A Way Out of the Euro Crisis: Fiscal Transfers Are Indispensable for Sustainability in a Union with Heterogeneous Members

Harashima, Taiji

Kanazawa Seiryo University

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Fiscal Transfers Are Indispensable for Sustainability in a Union with Heterogeneous Members

Taiji HARASHIMA*

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Abstract

This paper theoretically examines a way out of the euro crisis based on a model of inflation acceleration and differentials. The conclusion is that, unless more advantaged states (e.g., Germany) systematically transfer a necessary amount of money to less advantaged states (e.g., Greece) in every period, the euro area cannot necessarily reach equilibrium where all heterogeneous states achieve optimality. In this case, fiscal transfers are not a tool of risk-sharing or a buffer against asymmetric shocks; rather, they are indispensable for escaping from indefinite disparity acceleration within a union consisting of heterogeneous member states. Such fiscal transfers should not be viewed as alms for the less advantaged states but as a right these states should justly assert. The model indicates that the lack of a fiscal transfer mechanism inevitably generates inflation differentials and huge current account imbalances among member states. As a result, although relatively more advantaged member states obtain "extra" benefits from the euro, less advantaged member states eventually lose most of their capital ownership and their economies are devastated.

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Keywords: The euro; Monetary union; Inflation; Inflation differential; Current account imbalance; Fiscal transfer; Time preference

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*Correspondence: Taiji HARASHIMA, Kanazawa Seiryo University, 10-1 Goshomachi-Ushi, Kanazawa-shi, Ishikawa, 920-8620, Japan.
Email: harashim@seiryo-u.ac.jp or t-harashima@mve.biglobe.ne.jp.

1 INTRODUCTION

The euro crisis has been ongoing for several years and appears to be worsening (e.g., Obstfeld, 2013; Feldstein, 2015). The crisis began with persistent inflation differentials and current account imbalances among member states (ECB, 2003, 2007, 2008b; Gros et al., 2005; Angeloni and Ehrmann, 2007; de Grauwe, 2009; Decressin and Stavrev, 2009; EC, 2009; Fendel and Frenkel, 2009; Holinskia et al., 2010; Jaumotte and Sodsriwiboon, 2010; Gregoriou et al., 2011). The mechanism that causes these phenomena was examined by Harashima (2011). In this paper, I examine a way out of the crisis based on the model proposed in Harashima (2011).

Since its creation, the euro has been criticized for lacking a unified fiscal authority (e.g., ECB, 2008a). Harashima (2011) shows the mechanism for why the lack of a unified fiscal authority causes crises. Harashima's model shows that inflation accelerates if the time preference rate of the government is higher than that of the representative household. To stabilize inflation, therefore, the government's time preference rate needs to be controlled by delegating monetary policies to an independent central bank. The model in this paper indicates that if there is more than one national government but only one central bank (as is the case with the euro area), the central bank cannot sufficiently control the time preference rate of each member government. Thus, inflation differentials can be generated, and accordingly, current account imbalances are widened and fiscal balances are governed by complex non-linear processes.

The necessity of fiscal transfer has been argued by many researchers, because fiscal transfers are a tool of risk-sharing and a buffer against asymmetric shocks among member states (e.g., Kenen, 1969; Sala-i-Martin and Sachs, 1992; Kletzer and von Hagen, 2001). Although this reasoning is correct, it is a weak incentive in the euro case when trying to persuade more advantaged states to introduce a transfer mechanism. The more advantaged states will maintain that asymmetric shocks can be dealt with by many other measures and that the moral hazard generated as a by-product of the transfer is a much more serious problem. However, the model presented in this paper indicates that the transfer mechanism is indispensable for the euro area economy to reach equilibrium where all heterogeneous states achieve optimality. That is, without the transfer mechanism, the degree of disparity among heterogeneous member states accelerates indefinitely. Therefore, the fiscal transfers are not alms for the less advantaged states but a right they should justly assert. If the euro is of great value politically, socially, culturally, and ideologically and should be maintained, then the transfer mechanism is absolutely necessary.

2 THE MODEL

The model used in this paper is the same as that used by Harashima (2011). The details of the model are shown in the Appendix. The model indicates that because each member state behaves on the basis of its own intrinsic preferences, the law of motion for inflation in member state ρ becomes

$$\theta_{G,\rho} = \left[\frac{\varpi \alpha \sum_{q=1}^{N} \omega_q}{N \mu v (1-\alpha)} \right]^{\alpha} + \int_{t-1}^{t} \int_{s}^{s+1} \pi_{\rho,\upsilon} \, d\upsilon \, ds - \pi_{\rho,t} \quad ,$$

where $\theta_{G,\rho}$ is the time preference rate of the government of member state ρ, $\pi_{\rho,t}$ is the rate of inflation in member state ρ at time t, N is the number of member states, v, α, μ, ω_ρ, and ϖ are parameters, and $t-1<s\leq t+1$. Therefore, if

$$\theta_{G,\rho}=\theta_G=\left[\frac{\varpi\alpha\sum_{q=1}^{N}\omega_q}{N\mu v(1-\alpha)}\right]^{\alpha}$$

for any ρ, inflation does not accelerate in the monetary union (i.e., $\dot{\pi}_t=0$ where π_t is the rate of inflation in the euro area overall), but otherwise inflation differentials are generated.

3 THE EURO'S FLAW

In this section, I show the consequences if the essential euro scheme is unchanged; that is, I examine the mechanism by which inflation differentials and considerable current account imbalances are generated and the reasons why the euro fell into a crisis.

3.1 The basic structure

Suppose a euro-like monetary union consisting of N member states that are identical except for time preference. The time preference rate of the representative household in member state ρ is $\theta_{P,\rho}$. The integrated time preference rate of the representative household of all member states is $N^{-1}\sum_{q=1}^{N}\theta_{P,q}=\theta_P$, where $\theta_{G,\rho,t}$ is the actual time preference rate of the government of member state ρ in period t, and the rate is time variable because of control by the central bank of the monetary union. The intrinsic time preference rate of the government of member state ρ is $\bar{\theta}_{G,\rho}$ and $\bar{\theta}_{G,\rho}>\theta_{P,\rho}$. The integrated actual time preference rate of all member governments in period t is $N^{-1}\sum_{q=1}^{N}\theta_{G,\rho,t}=\theta_{G,t}$. Hence, the central bank of the monetary union needs to set $\theta_{G,t}$ equal to θ_P to not accelerate the overall rate of inflation in the area of the union. Because $\theta_{G,t}=\theta_P$ is kept by the central bank of the monetary union, each government is expected to adjust its time preference rate $\theta_{G,\rho,t}$ equal to θ_P. If a government does not sufficiently adjust its $\theta_{G,\rho,t}$ and sets $\theta_{G,\rho,t}>\theta_P$, it deviates from the expected behavior; such behavior is called a "deviation" hereafter.

3.2 Factors that generate the euro's flaw

3.2.1 Adherence to own preferences

The law of motion for inflation shown in equation (A20) in the Appendix indicates that inflation does not accelerate because a government acts in a stupid, foolish, or irrational manner, but rather because it behaves quite normally—it adheres to its intrinsic time preference unless an independent neutral institution (i.e., a central bank) forces it to stop doing so. However, a fundamental question arises. Even if the government is acting quite normally, is this behavior rational? In economics, rationality usually means that, given the available information, optimal decisions to achieve an objective are taken and rational behavior is generally assumed. However, can rational behavior still prevail when a government cannot optimize its behavior to achieve its objective? This special situation emerges if the central bank is perfectly independent and is firmly determined to stabilize inflation and if, at the same time, the intrinsic time preference rate of government is unchangeable. In this situation, the economy will destabilize and eventually collapse, as shown in Section A1 in the Appendix. Therefore, the government cannot achieve its objective (i.e., cannot maximize its expected utility) and can only behave irrationally in this case. Conversely, if the government wants to optimize its objective and behave rationally, it must change its time preference. Clearly, trade-offs between rationality and time preference exist in some situations, and either rationality or time preference must be endogenized.

Nevertheless, it is highly unlikely that people will not optimize their behavior to meet their objectives (i.e., maximize utility) if they have complete knowledge of the optimal path. Hence, rationality should prevail over preferences, and time preference should be endogenized when a clash between rationality and time preference occurs. If time preference is endogenized, rational decisions become possible.

Even though rationality should eventually prevail over preferences, governments will not easily change their own preferences. They will resist endogenizing them and search for options to avoid doing so—it is this stubborn nature that drives governments to deviate from the path specified by the central bank of the monetary union. Section A1 in the Appendix indicates that the inflation problem is equivalent to the deviation problem. The mechanism of inflation differentials in the euro area, therefore, must be fully examined considering the driving force of deviation.

Even though unfavorable consequences are expected if no change is made in this type of situation, it can be very difficult to change one's own preferences. Controlling preferences therefore usually requires the help of other people or institutions, which is one of the reasons why independent central banks were established to stabilize inflation. Nevertheless, as will be examined in the following sections, the question arises as to whether the central bank of the monetary union can fully control each member government's desire to adhere to its own time preference rate.

3.2.2 The limited capability of the central bank

The central bank of a monetary union (e.g., the European Central Bank or ECB) faces a problem that most other central banks do not face. There are an infinite number of combinations of $\theta_{G,\rho,t}$ that satisfy $\theta_{G,t} = N^{-1}\sum_{q=1}^{N}\theta_{G,\rho,t} = \theta_P$, but the central bank of the monetary union cannot force its member governments to select the combination that it wants them to select. That is, the central bank cannot separately control $\theta_{G,\rho,t}$, only $\theta_{G,t}$ collectively.

The central bank in a country that is not participating in a monetary union punishes the government's deviation by raising the nominal interest rate by ψ such that $i_t = \theta_{G,t} + \pi_t + \psi$. $\theta_{G,t} = i_t - \pi_t$ (equation [A22] in the Appendix) is not satisfied until the government obeys the central bank and lowers $\theta_{G,t}$. However, the central bank of the monetary union cannot effectively impose ψ separately on each member state; thereby, equation $\theta_{G,\rho,t} = i_{\rho,t} - \pi_{\rho,t}$ can be satisfied in a member state even though $\theta_{G,\rho,t} > \theta_{G,t} = \theta_P$. Even if a government behaves on the basis of its own $\bar{\theta}_{G,\rho}$ that is different from θ_P, the central bank of the monetary union can neither punish nor force the government to transition to $\theta_{G,\rho,t} = \theta_{G,t} = \theta_P$. As a result, the combination of $\theta_{G,\rho,t}$ is not selected only by the central bank of the monetary union but rather through conflict, negotiation, and cooperation among the member governments. Thereby, the possibility exists that, at the same time, $\theta_{G,\rho,t} > \theta_P$ for some member states and $\theta_{G,\rho,t} < \theta_P$ for others, whereas $\theta_{G,t} = \theta_P$ is maintained. Unlike the case for most central banks, independence by itself is not sufficient for a central bank of a monetary union to fully stabilize inflation.

3.2.3 Inflation differentials

Because all member states use the same currency, the price level would be identical across the union area by arbitrage if all goods and services were traded freely inside the union area. However, not all goods and services are tradable. If anything, the share of non-tradable goods and services in the area is large (e.g., Altissimo et al., 2005). Unlike the prices of tradable goods and services, those of non-tradable goods and services are not equalized by arbitrage. This price heterogeneity indicates that the rate of inflation in each member state can also be heterogeneous, and heterogeneous inflation indicates that governments may deviate from the path the central bank of the monetary union sets, at least temporarily. Member governments may enjoy periods when they behave on the basis of their own intrinsic $\overline{\theta}_{G,\rho}$, which is higher than $\theta_{G,t} = \theta_P$.

Note that even though inflation is heterogeneous, the marginal product of capital in every industry in every member state is kept identical by arbitrage; that is, the marginal return on capital is equal to θ_P, because capital flows freely within the euro area.

Because member states can choose to adhere to their own preferences and the central bank has limited enforcement capabilities, some member states will deviate from the path the central bank of the monetary union sets. By the law of motion for inflation, inflation will temporarily accelerate in the deviating member states with relatively high rates of time preference, even though, because of the central bank's control, overall inflation in the euro area does not accelerate. Because the deviations should be temporary, inflation acceleration will be on a small scale, but non-negligible inflation differentials will be observed. If the correction of the deviation is postponed to far future periods after the deviation ends, $\theta_{G,\rho,t} = \theta_{G,t} = \theta_P$ is kept and inflation does not accelerate during the interim period. However, because the deviation is left uncorrected, the relatively high rate of inflation continues in the deviating member state by the law of motion for inflation, and the inflation differentials thereby continue during this period. These features of inflation differentials predicted by the model are basically consistent with the observed persistent inflation differentials in the euro area.

3.2.4 Current account imbalances caused by inflation differentials

Inflation differentials will lead to current account imbalances (e.g., Blanchard, 2007; Arghyrou and Chortareas, 2008; EC, 2009). Although inflation rates diverge among the member states, the prices of tradable goods and services are still generally equalized across the euro area by arbitrage. The equalization is realized by outflows of cheaper tradable goods and services from member states with lower inflation to the states with higher inflation. Inflowing goods and services eventually will need to be purchased with money from the exporting member states (those with lower inflation), because the importing states (those with higher inflation) are not obtaining money by exporting either their higher priced tradable goods or their non-tradable goods and services. A large part of borrowed money, therefore, is used not for investment but for consumption in the higher inflation member states.

As a result, the trade and current account balances in member states with higher inflation will show continuous deficits: that is, the deviating member states with relatively high rates of time preference will show continuous deficits. Accordingly, relatively less competitive firms producing tradable goods or services in the deviating member states will disappear more rapidly because of the price differentials and the inflows of foreign goods and services. Industries providing tradable goods and services will decline, and the share of non-tradable goods and services industries will increase in the deviating member states. The features of current account imbalances predicted by the model are basically consistent with the observed current account imbalances in the euro area.

3.3 Disparity of inflation acceleration among member states

Because of inflation differentials and a "fixed" exchange rate among member states, relatively more advantaged (i.e. lower time preference rate) states will accumulate more capital than they would do if the monetary union (e.g., the euro) was not created. Such "extra" accumulated capital owned by the more advantaged states is utilized in less advantaged states because the real interest rate is kept equal within the area of the monetary union. More advantaged states receive all of the returns to this "extra" capital. As a result, more advantaged states become wealthier and less advantaged states become poorer than they would do if the monetary union was not created, and the magnitude of the disparity increases as time passes.

In theory, all capital will eventually be owned by the most advantaged state. Even the second most advantaged state eventually will lose ownership of capital, and extreme wealth and income inequality among member states will be generated. In this case, only the most advantaged state is better off, and all of the other states are worse off than they would be if the monetary union did not exist. Nevertheless, in the initial period after the creation of the monetary union, all states may experience an economic boom. Even so, however, the least advantaged states will eventually accumulate "extra" debts, and this process will continue successively until even the more advantaged states also begin to suffer "extra" debts. The magnitude of the suffering will be worst in the least advantaged states.

Although only the most advantaged state ultimately benefits from the union, most of relatively more advantaged states will enjoy benefits for quite some time after the creation of the monetary union. Therefore, in the early periods, most of the relatively more advantaged states will support the assertion that a transfer mechanism is an unwholesome, immoral, and unjustifiable concept. As time passes, however, and the crisis spreads across these relatively more advantaged states, even they may begin to oppose to that assertion.

4 A WAY OUT

4.1 The transfer mechanism

4.1.1 The two state transfer mechanism

The model provides us with a way to avoid the crises. It lies in the generation mechanism of inflation differentials. Equations (A16) and (A17) in the Appendix indicate that inflation differentials are generated by the term

$$\int_{t-1}^{t}\int_{s}^{s+1}\pi_{\upsilon}\, d\upsilon\ ds - \pi_t \quad .$$

If this term is eliminated from equations (A16) and (A17), inflation differentials are eliminated. The origin of this term lies in equation (A3) in the Appendix—that is, the budget constraint of the government. If additional revenue is added to the budget constraint, however, that term can also be eliminated. One example of additional revenue is the transfer of funds from relatively advantaged states to less advantaged states.

By using equation (A3), the government's budget constraint in member state ρ can be described as

$$\dot{b}_{\rho,t} = b_{\rho,t}\left(i_{\rho,t} - \pi_{\rho,t}\right) + g_{\rho,t} - x_{\rho,t} - \varphi_{\rho,t} \quad ,$$

where $b_{\rho,t}$, $i_{\rho,t}$, $g_{\rho,t}$, $x_{\rho,t}$, and $\varphi_{\rho,t}$ are the real obligation of government ρ to pay for its accumulated bonds and the real interest rate for government bonds, the real government expenditure, the real tax revenue, and the real amount of seigniorage, respectively, of member state ρ at time t. The real rate of interest r is common in member states at steady state. Suppose that there is no inflation acceleration and deceleration in the overall union area (i.e., $\theta_G = \theta_P$ and the rate of inflation is kept constant) because the central bank of the monetary union appropriately implements monetary policies. Suppose also, for simplicity, that a euro-like monetary union consists of two member states (state 1 and 2) that are identical except for their time preference rates ($\theta_{G,1} < \theta_{G,2}$ and $\theta_{P,1} < \theta_{P,2}$), and the central bank of the monetary union controls $\theta_{G,\rho,t}$ so as to achieve $\frac{\theta_{G,1} + \theta_{G,2}}{2} = \theta_G = \theta_P$.

Suppose also that there is a transfer mechanism such that the government of state 1 transfers an amount of money equivalent to $b_{2,t}\left(\int_{t-1}^{t}\int_{s}^{s+1} \tilde{\pi}_{2,\upsilon}\, d\upsilon\ ds - \tilde{\pi}_{2,t}\right)$ to the government of state 2 in any period where $\tilde{\pi}_{\rho,t}$ is the expected inflation in state ρ when the government of state ρ does not receive or provide transfers from or to other member states. Hence, $\int_{t-1}^{t}\int_{s}^{s+1} \tilde{\pi}_{2,\upsilon}\, d\upsilon\ ds - \tilde{\pi}_{2,t}$ is an exogenous variable, and $\tilde{\pi}_{1,t} < \pi_t$ and $\tilde{\pi}_{2,t} > \pi_t$.

The budget constraint of state 2's government is therefore

$$\dot{b}_{2,t} = b_{2,t}\left(i_{2,t} - \tilde{\pi}_{2,t}\right) - b_{2,t}\left(\int_{t-1}^{t}\int_{s}^{s+1} \tilde{\pi}_{2,\upsilon}\, d\upsilon\ ds - \tilde{\pi}_{2,t}\right) + g_{2,t} - x_{2,t} - \varphi_{2,t} \quad .$$

Because, by equation (A2) in the Appendix,

$$i_{2,t} = \int_{t-1}^{t}\int_{s}^{s+1} \pi_{2,\upsilon}\, d\upsilon\, ds + r \quad ,$$

the optimality conditions of the government of state 2 are, by equation (A16),

$$-g_{2,t}\left[\frac{\partial u_{G,2}\left(g_{2,t}, x_{2,t}\right)}{\partial g_{2,t}}\right]^{-1} \frac{\partial^2 u_{G,2}\left(g_{2,t}, x_{2,t}\right)}{\partial g_{2,t}^2}\frac{\dot{g}_{2,t}}{g_{2,t}} + \theta_{G,2} = i_{2,t} - \pi_{2,t} - \left(\int_{t-1}^{t}\int_{s}^{s+1} \tilde{\pi}_{2,\upsilon}\, d\upsilon\ ds - \tilde{\pi}_{2,t}\right) = r_t$$

and by equation (A17),

$$-\ x_{2,t}\left[\frac{\partial u_{G,2}\left(g_{2,t}, x_{2,t}\right)}{\partial x_{2,t}}\right]^{-1} \frac{\partial^2 u_{G,2}\left(g_{2,t}, x_{2,t}\right)}{\partial x_{2,t}^{\ 2}}\frac{\dot{x}_{2,t}}{x_{2,t}} + \theta_{G,2} = i_{2,t} - \pi_{2,t} - \left(\int_{t-1}^{t}\int_{s}^{s+1} \tilde{\pi}_{2,\upsilon}\, d\upsilon\ ds - \tilde{\pi}_{2,t}\right) = r_t \quad .$$

Here, $g_{2,t}\left[\frac{\partial u_{G,2}\left(g_{2,t}, x_{2,t}\right)}{\partial g_{2,t}}\right]^{-1} \frac{\partial^2 u_{G,2}\left(g_{2,t}, x_{2,t}\right)}{\partial g_{2,t}^2}\frac{\dot{g}_{2,t}}{g_{2,t}} = 0$ and $x_{2,t}\left[\frac{\partial u_{G,2}\left(g_{2,t}, x_{2,t}\right)}{\partial x_{2,t}}\right]^{-1} \frac{\partial^2 u_{G,2}\left(g_{2,t}, x_{2,t}\right)}{\partial x_{2,t}^{\ 2}}\frac{\dot{x}_{2,t}}{x_{2,t}} = 0$ at steady state such that $\dot{g}_{2,t} = 0$ and $\dot{x}_{2,t} = 0$; thus,

$$\theta_{G,2} - \pi_{2,t} + \int_{t-1}^{t}\int_{s}^{s+1} \pi_{2,\upsilon}\, d\upsilon\ ds + \tilde{\pi}_{2,t} - \int_{t-1}^{t}\int_{s}^{s+1} \tilde{\pi}_{2,\upsilon}\, d\upsilon\ ds = r_t$$

at steady state. By equation (A10) in the Appendix,

$$\theta_{G,2} - \pi_{2,t} + \int_{t-1}^{t}\int_{s}^{s+1} \pi_{2,\upsilon}\, d\upsilon\ ds + \tilde{\pi}_{2,t} - \int_{t-1}^{t}\int_{s}^{s+1} \tilde{\pi}_{2,\upsilon}\, d\upsilon\ ds = \theta_P \quad . \tag{1}$$

Because $\tilde{\pi}_{\rho,t}$ is the expected rate of inflation when the transfer mechanism does not exist, by equation (A19) in the Appendix,

$$\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{2,\upsilon}\,d\upsilon\,ds-\tilde{\pi}_{2,t}=\theta_{G,2}-\theta_{P} \tag{2}$$

at steady state. Therefore, by combining equations (1) and (2),

$$-\pi_{2,t}+\int_{t-1}^{t}\int_{s}^{s+1}\pi_{2,\upsilon}\,d\upsilon\;ds+\theta_{P}=\theta_{P}\quad ;$$

thus,

$$\pi_{2,t}=\int_{t-1}^{t}\int_{s}^{s+1}\pi_{2,\upsilon}\,d\upsilon\;ds \tag{3}$$

at steady state.

Equation (3) indicates that, if the transfer mechanism exists, inflation in state 2 ($\pi_{2,t}$) does not accelerate or decelerate (i.e., $\pi_{2,t}$ is kept constant).

At the same time, the government of state 1 reserves $-b_{1,t}\left(\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{1,\upsilon}\,d\upsilon\;ds-\tilde{\pi}_{1,t}\right)$ for the transfer of $b_{2,t}\left(\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{2,\upsilon}\,d\upsilon\;ds-\tilde{\pi}_{2,t}\right)$ to the government of state 2. The government of state 1 finances this amount of money by issuing bonds. Hence, the optimality conditions for government of state 1 are

$$\theta_{G,1}-\pi_{1,t}+\int_{t-1}^{t}\int_{s}^{s+1}\pi_{1,\upsilon}\,d\upsilon\;ds+\tilde{\pi}_{1,t}-\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{1,\upsilon}\,d\upsilon\;ds=\theta_{P}\quad ,$$

and

$$\pi_{1,t}=\int_{t-1}^{t}\int_{s}^{s+1}\pi_{1,\upsilon}\,d\upsilon\;ds$$

at steady state. Because

$$\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{\rho,\upsilon}\,d\upsilon\,ds-\tilde{\pi}_{\rho,t}=\theta_{G,\rho}-\theta_{P}\quad ,$$

if $b_{2,t}\left(\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{2,\upsilon}\,d\upsilon\;ds-\tilde{\pi}_{2,t}\right)=-b_{1,t}\left(\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{1,\upsilon}\,d\upsilon\;ds-\tilde{\pi}_{1,t}\right)$, then

$$\frac{b_{1,t}\theta_{G,1}+b_{2,t}\theta_{G,2}}{b_{1,t}+b_{2,t}}=\theta_{P}\quad .$$

Because $\frac{\theta_{G,1}+\theta_{G,2}}{2}=\theta_{G}=\theta_{P}$, then

$$\frac{2b_{1,t}}{b_{1,t}+b_{2,t}}\theta_{G,1}+\frac{2b_{2,t}}{b_{1,t}+b_{2,t}}\theta_{G,2}=\theta_{G,1}+\theta_{G,2}$$

and thereby

$$b_{1,t} = b_{2,t}$$

at steady state. That is, if the government of state 1 transfers an adequate amount of money to that of state 2 in any period such that $b_{2,t}\left(\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{2,v}\,dv\ ds - \tilde{\pi}_{2,t}\right) = -b_{1,t}\left(\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{1,v}\,dv\ ds - \tilde{\pi}_{1,t}\right)$ at steady state, the heterogeneous two states accumulate government debts in the same manner. Because of the transfer, the accumulation of government debt in state 2 is restrained and that in state 1 is enhanced; thereby, the debts of both governments are equalized. As a result, both states do not experience inflation acceleration or deceleration: that is, there are no inflation differentials and thereby no current account imbalances owing to inflation differentials.

4.1.2 The multi-state transfer mechanism

The transfer mechanism for two states can be easily extended to a multi-state transfer mechanism, but before examining the multi-state transfer mechanism, I examine the case of two states that are identical except for time preference and size. In this case, the size (the population) of state 1 is twice that of state 2:

$$\frac{2\theta_{G,1} + \theta_{G,2}}{3} = \theta_G = \theta_P \quad .$$

If $b_{2,t}\left(\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{2,v}\,dv\ ds - \tilde{\pi}_{2,t}\right) = -b_{1,t}\left(\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{1,v}\,dv\ ds - \tilde{\pi}_{1,t}\right)$, then

$$\frac{b_{1,t}\theta_{G,1} + b_{2,t}\theta_{G,2}}{b_{1,t} + b_{2,t}} = \theta_P \quad .$$

Because $\frac{2\theta_{G,1} + \theta_{G,2}}{3} = \theta_G = \theta_P$,

$$\frac{3b_{1,t}}{b_{1,t} + b_{2,t}}\theta_{G,1} + \frac{3b_{2,t}}{b_{1,t} + b_{2,t}}\theta_{G,2} = 2\theta_{G,1} + \theta_{G,2} \quad ;$$

thus,

$$b_{1,t} = 2b_{2,t} \quad .$$

That is, with the transfer mechanism shown above, two states that are identical except for size and time preference accumulate their government debts in the same manner. Both states do not experience inflation acceleration or deceleration: that is, there are no inflation differentials and thereby no current account imbalances caused by inflation differentials.

Next, suppose that a union consists of 3 states (state 1, 2, and 3) that are identical except for time preference (their sizes are identical). In this case,

$$\frac{\theta_{G,1} + \theta_{G,2} + \theta_{G,3}}{3} = \theta_G = \theta_P \quad .$$

If $b_{2,t}\left(\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{2,\upsilon}\,d\upsilon\ ds-\tilde{\pi}_{2,t}\right)=-b_{1,t}\left(\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{1,\upsilon}\,d\upsilon\ ds-\tilde{\pi}_{1,t}\right)$, the governments of state 1 and 2 can be seen as a combined government of state 1 + 2 with $b_{1+2,t}$. If there is a transfer mechanism between state 1+2 and state 3 such that

$$b_{3,t}\left(\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{3,\upsilon}\,d\upsilon\ ds-\tilde{\pi}_{3,t}\right)=-b_{1+2,t}\left(\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{1+2,\upsilon}\,d\upsilon\ ds-\tilde{\pi}_{1+2,t}\right) ,$$

then

$$b_{1+2,t} = 2b_{3,t}$$

for the same reasons discussed in the previous example. By iterating the same procedures used in the above cases, it can be shown that there is a transfer mechanism among H ($\in N$) member states that are identical except for size and time preference, by which government debts in all states are accumulated in the same manner and inflation differentials are not generated.

With such a multi-state transfer mechanism, member states with $\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{p,\upsilon}\,d\upsilon\ ds-\tilde{\pi}_{p,t}<0$ assist and transfer funds to member states with $\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{p,\upsilon}\,d\upsilon\ ds-\tilde{\pi}_{p,t}>0$. This mutual aid thereby makes the union sustainable. Conversely, if such a transfer mechanism is not established, inflation differentials and current account imbalances will be aggravated, and it is likely that a union with heterogeneous members will eventually collapse.

4.2 The transfer mechanism is just

The reason why the transfer mechanism is just can be understood by comparing the consequences when the transfer mechanism exists and when it does not. For simplicity, suppose the case of two states identical except for time preference—more advantaged state 1 (e.g., Germany) and less advantaged state 2 (e.g., Greece)—and suppose that $\theta_{G,1}<\theta_{G,2}$ and $\theta_{P,1}<\theta_{P,2}$. Suppose also that whether a euro-like monetary union exists or not, sustainable heterogeneity is established for both states. Therefore, whether a euro-like monetary union exists or not,

$$\frac{\theta_{P,1}+\theta_{P,2}}{2}=r \quad . \tag{4}$$

Finally, assume that the central bank of the monetary union and the central banks of states 1 and 2 are sufficiently independent from their governments.

First, I examine the case where a euro-like monetary union does not exist. In order to not accelerate inflation, the central banks of the two states control their governments' time preferences to satisfy

$$\theta_{G,1}=\frac{\theta_{P,1}+\theta_{P,2}}{2} \tag{5}$$

and

$$\theta_{G,2} = \frac{\theta_{P,1} + \theta_{P,2}}{2} \quad , \tag{6}$$

respectively. Both states' economies stabilize at steady state by equation (4) without inflation acceleration, because both central banks keep equations (5) and (6).

In the case where a euro-like monetary union is established and it is equipped with the transfer mechanism, the central bank of the monetary union controls the overall rate of governments' time preference such that

$$\frac{\theta_{G,1} + \theta_{G,2}}{2} = \frac{\theta_{P,1} + \theta_{P,2}}{2} \tag{7}$$

to not accelerate inflation in the union as a whole. Thanks to the transfer mechanism, inflation in both states does not accelerate as shown in Section 4.1. Therefore, both economies stabilize at steady state by equation (4) without inflation acceleration, because there is a transfer mechanism and the central banks maintain equation (7). The levels of production and consumption in both states are the same as in the case where a euro-like monetary union does not exist, because equation (4) is identical in both cases, and there is no current account imbalance owing to inflation differentials.

There is, however, a difference between these two cases. In the latter case, the government of member state 2 is not required to lower $\theta_{G,2}$ to $\theta_P = \frac{\theta_{P,1} + \theta_{P,2}}{2}$. Because the central bank of the monetary union keeps equation (7) satisfied, both governments reluctantly need to lower their rates of time preference. Faced with pressure from the central bank (i.e., it raises interest rates), both governments lower their rates of time preference similarly—that is, almost at the same rate—because they commonly adhere to their intrinsic time preferences unless a third party forces them to stop doing so. They will obey the third party to the same degree. As a result, through the actions of the central bank,

$$\theta_{G,2} > \frac{\theta_{G,1} + \theta_{G,2}}{2} = \frac{\theta_{P,1} + \theta_{P,2}}{2} > \theta_{G,1} \tag{8}$$

will be always held whether the transfer mechanism exists or not in a euro-like monetary union. Therefore, in the latter case, unlike with equations (5) and (6), the government of state 2 does not need to lower $\theta_{G,2}$ to $\frac{\theta_{P,1} + \theta_{P,2}}{2}$ but that of state 1 needs to make $\theta_{G,1}$ lower than $\frac{\theta_{P,1} + \theta_{P,2}}{2}$.

Inequality (8) means that the government of state 1 practically borrows money, replacing the borrowing of the government of state 2 by $b_{2,t}\left(\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{2,\upsilon}\,d\upsilon\ ds - \tilde{\pi}_{2,t}\right) = -b_{1,t}\left(\int_{t-1}^{t}\int_{s}^{s+1}\tilde{\pi}_{1,\upsilon}\,d\upsilon\ ds - \tilde{\pi}_{1,t}\right)$. Through this substitution, the debt increases of the government of state 2 are lowered and those of state 1 are raised. In other words, the more advantaged state is practically a guarantor for the less advantaged state's obligations. As a result, imbalances among member states are eliminated.

Finally, I examine the case where a euro-like monetary union is established but it is not equipped with a transfer mechanism. The central bank of the monetary union controls the overall rate of governments' time preference to satisfy equation (7). However, as shown in the model, each state's inflation cannot be fully controlled individually, and inequality (8) will always hold. Because there is no transfer mechanism, inequality (8) indicates that persistent inflation differentials are generated and they accelerate. Conversely, the levels of production in both states are the same as in the case where a euro-like monetary union does not exist because equation (4) is identical in both cases. The levels of consumption, however, differ between the two states because of the inflation differentials and the subsequent current account imbalances. As a result, the more advantaged state owns more capital than in the case where a euro-like monetary union does not exist, and conversely, the less advantaged state owns less. Therefore, consumption in state 1 is larger and that in state 2 is smaller than in the case where a euro-like monetary union does not exist. If the lack of a transfer mechanism continues, the magnitude of this disparity will accelerate as time passes.

Without the transfer mechanism, therefore, state 1 obtains "extra" gains because its consumption is larger, and state 2 suffers "extra" losses. If the lack of a transfer mechanism continues, the degree of disparity between the "extra" gains and losses will be aggravated to an extreme degree. Clearly, the transfer mechanism is just.

Some people may argue that the transfer mechanism is immoral and unjustifiable because the government of state 2 does not need to lower $\theta_{G,2}$ to θ_P, although satisfying equation (6) is required when a euro-like monetary union does not exist. It is true that if there is a transfer mechanism, the government of state 2 does not need to be as patient as in the case that a euro-like monetary union does not exist, but this does not necessarily mean that the transfer mechanism is immoral and unjustifiable. Not lowering $\theta_{G,2}$ to θ_P in and of itself does not signify profligacy. As argued above, because of the central bank's control, both governments lower their rates of time preference similarly—that is, almost at the same rate—and both governments naturally and equally adhere to their intrinsic time preferences. Why, then, should only the government of state 2 be condemned for being immoral? Furthermore, a union is formed because it is of great value politically, socially, culturally, or ideologically. If a union is important and should be maintained from many points of view, the transfer mechanism is justified because, with it, no member state enjoys "extra" benefits and no member state suffers "extra" losses.

The transfer mechanism can be seen as a necessity to achieve a kind of sustainable heterogeneity. Introducing the transfer mechanism means that a supranational authority assesses taxes on more advantaged member states and distributes the revenue from these taxes to less advantaged member states to establish and maintain unity and solidarity in a union consisting of heterogeneous member states.

4.3 Existing transfer mechanisms within Germany

The reason why the transfer mechanism is indispensable for justice in a union with heterogeneous members can be better understood by observing Germany's current existing intergovernmental transfer system. German municipalities are subject to a comprehensive fiscal equalization system: that is, they receive a huge amount of fiscal transfers from the federal and state governments. The fiscal equalization transfers in municipal finance in Germany amount to more than a quarter of total revenues on average. In addition, the transfer amounts vary greatly among municipalities. Municipalities with higher fiscal needs and a lower fiscal capacity can receive larger amounts.

This fiscal equalization system is usually justified on the grounds of redistribution to reduce disparities in fiscal capacity among municipalities and as insurance against asymmetric revenue shocks. However, it seems likely that Germans, like people in other nations, empirically know that, without appropriate fiscal transfers, a union consisting of heterogeneous members will collapse (by the mechanism shown in this paper); therefore, the more advantaged members should appropriately assist the less advantaged members. The assistance of less advantaged members by more advantaged members is quite reasonable if a union is to be maintained. If a union (e.g., Bundesrepublik Deutschland) is of great value politically, socially, culturally, or ideologically and needs transfers to be sustainable, then an appropriate transfer mechanism must be established.

5 CONCLUDING REMARKS

The euro crisis is ongoing and appears to be worsening. This paper theoretically examined the mechanism behind the crisis and proposed a solution. The model in this paper indicates that the essence of the euro's flaw is the lack of a transfer mechanism among heterogeneous member states. Unless, in every period, more advantaged states (e.g., Germany) systematically transfer a necessary amount of money to less advantaged states (e.g., Greece), the euro will eventually reach an impasse. Although the more advantaged member states obtain "extra" benefits from the euro system, less advantaged member states eventually lose most capital ownership and their economies are left devastated. This outcome is not a result of profligacy of some member states—rather, it stems from different degrees of intrinsic time preference among member states.

The necessity of fiscal transfer has been argued by many researchers because fiscal transfers are a tool of risk-sharing and a buffer against asymmetric shocks among member states. Although this reasoning is correct, it is a weak incentive to persuade more advantaged states to introduce a transfer mechanism. However, the model in this paper indicates that a transfer mechanism is indispensable for the euro area economy to reach equilibrium where all heterogeneous states achieve optimality. That is, without the transfer mechanism, the degree of disparity among heterogeneous member states accelerates indefinitely. Therefore, fiscal transfers should not be viewed as alms for the less advantaged states but as a right these states should justly assert. If the euro system is of great value politically, socially, culturally, and ideologically and it is desirable to maintain it from those perspectives, then the transfer mechanism is absolutely necessary.

Some people may argue that the solution presented in this paper is immoral and unjustifiable and generates serious moral hazard, but it is actually quite reasonable. If most members of a union with heterogeneous members do not want the union to collapse, the more advantaged members will usually help the less advantaged members. With this behavior, a spirit of unity and solidarity is established and maintained among the heterogeneous members. The model presented in this paper shows the mechanism supporting this statement from the perspective of theoretical economics.

APPENDIX

A1 The single-country model

The single-country model is based on the model of inflation by Harashima (2008). The single-country model will be extended to a multi-country model in Section A2.

A1.1 The optimal trend inflation

A1.1.1 The government

A1.1.1.1 The government budget constraint

The government budget constraint is

$$\dot{B}_t = B_t i_t + G_t - X_t - \vartheta_t \quad ,$$

where B_t is the nominal obligation of the government to pay for its accumulated bonds, i_t is the nominal interest rate for government bonds, G_t is the nominal government expenditure, X_t is the nominal tax revenue, and ϑ_t is the nominal amount of seigniorage at time t. The tax is assumed to be lump sum, the government bonds are long term, and the returns on the bonds are realized only after the bonds are held during a unit period (e.g., a year). The government bonds are redeemed in a unit period, and the government successively refinances the bonds by issuing new ones at each time t. Let $b_t = \frac{B_t}{P_t}$, $g_t = \frac{G_t}{P_t}$, $x_t = \frac{X_t}{P_t}$, and $\varphi_t = \frac{\vartheta_t}{P_t}$, where P_t is the price level at time t. Let also $\pi_t = \frac{\dot{P}_t}{P_t}$ be the inflation rate at time t. By dividing by P_t, the budget constraint is transformed to $\frac{\dot{B}_t}{P_t} = b_t i_t + g_t - x_t - \varphi_t$, which is equivalent to

$$\dot{b}_t = b_t i_t + g_t - x_t - \varphi_t - b_t \pi_t = b_t (i_t - \pi_t) + g_t - x_t - \varphi_t \quad . \tag{A1}$$

Because the returns on government bonds are realized only after holding the bonds during a unit period, investors buy the bonds if $\bar{i}_t \geq E_t \int_t^{t+1} (\pi_s + r_s) ds$ at time t, where $\bar{i}_t$ is the nominal interest rate for bonds bought at t and r_t is the real interest rate in markets at t. Hence, by arbitrage, $\bar{i}_t = E_t \int_t^{t+1} (\pi_s + r_s) ds$ and if r_t is constant such that $r_t = r$ (i.e., if it is at steady state), then

$$\bar{i}_t = E_t \int_t^{t+1} \pi_s \, ds + r \quad .$$

The nominal interest rate $\bar{i}_t = E_t \int_t^{t+1} \pi_s \, ds + r$ means that, during a sufficiently small period between t and $t + dt$, the government's obligation to pay for the bonds' return in the future increases not by $dt(\pi_t + r)$ but by $dt\left(E_t \int_t^{t+1} \pi_s \, ds + r \right)$. If π_t is constant, then $E_t \int_t^{t+1} \pi_s \, ds = \pi_t$ and $\bar{i}_t = \pi_t + r$, but if π_t is not constant, these equations do not necessarily hold.

Since bonds are redeemed in a unit period and successively refinanced, the bonds the government is holding at t have been issued between t - 1 and t. Hence, under perfect foresight, the average nominal interest rate for all government bonds at time t is the weighted sum of $\bar{i}_t$ such that

$$i_t = \int_{t-1}^{t} \bar{i}_s \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds + r \quad ,$$

where $\overline{B}_{s,t}$ is the nominal value of bonds at time t that were issued at time s. If the weights $\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv}$ between t - 1 and t are not so different from each other, then approximately $i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + r$. To be precise, if the absolute values of π_s for $t-1 < s \leq t+1$ are sufficiently smaller than unity, the differences among the weights are negligible and then approximately

$$i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + r \tag{A2}$$

(see Harashima, 2008). The average nominal interest rate for the total government bonds, therefore, develops by $i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + r$. If π_t is constant, then $\int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds$ $= \pi_t$; thus, $i_t = \pi_t + r$. If π_t is not constant, however, the equations $\int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds = \pi_t$ and $i_t = \pi_t + r$ do not necessarily hold.

A1.1.1.2 An economically Leviathan government

Under a proportional representation system, the government represents the median household whereas the representative household from an economic perspective represents the mean household.[1] Because of this difference, they usually have different preferences. To account for this essential difference, a Leviathan government is assumed in the model.[2] There are two extremely different views regarding government's behavior in the literature on political economy: the Leviathan view and the benevolent view (e.g., Downs 1957; Brennan and Buchanan 1980; Alesina and Cukierman 1990). From an economic point of view, a benevolent government maximizes the expected economic utility of the representative household, but a Leviathan government does not. Whereas the expenditure of a benevolent government is a tool used to maximize the economic utility of the representative household, the expenditure of a Leviathan government is a tool used to achieve the government's own policy objectives.[3] For example, if a Leviathan government considers national security to be the most important political issue, defense spending will increase greatly, but if improving social welfare is the top political priority, spending on social welfare will increase dramatically, even though the

[1] See the literature on the median voter theorem (e.g., Downs 1957). Also see the literature on the delay in reforms (e.g., Alesina and Drazen 1991; Cukierman et al. 1992).

[2] The most prominent reference to Leviathan governments is Brennan and Buchanan (1980).

[3] The government behavior assumed in the fiscal theory of the price level reflects an aspect of a Leviathan government. Christiano and Fitzgerald (2000) argue that non-Ricardian policies correspond to the type of policies in which governments are viewed as selecting policies and committing themselves to those policies in advance of prices being determined in markets.

increased expenditures may not necessarily increase the economic utility of the representative household.

Is it possible, however, for such a Leviathan government to hold office for a long period? Yes, because a government is generally chosen by the median of households under a proportional representation system (e.g., Downs 1957), whereas the representative household usually presumed in the economics literature is the mean household. The economically representative household is not usually identical to the politically representative household, and a majority of people could support a Leviathan government even if they know that the government does not necessarily pursue only the economic objectives of the economically representative household. In other words, the Leviathan government argued here is an economically Leviathan government that maximizes the political utility of people, whereas the conventional economically benevolent government maximizes the economic utility of people. In addition, because the politically and economically representative households are different (the median and mean households, respectively), the preferences of future governments will also be similarly different from those of the mean representative household. In this sense, the current and future governments presented in the model can be seen as a combined government that goes on indefinitely; that is, the economically Leviathan government always represents the median representative household.

The Leviathan view generally requires the explicit inclusion of government expenditure, tax revenue, or related activities in the government's political utility function (e.g., Edwards and Keen 1996). Because an economically Leviathan government derives political utility from expenditure for its political purposes, the larger the expenditure is, the happier the Leviathan government will be. But raising tax rates will provoke people's antipathy, which increases the probability of being replaced by the opposing party that also nearly represents the median household. Thus, the economically Leviathan government regards taxes as necessary costs to obtain freedom of expenditure for its own purposes. The government therefore will derive utility from expenditure and disutility from taxes. Expenditure and taxes in the political utility function of the government are analogous to consumption and labor hours in the economic utility function of the representative household. Consumption and labor hours are both control variables, and as such, the government's expenditure and tax revenue are also control variables. As a whole, the political utility function of economically Leviathan government can be expressed as $u_G(g_t, x_t)$.[4] In addition, it can be assumed on the basis of previously mentioned arguments that $\frac{\partial u_G}{\partial g_t} > 0$ and $\frac{\partial^2 u_G}{\partial g_t^{\ 2}} < 0$, and therefore that $\frac{\partial u_G}{\partial x_t} < 0$ and $\frac{\partial^2 u_G}{\partial x_t^{\ 2}} > 0$.[5] An economically Leviathan government therefore maximizes the expected sum of these utilities discounted by its time preference rate under the constraint of deficit financing.

A1.1.1.3 The optimization problem

[4] It is possible to assume that governments are partially benevolent. In this case, the utility function of a government can be assumed to be $u_G(g_t, x_t, c_t, l_t)$, where c_t is real consumption and l_t is the leisure hours of the representative household. However, if a lump-sum tax is imposed, the government's policies do not affect steady-state consumption and leisure hours. In this case, the utility function can be assumed to be $u_G(g_t, x_t)$.

[5] Some may argue that it is more likely that $\frac{\partial u_G}{\partial x_t} > 0$ and $\frac{\partial^2 u_G}{\partial x_t^{\ 2}} < 0$. However, the assumption used is not an important issue here because $-x_t\left[\frac{\partial u_G(g_t, x_t)}{\partial x_t}\right]^{-1}\frac{\partial^2 u_G(g_t, x_t)}{\partial x_t^{\ 2}}\frac{\dot{x}_t}{x_t} = 0$ at steady state, as will be shown in the solution to the optimization problem later in the paper. Thus, the results are not affected by which assumption is used.

The optimization problem of an economically Leviathan government is

$$Max\ E\int_0^{\infty} u_G(g_t, x_t)\exp(-\theta_G\, t)dt$$

subject to the budget constraint

$$\dot{b}_t = b_t(i_t - \pi_t) + g_t - x_t - \varphi_t \quad , \tag{A3}$$

where u_G is the constant relative risk aversion utility function of the government, θ_G is the government's rate of time preference, and E is the expectation operator. All variables are expressed in per capita terms, and population is assumed to be constant. The government maximizes its expected political utility considering the behavior of the economically representative household that is reflected in i_t in its budget constraint.

A1.1.2 Households

The economically representative household maximizes its expected economic utility. Sidrauski (1967)'s well-known money in the utility function model is used for the optimization problem. The representative household maximizes its expected utility

$$E\int_0^{\infty} u_P(c_t, m_t)\exp(-\theta_P\, t)dt$$

subject to the budget constraint

$$\dot{a}_t = (r_t\, a_t + w_t + \sigma_t) - [c_t + (\pi_t + r_t)m_t] - g_t \quad ,$$

where u_P and θ_P are the utility function and the time preference rate of the representative household, c_t is real consumption, w_t is real wage, σ_t is lump-sum real government transfers, m_t is real money, $a_t = k_t + m_t$, and k_t is real capital. It is assumed that $r_t = f'(k_t)$, $w_t = f(k_t) - k_t f'(k_t)$, $u_P' > 0$, $u_P'' < 0$, $\frac{\partial u_P(c_t, m_t)}{\partial m_t} > 0$, and $\frac{\partial^2 u_P(c_t, m_t)}{\partial m_t^{\,2}} < 0$, where $f(\cdot)$ is the production function. Government expenditure (g_t) is an exogenous variable for the representative household because it is an economically Leviathan government. It is also assumed that, although all households receive transfers from a government in equilibrium, when making decisions, each household takes the amount it receives as given, independent of its money holdings. Thus, the budget constraint means that the real output $f(k_t)$ at any time is demanded for the real consumption c_t, the real investment $\dot{k}_t$, and the real government expenditure g_t such that $f(k_t) = c_t + \dot{k}_t + g_t$. The representative household maximizes its expected economic utility considering the behavior of government reflected in g_t in the budget constraint. In this discussion, a central bank is not assumed to be independent of the government; thus, the functions of the government and the central bank are not separated. This assumption can be relaxed, and the roles of the government and the central bank are explicitly separated in Section A1.2.

Note that the time preference rate of government (θ_G) is not necessarily identical to that of the representative household (θ_P) because the government and the representative household represent different households (i.e., the median and mean households, respectively). In addition, the preferences will differ because (1) even though people want to choose a government that has the same time preference rate as the representative household, the rates may differ owing to errors in expectations (e.g., Alesina and Cukierman 1990); and (2) current voters cannot bind the choices of future voters and, if current voters are aware of this possibility, they may vote more myopically as compared with their own rates of impatience in private economic activities (e.g., Tabellini and Alesina 1990). Hence, it is highly likely that the time preference rates of a government and the representative household are heterogeneous. It should be also noted, however, that even though the rates of time preference are heterogeneous, an economically Leviathan government behaves based only on its own time preference rate, without hesitation.

A1.1.3 The simultaneous optimization

First, I examine the optimization problem of the representative household. Let Hamiltonian H_P be $H_P = u_P(c_t, m_t)\exp(-\theta_P t) + \lambda_{P,t}[r_t a_t + w_t + \sigma_t - c_t - (\pi_t + r_t)m_t - g_t]$, where $\lambda_{P,t}$ is a costate variable, c_t and m_t are control variables, and a_t is a state variable. The optimality conditions for the representative household are;

$$\frac{\partial u_P(c_t, m_t)}{\partial c_t}\exp(-\theta_P t) = \lambda_{P,t} \quad , \tag{A4}$$

$$\frac{\partial u_P(c_t, m_t)}{\partial m_t}\exp(-\theta_P t) = \lambda_{P,t}(\pi_t + r_t) \quad , \tag{A5}$$

$$\dot{\lambda}_{P,t} = -\lambda_{P,t} r_t \quad , \tag{A6}$$

$$\dot{a}_t = (r_t a_t + w_t + \sigma_t) - [c_t + (\pi_t + r_t)m_t - g_t] \quad , \tag{A7}$$

$$\lim_{t\to\infty} \lambda_{P,t} a_t = 0 \quad . \tag{A8}$$

By conditions (A4) and (A5), $\left[\frac{\partial u_P(c_t, m_t)}{\partial c_t}\right]^{-1} \frac{\partial u_P(c_t, m_t)}{\partial m_t} = \pi_t + r_t$, and by conditions (A4) and (A6),

$$-c_t\left[\frac{\partial u_P(c_t, m_t)}{\partial c_t}\right]^{-1} \frac{\partial^2 u_P(c_t, m_t)}{\partial c_t^2}\frac{\dot{c}_t}{c_t} + \theta_P = r_t \quad . \tag{A9}$$

Hence,

$$\theta_P = r_t = r \tag{A10}$$

at steady state such that $\dot{c}_t = 0$ and $\dot{k}_t = 0$.

Next, I examine the optimization problem of the economically Leviathan government. Let Hamiltonian H_G be $H_G = u_G(g_t, x_t)\exp(-\theta_G t) + \lambda_{G,t}[b_t(i_t - \pi_t) + g_t - x_t - \varphi_t]$, where $\lambda_{G,t}$ is a costate variable. The optimality conditions for the government are;

$$\frac{\partial u_G(g_t, x_t)}{\partial g_t} \exp(-\theta_G t) = -\lambda_{G,t} \quad , \tag{A11}$$

$$\frac{\partial u_G(g_t, x_t)}{\partial x_t} \exp(-\theta_G t) = \lambda_{G,t} \quad , \tag{A12}$$

$$\dot{\lambda}_{G,t} = -\lambda_{G,t}(i_t - \pi_t) \quad , \tag{A13}$$

$$\dot{b}_t = b_t(i_t - \pi_t) + g_t - x_t - \varphi_t \quad , \tag{A14}$$

$$\lim_{t \to \infty} \lambda_{G,t} b_t = 0 \quad . \tag{A15}$$

Combining conditions (A11), (A12), and (A13) and equation (A2) yields the following equations:

$$-g_t \left[\frac{\partial u_G(g_t, x_t)}{\partial g_t} \right]^{-1} \frac{\partial^2 u_G(g_t, x_t)}{\partial g_t^2} \frac{\dot{g}_t}{g_t} + \theta_G = i_t - \pi_t = r_t + \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \; ds - \pi_t \tag{A16}$$

and

$$-x_t \left[\frac{\partial u_G(g_t, x_t)}{\partial x_t} \right]^{-1} \frac{\partial^2 u_G(g_t, x_t)}{\partial x_t^2} \frac{\dot{x}_t}{x_t} + \theta_G = i_t - \pi_t = r_t + \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \; ds - \pi_t \quad . \tag{A17}$$

Here, $g_t \left[\frac{\partial u_G(g_t, x_t)}{\partial g_t} \right]^{-1} \frac{\partial^2 u_G(g_t, x_t)}{\partial g_t^2} \frac{\dot{g}_t}{g_t} = 0$ and $x_t \left[\frac{\partial u_G(g_t, x_t)}{\partial x_t} \right]^{-1} \frac{\partial^2 u_G(g_t, x_t)}{\partial x_t^2} \frac{\dot{x}_t}{x_t} = 0$ at steady state such that $\dot{g}_t = 0$ and $\dot{x}_t = 0$; thus,

$$\theta_G = r_t + \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds - \pi_t \quad . \tag{A18}$$

Hence, by equation (A10),

$$\int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds = \pi_t + \theta_G - \theta_P \tag{A19}$$

at steady state such that $\dot{g}_t = 0$, $\dot{x}_t = 0$, $\dot{c}_t = 0$, and $\dot{k}_t = 0$.[6]

Equation (A19) is a natural consequence of simultaneous optimization by the economically Leviathan government and the representative household. If the rates of time preference are heterogeneous between them, then

[6] If and only if $\theta_G = -\frac{g_t - x_t - \varphi_t}{b_t}$ at steady state, then the transversality condition (A15) $\lim_{t \to \infty} \lambda_{G,t} b_t = 0$ holds. The proof is shown in Harashima (2008).

$$i_t - r = \int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds \neq \pi_t \quad .$$

This result might seem surprising because it has been naturally conjectured that $i_t = \pi_t + r$. However, this is a simple misunderstanding because π_t indicates the instantaneous rate of inflation at a point such that $\pi_t = \frac{\dot{P}_t}{P_t}$, whereas $\int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds$ roughly indicates the average inflation rate in a period. Equation (A19) indicates that π_t develops according to the integral equation $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds - \theta_G + \theta_P$. If π_t is constant, the equations $i_t = \pi_t + r$ and $\int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds = \pi_t$ are true. However, if π_t is not constant, the equations do not necessarily hold. Equation (A19) indicates that the equations $i_t = \pi_t + r$ and $\int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds = \pi_t$ hold only in the case where $\theta_G = \theta_P$ (i.e., a homogeneous rate of time preference). It has been previously thought that a homogeneous rate of time preference naturally prevails; thus, the equation $i_t = \pi_t + r$ has not been questioned. As argued previously, however, a homogeneous rate of time preference is not usually guaranteed.

A1.1.4 The law of motion for trend inflation

Equation (A19) indicates that inflation accelerates or decelerates as a result of the government and the representative household reconciling the contradiction in heterogeneous rates of time preference. If π_t is constant, the equation $\pi_t = \int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds$ holds; conversely, if $\pi_t \neq \int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \; ds$, then π_t is not constant. Without the acceleration or deceleration of inflation, therefore, equation (A19) cannot hold in an economy in which $\theta_G \neq \theta_P$. In other words, it is not until $\theta_G \neq \theta_P$ that inflation can accelerate or decelerate. Heterogeneous time preferences ($\theta_G \neq \theta_P$) bend the path of inflation and enables inflation to accelerate or decelerate. The difference of time preference rates ($\theta_G - \theta_P$) at each time needs to be transformed to the accelerated or decelerated inflation rate π_t at each time.

Equation (A19) implies that inflation accelerates or decelerates nonlinearly in the case in which $\theta_G \neq \theta_P$. For a sufficiently small period dt, π_{t+1+dt} is determined with π_s $(t-1 < s \le t+1)$ that satisfies $\int_{t-1}^{t}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds - \pi_t = \theta_G - \theta_P$, so as to hold the equation $\int_{t}^{t+dt}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds$ $= \int_{t-1}^{t-1+dt}\int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + \pi_{t+dt} - \pi_t$. A solution of the integral equation (A19) for given θ_G and θ_P is

$$\pi_t = \pi_0 + 6(\theta_G - \theta_P)t^2 \quad . \tag{A20}$$

Generally, the path of inflation that satisfies equation (A19) for $0 \le t$ is expressed as

$$\pi_t = \pi_0 + 6(\theta_G - \theta_P)\exp[z_t \ln(t)] \quad ,$$

where z_t is a time dependent variable. The stream of z_t is various depending on the boundary condition, i.e., the past and present inflation during $-1 < t \le 0$ and the path of inflation during $0 < t \le 1$ that is set to make π_0 satisfy equation (A19). However, z_t has the following important property. If π_t satisfies equation (A19) for $0 \le t$, and $-\infty < \pi_t < \infty$ for $-1 < t \le 1$, then

$$\lim_{t \to \infty} z_t = 2 \quad .$$

Proof is shown in Harashima (2008). Any inflation path that satisfies equation (A19) for $0 \le t$ therefore asymptotically approaches the path of equation (A20). The mechanism behind the law of motion for inflation (equation [A20]) is examined more in detail in Harashima (2008).

A1.2 The central bank

A central bank manipulates the nominal interest rate according to the following Taylor-type instrument rule in the model;

$$i_t = \bar{\gamma} + \gamma_\pi \left(\pi_t - \pi^* \right) + \gamma_x x_t \quad , \tag{A21}$$

where π^* is the target rate of inflation and $\bar{\gamma}$, γ_π, and γ_x are constant coefficients. $\bar{\gamma} = \pi^* + r$ as is usually assumed.

In Section A1.1, central banks are not explicitly considered because they are not assumed to be independent of governments. However, in actuality, central banks are independent organizations in most countries even though some of them are not sufficiently independent. Furthermore, in the conventional inflation model, it is the central banks that control inflation and governments have no role in controlling inflation. Conventional inflation models show that the rate of inflation basically converges at the target rate of inflation set by a central bank. The target rate of inflation therefore is the key exogenous variable that determines the path of inflation in these models.

Both the government and the central bank can probably affect the development of inflation, but they would do so in different manners, as equation (A20) and conventional inflation models indicate. However, the objectives of the government and the central bank may not be the same. For example, if trend inflation is added to conventional models by replacing their aggregate supply equations with equation (A20), inflation cannot necessarily converge at the target rate of inflation because another key exogenous variable (θ_G) is included in the models. A government makes inflation develop consistently with the equation (A20), which implies that inflation will not necessarily converge at the target rate of inflation. Conversely, a central bank makes inflation converge at the target rate of inflation, which implies that inflation will not necessarily develop consistently with equation (A20). That is, unless either θ_G is adjusted to be consistent with the target rate of inflation or the target rate of inflation is adjusted to be consistent with θ_G, the path of inflation cannot necessarily be determined. Either θ_G or the target rate of inflation need be an endogenous variable. If a central bank dominates, the target rate of inflation remains as the key exogenous variable and θ_G should then be an endogenous variable. The reverse is also true.

A central bank will be regarded as truly independent if θ_G is forced to be adjusted to the one that is consistent with the target rate of inflation set by the central bank. For example, suppose that $\theta_G > \theta_P$ and a truly independent central bank manipulates the nominal interest rate according to the Taylor-type instrument rule (equation [A21]). Here,

$$i_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_\upsilon \, d\upsilon \, ds + r = \theta_G + \pi_t \tag{A22}$$

at steady state such that $\dot{g}_t = 0$, $\dot{x}_t = 0$, $\dot{c}_t = 0$, and $\dot{k}_t = 0$ by equations (A2), (A10), and (A19). If the accelerating inflation rate is higher than the target rate of inflation, the central bank can raise the nominal interest rate from $i_t = \theta_G + \pi_t$ (equation [A22]) to

$$i_t = \theta_G + \pi_t + \psi$$

by positive ψ by intervening in financial markets to lower the accelerating rate of inflation. In this case, the central bank keeps the initial target rate of inflation because it is truly independent. The government thus faces a rate of increase of real obligation that is higher than θ_G by the extra rate ψ.[7] If the government lowers θ_G so that $\theta_G < \theta_P$ and inflation stops accelerating, the central bank will accordingly reduce the extra rate ψ. If, however, the government does not accommodate θ_G to the target rate of inflation, the extra rate ψ will increase as time passes because of the gap between the accelerating inflation rate and the target rate of inflation widens by equation (A20) and γ_x in Taylor-type instrument rules is usually larger than unity, say 1.5. Because of the extra rate ψ, the government has no other way to achieve optimization unless it lowers θ_G to one that is consistent with the target rate of inflation. Once the government recognizes that the central bank is firmly determined to be independent and it is in vain to try to intervene in the central bank's decision makings, the government would not dare to attempt to raise θ_G again anymore.

Equation (A20) implies that a government allows inflation to accelerate because it acts to maximize its expected utility based only on its own preferences. A government is hardly the only entity that cannot easily control its own preferences even when these preferences may result in unfavorable consequences. It may not even be possible to manipulate one's own preferences at will. Thus, even though a government is fully rational and is not weak, foolish, or untruthful, it is difficult for it to self-regulate its preferences. Hence, an independent neutral organization is needed to help control θ_G. Delegating the authority to set and keep the target rate of inflation to an independent central bank is a way to control θ_G. The delegated independent central bank will control θ_G because it is not the central bank's preference to stabilize the price level—it is simply a duty delegated to it. An independent central bank is not the only possible choice. For example, pegging the local currency with a foreign currency can be seen as a kind of delegation to an independent neutral organization. In addition, the gold standard that prevailed before World War II can be also seen as a type of such delegation.

Note also that the delegation may not be viewed as bad from the Leviathan government's point of view because only its rate of time preference is changed, and the government can still pursue its political objectives. One criticism of the argument that central banks should be independent (e.g., Blinder 1998) is that, since the time-inconsistency problem argued in Kydland and Prescott (1977) or Barro and Gordon (1983) is more acute with fiscal policy, why is it not also necessary to delegate fiscal policies? An economically Leviathan government, however, will never allow fiscal policies to be delegated to an independent neutral organization because the Leviathan government would then not be able to pursue its political objectives, which in a sense would mean the death of the Leviathan government. The median household that backs the Leviathan government, but at the same time dislikes high inflation, will therefore support the delegation of authority but only if it concerns monetary policy. The independent central bank will then be given the authority to control θ_G and oblige the government to change θ_G in order to meet the target rate of inflation.

Without such a delegation of authority, it is likely that generally $\theta_G > \theta_P$ because θ_G represents the median household whereas θ_P represents the mean household. Empirical studies indicate that the rate of time preference negatively correlates with permanent income (e.g.,

[7] The extra rate ψ affects not only the behavior of government but also that of the representative household, in which the conventional inflation theory is particularly interested. In this sense, the central bank's instrument rule that concerns and simultaneously affects both behaviors of the government and the representative household is particularly important for price stability.

Lawrance 1991), and the permanent income of the median household is usually lower than that of the mean household. If generally $\theta_G > \theta_P$, that suggests that inflation will tend to accelerate unless a central bank is independent. The independence of the central bank is therefore very important in keeping the path of inflation stable.

Note also that the forced adjustments of θ_G by an independent central bank are exogenous shocks to both the government and the representative household because they are planned solely by the central bank. When a shock on θ_G is given, the government and the representative household must recalculate their optimal paths including the path of inflation by resetting θ_G, π_t, and φ.

A2 The multi-country model

In this section, the single-country model shown in Section A1 is extended to a multi-country model in the framework of endogenous growth, in which there is more than one government but only one central bank. More concretely, the single-country model is extended by combining it with the multi-country endogenous growth model by Harashima (2010). In addition, some technical modifications are made to the extended model so that the economics of monetary union can be analyzed.

A2.1 The optimization of households

A2.1.1 The base model

The production function is $Y_t = F(A_t, K_t, L_t)$, and the accumulation of capital is

$$\dot{K}_t = Y_t - C_t - v\dot{A}_t \quad ,$$

where Y_t is outputs, A_t is technology, K_t is capital inputs, L_t is labor inputs, C_t is consumption, $v(>0)$ is a constant, and a unit of K_t and v^{-1} of a unit of A_t are equivalent: that is, they are produced using the same quantities of inputs. All firms are identical and have the same size, and for any period,

$$\mu = \frac{M_t}{L_t} \quad , \tag{A23}$$

where M_t is the number of firms, and $\mu(>0)$ is a constant. In addition,

$$\frac{\partial Y_t}{\partial K_t} = \frac{\varpi}{M_t} \frac{\partial Y_t}{\partial (vA_t)} \quad ; \tag{A24}$$

thus,

$$\frac{\partial y_t}{\partial k_t} = \frac{\varpi}{\mu v} \frac{\partial y_t}{\partial A_t} \tag{A25}$$

is always kept, where y_t is output per capita, k_t is capital per capita, and $\varpi(>1)$ is a constant. For simplicity, the period of patent is assumed to be indefinite, and no capital depreciation is assumed. ϖ indicates the effect of patent protection. With patents, the income is distributed to not only capitals and labors but technologies. Equation (A23) indicates that population and number of firms are positively correlated. Equations (A24) and (A25) indicate that returns on investing in K_t and in A_t are kept equal and that a firm that produces a new technology cannot obtain all the returns on an investment in A_t. This means that investing in A_t increases Y_t, but the investing firm's return on the investment in A_t is only a fraction of the increase of Y_t, such that $\frac{\varpi}{M_t}\frac{\partial Y_t}{\partial(\nu A_t)} = \frac{\varpi}{\mu L_t}\frac{\partial Y_t}{\partial(\nu A_t)}$ because of uncompensated knowledge spillovers to other firms and complementarity of technologies.

A part of the knowledge generated as a result of an investment made by a firm spills over to other firms. Researchers in firms as well as universities and research institutions could not effectively generate innovations if they were isolated from other researchers. They contact and stimulate each other. Probably, mutual partial knowledge spillovers among researchers and firms give each other reciprocal benefits. Researchers take hints on their researches in exchange for spilled knowledge. Therefore, even though the investing firm wishes to keep its knowledge secret, some parts of it will spill over. In addition, many uncompensated knowledge spillovers occur because many technologies are regarded as so minor that they are not applied for patents and left unprotected by patents. Nevertheless, even if a technology that was generated as a byproduct is completely useless for the investing firm, it may be a treasure for firms in a different industry. A_t includes all these technologies, and an investment in technology generates many technologies that the investing firm cannot protect by patents.

Broadly speaking, there are two types of uncompensated knowledge spillovers: intra-sectoral knowledge spillovers (i.e., Marshall-Arrow-Romer [MAR] externalities; Marshall, 1890; Arrow, 1962; Romer, 1986) and inter-sectoral knowledge spillovers (i.e., Jacobs externalities; Jacobs, 1969). MAR theory assumes that knowledge spillovers between homogenous firms work out most effectively and that spillovers will therefore primarily emerge within one sector. As a result, uncompensated knowledge spillovers will be more active if the number of firms within a sector is larger. On the other hand, Jacobs (1969) argues that knowledge spillovers are most effective among firms that practice different activities and that diversification (i.e., a variety of sectors) is important for spillovers. As a result, uncompensated knowledge spillovers will be more active if the number of sectors in the economy is larger. Nevertheless, if all sectors have the same number of firms, an increase in the number of firms in the economy results in more active knowledge spillovers in any case, owing to either MAR externalities or Jacobs externalities.

Furthermore, as the volume of uncompensated knowledge spillovers increases, the investing firm's returns on the investment in A_t decrease. $\frac{\partial Y_t}{\partial A_t}$ indicates the total increase in Y_t in the economy by an increase in A_t, which consists of increases in both outputs in the firm that invested in the new technologies and outputs in other firms that utilize the newly invented technologies, whether the firms obtained the technologies by compensating the originating firm or by using uncompensated knowledge spillovers. If the number of firms becomes larger and uncompensated knowledge spillovers occur more actively, the compensated fraction in $\frac{\partial Y_t}{\partial A_t}$ that the investing firm can obtain becomes smaller, and the investing firm's returns on the investment in A_t also become smaller.

Complementarity of technologies also reduces the fraction of $\frac{\partial Y_t}{\partial A_t}$ that the investing firm can obtain. If a new technology is effective only if it is combined with some particular technologies, the return on the investment in technology will belong not only to the investing firm but to the firms that hold these particular technologies. For example, an innovation in software technology generated by a software company increases the sales and profits of computer hardware companies. The economy's productivity increases because of the innovation but the increased incomes are attributed not only to the firm that generated the innovation but also to the firms that hold complementary technologies. A part of $\frac{\partial Y_t}{\partial A_t}$ leaks to these firms. For them, the leaked income is a kind of rent revenue unexpectedly become obtainable thanks to the innovation. Most new technologies will have complementary technologies. In addition, as the number of firms increases, the number of firms that holds complementary technologies will also increase, and thereby these leaks will also increase.

Because of the uncompensated knowledge spillovers and the complementarity of technologies, therefore, the fraction of $\frac{\partial Y_t}{\partial A_t}$ that an investing firm can obtain on average will be comparatively small, i.e., ϖ will be far smaller than M_t except that M_t is very small,[8] and the fraction will decrease as M_t increases.

The production function is specified as $Y_t = A_t^{\alpha} f(K_t, L_t)$ where α $(0 < \alpha < 1)$ is a constant. Let $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$, $c_t = \frac{C_t}{L_t}$, and $n_t = \frac{\dot{L}_t}{L_t}$, and assume that $f(K_t, L_t)$ is homogenous of degree one. Thus $y_t = A_t^{\alpha} f(k_t)$ and $\dot{k}_t = y_t - c_t - \frac{v\dot{A}_t}{L_t} - n_t k_t$. By equation (A25), $A_t = \frac{\varpi \alpha f(k_t)}{\mu v f'(k_t)}$ because $\frac{\varpi \partial y_t}{\mu v \partial A_t} = \frac{\partial y_t}{\partial k_t} \Leftrightarrow \frac{\varpi \alpha}{\mu v} A_t^{\alpha - 1} f(k_t) = A_t^{\alpha} f'(k_t)$.

A2.1.2 Models with heterogeneous households

Three heterogeneities—heterogeneous time preference, risk aversion, and productivity—are examined in endogenous growth models, which are modified versions of the model shown in Section A2.1.1. First, suppose that there are two economies— economy 1 and economy 2—that are identical except for time preference, risk aversion, or productivity. The population growth rate is zero (i.e., $n_t = 0$). The economies are fully open to each other, and goods, services, and capital are freely transacted between them, but labor is immobilized in each economy. Note that the two-country models shown in this section can be extended to include numerous economies that have differing degrees of heterogeneity, which will be constructed in Section A2.2.

A2.1.2.1 Heterogeneous time preference model

First, a model in which the two economies are identical except for time preference is constructed. The rate of time preference of the representative household in economy 1 is $\theta_{P,1}$ and that in economy 2 is $\theta_{P,2}$, and $\theta_{P,1} < \theta_{P,2}$. The production function in economy 1 is $y_{1,t} = A_t^{\alpha} f(k_{1,t})$ and that in economy 2 is $y_{2,t} = A_t^{\alpha} f(k_{2,t})$, where $y_{\rho,t}$ and $k_{\rho,t}$ are, respectively, output and capital per capita in economy ρ in period t for ρ = 1, 2. The population of each economy is $\frac{L_t}{2}$; thus, the total for both is L_t, which is sufficiently large. Firms operate in both economies, and the number of firms is M_t. The current account balance in economy 1 is τ_t and

[8] If M_t is very small, the value of ϖ will be far smaller than that for sufficiently large M_t, because the number of firms that can benefit from an innovation is constrained owing to very small M_t. The very small number of firms indicates that the economy is not sufficiently sophisticated, and thereby the benefit of an innovation can not be fully realized in the economy. This constraint can be modeled as $\varpi = \tilde{\varpi}\left[1 - \left(1 - \tilde{\varpi}^{-1}\right)^{M_t}\right]$ where $\tilde{\varpi}(\geq 1)$ is a constant. Nevertheless, for sufficiently large M_t (i.e., in sufficiently sophisticated economies), the constraint is removed such that $\lim_{M_t \to \infty} \tilde{\varpi}\left[1 - \left(1 - \tilde{\varpi}^{-1}\right)^{M_t}\right] = \tilde{\varpi} = \varpi$.

that in economy 2 is $-\tau_t$. Because a balanced growth path requires Harrod neutral technological progress, the production functions are further specified as

$$y_{\rho,t} = A_t^{\alpha} k_{\rho,t}^{1-\alpha} \quad ;$$

thus, $Y_{\rho,t} = K_{\rho,t}^{1-\alpha}(A_t L_t)^{\alpha} \; (\rho = 1,2)$.

Because both economies are fully open, returns on investments in each economy are kept equal through arbitration such that

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} = \frac{\varpi}{2\mu v} \frac{\partial (y_{1,t} + y_{2,t})}{\partial A_t} = \frac{\partial y_{2,t}}{\partial k_{2,t}} \quad . \tag{A26}$$

Equation (A26) indicates that an increase in A_t enhances outputs in both economies such that $\frac{\partial Y_{i,t}}{\partial K_{i,t}} = \frac{\varpi}{M_t} \frac{\partial (Y_{1,t} + Y_{2,t})}{\partial (v A_t)}$, and because the population is equal ($\frac{L_t}{2}$), $\frac{\partial Y_{i,t}}{\partial K_{i,t}} = \frac{\partial y_{i,t}}{\partial k_{i,t}} = \frac{\varpi}{M_t} \frac{\partial (Y_{1,t} + Y_{2,t})}{\partial (v A_t)} = \frac{\varpi}{\mu L_t} \frac{\partial (y_{1,t} + y_{2,t})}{\partial (v A_t)} \frac{L_t}{2} = \frac{\varpi}{2\mu v} \frac{\partial (y_{1,t} + y_{2,t})}{\partial A_t}$. Therefore,

$$A_t = \frac{\varpi \alpha [f(k_{1,t}) + f(k_{2,t})]}{2\mu v f'(k_{1,t})} = \frac{\varpi \alpha [f(k_{1,t}) + f(k_{2,t})]}{2\mu v f'(k_{2,t})} \quad .$$

Because equation (A26) is always held through arbitration, equations $k_{1,t} = k_{2,t}$, $\dot{k}_{1,t} = \dot{k}_{2,t}$, $y_{1,t} = y_{2,t}$ and $\dot{y}_{1,t} = \dot{y}_{2,t}$ are also held. Hence,

$$A_t = \frac{\varpi \alpha f(k_{1,t})}{\mu v f'(k_{1,t})} = \frac{\varpi \alpha f(k_{2,t})}{\mu v f'(k_{2,t})} \quad .$$

In addition, because $\frac{\partial (y_{1,t} + y_{2,t})}{\partial A_{1,t}} = \frac{\partial (y_{1,t} + y_{2,t})}{\partial A_{2,t}}$ through arbitration, then $\dot{A}_{1,t} = \dot{A}_{2,t}$ is held.

The accumulated current account balance $\int_0^t \tau_s ds$ mirrors capital flows between the two economies. The economy with current account surpluses invests them in the other economy. Since $\frac{\partial y_{1,t}}{\partial k_{1,t}} \left(= \frac{\partial y_{2,t}}{\partial k_{2,t}} \right)$ are returns on investments, $\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds$ and $\frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$ represent income receipts or payments on the assets that an economy owns in the other economy. Hence,

$$\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

is the balance on goods and services of economy 1, and

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t$$

is that of economy 2. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies such that

$$\tau_t = g(k_{1,t}, k_{2,t}) \quad .$$

The representative household in economy 1 maximizes its expected utility

$$E\int_0^\infty u_1(c_{1,t}) \exp(-\theta_{P,1}\, t) dt \quad ,$$

subject to

$$\dot{k}_{1,t} = y_{1,t} + \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s\, ds - \tau_t - c_{1,t} - v\dot{A}_{1,t} \left(\frac{L_t}{2}\right)^{-1} \quad , \tag{A27}$$

and the representative household in economy 2 maximizes its expected utility

$$E\int_0^\infty u_2(c_{2,t}) \exp(-\theta_{P,2} t) dt \quad ,$$

subject to

$$\dot{k}_{2,t} = y_{2,t} - \frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds + \tau_t - c_{2,t} - v\dot{A}_{2,t} \left(\frac{L_t}{2}\right)^{-1} \quad , \tag{A28}$$

where $u_{\rho,t}$, $c_{\rho,t}$, and $\dot{A}_{\rho,t}$, respectively, are the utility function, per capita consumption, and the increase in A_t by R&D activities in economy ρ in period t for ρ = 1, 2; E is the expectation operator; and $\dot{A}_t = \dot{A}_{1,t} + \dot{A}_{2,t}$. Equations (A27) and (A28) implicitly assume that each economy does not have foreign assets or debt in period t = 0.

Because the production function is Harrod neutral and because $A_t = \frac{\varpi \alpha f(k_{1,t})}{\mu v f'(k_{1,t})}$ $= \frac{\varpi \alpha f(k_{2,t})}{\mu v f'(k_{2,t})}$ and $f = k_{\rho,t}^{1-\alpha}$, then

$$A_t = \frac{\varpi \alpha}{\mu v (1-\alpha)} k_{\rho,t}$$

and

$$\frac{\partial y_{\rho,t}}{\partial k_{\rho,t}} = \left(\frac{\varpi \alpha}{\mu v}\right)^\alpha (1-\alpha)^{1-\alpha} \quad .$$

Since $\dot{A}_{1,t} = \dot{A}_{2,t}$ and $\frac{\partial y_{1,t}}{\partial k_{1,t}} = \frac{\partial y_{2,t}}{\partial k_{2,t}}$, then

$$\dot{k}_{1,t} = y_{1,t} + \frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t - c_{1,t} - \frac{v\dot{A}_t}{2}\left(\frac{L_t}{2}\right)^{-1}$$
$$= \left(\frac{\varpi\alpha}{\mu v}\right)^{\alpha}(1-\alpha)^{-\alpha}k_{1,t} + \left(\frac{\varpi\alpha}{\mu v}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds - \tau_t - c_{1,t} - \frac{\varpi\alpha}{\mu L_t(1-\alpha)}\dot{k}_{1,t}$$

and

$$\dot{k}_{1,t} = \frac{\mu L_t(1-\alpha)}{\mu L_t(1-\alpha)+\varpi\alpha}\left[\left(\frac{\varpi\alpha}{\mu v}\right)^{\alpha}(1-\alpha)^{-\alpha}k_{1,t} + \left(\frac{\varpi\alpha}{\mu v}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds - \tau_t - c_{1,t}\right] .$$

Because L_t is sufficiently large and ϖ is far smaller than M_t, the problem of scale effects vanishes and thereby $\frac{\mu L_t(1-\alpha)}{\mu L_t(1-\alpha)+\varpi\alpha} = 1$.

Putting the above elements together, the optimization problem of economy 1 can be rewritten as

$$Max\ E\int_0^{\infty} u_1(c_{1,t})\exp(-\theta_{P,1}t)dt \quad ,$$

subject to

$$\dot{k}_{1,t} = \left(\frac{\varpi\alpha}{\mu v}\right)^{\alpha}(1-\alpha)^{-\alpha}k_{1,t} + \left(\frac{\varpi\alpha}{\mu v}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds - \tau_t - c_{1,t} \quad .$$

Similarly, that of economy 2 can be rewritten as

$$Max\ E\int_0^{\infty} u_2(c_{2,t})\exp(-\theta_{P,2}t)dt \quad ,$$

subject to

$$\dot{k}_{2,t} = \left(\frac{\varpi\alpha}{\mu v}\right)^{\alpha}(1-\alpha)^{-\alpha}k_{2,t} - \left(\frac{\varpi\alpha}{\mu v}\right)^{\alpha}(1-\alpha)^{1-\alpha}\int_0^t \tau_s ds + \tau_t - c_{2,t} \quad .$$

A2.1.2.2 Heterogeneous risk aversion model

The basic structure of the model with heterogeneous risk aversion is the same as that of heterogeneous time preference. The two economies are identical except in regard to risk aversion. The degree of relative risk aversion of economy 1 is $\varepsilon_1 = -\frac{c_{1,t}\, u_1''}{u_1'}$ and that of economy 2 is $\varepsilon_2 = -\frac{c_{2,t}\, u_2''}{u_2'}$, which are constant, and $\varepsilon_1 < \varepsilon_2$. The optimization problem of economy 1 is

$$Max\ E\int_0^\infty u_1(c_{1,t})\exp(-\theta_P t)dt \quad ,$$

subject to

$$\dot{k}_{1,t} = \left(\frac{\varpi\alpha}{\mu\nu}\right)^\alpha (1-\alpha)^{-\alpha} k_{1,t} + \left(\frac{\varpi\alpha}{\mu\nu}\right)^\alpha (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds - \tau_t - c_{1,t} \quad ,$$

and that of economy 2 is

$$Max\ E\int_0^\infty u_2(c_{2,t})\exp(-\theta_P t)dt \quad ,$$

subject to

$$\dot{k}_{2,t} = \left(\frac{\varpi\alpha}{\mu\nu}\right)^\alpha (1-\alpha)^{-\alpha} k_{2,t} - \left(\frac{\varpi\alpha}{\mu\nu}\right)^\alpha (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds + \tau_t - c_{2,t} \quad .$$

A2.1.2.3 Heterogeneous productivity model

With heterogeneous productivity, the production function is heterogeneous, not the utility function. Because technology A_t is common to both economies, a heterogeneous production function requires heterogeneity in elements other than technology. Prescott (1998) argues that unknown factors other than technology have made total factor productivity (TFP) heterogeneous across countries. Harashima (2009) argues that average workers' innovative activities are an essential element of productivity and make TFP heterogeneous across workers, firms, and economies. Since average workers are human and capable of creative intellectual activities, they can create innovations even if their innovations are minor. It is rational for firms to exploit all the opportunities that these ordinary workers' innovative activities offer. Furthermore, innovations created by ordinary workers are indispensable for efficient production. A production function incorporating average workers' innovations has been shown to have a Cobb-Douglas functional form with a labor share of about 70% (Harashima 2009), such that

$$Y_t = \bar{\sigma}\omega_A\omega_L A_t^\alpha K_t^{1-\alpha} L_t^\alpha \quad , \tag{A29}$$

where ω_A and ω_L are positive constant parameters with regard to average workers' creative activities, and $\bar{\sigma}$ is a parameter that represents a worker's accessibility limit to capital with regard to location. The parameters ω_A and ω_L are independent of A_t but are dependent on the creative activities of average workers. Thereby, unlike with technology A_t, these parameters can be heterogeneous across workers, firms, and economies.

In this model of heterogeneous productivity, it is assumed that workers whose households belong to different economies have different values of ω_A and ω_L. In addition, only productivity that is represented by $\bar{\sigma}\omega_A\omega_L A_t^\alpha$ in equation (A29) is heterogeneous between the two economies. The production function of economy 1 is $y_{1,t} = \omega_1^\alpha A_t^\alpha f(k_{1,t})$ and that of economy 2 is $y_{2,t} = \omega_2^\alpha A_t^\alpha f(k_{2,t})$, where $\omega_1\ (0 < \omega_1 \le 1)$ and $\omega_2\ (0 < \omega_2 \le 1)$ are constants and

$\omega_2 < \omega_1$. Since $\frac{\partial Y_{\rho,t}}{\partial K_{\rho,t}} = \frac{\partial y_{\rho,t}}{\partial k_{\rho,t}} = M_t^{-1}\frac{\partial(Y_{1,t}+Y_{2,t})}{\partial(vA_t)} = \frac{\varpi}{\mu L_t}\frac{\partial(y_{1,t}+y_{2,t})}{\partial(vA_t)}\frac{L_t}{2} = \frac{\varpi}{2\mu L_t}\frac{\partial(y_{1,t}+y_{2,t})}{\partial A_t}$ by equation (A26), then

$$A_t = \frac{\varpi\alpha[\omega_1^\alpha f(k_{1,t})+\omega_2^\alpha f(k_{2,t})]}{2\mu v\,\omega_1^\alpha f'(k_{1,t})} = \frac{\varpi\alpha[\omega_1^\alpha f(k_{1,t})+\omega_2^\alpha f(k_{2,t})]}{2\mu v\,\omega_2^\alpha f'(k_{2,t})} \quad . \tag{A30}$$

Because equation (A26) is always held through arbitration, equations $k_{1,t} = \frac{\omega_1}{\omega_2}k_{2,t}$, $\dot{k}_{1,t} = \frac{\omega_1}{\omega_2}\dot{k}_{2,t}$, $y_{1,t} = \frac{\omega_1}{\omega_2}y_{2,t}$, and $\dot{y}_{1,t} = \frac{\omega_1}{\omega_2}\dot{y}_{2,t}$ are also held. In addition, since $\frac{\partial(y_{1,t}+y_{2,t})}{\partial A_{1,t}} = \frac{\partial(y_{1,t}+y_{2,t})}{\partial A_{2,t}}$ by arbitration, $\dot{A}_{1,t} = \frac{\omega_1}{\omega_2}\dot{A}_{2,t}$ is held. Because of equation (A30) and $f = \omega_\rho^\alpha k_{\rho,t}^{1-\alpha}$, then $A_t = \frac{\varpi\alpha}{2\mu v(1-\alpha)\omega_1^\alpha}\left(\omega_1^\alpha k_1 + \omega_2^\alpha k_1^\alpha k_2^{1-\alpha}\right) = \frac{\varpi\alpha}{2\mu v(1-\alpha)\omega_2^\alpha}\left(\omega_1^\alpha k_1^{1-\alpha}k_2^\alpha + \omega_2^\alpha k_2\right)$, $\frac{\omega_1^\alpha k_1 + \omega_2^\alpha k_1^\alpha k_2^{1-\alpha}}{\omega_1^\alpha} = \frac{\omega_1^\alpha k_1^{1-\alpha}k_2^\alpha + \omega_2^\alpha k_2}{\omega_2^\alpha}$, and $\frac{\partial y_{\rho,t}}{\partial k_{\rho,t}} = \left(\frac{\varpi\alpha}{2\mu v}\right)^\alpha (1-\alpha)^{1-\alpha}\left(\omega_1^\alpha k_1 + \omega_2^\alpha k_1^\alpha k_2^{1-\alpha}\right)^\alpha k_1^{-\alpha} = \left(\frac{\varpi\alpha}{2\mu v}\right)^\alpha (1-\alpha)^{1-\alpha}\left(\omega_1^\alpha k_1^{1-\alpha}k_2^\alpha + \omega_2^\alpha k_2\right)^\alpha k_2^{-\alpha}$. Since $\frac{\omega_2}{\omega_1}k_{1,t} = k_{2,t}$, then $\frac{\omega_1^\alpha k_1 + \omega_2^\alpha k_1^\alpha k_2^{1-\alpha}}{\omega_1^\alpha} = \frac{\omega_1^\alpha k_1 + \omega_2^\alpha k_1^\alpha\left(\frac{\omega_2}{\omega_1}\right)^{1-\alpha} k_1^{1-\alpha}}{\omega_1^\alpha} = k_1\left(1+\omega_1^{-1}\omega_2\right)$ and $\frac{\omega_1^\alpha k_1^{1-\alpha}k_2^\alpha + \omega_2^\alpha k_2}{\omega_2^\alpha} = \frac{\omega_1^\alpha k_1^{1-\alpha}\left(\frac{\omega_2}{\omega_1}\right)^\alpha k_1^\alpha + \omega_2^\alpha\frac{\omega_2}{\omega_1}k_1}{\omega_2^\alpha} = k_1 + \frac{\omega_2}{\omega_1}k_1 = k_1\left(1+\omega_1^{-1}\omega_2\right) = k_2\left(1+\omega_1\omega_2^{-1}\right)$. Hence,

$$A_t = k_1\frac{\varpi\alpha\left(1+\omega_1^{-1}\omega_2\right)}{2\mu v(1-\alpha)} = k_2\frac{\varpi\alpha\left(1+\omega_1\omega_2^{-1}\right)}{2\mu v(1-\alpha)} \quad ,$$

and

$$\frac{\partial y_{\rho,t}}{\partial k_{\rho,t}} = \left(\frac{\omega_1+\omega_2}{2}\right)^\alpha\left(\frac{\varpi\alpha}{\mu v}\right)^\alpha(1-\alpha)^{1-\alpha}$$

for ρ = 1, 2. Because $\dot{A}_{1,t} = \left(\frac{\omega_2}{\omega_1}\right)^{-\frac{1}{\alpha}}\dot{A}_{2,t}$ (i.e., $\dot{A}_t = \dot{A}_{1,t} + \dot{A}_{2,t} = \left(1+\omega_1^{-1}\omega_2\right)\dot{A}_{1,t}$) and $\frac{\partial y_{1,t}}{\partial k_{1,t}} = \frac{\partial y_{2,t}}{\partial k_{2,t}}$, then

$$\dot{k}_{1,t} = y_{1,t} + \frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t - c_{1,t} - v\dot{A}_{1,t} \left(\frac{L_t}{2}\right)^{-1}$$

$$= y_{1,t} + \frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t - c_{1,t} - v\dot{A}_t \left(1 + \omega_1^{-1}\omega_2\right)^{-1} \left(\frac{L_t}{2}\right)^{-1}$$

$$= \omega_1^{\alpha} \left[\frac{\left(1+\omega_1^{-1}\omega_2\right)\varpi\alpha}{2\mu v(1-\alpha)}\right]^{\alpha} k_{1,t} + \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2\mu v}\right]^{\alpha} (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds - \tau_t - c_{1,t} - \frac{\varpi\alpha}{\mu L_t(1-\alpha)} \dot{k}_{1,t} \quad ,$$

and

$$\dot{k}_{1,t} = \frac{\mu L_t(1-\alpha)}{\mu L_t(1-\alpha) + \varpi\alpha} \left\{ \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2\mu v(1-\alpha)}\right]^{\alpha} k_{1,t} + \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2\mu v}\right]^{\alpha} (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds - \tau_t - c_{1,t} \right\} \quad .$$

Because L_t is sufficiently large and ϖ is far smaller than M_t and thus $\frac{\mu L_t(1-\alpha)}{\mu L_t(1-\alpha)+\varpi\alpha} = 1$, the optimization problem of economy 1 is

$$Max\ E\int_0^{\infty} u_1(c_{1,t}) \exp(-\theta_P t) dt \quad ,$$

subject to

$$\dot{k}_{1,t} = \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2\mu v(1-\alpha)}\right]^{\alpha} k_{1,t} + \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2\mu v}\right]^{\alpha} (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds - \tau_t - c_{1,t} \quad ,$$

and similarly, that of economy 2 is

$$Max\ E\int_0^{\infty} u_2(c_{2,t}) \exp(-\theta_P t) dt \quad ,$$

subject to

$$\dot{k}_{2,t} = \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2\mu v(1-\alpha)}\right]^{\alpha} k_{2,t} - \left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2\mu v}\right]^{\alpha} (1-\alpha)^{1-\alpha} \int_0^t \tau_s ds + \tau_t - c_{2,t} \quad .$$

A2.1.3 Sustainable heterogeneity

Heterogeneity is defined as being sustainable if all the optimality conditions of all heterogeneous households are satisfied indefinitely. The nature of sustainable heterogeneity is examined in a multi-country model of heterogeneous time preference, risk aversion and productivity, which is constructed by combining the three models in the previous section (see Harashima, 2010).

Suppose that there are N economies with identical population, and let $\tau_{\rho,\varsigma,t}$ be the current account balance of economy ρ with economy ς, where $\rho = 1, 2, \ldots, N$, $\varsigma = 1, 2, \ldots, N$, and $\rho \neq \varsigma$.

Proposition: If and only if

$$\lim_{t\to\infty}\frac{\dot{c}_{\rho,t}}{c_{\rho,t}}=\left(\frac{\sum_{q=1}^{N}\varepsilon_q\omega_q}{\sum_{q=1}^{N}\omega_q}\right)^{-1}\left\{\left[\frac{\varpi\alpha\sum_{q=1}^{N}\omega_q}{N\mu v(1-\alpha)}\right]^{\alpha}-\frac{\sum_{q=1}^{N}\theta_{P,q}\omega_q}{\sum_{q=1}^{N}\omega_q}\right\} \tag{A31}$$

for any ρ (= 1, 2, … , N), all the optimality conditions of all heterogeneous economies are satisfied at steady state such that $\lim_{t\to\infty}\frac{\dot{c}_{\rho,t}}{c_{\rho,t}}$, $\lim_{t\to\infty}\frac{\dot{k}_{\rho,t}}{k_{\rho,t}}$, and $\lim_{t\to\infty}\frac{\dot{\tau}_{\rho,\varsigma,t}}{\tau_{\rho,\varsigma,t}}$ are constants, and

$$\lim_{t\to\infty}\frac{\dot{c}_{\rho,t}}{c_{\rho,t}}=\lim_{t\to\infty}\frac{\dot{k}_{\rho,t}}{k_{\rho,t}}=\lim_{t\to\infty}\frac{\dot{y}_{\rho,t}}{y_{\rho,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\lim_{t\to\infty}\frac{\dot{\tau}_{\rho,\varsigma,t}}{\tau_{\rho,\varsigma,t}}=\lim_{t\to\infty}\frac{\frac{d\int_0^t\tau_{\rho,\varsigma,s}ds}{dt}}{\int_0^t\tau_{\rho,\varsigma,s}ds}$$

for any ρ and ς ($\rho\neq\varsigma$).
Proof: See Harashima (2010).

On the balanced growth path satisfying the condition shown in Proposition, heterogeneities in time preference, risk aversion and productivity are sustainable by definition because all the optimality conditions of the two economies are indefinitely satisfied. Economies that keep sustainable heterogeneity constitute a combined economy with productivity $\left[N^{-1}\sum_{q=1}^{N}\omega_q\right]^{\alpha}A_t^{\alpha}$ and the time preference rate $\left(\sum_{q=1}^{N}\omega_q\right)^{-1}\sum_{q=1}^{N}\theta_{P,q}\omega_q$ and degree of risk aversion $\left(\sum_{q=1}^{N}\omega_q\right)^{-1}\sum_{q=1}^{N}\varepsilon_q\omega_q$ of the representative household. The nature of sustainable heterogeneity is examined more in detail in Harashima (2010).

A2.2 The law of motion for inflation in the multi-country endogenous growth model

A2.2.1 The monetary union

Suppose that there is a monetary union that consists of N member states and one currency. There is no federal government in the monetary union, and fiscal policies are therefore implemented separately by each member. Monetary policies are unified and implemented only by the central bank of the monetary union, which is sufficiently independent of the member states. For simplicity, population in each member state is assumed to be identical and constant, and the total population in the monetary union is sufficiently large. Sustainable heterogeneity, as shown in Section A2.1.3, is kept. The time preference rate of the representative household in member state ρ is $\theta_{P,\rho}$ for ρ = 1, 2, …, N. The time preference rate of the government of member state ρ is $\theta_{G,\rho}$ and $\theta_{G,\rho}>\theta_{P,\rho}$. Suppose for simplicity that $-g_{\rho,t}\left[\frac{\partial u_{G,\rho}(g_{\rho,t},x_{\rho,t})}{\partial g_{\rho,t}}\right]^{-1}\frac{\partial^2 u_{G,\rho}(g_{\rho,t},x_{\rho,t})}{\partial g_{\rho,t}^2}$

$= \varepsilon_{g,\rho}$ and $-x_{\rho,t}\left[\frac{\partial u_{G,\rho}\left(g_{\rho,t},x_{\rho,t}\right)}{\partial x_{\rho,t}}\right]^{-1}\frac{\partial^2 u_{G,\rho}\left(g_{\rho,t},x_{\rho,t}\right)}{\partial x_{\rho,t}^{\ 2}} = \varepsilon_{x,\rho}$ are positive and constant, similar to the degree of relative risk aversion of households. ε_g and ε_x are, respectively, identical across member governments, such that $\varepsilon_{g,\rho} = \varepsilon_g$ and $\varepsilon_{x,\rho} = \varepsilon_x$ for any ρ.

As shown in Section A2.1.3, economies that keep sustainable heterogeneity constitute a combined economy with productivity $\left[N^{-1}\sum_{q=1}^{N}\omega_q\right]^{\alpha} A_t^{\alpha}$ and the time preference rate $\left(\sum_{q=1}^{N}\omega_q\right)^{-1}\sum_{q=1}^{N}\theta_{P,q}\omega_q$ and degree of risk aversion $\left(\sum_{q=1}^{N}\omega_q\right)^{-1}\sum_{q=1}^{N}\varepsilon_q\omega_q$ of the representative household. The combined economy grows at the constant rate

$$\lim_{t\to\infty}\frac{\dot{c}_t}{c_t} = \left(\frac{\sum_{q=1}^{N}\varepsilon_q\omega_q}{\sum_{q=1}^{N}\omega_q}\right)^{-1}\left\{\left[\frac{\varpi\alpha\sum_{q=1}^{N}\omega_q}{N\mu\nu(1-\alpha)}\right]^{\alpha} - \frac{\sum_{q=1}^{N}\theta_{P,q}\omega_q}{\sum_{q=1}^{N}\omega_q}\right\}$$

(equation [A31]) where c_t is the consumption of the representative household of the monetary union. Because sustainable heterogeneity is kept in the monetary union, the time preference rate of the representative household of the monetary union is $\left(\sum_{q=1}^{N}\omega_q\right)^{-1}\sum_{q=1}^{N}\theta_{P,q}\omega_q = \theta_P$. Because the size of an economy in the monetary union is measured by the productivity differential parameter ω_ρ, the integrated time preference rate of government in the monetary union is $\left(\sum_{q=1}^{N}\omega_q\right)^{-1}\sum_{q=1}^{N}\theta_{G,\rho}\omega_q = \theta_G$. The overall rate of inflation in the monetary union is π_t. The central bank of the monetary union sets θ_G equal to θ_P so that π_t will not accelerate.

Because

$$\frac{\partial y_t}{\partial k_t} = \left(N^{-1}\sum_{q=1}^{N}\omega_q\right)^{\alpha}\left(\frac{\varpi\alpha}{\mu\nu}\right)^{\alpha}(1-\alpha)^{1-\alpha} = \text{constant},$$

then

$$r_t = \left(N^{-1}\sum_{q=1}^{N}\omega_q\right)^{\alpha}\left(\frac{\varpi\alpha}{\mu\nu}\right)^{\alpha}(1-\alpha)^{1-\alpha} = \bar{r} \qquad \text{(A32)}$$

across the monetary union, where y_t, k_t, and r_t are the per capita outputs, per capita capital inputs, and the real interest rate in the monetary union, respectively, and $\bar{r}$ is a constant. By equation (A9),

$$\left(\sum_{q=1}^{N}\omega_q\right)^{-1}\sum_{q=1}^{N}\varepsilon_q\omega_q\frac{\dot{c}_t}{c_t} + \theta_P = \bar{r} \qquad \text{(A33)}$$

in the monetary union. By equations (A31), (A32), and (A33),

$$\theta_p = \bar{r} - \left(\sum_{q=1}^{N}\omega_q\right)^{-1}\sum_{q=1}^{N}\varepsilon_q\omega_q\frac{\dot{c}_t}{c_t} = \bar{r} - \left[\frac{\varpi\alpha\sum_{q=1}^{N}\omega_q}{N\mu v(1-\alpha)}\right]^{\alpha} + \theta_p = \text{constant} \tag{A34}$$

if sustainable heterogeneity is kept.

A2.2.2 The law of motion for inflation

Each member government maximizes its expected utility subject to constraints. However, unlike the single-country model, the constraint is not limited to equation (A3). The real current account balance ($\eta_{\rho,t}$) within the monetary union in each member state must be stable at steady state; thus,

$$\lim_{t\to\infty}\frac{\dot{\eta}_{\rho,t}}{\eta_{\rho,t}} = \lim_{t\to\infty}\frac{\dot{c}_{\rho,t}}{c_{\rho,t}} \tag{A35}$$

because the economy of the monetary union otherwise eventually collapses. As will be discussed in Section 4.2.4, current account balances depend on inflation differentials; thereby, $\eta_{\rho,t}$ is a function of $\pi_{1,t},\pi_{2,t}, \ldots, \pi_{N,t}$ and thus a function of $\theta_{G,1,t},\theta_{G,2,t}, \ldots, \theta_{G,N,t}$ and θ_P, such that

$$\eta_{i,t} = h\left(\theta_{G,1,t}, \theta_{G,2,t}, \ldots, \theta_{G,N,t}, \theta_P\right) ,$$

where $\pi_{\rho,t}$ is the rate of inflation in member state ρ. Each member government therefore maximizes its expected utility subject to not only equation (A3) but also equation (A35).

By equations (A18), (A32) and (A34), the law of motion for inflation is

$$\theta_{G,\rho} = \left[\frac{\varpi\alpha\sum_{q=1}^{N}\omega_q}{N\mu v(1-\alpha)}\right]^{\alpha} + \int_{t-1}^{t}\int_{s}^{s+1}\pi_{\rho,\upsilon}\, d\upsilon\, ds - \pi_{\rho,t} \quad .$$

Therefore, if

$$\theta_{G,\rho} = \theta_G = \left[\frac{\varpi\alpha\sum_{q=1}^{N}\omega_q}{N\mu v(1-\alpha)}\right]^{\alpha} \tag{A36}$$

for any ρ, inflation does not accelerate in the monetary union (i.e., $\dot{\pi}_t = 0$). Because

$\left[\dfrac{\varpi\alpha\sum_{q=1}^{N}\omega_q}{N\mu v(1-\alpha)}\right]^{\alpha}-\theta_P>0$, equation (A36) indicates that, unlike the exogenous growth model shown in Section A1 in which inflation does not accelerate if $\theta_G=\theta_P$, inflation does not accelerate in the framework of endogenous growth if $\theta_G>\theta_P$.

However, an additional element in the behavior of government should also be considered in the framework of endogenous growth. Unlike the exogenous growth model, the economy grows at a constant rate in the endogenous growth model. As y_t increases, x_t and g_t will not remain at the same level as before because, as the economy grows, the capability of the government to collect and spend money also increases. Conversely, as the economy grows, the utility obtained by spending a unit of g_t and disutility generated by imposing a unit of x_t decrease. Considering this scale effect, the utility function of government is replaced with

$$u_{G,\rho}\left(g_{\rho,t},x_{\rho,t},y_{\rho,t}\right) .$$

Notice that $y_{\rho,t}$ is exogenous for the government, although it is endogenous for households. The constant endogenous growth of $y_{\rho,t}$ is perceived by the government as successive exogenous shocks on $y_{\rho,t}$ in $u_{G,\rho}\left(g_{\rho,t},x_{\rho,t},y_{\rho,t}\right)$. When an exogenous upward shock of $y_{\rho,t}$ occurs, larger $g_{\rho,t}$ and $x_{\rho,t}$ are optimal for the government because of the scale effect; thus, $g_{\rho,t}$ and $x_{\rho,t}$ begin to increase on the transition path to the new steady state. The government perceives that exogenous shocks on $y_{\rho,t}$ continuously occur because of the constant endogenous growth of $y_{\rho,t}$; thereby, $g_{\rho,t}$ and $x_{\rho,t}$ continuously move to new transition paths. Because $\varepsilon_{g,\rho}\dfrac{\dot{g}_{\rho,t}}{g_{\rho,t}}+\theta_{G,\rho}=r_t+\int_{t-1}^{t}\int_{s}^{s+1}\pi_{\rho,\upsilon}\,d\upsilon\;ds-\pi_{\rho,t}$, $\varepsilon_{x,\rho}\dfrac{\dot{x}_{\rho,t}}{x_{\rho,t}}+\theta_{G,\rho}=r_t+\int_{t-1}^{t}\int_{s}^{s+1}\pi_{\rho,\upsilon}\,d\upsilon\;ds-\pi_{\rho,t}$, $\dfrac{\dot{g}_{\rho,t}}{g_{\rho,t}}>0$, and $\dfrac{\dot{x}_{\rho,t}}{x_{\rho,t}}>0$ on transition paths, and because $\varepsilon_{g,\rho}>0$ and $\varepsilon_{x,\rho}>0$, then by equations (A32) and (A34), if

$$\theta_{G,\rho}+\Omega=\theta_G+\Omega=\left[\frac{\varpi\alpha\sum_{q=1}^{N}\omega_q}{N\mu v(1-\alpha)}\right]^{\alpha} \tag{A37}$$

is always satisfied for any ρ, inflation does not accelerate where Ω is a positive variable. Hence, if $\Omega=\left[\dfrac{\varpi\alpha\sum_{q=1}^{N}\omega_q}{N\mu v(1-\alpha)}\right]^{\alpha}-\theta_P$, then when the central bank of the monetary union keeps $\theta_G=\theta_P$, inflation does not accelerate even in the framework of endogenous growth.

A3 THE EURO'S FLAW

A3.1 The basic structure

The euro is examined in this section using the model of monetary union constructed in Section 3.

For simplicity, the degree of relative risk aversion of the representative household is assumed to be identical in all euro member states. Productivity and the time preference rate of the representative household, however, are assumed to be heterogeneous across member states. The productivity differential parameter in member state ρ (ω_ρ) is given exogenously and is constant for any ρ. The time preference rate of the representative household in member state ρ is $\theta_{P,\rho}$ and is inversely correlated to the productivity differential parameter ω_ρ (see, e.g., Lawrance, 1991; Samwick, 1998; Ventura, 2003). The integrated time preference rate of the representative household of all member states is $\left(\sum_{q=1}^{N}\omega_q\right)^{-1}\sum_{q=1}^{N}\theta_{P,q}\omega_q = \theta_P$. The intrinsic time preference rate of the government of member state ρ is $\bar{\theta}_{G,\rho}$ and $\bar{\theta}_{G,\rho} > \theta_{P,\rho}$. The actual time preference rate of the government of member state ρ in period t is $\theta_{G,\rho,t}$, and it is time variable because of control by the ECB. The integrated actual time preference rate of all member governments in period t is $\left(\sum_{q=1}^{N}\omega_q\right)^{-1}\sum_{q=1}^{N}\theta_{G,\rho,t}\omega_q = \theta_{G,t}$. For simplicity, suppose that, because of the scale effect shown in Section 3.2.2, $\Omega = \left[\frac{\varpi\alpha\sum_{q=1}^{N}\omega_q}{N\mu\nu(1-\alpha)}\right]^{\alpha} - \theta_P$ is always satisfied. Hence, the ECB needs to set $\theta_{G,t}$ equal to θ_P not to accelerate the overall rate of inflation in the euro area. Because $\theta_{G,t} = \theta_P$ is kept by the ECB, each government is expected to adjust its time preference rate $\theta_{G,\rho,t}$ equal to θ_P. If a government does not sufficiently adjust its $\theta_{G,\rho,t}$ and sets $\theta_{G,\rho,t} > \theta_P$, it deviates from the expected behavior; such behavior is called a “deviation” hereafter.

A3.2 Factors that generate the flaw

A3.2.1 Adhering to own preferences

The law of motion for inflation shown in equation (A20) indicates that inflation does not accelerate because a government acts in a stupid, foolish, or irrational manner, but rather because it behaves quite normally—it adheres to its intrinsic time preference unless an independent neutral institution (i.e., a central bank) forces it to stop doing so. However, a fundamental question arises. Even if the government is acting quite normally, is this behavior rational? In economics, rationality usually means that, given the available information, optimal decisions to achieve an objective are taken and rational behavior is generally assumed. However, can rational behavior still prevail when a government cannot optimize its behavior to achieve its objective? This special situation emerges if the central bank is perfectly independent and is firmly determined to stabilize inflation and if, at the same time, the intrinsic time preference rate of government is unchangeable. In this situation, the economy will destabilize and eventually collapse as shown in Section 2. Therefore, the government cannot achieve its objective (i.e., cannot maximize its expected utility) and can only behave irrationally in this case. Conversely, if the government wants to optimize its objective and behave rationally, it must change its time preference. Clearly, trade-offs between rationality and time preference exist in some situations, and either rationality or time preference must be endogenized.

Nevertheless, it is highly unlikely that people will not optimize their behavior to meet their objectives (i.e., maximize utility) if they have complete knowledge of the optimal path. Hence, rationality should prevail over preferences, and time preference will be endogenized when a clash between rationality and time preference occurs. If time preference is endogenized, rational decisions become possible.

Even though rationality should eventually prevail over preferences, governments will not easily change their own preferences. They will resist endogenizing them and search for options to escape from doing so—it is this stubborn nature that drives governments to deviate from the path specified by the ECB. Section 2 indicated that the inflation problem is equivalent to the deviation problem. The mechanism of inflation differentials in the euro area, therefore, must be fully examined considering this driving force of deviation.

Even though unfavorable consequences are expected if no change is made, it can be very difficult to change one's own preferences alone. Controlling preferences therefore usually requires the help of other people or institutions, which is one of the reasons why independent central banks were established to stabilize inflation. Nevertheless, as will be examined in the following sections, the question arises whether the ECB can fully control each member government's desire to adhere to its own time preference rate.

A3.2.2 The limited capability of the ECB

The ECB faces a problem that most other central banks do not face. There are an infinite number of combinations of $\theta_{G,\rho,t}$ that satisfy $\theta_{G,t} = \left(\sum_{q=1}^{N} \omega_q \right)^{-1} \sum_{q=1}^{N} \theta_{G,\rho,t} \omega_q = \theta_P$, but the ECB cannot force its member governments to select the combination that it wants them to select. That is, the ECB cannot separately control $\theta_{G,\rho,t}$, only $\theta_{G,t}$ collectively.

As shown in Section 2.2, the central bank in the single-country model punishes the government's deviation by raising the nominal interest rate such that $i_t = \theta_{G,t} + \pi_t + \psi$. Equation $\theta_{G,t} = i_t - \pi_t$ (equation [A22]) is not satisfied until the government obeys the central bank and lowers $\theta_{G,t}$. However, the ECB cannot effectively impose ψ separately on each member state; thereby, equation $\theta_{G,\rho,t} = i_{\rho,t} - \pi_{\rho,t}$ can be satisfied in a member state even though $\theta_{G,\rho,t} > \theta_{G,t} = \theta_P$. Even if a government behaves based on its own $\bar{\theta}_{G,\rho}$ that is different from θ_P, the ECB can neither punish nor force the government to transition to $\theta_{G,\rho,t} = \theta_{G,t} = \theta_P$. As a result, the combination of $\theta_{G,\rho,t}$ is not selected only by the ECB but rather through conflict, negotiation, and cooperation among the member governments. Thereby, the possibility exists that, at the same time, $\theta_{G,\rho,t} > \theta_P$ for some member states and $\theta_{G,\rho,t} < \theta_P$ for others and $\theta_{G,t} = \theta_P$ is kept. Unlike most other central banks, independence is not sufficient for the ECB to fully stabilize inflation.

A3.2.3 Diverse inflation rates owing to non-tradability

Because all member states use the same currency, the price level would be identical across the euro area by arbitrage if all goods and services were traded freely inside the euro area. However, not all goods and services are tradable. If anything, the share of non-tradable goods and services in the euro area is large (e.g., Altissimo et al., 2005). Unlike tradable goods and services, the prices of non-tradable goods and services are not equalized by arbitrage. This price heterogeneity indicates that the rate of inflation in each member state can also be heterogeneous, and heterogeneous inflation indicates that governments may deviate from the path the ECB sets, at least temporarily. Member governments may enjoy periods when they behave based on their own intrinsic $\bar{\theta}_{G,\rho}$ that is higher than $\theta_{G,t} = \theta_P$.

Note that even though inflation is heterogeneous, the marginal product of capital in every industry in every member state is kept identical by arbitrage; that is, $\frac{\partial y_{\rho,t}}{\partial k_{\rho,t}} = \theta_P$, because capital flows freely within the euro area.

A3.2.4 Current account imbalances owing to inflation differentials

Inflation differentials will lead to current account imbalances (e.g., Blanchard, 2007; Arghyrou and Chortareas, 2008; EC, 2009). Although inflation rates diverge among the member states, the prices of tradable goods and services are still generally equalized across the euro area by arbitrage. The equalization is realized by outflows of cheaper tradable goods and services from member states with lower inflation member states to the states with higher inflation. The inflowing goods and services eventually will need to be purchased with money from the exporting member states (lower inflation states) because the importing states (higher inflation states) are not obtaining money by exporting either their higher priced tradable goods or their non-tradable goods and services. A large part of borrowed money, therefore, is used not for investment but for consumption in the higher inflation member states. As a result, the trade and current account balances in member states with higher inflation will show continuous deficits.

A3.3 The mechanism of the flaw

A3.3.1 The utility functional

Considering the conflict between rationality and preference, the government's utility function $u_{G,\rho}(g_{\rho,t}, x_{\rho,t})$ is extended to the functional consisting of the utility function $u_{G,\rho}(g_{\rho,t}, x_{\rho,t})$ and a variable $\tilde{\theta}_{G,\rho,t}$ such that

$$\tilde{u}_{G,\rho}\left[u_{G,\rho}(g_{\rho,t}, x_{\rho,t}), \tilde{\theta}_{G,\rho,t}\right] ,$$

where $\tilde{\theta}_{G,\rho,t} = \bar{\theta}_{G,\rho} - \theta_{G,\rho,t}$.[9] The government has a strong desire to behave based on its intrinsic time preference rate but it has to change its rate for it to behave rationally under the control of the ECB. $\tilde{\theta}_{G,\rho,t}$ represents the gap between the reality ($\theta_{G,\rho,t}$) and the desire ($\bar{\theta}_{G,\rho}$). For simplicity, only the case $\tilde{\theta}_{G,\rho,t} \geq 0$ is examined. The functional has the following properties:

$$\frac{\partial \tilde{u}_{G,\rho}\left[u_{G,\rho}(g_{\rho,t}, x_{\rho,t}), \tilde{\theta}_{G,\rho,t}\right]}{\partial \tilde{\theta}_{G,\rho,t}} < 0 \quad \text{and} \quad \frac{\partial^2 \tilde{u}_{G,\rho}\left[u_{G,\rho}(g_{\rho,t}, x_{\rho,t}), \tilde{\theta}_{G,\rho,t}\right]}{\partial \tilde{\theta}_{G,\rho,t}^2} > 0 \ .$$

The more $\theta_{G,\rho,t}$ is forced to decrease, the more utility decreases, but the magnitude of the decrease diminishes as the scale of the forced decrease increases. As a whole, each member government maximizes its expected utility

$$E\int_0^\infty \tilde{u}_{G,\rho}\left[u_{G,\rho}(g_{\rho,t}, x_{\rho,t}), \tilde{\theta}_{G,\rho,t}\right]\int_0^t \exp\left(-\theta_{G,\rho,s}\right) ds\, dt$$

subject to equations (A3) and (A35). In addition, the ECB always keeps $\theta_{G,t} = \theta_P$ for any t, by which $\theta_{G,\rho,t}$ is endogenized.

If there is an inflation differential, equation (A35) cannot be satisfied. Hence, a necessary condition for satisfying equation (A35) is $\theta_{G,\rho,t} = \theta_{G,t} = \theta_P$ for any ρ, and an indefinite deviation is therefore impossible. Although an indefinite deviation is impossible, temporary and intermittent deviations may be possible. However, because of equation (A35) and $\theta_{G,t} = \theta_P$, deviations necessitate future corrections. Because deviations increase current

[9] The utility function of government is $u_{G,\rho}(g_{\rho,t}, x_{\rho,t}, y_{\rho,t})$ if the scale effect shown in Section 3.2.2 is explicitly considered. However, for simplicity, the scale effect is made implicit and $u_{G,\rho}(g_{\rho,t}, x_{\rho,t})$ is used.

account deficits and, consequently, external debt burdens (see Section A3.2.4), $\theta_{G,\rho,t}$ must be made temporarily lower than θ_P in some future periods to decrease $\pi_{\rho,t}$ and external debt burdens before current account imbalances can stabilize (i.e., before $\theta_{G,\rho,\rho} = \theta_{G,t} = \theta_P$ for any ρ is achieved). If the utility gains resulting from a temporary deviation exceed the discounted sum of disutility caused by the future correction of the deviation, the deviation will be selected as a rational choice.

Note that if a member government temporarily behaves based on a $\theta_{G,\rho,t}$ that is higher than $\theta_{G,t} = \theta_P$, then at least one of the other member states has to set its $\theta_{G,\rho}$ below $\theta_{G,t}$ during the deviating period because the ECB keeps $\theta_{G,t} = \theta_P$.

A3.3.2 Rational deviations

Suppose that a member government deviates by discontinuously increasing (hereafter "jumping") its $\theta_{G,\rho,t}$ from the ECB's target rate ($\theta_{G,t} = \theta_P$) to $\bar{\theta}_{G,\rho}(> \theta_P)$ and keeping it until $t = t_1$. The government corrects the deviation after t_1 by jumping $\theta_{G,\rho,t}$ downwards to $\hat{\theta}_{G,\rho}(< \theta_P)$ and keeping it there during $t_1 \le t < t_2$. After t_2, the government keeps $\theta_{G,\rho,t} = \theta_{G,t} = \theta_P$. Hence, the government's expected utility when it deviates and later corrects the deviation is

$$
\begin{aligned}
\Lambda_D = & E\int_0^{t_1} \tilde{u}_{G,\rho}\left[u_{G,\rho}\left(g_{\rho t}, x_{\rho,t}\right), 0\right]\exp\left(-\bar{\theta}_{G,\rho}\, t\right)dt \\
& + E\int_0^{t_1} \exp\left(-\bar{\theta}_{G,\rho}\, t\right)dt \int_{t_1}^{t_2} \tilde{u}_{G,\rho}\left[u_{G,\rho}\left(g_{\rho,t}, x_{\rho,t}\right), \left(\bar{\theta}_{G,\rho} - \hat{\theta}_{G,\rho}\right)\right]\exp\left(-\hat{\theta}_{G,\rho}\, t\right)dt \\
& + E\int_0^{t_1} \exp\left(-\bar{\theta}_{G,\rho}\, t\right)dt \int_{t_1}^{t_2} \exp\left(-\hat{\theta}_{G,\rho}\, t\right)dt \int_{t_2}^{\infty} \tilde{u}_{G,\rho}\left[u_{G,\rho}\left(g_{\rho,t}, x_{\rho,t}\right), \left(\bar{\theta}_{G,\rho} - \theta_{G,t}\right)\right]\exp\left(-\theta_{G,t} t\right)dt \quad .
\end{aligned}
$$

Here, if the government does not deviate, its expected utility is

$$
\Lambda_N = E\int_0^{\infty} \tilde{u}_{G,\rho}\left[u_{G,\rho}\left(g_{\rho,t}, x_{\rho,t}\right), \left(\bar{\theta}_{G,\rho} - \theta_{G,t}\right)\right]\exp\left(-\theta_{G,t} t\right)dt \quad .
$$

Hence,

$$
\begin{aligned}
\Lambda_D - \Lambda_N = & E\int_0^{t_1} \tilde{u}_{G,\rho}\left[u_{G,\rho}\left(g_{\rho,t}, x_{\rho,t}\right), 0\right]\exp\left(-\bar{\theta}_{G,\rho}\, t\right)dt \\
& - E\int_0^{t_1} \tilde{u}_{G,\rho}\left[u_{G,\rho}\left(g_{\rho,t}, x_{\rho,t}\right), \left(\bar{\theta}_{G,\rho} - \theta_{G,t}\right)\right]\exp\left(-\theta_{G,t} t\right)dt \\
& + E\int_0^{t_1} \exp\left(-\bar{\theta}_{G,\rho}\, t\right)dt \int_{t_1}^{t_2} \tilde{u}_{G,\rho}\left[u_{G,\rho}\left(g_{\rho,t}, x_{\rho,t}\right), \left(\bar{\theta}_{G,\rho} - \hat{\theta}_{G,\rho}\right)\right]\exp\left(-\hat{\theta}_{G,\rho}\, t\right)dt \\
& - E\int_0^{t_1} \exp\left(-\theta_{G,t}\right)dt \int_{t_1}^{t_2} \tilde{u}_{G,\rho}\left[u_{G,\rho}\left(g_{\rho,t}, x_{\rho,t}\right), \left(\bar{\theta}_{G,\rho} - \theta_{G,t}\right)\right]\exp\left(-\theta_{G,t}\right)dt \\
& + E\int_0^{t_1} \exp\left(-\bar{\theta}_{G,\rho}\, t\right)dt \int_{t_1}^{t_2} \exp\left(-\hat{\theta}_{G,\rho}\, t\right)dt \int_{t_2}^{\infty} \tilde{u}_{G,\rho}\left[u_{G,\rho}\left(g_{\rho,t}, x_{\rho,t}\right), \left(\bar{\theta}_{G,\rho} - \theta_{G,t}\right)\right]\exp\left(-\theta_{G,t}\right)dt \\
& - E\int_0^{t_2} \exp\left(-\theta_{G,t}\right)dt \int_{t_2}^{\infty} \tilde{u}_{G,\rho}\left[u_{G,\rho}\left(g_{\rho,t}, x_{\rho,t}\right), \left(\bar{\theta}_{G,\rho} - \theta_{G,t}\right)\right]\exp\left(-\theta_{G,t}\right)dt \, .
\end{aligned}
\tag{A38}
$$

Let Λ_1, Λ_2, …, Λ_6 be the first, second, …, sixth terms of the right side of equation (A38), respectively. $\Lambda_1 + \Lambda_2$ indicates the utility gain owing to the deviation, and $\Lambda_3 + \Lambda_4$ indicates the utility loss owing to the future correction. $\Lambda_5 + \Lambda_6 \approx 0$ and $\Lambda_D - \Lambda_N \approx \Lambda_1 + \Lambda_2 + \Lambda_3 + \Lambda_4$

because $\int_0^{t_1} \exp\left(-\bar{\theta}_{G,\rho}\, t\right)dt \int_{t_1}^{t_2} \exp\left(-\hat{\theta}_{G,\rho}\, t\right)dt - \int_0^{t_2} \exp\left(-\theta_{G,t}\right)dt \approx 0$.

If the correction is implemented in a far shorter period than the deviating period and thus the scale of correction in each period is far larger than the scale of deviation in each period, then $\Lambda_1 + \Lambda_2 > -(\Lambda_3 + \Lambda_4)$ because $\frac{\partial^2 \tilde{u}_{G,\rho}\left[u_{G,\rho}\left(g_{\rho,t}, x_{\rho,t}\right), \tilde{\theta}_{G,\rho,t}\right]}{\partial \tilde{\theta}_{G,\rho,t}^2} < 0$ and of the effect of the discount factor. Hence, if $\frac{\partial^2 \tilde{u}_{G,\rho}\left[u_{G,\rho}\left(g_{\rho,t}, x_{\rho,t}\right), \tilde{\theta}_{G,\rho,t}\right]}{\partial \tilde{\theta}_{G,\rho,t}^2}$ is sufficiently large, $\Lambda_D - \Lambda_N > 0$; that is, the expected utility gains owing to the early deviation will exceed the discounted sum of the expected utility losses resulting from the future correction. This means that a government will rationally choose to deviate. Therefore, in a euro-type monetary union in which the central bank has only limited enforcement power, substantial deviations of member governments will happen and be left unchecked for a relatively long period.

Notice that deviation paths are not limited to the type shown above. It was assumed in the above examination that the correction is taken just after the deviation ends. However, it is possible to postpone the correction to the far future. During the period between the deviation and the postponed corrections, $\theta_{G,\rho,t} = \theta_{G,t} = \theta_P$ is kept.

A3.3.3 Inflation differentials

By the law of motion for inflation, inflation will temporarily accelerate in the deviating member state even though overall inflation in the euro area does not accelerate because of ECB control. Because the deviations should be temporary, inflation acceleration will be small scale, but non-negligible inflation differentials will be observed. If the correction is postponed to far future periods after the deviation ends, $\theta_{G,\rho,t} = \theta_{G,t} = \theta_P$ is kept and inflation does not accelerate during the interim period. However, because the deviation is left uncorrected, the relatively high rate of inflation continues in the deviating member state by the law of motion for inflation, and the inflation differentials thereby continue during this period. In addition, because the scale of deviation increases as the government's intrinsic time preference rate ($\bar{\theta}_{G,\rho}$) increases and the rate of time preference is empirically inversely correlated to productivity (see, e.g., Lawrance, 1991; Samwick, 1998; Ventura, 2003), relatively less productive member states (that have a relatively higher intrinsic time preference rate) will experience larger scale deviations and consequently higher inflation than relatively more productive member states. These features of inflation differentials predicted by the model are basically consistent with the observed persistent inflation differentials in the euro area.

A3.3.4 Fiscal deficits

When $\theta_{G,\rho,t}$ jumps upwards, the government's fiscal balance and debts are governed by non-liner complex processes. For simplicity, these processes are examined here based on the exogenous growth model used in Section 2. The thick solid line in Figure 1 indicates the steady state values of $\frac{x_{\rho,t} - g_{\rho,t} + \varphi_{\rho,t}}{b_{\rho,t}}$ corresponding to those of $i_{\rho,t} - \pi_{\rho,t}$. Suppose that the economy is first at steady state (point *E* in Figure 1). Then, by the upwards jump of $\theta_{G,\rho,t}$, the steady state moves from *E* to $\tilde{E}$. Because of the steady state shift, $\frac{x_{\rho,t} - g_{\rho,t} + \varphi_{\rho,t}}{b_{\rho,t}}$ jumps downwards from *E* to *J* and then moves upwards on the transition path to $\tilde{E}$ (along the thick dotted line in Figure 1). Accordingly, $g_{\rho,t}$ and $x_{\rho,t}$ jump to their transition paths and proceed on them to the new steady state. Note that, because seigniorage ($\varphi_{\rho,t}$) plays a limited role in modern economies, it is assumed here for simplicity that $\frac{\dot{\varphi}_{\rho,t}}{\varphi_{\rho,t}} = 0$ even after the jump.

By equations (A14), (A16) and (A17),

$$\theta_{G,\rho,t} = \frac{x_{\rho,t} - g_{\rho,t} + \varphi_{\rho,t}}{b_{\rho,t}} \tag{A39}$$

at any steady state (see Harashima, 2006, 2008). Hence, the upwards jump of $\theta_{G,\rho,t}$ requires a decrease of steady state $b_{\rho,t}$ and/or an increase of steady state $x_{\rho,t} - g_{\rho,t}$ to satisfy equation (A39). Because both an increase of steady state $x_{\rho,t}$ and a decrease of steady state $g_{\rho,t}$ reduce the utility at steady state, the steady state $b_{\rho,t}$ is expected to decrease to satisfy equation (A39). Hence, the government's real debts $b_{\rho,t}$ are smaller at the new steady state $\tilde{E}$ than at the previous steady state E. Note that the level of $b_{\rho,t}$ is not determined only by equation (A39) but also by the initial level of $b_{\rho,t}$ and exogenous shocks on $b_{\rho,t}$ (e.g., discretionary fiscal policies in case of a recession).

Because $b_{\rho,t}$ is a stock variable and cannot move drastically and discontinuously, $x_{\rho,t} - g_{\rho,t}$ (not $b_{\rho,t}$) jumps downward substantially as $\frac{x_{\rho,t} - g_{\rho,t} + \varphi_{\rho,t}}{b_{\rho,t}}$ jumps downward to J. Here, $\frac{\dot{b}_{\rho,t}}{b_{\rho,t}} = i_{\rho,t} - \pi_{\rho,t} - \frac{x_{\rho,t} - g_{\rho,t} + \varphi_{\rho,t}}{b_{\rho,t}}$ by equation (A1). In addition, $\pi_{\rho,t}$ and $i_{\rho,t}$ cannot jump and remain at almost the same values just after the jump by the law of motion for inflation. Therefore, if the downward jump of $x_{\rho,t} - g_{\rho,t}$ is sufficiently large, $\frac{\dot{b}_{\rho,t}}{b_{\rho,t}} > 0$ and the fiscal balance ($\dot{b}_{\rho,t}$) shows deficits initially after the jump. However, as $b_{\rho,t}$ gradually increases because of the fiscal deficits, $x_{\rho,t} - g_{\rho,t}$ also gradually increases but more rapidly than $b_{\rho,t}$ to satisfy equation (A39) (i.e., $\frac{x_{\rho,t} - g_{\rho,t} + \varphi_{\rho,t}}{b_{\rho,t}}$ gradually increases on the transition path from J to $\tilde{E}$ in Figure 1), and after a certain period, fiscal deficits turn to surpluses. The government's real debts $b_{\rho,t}$ that initially increased after the jump eventually then start to decrease and move to a lower level than the previous steady state $b_{\rho,t}$ at E (Figure 2). The fiscal balance and government debt after the jump therefore are governed by non-linear complex processes. If the deviation is not a single jump but intermittently repeated, the process of fiscal balance will become substantially more complex.

The amount by which fiscal deficits initially increase depends on the values of $\varepsilon_{g,\rho}$ and $\varepsilon_{x,\rho}$ and other conditions, including the scale of deviation. The values of $\varepsilon_{g,\rho}$ and $\varepsilon_{x,\rho}$ will be heterogeneous among the member states, similar to $\theta_{G,\rho,t}$, and the scale of deviation will be also heterogeneous. Therefore, fiscal balances after deviations will be governed by different processes across the member states. The features of non-linearity, complexity, and heterogeneity indicate that the SGP requirement that budget deficits of less than 3% of GDP are allowable whereas those over 3% must be punished in any period for any member state may not be reasonable. These features suggest that focusing only on fiscal deficits and setting an inflexible and non-country-specific ceiling on fiscal deficits and debts are not necessarily an appropriate way to prevent deviations.

A3.3.5 Current account deficits

As discussed in Section A3.2.3, although inflation diverges among the member states owing to deviations, the prices of tradable goods and services are still equalized generally across the euro area by arbitrage. As a result, the trade and current account balances in the deviating member state will show continuous deficits. Accordingly, relatively less competitive firms producing tradable goods or services in deviating member states will disappear more rapidly because of the price differentials and the inflows of foreign goods and services. Industries of tradable goods and services will decline and the share of non-tradable goods and services industries will increase in deviating member states. The features of current account imbalances predicted by the model are basically consistent with the observed current account imbalances in the euro area.

If floating exchange rates were working, current account imbalances would be adjusted substantially through currency depreciation in deviating member states, but there is no such mechanism within the euro area. As a result, external debts of the deviating member states will accumulate continuously until the distortion caused by the deviation is corrected. The accumulation of external debts may not immediately threaten the euro. However, as more external debts accumulate, the economies of the deviating member states will become more vulnerable to various shocks, and because the member states' economies are closely linked, the entire economy of the euro area also becomes more vulnerable to various shocks.

A3.4 Comparison with a currency peg

A foreign currency peg is similar to the situation with the euro because, in essence, more than one state uses the same currency. However, there is a fundamental difference between these situations. In the case of a currency peg, there are not only multiple governments but also multiple central banks. Hence, the central bank can directly control its government's behavior in each country. Nevertheless, the fixed exchange ratio can only be maintained if inflation is stabilized in the pegging and pegged countries. The fixed exchange rate is not automatically kept—it is the result of work to stabilize inflation in both countries. The exchange rate will soon destabilize if the efforts to stabilize inflation are relaxed. Central banks in countries adopting a currency peg need to be sufficiently independent or the peg cannot be maintained. This is one reason why a currency peg is usually introduced as a tool to stabilize ongoing high inflation in pegging countries.

On the other hand, the euro unconditionally guarantees the same value across member states regardless of efforts to stabilize inflation in each state. Combining central banks deprives the incentive to and removes the tool to locally stabilize inflation. It may have been presumed that, if the overall inflation in the euro area is stabilized, local inflation will also naturally stabilize even though member governments are heterogeneous and independent. However, as shown in the previous sections, local inflation can be differentiated if governments are heterogeneous and independent.

References

Alesina, Alberto and Alex Cukierman (1990) "The Politics of Ambiguity," *The Quarterly Journal of Economics*, Vol. 105, No. 4, pp. 829–850.

Alesina, Alberto and Allen Drazen (1991) "Why Are Stabilizations Delayed?" *International Economic Review,* Vol. 81, No. 5, pp. 1170–88.

Altissimo, Filippo, Pierpaolo Benigno and Diego Rodriguez Palenzuela (2005) "Long Run Determinants of Inflation Differentials in a Monetary Union," *NBER Working Paper*, No. 11473.

Angeloni, Ignazio and Michael Ehrmann (2007) "Euro Area Inflation Differentials," *The B.E. Journal of Macroeconomics*, Vol. 7, No. 1.

Arghyrou, Michael G. and Georgios Chortareas (2008) "Current Account Imbalances and real Exchange. Rates in the Euro Area," *Review of International Economics*, Vol. 9, No. 5, pp. 747–764

Arrow, Kenneth J. (1962) "Economic Welfare and the Allocation of Resources for Invention," in *The Rate and Direction of Inventive Activity: Economic and Social Factors,* pp. 609–626, Princeton University Press, Princeton.

Barro, Robert J. and David B. Gordon (1983) "A Positive Theory of Monetary Policy in a Natural Rate Model," *The Journal of Political Economy*, Vol. 91, No. 4. pp. 589–610.

Blanchard, Olivier (2007) "Adjustment within the Euro. The Difficult Case of Portugal," *Portuguese Economic Journal*, Vol. 6, No. 1, pp. 1–21.

Blinder, Alan S. (1998) *Central Banking in Theory and Practice*, The MIT Press, Cambridge.

Brennan, Geoffrey and James M. Buchanan (1980) *The Power to Tax: Analytical Foundations of a Fiscal Constitution*, Cambridge MA, Cambridge University Press.

Christiano, Lawrence and Terry J. Fitzgerald (2000) "Understanding the Fiscal Theory of the Price Level," *NBER Working Paper* No. 7668.

Cukierman, Alex, Steven B. Webb, and Bilin Neyapti (1992) "Measuring the Independence of Central Banks and Its Effect on Policy Outcomes," *World Bank Economic Review*, Vol.6, No.3, pp. 353-398.

Decressin Jörg and Emil Stavrev (2009) "Current Accounts in a Currency Union," *IMF Working Paper*, No. 127.

de Grauwe. Paul (2009) "The Euro at Ten: Achievements and Challenges," *Empirica*, Vol. 36, pp. 5–20.

Downs, Anthony (1957) *An Economic Theory of Democracy*, Harper, New York.

Edwards, Jeremy and Michael Keen (1996) "Tax Competition and Leviathan," *European Economic Review*, Vol. 40, No. 1, pp. 113–134.

European Central Bank (ECB) (2003) *Inflation Differentials in the Euro Area: Potential Causes and Policy Implications*, Frankfurt am Main, European Central Bank.

European Central Bank (ECB) (2007) "Challenges to Fiscal Sustainability in the Euro Area," *ECB Monthly Bulletin*, February. pp. 59-72.

European Central Bank (ECB) (2008a) "One Monetary Policy and Many Fiscal Policies: Ensuring a Smooth Functioning of EMU," *ECB Monthly Bulletin*, July, pp. 65-78.

European Central Bank (ECB) (2008b) "Ten Years of the Stability and Growth Pact," *ECB Monthly Bulletin,* October, pp. 53-66.

European Commission (EC) (2009) *Quarterly Report on the Euro Area*, Vol. 8, No. 1.

Feldstein, Martin (2015) "Ending the Euro Crisis?" *NBER Working Papers*, No. 20862

Fendel, Ralf and Michael Frenkel (2009) "Inflation Differentials in the Euro Area: Did the ECB Care?" *Applied Economics*, Vol. 41, No. 10, pp. 1293–1302.

Gregoriou, Andros, Alexandros Montagnoli and Alberto Kontonikas (2011) "Euro Area Inflation Differentials: Unit Roots and Non-linear Adjustment," *Journal of Common Market Studies*, (In Press).

Gros, Daniel, Thomas Mayer and Angel Ubide (2005) *EMU at Risk,* 7th Annual Report of the CEPS Macroeconomic Policy Group. Centre for European Policy Studies (CEPS), Brussels.
Harashima, Taiji (2008) "A Microfounded Mechanism of Observed Substantial Inflation Persistence," *MPRA* (*The Munich Personal RePEc Archive*) *Paper*, No. 10668.
Harashima, Taiji (2009) "A Theory of Total Factor Productivity and the Convergence Hypothesis: Workers' Innovations as an Essential Element," *MPRA* (*The Munich Personal RePEc Archive*) *Paper*, No. 15508.
Harashima, Taiji (2010) "Sustainable Heterogeneity: Inequality, Growth, and Social Welfare in a Heterogeneous Population," *MPRA* (*The Munich Personal RePEc Archive*) *Paper*, No. 24233.
Harashima, Taiji (2011) "A Mechanism of Inflation Differentials and Current Account Imbalances in the Euro Area," *MPRA* (*The Munich Personal RePEc Archive*) *Paper*, No. 28121.
Holinskia, Nils, Clemens Koolb and Joan Muyskena (2010) "Origins of Persistent Macroeconomic Imbalances in the Euro Area," *Utrecht School of Economics Working Papers*, No. 10-12.
Jacobs, Jane (1969) *The Economy of Cities,* Random House, New York.
Jaumotte, Florence and Piyaporn Sodsriwiboon (2010) "Current Account Imbalances in the Southern Euro Area," *IMF Working Paper* WP/10/139.
Kenen, Peter B. (1969) "The Optimum Currency Area: An Eclectic View," in R. A. Mundell and A. Swoboda (eds.), *Monetary Problems of the International Economy*, University of Chicago Press, Chicago.
Kletzer, Kenneth and Jürgen von Hagen (2001) "Monetary Union and Fiscal Federalism," in Charles Wyplosz (ed.), *The Impact of EMU on Europe and the Developing Countries*, Oxford University Press, Oxford.
Kydland, Finn E. and Edward C. Prescott (1977) "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy*, Vol. 85, No. 3, pp. 473–491.
Lawrance, Emily C. (1991) "Poverty and the Rate of Time Preference: Evidence from Panel Data," *Journal of Political Economy*, Vol. 99, No. 1, pp. 54–77.
Marshall, Alfred (1890) *Principles of Economics*, MacMillan, London.
Obstfeld, Maurice (2013) "Finance at Center Stage: Some Lessons of the Euro Crisis," *European Commission Economic Papers*, No. 493.
Prescott, Edward C. (1998) "Needed: A Theory of Total Factor Productivity," *International Economic Review*, Vol. 39, No. 3, pp. 525–551.
Romer, Paul M. (1986) "Increasing Returns and Long-run Growth," *Journal of Political Economy*, Vol. 94, No. 5, pp. 1002–1037.
Sala-i-Martín, Xavier X. and Jeffrey D. Sachs (1992) "Fiscal Federalism and Optimum Currency Areas: Evidence for Europe from the United States" in M. B. Canzoneri, V. Grilli and P. R. Masson (eds.), *Establishing a Central Bank: Issues in Europe and Lessons from the US*, Cambridge University Press, Cambridge.
Samuelson, Paul A. (1994) "Facets of Balassa-Samuelson Thirty Years Later," *Review of International Economics*, Vo. 2, No. 3, pp. 201–226.
Sidrauski, Miguel (1967) "Rational Choice and Patterns of Growth in a Monetary Economy," *American Economic Review*, Vol. 57, No. 2, pp. 387–393.
Tabellini, Guido and Alberto Alesina (1990) "Voting on the Budget Deficit," *American Economic Review*, Vol. 80, No. 1, pp. 37–49.

Figure 1 ***The transition path after the jump of $\theta_{G,\rho}$***

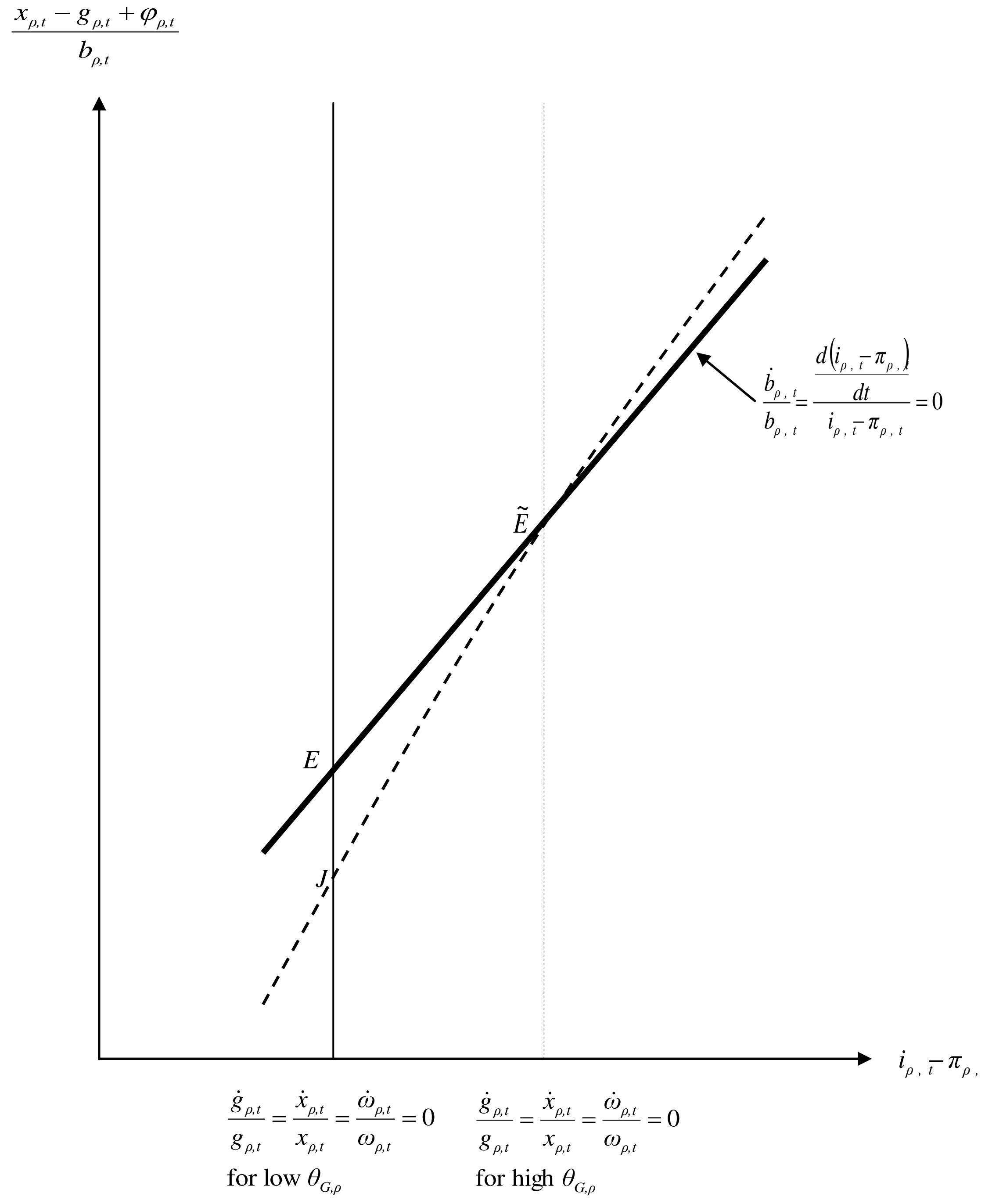

Figure 2 ***The government's real debts after the jump of $\theta_{G,\rho}$***