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CRITICISM OF THE BLACK-SCHOLES MODEL: BUT WHY IS IT STILL USED? (THE ANSWER IS SIMPLER THAN THE FORMULA)

Abstract
The Black Scholes Model (BSM) is one of the most important concepts in modern financial theory both in terms of approach and applicability. The BSM is considered the standard model for valuing options; a model of price variation over time of financial instruments such as stocks that can, among other things, be used to determine the price of a European call option. However, while the formula has been subject to repeated criticism for its shortcomings, it is still in widespread use. This paper provides a brief overview of BSM, its foundational underpinnings, as well as discusses these shortcomings vis-à-vis alternative models. This paper was originally written as a part of the course "Derivatives & Capital Markets" in 2004 during my time at New York University under exam conditions. This present paper is an updated, expanded and re-formatted version with references.

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Introduction

i. Background

The Black Scholes Model (hereinafter 'BSM' or 'Black-Scholes theorem') is one of the most important concepts in modern financial theory both in terms of approach and applicability.[1] The BSM is considered the standard model for valuing options; a model of price variation over time of financial instruments such as stocks that can, among other things, be used to determine the price of a European call option.[2] For example, consider a European call option for a stock.[3] This is the right to buy a specific number of shares of a specific stock on a specific date in the future, at a specific price (the exercise price, also called the strike price).[4] If all these quantities are fixed, the question becomes: what is a fair price to charge for the option? The Black-Scholes formula gives the price of the option, in terms of other quantities, which are assumed known.[5] These include the exercise price and the current price of the stock.[6] The formula is derived under the assumption that the time interval between observations is very small, and that the log prices follow a random walk with normally distributed innovations. The formula is not affected by any linear drift in the random walk.[7] The model for the stock prices themselves is called geometric Brownian motion.[8] A key input to the Black-Scholes formula is σ, the standard deviation of the stock's continuously compounded rate of return.[9] If the time interval between observations is sufficiently small, σ is just the standard deviation of the innovations in the random walk, so σ can be viewed as a measure of the volatility of the stock price.[10] According to the random walk model, σ must remain constant over time.[11] Its value will not be known, however, so it is usually estimated from the available data.[12] In essence, the BSM assumes that the price of heavily traded assets follow a geometric Brownian motion with constant drift and volatility, which can be seen as continuous time limits of random walks. The model can also be derived from additive binomial tree models.[13] When applied to a stock option, the model incorporates the constant price variation of the stock, the time value of money, the option's strike price and the time to the option's expiry.[14] However, since the advent of this Nobel Prize winning formula, it has become apparent that several of the assumptions used in the Black-Scholes method may be unrealistic.[15]

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[1] The BSM has been around for approximately 30 years and the authors have won a Nobel Prize for the formula. *See* Scholes, M., "Autobiography" in Frangsmyr, T., *The Nobel Prizes 1997*, (Nobel Prize Foundation, Stockholm 1998).

[2] Hull, J.C., *Options Futures and Other Derivatives* (5th edn., Prentice Hall, 2002); *see also* Chris, N.A., '*Black-Scholes and Beyond: Options Pricing Models*' (McGraw Hill, 1997).

[3] *Ibid.* (*Chris*)

[4] *id.*

[5] *id.*

[6] *id*.

[7] *id*.

[8] *id.*

[9] Hull (2002), n.2 *supra*. (the continuously compounded rate of return is the change in the log price divided by the length of time elapsed.)

[10] *id.*

[11] *id.*

[12] *id.*

[13] *id.*

[14] *id.*

[15] *id.*; *see also* Chris (1997), n.2 *supra*; and Teneng, D., *Limitations of the Black-Scholes Model*, (2011) 68 Int'l.R.J.F.E. 99-102; *see also* Taleb, N., Haug, E.G., *Option Traders Use (very) Sophisticated Heuristics, Never*

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On the one hand, the BSM's biggest strength is the possibility of estimating market volatility of an underlying asset generally as a function of price and time without, for example, direct reference to expected yield, risk aversion measures or utility functions.[16] The second strongest strength aspect is its self-replicating strategy or hedging: explicit trading strategy in underlying assets and risk-less bonds whose terminal payoff, which equals the payoff of a derivative security at maturity.[17] In other words, theoretically an investor can continuously buy and sell derivatives by the strategy and never incur loss. It is also simple and mathematically tractable as compared to some of its more recent variations.[18] On the other hand, over the past three decades the shortcomings (and in some cases the failure) of the BSM have become increasingly clear, with some academic commentators repeatedly ringing the "death-knell" of the formula as its weaknesses become more apparent.[19] The BSM is based on certain explicit assumptions, including but not limited to: (i) assuming constant volatility (it does not reflect the volatility smile prevalent in options since the 1987 crash); (ii) it assumes asset prices follow a random walk; (iii) it assumes that stock that's moves are normally distributed; (iv) it assumes that interest rates (risk free interest rate) are constant; (v) the asset (i.e. stock) does not pay any dividends; (vi) it assumes no commissions and transactions costs; and (vii) it assumes markets are perfectly liquid.[20] Nevertheless, despite of some of these assumptions, the formula remains in wide spread use and despite its harshest critics, e.g., Nassim Taleb, a formula that is still integral to options pricing can hardly be called "dead" or "dying", although, nothing in this paper should be construed as marginalizing the shortcomings of these assumptions. The fact remains, as will be shown in this paper, there is no one-size fits all model that cures all of the problems and the simplicity of the BSM gives way to continued, wide-spread use.

ii. Structure

This paper is divided into four parts and narrows the discussion to focus on these shortcomings. Part I briefly explains Brownian motion. Part II provides the simplest form of the Black Scholes formula. Building on the previous two sections, Part III identifies and critiques the shortcomings of the Black-Scholes theorem and briefly discusses the the Lévy Process and other academic developments in this field. However, given the depth of recent research, the discussion on these new developments is strictly limited. Part IV offers some concluding remarks.

I.
Brownian Motion: Random Walk Assumption

Brownian motion is important because it provides a framework to capture movements in stock prices which rise and fall due to unforeseen circumstances.[21] Brownian motion is closely linked to normal distribution.[22] For

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the Black–Scholes–Merton Formula, found at < http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1012075> last accessed on 30 June, 2012.

[16] *Ibid.* (*Teneng*)(citing Black, F., Scholes, M., "*The Pricing of Options and Corporate Liabilities*," (1973) 81(3) The Journal of Political Economy 637-654).

[17] *id.*

[18] *id.*

[19] *id.*

[20] *id*.

[21] For an in depth discussion of Brownian Motion, *see* Morters, P., Peres, Y., Schramm, Werner, W., *Brownian Motion* (Cambridge University Press: Cambridge, 2010).

[22] *id.*

purposes of this part, given the brevity of this paper, it is assumed that the reader is familiar with the definition of Brownian motion and thus the discussion will be limited to its relationship vis-à-vis aspects of real asset price data.

Asset prices modelled by a geometric Brownian motion are represented by:[23]

$$S_t = S_0\, e^{ut+\sigma W_t},$$

where,

: drift

σ : volatility,

W : Brownian motion.

and

$W_0 = 0$,

$W_t - W_s$ is independent of $(W_u : u \leq s)$ for all $t \geq s$,

$W_t - W_s \sim N(0, t - s)$

$T \rightarrow W_t$ continuous

In other words, say uncertainty in the economy, or particular or specific data set is represented by a filtered probability space $(\Omega F, F_t, P)$ where F_t, is the filtration of information available at time t and P is the real probability measure. Let (Ω, F, F_t, P) be a filtered probability space. The process $\{B(t, \omega), t \geq 0\ \omega in\ \Omega\}$ is called Brownian motion if:[24]

1. For each t, $B_t = B(t.,)$ is a random variable,
2. For each w, the path $B_t = B(t.,)$ is continuous,
3. $B_t \sim N(0, \sqrt{t})$ where $\sqrt{t}$ is the standard deviation.

If $0 \geq t_1 < t_2 < \ldots < t_n$, then B_{t1}, $B_{t2} - B_{t1}, \ldots, B_{tn} - B_{t\,n-1}$ are independent and $B_{tn} - B_{tn-1} \sim N(0, \sqrt{t_i - t_{i-1}})$ i.e., the process has stationary and independent increments. Thus, the moment generating function of such a process is given by:[25]

1. $$E(e^{yB_t}) = \int_{-\infty}^{+\infty} \left(e^{rx} \frac{1}{\sqrt{2\pi t}}\ e^{x\wedge 2/2t}\right) dx = e^{r\,(r\wedge 2/2)}.$$

[23] Teneng (2011) n.15 *supra*.
[24] *id.*
[25] *id.*

However, typical features of asset price data don't exactly fit neatly in this mould because they display, amongst other things:[26]

(a) heavy-tailed returns;
(b) squared returns are positively correlated;
(c) variable volatility; and
(d) volatility clustering.

As shown by Teneng (2011), the BSM does not always describe all aspects of real asset price data. And while stochastic volatility or fractional Brownian Motion (*f*BM) models typically do better, it has its own problems: (a) stochastic models, in relation to volatility, raise the issue of incompleteness and thus how to derive derivative prices? And (b) *f*BM models raise the issue arbitrage, how to deal with it?[27] Given the significant depth of the debates surrounding these issues and given the brevity of this paper, these issues are excluded; however, while there are an increasing number of new models to account for some of these shortcomings, none of them have gained the popularity of the BSM. The next part of this paper provides the mathematical foundation of BSM.

II.
Classic Black-Scholes Model

The basic form of the BSM involves only two assets:[28]

-a riskless asset (i.e. a cash bond),
-a risky asset (i.e. a stock).

The cash bond appreciates at a riskless rate of return r, which can be time varying but non-random in this classical case. The price Z_t of this riskless cash bond at time t is assumed to satisfy:[29]

$$2. \qquad \frac{dZ_t}{dt} = r_t Z_t$$

with unique solution:

$$3. \quad Z_t = Z_0 \left(e^{\int_0^t (r_s\, ds)} \right)$$

Where Z_0 is the price of the riskless asset at time t=0. Let S be the price of the risky asset. After a short time interval of time dt, the asset price changes by dS to $S+dS$. Rather than measuring the absolute change S, we measure the return on the risky asset defined by:[30]

[26] *id.*
[27] *id.*
[28] *id.*
[29] *id.*
[30] *id.*

$$\frac{dS}{S}$$

i.e. returns measure a change in the risky asset price as a proportion of the original risky asset price. Since this is a risky asset, we assume the risk can be generated from both predictable and unpredictable sources/circumstances.[31] That from the predictable circumstances is assumed to be almost equal to the risk free rate of the bank i.e. the rate of appreciation of the riskless cash bond. We denote this μ also called the drift which measures the average growth of the asset price.[32] The second contribution that comes from unpredictable factors is denoted σ - the volatility, which is a measure of the standard deviation of the returns.[33] It is generally accepted to be of form σdB where B is standard Brownian motion and dB is its stochastic differential.[34] The parameters μ and s can always be estimated from historical data. Bringing the two components together gives us the equation:[35]

4. $$\frac{dS}{S} = \mu\, dt + dB$$

This equation describes the evolution of the risky asset price and is called Black-Scholes market model. If σ=0, then dS/S = μdt and $S_t = S_0 e^{ut}$ which is a purely deterministic asset price.[36]

The stochastic differential equation presented above can be solved for S_t using Ito's formula:

5. $$dF(B_t, t) = F_t\, dt + F_{B_t}\, dB_t + ½\, F_{B_tB_t}\, d_B(<m>)$$

where $F_t = \partial \mathrm{F}/\partial t$, $F_{B_t} = F/B_t$, $F_{B_t}F_{B_t} = F/B_t^2$ and $<m_t> = t$ is the quadratic variation of B_t. For example,

6. $$S_t = S_0 \left(e^{\wedge (\sigma B_t - t/2\ \sigma^{\wedge 2 + tu})} \right)$$

is a solution to equation 4, above, *see* e.g.,

7. $$F(B_t, t) = S_0 \left(e^{(B - ½ (\sigma + t\mu))} \right)$$

then

$F_t = (\mu_S - \sigma^2/2)\ F(B_t, t)$, $F_{B_t} = \sigma F(B_t, t)$, $F_{B_tB_t} = \sigma^2\ F(B_t, t)$ and substituting these in to the Black-Scholes formula gives:

8. $$\Delta F(B_t, t) = (\mu\text{-}\sigma^2/2)\ F(B_t, t) dt + \sigma F(B_t, t)\ dB_t + (\sigma^2/2)\ F(B_t, t)\ d(<m>)$$

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[31] *id.*
[32] *id.*
[33] *id.*
[34] *id.*
[35] *id.*
[36] *id.*

Now, $d(<m>) = dt$ by approximation leading to:

9. $dS_t = S_t(dt + \sigma dB_t)$, e.g., same as equation 6 above.[37]

III.
Criticism of Implicit Properties of Black-Scholes

To provide one motivation for the more recent models, e.g. Lévy Process, it should be noted that despite the Black Scholes model's popularity and wide spread use, the model is built on some non-real life assumptions.[38] This section is divided into seven parts. Sections one through six examine some of these non-real life assumptions; section seven examines the Lévy process.

3.1 Brownian Motion

Firstly, the geometric Brownian motion model, examined in Part I implies that the series of first differences of the log prices must be uncorrelated.[39] But for the S&P 500 as a whole, observed over several decades, daily from 1 July 1962 to 29 Dec 1995, there are in fact small but statistically significant correlations in the differences of the logs at short time lags.[40] At its core, neither people nor a model can consistently predict the direction of the market or an individual stock. The Black Scholes theorem assumes stocks move in a manner referred to as a random walk; random walk means that at any given moment in time, the price of the underlying stock can go up or down with the same probability. However, this assumption does not hold as stock prices are determined by many factors that cannot be assigned the same probability in the way they will affect the movement of stock prices. Moreover, the price of a stock in time t+1 is independent from the price in time t (i.e. martingale property of Brownian motion). Next, there may not be a single source or factor driving two assets even if one is a derivative of the other as is stated in the martingale representation theorem.

3.2 Asset Returns Are Not Normally Distributed

Secondly, the model assumes that log normally distributed underlying stock prices are normally distributed. However, as observed by Clark, asset returns have a finite variance and semi-heavy tails contrary to stable distributions like log normal with infinite variance and heavy tails. As noted by Hull, experience has shown that returns are *leptokurtic*, i.e., have much more of a tendency to exhibit outliers than would be the case if they were normally distributed. An example is provided by the returns on the S&P 500 series. There is overwhelming evidence that the returns are not normal, but instead have a leptokurtic (i.e., long-tailed) distribution.[41]

3.3 Volatility is Not Constant

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[37] *id.*
[38] *id.*
[39] *id.*
[40] *id.*
[41] Hull (2002) n.9 *supra*; Teneng, n.15 *supra* at 101 (citing Cont, R., Tankov, P., "*Financial Modelling with Jump Processes*" (CRC Press: 2003)(noting that, despite the fact that auto-correlation of asset prices are often insignificant, as the time scale over which returns of assets are calculated increases, the distribution of asset prices looks more like the normal-distribution).

Thirdly, the model assumes a constant volatility. However, ever since the 1987 stock market crash, this assumption has proven false. While volatility can be relatively constant in very short term periods, it is never constant in the long term. In other words, it is often found that for financial time series, after taking logs (if needed) and first differences, the level of volatility (i.e., fluctuation) seems to change with time.[42] Often, periods of high volatility follow immediately after a large change (often downward) in the level of the original series.[43] It may take quite some time for this heightened volatility to subside.[44] For example, the plot of differences of the logs of the S&P 500 shows very long periods of high volatility interspersed with periods of relative calm.[45] This type of pattern is often referred to as volatility clustering. A way to measure this clustering is to look at the autocorrelations of the squares of the differences of the logs (or of the squared returns). If there is volatility clustering, these autocorrelations should be significant for many lags, so that a shock to the volatility persists for many periods into the future. This is the case for the S&P 500. Similar findings of leptokurtosis and volatility clustering have been obtained for the differences of the logs of individual stock prices, exchange rates, and interest rates. Since measures of volatilities are negatively correlated with asset price returns, while trading volumes or the number of trades are positively correlated, the constant volatility assumption of the theorem is unrealistic over time.[46] In particular, an estimate of variance based on historical data may severely detract from the fairness of the computed options price if the stock has just entered a highly volatile phase.[47] As noted by Hull (2002), leptokurtosis in the innovations can produce further pricing biases and how long tails in the original stock price distribution relative to a log-normal distribution can cause Black-Scholes to systematically under-price or over-price an option.[48] Consequently, more recent option valuation models substitute Black-Scholes's constant volatility with a stochastic-process generated estimates.[49] However, given its simplicity and mathematical tractability as compared to some of its more recent variations, the Black-Scholes model continues to be in widespread use.[50]

3.4 Interest Rates are Not Constant

Fourth, similar to the Black-Scholes theorem's assumption vis-à-vis constant volatility, the model assumes that interest rates are constant and known. This assumption is also unrealistic.[51] The model uses the risk-free rate to represent this constant and known rate. While the U.S. Government Treasury Bill 30-day rate is often used as a part of the model, as demonstrated by the recent downgrade of U.S. creditworthiness,[52] there is no such thing as a risk-free rate.[53] Next, treasury rates can and do change in times of increased volatility.[54]

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[42] *Ibid.* (*Hull*); *see also* Teneng (2011).
[43] *id.*
[44] *id.*
[45] *id.*
[46] Carr, P., Wu, L., "*Time Change Lévy Processes and Option Pricing*" (2002), found at <http://www.math.nyu.edu/research/carrp/papers/pdf/jfetchgepaper.pdf> last accessed 28th June, 2012.
[47] Hull (2002); and Teneng (2011).
[48] *id.*
[49] *id.*
[50] *id.*
[51] *id.*
[52] *id.*
[53] *id.*
[54] *id.*

3.5 Dividends

Fifthly, the model assumes that the underlying stock does not pay dividends during the option's life. However, this assumption does not apply in all, or actually, most cases since most public companies pay dividends to their shareholders.[55] This assumption relates to the basic Black-Scholes formula and typically the model is adjusted by subtracting the discounted value of a future dividend from stock prices to account for dividends.[56]

3.6 Ignorance of Transaction Costs, Perfect Liquidity, Constant Trading

Finally, one of the most significant assumptions of the theorem is its assumption that there are no fees for buying and selling options and stocks and no barriers to trading.[57] However, this is hardly the case in the real world.[58] More crucially, the model assumes that markets are perfectly liquid and it is possible to purchase or sell any amount of stock or options or their fractions at any given time.[59] This assumption is not only implausible but can be fatal. For one thing, as demonstrated by the events of 1987,[60] 1998,[61] 2007-2008[62] markets are not perfectly liquid.[63] Next, in most instances, due to company policies, or other factors, investors are limited by the amount of money they can invest and, it is not possible to sell fractions of options.[64]

3.7 Lévy Process Based Heath-Jarrow-Morton Framework to the Rescue?

While the above discussion was focused on stock prices, the discussion can be extended to other markets. As noted by Andersen (2008), in examining the market prices of interest derivatives, or rather *the implied Black volatilities of these derivatives*, there are inconsistencies with most short rate models.[65] For example, while recent developments in interest rate modelling, e.g., LIBOR market model, have tried to model interest rate derivatives by modelling simple forward rates as opposed to instantaneous short- or forward rates, mispricing still occurs for in-the-money or out-of-the-money derivatives.[66] As concluded by Andersen's study, while more recent models based on the Lévy process outperform Gaussian models, this process is dependent on the specific multivariate Lévy processes. In other words, if the dependence between the driving factors is restrictive, the

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[55] *id.*
[56] *id.*
[57] *id.*
[58] *id.*
[59] *id.*
[60] Derman, E., "*Laughter in the Dark – The Problem of the Volatility Smile*," 26th May, 2003, found at <http://www.ederman.com/new/docs/euronext-volatility_smile.pdf> last accessed 28th June, 2012.
[61] Jorion, P., "*Risk Management Lessons from Long-Term Capital Management*" (1999).
[62] Millo, Y., MacKenzie, D., "*The Usefulness of Inaccurate Models: Financial Risk Management 'In the Wild.'*" (2009) 3(1) The Journal of Risk Model Valuation 23-49, found at <http://www.risk.net/digital_assets/5044/jrm_v3n1a2.pdf> last accessed 28th June, 2012.
[63] Teneng (2011), n.15 *supra*.
[64] *id.*
[65] Andersen, A.S.T., "*Lévy Process in Finance: Lévy Heath-Jarrow-Morton Models*," MSc Business Admin, & Mgmt Science Master Thesis, Copenhagen Business School (2008), found at <http://www.allan-andersen.dk/thesis/MScThesis.pdf>, last accessed 30th June, 2012.
[66] *id.*

effect from adding additional driving factors are hardly visible; however, a model based on independent factors seems to perform far better.[67]

IV.
Conclusion

As demonstrated in this paper, and is widely accepted in the financial industry, most of the above identified limitations of the Black-Scholes model are fundamental and, thus, it is necessary to come up with models that will take into consideration some of the assumptions not addressed by Black-Scholes models. And, not surprisingly, there is no shortage in the academic literature, which proposes alternative models, all attempting to mimic the characteristics of the market fully.[68] However, basic common sense, and recent history in the wake of numerous market events, dictates that every aspect of the market cannot be considered in any given model, as every factor affecting the price of a financial security cannot be captured mathematically.[69] Mathematical models do, and can only attempt, to capture most of the aspects, which is what is proposed by Lévy models; however, even that model has its limitations, which relate to fundamental aspects of the market.[70]

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[67] *id.*

[68] Fortune, P., "*Anomalies in Options Pricing: the Black-Scholes Model Revisited*" (1986) Federal Reserve Bank of Boston, found at <http://www.bos.frb.org/economic/neer/neer1996/neer296b.htm>, last accessed 30th June, 2012; Schwartz, R.J., & Smith, C.W., "*Derivatives Handbook: Risk Management & Control*" (John Wiley & Sons, Inc., 1997); Heston, S.L., Nandi, S., (1997) *A Closed-Form GARCH Option Pricing Model*, found at <http://papers.ssrn.com/sol3/papers.cfm?abstract_id=96651>, last accessed 28th June, 2012; Hallerbach, W.G., "*An Improved Estimator of Black-Scholes-Merton Implied Volatility*" (2004), found at <http://papers.ssrn.com/sol3/papers.cfm?abstract_id=567721>, last accessed on 30th June, 2012; Sattayatham, P., Intarasit, A., and Chaiyasema, A.P., (2006) "*A Fractional Black-Scholes Model With Jumps*", found at <http://science.sut.ac.th/mathematics/pairote/uploadfiles/baibao7_AJ.PDF>, last accessed on 1st July, 2012; Tomas, B., Henrik, H., (2005) "*A Note on Wick Products and Fractional Black Scholes Model*", found at <http://ideas.repec.org/p/hhs/hastef/0596.html>, last accessed on 1st July, 2012; and see generally, Chris (1997), n.2 *supra*, at 61-74.

[69] Andersen (2008), n.65 *supra*.

[70] *id.*

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