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THEORETICAL FLAWS IN THE USE OF THE CAPM FOR INVESTMENT DECISIONS

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Abstract. This paper uses counterexamples and simple formalization to show that the standard CAPM-based Net Present Value may not be used for investment valuations. The reason is that the standard CAPM-based capital budgeting criterion implies a notion of value which does not comply with the principle of additivity. Framing effects arise in decisions so that different descriptions of the same problem lead to different choices. As a result, the CAPM-based NPV as a tool for valuing projects and making investment decisions is theoretically unsound, even if the CAPM assumptions are met.

Keywords. Capital budgeting, CAPM, investment decisions, nonadditivity, framing effects

Introduction

The Capital Asset Pricing Model (henceforth CAPM) is a consolidated paradigm in financial economics since its presentation in Sharpe (1964) and Lintner (1965). Its empirical confirmation has been debated for years (Black, Jensen and Scholes, 1972; Fama and MacBeth, 1973; Roll, 1977; Gibbons, 1982; Fama and French, 1992) but its theoretical soundness as an equilibrium model is undisputed. As a tool for valuing and selecting projects, its use is considered theoretically correct, once the CAPM assumptions are met in the relevant security market (see Mossin, 1969; Rubinstein, 1973; Weston, 1973; Copeland and Weston, 1988; Brealey and Myers, 2000):

> The capital-asset-pricing model represents one of the most important advances in financial economics.… it is useful in corporate finance, since the discount rate on a project is a function of the project's beta. (Ross, Westerfield, and Jaffe, 1999, p. 269).

Numerous contributions have been devoted to verifying whether real-life decision makers comply with the CAPM paradigm, and there is evidence that it is often violated by managers and practitioners in capital budgeting decisions (Poterba and Summers, 1995; Graham and Harvey, 2001, 2002). Some scholars are trying to provide reasons for explaining this incorrect choice behavior (McDonald, 2000; Jagannathan and Meier, 2002).

This paper does not address the problem of corroborating or disconfirming the CAPM+NPV methodology from a descriptive or empirical point of view, and does not either deal with the CAPM as an equilibrium model; it copes with the *standard* use of the CAPM and NPV for valuing risky projects and for making decisions. In particular, this work aims at showing that the standard CAPM-based capital budgeting criterion is unsound for both valuation and decision. In other terms, while the current behavioral literature's efforts are addressed to show that real-life decision makers are biased against the CAPM in making decisions, this paper advocates the dismissal of the *standard* CAPM-NPV procedure for project valuation, since it is biased against two principles of rationality: The principle of value additivity and that of description invariance. In essence, this paper shows that (i) value additivity is infringed, (ii) the net present value is an ambiguous notion, (iii) framing effects arise in valuation and decision.

The paper is structured as follows. Section 1 summarizes the standard use of the CAPM for valuing projects and making investment decisions by briefly reviewing the classical literature on capital budgeting. Section 2 relies on an example of Copeland and Weston (1988) to show that the use of the CAPM-based NPV leads to nonadditivity in valuation. Section 3 presents a counterexample where nonadditivity implies different decisions for the same project. Section 4, relying on a simple counterexample, suggests the idea that one may part projects in the way one wants and a different value is obtained for a different partition. While the counterexamples are sufficient to prove value nonadditivity, a formalization of the results is given in section 5, where it is also highlighted the ambiguity of the notions of value and net present value: They may be any real number. (At the end of the paper, Table 13 collects the notational conventions used throughout the paper. All examples refer to one-period projects and rates of return in all Tables are expressed as percentages.)

1. The use of the CAPM for investment decisions

In the corporate finance literature it is given for granted that the CAPM, originated as an equilibrium model, may be unambiguously and safely used as a tool for valuing projects and making decisions, provided that the assumptions of the model are met. The procedure for valuing projects and take decisions is very simple and has been presented and proved in several papers, all of which assume that the CAPM assumptions are met. In particular, assuming that a security market is in equilibrium, any asset *i* traded in the security market lies on the Security Market Line (SML) and his expected rate of return is given by the relation

$$\bar{r}_i = r_f + \lambda \operatorname{cov}(r_i, r_m)$$

where $\lambda := \dfrac{\bar{r}_m - r_f}{\sigma_m^2}$ is the so-called *market price of risk* and $r_i = \dfrac{\mathrm{F}_i - V(i)}{V(i)}$ is the equilibrium rate of return of the asset. Starting from this classical relation, Rubinstein (1973) proves that a firm *i* facing a project *j* should undertake the project if

$$\bar{r}_j > r_f + \lambda \operatorname{cov}(r_j, r_m) \tag{1}$$

where

$$r_j = \frac{\mathrm{F}_j - I_j}{I_j} \tag{2}$$

is the (disequilibrium) expected rate of return of project *j* (Rubinstein, 1973, pp. 171-172 and, in particular, footnote (10)). Eq. (1) above may be equivalently rephrased as

$$\bar{r}_j > r_f + \beta_j(\bar{r}_m - r_f) \tag{3}$$

where

$$\beta_j := \frac{\mathrm{cov}(r_j, r_m)}{\sigma_m^2} \tag{4}$$

is the systematic risk of project *j* (see eq. (3) in Rubinstein, 1973, p. 170, and eq. (2) in Weston, 1973, p. 26). Henceforth, we let $i_j := r_f + \beta_j(\bar{r}_m - r_f)$, so that eq. (3) becomes

$$\bar{r}_j > i_j\,. \tag{5}$$

Eq. (5) means that a project is worth undertaking if the (expected) rate of return of the project is greater than the (expected) rate of return of a security with the same risk. In Rubinstein's words, the project is worth undertaking if

> its expected internal rate of return … exceeds the appropriate risk-adjusted discount rate for the project …; this discount rate is equal to the expected rate of return on a security with the same risk as the project. (Rubinstein, 1973, p. 172)

An alternative present-value formulation of eqs. (3) and (5) is easily derived:

$$-I_j + \frac{I_j(1+\bar{r}_j)}{1+i_j} = -I_j + \frac{\overline{\mathrm{F}}_j}{1+i_j} > 0\,. \tag{6}$$

In a nutshell, the decision maker should accept the project if its NPV, calculated at the risk-adjusted cost of capital, is positive. Note that the second addend in (6) is the *disequilibrium* value of project *j*:

$$V(j) = \frac{\overline{\mathrm{F}}_j}{1+i_j} = \frac{\overline{\mathrm{F}}_j}{1+r_f+\beta_j(\bar{r}_m - r_f)}\,. \tag{7}$$

We may then say that a project is worth undertaking if its *disequilibrium* value exceeds its cost: $V(j) > I_j$.[1]

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[1] If the project lies on the Security Market Line, then $V(j) = I_j$, i.e. $\bar{r}_j = i_j = r_f + \beta_j(\bar{r}_m - r_f)$.

Other outstanding scholars have formulated capital budgeting criteria: Tuttle and Litzenberger (1968), Hamada (1969), Mossin (1969), Stapleton (1971), Bierman and Hass (1973), Bogue and Roll (1974), starting from the CAPM relations as well, present criteria for valuing project profitability. These criteria are actually equivalent to Rubinstein's (and therefore equivalent one another). In particular, Litzenberger and Budd (1970) acknowledge the equivalence of the criteria proposed by Tuttle and Litzenberger, Hamada, and Mossin. Stapleton (1974) recognizes that his own criterion is equivalent to that proposed by Bierman and Hass (see also Bierman and Hass, 1974). Rubinstein (1973) acknowledges the equivalence of his criterion and Mossin's criterion (however, Rubinstein does not directly prove this equivalence so the proof is provided in the appendix of this paper). The beautiful paper of Sebnet and Thompson (1978) proves the equivalence of the criteria formulated by Hamada, Rubinstein, Bierman and Hass, Bogue and Roll.

As a result, in the literature there is a unanimous agreement that eqs. (1)-(7) represent a theoretically impeccable capital budgeting criterion. Despite the universal agreement on this criterion, eqs. (2) and (4) are critical in understanding the theoretical flaws a decision maker incurs by using this criterion for investment valuation and decision. As for now, suffice it to say that placing eq. (2) in eq. (4) and reminding that the covariance operator is homogenous and additive, we have

$$\beta_j = \frac{\operatorname{cov}(\mathrm{F}_j / I_j - 1, r_m)}{\sigma_m^2} = \frac{\operatorname{cov}(\mathrm{F}_j, r_m)}{I_j \sigma_m^2}. \tag{8}$$

Note that the project beta does not depend on the equilibrium value of the project, but on project cost. This boils down to saying that the project beta is a *disequilibrium* beta, not an *equilibrium* beta. The difference between the equilibrium covariance term and the disequilibrium covariance term is not always appreciated. Magni (2007a) argues that the standard use of the CAPM for project appraisal in the corporate finance literature is grounded, implicitly or explicitly, on disequilibrium values, as opposed to equilibrium values (see also Ekern, 2006).[2]

The following sections deal with value additivity. If the CAPM capital budgeting criterion and the resulting disequilibrium NPV are consistent with value additivity, then, given any two projects A and B, we will have

$$V(A) + V(B) = V(A+B) \tag{9}$$

[2] Some authors use equilibrium betas for valuing projects (e.g. De Reyck, 2005), but this procedure does not always guarantee rational valuations (see Magni, 2007a, 2007b).

as well as

$$\text{NPV}(A)+\text{NPV}(B)=-I_A+V(A)-I_B+V(B)=-I_{A+B}+V(A+B)=\text{NPV}(A+B). \qquad (10)$$

If, instead, it is possible to find one counterexample that violates (9) and/or (10), then value additivity is infringed and we may claim that the CAPM-based criterion is biased or, equivalently, that the disequilibrium NPV may not be used for project valuation and selection.

2. Nonadditivity: Different valuations

This section presents an example provided by Copeland and Weston (1988, pp. 414–418). The authors present two projects (we here rename them A and B) and a security market whose rate of return varies across states. In particular, the market rate of return is 26%, 14%, 20% in state 1, state 2, state 3 respectively. The probability of each state is 1/3. The authors assume the risk-free rate is 4%. After having reminded the obvious relation between cash flows and rates of return (Copeland and Weston, 1988, p. 416)

$$r_j=\frac{\text{F}_j-I_j}{I_j}$$

the authors value the project using the CAPM-based notion of value (see (7)-(8) above). Table 1 collects data about the cash flows and the rates of return of the two projects and the market. Table 2 collects the relevant statistics, the values computed by the authors, and provides the net present values as well. The values the authors arrive to are indeed the correct values an evaluator should get if he uses the CAPM-based criterion we have seen in the previous section.

Table 1. Copeland and Weston's example –cash flows and rates of return

		Market		**Project A** $(I_A=100)$		**Project B** $(I_B=100)$	
	Probability	r_m	r_f	F_A	r_A	F_B	r_B
State 1	0.333	26	4	105	5	107.5	7.5
State 2	0.333	14	4	115	15	100	0
State 3	0.333	20	4	95	–5	102.5	2.5

Table 2. Copeland and Weston's example –relevant statistics and values						
	$\bar{r}_j$	$\text{cov}(r_j, r_m)$	β_j	i_j	$V(j)$	$\text{NPV}(j)$
Project A	5.00	−0.002	−0.833	−9.33	115.808	15.808
Project B	3.33	0.0015	0.625	14.00	90.643	−9.356
Market	20.00	0.0024	1.000			

Now we will see that this example is actually a counterexample showing that value additivity is infringed. To this end, let us assume that a third project is available to the investor, say project C. Its cash flows and rates of return are summarized in Table 3. Applying the same formulas used by Copeland and Weston for valuing A and B we can value project C (Table 4). As a result, we find three values for A, B and C: $V(A)$=115.808, $V(B)$=90.643, $V(C)$=203.583. But note that project C's cash flows are such that

$$\mathrm{F}_A + \mathrm{F}_B = \mathrm{F}_C$$

and the costs are such

$$I_A + I_B = I_C .$$

Table 3. Introducing Project C –cash flow and rates of return			
		Project C ($I_C = 200$)	
	Probability	F_C	r_C
State 1	0.333	212.5	6.25
State 2	0.333	215	7.50
State 3	0.333	197.5	1.25

Table 4. Introducing Project C –relevant statistics and values						
	$\bar{r}_C$	$\text{cov}(r_C, r_m)$	β_C	i_C	$V(C)$	$\text{NPV}(C)$
Project C	4.166	−0.00025	−0.104	2.33	203.583	3.583

In other terms, we have C=A+B so that $V(C) = V(A+B)$. If value additivity held, we would have

$$V(A) + V(B) = V(A+B).$$

By contrast, we have

$$V(A) + V(B) = 115.808 + 90.643 = 206.451 \neq 203.583 = V(A+B).$$

We also have

$$\text{NPV}(A) + \text{NPV}(B) = 15.808 - 9.356 = 6.452 \neq 3.583 = \text{NPV}(A+B).$$

This counterexample is itself sufficient to invalidate the CAPM-based capital budgeting criterion, for additivity implies $V(A) + V(B) = V(A+B)$ for *any* project, while we have found two projects A and B for which the equality does not hold.

3. Nonadditivity: Different decisions

Nonadditivity means that different evaluations are made for the same asset (C=A+B). However, there are even cases where decision as well as valuation is different. Suppose, other things equal, that the cost of project A is 104.6 and rename the modified project by labeling it A*. The value of project A* is easily computed (see Tables 5 and 6): $V(A^*) = 115.064$. Accordingly, the net present value is $\text{NPV}(A^*) = 115.064 - 104.6 = 10.464$.

Table 5. Changing the cost of project A –cash flows and rates of return

		Project A* ($I_{A^*} = 104.6$)	
	Probability	F_{A^*}	r_{A^*}
State 1	0.333	105	0.382
State 2	0.333	115	9.942
State 3	0.333	95	−9.177

Table 6. Changing the cost of project A –relevant statistics and values

	$\bar{r}_j$	$\mathrm{cov}(r_j, r_m)$	β_j	i_j	$V(j)$	$\mathrm{NPV}(j)$
Project A*	0.382	−0.0019	−0.796	−8.74	115.064	10.464
Project B	3.330	0.0015	0.625	14.00	90.643	−9.356
Market	20.000	0.0024	1.000			

Let us introduce a third project, say C*, with the same end-of-period cash flow as project C but with a cost of $I_{C*} = 204.6$ (see Table 7). We have $104.6 + 100 = I_{A*} + I_B = I_{C*} = 204.6$ as well as $\mathrm{F}_{A*} + \mathrm{F}_B = \mathrm{F}_{C*}$. As a result, C*=A*+B so that we should have $V(A^*) + V(B) = V(A^* + B)$. Instead, we have (see Table 8)

$$V(A^*) + V(B) = 115.064 + 90.643 = 205.707 \neq 203.508 = V(A^* + B).$$

Value additivity is violated again. Beside the different evaluations we have arrived to, there is here a striking implication for decision making. We find

$$\mathrm{NPV}(A^*) + \mathrm{NPV}(B) = 10.464 - 9.356 = 1.108 \neq -1.091 = \mathrm{NPV}(A^* + B).$$

Net present value additivity is contravened and, in addition, the final decision changes under changes in the description of the project: The investor accepts it or rejects it depending on how he computes the NPV. In particular, if he regards the course of action as a single investment paying off the cash flows 212.5, 215, 197.5 in the three states respectively, he will reject it; if instead he considers it a bundle of two separate investments to be undertaken simultaneously, he will sum the two NPVs and will accept the portfolio.

Table 7. Changing the cost of project C –cash flow and rates of return

		Project C* ($I_{C*} = 204.6$)	
	Probability	F_{C*}	r_{C*}
State 1	0.333	212.5	3.861
State 2	0.333	215	5.083
State 3	0.333	197.5	−3.470

Table 8. Changing the cost of project C –relevant statistics and values						
	$\bar{r}_{C*}$	$\mathrm{cov}(r_{C*}, r_m)$	β_{C*}	i_{C*}	$V(C^*)$	$\mathrm{NPV}(C^*)$
Project C*	1.824	−0.000244	−0.101	2.37	203.508	−1.091

4. Nonadditivity: A different partition

Let us back to projects A and B. Suppose two other projects, D and E, are available to the investor. Their costs are, respectively, $I_D = 80$ and $I_E = 120$. Tables 9 and 10 collect the data for these projects and provide the values $V(D) = 69.513$ and $V(E) = 120.919$. But E+D=A+B, as is easily verified. We should then have

$$V(D) + V(E) = V(E + D) = V(A + B) = V(A) + V(B).$$

Quite the contrary, we have three different values:

$$\begin{aligned} V(D) + V(E) &= 190.433 \\ V(A) + V(B) &= 206.451 \\ V(A + B) = V(C) &= 203.583. \end{aligned}$$

This result suggests that a project C may be arbitrarily partitioned in two projects and the value obtained differs according to the partition selected. In terms of decision making, the situation is chaotic. The net present values are

$$\begin{aligned} \mathrm{NPV}(D) + \mathrm{NPV}(E) &= -9.566 \\ \mathrm{NPV}(A) + \mathrm{NPV}(B) &= 6.452 \\ \mathrm{NPV}(A + B) = \mathrm{NPV}(C) &= 3.583 \end{aligned}$$

which implies that D+E is not profitable while a portfolio of projects A and B is profitable but the profitability is quantitatively ambiguous, depending on whether the evaluator considers the course of action as a single alternative or a bundle of two separate alternatives (to be undertaken in conjunction).

This section evidently suggests that we may partition the cash flow of a project in the way we prefer, and a different value for each partition is obtained. This intuition will be formally proved in section 6.

Table 9. Introducing projects D and E –cash flows and rates of return

		Market		**Project D** ($I_D = 80$)		**Project E** ($I_E = 120$)	
	Probability	r_m	r_f	CF_D	r_D	CF_E	r_E
State 1	0.333	26	4	84.875	6.093	127.625	6.354
State 2	0.333	14	4	86.750	8.437	128.250	6.875
State 3	0.333	20	4	78.625	−1.718	118.875	−0.937

Table 10. Introducing projects D and E – relevant statistics and values

	$\bar{r}_j$	$\mathrm{cov}(r_j, r_m)$	β_j	i_j	$V(j)$	$\mathrm{NPV}(j)$
Project D	4.270	−0.000469	−0.1953	0.875	69.513	−10.486
Project E	4.097	−0.000104	−0.0434	3.305	120.919	0.919
Market	20	0.0024	1.000			

Copeland and Weston do not provide us with a detailed description of the security market the decision maker refers to. We are only offered the risk-free rate and the market rates of return in the various states alongside the corresponding probabilities. We now propose one of many infinite markets that are consistent with the data chosen by Copeland and Weston. Tables 11 and 12 show a simple market consisting of one risky security and a risk-free asset.[3] The market is in equilibrium and all the marketed assets lie on the Security Market Line. This implies (see footnote 1 above) $V(j) = I_j$ and

$$\bar{r}_j = \frac{\overline{\mathrm{F}}_j - V_j}{V_j} = r_f + \beta_j(\bar{r}_m - r_f)$$

[3] Given the assumption of a single risky security, the latter's rate of return is just the market rate of return (it is worth remind that the term "market" in the expression "market rate of return" refers only to risky securities).

for all the assets traded in the security market. The market is not complete since we have two linearly independent securities and three states. In particular, neither project A nor project B can be replicated by a portfolio of the risky security and the risk-free asset, since

$$\det\begin{bmatrix}126 & 104 & 105\\ 114 & 104 & 115\\ 120 & 104 & 95\end{bmatrix}\neq 0 \quad \text{and} \quad \det\begin{bmatrix}126 & 104 & 107.5\\ 114 & 104 & 100\\ 120 & 104 & 102.5\end{bmatrix}\neq 0.$$

Therefore, we cannot use arbitrage pricing to value our projects.[4] But if we use the CAPM-based NPV, value additivity is not fulfilled.

Table 11. A possible market for Copeland and Weston's example –cash flows and rates of return

		Risky security		Risk-free security		
	Probability	F_R	r_R	F_f	r_f	r_m
State 1	0.333	126	26	104	4	26
State 2	0.333	114	14	104	4	14
State 3	0.333	120	20	104	4	20

Table 12. A possible market for Copeland and Weston's example –relevant statistics and values

	$\text{cov}(r_j, r_m)$	β_j	Outstanding shares	$V(j)$
Risky security	0.0024	1	3,000	100
Risk-free security	0	0		
Market	0.0024	1	3,000	300,000

5. Formalization

This section formalizes the results just obtained showing that the CAPM-based disequilibrium NPV is intrinsically unable to value projects consistently. As we may always write $F_j = F_j - k + k$ and $I_j = I_j - h + h$, with $h, k \in R$, we may consider project *j* as a portfolio of two investments, one of which is risky, the second one is riskless. The risky investment pays off the

[4] In other terms, neither A and B may be expressed as a linear combination of the risky security and the risk-free asset.

sum $\mathrm{F}_j - k$ and costs $I_j - h$, the riskless investment pays off the sum k and costs h. Letting $V(h,k)$ be the value of project *j* as a function of *h* and *k*, value additivity implies that $V(h,k)$ is constant under changes in *h*, *k*:

$$V(h_1,k_1) = V(h_2,k_2) \qquad \text{for any} \quad h_1,k_1,h_2,k_2 \in R. \tag{11}$$

Using (7) and (8), we have

$$\begin{aligned} V(h,k) &= \frac{\overline{\mathrm{F}}_j - k}{1 + r_f + \dfrac{\bar{r}_m - r_f}{\sigma_m^2}\operatorname{cov}\left(\dfrac{\mathrm{F}_j - k}{I_j - h} - 1,\, r_m\right)} + \frac{k}{1+r_f} \\ &= \frac{\overline{\mathrm{F}}_j - k}{1 + r_f + \dfrac{\bar{r}_m - r_f}{\sigma_m^2 (I_j - h)}\operatorname{cov}\left(\mathrm{F}_j,\, r_m\right)} + \frac{k}{1+r_f}. \end{aligned} \tag{12}$$

Taking the partial derivatives, we have

$$\frac{\partial V(h,k)}{\partial h} = \frac{-(\overline{\mathrm{F}}_j - k)\dfrac{(\bar{r}_m - r_f)\operatorname{cov}\left(\mathrm{F}_j,\, r_m\right)}{\sigma_m^2 (I_j - h)^2}}{\left[1 + r_f + \dfrac{\bar{r}_m - r_f}{\sigma_m^2 (I_j - h)}\operatorname{cov}\left(\mathrm{F}_j,\, r_m\right)\right]^2} \tag{13}$$

whence

$$\frac{\partial V(h,k)}{\partial h} = \frac{(r_f - \bar{r}_m)\dfrac{\operatorname{cov}\left(\mathrm{F}_j,\, r_m\right)}{\sigma_m^2}(\overline{\mathrm{F}}_j - k)}{\left[\left(1 + r_f\right)\left(I_j - h\right) + \dfrac{(\bar{r}_m - r_f)\operatorname{cov}\left(\mathrm{F}_j,\, r_m\right)}{\sigma_m^2}\right]^2}, \tag{14}$$

and

$$\frac{\partial V(h,k)}{\partial k}=\frac{1}{1+r_f}-\frac{1}{1+r_f+\dfrac{\bar{r}_m-r_f}{\sigma_m^2(I_j-h)}\operatorname{cov}\left(\mathrm{F}_j,\,r_m\right)}. \tag{15}$$

It is evident that both $\dfrac{\partial V(h,k)}{\partial h}$ and $\dfrac{\partial V(h,k)}{\partial k}$ are non identically zero. This means that the function $V(h,k)$ is not constant under changes in *h* and *k*, so that (11) is not fulfilled.[5]

As for the NPV as a function of *h* and *k*, we have

$$\mathrm{NPV}(h,k)=V(h,k)-I_j=\left(-(I_j-h)+\frac{\bar{\mathrm{F}}_j-k}{1+r_f+\dfrac{r_m-r_f}{\sigma_m^2(I_j-h)}\operatorname{cov}\left(\mathrm{F}_j,\,r_m\right)}\right)+\left(-h+\frac{k}{1+r_f}\right)$$

and it is obvious that both $\dfrac{\partial \mathrm{NPV}(h,k)}{\partial h}$ and $\dfrac{\partial \mathrm{NPV}(h,k)}{\partial k}$ are not identically zero, as they should be.

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[5] While this proof is self-sufficient, the use of a riskless project is not restrictive at all. The counterexamples presented in the previous sections show a project as a portfolio of two risky projects. In general, if a project is seen as a portfolio of two risky projects A and B, we have

$$V(A)=\bar{\mathrm{F}}_A\Big/\left(1+r_f+(\bar{r}_m-r_f)\operatorname{cov}(\mathrm{F}_A,r_m)/(\sigma_m^2 I_A)\right)$$

and

$$V(B)=\bar{\mathrm{F}}_B\Big/\left(1+r_f+(\bar{r}_m-r_f)\operatorname{cov}(\mathrm{F}_B,r_m)/(\sigma_m^2 I_B)\right)$$

as well as

$$V(A+B)=\bar{\mathrm{F}}_{A+B}\Big/\left(1+r_f+(\bar{r}_m-r_f)\operatorname{cov}(\mathrm{F}_{A+B},r_m)/(\sigma_m^2\, I_{A+B})\right).$$

As $\mathrm{F}_{A+B}=\mathrm{F}_A+\mathrm{F}_B$ and $I_{A+B}=I_A+I_B$, we obtain

$$V(A+B)=(\bar{\mathrm{F}}_A+\bar{\mathrm{F}}_B)\Big/\left(1+r_f+(\bar{r}_m-r_f)\operatorname{cov}(\mathrm{F}_A+\mathrm{F}_B,r_m)/(\sigma_m^2\,(I_A+I_B))\right).$$

It is evident that, in general, $V(A)+V(B)\neq V(A+B)$.

This boils down to saying that the NPV of a project is any real number. Suppose a decision maker faces project *j* and he is willing to reach a NPV of $L \in R$. He just has to solve the equation

$$\mathrm{NPV}(h,k) = L \tag{16}$$

which has infinite solutions. The decision maker may then always justify acceptance or rejection of a project, as he can part cash flows so as satisfy eq. (16). In general, dealing with a bundle of projects to be accepted or rejected, managers may present the alternatives collecting some projects, or splitting them into several ones, so as to accept and reject the ones they prefer on a subjective basis. This resolves in a distortion and in an arbitrariness left to managers. And if managers are unaware of this distortion, this means that decision on acceptance or rejection is left to chance: Depending on how courses of action are described by (or to) managers, professionals, analysts, clerks, practitioners,[6] the decision process changes and turns to a random decision process where solution is established by a flip-of-a-coin-like procedure.

In formal terms, the notion of (disequilibrium) net present value is an ambiguous notion, since it is any number a decision maker wants it to be (or chance wants it to be). Putting it in behavioral terms, valuations and decisions should not be dependent on how they are described (principle of description invariance). If different description of the same situation lead to different valuations and/or choice behaviors, then "framing effects" arise. (Tversky and Kahneman, 1981; Kahneman and Tversky, 1984; Thaler, 1985, 1999; Soman, 2004). Such a CAPM-minded decision maker just falls prey to framing effects in both valuation and decision. As the violation of the principle of description invariance is considered to be a violation of rationality, we must admit that the CAPM-minded decision maker is an irrational decision maker.

Conclusions

This paper makes use of counterexamples and simple formalization to show that the *standard* use of the CAPM-based capital budgeting criterion is theoretically biased for both valuation and decision. In particular, the use of disequilibrium NPV implies that the principle of value (and net present value) additivity is violated and framing effects arise in both valuations and choice behaviors. This implies that value and net present value are not unambiguous notions, since they may be any real number depending on the way one is willing to frame the project. It is also well-known that arbitrage pricing is consistent with value additivity (see Modigliani and Miller,

[6] The agent may come across an exogenously framed investment or, rather, may frame it herself as she subjectively perceives it: "The choice [of a particular framing] depends on the economic conditions giving rise to that particular net cash flow and on the psychological factors that influence the cognitive perception of the decision maker" (Magni, 2002, p. 211).

1958, whose Proposition I shows that an asset has the same value regardless of how it is framed in terms of equity and debt). But the *standard* CAPM-based NPV (which implies the use of disequilibrium betas) is not consistent with arbitrage pricing for project valuation.[7] It is worth noting that if disequilibrium betas are replaced by equilibrium betas, things improve but in some cases even equilibrium betas lead to irrational valuations and decisions (see Magni, 2007a, 2007b).
Brealey and Myers (2000, ch. 34) claim that three of the seven most importance ideas in finance are Net Present Value, Capital Asset Pricing Model, value additivity; in fact, the use of the NPV+CAPM procedure should be carefully revisited.

References

Bierman, H. and Hass, J. E. (1973), Capital budgeting under uncertainty: a reformulation. *Journal of Finance*, 28(1), 119–129.

Bierman, H. and Hass, J. E. (1974), Reply, *Journal of Finance*, 29(5), 1585.

Bogue, M.C. and Roll, R. (1974), Capital budgeting of risky projects with 'imperfect markets' for physical capital, *Journal of Finance*, 601–613, May.

Black, F., Jensen, M. C., and Scholes, M. (1972), The Capital Asset Pricing Model: Some empirical tests, in Jensen, M. C., *Studies in the Theory of Capital Markets*, New York: Praeger

Brealey, R. A. and Myers, S. C. (2000), *Principles of Corporate Finance*, sixth edition, New York: McGraw-Hill.

Copeland, T. and Weston, J. F. (1988), *Financial Theory and Corporate Finance*. Addison-Wesley Publishing Company, Inc.

Dybvig, P. H. and Ingersoll, J. E. (1982), Mean-variance theory in complete markets, *Journal of Business*, 55 (2), 233–251.

Ekern, S., 2006. A dozen consistent CAPM-related valuation models – so why use the incorrect one? Department of Finance and Management Science, Norwegian School of Economics and Business Administration (NHH). Bergen, Norway. Available at <http://www.nhh.no/for/dp/2006/0606.pdf>.

Fama, E. F. (1977), Risk-adjusted discount rates and capital budgeting under uncertainty, *Journal of Financial Economics*, 5, 3–24.

Fama, E. F. and French, K. R. (1992), The cross-section if expected stock returns, *Journal of Finance*, 47, 427–466.

[7] See Dybvig and Ingersoll (1982) for conditions where CAPM pricing infringes the arbitrage principle. See Magni (2007c) for incompatibility of CAPM-based valuation and arbitrage-based valuation even in those situations where Dybvig and Ingersoll's conditions do not hold.

Fama, E. F. and MacBeth, J. D. (1973), Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy,* 81, 607–636.

Gibbons M. R. (1982), Multivariate test of financial models, *Journal of Financial Economics*, 10, 3–27.

Graham, J. and Harvey, C. (2001), The theory and practice of corporate finance: Evidence from the field. *Journal of Financial Economics, 60*, 187–243.

Graham, J. and Harvey, C. (2002), How do CFOs make capital budgeting and capital structure decisions?. *Journal of Applied Corporate Finance*, *15* (1), 8–22.

Hamada, R. S. (1969), Portfolio analysis, market equilibrium and corporation finance, *Journal of Finance*, 24(1), 13–31, March.

Jagannathan, R. and Meier, I. (2002), Do we need CAPM for Capital Budgeting?, *Financial Management*, 31 (4), 55–77. Available online: <http://www.nber.org/papers/w8719>.

Kahneman, D. and Tversky, A. (1984), Choices, values and frames, *American Psychologist*, 39, 341–350.

Lintner, J. (1965), The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economic and Statistics*, 47, 13–37.

Litzenberger, R. H. and Budd, A. P. (1970), Corporate investment criteria and the validation of risk assets, *Journal of Financial and Quantitative Analysis*, 5(4), 395–418, December.

Magni, C. A. (2002), Investment decisions in the theory of finance: Some antinomies and inconsistencies, *European Journal of Operational Research*, 137/1, 206–217.

Magni, C. A. (2007a), Correct or incorrect application of CAPM? Correct or incorrect decisions with CAPM? forthcoming in *European Journal of Operational Research*, Available at <http://ssrn.com/abstract=1025998>.

Magni, C. A. (2007b), CAPM and capital budgeting: present versus future, equilibrium versus disequilibrium, decision versus valuation. Available at <http://ssrn.com/abstract=1024716>.

Magni, C. A. (2007c), Project valuation and investment decisions: CAPM versus arbitrage, *Applied Financial Economics Letters*. 3(1), 137–140, March.

McDonald, R. L. (2000), Real Options and Rules of Thumb in Capital Budgeting. In M. J. Brennan and L. Trigeorgis, Eds., Project flexibility, agency and competition: new developments in the theory and application of real options. New York: Oxford University Press.

Modigliani, F. and Miller, M. (1958), The cost of capital, corporation finance, and the theory of investment, *American Economic Review*, 48, 261–297.

Mossin, J. (1969), Security pricing and investment criteria in competitive markets, *American Economic Review*, 59 (5), 749–756, December.

Poterba, J. M. and Summers, L. H. (1995), A CEO survey of US companies' time horizons and hurdles rates, *Sloan Management Review*, 37 (1), 43–53, Fall.

Ross, S. A., Westerfield, R. W., and Jaffe, J. (1999). *Corporate Finance*, 5th ed.. Singapore: McGraw-Hill

Rubinstein, M. E. (1973), A mean-variance synthesis of corporate financial theory, *Journal of Finance*, 28, 167–182, March.

Sebnet, L. W. and Thompson, H. E. (1978), The equivalence of mean-variance capital budgeting models, *Journal of Finance*, 23(29), 395–401, May.

Sharpe, W. (1964), Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance*, 19, 425–442.

Soman, D. (2004), Framing, loss aversion, and mental accounting. In D. J. Koehler and N. Harvey, Eds., *Blackwell Handbook of Judgment and Decision Making*, pp. 379–398. Oxford, UK: Blackwell.

Stapleton, R. C. (1971), Portfolio analysis, stock valuation and capital budgeting decision rule for risky projects, *Journal of Finance*, 26(1), 95–117, March.

Stapleton, R. C. (1974), Capital Budgeting under Uncertainty: A Reformation: Comment, *Journal of Finance*, 29(5), 1583–1584.

Thaler, R. H. (1985), Mental accounting and consumer choice, *Marketing Science*, 4 (3), 199–214.

Thaler, R. H. (1999), Mental accounting matters, *Journal of Behavioral Decision Making*, 12, 183–206.

Tuttle, D. L. and Litzenberger, R. H. (1968), Leverage, diversification and capital market effects on a risk-adjusted capital budgeting framework, *Journal of Finance,* 23(3), 427–443.

Tversky, A. and Kahneman, D. (1981), The framing of decisions and the psychology of choice, *Science*, 211, 453–458.

Weston, F. (1973), Investment decisions using the Capital Asset Pricing Model, *Financial Management*, 2 (1), 25–33, Spring..

Appendix

Mossin (1969, p. 755, left column) shows that, assuming the market is in equilibrium, an investment Z will be undertaken by a firm *l* if and only if

$$\frac{1}{1+r_f}(\overline{\mathrm{F}}_Z - R\,\mathrm{cov}(\mathrm{F}_Z, V_m)) > I_Z \tag{A.1}$$

where F_Z is the cash flow generated by the project, V_m is the end-of-period value of the security market, I_Z is the investment cost, and

$$R = \frac{\overline{\mathrm{F}}_l - (1+r_f)V_l}{\mathrm{cov}(\mathrm{F}_l, V_m)} \tag{A.2}$$

with $\overline{\mathrm{F}}_l$=free cash flow of firm *l* (the overbar represents expectation), V_l=market value of firm *l*. Dividing both sides of (A.1) by I_Z we have

$$\frac{\overline{\mathrm{F}}_Z}{I_Z} - R\,\mathrm{cov}(r_Z, V_m) > 1 + r_f \,. \tag{A.3}$$

As $\frac{\overline{\mathrm{F}}_z}{I_z} - 1 = \overline{r}_z$, eq. (A.3) becomes

$$\overline{r}_z > r_f + R\,\mathrm{cov}(r_Z, V_m)\,. \tag{A.4}$$

Letting V_0 be the current value of the market, we have $\mathrm{cov}(r_Z, V_m) = V_0\,\mathrm{cov}(r_Z, r_m)$. Therefore, we have, using (A.2) ,

$$\overline{r}_z > r_f + \frac{\overline{\mathrm{F}}_l - (1+r_f)V_l}{\mathrm{cov}(\mathrm{F}_l, V_m)} V_0\,\mathrm{cov}(r_Z, r_m) \tag{A.5}$$

which boils down to

$$\overline{r}_z > r_f + \frac{\overline{\mathrm{F}}_l - (1+r_f)V_l}{\mathrm{cov}(\mathrm{F}_l, r_m)} \mathrm{cov}(r_Z, r_m) \tag{A.6}$$

whence

$$\overline{r}_z > r_f + \frac{\overline{r}_l - r_f}{\mathrm{cov}(r_l, r_m)} \mathrm{cov}(r_Z, r_m) \tag{A.7}$$

where r_l is the rate of return on firm *l*.[8] The term $\frac{\overline{r}_l - r_f}{\mathrm{cov}(r_l, r_m)}$ "is the same for all companies" (Mossin, 1969, p. 755, right column), so that

$$\frac{\overline{r}_l - r_f}{\mathrm{cov}(r_l, r_m)} = \frac{\overline{r}_m - r_f}{\sigma_m^2}\,.$$

As a result, eq. (A.7) becomes

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[8] V_l and r_l refer to the value and rate of return of firm *l* prior to investment Z.

$$\bar{r}_z > r_f + \beta_Z(\bar{r}_m - r_f)$$

or

$$-I_Z + \frac{\overline{\mathrm{F}}_Z}{1 + r_f + \beta_Z(\bar{r}_m - r_f)} = -I_Z + \frac{I_Z(1 + \bar{r}_z)}{1 + r_f + \beta_Z(\bar{r}_m - r_f)} > 0$$

which coincide with (3) and (6) respectively, with *j*=*Z*. [9] **Q.E.D.**

Table 13. Notations

F_j	End-of-period cash flow of project j
$\overline{\mathrm{F}}_j$	End-of-period expected cash flow of project j
I_j	Investment cost of project j
r_i, r_j	Rates of return of firm *i* and project j
$\bar{r}_i$, $\bar{r}_j$	Expected rates of return of firm *i* and project j
r_m	Market rate of return
$\bar{r}_m$	Expected market rate of return
σ_m^2	Variance of market rate of return
r_f	Risk-free rate
β_j	Beta of project *j*
i_j	Risk-adjusted cost of capital
$V(i), V(j)$	Values of firm *i* and project *j*
NPV(j)	Net Present Value of project *j*
Cov	covariance

[9] Again, one should bear in mind that the beta is a disequilibrium systematic risk, as previously seen.