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Abstract

The most popular model for pricing options, both in financial literature as well as in practice has been the Black-Scholes model. In spite of its wide spread use the model appears to be deficient in pricing deep in the money and deep out of the money options using statistical estimates of volatility. This limitation has been taken into account by practitioners using the concept of implied volatility. The value of implied volatility for different strike prices should theoretically be identical, but is usually seen in the market to vary. In most markets across the world it has been observed that the implied volatilities of different strike prices form a pattern of either a 'smile' or 'skew'. Theoretically, since volatility is a property of the underlying asset it should be predicted by the pricing formula to be identical for all derivatives based on that same asset. Hull [1993] and Nattenburg [1994] have attributed the volatility smile to the non normal Skewness and Kurtosis of stock returns.

Many improvements to the Black-Scholes formula have been suggested in academic literature for addressing the issue of volatility smile. This paper studies the effect of using a variation of the BS model (suggested by Corrado & Sue [1996] incorporating non-normal skewness and kurtosis) to price call options on S&P CNX Nifty.

The results strongly suggest that the incorporation of skewness and kurtosis into the option pricing formula yields values much closer to market prices. Based on this result and the fact that this approach does not add any further complexities to the option pricing formula, we suggest that this modified approach should be considered as a better alternative.

Introduction

The most popular model for pricing options, both in financial literature as well as in practice has been the one developed by Fischer Black & Myron Scholes. In spite of its wide spread use the model appears to be deficient in pricing deep in the money and deep out of the money options using statistical estimates of volatility. This limitation has been taken into account by practitioners using the concept of implied volatility. Implied volatility is the value of statistical volatility needed to be used in the standard Black-Scholes pricing formula for a given strike price to yield the market price of that option. The value of implied volatility for different strike prices should theoretically be identical, but is usually seen in the market to vary. In most markets across the world it has been observed that the implied volatilities of different strike prices form a pattern of either a 'smile' or 'skew'. Theoretically, since volatility is a property of the underlying asset it should be predicted by the pricing formula to be identical for all derivatives based on that same asset. Hull [1993] and Nattenburg [1994] have attributed the volatility smile to the non normal Skewness and Kurtosis of stock returns which is contrary to the assumption of Black-Sholes Model.

Many improvements to the Black-Scholes formula have been suggested in academic literature for addressing the issue of volatility smile. Corrado and Su [1996] have extended the Black–Scholes formula to account for non-normal skewness and kurtosis in stock return distributions. Their assumption is that if the volatility smile is due to non-normal skewness and kurtosis of the distribution of asset returns, this should be removed if the effect of this deviations is included in the pricing formula. The method suggested by them for incorporation of this deviation is based on fitting of the first four moments of stock return distribution on to a pattern of empirically observed option prices. The values of implied volatility, implied skewness and implied kurtosis have been estimated by minimizing the error between predicted and actual market option prices for all available strike prices.

In this paper we address the volatility smile pattern as observed in the NSE Nifty index options. This paper is organized into four sections. The first section discusses current and recent literature on this topic. The second and third sections present the applied model and the implementation details. The last section presents the conclusion of this study and the managerial implications thereof.

Literature Review

In all major markets across the world differing implied volatilities of options on the same underlying asset across different exercise prices and terms to maturity have been observed. In a recent study on the NSE NIFTY, Misra, Kannan and Misra [2006] have reported a significant volatility smile on NIFTY options. The results of their study show that deep in the money and deep out of the money options have higher volatility than at the money options and that the implied volatility of OTM call options is greater than ITM calls. Daily returns of the NSE NIFTY have been found to follow normal distribution with some Skewness and Kurtosis. These results suggest that the volatility smile observed in the NSE NIFTY options can be explained in some measure by the observed Skewness and Kurtosis.

To incorporate the effects of non-normal skewness and kurtosis into the Black-Scholes option pricing formula, Hermite polynomials have been used to get an expansion of the probability density function adjusted for skewness and kurtosis. Usually, this series is called Gram-Charlier. For practical purposes, only the first few terms of this expansion are taken into consideration. The resulting truncated series may be viewed as the normal probability density function multiplied by a polynomial that accounts for the effects of departure from normality. The Gram-Charlier series uses the moments of the real distribution. The Edgeworth series is similar to Gram-Charlier but uses cumulants instead of moments. Although the series are equivalent, for computational purposes the Gram-Charlier series seems to perform better than the Edgeworth series [Johnson et al., 1994].

This approach was introduced in financial economics by Jarrow and Rudd [1982], and it has been applied by Madan and Milne [1994], Longstaff [1995], Abken et al. [1996a ; 1996b], Brenner and Eom [1997], Knight and Satchell [1997], Backus et al. [1997], Corrado and Su [1997].

Jarrow and Rudd [1982] proposed a semi-parametric option pricing model to account for observed strike price biases in the Black-Scholes model. They derived an option pricing formula from an Edgeworth expansion of the lognormal probability density function to model the distribution of stock prices. Corrado and Su [1996] have adapted this extension developed by Jarrow and Rudd to extend the Black-Scholes formula to account for non-normal skewness and kurtosis in stock returns. While following the same methodology they used a Gram-Charlier series expansion of the normal probability density function to model the distribution of stock log prices. This method fits the first four moments of a distribution to a pattern of empirically observed option prices. The mean of this distribution is determined by option pricing theory, but an estimation procedure is employed to yield implied values for the standard deviation, skewness and kurtosis of the distribution of stock index prices. We have used the extended formula adapted by Corrado and Su [1996] for the NSE NIFTY index options to address the volatility smile reported.

Framework/Model Applied

Corrado and Su [1996] have developed a method to incorporate effects of non-normal skewness and kurtosis of asset returns into an expanded Black-Scholes option pricing formula (Brown et. al. [2002] suggested a correction to this approach which was incorporated in Corrado and Su [1997]). Their method adapts a Gram–Charlier series expansion of the standard normal density function to yield an option price formula which is the sum of Black–Scholes option price plus two adjustment terms for non-normal skewness and kurtosis. Specifically, the density function g(z) defined below accounts for non-normal skewness and kurtosis, denoted by μ_3 and μ_4, respectively, where n(z) represents the standard normal density function.

$$g(z) = n(z)\left[1 + \frac{\mu_3}{3!}(z^3 - 3z) + \frac{\mu_4 - 3}{4!}(z^4 - 6z^2 + 3)\right]$$

Where,

$$z = \frac{\ln\left({S_t}/{S_o}\right) - \left(r - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

And,

S_o is the current asset price

S_t is the stochastic asset price at time t

r is the risk free rate of interest

σ is the standard deviation of the returns for the underlying asset

In the formula above, skewness μ_3 and kurtosis μ_4 have been explicitly used in the density function g(z) in the functional form. For the normal distribution curve the values of these coefficients are: skewness $\mu_3 = 0$; and kurtosis $\mu_4 = 3$.

Using the function g(z), the value of the theoretical call price as the present value of the expected payoff at option expiration has been found out to be:

$$C = e^{-rt} \int_K^{\infty} (S_t - K) g\big(z(S_t)\big) dz(S_t)$$

Where,

K is the strike price

$z(S_t) = (\log S_t - \mu)/\ \sigma\sqrt{t}$

$\mu = \log S_0 + \left(r - \frac{\sigma^2}{2}\right) t$

The above integral can be evaluated using the Gram-Charlier density expansion, and the evaluated option price is denoted as C_{GC} while the Black-Scholes option price formula is denoted as C_{BS}.

$$C_{BS} = S_o N(d) - K e^{-rt} N(d - \sigma\sqrt{t})$$

$$d = \frac{\ln\left({}^{S_o}/_{K}\right) + \left(\mathrm{r} + \frac{\sigma^2}{2}\right)\mathrm{t}}{\sigma\sqrt{t}}$$

$$C_{GC} = C_{BS} + \mu_3 Q_3 + (\mu_4 - 3) Q_4$$

Where,

$$Q_3 = \frac{1}{3!} S_o \sigma\sqrt{t} \left((2\sigma\sqrt{t} - d) n(d) + \sigma^2 t N(d) \right)$$

$$Q_4 = \frac{1}{4!} S_o \sigma\sqrt{t} \left((d^2 - 1 - 3\sigma\sqrt{t}(d - \sigma\sqrt{t}) n(d) + \sigma^3 t^{3/2} N(d) \right)$$

In the adjusted formula, the terms $\mu_3 Q_3$ and $(\mu_4 - 3)Q_4$ measure the effect of the non-normal skewness and kurtosis on the option price C_{GC}.

For this study, we have used both the Black-Scholes formula and the modified formula as suggested by Corrado and Su [1996] to calculate option prices. Thereafter, the error in both cases has been calculated and tested for statistically significant difference using paired 't' test.

Empirical Study

We started the study by testing the fact that the Nifty closing values do not follow a lognormal distribution and that Nifty returns do not conform to the normal distribution. **Figure 1** shows the frequency plot of Nifty closing values.

For the examination, we have computed the mean, standard deviation, skewness and kurtosis of the daily and weekly NSE Nifty returns since its inception. The values are reported in **Table 1**. The distribution of the returns is shown in **Figure 2 & 3** and is superimposed with the normal distribution of identical mean and variance for better comparison.

From this analysis it can be observed that the distribution of Nifty returns has a negative skew and positive kurtosis as expected and reported for most markets across the world. After confirming significant skewness and kurtosis in the returns, the Black-Scholes formula and the skewness and kurtosis adjusted formula have been applied for prediction of option prices.

Thereafter, the analysis was based on call prices of Nifty Options (European type) for the period of about three months from 1st August 2007 to 24th October 2007. Data for thinly traded options (less than 100 contracts on a given day) was excluded from the study.

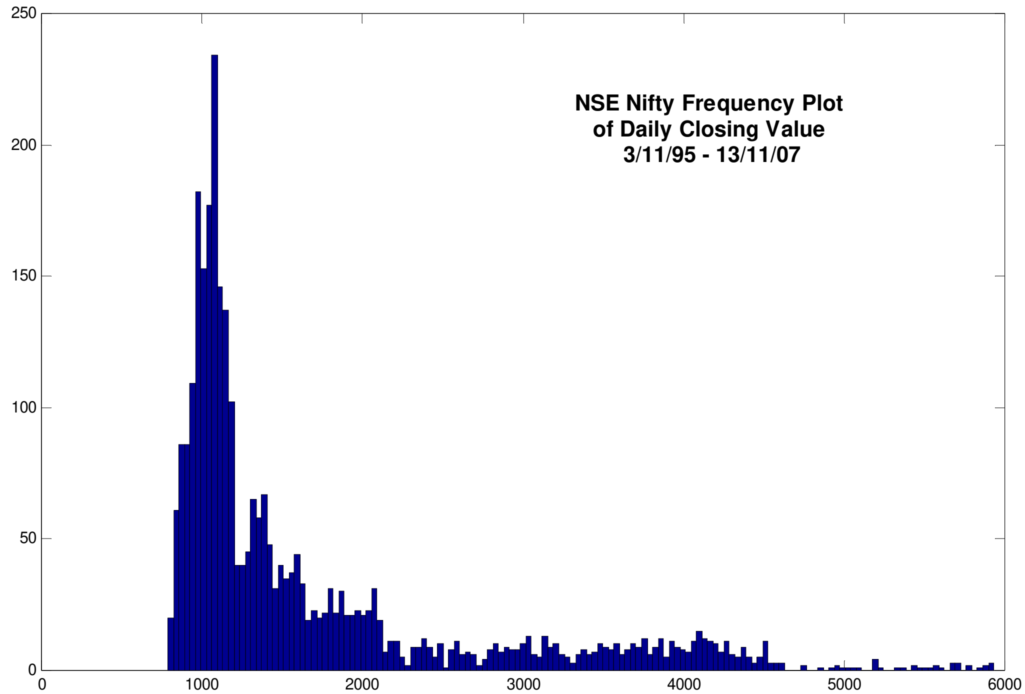

Figure 1: NSE Nifty Frequency of Closing Values

NSE Nifty	Daily Returns	Weekly Returns
Mean	0.0006	0.0023
Std. Dev.	0.016	0.033
Skewness	-0.3168	-0.3334
Kurtosis	4.4682	1.7953

Table 1: NSE Nifty Returns Data

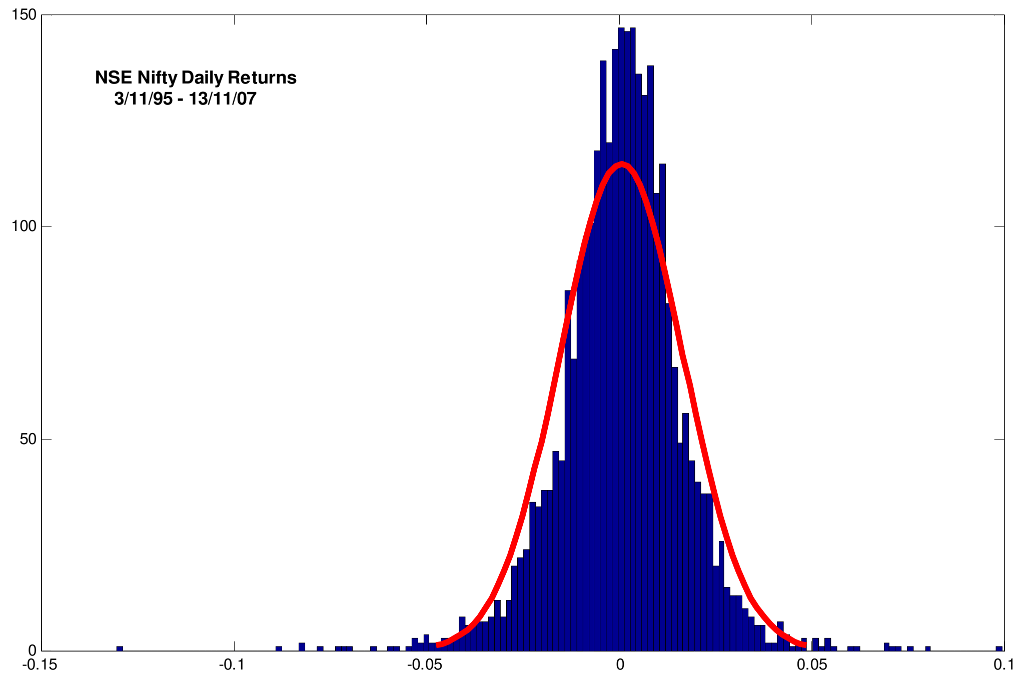

Fig.: 2: NSE Nifty Frequency Plot of Daily Returns (3/11/95 – 13/11/97)

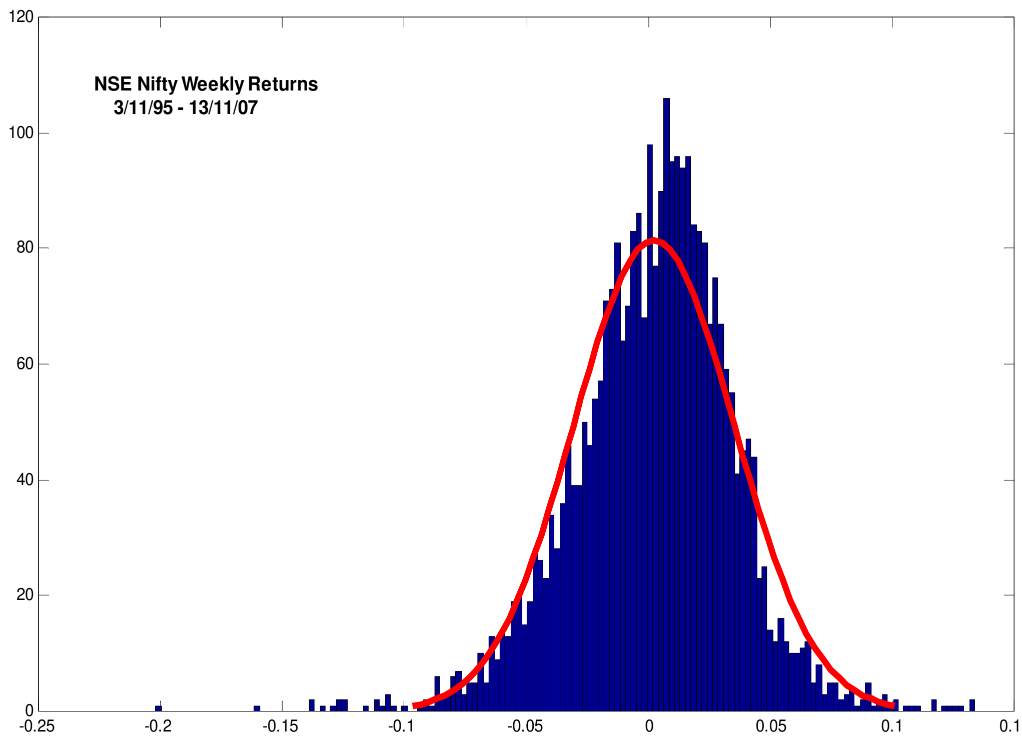

Fig.: 3: NSE Nifty Frequency Plot of Weekly Returns (3/11/95 – 13/11/97)

Methodology

The Black Scholes option pricing formula

The four parameters of Black-Scholes option pricing formula namely, stock price, strike price, time to option maturity and the risk free interest rate have been directly observed form the market. The daily MIBOR (Mumbai Inter Bank Offer Rate) rate has been taken as the risk free interest rate. Another input to the formula is the standard deviation of stock price. This should theoretically be identical for options of all strike prices because the underlying asset is the same in each case. But, since this is not directly observable, it has been estimated using the following method -

Using option prices for all contracts within a given maturity series observed on a given day, we estimate a single implied standard deviation to minimize the total error sum of squares between the predicted and the market prices of options of various strike prices. This has been calculated using Microsoft Excel Solver function by minimizing the following function by iteratively changing the implied standard deviation.

$$\min_{BSISD} \sum_{j=1}^{N} \left[C_{OBS,j} - C_{BS,j}(BSISD)\right]^2$$

Where BSISD stands for the Black-Scholes Implied Standard Deviation

After all the input variables for the model are obtained, they are used to calculate theoretical option prices for all strikes within the same maturity series for the following day. Thus theoretical option prices for a given day are based on a prior-day, out-of sample implied standard deviation estimate. We then compare these theoretical prices with the actual market prices observed on that day.

Skewness and Kurtosis adjusted Black-Sholes option pricing formula

Next, we assess the skewness and kurtosis adjusted Black-Scholes option pricing formula developed by Jarrow and Rudd [1982] using an analogous procedure. Specifically, on a given day we estimate a single implied standard deviation, a single skewness coefficient, and a single excess kurtosis coefficient by minimizing once again the error sum of squares represented by the following formula.

$$\min_{ISD,ISK,IST} \sum_{j=1}^{N} \left[C_{OBS,j} - \left(C_{BS,j}(ISD) + ISK * Q_3 + (IKT - 3) * Q_4 \right) \right]^2$$

Where ISD, ISK and IKT represent estimates of the implied standard deviation, implied skewness and implied kurtosis parameters based on N price observations.

We then use these three parameter estimates as inputs to the Jarrow-Rudd formula to calculate theoretical option prices corresponding to all option prices within the same maturity series observed on the following day. Thus these theoretical option prices for a given day are based on prior-day, out-of-sample implied standard deviation, skewness, and excess kurtosis estimates. We then compare these theoretical prices with the actual market prices observed on that day.

Hypothesis:

The total error in prediction of option prices for various strike prices by the modified Black Scholes method is less than that by the original Black Scholes method.

Comparison:

The theoretical option prices thus generated using the two approaches are then compared with their actual market prices. For comparison, we compute the Error Sum of Squares (ESS) for the two approaches by summing the square of the difference between the predicted and the actual prices. These two ESS are then compared for statistically significant difference using the paired 't' test.

Results:

The paired 't' test of the samples of ESS results in a 't' statistic value of 7.57 which overwhelmingly rejects the hypothesis that the errors in prediction of option prices by the two methods are not significantly different.

We also do the comparison test independently for options of all four maturities. The 't' statistic values in case of each of the four separate tests are greater than the critical value for 95% confidence level. Hence we can see that the modified Black-Scholes formula appears to price Nifty options much closer to the actual market prices.

Figure 4 shows the calculated implied volatilities using the two methods. Black-Scholes implied volatilities are the usual volatilities required to be inserted into the BS formula so that it gives the market price of the option. For the modified Black-Scholes method, the skewness and kurtosis have been kept constant and equal to that obtained upon reducing the total error in pricing of options of all strikes for a given maturity for that day (as explained in methodology), and then the volatilities have been calculated as those required to be inserted into the modified BS formula so that it gives the market price of the option.

The volatility smile as observed for the BS model is significant, while that for the modified model the implied volatility curve is almost flat.

The detailed results tables are as follows:

t-Test: Paired Two Sample for Means		
	BS Original	*BS Modified*
Mean	20598.15661	2307.040835
Variance	342544637.2	10664366.58
Observations	60	60
Pearson Correlation	0.024857351	
Hypothesized Mean Difference	0	
Df	59	
t Stat	**7.571032779**	
P(T<=t) one-tail	1.45726E-10	
t Critical one-tail	1.671093033	
P(T<=t) two-tail	2.91451E-10	
t Critical two-tail	2.000995361	

Table 2: Paired ‘t’ test results (combined for all strike prices)

t-Test: Paired Two Sample for Means - 30th Aug		
	BS Original	*BS Modified*
Mean	20340.74418	1302.505601
Variance	424366615.5	1382374.835
Observations	10	10
Pearson Correlation	-0.55641946	
Hypothesized Mean Difference	0	
Df	9	
t Stat	**2.829569097**	
P(T<=t) one-tail	0.009868142	
t Critical one-tail	1.833112923	
P(T<=t) two-tail	0.019736283	
t Critical two-tail	2.262157158	

t-Test: Paired Two Sample for Means - 27th Sep		
	BS Original	*BS Modified*
Mean	30932.3018	1982.878403
Variance	389184830	5497910.873
Observations	20	20
Pearson Correlation	0.2606916	
Hypothesized Mean Difference	0	
df	19	
t Stat	**6.72546979**	
P(T<=t) one-tail	9.958E-07	
t Critical one-tail	1.72913279	
P(T<=t) two-tail	1.9916E-06	
t Critical two-tail	2.09302405	

t-Test: Paired Two Sample for Means - 25th Oct		
	BS Original	*BS Modified*
Mean	18908.97069	3382.171758
Variance	192781467.8	19322125.72
Observations	20	20
Pearson Correlation	-0.13652606	
Hypothesized Mean Difference	0	
Df	19	
t Stat	**4.59090931**	
P(T<=t) one-tail	9.97638E-05	
t Critical one-tail	1.729132792	
P(T<=t) two-tail	0.000199528	
t Critical two-tail	2.09302405	

t-Test: Paired Two Sample for Means - 29th Nov		
	BS Original	*BS Modified*
Mean	3565.65045	1809.639087
Variance	26532333.2	11932312.04
Observations	10	10
Pearson Correlation	0.96883806	
Hypothesized Mean Difference	0	
df	9	
t Stat	**2.78084378**	
P(T<=t) one-tail	0.01068564	
t Critical one-tail	1.83311292	
P(T<=t) two-tail	0.02137127	
t Critical two-tail	2.26215716	

Table 3: Paired ‘t’ test results (separately for different strike prices)

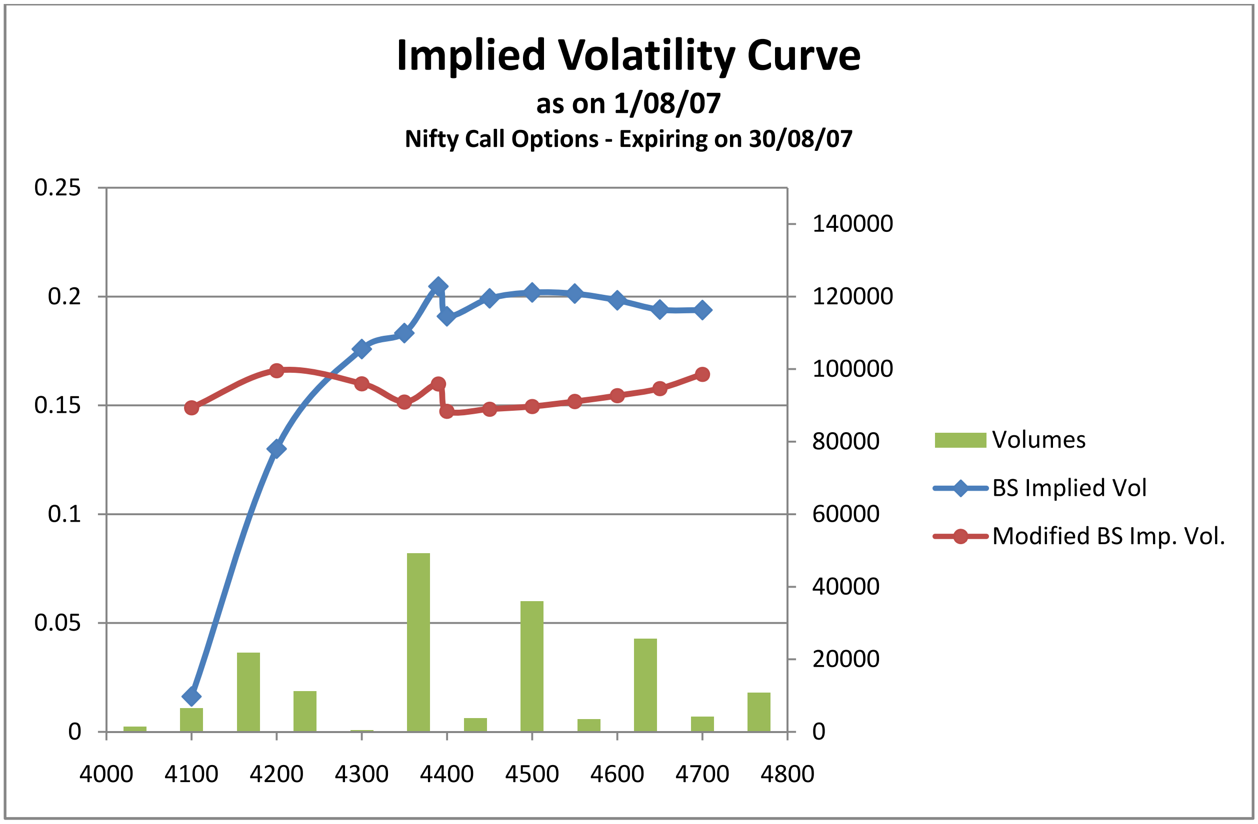

Figure 4: Implied Volatility Curve (BS and Modified BS) for Nifty Call Options on 1/08/07

For the Modified BS curve, Skewness = 2.13 & Kurtosis = -2.44

Conclusion and managerial/policy implications

The results obtained from the analysis confirm our hypothesis that the modified Black-Scholes model as put forward by Corrado & Su performs significantly better for NSE Nifty option prices. We also see that the calculation process in the modified model is not very different from the Black-Scholes model. Since it does not add unnecessary complexity and still gives significantly better predictions of option prices, we recommend that this modified model should be looked at as a better alternative to the existing method.

This study also confirms that fitting of higher order moments of the distribution of returns is important, especially for options away from the money.

Figure 4 shows the implied volatility curve for both the methods. It graphically shows that the 'smile' of the implied volatility curve can be significantly explained by the implied skewness and kurtosis of the reduced by using the modified BS formula.

This study also shows a way towards further work in this area. In this study, we have calculated implied volatility, skewness and kurtosis based on today's data to predict tomorrow's prices. This can be extended to explore whether the modified approach gives significantly better prices for longer durations or not. A related study could be regarding comparison of returns achieved using trading strategies based on these two different models. It would be interesting to see if significant gains can be made using the modified BS model over the original model.

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Appendix 'A' – Data on Volatility Smile

Date	Expiry	Strike Price	BS Implied Vol	Modified BS Imp. Vol.	Volumes	BS Mod Imp Skew	BS Mod Imp Kurt
01-Aug-07	30-Aug-07	4100	0.0162	0.148910961	1448	4.63819567	1.138337607
01-Aug-07	30-Aug-07	4200	0.130055461	0.165977733	6628	1.997287203	3.001311393
01-Aug-07	30-Aug-07	4300	0.175867395	0.159943408	21800	2.009064915	2.986649372
01-Aug-07	30-Aug-07	4350	0.183203284	0.151586922	11183	2.000371554	2.999023259
01-Aug-07	30-Aug-07	4390	0.204689112	0.159854501	443	2.006943806	2.989818834
01-Aug-07	30-Aug-07	4400	0.190901711	0.147323983	49270	2.000230589	2.999689689
01-Aug-07	30-Aug-07	4450	0.19914362	0.14826732	3789	1.997030102	3.005560229
01-Aug-07	30-Aug-07	4500	0.201871418	0.149495946	35965	1.95800107	3.057268601
01-Aug-07	30-Aug-07	4550	0.201336728	0.151756237	3497	1.801023346	3.137984053
01-Aug-07	30-Aug-07	4600	0.198280889	0.154485538	25647	1.649510342	4.0545701
01-Aug-07	30-Aug-07	4650	0.193998608	0.157726943	4129	1.386723803	5.506102777
01-Aug-07	30-Aug-07	4700	0.193757333	0.164291497	10814	1.181957805	4.921665971

Appendix 'B' – Error Sum of Squares (both approaches) for the days on which option prices are predicted based on last day's data

			Black Scholes Method			Modified Black Scholes Method			
Date	**Expiry**		**Error**	**Volatility**		**Error**	**Volatility**	**Skewness**	**Kurtosis**
02-Aug-07	30-Aug-07		60927.04942	0.185121765		97.32835152	0.153913651	2.133776387	-2.441662433
06-Aug-07	30-Aug-07		39604.39956	0.173739545		914.3397652	0.160598894	1.210660684	-4.225859386
08-Aug-07	30-Aug-07		38326.3267	0.196681537		571.4732793	0.183089493	1.271304849	-2.984777628
10-Aug-07	30-Aug-07		30481.28412	0.213455491		898.2077131	0.16358353	1.028276584	-4.059784161
14-Aug-07	30-Aug-07		13291.71849	0.237645487		459.5281112	0.204639867	0.950000204	-0.195634945
17-Aug-07	30-Aug-07		5052.086997	0.285664629		4257.804148	0.213343235	0.304324055	0.216262002
21-Aug-07	30-Aug-07		1991.894706	0.31349603		1041.299202	0.305517736	0.297634462	2.867832614
23-Aug-07	30-Aug-07		2495.495556	0.324967867		1201.377308	0.318174196	-0.14275884	4.630263535
27-Aug-07	30-Aug-07		4477.808672	0.252838284		2055.110569	0.300983547	-0.030773994	3.524009033
29-Aug-07	30-Aug-07		6759.377573	0.203734698		1528.587567	0.221626403	0.268942068	2.639804032
02-Aug-07	27-Sep-07		24545.5972	0.191765495		1273.51681	0.187315757	2.009737929	2.989815228
06-Aug-07	27-Sep-07		27625.4738	0.188598911		495.5861234	0.171705501	1.997653383	3.003895318
08-Aug-07	27-Sep-07		14389.47524	0.20624162		134.9406774	0.172402576	1.270817244	-1.367706694
10-Aug-07	27-Sep-07		30918.05662	0.208012031		1416.435846	0.152661136	-0.073796567	-2.463725386
14-Aug-07	27-Sep-07		14957.31591	0.227459773		540.2406074	0.203200152	0.410949443	-0.211781326
17-Aug-07	27-Sep-07		7883.952548	0.279622797		2727.785768	0.331290893	-0.652350316	10.86069605
21-Aug-07	27-Sep-07		5047.711927	0.28935602		637.1112978	0.322711979	-0.493129007	4.556720639
23-Aug-07	27-Sep-07		9241.406586	0.269242499		1369.384506	0.357319115	-0.876827836	7.276004416
27-Aug-07	27-Sep-07		13192.93678	0.241965057		928.7633029	0.355687202	-0.977988769	8.576923242
29-Aug-07	27-Sep-07		19683.2513	0.212769884		1729.637639	0.690747595	-2.266262029	17.21665822
31-Aug-07	27-Sep-07		50487.60887	0.187153381		2617.945979	0.186128974	1.051971388	-0.494280462
04-Sep-07	27-Sep-07		79457.12284	0.187578787		2893.348446	0.174480048	0.465772816	-1.484379269
06-Sep-07	27-Sep-07		47443.87677	0.168655006		3376.0764	0.162594612	1.041245525	-2.644880322
10-Sep-07	27-Sep-07		46967.91405	0.169294233		913.9482611	0.171923206	-0.110640111	1.387471659
12-Sep-07	27-Sep-07		28298.00761	0.211085038		371.1404513	0.206896848	-0.339116563	2.285022148
14-Sep-07	27-Sep-07		34601.81409	0.218454552		260.3347116	0.201178654	-0.150590446	0.523953447
18-Sep-07	27-Sep-07		30979.72138	0.25467181		975.4562652	0.212364379	-0.684492897	2.893715221
20-Sep-07	27-Sep-07		40782.52379	0.23760712		10487.03638	0.281737588	-0.940859816	7.669125836
24-Sep-07	27-Sep-07		66787.28664	0.230107453		1587.041933	0.174691159	0.20653119	0.47705759
26-Sep-07	27-Sep-07		25354.98234	0.313028904		4921.836667	0.196094293	0.185791029	1.278697089
06-Aug-07	25-Oct-07		13572.33457	2.26909E-24		54.42775868	0.182696846	0.886052145	3.047513507
27-Aug-07	25-Oct-07		34.31395316	0.235429191		49.13358529	0.318038088	-0.818132226	1.740351362
30-Aug-07	25-Oct-07		12907.07854	0.201130594		15.56056892	0.305265361	2.260098004	2.837924755
03-Sep-07	25-Oct-07		8358.235683	0.189943518		78.77873965	0.111195223	-2.49134533	-16.92834634
05-Sep-07	25-Oct-07		15024.16326	0.182693215		113.8009676	0.189859232	-0.300374473	2.08864019

			Black Scholes Method			Modified Black Scholes Method			
Date	Expiry		Error	Volatility		Error	Volatility	Skewness	Kurtosis
07-Sep-07	25-Oct-07		15512.35963	0.17962645		16.74290491	0.162033309	-0.150605651	-0.410715423
11-Sep-07	25-Oct-07		16588.19845	0.198409735		1186.875185	0.182954322	0.508223368	4.764283899
13-Sep-07	25-Oct-07		17295.41729	0.194706798		325.6152478	0.297277594	-0.983474273	10.07140551
17-Sep-07	25-Oct-07		23140.15157	0.193442793		9.261525815	0.185361413	-0.270486098	1.09183015
19-Sep-07	25-Oct-07		42622.0652	0.210323665		1043.873968	0.186192854	0.223479441	-0.515184411
21-Sep-07	25-Oct-07		49777.50688	0.187335924		3959.685787	0.20636702	0.845542987	7.771007336
25-Sep-07	25-Oct-07		26850.46777	0.220757391		3632.730714	0.630504493	1.636151396	13.64128097
01-Oct-07	25-Oct-07		51159.54959	0.23680282		652.0953917	0.230711499	-0.891869967	-0.300833878
04-Oct-07	25-Oct-07		16827.83974	0.257052229		3783.990284	0.227174056	-0.564526729	2.663369987
08-Oct-07	25-Oct-07		4619.269149	0.308464606		3270.476348	0.28156063	-0.874079836	3.624433127
10-Oct-07	25-Oct-07		15873.11113	0.316287833		14587.7481	0.310515665	-0.669919981	2.314873149
12-Oct-07	25-Oct-07		10227.90974	0.35044309		9994.201691	0.304810038	-0.652502455	5.641813353
16-Oct-07	25-Oct-07		9071.745112	0.382296023		6002.89684	0.367113463	-1.537935272	4.662973066
18-Oct-07	25-Oct-07		10575.53459	0.407934706		11910.81317	0.313228748	-0.567909754	-0.003820752
22-Oct-07	25-Oct-07		18142.16201	0.506001568		6954.726385	0.373287352	0.034676262	3.557241188
05-Sep-07	29-Nov-07		4432.286627	0.170201607		496.7397359	0.232568702	2.185911954	2.813007118
11-Sep-07	29-Nov-07		1.26963E-18	13338.55904		0	0.188072868	2.011495679	2.987036955
14-Sep-07	29-Nov-07		4210.198137	0.197395971		1057.314135	0.16114617	1.820778937	3.187688372
01-Oct-07	29-Nov-07		88.8369657	0.220402452		88.71934226	0.201692046	-0.790082127	-0.076654269
04-Oct-07	29-Nov-07		330.4163573	0.250588352		77.65432153	0.189138562	-1.024863272	-1.073092029
12-Oct-07	29-Nov-07		680.0303735	0.291539505		209.29181	0.435122662	-0.501176589	9.865252808
16-Oct-07	29-Nov-07		1450.441529	0.331122876		825.5637678	0.381138556	-0.596131803	3.940308769
18-Oct-07	29-Nov-07		1527.895054	0.364658683		894.8321757	0.34959633	-1.196699546	0.795316114
22-Oct-07	29-Nov-07		5968.120859	0.387403218		3168.100335	0.813332739	-1.49727559	12.22853378
24-Oct-07	29-Nov-07		16968.27858	0.345191171		11278.17524	0.626406123	-0.796780204	5.925996473