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Endogenous Fixprices and Sticky Price Adjustment of Risk-averse Firms

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A risk-averse price-setting firm which knows the quantity demanded at the status quo price but has imperfect information otherwise may choose not to change it although an otherwise identical risk-neutral firm would do so, provided the variance of the firm's subjective probability distribution over quantities demanded as a function of price displays a kink at the status quo. This is equivalent to risk aversion of order one. When no such endogenous fixprice exists, the size of price adjustment still tends to zero as risk aversion tends to infinity, and to any arbitrarily small menu cost there exists a degree of risk aversion so that the firm will not adjust.

Keywords: fixed prices, price adjustment, risk aversion, menu cost.

JEL classification: D21, D81, D83.

1 Introduction

The nominal rigidity of prices is still an intriguing economic problem. From empirical studies like Carlton's (1986) it is clear that price stickiness is a phenomenon that cannot be ignored as part of business reality. However, theoretical models so far fail to fully account for the degree of rigidity empirically observed.[1] The problem is that nominal rigidities appear to contradict the rationality of the economic agents involved. Accordingly there have been theoretical investigations that ascribe rigidities to the presence of not perfectly maximizing behavior (near rationality), but also to costs related to the adjustment of prices

1 As for example Mankiw and Romer (1991, p. 5) put it: "Empirical studies of price adjustment by individual firms ... suggest that the extent of nominal stickiness at the microeconomic level is large and that price adjustment policies are quite complex. To date the reasons for those policies have defied explanation."

(menu costs), to strategic interactions among oligopolists and missing coordination among monopolistic competitors (coordination failure), to certain characteristics of the production function and of the demand function, and to imperfect and asymmetric information. The present paper takes up a relatively new reason for nominal price rigidities, namely risk aversion by price-setting firms.

The main idea is simple. When a monopolist[2] contemplates to adjust his price, he typically has a better idea of the quantity demanded at the current price than of that at any new one. Moreover, uncertainty about the variation in profit increases with the size of the price change. This does not necessarily matter as long as he is risk neutral but if uncertainty is undesirable due to risk aversion it will give rise to an incomplete adjustment only.

Moreover, and this is less immediate, a risk-averse firm may choose not to adjust its price at all although an otherwise identical risk-neutral firm would do so. When a status quo price has this property we call it an endogenous fixprice. As we show, whether or not such prices exist depends on the behavior of the variance associated with the random quantity demanded at any price. This defines a functional relationship between price and variance. Loosely speaking, endogenous fixprices exist if, and only if, this relationship displays a kink at the status quo price. Such kinks are compatible with weak convergence, traditionally assumed in decision theory under uncertainty, of the probability distributions involved, and thus with smooth expected-demand curves.

This fact can also be expressed using terminology recently introduced in the theory of decision making under uncertainty. More precisely, Segal and Spivak (1990) have distinguished between risk aversion of order one and risk aversion of order two. It turns out that the case of existence of endogenous fixprices corresponds exactly to risk aversion of order one.

When no endogenous fixprices exist, risk aversion is still crucial because the optimal size of price change decreases with increasing risk aversion. We show this for the case in which quantity demanded is normally distributed and the utility function of profit of the firm displays constant absolute risk aversion. Specifically, as the risk-aversion parameter goes to infinity, price adjustment tends to zero.

When price adjustment tends to zero, so does the difference between the maximum expected utility and the utility at the status quo. Therefore, if there is a small cost of price adjustment, it is less convenient

2 Although we will speak in the sequel mostly of a monopolist, this is mainly for abbreviation of "price-setting firm acting in an imperfectly competitive market."

for a risk-averse firm than for a risk-neutral one to change its price. This substantially reinforces the menu-cost argument. More precisely we show that, however small the cost is, there exists a threshold degree of risk aversion such that a firm with at least this risk aversion sticks to its price whereas a less risk-averse (and a fortiori a risk-neutral) firm adjusts it.

Although risk aversion of firms is a hypothesis not unfamiliar to macroeconomists,[3] there seem to be few works that have investigated its consequences in a formal model. These include Drèze (1979), Greenwald and Stiglitz (1989), and Frank (1990). The paper closest to the present is the one by Drèze who shows that, if the monopolist uses the "truncated minimax" decision criterion (see van Moeseke, 1965, for a discussion), which calls for maximizing expected value of profit minus a multiple of profit's standard deviation, then uncertainty about the price elasticity of a linear demand function and risk aversion have an effect comparable to that of a kink in the demand curve at the status quo point. Moreover, if the firm maximizes expected utility of profit, then no kink appears but a risk-averse monopolist behaves as if he had to face a concave demand function. Therefore, if changes in price had to take place in multiples of a certain basic unit, then this concavity still implies that he behaves as if his demand function had a kink at the status quo.

From the description of our results above it should be clear in which way the present paper differs from Drèze (1979). In particular we do assume throughout the paper that the price-setting firm maximizes expected utility of profit. Moreover, no specific functional form of the demand function is postulated and the firm is not committed a priori to adjust prices in steps of predetermined size.

Whether firms are risk averse or not is a controversial issue. We do not elaborate on this discussion here but maintain that capital-market imperfections, the impact of performance-based compensation schemes on risk-averse managers, and the possibility of significant bankruptcy costs (see Greenwald and Stiglitz, 1993) lend support to the view that many firms are risk averse and thus warrant the analysis of the implications of risk aversion for price-setting behavior.[4]

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3 For example van de Klundert and van Schaik (1990, p. 365) write: "In an economywide recession, firms may increase sales by lowering the price of output, but they may be highly uncertain whether this would entail a rise in revenue as competitors may lower their prices, too. Under these circumstances it could be rational for risk-averse firms to stick to the prevailing price level."

4 While this is definitely plausible when a firm is owned and run by an individual entrepreneur, the assumption of risk aversion might seem less so for

In Sect. 2 we set out the model and prove that for a risk-averse monopolist his expected utility function is kinked if, and only if, the same holds for his corresponding variance function. In Sect. 3 we characterize formally the circumstances under which endogenous fixprices exist and illustrate the significance of the main theorem by means of examples. More precisely we show that, somewhat surprisingly, an apparently smooth specification of the firm's beliefs may induce it to change its behavior from flexible to rigid when a single parameter is allowed to vary in a continuous way. This fact is related to risk aversion of order one in Sect. 4.

In Sect. 5 we study a scenario in which no fixed prices exist and establish the convergence of the optimally chosen price by a risk-averse monopolist to the status quo price if risk aversion tends to infinity. In Sect. 6 we introduce a small cost of price adjustment into the model where no endogenous fixprices exist and show that this reestablishes the qualitative distinction between risk neutrality and risk aversion in that menu cost may imply that a risk-averse firm's behavior is rigid where a risk-neutral one's is flexible. Section 7 contains concluding remarks, and the more complex proofs are given in an appendix.

2 The Model

We consider a monopolist who is uncertain about how quantity demanded is related to the product's price. However, he has experienced, and thus has perfect knowledge of, the status quo relationship $p_0 \mapsto y_0$ where we assume $p_0 > 0$.[5] Moreover, the beliefs he has about the price–quantity relationship for $p \neq p_0$ are captured by a subjective probability distribution $\mu(p|p_0) \in \Delta(\mathbb{R}_+)$, where $\Delta(\mathbb{R}_+)$ is the set of probability distributions over quantities demanded $y \in \mathbb{R}_+$. Denoting with $\bar{y}(p|p_0)$ the expected value of the random variable having distribution $\mu(p|p_0)$ and with $\delta(y) \in \Delta(\mathbb{R}_+)$ the degenerate distribution which is concentrated in y, we assume

publicly traded firms. But then a further argument can be added, provided the uncertainty is aggregate in nature, because in that case the discussion of the legitimacy of risk aversion in Grossman and Hart's (1981) labor-contracting model is pertinent here: when the uncertainty is aggregate in nature, the risk facing a firm is nondiversifiable by shareholders, and therefore the risk aversion of shareholders may carry over to the firm's optimal behavior.

5 We indicate in the concluding remarks how the analysis can be generalized to the case where there is uncertainty also at the status quo.

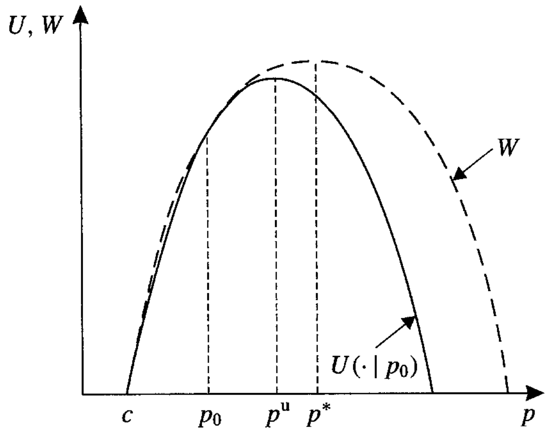

Fig. 1

A1. $\mu(p_0|p_0) = \delta(y_0)$; for $p \neq p_0$, $\mu(p|p_0)$ is given by means of a density function $f(y, p|p_0)$ continuous in y and differentiable in p;

A2. $\bar{y}(p|p_0)$ is decreasing and continuously differentiable in p for all $p > 0$ and $p_0 > 0$; moreover, if $p \to p_0$, then $\mu(p|p_0) \to \mu(p_0|p_0)$ with respect to the topology of weak convergence on $\Delta(\mathbb{R}_+)$.[6]

A1 states that for $p \neq p_0$ the distribution $\mu(p|p_0)$ can be represented by means of a density function whereas for $p = p_0$ it is concentrated in y_0. A2 expresses a regularity assumption on the monopolist's beliefs which, if not made, would make the analysis quite uninteresting in that there could be a kink in the expected-demand curve in which case it is of course known that there could be fixed prices. Since in most of the following p_0 will be fixed we shall suppress it as argument in the functions $\bar{y}(\cdot)$, $f(\cdot)$, etc. whenever this can be done without causing confusion.

Regarding the firm's technology we assume zero fixed cost and constant marginal cost c, so its profit is $\pi = (p - c)y$, if (p, y) is an actual price–quantity combination. We also write $\bar{\pi}(p) = (p - c)\bar{y}(p)$ for the expected profit in dependence of p. Moreover, we will use the notation $\pi_0 = (p_0 - c)y_0$ for the status quo profit.

Now let $V(\pi)$ denote the monopolist's von Neumann–Morgenstern

6 For $\mu, \nu \in \Delta(\mathbb{R}_+)$, $\mu \to \nu$ with respect to the topology of weak convergence iff, for every continuous and bounded function $f\colon \mathbb{R}_+ \to \mathbb{R}$, $\int f \, d\mu \to \int f \, d\nu$.

utility function of profit. We assume that it is real analytic and strictly increasing. V gives rise to the expected-utility function $U(p|p_0) = \int V((p-c)y)f(y,p)\,\mathrm{d}y$. Consider next the function $W(p) = V(\bar{\pi}(p))$. By assumption it is differentiable and it coincides with the expected-utility function in case the firm is risk neutral. In case it is risk averse $W(p) \geq U(p|p_0)$ for all p whereas $W(p_0) = V(\pi_0) = U(p_0|p_0)$ since for $p = p_0$ there is no uncertainty (Fig. 1).[7] From this it is obvious that $U'^{-}(p_0) := \lim_{p\to p_0^-} U'(p|p_0) \geq \lim_{p\to p_0^+} U'(p|p_0) =: U'^{+}(p_0)$. Moreover, the firm's optimal uncertainty price $p^{\mathrm{u}} = \arg\max_p U(p|p_0)$ will differ from its status quo price p_0 as long as W and $U(\cdot|p_0)$ are tangent at $p = p_0$ and p_0 is different from $p^* = \arg\max_p W(p)$, the price that maximizes expected profit (and expected utility in case of risk neutrality). Therefore, to obtain fixed prices, it is decisive to characterize the circumstances under which the function $U(\cdot|p_0)$ has a kink at $p = p_0$. Whether $U'^{-}(p_0)$ and $U'^{+}(p_0)$ differ from $W'(p_0)$ will turn out to be related to the behavior at $p = p_0$ of the function $p \mapsto \operatorname{var}\tilde{y}(p|p_0)$, where $\tilde{y}(p|p_0)$ indicates the random variable having distribution $\mu(p|p_0)$ and $\operatorname{var}\tilde{y}(p|p_0)$ as its variance. Defining $\operatorname{var}'^{-}(p_0) = \lim_{p\to p_0^-} \mathrm{d}\operatorname{var}\tilde{y}(p|p_0)/\mathrm{d}p$ and $\operatorname{var}'^{+}(p_0) = \lim_{p\to p_0^+} \mathrm{d}\operatorname{var}\tilde{y}(p|p_0)/\mathrm{d}p$, the following result is obtained the proof of which is given in the appendix.

Proposition 1: Assume A1 and A2. Then $U'^{-}(p_0) > W'(p_0)$ iff the firm is risk averse and $\operatorname{var}'^{-}(p_0) < 0$; similarly, $U'^{+}(p_0) < W'(p_0)$ iff the firm is risk averse and $\operatorname{var}'^{+}(p_0) > 0$.

A corollary of Proposition 1 is that the expected-utility function $U(\cdot|p_0)$ has a kink at $p = p_0$ if, and only if, the firm is risk averse and the function $\operatorname{var}\tilde{y}(\cdot|p_0)$ is kinked at $p = p_0$ (Fig. 2).

To understand this result intuitively, consider the special case where the von Neumann–Morgenstern utility V and/or the distributions $\mu(p|p_0)$ are such that the expected utility can be expressed as a function of $\bar{\pi}(p)$ and $\operatorname{var}\tilde{\pi}(p)$ only, i.e., $U(p|p_0) = T(\bar{\pi}(p), \operatorname{var}\tilde{\pi}(p))$, where $\tilde{\pi}(p) = (p-c)\tilde{y}(p)$. If T is differentiable in its arguments $\bar{\pi}$ and

7 We work here with expected rather than with rank-dependent or other nonexpected utility because the former is still the most commonly known and used approach among economists for dealing with decisions under uncertainty. This implies, however, that risk aversion is entirely modeled through decreasing marginal von Neumann–Morgenstern utility. Rank-dependent utility, on the other hand, allows to separate probabilistic risk attitudes from marginal utility (see, e.g., Wakker, 1994).

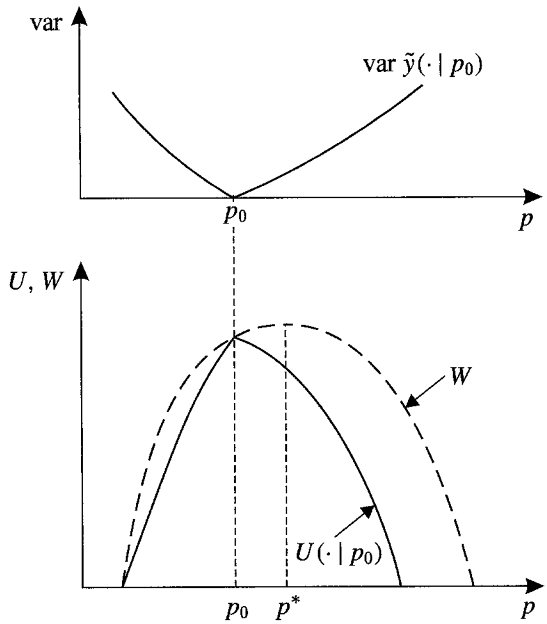

Fig. 2

$\operatorname{var}\tilde{\pi}$, then it is clear that a kink occurs in $U(\cdot|p_0)$ only if it occurs in $\operatorname{var}\tilde{\pi}(p) = (p-c)^2 \operatorname{var}\tilde{y}(p|p_0)$ since $\bar{\pi}(p) = (p-c)\bar{y}(p)$ is by assumption differentiable.

3 Endogenous Fixprices

We now discuss the relevance of the above result for the existence of fixed prices. To this end consider the assumption

A3. The optimal price under risk neutrality p^* exists, is unique, and is independent of the status quo price p_0.

Although it is quite easy to conceive of reasonable cases in which p^* does not exist (e.g., isoelastic expected demand with elasticity smaller, in absolute value, than one), it is better not to tackle that problem here.[8] To assume that p^* is unique and not depending on p_0 will facilitate

8 For a systematic treatment, see Weinrich (1997).

the argument; the latter is for example fulfilled whenever expectations are unbiased, i.e., $\bar{y}(p|p_0) = y(p)$ for any p and p_0 where $y(p)$ is an underlying true (but unknown) demand function.

Consider now the case that the variance function $\operatorname{var} \tilde{y}(\cdot|p_0)$ is kinked at $p = p_0$ and more precisely that $\operatorname{var}'^-(p_0) < 0$ and $\operatorname{var}'^+(p_0) > 0$ for all p_0 in a neighborhood of p^*. Then, since $W'(p^*) = 0$, Proposition 1 not only implies that $U'^-(p^*) > 0 > U'^+(p^*)$ but also that there exists a neighborhood N of p^* such that $U'^-(p_0) > 0 > U'^+(p_0)$ for all $p_0 \in N$, provided the following holds:

A4. $U'^-(p_0) \to U'^-(p^*) \in \bar{\mathbb{R}}$ and $U'^+(p_0) \to U'^+(p^*) \in \bar{\mathbb{R}}$ when $p_0 \to p^*$.

But then, whenever $p_0 \in N$, it is optimal for the risk-averse firm not to change it and thus p_0, if different from p^*, is an endogenous fixprice in the following sense:

Definition 1: A status quo price $p_0 \neq p^*$ is an endogenous fixprice if it is suboptimal in case of risk neutrality but optimal in case of risk aversion.

The following theorem summarizes our analysis.

Theorem 1: Assume A1, A2, and A3. If p_0 is an endogenous fixprice, then the variance function $\operatorname{var} \tilde{y}(\cdot|p_0)$ is kinked at $p = p_0$. Conversely, if $\operatorname{var}'^-(p_0) < 0$ and $\operatorname{var}'^+(p_0) > 0$ for all p_0 in a neighborhood of p^* and if in addition A4 holds, then there exist endogenous fixprices.

Loosely speaking, there exist endogenous fixprices if, and only if, the variance function of $\tilde{y}(\cdot|p_0)$ is kinked at $p = p_0$ for all p_0 in a neighborhood of p^*.

We illustrate the significance of this result in the following two examples.

Example 1: Assume that there is a true, but unknown, demand function of the form $y(p) = a - p$, $a > 0$. Thus any observation / status quo (p_0, y_0) meets $y_0 + p_0 = a$. Assume moreover that the firm believes that $\tilde{y}(p_0|p_0) = y_0$ with probability one and the change in y for $p \neq p_0$, $z = y - y_0$, is normally distributed with parameters $\nu(\rho)$ and $\sigma(\rho)$, where $\rho = p - p_0$. More precisely, setting $g(z; \nu, \sigma)$

$= [\sigma\sqrt{2\pi}]^{-1}\mathrm{e}^{-(z-\nu)^2/2\sigma^2}$, $\tilde{y}(p|p_0)$ has density $f(y_0+z, p_0+\rho) = g(z; \nu(\rho), \sigma(\rho))$. Then $\bar{y}(p_0+\rho) = y_0+\nu(\rho)$ and $\operatorname{var}\tilde{y}(p_0+\rho|p_0) = \sigma^2(\rho)$.

Now specify the functions ν and σ to be $\nu(\rho) = -\rho$ and $\sigma(\rho) = \beta|\rho|^\gamma$, with β and γ given positive parameters. Then $\bar{y}(p_0+\rho) = y_0-\rho = a-p$, $p^* = (a+c)/2$ is unique and independent of p_0, $\operatorname{var}\tilde{y}(p_0+\rho| p_0) = \beta^2|\rho|^{2\gamma}$, and $\mu(p|p_0) \to \delta(y_0)$ weakly for $p \to p_0$. The absolute value of the derivative of $\operatorname{var}\tilde{y}(p_0+\rho|p_0)$ is $2\beta^2\gamma|\rho|^{2\gamma-1}$ and tends to zero for $p \to p_0$ iff $\gamma > 1/2$. For $\gamma \leq 1/2$ Proposition 1 therefore implies that the expected-utility function is kinked at the status quo.

To be still more specific, assume $V(\pi) = -\mathrm{e}^{-\pi}$. Then, using the well-known result in distribution theory (see, e.g., Meyer, 1970) that

$$\int \mathrm{e}^{tx} g(x; \nu, \sigma)\, \mathrm{d}x = \mathrm{e}^{t\nu+t^2\sigma^2/2}\ , \tag{1}$$

the monopolist's expected-utility function becomes

$$\begin{aligned} U(p_0+\rho|p_0) &= -\mathrm{e}^{-(p_0+\rho-c)y_0} \int \mathrm{e}^{-(p_0+\rho-c)z} g(z; -\rho, \beta|\rho|^\gamma)\, \mathrm{d}z \\ &= -\mathrm{e}^{-\tau(\rho|p_0)} \end{aligned} \tag{2}$$

with

$$\begin{aligned} \tau(\rho|p_0) &= (p_0+\rho-c)(y_0-\rho) - \tfrac{1}{2}(p_0+\rho-c)^2\beta^2|\rho|^{2\gamma} \\ &= \bar{\pi}(p_0+\rho|p_0) - \tfrac{1}{2}\operatorname{var}\tilde{\pi}(p_0+\rho|p_0)\ . \end{aligned}$$

Then $U'(p|p_0) = \mathrm{d}U(p_0+\rho|p_0)/\mathrm{d}\rho = \mathrm{e}^{-\tau(\rho|p_0)}\tau'(\rho|p_0)$. Moreover, $\lim_{\rho\to 0}\tau(\rho|p_0) = \pi_0 = (p_0-c)\bar{y}(p_0)$, whereas

$$\begin{aligned} \tau'(\rho|p_0 &= y_0 - p_0 + c - 2\rho \\ &\quad - \beta^2(p_0+\rho-c)|\rho|^{2\gamma-1}[|\rho| + \operatorname{sgn}(\rho)\gamma(p_0+\rho-c)] \end{aligned}$$

and thus

$$\lim_{\rho\to 0^-}\tau'(\rho|p_0) = y_0 - p_0 + c + \beta^2\gamma(p_0-c)^2 \lim_{\rho\to 0^-}(-\rho)^{2\gamma-1}\ .$$

This yields

$$U'^{-}(p_0)$$
$$= e^{-(p_0-c)\bar{y}(p_0)} \times \begin{cases} \bar{y}(p_0) - p_0 + c & \text{if } \gamma > 1/2, \\ \bar{y}(p_0) - p_0 + c + \beta^2\gamma(p_0 - c)^2 & \text{if } \gamma = 1/2, \\ +\infty & \text{if } \gamma < 1/2. \end{cases}$$

From this it is clear that, for any given γ, $U'^{-}(p_0) \to U'^{-}(p^*)$ in $\bar{\mathbb{R}}$ when $p_0 \to p^*$. Analogously follows $U'^{+}(p_0) \to U'^{+}(p^*)$ when $p_0 \to p^*$; thus A4 is fulfilled. Therefore Theorem 1 implies the existence of endogenous fixprices when $\gamma \leq 1/2$. Note also that $\bar{y}(p_0) - p_0 + c \geq 0 \iff a - 2p_0 + c \geq 0 \iff p_0 \leq (a + c)/2 = p^*$.

In the above example the monopolist does not exclude that an increase (decrease) in his price results in an increase (decrease) of the quantity demanded.[9] Although this is an event not incompatible with consumer theory, it may be considered unlikely. This can be easily taken care of by, for example, assuming the parameter β small or postulating lognormal densities; the latter is done in the next example.

Example 2: We consider a similar set-up as in Example 1 but assume now that $h(z; \nu, \sigma) = [z\sigma\sqrt{2\pi}]^{-1}e^{-(\log z-\nu)^2/2\sigma^2}$ and $\tilde{y}(p|p_0)$ has density

$$f(y_0 + z, p_0 + \rho) = \begin{cases} h(|z|; \nu(\rho), \sigma(\rho)) & \text{if } \rho z < 0, \\ 0 & \text{if } \rho z \geq 0. \end{cases}$$

Then

$$\bar{y}(p_0 + \rho) = \begin{cases} y_0 - \operatorname{sgn}(\rho)e^{\nu(\rho)+\sigma^2(\rho)/2} & \text{if } \rho \neq 0, \\ y_0 & \text{if } \rho = 0, \end{cases}$$

and

$$\operatorname{var}\tilde{y}(p_0 + \rho|p_0) = e^{2\nu(\rho)}\{e^{2\sigma^2(\rho)} - e^{\sigma^2(\rho)}\} \quad \text{if } \rho \neq 0\ ,$$

9 Actually, even a negative quantity is not excluded. This could be easily overcome, however, by working with truncated normal distributions, without changing anything substantial but adding to technical complexity.

whereas $\operatorname{var}\tilde{y}(p_0|p_0) = 0$. $\nu(\cdot)$ and $\sigma(\cdot)$ are assumed to be

$$\nu(\rho) = \log|\rho| - \tfrac{1}{2}\log(|\rho|^{2\gamma-2}+1), \quad \sigma(\rho) = \sqrt{\log(|\rho|^{2\gamma-2}+1)}$$

with $\gamma > 0$. Inserting this in the above expressions for $\bar{y}$ and $\operatorname{var}\tilde{y}$ one finds $\bar{y}(p_0+\rho) = y_0 - \rho$, $\operatorname{var}\tilde{y}(p_0+\rho|p_0) = |\rho|^{2\gamma}$. Then $\mu(p|p_0) \to \delta(y_0)$ weakly for $p \to p_0$ and the derivative of $\operatorname{var}\tilde{y}(p_0+\rho|p_0)$ again tends to zero for $p \to p_0$ iff $\gamma > 1/2$. Thus the expected-utility function is kinked at the status quo iff $\gamma \leq 1/2$.

Note that in the present examples an apparently smooth specification of the firm's beliefs may result in a nonsmooth behavior of the firm due to a continuous variation of a single parameter. This does not appear to be exactly what intuition would have suggested, and it indicates that the origins of price rigidities may be quite intricate.

4 Endogenous Fixprices and Risk Aversion of Order One

We now relate the above results to the concept of risk aversion of order one which has been introduced by Segal and Spivak (1990) to explain full insurance of a risk-averse agent in presence of positive marginal loading. Expected-utility theory with the traditional notion of risk aversion has not been able to achieve this but, on the contrary, as Borch (1974, pp. 27f) has asserted, its prediction of partial insurance only is "against all observation."

To bridge the present model to Segal and Spivak's framework we define the certainty equivalent $\pi^c(\rho)$ of the random variable $\tilde{\pi}(p_0+\rho|p_0)$ by $V(\pi^c(\rho)) = U(p_0+\rho|p_0)$ and its risk premium by $R(\rho) = \bar{\pi}(\rho) - \pi^c(\rho)$ where, by slight abuse of notation, $\bar{\pi}(\rho) = \bar{\pi}(p_0+\rho|p_0)$. Then $R(0) = 0$, $R(\rho) \geq 0$ for all ρ when V is concave, and we can adapt Segal and Spivak's definition to the present context as follows.

Definition 2: A firm is risk averse of order one at p_0 if $\lim_{\rho\to 0^+} R'(\rho) > 0$ and $\lim_{\rho\to 0^-} R'(\rho) < 0$. It is risk averse of order two if $\lim_{\rho\to 0} R'(\rho) = 0$ but $\lim_{\rho\to 0^+} R''(\rho) > 0$ and $\lim_{\rho\to 0^-} R''(\rho) > 0$.

Since V is by assumption differentiable, $U(p|p_0)$ has a kink at $p = p_0$ iff $\pi^c(\rho)$ has one at $\rho = 0$. But a kink in $U(p|p_0)$ is equivalent to a kink in $\operatorname{var}\tilde{y}(p|p_0)$ whereas a kink in $\pi^c(\rho)$ is equivalent to a kink in $R(\rho)$. Thus Theorem 1 implies the following

Corollary 1: Assume A1 to A4. If the monopolist is risk averse of order one at p_0 for all p_0 in a neighborhood of p^*, then there exist endogenous fixprices. Conversely, if p_0 is an endogenous fixprice, then the monopolist is risk averse of order one at p_0.

Example 1 (continued): From $V(\pi) = -\mathrm{e}^{-\pi}$ and the definition of $\pi^c(\rho)$ it follows that $U(p_0+\rho|p_0) = -\mathrm{e}^{-\pi^c(\rho)}$, so from Eq. (2) $\pi^c = \tau$ and $R(\rho) = \frac{1}{2}(p_0 + \rho - c)^2 \operatorname{var} \tilde{y}(p_0 + \rho|p_0)$. From this it is evident that for $p_0 > c$ the monopolist is risk averse of order one iff $\operatorname{var} \tilde{y}(p|p_0)$ is kinked at $p = p_0$. This is the case when $\gamma \in [0, 1/2]$. Instead when $\gamma \in (1/2, 1]$ he is risk averse of order two.[10]

In the present context, risk aversion of order one is compatible with V being differentiable. This is at variance with what Segal and Spivak (1990, pp. 117f) find. The reason is that the random variables they consider are of the form $\tilde{x}(t) = x + t\tilde{\varepsilon}$, with $\tilde{\varepsilon}$ being independent of t. This is equivalent to saying that there hold stochastic constant returns to scale (i.e., constant distributions over the rate of return). Therefore $\operatorname{var} \tilde{x}(t) = t^2 \operatorname{var} \tilde{\varepsilon}$, which can never be kinked (so to obtain risk aversion of order one V needs to be kinked or expected utility to be abandoned). In the present model, on the other hand, $\tilde{\pi}(\rho) = \pi_0 + \tilde{\varepsilon}(\rho)$ and, although $\tilde{\varepsilon}(0) = 0$ and the distribution of $\tilde{\varepsilon}(\rho)$ is weakly continuous in ρ at $\rho = 0$, stochastic returns to scale typically are not constant. In fact, in the present case of a monopolist there is no reason why they should be.

5 Price Inertia When No Endogenous Fixprices Exist

As seen in the previous sections, risk aversion can give rise to endogenous fixprices in the sense that a risk-averse monopolist may stick to a price which is not optimal under risk neutrality. The crucial condition for this to happen is that the variance function of the perceived random variable quantity demanded has a kink at the status quo price. But even if this is not the case, risk aversion may make an important difference: although there may be no fixed prices, the adjustment of prices under risk aversion tends to be more sluggish than under risk neutrality, or in other words, risk aversion implies price inertia. To see this, we shall be

10 Note also that for $\gamma = 1/2$ maximizing $\tau(\rho)$ [and thus $U(p|p_0)$] is comparable to applying van Moeseke's (1965) truncated minimax decision criterion as done in Drèze (1979, eq. (2.7)).

more specific now in our assumptions regarding the firm's subjective probability distributions and utility function.

Let as before (p_0, y_0) denote the status quo where we suppose $p_0 \geq c$ so that the firm is not making losses. Using the specifications already introduced in Example 1, we now assume

B1. For $p \neq p_0$, $\tilde{y}(p|p_0)$ is normally distributed with density $f(y_0+z, p_0+\rho) = g(z; -\rho, |\rho|) = [|\rho|\sqrt{2\pi}]^{-1}\mathrm{e}^{-(z+\rho)^2/2|\rho|^2}$, where $z = y - y_0$ and $\rho = p - p_0$; $\mu(p_0|p_0) = \delta(y_0)$.

From this it is immediate that $\bar{y}(p) = y_0 + p_0 - p$. Moreover, from Example 1, no endogenous fixprices exist.

Regarding the monopolist's utility function for profit we assume:

B2. The monopolist's utility function is of type CARA, that is $V_\alpha(\pi) = -\mathrm{e}^{-\alpha\pi}$, when $\alpha > 0$, whereas $V_0(\pi) = \pi$.

Accordingly, he has constant absolute risk aversion α and for $\alpha \to 0$ he becomes risk neutral.

B1 and B2 give rise, when $\alpha > 0$, to the expected-utility function $EV_\alpha\pi = -\int \mathrm{e}^{-\alpha(p_0+\rho-c)(y_0+z)}g(z; -\rho, |\rho|)\,\mathrm{d}z =: U(\rho, \alpha)$ while $EV_0\pi = \bar{\pi}(\rho)$. Applying again formula (1) one finds for $\alpha > 0$, $U(\rho, \alpha) = -\mathrm{e}^{-\alpha\tau(\rho,\alpha)}$, with

$$\begin{aligned}\tau(\rho, \alpha) &= (p_0 + \rho - c)(y_0 - \rho) - (\alpha/2)(p_0 + \rho - c)^2|\rho|^2 \\ &= \bar{\pi}(\rho) - (\alpha/2)\,\mathrm{var}\,\tilde{\pi}(\rho)\ .\end{aligned} \tag{3}$$

Since $U(\cdot)$ is strictly increasing in τ, the optimal deviation from the status quo price $\rho^*(\alpha) = \arg\max_\rho U(\rho, \alpha)$ is determined by the condition $\partial\tau(\rho^*, \alpha)/\partial\rho = 0$. Note that this can be extended to the case $\alpha = 0$ because $EV_0\pi = \tau(\rho, 0)$. Thus $\tau(\rho, \alpha)$ can effectively be taken as the firm's objective function; it is well defined and continuous in α for all $\alpha \geq 0$.

Proposition 2: Assume B1, B2, and $p_0 \neq p^*$. Then (a) the absolute value of the optimal deviation from the status quo price is a decreasing function of the firm's absolute risk aversion, i.e., $\mathrm{d}|\rho^*(\alpha)|/\mathrm{d}\alpha < 0$, and (b) $\lim_{\alpha\to\infty} \rho^*(\alpha) = 0$.

Proof: a. We prove the result for the case $p^* > p_0$ only; the one for p^*

$< p_0$ is analogous. If $p^* > p_0$, then $\partial U(0,\alpha)/\partial\rho = W'(p_0) > 0$ and therefore $\rho^*(\alpha) > 0$. Thus the absolute value in (3) can be neglected and condition $\partial\tau(\rho^*,\alpha)/\partial\rho = 0$ becomes

$$y_0 + c - p_0 - 2\rho^* - \alpha\rho^*(p_0+\rho^*-c)(p_0+2\rho^*-c) = 0\ . \qquad (4)$$

This allows to calculate

$$\begin{aligned} \mathrm{d}\rho^*(\alpha)/\mathrm{d}\alpha = &-\{\rho^*(p_0+\rho^*-c)(p_0+2\rho^*-c)\}\times \\ &\times\big\{2+\alpha(p_0+\rho^*-c)(p_0+2\rho^*-c) \\ &+\alpha\rho^*[(p_0+2\rho^*-c)+2(p_0+\rho^*-c)]\big\}^{-1} \end{aligned}$$

which is negative since $p_0 - c \geq 0$ and $\rho^* > 0$.

b. We show the case $p^* > p_0$ only. Since $\partial U(0,\alpha)/\partial\rho = W'(p_0) > 0$, $\rho^*(\alpha) > 0$. We show first:

$$\text{For any } \rho > 0 \text{ there exists } \alpha > 0 \text{ such that } U(\rho,\alpha) < U(0,\alpha)\ . \quad (5)$$

Indeed, $U(\rho,\alpha) < U(0,\alpha)$ is equivalent to $\tau(\rho,\alpha) < \tau(0,\alpha)$ which means $(p_0+\rho-c)(y_0-\rho) - (\alpha/2)(p_0+\rho-c)^2\rho^2/2 < \pi_0$. For this inequality it is immediate that the expression on the left-hand side can be made smaller than the one on the right-hand side by choosing α big enough. This proves (5).

By (5) $\rho^*(\alpha)$ cannot be bounded away from 0 as $\alpha \to \infty$. Since $\mathrm{d}|\rho^*(\alpha)|/\mathrm{d}\alpha < 0$, this proves the claim. □

6 Risk Aversion and Menu Cost

The fact that the optimal deviation from the status quo price tends to zero when risk aversion goes to infinity implies that the maximum expected utility under risk aversion tends to the status quo utility, too. This suggests that small menu costs have a bigger impact on the firm's behavior when it is risk averse than when it is risk neutral.

To make this idea more precise assume now that profit is $\pi = (p-c)y - \lambda$, where $\lambda \geq 0$ is a price-adjustment cost parameter, when $p \neq p_0$, and $\pi = (p_0 - c)y_0 = \pi_0$ otherwise. Then it is easy to see that expected utility is, when $\rho \neq 0$ and $\alpha > 0$, $U(\rho,\alpha,\lambda) = -\mathrm{e}^{-\alpha[\tau(\rho,\alpha)-\lambda]}$, where τ is as in (3). When $\rho \neq 0$ and $\alpha = 0$ we set $U(\rho,0,\lambda) = \bar{\pi}(\rho) - \lambda$. In any case this is to be compared with $V_\alpha(\pi_0)$, the status quo utility. The firm will not adjust its price iff $U(\rho^*(\alpha),\alpha,\lambda) \leq V_\alpha(\pi_0)$.

Since, for $p_0 \neq p^*$, $V_\alpha(\pi_0) < U(\rho^*(\alpha), \alpha, 0) < 0$ and $U(\rho^*(\alpha), \alpha, \lambda) = U(\rho^*(\alpha), \alpha, 0)e^{\alpha\lambda}$ for $\alpha > 0$ and $\lambda \geq 0$, it follows that there exists, for any $\alpha > 0$, $\bar{\lambda}(\alpha)$ such that $U(\rho^*(\alpha), \alpha, \bar{\lambda}(\alpha)) = V_\alpha(\pi_0)$. Moreover, we can set $\bar{\lambda}(0) = (p_0 + \rho^*(0) - c)(y_0 - \rho^*(0)) - \pi_0$. Then $\bar{\lambda}(\alpha) > 0$ for all α and the firm adjusts its price iff $U(\rho^*(\alpha), \alpha, 0)e^{\alpha\lambda} > U(\rho^*(\alpha), \alpha, 0)e^{\alpha\bar{\lambda}(\alpha)}$, if $\alpha > 0$, and $(p_0 + \rho^*(0) - c)(y_0 - \rho^*(0)) > \pi_0 + \lambda$, if $\alpha = 0$. In any case this is equivalent to $\lambda < \bar{\lambda}(\alpha)$. This proves the first part of the following result.

Proposition 3: Assume B1, B2, and $p_0 \neq p^*$. Then, for any degree of risk aversion $\alpha \geq 0$ there exists $\bar{\lambda}(\alpha) > 0$ such that the monopolist adjusts his price iff his adjustment cost λ is smaller than $\bar{\lambda}(\alpha)$. Conversely, for any adjustment cost $\lambda > 0$ there exists a finite $\bar{\alpha}(\lambda) \geq 0$ such that he adjusts his price iff his degree of risk aversion α is smaller than $\bar{\alpha}(\lambda)$.

Proof: See appendix.

The significance of Proposition 3 should be straightforward. The menu-cost argument is so far the most frequently given justification for assuming nominal price stickiness, in particular in New Keynesian Economics. However, a given menu cost may be too small to prevent a firm from adjusting. In that case, admitting uncertainty, there exists a degree of risk aversion so that the monopolist sticks to his current price. In other words, with uncertainty and risk aversion much smaller menu cost are needed to obtain nominal rigidity. Indeed, according to the above result *any* positive adjustment cost does the job, provided risk aversion is sufficiently big.

To get an idea of the size of the effect of risk aversion on minimum menu cost needed, observe that, by definition of $\bar{\lambda}(\alpha)$, $e^{\alpha\bar{\lambda}(\alpha)} = V_\alpha(\pi_0)/U(\rho^*(\alpha), \alpha, 0) = -e^{-\alpha\pi_0}/-e^{-\alpha\tau(\rho^*(\alpha),\alpha)}$ which is, taking logarithms, dividing by α, and using (3), equivalent to

$$\begin{aligned} \bar{\lambda}(\alpha) &= -\pi_0 + \tau(\rho^*(\alpha), \alpha) \\ &= \bar{\pi}(\rho^*(\alpha)) - (\alpha/2)\operatorname{var}\tilde{\pi}(\rho^*(\alpha)) - \pi_0 \,. \end{aligned} \tag{6}$$

Now set

$$\Lambda(\rho, \alpha) = \bar{\pi}(\rho) - (\alpha/2)\operatorname{var}\tilde{\pi}(\rho) - \pi_0 \,. \tag{7}$$

Then $\bar{\lambda}(\alpha) = \Lambda(\rho^*(\alpha), \alpha)$. This yields

$$\bar{\lambda}'(\alpha) = \frac{\partial \Lambda}{\partial \rho}(\rho^*(\alpha), \alpha)\frac{d\rho^*}{d\alpha}(\alpha) + \frac{\partial \Lambda}{\partial \alpha}(\rho^*(\alpha), \alpha) \ .$$

But from (3) and (7) $\rho^*(\alpha) = \arg\max_\rho \tau(\rho, \alpha) = \arg\max_\rho \Lambda(\rho, \alpha)$ and so $(\partial\Lambda/\partial\rho)(\rho^*(\alpha), \alpha) = 0$. This implies

$$\bar{\lambda}'(\alpha) = -\tfrac{1}{2} \operatorname{var} \tilde{\pi}(\rho^*(\alpha)) \ . \tag{8}$$

Whenever $p_0 \neq p^*$, $\rho^*(\alpha) \neq 0$ for all α, and thus from (8) $\bar{\lambda}'(\alpha) < 0$ for all α. Since this holds in particular for $\alpha = 0$, it means that there is always a first-order effect of abandoning risk neutrality in favor of risk aversion on the minimum menu cost required for fixed prices.

Example 3: Assume $c = 1$, $p_0 = 2$, and $y_0 = 3$. Then $\pi_0 = 3$ and (3) yields

$$\bar{\pi}(\rho) = (1+\rho)(3-\rho) \Rightarrow \bar{\pi}'(\rho) = 2(1-\rho) = 0 \Rightarrow \rho^*(0) = 1 \ .$$

Thus the optimal price in case of risk neutrality is $p^* = 3$ and maximum profit is $\bar{\pi}(1) = 4$. From this and (6) follows $\bar{\lambda}(0) = 1$. Moreover, from (3) and $\operatorname{var}\tilde{\pi}(\rho) = (1+\rho)^2|\rho|^2$ we can derive the optimality condition for $\rho^*(\alpha)$, $(\partial\tau/\partial\rho)(\rho, \alpha) = 2(1-\rho) - \alpha\rho(1+\rho)(1+2\rho) = 0$. From this and (6) one calculates for example $\rho^*(1) \approx 0.429$, $\bar{\lambda}(1) \approx 0.486$, $\rho^*(2) \approx 0.317$, $\bar{\lambda}(2) \approx 0.359$, $\rho^*(10) \approx 0.125$, and $\bar{\lambda}(10) \approx 0.135$. The graph of the function $\bar{\lambda}(\cdot)$ is shown in Fig. 3. From this it is evident that even small degrees of risk aversion substantially reduce minimum menu cost needed.

7 Concluding Remarks

In this paper we have shown that risk aversion of firms may be a crucial element in explaining price rigidity. We have identified circumstances under which a risk-averse monopolist sticks to his current price although an otherwise identical risk-neutral firm would change it. Such prices, which we have called endogenous fixprices, exist if the monopolist's perceived variance of quantity demanded displays a kink at any status quo price sufficiently close to the optimal price under risk neutrality. Equivalently, it means that he is risk averse of order one. The examples given suggest that this may occur quite easily. Moreover,

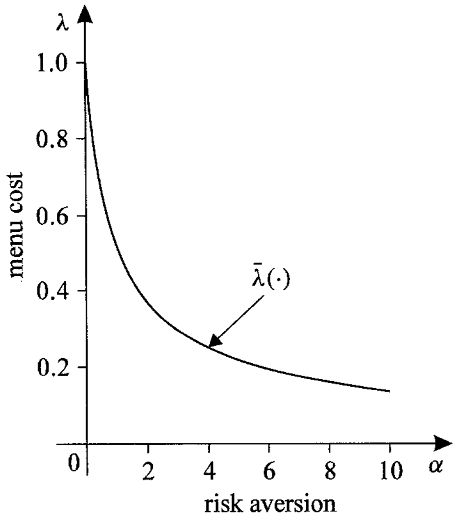

Fig. 3

since kinks in the variance function are compatible with weak convergence of the probability distributions involved, the expected-demand function may be perfectly smooth.

This result can also be seen as an extension of Sweezy's (1939) on the existence of fixed prices in a kinky demand-curve model: under uncertainty with risk aversion, to obtain fixed prices there is no need for a kink in the demand curve itself but only in the variance function arising from the firm's subjective-probability distributions.

In the second part of the paper we have adopted a scenario which excludes the existence of endogenous fixprices so as to show that even in that case risk aversion makes a difference relative to risk neutrality. In fact, under our assumptions, risk aversion not only implies that the size of price adjustment is smaller than under risk neutrality but in the limit, as risk aversion tends to infinity, price adjustment tends to zero.

The introduction of menu cost in this context plays a role similar to that of the kink in the variance function: firms risk averse enough stick to their status quo price whereas risk-neutral firms abandon it. This may at first glance appear surprising because it conflicts with the intuition from the well-known local risk-neutrality theorem (Samuelson, 1961; Arrow, 1965) according to which a risk-averse decision maker behaves locally, i.e., for small amounts at risk, as if he were risk neutral. In the present context this would require that a risk-averse monopolist adjust his price whenever a risk-neutral one does, albeit by a smaller amount.

The reason for the invalidity of that result in the present model is of course the presence of menu cost or of a kink in the variance function. Both factors are in contrast to stochastic constant returns to scale, which is one of the hypotheses on which the local risk-neutrality theorem builds. In the present model of a monopolist there are no reasons for stochastic constant rates of return to hold.

Regarding the assumption that there be no uncertainty at the status quo, this is no substantial limitation of generality. In fact, it would be easy to generalize the model to the case where there is uncertainty also at the status quo: the only change would be to have the variance at the status quo positive instead of zero. As long as it takes on a minimum there, to obtain endogenous fixprices it would still be necessary and sufficient that the variance function be kinked at the status quo.

Apart from aiming to contribute to the explanation of price stickiness per se, this paper's results may be linked to research going on in other areas, like Keynesian-type macroeconomics, and, in particular, to New Keynesian Economics. There, the assumption of monopolistically competitive industries is combined with price rigidities to produce real effects of nominal money shocks. The rigidities are mostly rationalized through the assumption of menu cost. The present results suggest that risk aversion and kinked variance functions – or, equivalently, risk aversion of order one – can play a similar role. More importantly, according to our result in Sect. 6, the relevance of the menu-cost argument is revalorized through the presence of risk aversion because menu cost too small to matter to risk-neutral firms may well have an impact on risk-averse ones. As we have illustrated in a numerical example, the difference may be substantial.

Moreover, in order that nominal money shocks have a real effect it is sufficient that a nonnegligible part of the industry's firms keep their prices unchanged. But this may in fact happen as different firms may have different attitudes towards risk, even if they share the same technology, because they are directed by managers with possibly different degrees of risk aversion. The above result then establishes that there is, for any arbitrarily small menu cost, a threshold degree of risk aversion which separates the flexprice firms from their fixprice companions. Thus for nominal money shocks to be effective it is sufficient that some firms' degrees of risk aversion reach the threshold level.

A further aspect of this result is that, although it has been obtained under the assumption of utility functions that display constant absolute risk aversion, it is likely that in reality the degree of risk aversion is influenced by the firm's current and expected profits and thus by the general business climate: with decreasing absolute risk aversion entrepreneurs tend to be more risk averse in recessions than in booms.

Thus effective demand management should work better in recessions and particularly well in depressions – a thesis already advanced by Keynes himself. The formalization of this conjecture could be the subject of further research.

The changes in economic conditions that induce a firm to adjust its price are not an explicit part of the present model. Although we are all the same convinced that its results are significant enough to support the view that risk aversion contributes to price stickiness, we also think that further work should be directed to reformulate the setting so as to explicitly comprise factors like inflation and other prices. In particular it should then be interesting to show that, in presence of menu costs, risk aversion leads to less frequent price adjustment than risk neutrality also in an explicitly dynamic model, as strongly suggested by the present results.

Appendix

Proof of Proposition 1

Since the variance of $\tilde{y}(p|p_0)$ is $\operatorname{var}\tilde{y}(p|p_0) = \int y^2 f(y,p)\,\mathrm{d}y - (\int y f(y,p)\,\mathrm{d}y)^2$, its derivative, for $p \neq p_0$, is

$$\frac{\mathrm{d}\operatorname{var}\tilde{y}(p|p_0)}{\mathrm{d}p} = \int [y^2 - 2\bar{y}(p)y]\frac{\partial f}{\partial p}(y,p)\,\mathrm{d}y\ .$$

Taking limits as $p \to p_0^-$, this yields

$$\operatorname{var}'^-(p_0) = \int (y^2 - 2y_0 y)\frac{\partial f^-}{\partial p}(y,p_0)\,\mathrm{d}y\ ,$$

where $(\partial f^-/\partial p)(y,p_0) = \lim_{p\to p_0^-}(\partial f/\partial p)(y,p)$. Observing that $\int f(y,p)\,\mathrm{d}y = 1$ for all p implies $y_0^2 \int (\partial f^-/\partial p)(y,p_0)\,\mathrm{d}y = 0$ one obtains $\operatorname{var}'^-(p_0) = \int (y^2 - 2y_0 y + y_0^2)(\partial f^-/\partial p)(y,p_0)\,\mathrm{d}y$ which can be written

$$\operatorname{var}'^-(p_0) = \int (y - y_0)^2 \frac{\partial f^-}{\partial p}(y,p_0)\,\mathrm{d}y\ . \qquad (9)$$

Since $\mu(p|p_0) \to \delta(y_0)$ weakly and f is continuous in y, $f(y,p) \to 0$ for $p \to p_0$ whenever $y \neq y_0$. Therefore, and since $f(y,p) \geq 0$ for all y and p, $(\partial f^-/\partial p)(y,p_0) \leq 0$ for all $y \neq y_0$.

Next consider the derivative of the expected-utility function for $p \neq p_0$,

$$U'(p|p_0) = \int V'((p-c)y)yf(y,p)\,\mathrm{d}y + \int V((p-c)y)\frac{\partial f}{\partial p}(y,p)\,\mathrm{d}y\ .$$

Since $\mu(p|p_0) \to \delta(y_0)$ weakly for $p \to p_0$, the first integral tends to $V'(\pi_0)y_0$. Therefore

$$U'^{-}(p_0) = V'(\pi_0)y_0 + \int V((p_0-c)y)\frac{\partial f^{-}}{\partial p}(y,p_0)\,\mathrm{d}y\ .$$

The function $W(p) = V(\bar{\pi}(p))$ gives rise to

$$W'(p_0) = V'(\pi_0)y_0 + V'(\pi_0)(p_0-c)\bar{y}'(p_0)\ .$$

This entails

$$\begin{aligned}
&U'^{-}(p_0) - W'(p_0)\\
&= \int [V((p_0-c)y) - V'(\pi_0)(p_0-c)y]\frac{\partial f^{-}}{\partial p}(y,p_0)\,\mathrm{d}y\\
&= \int \Big[V(\pi_0) - V'(\pi_0)\pi_0\\
&\quad + \sum_{n=2}^{\infty} V^{(n)}(\pi_0)\frac{[(p_0-c)y-\pi_0]^n}{n!}\Big]\frac{\partial f^{-}}{\partial p}(y,p_0)\,\mathrm{d}y\ ,
\end{aligned}$$

which yields

$$\begin{aligned}
&U'^{-}(p_0) - W'(p_0)\\
&= \int \Big[\sum_{n=2}^{\infty} V^{(n)}(\pi_0)\frac{[(p_0-c)y-\pi_0]^n}{n!}\Big]\frac{\partial f^{-}}{\partial p}(y,p_0)\,\mathrm{d}y
\end{aligned} \tag{10}$$

since $\int f(y,p)\,\mathrm{d}y = 1$ for all p implies

$$[V(\pi_0) - V'(\pi_0)\pi_0]\int \frac{\partial f^{-}}{\partial p}(y,p_0)\,\mathrm{d}y = 0\ .$$

Comparing (9) and (10) one sees that, for a risk-averse firm, the integral in (9) is nil iff the one in (10) is nil. This proves the claim for $U'^{-}(p_0)$. The one for $U'^{+}(p_0)$ follows symmetrically. □

Proof of Proposition 3

We only need to prove the second part of the assertion. We show first that range $\bar{\lambda}(\cdot) = \left]0, \bar{\lambda}(0)\right]$. Since $\bar{\lambda}$ is continuous and from (8) decreasing in α, it is sufficient to establish that $\lim_{\alpha\to\infty} \bar{\lambda}(\alpha) = 0$. For convenience we reproduce (6) as

$$\begin{aligned}\bar{\lambda}(\alpha) &= -\pi_0 + \tau(\rho^*(\alpha), \alpha) \\ &= \bar{\pi}(\rho^*(\alpha)) - (\alpha/2)\operatorname{var}\tilde{\pi}(\rho^*(\alpha)) - \pi_0 \,.\end{aligned} \tag{11}$$

By (3) and Proposition 2,

$$\lim_{\alpha\to\infty} \tau(\rho^*(\alpha), \alpha) = \pi_0 - \frac{(p_0 - c)^2}{2} \lim_{\alpha\to\infty} \alpha|\rho^*(\alpha)|^2 \,.$$

To determine the latter limit, use (4) to derive

$$\alpha\rho^*(\alpha) = (y_0 + c - p_0 - 2\rho^*(\alpha))/(p_0 + \rho^*(\alpha) - c)(p_0 + 2\rho^*(\alpha) - c) \,,$$

which implies $\lim_{\alpha\to\infty} \alpha\rho^*(\alpha) = (y_0 + c - p_0)/(p_0 - c)^2 =: k \in \mathbb{R}$. Accordingly, $\lim_{\alpha\to\infty} \alpha|\rho^*(\alpha)|^2 = \lim_{\alpha\to\infty} |\rho^*(\alpha)| \cdot |k| = 0$ by Proposition 2, and hence $\lim_{\alpha\to\infty} \tau(\rho^*(\alpha), \alpha) = \pi_0$. But this yields $\lim_{\alpha\to\infty} \bar{\lambda}(\alpha) = 0$. Therefore, for any $\lambda > 0$ smaller than $\bar{\lambda}(0)$ there exists $\bar{\alpha}(\lambda) > 0$ such that

$$U(\rho^*(\bar{\alpha}(\lambda)), \bar{\alpha}(\lambda), \lambda) = V_{\bar{\alpha}(\lambda)}(\pi_0) \,. \tag{12}$$

Next consider the derivative of $U(\rho^*(\alpha), \alpha, \lambda) - V_\alpha(\pi_0) =: G(\alpha)$ with respect to $\alpha > 0$. We will show that it is negative at $\alpha = \bar{\alpha}(\lambda)$ which will imply the claim.

Since $V_\alpha(\pi_0) = -\mathrm{e}^{-\alpha\pi_0}$, $G'(\alpha)$ is

$$\frac{\partial U}{\partial \rho}(\rho^*(\alpha), \alpha, \lambda)\rho^{*\prime}(\alpha) + \frac{\partial U}{\partial \alpha}(\rho^*(\alpha), \alpha, \lambda) + V_\alpha(\pi_0)\pi_0 \,.$$

Recalling that $(\partial U/\partial \rho)(\rho^*(\alpha), \alpha, \lambda) = 0$, this becomes

$$U(\rho^*(\alpha), \alpha, \lambda)\Big[-\tau(\rho^*(\alpha), \alpha) + \lambda - \alpha\frac{\partial \tau}{\partial \alpha}(\rho^*(\alpha), \alpha)\Big] + V_\alpha(\pi_0)\pi_0 \,.$$

Using (12), this yields

$$G'(\bar{\alpha}(\lambda)) = U(\rho^*(\bar{\alpha}(\lambda)), \bar{\alpha}(\lambda), \lambda)\Big[-\tau(\rho^*(\bar{\alpha}(\lambda)), \bar{\alpha}(\lambda)) + \lambda$$
$$- \alpha\frac{\partial\tau}{\partial\alpha}(\rho^*(\bar{\alpha}(\lambda)), \bar{\alpha}(\lambda)) + \pi_0\Big] ,$$

which by (11) becomes

$$\begin{aligned} G'(\bar{\alpha}(\lambda)) &= U(\rho^*(\bar{\alpha}(\lambda)), \bar{\alpha}(\lambda), \lambda)\Big[-\alpha\frac{\partial\tau}{\partial\alpha}(\rho^*(\bar{\alpha}(\lambda)), \bar{\alpha}(\lambda))\Big] \\ &= U(\rho^*(\bar{\alpha}(\lambda)), \bar{\alpha}(\lambda), \lambda)\frac{\bar{\alpha}(\lambda)}{2}\operatorname{var}\tilde{\pi}(\rho^*(\bar{\alpha}(\lambda))) , \end{aligned}$$

where we have used (3). Since $U(\cdot)$ is negative, this proves the claim. □

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