to the variance of the fair value of the hedged item (Lipe, 1996). Nonetheless, the Dollar-Offset method compares the changes in the fair value of the hedged item and the derivative (Kawaller and Koch, 2000).

Unfortunately, a number of studies estimating the optimal hedge ratio do not make use of the test of hedge effectiveness for the selection of the relevant econometric technique inherent to the estimation of the optimal hedge ratio or only make use of a single test of hedge effectiveness. For example, the study by Degiannakis and Floros (2010), being among the few that are conducted on estimating the hedge ratio in the South African equity market, did not compare the hedge effectiveness of the estimated optimal hedge ratios. To remedy to this shortage, this paper performs an empirical estimation of the hedge ratios in the South African equity market during the period of global financial crisis and compares the hedge effectiveness of four econometric methods used in the estimation of these ratios. The four econometric methods used by this paper in the estimation of the optimal hedge ratio include the Single Equation Method estimated via Ordinary Least Squares (SEMOLS); the bivariate Vector Auto Regression (VAR) method; the Vector Error Correction method (VECM); and the Multivariate Generalised Autoregressive Conditional Heteroskedasticity (GARCH). Analyses of the effectiveness of these methods will include measures based on portfolio variance (volatility) reduction, minimisation of coefficients of variation and utility maximisation. The sample period include the time of global financial crisis, which justifies the most hedging strategy by investors.

The remains of the paper is structured as follows; section 2 presents the literature review of the optimal hedge ratio and the test of hedge effectiveness. Section three discusses the econometric methodologies used in the paper. Section analyses of the data, highlighting the properties of the data set used in the paper. Section five provides the estimation of constant and time varying optimal hedge ratios using different econometric methods. Section six contains a brief discussion of the methodology used for the assessment of hedging effectiveness. Moreover, results of the evaluations of the different hedging effectiveness are provided in this same section. Finally, section seven presents the conclusion of the paper.

2. Literature review

The ideas that led to the development of the optimal hedge ratio started with Stein (1961) and Johnson (1960). Johnson (1960) showed that hedging spot exposure with futures contracts could maximise the utility of portfolio holders by minimising the unconditional variance of portfolio returns. This is an important development, it showed how to apply futures contracts effectively for hedging underlying exposure. Stein (1961) analysed the relationship between spot and futures prices and considered that some portion of exposure should be hedged and some should remain un-hedged. Stein (1961) discussed a ratio of one-to-one of futures exposure to spot exposure on the hedged portion of portfolios; in subsequent literature, this ratio is the “naïve hedge ratio” or part of a “naïve hedging strategy”. It is termed “naïve” because one could hold futures contracts covering less exposure than the spot exposure held whilst, still reducing the variance of portfolio returns and maximising utility. However, the optimal amount of futures exposure required per unit of spot of exposure was often hard to determine.

In a pioneering study, Ederington (1979) sought to find the optimal hedge ratio that would maximise utility. Ederington (1979) applied a constant hedge ratio from the Single Equation Method, with parameters estimated by the method of Ordinary Least Squares (SEMOLS), to estimate the optimal hedge ratio. His study concluded that the SEMOLS outperforms the naïve (one-to-one) methodology by providing lower portfolio variance. Ederington’s (1979) study is important as it introduced a simple yet theoretically sound method of determining the optimal hedge ratio, which is widely applied in empirical studies such as those by Franckle (1980) and Figlewski (1985).

Franckle (1980) expanded on the work done by Ederington (1979). Franckle (1980) agreed that matching a long T-Bill spot position with a short T-Bill futures position, proportional to the hedge ratio from the SEMOLS, improves hedging effectiveness compared to the naïve strategy. Figlewski's (1985) study assessed the effectiveness of stock index futures hedging, when they were first introduced, on the Kansas City Board of Trade in 1982. The study analysed various stock portfolios over multiple investment horizons with hedges using stock index futures. Figlewski (1985) identified the important phenomenon of basis risk in stock portfolios hedged by futures and highlighted how the risk needed to be managed by reducing hedging periods. This study is significant as it indicated that static hedges are not effective over long hedging periods. After the studies of Franckle (1980) and Figlewski (1985), almost all studies on the optimal hedge ratio applied Ederington's (1979) SEMOLS including: Vishwanath (1993); Ghosh (1993), Chou et al. (1996); Myers (1991); Park and Switzer (1995); Wahab (1995); Holmes (1996); Choudhry (2003); Laws and Thompson (2005); Yang and Allen (2005); as well as Bhaduria and Duraia (2008).

Figlewski's (1985) study prompted an important question in financial economics literature: can one's portfolio remain hedged *ad infinitum* with the same hedge ratio? Myers and Thompson (1989) in expanding on Figlewski's (1985) assertions on basis risk, indicated that basis risk exists as changes in spot prices and changes in futures prices are not perfectly correlated. This inferred that optimal hedge ratios would have to be frequently recalculated whenever futures contracts expire and sometimes before expiry. Myers and Thompson's (1989) study was based on soft commodities futures hedging in Michigan, USA. Their study applied the bivariate VAR as well as the SEMOLS. Myers and Thompson (1989) further indicated that futures prices do not necessarily predict future spot prices, refuting earlier assertions of Stein (1961).

Another challenge faced in the estimation of the optimal hedge ratio using SEMOLS is the issue of cointegration. As most financial time series are non-stationary, their volatility changes over time. However, if changes in two financial time series, such as spot and futures prices based on the same underlying financial instrument have related changes in volatility, they are cointegrated and thus have a related or common trend. When cointegration exists, a Vector Error-Correction Method such as those expounded upon by Vishwanath (1993), Ghosh (1993), and Chou et al. (1996) has been applied to address the two limitations of hedge ratios from the SEMOLS, cointegration and serial correlation. The above studies indicated that the theoretically superior method of vector error-correction further improved utility. However, the challenge of frequent re-estimation of hedge ratios was still a challenge faced by users of this improved methodology.

In the late 1980's, conditional time-varying volatility (heteroskedasticity), often observed in financial time series, became topical in literature after the ground breaking studies by the Nobel economics laureate Robert Engle (1982) and Bollerslev, Engle and Wooldridge (1988) who extolled the value of Autoregressive Conditional Heteroskedasticity (ARCH) and Generalised Autoregressive Conditional Heteroskedasticity (GARCH) methods in dealing with conditional heteroskedasticity. Univariate GARCH methodology would consider changes in volatility in spot or futures returns series independently, assuming a constant covariance. Multivariate GARCH methodology would consider changes in volatility in both spot and futures returns and the changes in their covariance. Wahab (1995) purported that Multivariate GARCH methodology provides unbiased parameter estimates compared to the univariate GARCH models for each variable. Wahab's (1995) use of a Multivariate GARCH method to estimate hedge ratios for precious metals, such as gold and silver, was well specified, as it took into account the related time dependent variation of the two time series used for index futures hedging, the spot time series and the corresponding futures time series.

Multivariate GARCH methodology led to the estimation of time-varying optimal hedge ratios that provided the user with greater facility in the application of dynamic hedging. Studies that applied this methodology such as those by Myers (1991), Park and Switzer (1995) noted the superiority of hedge ratios from GARCH methods over the constant hedge ratios from methods such as the SEMOLS, VAR and VECM. Myers (1991) estimated time-varying hedge ratios and compared these estimates to constant estimates of the optimal hedge ratio, in terms of volatility reduction. These studies indicated that GARCH methods reduced hedged portfolio variance more than other constant hedge ratio methods such as the SEMOLS and VAR. Not all scholars, however, hold this view; Lien (2005) contested the theoretical basis for the VECM superiority over the SEMOLS. Lien and Shrestha (2008) controversially asserted that the simple SEMOLS method is more effective than VECM and Multivariate GARCH methods. Lien and Shrestha (2008) further asserted that the computational complexity of GARCH methods were not commensurate with the often marginal gains in variance reduction. The empirical nature of these studies leaves much room for divergent results depending on the subject market and the nature of the underlying financial instrument. Lien (2005) and Lien and Shrestha (2008) contributed to the growing body of literature analysing the comparative hedging effectiveness of various methods. This is increasingly important, as there are a variety of methods of estimating optimal hedge ratios.

Multiple studies have considered the effectiveness of hedging methods, such as those by Holmes (1996), Laws and Thompson (2005), and Yang and Allen (2005). Holmes (1996) chronicled the dynamics of stock index futures hedging in the United Kingdom. Holmes (1996) applied three econometric methods: the SEMOLS, VECM and GARCH. Holmes (1996) compared the effectiveness of these methods by assessing the reduction in the portfolio variance by applying each method over various hedging periods. The author concluded that the SEMOLS outperformed the more complex methods by reducing portfolio variance more than the other methods. Laws and Thompson (2005) compared the hedging effectiveness of stock index futures from the London International Financial Futures and Options Exchange on portfolios of stocks listed on the London Stock Exchange. In their study, the effectiveness of hedging is based on the reduction in portfolio variance. Laws and Thompson (2005) estimated the optimal hedge ratio with the SEMOLS and methods taking into account ARCH effects and effects Exponential Weighted Moving Average (EWMA). This study concluded that the method that takes into account EWMA effects was the most effective in reducing the portfolio variance. Yang and Allen (2005) performed an analysis of Multivariate GARCH methods for estimating hedge ratios in the Australian futures market. Yang and Allen (2005) compared the time varying Multivariate GARCH methods to constant estimation methods using comparative reductions in portfolio variance and risk aversion based utility maximisation. This study concluded that the Multivariate GARCH method was the most effective hedging technique based on portfolio variance reduction and risk aversion based utility maximisation for out-of-sample data.

The studies on the optimal hedge ratio detailed above, primarily apply evidence from developed markets. The body of knowledge therein is quite extensive, as developed markets have been the focus of most twentieth century financial research. Research focusing on emerging markets is considerably less abundant. Lien and Zhang (2008) surveyed 28 emerging markets and investigated the plausibility of effective hedging of financial market exposure in emerging markets. Lien and Zhang (2008) analysed several types of derivatives in multiple emerging markets in order to assess the effectiveness of hedging in volatile markets. This study showed that if structured strategies are affected, significant risk reduction is possible in various emerging markets. Olgun et al. (2012) produced a study covering the Turkish markets, where they estimated constant and time-varying hedge ratios for stock index futures on the Istanbul Stock Exchange's ISE-30 index. Choudhry (2003) compared hedging effectiveness in three emerging markets to three developed markets. This study estimated hedge ratios with the traditional (naïve) method, SEMOLS, as well as time varying methods, such as the DVEC Multivariate GARCH and DVEC GARCH-X methods. Choudhry (2003) concluded that the DVEC GARCH method is the most effective method when assessed on out-of-sample data.

The above studies are among the small body of literature on stock index futures hedging in emerging markets. Studies that estimate optimal hedge ratios for stock index futures and analyse the effectiveness of methods are rare. One such study conducted by Bhaduria and Duraia (2008) applied data from India's National Stock Exchange to assess the effectiveness of methods, such as the SEMOLS, VAR, VECM and the DVEC GARCH method. This study concluded that the GARCH model was moderately more effective than the SEMOLS over longer hedging periods and less effective over shorter periods. However, this study fails to provide different measures of hedging effectiveness.

METHODOLOGY AND ESTIMATION OF THE HEDGING RATIO

This paper applies multiple methods to estimate the optimal hedge ratio for the exposure on FTSE/JSE Top 40 index between 2008 and 2010. The choice of this index is justified by the fact that being a market index the FTSE/JSE Top 40 index has only a systemic risk to be hedged. Thus, the focus of this paper will be on how to apply different econometric and hedge effectiveness methods to determine the hedge ratio that minimises the systemic risk of investing in the JSE market index. The paper generates point estimates for the hedge ratio using the Single Equation Method via Ordinary Least Squares regression (SEMOLS), Bivariate Vector Autoregression (VAR) and Bivariate Vector Autoregression with Vector Error-Correction Methodology (VECM). Following the point estimates, this study generates time varying estimates of the optimal hedge ratio by utilising a Multivariate Generalised Autoregressive Conditional Heteroskedasticity (MGARCH) approach.

Subsequent to the estimation of the hedge ratios, this paper assesses hedging effectiveness through a number of methodologies such as volatility reduction, coefficient of variation minimisation and utility maximisation. It assesses the effectiveness of hedging against in-sample data as well as out-of-sample data over one, five and twenty day hedging periods. These different hedging periods represent daily, weekly and monthly investment horizons.

2.1. SINGLE EQUATION METHOD

This is the most commonly applied method of estimating the optimal hedge ratio first executed by Ederington (1979) in his pioneering study.

This simple yet conventional method of determining the optimal hedge ratio applies the following regression model, estimated by the method of Ordinary Least Squares:

$$r_{st} = \alpha + \beta r_{ft} + \varepsilon_t, \qquad (1)$$

where ε_t represents the error term from Ordinary Least Squares regression estimation. r_{st} and r_{ft} respectively represent continuously compounded returns on spot and futures indices. β represents the estimate for the optimal hedge ratio, h^*[1].

The continuously compounded returns are calculated as follows:

$$r_{st} = ln(S_t) - ln(S_{t-1})$$
$$r_{ft} = ln(F_t) - ln(F_{t-1}), \qquad (2)$$

where S_t and F_t are spot and futures prices of the FTSE/JSE Top 40 index at time t.

To simplify the expression, returns are expressed as follows:

$$r_{st} = s_t - s_{t-1}$$
$$r_{ft} = f_t - f_{t-1}, \qquad (3)$$

where $s_t = ln(S_t)$ and $f_t = ln(F_t)$.

2.2. BIVARIATE VECTOR AUTOREGRESSION (VAR) METHOD

A major shortcoming of the Single Equation Method using OLS, as argued by Herbst et al. (1989), is that it does not address the problem of serial correlation among the residuals of the endogenous variables, which, in this case, are the returns series. The bivariate VAR addresses the problem faced by serial correlation by modelling the different endogenous variables utilising the bivariate VAR structure as follows:

$$\Delta s_t = \alpha_s + \sum_{i=1}^{m} \beta_{si} \Delta s_{t-i} + \sum_{i=1}^{m} \gamma_{si} \Delta f_{t-i} + \lambda_s Z_{t-1} + \varepsilon_{st}$$

$$\Delta f_t = \alpha_f + \sum_{i=1}^{m} \beta_{fi} \Delta s_{t-i} + \sum_{i=1}^{m} \gamma_{fi} \Delta f_{t-i} + \lambda_f Z_{t-1} + \varepsilon_{ft}, \qquad (11)$$

where $\Delta s_t = ln(S_t) - ln(S_{t-1})$ and $\Delta f_t = ln(F_t) - ln(F_{t-1})$. α_s and α_f are the intercepts, β_{si}, β_{fi}, γ_{si} and γ_{fi} are parameters and ε_{st} and ε_{ft} are independently identically distributed random vectors.

The optimal lag length, m, is determined by repeating the model using different lags and selecting the optimal lag length based on Schwarz Bayesian Criterion developed by Schwarz (1978). Once the optimal lag length is determined, residual time series of Δs_t and Δf_t are estimated and used to estimate the optimal hedge ratio.

The estimate of the optimal hedge ratio in this case is defined as the ratio that yields the minimum variance, or the minimum variance hedge ratio, h^*, defined as follows:

$$h^* = \sigma_{sf} / \sigma_f^2, \qquad (12)$$

where σ_{sf} is the covariance between the residuals of Δs_t and the residuals of Δf_t and σ_f^2 is the variance of the residuals of Δf_t. This is in line with the derivation applied earlier that resulted in equation (8).

2.3. VECTOR ERROR CORRECTION METHOD (VECM)

Vishwanath (1993), Ghosh (1993) and Chou et al. (1996) discussed a shortcoming of the SEMOLS and VAR, cointegration. This occurs when the levels of two time series, such as spot and futures prices, are non-stationary and are integrated of order one. When this occurs, their variances vary with time, however, this variation is related, and this related time dependent variation is called cointegration.

The Vector Error-Correction Method addresses this by adding an error-correction to the error terms of each variable in the vector autoregression to account for the long-term equilibrium relationship of spot and futures price movements. The execution of the Vector Error-Correction Method is as follows:

$$\Delta s_t = \alpha_s + \sum_{i=1}^{m} \beta_{si} \Delta s_{t-i} + \sum_{i=1}^{m} \gamma_{si} \Delta f_{t-i} + \lambda_s Z_{t-1} + \varepsilon_{st}$$

[1] See appendix for rationale

$$\Delta f_t = \alpha_f + \sum_{i=1}^{m} \beta_{fi}\Delta s_{t-i} + \sum_{i=1}^{m} \gamma_{fi}\Delta f_{t-i} + \lambda_f Z_{t-1} + \varepsilon_{ft}, \qquad (13)$$

where $\Delta s_t = ln(S_t) - ln(S_{t-1})$ and $\Delta f_t = ln(F_t) - ln(F_{t-1})$. $Z_{t-1} = s_{t-1} - C - \delta f_{t-1}$ is the error correction term with δ as the cointegrating factor, C, as a constant and λ_s, λ_f as adjustment parameters.

Following the estimation of parameters in the VECM, the hedge ratio is estimated using a similar procedure to the standard Vector Autoregression Method.[2] The residual series are generated and their variances and covariances are utilised to estimate the optimal hedge ratio as depicted in equation (12).

2.4. MULTIVARIATE GENERALISED AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY (GARCH) METHOD

The major shortcoming shared by the SEMOLS, the VAR, and the VECM is the risk of Autoregressive Conditional Heteroskedasticity (ARCH) effects in the residuals, which occurs when the variance of a time-series changes significantly with the passage of time. This property is common among financial time series as indicated by Bollerslev, Engle and Wooldridge (1988).

Heteroskedasticity, as indicated by Myers (1991) as well as Park and Switzer (1995), invalidates hedge ratio estimates made by the first three methods used in this study, due to model misspecification. Myers (1991), Park and Switzer (1995), as well as Wahab (1995) purported that the application of Multivariate Generalised Autoregressive Conditional Heteroskedasticity (GARCH) methods provide more effective and unbiased estimates of the optimal hedge ratio.

Bollerslev, Engle and Wooldridge (1988) proposed the Multivariate GARCH method, which deals with ARCH effects in multiple time series by generating time-varying second order moments to allow for time series of variances.

The Multivariate GARCH method is a generalised version of the univariate GARCH method that takes into account the related time-varying variation in the analysed time series. A conventional MGARCH with ARCH components of order one and GARCH components of order one, MGARCH(1,1) is formulated as follows:

$$\begin{bmatrix} h_{ss} \\ h_{sf} \\ h_{ff} \end{bmatrix}_t = \begin{bmatrix} \gamma_{ss} \\ \gamma_{sf} \\ \gamma_{ff} \end{bmatrix}_t + \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_s^2 \\ \varepsilon_s \varepsilon_f \\ \varepsilon_f^2 \end{bmatrix}_{t-1} + \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} \begin{bmatrix} h_{ss} \\ h_{sf} \\ h_{ff} \end{bmatrix}_{t-1}, (14)$$

where $\mathrm{h_{ss}}$, $\mathrm{h_{ff}}$ and $\mathrm{h_{sf}}$ are the respective conditional variances of the spot and futures returns' residuals and their conditional covariance with a constant mean model as the mean equation.

Equation (7) can be represented in matrix form as follows:

$$H_t = \Gamma + \mathrm{A}E_{t-1} + \mathrm{B}H_{t-1} \qquad (15)$$

Where Γ, A and B are matrices of constants. E_{t-1} and H_t are past shocks and conditional variances/covariances, respectively.

Due to the large number of parameters that are required to apply the estimate the model for $\mathrm{H_t}$, Bollerslev, Engle and Wooldridge (1988) suggested a parameterisation of the conditional variance equation in the GARCH model, H_t, the DVEC MGARCH model that allows for a less computationally intensive method of estimating the variances and covariances. The DVEC model sets the off-diagonal elements of A and B from equation (14) to zero, which reduces the number of parameters to be estimated and reduces the system of linear equations below:

$$h_{sst} = \gamma_{ss} + \alpha_{11}\varepsilon_{s,t-1}^2 + \beta_{11}h_{ss,t-1}$$

$$h_{sft} = \gamma_{sf} + \alpha_{22}\varepsilon_s\varepsilon_f + \beta_{22}h_{sf,t-1}$$

[2] See section 3.3

$$h_{fft} = \gamma_{ff} + \alpha_{33}\varepsilon_{f,t-1}^2 + \beta_{33}h_{ff,t-1}, \qquad (16)$$

With the above estimated, the time varying hedge ratio, h_t^*, is defined in a similar way to the way defined in the bivariate VAR model with a time varying subscript added, indicating time-varying variances and covariances.

$$h_t^* = h_{sft}/h_{fft}\,, \qquad (17)$$

This DVEC MGARCH model take into account time-varying conditional variances and correlation between spot and futures prices and thus produces time varying estimates of the optimal hedge ratio.

3. DATA ANALYSIS

3.1. DATA DESCRIPTION

This study assesses the effectiveness of methodology for estimating the optimal hedge ratio during the financial crisis period of the late 2000s and its aftermath, where irregular market volatility was present. For this reason, the sample commences in 2008. The persistent equity market volatility resulting from the prolonged European debt crisis justifies the extension of the sample to 2010.

The data applied in this study is daily closing spot prices for the FTSE/JSE All Share Top 40 index (Top 40) traded on the Johannesburg Stock Exchange (JSE), as well as daily closing prices for the ALSI Top 40 future with the nearest maturity, traded on the South African Futures Exchange (SAFEX). The data period ranges from the 2nd of January 2008 to the 31st of December 2010 for in-sample data. Out-of-sample data covers sample data from the 3rd of January 2011 to 15th of June 2011.

The data applied is from the I-Net Bridge as well as BFA McGregor databases. Figure 1 presents the spot index returns for in-sample data. There appears to be a surge in volatility in 2008, which is likely a result of the various financial crisis events of 2008, such as the collapse of Lehman Brothers. A lesser surge in volatility occurs in 2010, which likely represents the market response to the European debt crisis.

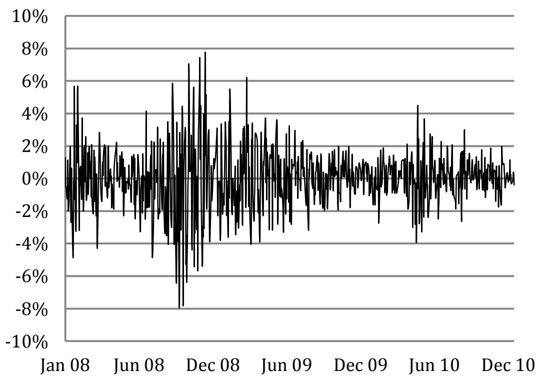

FIGURE 1: SPOT RETURNS

The futures returns of in-sample data appear highly correlated to the spot returns, depicted in Figure 2.

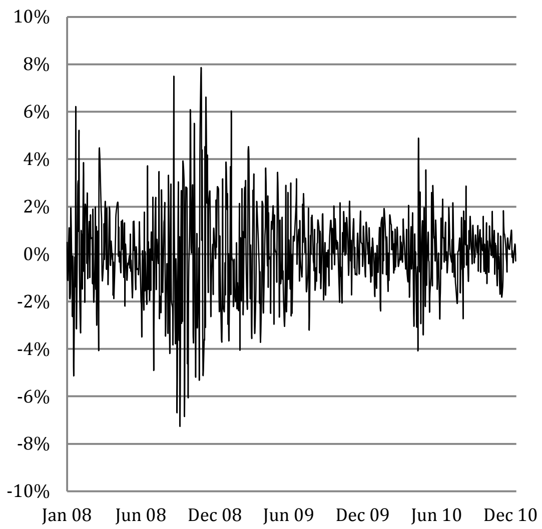

FIGURE 2: FUTURES RETURNS

Figures 1 and 2 indicate that the volatility of the spot and futures returns series change with time in a similar manner; this is a potential indication of stationarity and cointegration. This could affect the specification of the models applied to estimate the optimal hedge ratio. Thus, tests for stationarity and cointegration, prior to estimating models, would limit model misspecification.

3.2. TEST FOR STATIONARITY

This section performs tests for stationarity in order to ensure correct model specification. Time varying probability distributions for returns may lead to biased estimates of the optimal hedge ratio with methods, such as the Single Equation Method and vector autoregression. The vector error correction method should be applied when the presence of non-stationarity is detected.

Table 1 presents the results of unit root tests for the natural logarithms of spot (lnS) and futures prices (lnF) as well as their first differences, which are continuously compounded spot return (R_s) and futures return (R_f).

TABLE 1: TESTS FOR UNIT ROOTS

	ADF Test statistic	KPSS test statics	
		Level	Trend
lnS	-1.6855	0.6319*	0.5796*
lnF	-1.6890	0.6227*	0.5831*
R_s	-26.5414*	0.2336	0.0687
R_f	-26.3574*	0.2445	0.0676

The asterisk (*) represents the rejection of the null hypothesis at a 5% level of significance

The statistics in the Table 1 include augmented Dickey-Fuller (ADF) test statistics as well as Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test statistics.

The ADF test, developed by Dickey and Fuller (1981), has a null hypothesis that the variables contain a unit root and are thus non-stationary. The results in Table 1 indicate that the natural logarithms of spot and futures prices are non-stationary according to the ADF test (null hypothesis is not rejected). The continuously compounded returns are however, stationary at a 5% level of significance.

The power of unit root tests with null hypotheses of non-stationarity was challenged by Schwert (1987), as well as DeJong and Whiteman (1991), whose studies indicated that these tests often experience Type II errors. Type II errors infer that these tests tend to accept the null hypothesis of non-stationarity when the alternative hypothesis of stationarity is true. The prevalence of Type II errors is attributed to low power of the test against stable autoregressive alternatives with roots close to unity. The ADF test has a null hypothesis of non-stationarity, for this reason, the study also performed different tests for stationarity, KPSS tests. The KPSS tests, developed by Kwiatkowski et al. (1992) have a null hypothesis of that a series is stationary around a deterministic level or a deterministic trend. One rejects the null hypothesis of stationarity if the KPSS statistics are large relative to asymptotic critical values.

Table 1 indicates that the natural logarithms of spot and futures prices are level and trend non-stationary at a 5% level of significance, according to the test statistics of the KPSS test, thus confirming the results of the ADF test. The continuously compounded returns are level stationary and trend stationary, as their KPSS test statistics are small, at a 5% level of significance.

The natural logarithms of the price series are thus non-stationary and their first differences, the continuously compounded returns series, are non-stationary. This lessens the bias of estimating the optimal hedge ratio using the Single Equation Method and the Vector Autoregression Methods.

The elimination of unit roots by differencing may indicate that the spot and futures price processes are integrated to order one.

3.3. TESTS FOR COINTEGRATION

Given that the spot and futures price series are integrated to order one, there may be a cointegrating relationship between them. A cointegrating relationship exists when variables are integrated of the same order greater than zero. If a cointegrating relationship exists, time series analysis of non-stationary variables can yield valid econometric results (Engle & Granger, 1987).

Table 2 presents the results from the tests for cointegration proposed by Johansen (1991):

TABLE 2: JOHANESEN'S TEST FOR COINTEGRATION

H_0	H_1	Max. eigen-value	Max. eigen-value statistic	Trace statistic	Max. eigen-value test p-value	Trace test p-value
r = 0	r < 1	0.0306	23.2743*	25.1833*	0.0015	0.0013
r = 1	r < 2	0.0025	1.9091	1.9091	0.1671	0.1671

The asterisk (*) represents the presence of stationarity at a 5% level of significance

The cointegration tests applied in this study are unrestricted cointegration rank tests (trace and maximum eigenvalue) that use likelihood ratios based on the maximum eigenvalue or the trace of the stochastic matrix. r represents the number of linearly independent cointegrating vectors (relationships). The test based on the maximum eigenvalue statistic tests a null hypothesis of r cointergrating relations against an alternative hypothesis of $(r+1)$ cointergrating relations. The maximum eigenvalue statistic is computed as follows:

$$LR_{max}(r|r+1) = -TLn(1-\lambda_i), \qquad (18)$$

where LR is the likelihood ratio, T is the number of observations, and λ_i represents the i^{th} smallest squared canonical correlation derived by Johansen (1991).

The null hypotheses of Johanesen's tests for cointegration are that there are at most a fixed number of cointegrating vectors (in this case zero or one). The alternative hypothesis is that the number of cointegrating vectors is less than the next largest integer (in this case one and two). As the p-values for both the trace and maximum eigenvalue tests in Table 2 indicate, the null hypothesis of no cointegrating vectors is rejected at a 1% level of significance and the null hypothesis of one cointegrating vector is not rejected at a 1% level of significance. One can conclude that the time series have a cointegrating relationship with a rank of one.

The presence of cointegrating relationships between the price series indicated a benefit in the application of Vector Error-Correction Methods to estimate the optimal hedge ratio.

4. ESTIMATION OF THE OPTIMAL HEDGE RATIO

4.1. Estimating the Optimal Hedge Ratio Using the Single Equation Method

The first method applied to estimate the optimal hedge ratio is the Single Equation Method via Ordinary Least Squares (SEMOLS) regression of the continuously compounded spot and futures returns as earlier denoted in equation (1). The β coefficient of the regression is the estimate of the optimal hedge ratio, as derived earlier.

TABLE 3: ESTIMATES USING THE SINGLE EQUATION METHOD

	Coefficient	Standard Error	P-value
α	0.0000	0.0001	0.8861
β	0.9841*	0.0075	0.0000
R^2	0.9584		

Table 3 indicates the at the hedge ratio in this context is 0.9584 futures contracts per unit of underlying spot diversified portfolio (FTSE/JSE Top40) held. It is important to note that the β coefficient is statistically significant at a 5% level of significance, based on the coefficient's p-value, which is less than 5%.

A challenge faced by the SEMOLS is that it does not account for serial correlation of the residuals. Table 4 indicates that the Q-statistics of the residuals from the Single Equation Method are statistically significant indicating the presence of serial correlation.

TABLE 4: THE AUTOCORRELATION FUNCTION OF THE RESIDUALS FROM THE SINGLE EQUATION METHOD, ESTIMATED BY OLS

Lags	Autocorrelation coefficient	Q-Statistic	p-value
1	-0.3890	114.1900*	0.0000
5	-0.0490	122.8800*	0.0000
20	0.0250	139.0800*	0.0000

The asterisk (*) represents statistical significance at a 5% level of significance

This challenge of serial correlation can be addressed by applying vector autoregression.
Another challenge faced by the SEMOLS is that it assumes that the spot return is a function of the futures return. This challenge of having to determine whether the futures or spot return is the endogenous variable can also be addressed by applying vector autoregression.

4.2. ESTIMATING THE OPTIMAL HEDGE RATIO USING THE BIVARIATE VECTOR AUTOREGRESSION (VAR) METHOD

The second method applied to estimate the optimal hedge ratio is the Bivariate Vector Autoregression Method. This method assumes all variables are endogenous. The continuously compounded returns of the spot and futures prices are the endogenous variables for this VAR model, as earlier depicted in equation (11). The optimal lag length, m, is two, as determined by the Bayesian information criterion developed by Schwarz (1978).

Table 5 contains the estimates for equation (11) for the optimal lag length of three.

TABLE 5: ESTIMATES USING THE BIVARIATE VAR

	r_{st}	r_{ft}
β_1	**-0.1762**	**0.2515**
	(0.1963)	(0.1952)
β_2	**-0.1426**	**-0.0396**
	(0.1952)	(0.1940)
γ_1	**0.2138**	**-0.2076**
	(0.1975)	(0.1964)
γ_2	**0.1197**	**0.0132**
	(0.1954)	(0.1943)
α	**0.0001**	**0.0001**
	(0.0007)	(0.0007)

The values of the coefficients are in bold and the standard errors are parenthesis
The asterisk (*) represents statistical significance at a 5% level of significance

The residuals of the VAR model, rather than the parameter estimates, are the important data points required to estimate the optimal hedge ratio. The time series of the residuals of ε_{st} and ε_{ft} from equation (11) are generated by taking the difference of the returns estimated by the VAR model and the actual returns from the in-sample data set.

Upon generation of the time series of the residuals of the bivariate VAR model, equation (12) is applied to estimate the optimal hedge ratio. The inputs for equation (12) are the variance of the residuals of the futures returns and the covariance of the residuals of the spot and futures returns. Table 6 contains the estimate for the optimal hedge ratio using the bivariate VAR model, as well as the direct inputs to equation (12).

TABLE 6: HEDGE RATIO ESTIMATES USING THE BIVARIATE VAR

Statistic	Value
σ_{sf}	0.0003
σ_f^2	0.0003
h^*	0.9885

The tests in section 4.3 indicate the presence of a cointegrating relationship between spot and futures prices. This indicates that the Vector Error-Correction Method can improve the model specification and reduce the bias of the estimate of the optimal hedge ratio.

4.3. VECTOR ERROR-CORRECTION (VECM) METHOD

The Vector Error-Correction Method is the next method applied to estimate the optimal hedge ratio. This method adds error-correction terms to VAR model in equation (11) to yield equation (13). The estimates for the VECM are presented in Table 7.

From the speed of the adjustment parameters, λ_s and λ_f, only the adjustment parameter for the futures equation, λ_f, is significant, indicating that the futures index is converging to movements in the spot index and not vice versa. This indicates that the futures price series adjusts more rapidly to the previous period's deviation from the long-run equilibrium than the spot price series.

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TABLE 7: ESTIMATES USING THE VECM BIVARIATE VAR

	r_{st}	r_{ft}
λ	**-0.1809**	**1.4902***
	(0.5208)	(0.5154)
β_1	**-0.6241**	**-0.8646**
	(0.4024)	(0.3982)
β_2	**-0.3276**	**-0.4257**
	(0.2310)	(0.2286)
γ_1	**0.0270**	**0.2882**
	(0.4065)	(0.4022)
γ_2	**0.0586**	**0.1665**
	(0.2327)	(0.2303)
α	**0.0000**	**0.0000**
	(0.0008)	(0.0008)
Cointegrating equation		
S_{t-1}	**1.0000**	
F_{t-1}	**-1.0129***	
C	**-0.0000**	

Similar to the bivariate VAR, the VECM residuals are generated to estimate the optimal hedge ratio using equation (12). Table 8 presents the estimate for the optimal hedge ratio and the direct inputs.

TABLE 8: HEDGE RATIO ESTIMATES USING THE VECM BIVARIATE VAR

Statistic	Value
σ_{sf}	0.0005
σ_f^2	0.0005
h^*	0.9981

Figure 3 and Figure 4 depict the spot and futures residuals of the VAR model. Prior to testing for conditional heteroskedasticity, Figure 3 and Figure 4 suggest the residuals of both series appear to exhibit time varying volatility or ARCH effects.

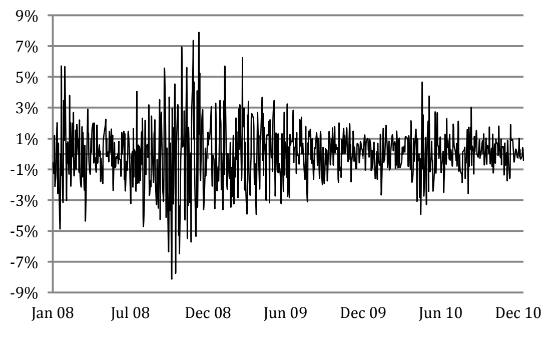

FIGURE 3: VAR SPOT RETURN RESIDUALS

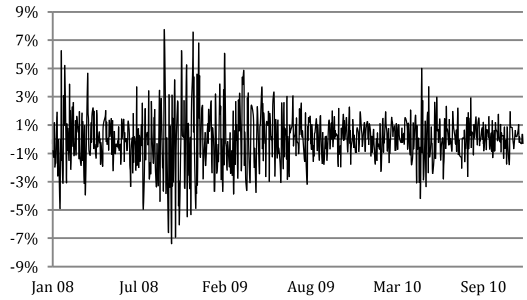

FIGURE 4: VAR FUTURES RETURNS RESIDUALS

This study uses White's (1980) method to test for the presence of conditional heteroskedasticity (ARCH effects). Table 9 presents the results for White's test (with and without cross-terms). The null hypothesis for White's test is homoskedasticity and the alternative is heteroskedasticity. As the p-values are zero at five decimal places, one can reject null hypothesis in favour of the alternative of heteroskedasticity.

TABLE 9: WHITE TEST FOR HETEROSKEDASTICITY

White test type	χ^2 statistic	Degrees of freedom	p-value
Cross-terms	298.3345	42.0000	0.00000
No cross-terms	107.8269	24.0000	0.00000

The ARCH effects present in the data can be addressed by estimating the optimal hedge ratio using a Multivariate GARCH approach.

4.4. ESTIMATING THE OPTIMAL HEDGE RATIO USING THE MULTIVARIATE GENERALISED AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY (GARCH) METHOD

The Multivariate GARCH Method applied herein is a DVEC GARCH model that addressed ARCH effects by explicitly modelling the conditional variance and covariances of the residuals of the spot and future returns. Studies by Yang and Allen (2005), Bhaduria and Duraia (2008), and Olgun and Yetkiner (2012) have applied the DVEC GARCH model to estimate time-varying hedge ratios because the DVEC GARCH model allows for time-varying conditional covariances (Bollerslev et al., 1988). The time series of conditional variances and covariances are the series data required to estimate time-varying hedge ratios.

Table 10 presents the estimation results from equation (16) the DVEC Multivariate GARCH Model. The constant mean function is used as the mean equation for this GARCH model.

TABLE 10: ESTIMATES USING THE DVEC MGARCH

Parameter	Coefficient	Std. Error
γ_{ss}	0.0000	0.0000
γ_{sf}	0.0000	0.0000
γ_{ff}	0.0000	0.0000
α_{11}	0.2000	0.0000
α_{22}	0.2088	0.0000
α_{33}	0.2221	0.0000
β_{11}	0.7791	0.0000
β_{22}	0.7624	0.0000
β_{33}	0.7421	0.0165

* Statistically significant at a 5% level of significance

Dynamic hedge ratios are estimated by applying the variances and covariances of the residuals of the GARCH model to equation (17) to determine time-varying hedge ratios. These time-varying hedge ratios are presented in Figure 5.

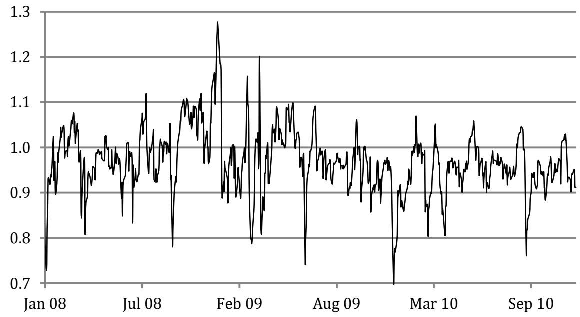

FIGURE 5: DVEC MGARCH HEDGE RATIOS

4.5. Summary of Estimates of the Optimal Hedge Ratio

The hedge ratio estimates from the different methods are summarised in Table 11. The VECM method produces the most conservative hedge ratio of 99.8% compared to the more aggressive hedge ratios of the OLS and VAR methods.

TABLE 11: SUMMARY OF HEDGE RATIO ESTIMATES

Method	h^*
Single equation (OLS)	0.9841
Bivariate VAR	0.9885
VECM	0.9981
DVEC-MGARCH	Time-varying

5. EFFECTIVENESS OF HEDGING METHODS

5.1. DISCUSSION OF METHODOLOGY

Hedging effectiveness is important as it allows users of hedging methods to assess the quality of their hedges. Following the application of one of the econometric methods discussed earlier to estimate the optimal hedge ratio (whether static or dynamic), the hedge ratios estimated are used to create hedging strategies. The hedging strategy involves a portfolio comprised of a long position in the underlying equity index as well as a short position in futures contracts proportionate to the estimate of the optimal hedge ratio. A portfolio comprised as such is a hedged portfolio; a portfolio comprising of an equivalent long spot exposure to the hedged portfolio is an un-hedged portfolio.

In order to perceive effectiveness, a comparison of some kind must be made. This comparison is between the hedged and un-hedged portfolio. In order to compare these two portfolios the key benefits that the hedged portfolio provides must be quantified.

The first way the benefits of hedging can be quantified is in terms of the reduction in volatility as intimated by Johnson(1960) and Ederington(1979). In this study, volatility is expressed by the standard deviation of the returns. Volatility presents the investor with uncertainty regarding the portfolio's value and therefore, the lower the volatility (standard deviation) of the hedged portfolio, relative to the un-hedged portfolio, the more effective the hedge.

The returns of the hedged and un-hedged portfolios are defined as follows:

$$r_{Ut} = r_{st}$$
$$r_{Ht} = r_{st} - h^* \times r_{ft}, \qquad (21)$$

where r_{Ut} is the return on the un-hedged portfolio that only comprises of the underlying stock index, r_{Ht} is the return on the hedged portfolio as derived earlier in equation (6), h^* is the optimal hedge ratio, and r_{st} and r_{ft} are defined as per equation (2) .

Based on return estimated in equation (21) one can determine the equations for the variances of the un-hedged and hedged portfolios as follows:

$\sigma_U^2 = \sigma_s^2$
$\sigma_H^2 = \sigma_s^2 - 2h^* \times \sigma_{sf} + h^{*2} \times \sigma_f^2, \qquad (22)$
where σ_U^2 and σ_H^2 are the variance of the un-hedged and hedged portfolios; σ_s^2, σ_f^2 and σ_{sf} are variances and covariance of the spot and futures returns respectively.

Based on the definitions of the variances of the hedged and un-hedged portfolio, in equation (22), one can compare the standard deviations (volatilities) of the hedged portfolio against the un-hedged portfolio. The volatility reduction of the hedged portfolio relative to the un-hedged portfolio is a measure of hedging effectiveness, as this indicates how much uncertainty has been removed from the portfolio by hedging. The volatility reduction percentage is determined by taking the difference in volatility of the two portfolios' and dividing the difference by the volatility of the un-hedged portfolio, as depicted in equation (23):

$$\frac{\sigma_U - \sigma_H}{\sigma_U}, \qquad (23)$$

In accordance with Ederington's (1979) work, a higher ratio in equation (23) infers a more effective hedge, because the reduction in hedged portfolio's volatility is higher relative to the un-hedged portfolio.

Another method of comparing the effectiveness of hedging is comparing the coefficient of variation of the hedged portfolio against that of another hedged portfolio (Jianru & Jinghua, 2011). The coefficient of variation is the ratio of the standard deviation over the expected return. In this context, it represents the amount of risk that needs to be taken per unit of return. Thus, a portfolio with a small coefficient of variation is less risky than a portfolio with a larger coefficient of variation, relative to their respective levels of returns.

The coefficient of variation for a hedged portfolio is defined in equation (24):

$$\frac{\sigma_H}{r_H}, \qquad (24)$$

Although a coefficient of variation comparison takes into account the relative level of return for when comparing volatility reduction, it does not take investors' level of risk aversion into account. This study uses a utility-based comparison as applied by Gagnon et al. (1998) as well as Yang and Allen (2005), to take an investors' degree of risk aversion into account in the comparison of hedging effectiveness.

Gagnon et al. (1998) represented the utility problem as follows:

$$\max_h \left[E(r_{Ht}|\Omega_{t-1}) - \frac{1}{2}\phi Var(r_{Ht}|\Omega_{t-1}) \right], \qquad (25)$$

where r_{Ht} is defined in equation (21), ϕ is the level of risk aversion and Ω_{t-1} is the historical data available at time $t-1$.

For an investor that is a mean-variance optimiser, if ϕ is large ($\phi > 1$) there is a high level of risk aversion and if ϕ is low ($0 < \phi < 1$) there is a low level of risk aversion. If ϕ is zero, the investor is risk neutral thus for a given level of return, the investor neither loses utility by taking on higher levels of risk nor does the investor gain utility by reducing levels of risk. Risk in this context is represented by the variance of returns on the hedge portfolio, $Var(r_{Ht}|\Omega_{t-1})$. As ϕ increases positively from zero, a risk-averse investor experiences a higher penalty on utility by taking on higher levels of risk to gain a particular return. A risk-seeking investor would have a ϕ less than zero. Such an investor would gain utility by experiencing higher levels of risk to obtain a given return.

The utility maximisation method generally allows some flexibility in terms of how it addresses the investors' degree of risk aversion by allowing investors' to choose an optimal hedge ratio that will maximise utility at varying levels of risk aversion. In this study, the focus in on applying the utility-maximisation method to compare the level of investors' utility when there is no hedging and when hedging is executed with the estimates of the optimal hedge ratio developed earlier in the study. At specified degrees of risk aversion, the hedge ratios from different estimation methods are compared to assess the degree of utility improvement experienced by holding the hedged portfolio compared to holding the un-hedged portfolio.

The above methods of comparing the effectiveness of hedging were applied to in-sample, as well as out-of-sample data over one, five and twenty day hedging periods. These different hedging periods represent daily, weekly and monthly investment horizons, in line with Lien and Tse's (1998) indication that hedging effectives differs over different hedging periods.

The data period ranges from the 2nd of January 2008 to the 31st of December 2010 for in-sample data. Out-of-sample data covers sample data from the 3rd of January 2011 to 15th of June 2011.

5.2. STANDARD DEVIATION ANALYSIS FOR HEDGING EFFECTIVENESS

The first method applied to compare effectiveness of hedging, initially proposed by Ederington (1979), is the most widespread method of comparing hedging effectiveness. The standard deviation is used as a measure of volatility. Table 12 presents the standard deviation reduction of the various hedging methods.

TABLE 12: STANDARD DEVIATION REDUCTION FOR IN-SAMPLE DATA

Method	h*	1-day	5-day	20-day
OLS	0.9841	**-79.5990%**	-86.7484%	-88.9345%
VAR	0.9885	-79.5943%	-86.7191%	-88.9924%
VECM	0.9981	-79.5514%	-86.6036%	-89.0588%
MGARCH	Varying	-78.7276%	**-87.3831%**	**-90.1637%**

As the results in Table 12 depict, for in-sample data, the SEMOLS is the most effective method for daily hedging, however over longer periods the Multivariate GARCH method is the most effective method. The VEC and VECM are less effective than the GARCH over five and twenty day periods but more effective over one day hedging periods.

Table 13 presents the standard deviation reduction of the four methods on out-of-sample data.

TABLE 13: STANDARD DEVIATION REDUCTION FOR OUT-OF-SAMPLE DATA

Method	h*	1-day	5-day	20-day
OLS	0.9841	**-82.5452%**	-92.9610%	-83.9640%
VAR	0.9885	-82.4683%	-92.7690%	-84.1309%
VECM	0.9981	-82.2619%	-92.2726%	**-84.4621%**
MGARCH	Varying	-82.4512%	**-93.2351%**	-82.0771%

The results in Table 13 indicate that over a one-day hedging period, the SEMOLS is the most effective, the Multivariate GARCH is most effective over a five day hedging period and the VECM is most effective over a twenty day hedging period.
However, it is important to note that all the methods provide in excess of 80% variance reduction, which is rather high; therefore, most investors should be willing to apply all the methods to hedge their exposure.

Based on the empirical data, the Multivariate GARCH method is best applied over hedging periods longer than one day and the Single Equation Method is best applied over a one-day hedging period.

5.3. COEFFICIENT OF VARIATION ANALYSIS FOR HEDGING EFFECTIVENESS

The second method applied to compare the effectiveness of hedging is the coefficient of variation.

Table 14 presents the in-sample coefficients of variation for the various hedging methods.

Table 14: COEFFICIENTS OF VARIATION FOR IN-sample data

Method	h*	1-day	5-day	20-day
OLS	0.9841	**19131.9554%**	4822.8101%	4212.1783%
VAR	0.9885	19560.2425%	4930.3089%	4143.0474%
VECM	0.9981	20594.0521%	5199.8224%	4019.8061%
MGARCH	Varying	-165237.3002%	**1605.1694%**	**604.7692%**

The smallest coefficient of variation observed for a one-day investment horizon is that of the SEMOLS (the negative coefficient of variation for the MGARCH is not meaningful). However, over five and twenty day hedging periods the MGARCH is the most effective method. These results are in line with the results from the standard deviation reduction analysis for in-sample data.

Table 15 presents the coefficients of variation for out-of-sample data.

TABLE 15: COEFFICIENTS OF VARIATION FOR OUT-OF-SAMPLE DATA

Method	h*	1-day	5-day	20-day
OLS	0.9841	-3599.1617%	776.2876%	7158.5948%
VAR	0.9885	-3691.1272%	**776.2382%**	5820.6908%
VECM	0.9981	-3913.9175%	784.1066%	**4105.6818%**
MGARCH	Varying	-9692.0596%	2329.0006%	-2707.1078%

According to results in Table 15, no meaningful comparison of hedging effectiveness can be made for one-day hedging periods due to negative coefficients of variation that are not meaningful. Over a five day hedging period the VAR is the most effective method. Over a twenty day hedging period the VECM is the most effective method.

The coefficient of variation analysis neither validates the outcomes of the standard deviation analysis for out-of-sample data nor does it identify one method that is generally superior over the various hedging periods.

5.4. Risk-Aversion Based Utility Maximisation Analysis for Hedging Effectiveness.

The final method for comparing hedging effectiveness is the risk aversion based utility maximisation analysis. Table 16, Table 17, and Table 18 present the risk aversion based utility of the various methods over one, five and twenty day hedging periods for in-sample data.

Table 16 indicates that for the given levels of risk aversion, in this study, the Single Equation Method estimated by OLS is most effective strategy over a one-day period for investors with low risk tolerance, investors with higher risk tolerance may prefer not to hedge at all over a one-day investment horizon. This is consistent with the findings for the standard deviation reduction and coefficient of variation analyses for in-sample data. The reason behind the effectiveness of the Single Equation Method should be attributed to its straightforward estimation of the hedge ration, whereby the slope coefficient of the bivariate relationship between spot and futures prices is interpreted as the hedge ratio. The estimation of the single equation by OLS can be robust in the presence of cointegration as it can be formulated in the form of Engle-Granger cointegrating equation.

Table 17 indicates that for the given levels of risk aversion, in this study, the Multivariate GARCH method is the only hedging strategy preferred to the un-hedged strategy over a five-day period for investors with varying levels of risk tolerance. Investors with high levels risk tolerance may prefer not to hedge at all. This is consistent with the findings for the standard deviation reduction and coefficient of variation analyses for in-sample data.

Table 18 indicates that, in this study, for the given levels of risk aversion the Multivariate GARCH strategy is the most effective hedging method over a twenty-day period for in-sample data. This is consistent with the findings of the standard deviation reduction and coefficient of variation analyses for in-sample data.

TABLE 16: UTILITY COMPARISON OVER A ONE-DAY PERIOD FOR IN-SAMPLE DATA

	Un-hedged	**OLS**	**VAR**	**VECM**	**DVECH GARCH**
ϕ	1 - day	1 - day	1 - day	1 - day	1 - day
0.10	**0.0001**	0.0000	0.0000	0.0000	0.0000
0.50	**0.0000**	0.0000	0.0000	0.0000	-0.0000
0.01	**0.0001**	0.0000	0.0000	0.0000	0.0000
1.50	-0.0001	**0.0000**	0.0000	0.0000	-0.0000
2.00	-0.0002	**0.0000**	0.0000	0.0000	-0.0000
2.50	-0.0003	**0.0000**	0.0000	0.0000	-0.0000
3.00	-0.0004	**0.0000**	0.0000	0.0000	-0.0000

Maximum utility for given levels of risk aversion in bold

TABLE 17: UTILITY COMPARISON OVER A FIVE-DAY PERIOD FOR IN-SAMPLE DATA

	Un-hedged	**OLS**	**VAR**	**VECM**	**DVECH GARCH**
ϕ	5-day	5-day	5-day	5-day	5-day
0.10	**0.0005**	0.0001	0.00011	0.0001	0.0003
0.50	0.0002	0.0001	0.00010	0.0001	**0.0003**
0.01	**0.0006**	0.0001	0.00011	0.0001	0.0003
1.50	-0.0006	0.0001	0.00009	0.0001	**0.0003**
2.00	-0.0010	0.0001	0.00008	0.0001	**0.0003**
2.50	-0.0014	0.0001	0.00007	0.0001	**0.0003**
3.00	-0.0018	0.0001	0.00007	0.0001	**0.0003**

Maximum utility for given levels of risk aversion in bold

TABLE 18: UTILILTY COMPARISON OVER A TWENTY-DAY PERIOD FOR IN-SAMPLE DATA

	Un-hedged	**OLS**	**VAR**	**VECM**	**DVECH GARCH**
ϕ	20-day	20-day	20-day	20-day	20-day
0.10	-0.0005	0.0002	0.0002	0.0002	**0.0012**
0.50	-0.0015	0.0002	0.0002	0.0002	**0.0011**
0.01	-0.0003	0.0002	0.0002	0.0002	**0.0012**
1.50	-0.0040	0.0001	0.0001	0.0002	**0.0011**
2.00	-0.0052	0.0001	0.0001	0.0001	**0.0011**
2.50	-0.0064	0.0001	0.0001	0.0001	**0.0011**
3.00	-0.0077	0.0001	0.0001	0.0001	**0.0011**

Maximum utility for given levels of risk aversion in bold

Table 19, Table 20 and

Table 21 present the risk aversion based utility of the various methods over one, five and twenty day hedging periods for in-sample data.

TABLE 19: UTILITY COMPARISON OVER A ONE-DAY PERIOD FOR OUT-OF-SAMPLE DATA

	Un-hedged	**OLS**	**VAR**	**VECM**	**DVECH GARCH**
ϕ	1 - day	1 - day	1 - day		1 - day
0.10	-0.0003	-0.0001	-0.0001	-0.0001	**-0.0000**
0.50	-0.0003	-0.0001	-0.0001	-0.0001	**-0.0000**
0.01	-0.0003	-0.0001	-0.0001	-0.0001	**-0.0000**
1.50	-0.0004	-0.0001	-0.0001	-0.0001	**-0.0000**
2.00	-0.0004	-0.0001	-0.0001	-0.0001	**-0.0000**
2.50	-0.0005	-0.0001	-0.0001	-0.0001	**-0.0000**
3.00	-0.0005	-0.0001	-0.0001	-0.0001	**-0.0000**

Maximum utility for given levels of risk aversion in bold

TABLE 20: UTILITY COMPARISON OVER A FIVE-DAY PERIOD FOR OUT-OF-SAMPLE DATA

	Un-hedged	**OLS**	**VAR**	**VECM**	**DVECH GARCH**
ϕ	5-day	5-day	5-day	5-day	5-day
0.10	-0.0013	0.0003	0.0003	**0.0003**	0.0001
0.50	-0.0015	0.0003	0.0003	**0.0003**	0.0001
0.01	-0.0013	0.0003	0.0003	**0.0003**	0.0001
1.50	-0.0019	0.0003	0.0003	**0.0003**	0.0001
2.00	-0.0021	0.0003	0.0003	**0.0003**	0.0001
2.50	-0.0023	0.0003	0.0003	**0.0003**	0.0001
3.00	-0.0025	0.0003	0.0003	**0.0003**	0.0001

Maximum utility for given levels of risk aversion in bold

TABLE 21: UTILITY COMPARISON OVER A TWENTY-DAY PERIOD FOR OUT-OF-SAMPLE DATA

	Un-hedged	**OLS**	**VAR**	**VECM**	**DVECH GARCH**
ϕ	20-day	20-day	20-day	20-day	20-day
0.10	-0.0028	0.0001	0.0001	**0.0001**	-0.0002
0.50	-0.0029	0.0001	0.0001	**0.0001**	-0.0002
0.01	-0.0028	0.0001	0.0001	**0.0001**	-0.0002
1.50	-0.0033	0.0001	0.0001	**0.0001**	-0.0002
2.00	-0.0034	0.0000	0.0001	**0.0001**	-0.0002
2.50	-0.0036	0.0000	0.0001	**0.0001**	-0.0002
3.00	-0.0038	0.0000	0.0001	**0.0001**	-0.0002

Maximum utility for given levels of risk aversion in bold

Table 19 indicates that for the given levels of risk aversion, in this study, the Multivariate GARCH strategy is the most effective hedging method over a one-day period for out-of-sample data. This is inconsistent with the findings of the standard deviation reduction and coefficient of variation analyses for out-of-sample data. The model accounts for conditional volatility consistent with turmoil periods.

Table 20 indicates that, in this study, for the given levels of risk aversion the VECM strategy is the most effective hedging method over a five-day period for in-sample data. This is inconsistent with the coefficient of variation and standard deviation reduction analysis for out-of-sample data.

Table 21 indicates that for the given levels of risk aversion, in this study, the VECM strategy is the most effective hedging method over a twenty-day period for in-sample data. This is consistent with the findings of the coefficient of variation analysis and consistent with the standard deviation reduction analysis for out-of-sample data.
It is important to note that investors with different levels of risk aversion may rank the effectiveness of the four methods differently, as their utility functions may differ.

6. CONCLUSION

This paper provides a comparative assessment of the effectiveness of various hedging methods in the South African equities market during the crisis period of 2008-2010. It assesses the effectiveness of hedging by the degree of standard deviation reduction, the minimisation of the coefficient of variation, and the maximisation of utility.

Studies on the optimal hedge ratio have evolved from using the Single Equation Method, estimated by OLS (SEMOLS), to more advanced methods that address the Single Equation Method's shortcomings. The shortcomings addressed by Vector Autoregression (VAR) model include endogeneity of the variables and serial correlation. The VAR, however, does not address cointegration that is often present in financial time series. Thus, the Vector Error-Correction (VECM) is used to deal with cointegration. The first three methods highlighted above, in the evolution of studies on the optimal hedge ratio, do not take into account ARCH effects that are prevalent in financial time series. Multivariate GARCH methods address the challenge of ARCH effects. Studies on the optimal hedge ratio have evolved to cover a variety of methods. This study applies four such methods: the SEMOLS, the VAR method, a VECM with a VAR base, and a DVEC Multivariate GARCH method.

The evolution of studies on the optimal hedge ratio led to the application of the above stated methods in emerging markets. However, the numbers of studies on the optimal hedge ratio for stock index futures in emerging markets are limited. This study is part of a contemporary set of studies that explore the application of the optimal hedge ratio in emerging markets. Furthermore, the evolution of studies on the optimal hedge ratio has also led to more advanced assessments of hedging effectiveness. This study continues in this vein by applying three methods to assess hedging effectiveness, particularly, standard deviation reduction, coefficient of variation minimisation, and risk aversion based utility maximisation. This assessment of hedging effectiveness compares the four econometric estimation methods applied in this study to estimate the optimal hedge ratio in the South African equity market.

The data applied in this study are daily closing spot prices for the FTSE/JSE. All Share Top 40 index (ALSI Top 40) traded on the Johannesburg Stock Exchange (JSE), as well as daily closing prices for the ALSI Top 40 future with the nearest maturity, traded on the South African Futures Exchange (SAFEX). The data period ranges from the 2^{nd} of January 2008 to the 31^{st} of December 2010 for in-sample data. Out-of-sample data covers sample data from the 3^{rd} of January 2011 to 15th of June 2011.

The empirical results indicate there is not one econometric hedging method that is persistently the most effective but rather that the most effective method is a function of the hedging period. The SEMOLS is generally more effective over a one-day hedging period and the VECM and Multivariate GARCH are effective over five and twenty day hedging periods. This is likely explained by the SEMOLS straightforward estimation of the hedge ratio interpreted as the slope of the bivariate regression between spot and futures variables. Moreover, volatilities are accounted in a longer period than a short horizon. This reality makes a multivariate GARCH method a preferred method for hedge ratio in long horizon.

There remains scope for continued research to expand on the works of this study. In particular, the use of other methods to assess hedging effectiveness in South Africa and other emerging markets and the currency effects for international investors investing in emerging markets. Further investigation into the aforementioned areas would contribute to effective management of the risks associated with investing in emerging equity markets.

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APPENDIX:

The rationale behind using the β coefficient from the simple linear regression in equation (1) as an estimate for the optimal hedge ratio h^* is as follows:

The value a long spot portfolio hedged with a short futures position can be represented in equation (4).

$$V_t = Q_S S_t - Q_F F_t, \qquad (4)$$

where V_t is the value of the hedged portfolio, Q_S and Q_F are the respective quantities of long spot units and futures units held in the portfolio. S_t and F_t represent the respective prices of one unit of spot and futures exposure.

The hedge ratio, h, is the ratio of the quantity of futures units held for each spot unit held. Equation (4) can be restated per unit of spot exposure in terms of h, as depicted in equation (5).

$$v_t = S_t - hF_t, \qquad (5)$$

where v_t is the portfolio value per share and $h = \frac{Q_F}{Q_S}$ is the hedge ratio.

Given the structure of equation (5), portfolio returns can be expressed in terms of the hedge ratio, as depicted in equation (6).

$$r_t = r_{st} - hr_{ft}, \qquad (6)$$

The variance of portfolio returns is as expressed in equation (7).

$$Var(r_t) = Var\left(r_{st} - hr_{ft}\right)$$
$$\therefore Var(r_t) = Var(r_{st}) - 2Cov(r_{st}, hr_{ft}) + Var\left(hr_{ft}\right)$$
$$\therefore Var(r_t) = Var(r_{st}) - 2hCov(r_{st}, r_{ft}) + h^2 Var\left(r_{ft}\right)$$
$$\sigma_P^2 = \sigma_s^2 - 2h\sigma_{sf} + h^2\sigma_f^2, \qquad (7)$$

where $\sigma_P^2 = Var(r_t)$, $\sigma_s^2 = Var(r_{st})$, $\sigma_f^2 = Var\left(r_{ft}\right)$ and $\sigma_{sf} = Cov(r_{st}, r_{ft})$.

The next step involves minimising the portfolio variance, σ_P^2, by choosing an optimal hedge ratio, h^*.
The minimisation problem is solved via differential calculus by setting ${\partial \sigma_P^2}/{\partial h}$ to zero, and solving for h. The optimal hedge ratio, h^*, is the solution to this partial differential equation as depicted in equation (8).

$${\partial \sigma_P^2}/{\partial h} = -2\sigma_{sf} + 2h\sigma_f^2$$
$$\because {\partial \sigma_P^2}/{\partial h} = 0$$
$$\Rightarrow h^* = \frac{\sigma_{sf}}{\sigma_f^2}, \qquad (8)$$

The method of Ordinary Least Squares is used to estimate the parameters in equation (1). This method seeks to determine α and β such in a manner that minimises the sum of the squared errors, $Q = \sum_{\forall t} {\varepsilon_t}^2$.

The minimisation problem is expressed in equation (9).

$$\min_{\alpha,\beta} Q(\alpha, \beta)$$
$$\min_{\alpha,\beta} \sum_{\forall t} {\varepsilon_t}^2$$
$$\min_{\alpha,\beta} \sum_{\forall t} (r_{st} - \alpha - \beta r_{ft})^2, \qquad (9)$$

The variables in equation (9) are defined in equation (1).

The estimated solutions to equation (9) via differential calculus are depicted in equation (10):

$$\beta = {\sigma_{sf}}/{\sigma_f^2}$$
$$\alpha = E[r_{st}] - \beta E\left[r_{ft}\right], \qquad (10)$$

As equations (8) and (10) indicate, $\beta = h^*$ therefore, β in equation (1) represents an estimate for the optimal hedge ratio.