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Abstract

Economic growth and unemployment exhibit an ambiguous relationship – according to empirical studies. This ambiguity can be investigated by observing the role of the underground economy in shaping the productivity of firms. Indeed, unemployment may be absorbed by underground firms, which adopt backward technology, at the cost of reduced economic growth. Alternatively, unemployment diminishes because productivity grows by employing workers who prefer to become skilled, and thus not to work in underground firms. This paper develops these arguments by using a matching model with underground firms and heterogeneous entrepreneurial ability, and by assuming skill-driven growth. Economic growth thus becomes endogenous, and both the underground sector and unemployment become persistent. The main result is that, under conditions of strict monitoring of the regularity of firms, the underground economy is squeezed, unemployment is reduced, and growth is high, whereas in the case of lax monitoring, the underground economy expands, unemployment is absorbed, and growth is low.

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1. Introduction

Economic growth and unemployment are not always inversely related (Pissarides and Vallanti 2004, 2007) because an ambiguous relationship has also been found both across countries and across long periods of time in the same country (Aghion and Howitt, 1994; Bean and Pissarides, 1993; Caballero, 1993; Hoon and Phelps, 1997; Muscatelli and Tirelli, 2001). Theoretical explanations have usually focused on technological progress in order to determine the conditions that make productivity growth employment-friendly or employment-displacing.[1] This paper takes a different approach by exploring the role of the underground economy in shaping the productivity of firms, so that growth and unemployment can be affected in different ways. Indeed, unemployment may be absorbed by underground firms, which adopt backward technology (see, e.g., La Porta and Shleifer, 2008), at the cost of reduced economic growth. In this case, the relationship between growth and unemployment emerges as positive, and the relationships between underground employment and unemployment emerges as negative. But both signs are reversed in a different case, i.e. when unemployment and the underground economy diminish because productivity grows by employing workers who prefer to become skilled, and thus not to work in underground firms.[2]

Studying the ambiguous relationship between economic growth and unemployment by focusing on the underground economy is especially interesting for policy purposes. In fact, new policies can be envisaged to increase economic growth and to reduce unemployment. Moreover, combating the underground economy, which is undesirable for fiscal and moral reasons, may avoid the counter-indication that unemployment could increase.

This paper studies these issues by adopting a matching-type of model, which has been used in the literature to study both the relationship between growth and unemployment (Laing et al., 1995; Aghion and Howitt, 1994, 1998; Mortensen and Pissarides, 1998; Pissarides, 1990, 2000; Mortensen, 2005), and the relationship between the underground employment and unemployment (Bouev, 2002, 2005; Kolm and Larsen, 2003, 2010; Fugazza and Jacques,

[1] If the technological progress is disembodied, i.e. both old and new jobs benefit from higher labour productivity, the "capitalisation" effect yields a lower unemployment in the steady-state (Pissarides, 1990); whereas, if the technological progress is embodied, i.e. only the new jobs benefit from higher labour productivity, the "creative destruction" effect yields an increase in steady-state unemployment (Aghion and Howitt, 1994, 1998).

[2] A variety of evidence shows that regular firms employ skilled labour, while underground firms employ unskilled labour (Agénor and Aizenman, 1999; Boeri and Garibaldi, 2002, 2006; Bosch and Esteban-Pretel, 2009; Cimoli, Primi and Pugno, 2006; Kolm and Larsen, 2010), and that employment in the underground sector and the education level within countries appear to be negatively correlated (Albrecht et. al., 2009; Cappariello and Zizza, 2009).

2004; Bosch and Esteban-Pretel, 2009; Albrecht et al., 2009).[3] In particular, this second body of studies discusses a further connected issue called the 'shadow puzzle', i.e. the persistence of the underground economy in a variety of contexts and times (Boeri and Garibaldi, 2002, 2006). However, as far as we are aware, no study has attempted to deal with all these issues at the same time.

This paper assumes that entrepreneurial ability is heterogeneous among individuals. This assumption follows the approach of heterogeneous talent allocation, which allows the partition of the economy between regular and underground firms (Lucas, 1978; Baumol, 1990; Rauch, 1991; van Praag and Cramer, 2001). Secondly, the paper assumes that participating to the underground economy involves a moral cost, thus following the idea that both economic incentives and social norms drive individual behaviour (Traxler 2010; Kolm and Larsen 2002; Elster 1989). This assumption allows to separate the individuals according to which they search for a job in the underground or in the regular economy. The third key assumption is that education and skill are crucial for economic growth (Laing et al., 1995), so that endogenous economic growth can emerge. Also this assumption has received a great deal of attention in the literature (Romer, 1986, 1988, 1989; Lucas, 1988; Rebelo, 1991; Stokey, 1991; Savvides and Stengos, 2009).

The conclusions will show the importance of monitoring the regularity of firms to determine the allocation of entrepreneurial ability, the size of the underground economy, the investment in education and skills, and economic growth. Specifically, under conditions of strict monitoring, the underground economy is squeezed, unemployment is reduced, and growth is high, whereas in case of lax monitoring, the underground economy expands, unemployment is thus absorbed, and growth is low. These conclusions, which cover different issues at the same time, can contribute to the debate on the role of the underground economy in economic development and on the policy implications (de Soto, 1989; Johnson et al., 2000; Friedman et al., 2000; Farrell, 2004; Carillo and Pugno, 2004; Banerjee and Duflo, 2005; Cimoli, Primi and Pugno, 2006; La Porta and Shleifer, 2008).

The paper is organised as follows. Section 2 presents the static version of the model and finds the steady-state solutions, thus contributing to better understanding of the persistence of the underground economy (called issue (*i*)), and the relationship between the underground employment and unemployment (issue (*ii*)). Section 3 makes the model dynamic

[3] According to Bouev's (2002, 2005) matching model, scaling down the underground sector may lead to a decrease in unemployment, whereas, according to Boeri and Garibaldi's (2002, 2006) matching model, attempts to reduce shadow employment will result in higher open unemployment.

by introducing investments in education, so that the steady-growth solutions can be identified, hence contributing to a better understanding of the relationship between growth and unemployment (issue (*iii*)). This section also provides some evidence on the role of monitoring the regularity of firms in the relationship between growth and unemployment by considering a broad cross-section of countries. Section 4 concludes the paper with some remarks on policy implications, while the appendices set out the relevant proofs and mathematical details.

2. The model with underground sector and unemployment

This section will show how the differentiation of the economy in two sectors, i.e. the regular and the underground sector, emerges essentially because the population of the economy has different levels of ability, and exploits the different production opportunities that are available. The presentation of the model starts with the matching equations that describe both how firms emerge from matching the vacancies with job seekers, and the possibility of the economy to differentiate in the two sectors.

2.1 The matching framework

The proposed model is of general equilibrium unemployment (Mortensen and Pissarides, 1994; Pissarides, 2000), where numerous firms produce the same type of good, but adopt different institutional set-ups and production techniques. They may be registered, and therefore pay a production tax and adopt a relatively advanced technique; or they may be non-registered, and therefore evade taxes and adopt a less efficient technique. Hence non-registered firms form the underground or shadow sector of the economy, which is illegal because of the process employed, not because of the goods being produced.

As is usual in matching-type models (Pissarides, 2000; Petrongolo and Pissarides, 2001), the meeting of vacant jobs and unemployed workers is regulated by an aggregate matching function $m = m(v, u)$, where v measures the vacancies, and u is the unemployed rate, thus excluding on-the-job-search. The entrepreneurs open vacancies (one-job firms), that can be of two different types, either regular or underground, thus forming an economy with two sectors. Unemployed workers can search for a job only in one of the two sectors, so that v_i and u_i are sector-specific, namely 'directed search' is assumed.[4] The matching function

[4] If the unemployed could search for a job in both sectors simultaneously, it is said that the assumption is of 'random search'.

can thus be written as $m_i = m_i(v_i, u_i)$, where $i \in \{r, s\}$ denotes the sector (r = regular sector, s = shadow sector). This function has the usual properties of being non-negative, increasing and concave in both arguments and performing constant returns to scale. Therefore, the job-finding rate, $g(\theta_i) = m(v_i, u_i)/u_i = m(\theta_i, 1)$, is positive, increasing and concave in the so-called market tightness, i.e. $\theta_i \equiv v_i / u_i$, and the vacancy-filling rate, $f(\theta_i) = m(v_i, u_i)/v_i = m(1, \theta_i^{-1})$, is a positive, decreasing and convex function in θ_i. Furthermore, the *Inada*-type conditions hold: $\lim_{\theta_i \to 0} f(\theta_i) = \lim_{\theta_i \to \infty} g(\theta_i) = \infty$; $\lim_{\theta_i \to \infty} f(\theta_i) = \lim_{\theta_i \to 0} g(\theta_i) = 0$. Indeed, the matching functions of the two sectors may be different, but evidence is lacking in this regard.

For ease of presentation, the Bellman equations are written by already differentiating the two sectors (and specified to find infinite horizon steady-state solutions), i.e.:[5]

$$rV_s = -c_s + f(\theta_s)[J_s - V_s] \qquad rV_r = -c_r + f(\theta_r)[J_r - V_r]$$

$$rJ_s = xy_s - w_s + (\delta + \rho)[V_s - J_s] \qquad rJ_r = xy_r(h) - w_r - \tau + \delta[V_r - J_r]$$

$$rW_s = w_s + (\delta + \rho)[U_s - W_s] \qquad rW_r = w_r + \delta[U_r - W_r]$$

$$rU_s = z + g(\theta_s)[W_s - U_s] \qquad rU_r = z + g(\theta_r)[W_r - U_r]$$

where V_i is the value of a vacancy; J_i is the value of a filled job; U_i is the value for seeking a job; W_i is the value for being employed; c_i is the cost to post a vacancy; x is entrepreneurial ability; y_i is labour productivity, which depends – in the regular sector – on human capital of workers, h; w_i is the wage rate; τ is an exogenous production tax; ρ is the monitoring rate, i.e. the exogenous instantaneous probability of a firm being discovered (and destroyed) as unregistered; δ is the exogenous destruction rate; z is the opportunity cost of employment. The parameters r, ρ and δ are always considered as positive and given, while c_i, y_i, τ and z are positive and given but they will change endogenously in section 3. Similarly, h is assumed as temporarily given, since it will be determined in section 3, thus also affecting w_i. The entrepreneurial ability x is heterogeneous among individuals, as discussed in the next subsection, so that the solutions of the equations will depend on x.

Precisely, the wage rate in both sectors is assumed to be the outcome of bargaining between one of the workers who seeks a job in the sector and the entrepreneur endowed with the minimum level of ability to open vacancies, i.e. $x_{\min}$ (see subsection 2.2):

[5] It is further assumed that time is continuous and individuals are risk neutral, live infinitely and discount the future at the exogenous and positive interest rate r.

$$w_s = \arg\max\left\{(W_s - U_s)^{\beta_s} \cdot (J_s - V_s)^{1-\beta_s}\right\} \Rightarrow (W_s - U_s) = \frac{\beta_s}{(1-\beta_s)} \cdot (J_s - V_s)$$

$$w_r = \arg\max\left\{(W_r - U_r)^{\beta_r} \cdot (J_r - V_r)^{1-\beta_r}\right\} \Rightarrow (W_r - U_r) = \frac{\beta_r}{(1-\beta_r)} \cdot (J_r - V_r)$$

where the parameter $\beta_i \in (0,1)$, with $i \in \{r,s\}$, is the worker's bargaining power. Simple manipulations thus yield:

$$w_s = (1-\beta_s) \cdot rU_s(\theta_s) + \beta_s \cdot (x_{\min} \cdot y_s - rV_s(\theta_s))$$

$$w_r = (1-\beta_r) \cdot rU_r(\theta_r) + \beta_r \cdot (x_{\min} \cdot y_r(h) - rV_r(\theta_r))$$

The property that $w_i'(\theta_i) > 0 \ \ \forall i$ holds, since $V_i'(\theta_i) < 0$, and $U_i'(\theta_i) > 0 \ \ \forall i$.

Therefore, the wage rate depends, in particular, on the labour productivity, which is homogeneous within each sector, i.e. y_i, with $i \in \{r,s\}$, but not on the heterogeneity of the entrepreneurial ability *x*. All entrepreneurs of each sector thus adopt the same wage rate, i.e. w_i. This result, which may appear unusual, can be justified by the following grounds: first, the entrepreneurial ability may include the ability to bargain the wage, so that the greater is the entrepreneurial x with respect to the minimum $x_{\min}$, the greater is the ability to put an upward limit to wages, secondly, workers may be unable to distinguish entrepreneurs for their ability during wage bargaining, while it is common knowledge that labour productivity is homogeneous within each sector.

The surplus of a job in each sector is defined as the sum of the worker's and firm's value of being on the job, net of the respective outside options, so that $S_i = J_i - V_i + W_i - U_i$. Using the Bellman equations, we get:

$$S_s = \frac{x_s \cdot y_s - z + c_s}{r + \delta + \rho + (1-\beta_s) \cdot f(\theta_s) + \beta_s \cdot g(\theta_s)}; \quad S_r = \frac{x_r \cdot y_r - \tau - z + c_r}{r + \delta + (1-\beta_r) \cdot f(\theta_r) + \beta_r \cdot g(\theta_r)}.$$

Note that the heterogeneity of the surpluses is due to the overall heterogeneity of entrepreneurial ability *x*. The expected present values of vacancies for firms can be also obtained, since $(J_s - V_s) = (1-\beta_s) \cdot S_s$ and $(J_r - V_r) = (1-\beta_r) \cdot S_r$, i.e.:

$$rV_s = \frac{f(\theta_s) \cdot (1-\beta_s) \cdot (x_s \cdot y_s - z) - c_s \cdot (r + \delta + \rho + \beta_s \cdot g(\theta_s))}{r + \delta + \rho + (1-\beta_s) \cdot f(\theta_s) + \beta_s \cdot g(\theta_s)} \qquad [1]$$

$$rV_r = \frac{f(\theta_r) \cdot (1-\beta_r) \cdot (x_r \cdot y_r - \tau - z) - c_r \cdot (r + \delta + \beta_r \cdot g(\theta_r))}{r + \delta + (1-\beta_r) \cdot f(\theta_r) + \beta_r \cdot g(\theta_r)} \qquad [2]$$

As in Fonseca et al. (2001), we ignore the range beyond which θ_i ($\equiv v_i/u_i$) is large enough to turn rV_i negative. Hence, *u* must remain positive, and v_i must be restricted below some bound, say $\tilde{v}_i$. Furthermore, in order to exclude the case of $\theta_i = 0$, where the vacancy would be always filled, $v_i \neq 0$ being $u_i \neq 0$, so that the relevant interval for v_i becomes $v_i \in]0, \tilde{v}_i[$, $\forall\, i$. Therefore, the wages of both sectors, which positively depend on θ_i, are upper bounded.

Empirical evidence suggests that underground employment regards low productivity jobs (Agénor and Aizenman, 1999; Boeri and Garibaldi, 2002, 2006; Cimoli, Primi and Pugno, 2006; Bosch and Esteban-Pretel, 2009; Mattos and Ogura, 2009). Therefore, our first key assumption is the following:

Assumption 1. *Labour productivity is much higher in the regular sector with respect to the underground sector:* $y_s << y_r$.

If labour productivity in the regular sector is sufficiently higher, then $w_r > w_s$ for any size of the two sectors, because, in particular, w_s is upper bounded. Higher wages in the regular sector are in fact rather common in the literature (Rauch, 1991; Fugazza and Jacques, 2003; Kolm and Larsen, 2003; Amaral and Quintin, 2006; Boeri and Garibaldi, 2006; Albrecht et al., 2009).

2.2 Moral cost, entrepreneurial ability and the underground sector

Following the idea that both economic incentives and social norms drive individual behaviour, we assume that participating to the shadow economy involves a moral cost, because the individual may feel a sense of guilt or remorse to deviate from the social norm (Traxler 2010; Kolm and Larsen 2002; Elster 1989). Indeed, different individuals may bear a moral cost as a non-pecuniary cost and to different extents. Let us call define this moral cost with the variable *R* according to the following assumption:

Assumption 2. *The moral cost R is distributed over a unitary set of a continuum of infinitely-living individuals who expect to participate to the shadow economy. This cost can be measured in a continuous manner,* $R \in [0, R_{\max}]$, *following the known c.d.f.* $G : [0, R_{\max}] \rightarrow [0,1]$.

This cost is assumed independent from the present values, i.e. $r \cdot (U_s - R)$ and $r \cdot (V_s - R)$. While this condition simplifies the analysis without loss of generality, the heterogeneity of *R* makes it possible to separate individuals who search for a job in the underground sector or not, thus substantiating the assumption of 'directed search'. In fact, the condition:

$$r \cdot (U_s - R) = r \cdot U_r \Rightarrow \tilde{R} = U_S - U_r \qquad [3]$$

yields a threshold (reservation) value $\tilde{R}$ which separates the individuals whose moral cost is relatively low, thus inducing him to search for an irregular job, i.e. $R < \tilde{R}$, from the individuals who feel a high moral cost, and prefer to search for a job in the regular sector, i.e. $R \geq \tilde{R}$. As a consequence, more pervasive is the underground sector (i.e. the higher is θ_s with respect to θ_r), more people search for a job in this sector (i.e. the higher is the threshold value $\tilde{R}$), since the outside opportunity is higher.

A more important role in the model is played by the entrepreneurial ability of individuals *x*, because it determines whether they post a vacancy in one or the other sector. More precisely, let us assume the following:

Assumption 3. *Entrepreneurial ability x is distributed over the whole set of individuals who expect to participate in production activity either as entrepreneurs or as workers. This ability can be measured in a continuous manner,* $x \in [0, x_{\max}]$*, following the known c.d.f.* $F : [0, x_{\max}] \rightarrow [0,1]$.

The individual must be endowed with a minimum level of entrepreneurial ability in order to open a vacancy, thus becoming an entrepreneur. As will shortly be made clear, this minimum level is required to enter the underground sector only, because the level of ability required to enter the regular sector is even higher. Thus, the minimum ability required to become an entrepreneur ($x_{\min}$) can be obtained from the zero-profit condition in the underground sector, because an individual with entrepreneurial ability *x* can always choose between posting a vacancy and searching for a job:

$$r \cdot (U_s - R) = r \cdot (V_s - R) \Rightarrow U_S = V_r \Rightarrow \lim_{v_s \to 0} (U_S = V_r) = x_{\min} = \frac{2 \cdot z}{y_s} > 0 \qquad [4]$$

This result is due to the following: $\lim_{\theta_s \to 0} U_S = z$, which is obtained in a straightforward manner from the Bellman equation for U_s, and $\lim_{\theta_s \to 0} V_S = x \cdot y_s - z$ by applying the *l'Hôpital rule* to the Bellman equation for V_s. Therefore, the zero-profit condition can be used to distinguish entrepreneurs from workers.[6] Since for $r > 0$, $w_i \geq z, \forall i$, then $W_i \geq U_i$. Indeed, from equation [4], we obtain that the productivity level of the less able entrepreneur is twice the

[6] In a framework with a fixed number of firms, the zero-profit condition is no longer used to determine the labour-market tightness (see Fonseca et al., 2001, and Pissarides, 2002). Note that entrepreneurs will earn extra-profit as a rent in posting vacancies, because ability is not tradable.

opportunity cost of employment, $y_s \cdot x_{\min} = 2 \cdot z$. Hence, the worker always finds it optimal to work for the current employer instead of searching for a new one.

Lemma 1. *All the individuals endowed with* $x \geq x_{\min}$ *expect to profitably open a vacancy, thus becoming entrepreneurs, while the individuals, labelled with l and endowed with* $x < x_{\min}$, *will not post any vacancy, thus becoming workers.*

The distinction between individuals who post a vacancy in the underground sector and those who post a vacancy in the regular sector must take into account their entrepreneurial ability and their moral cost if they participate to the underground economy. It rather natural to assume that the higher is entrepreneurial ability, the higher is moral cost, because entrepreneurial ability can be better displayed if it does not violate social norms. In formulae:

$$R = R(x) = \kappa \cdot x \qquad [5]$$

with $\kappa > 0$, so that $R_{\max} > R(x_{\max})$. Therefore, since the level of entrepreneurial ability is lower in the underground sector, the moral cost becomes an increasing function of the size of the regular sector (Kolm and Larsen, 2002; Pugno and Lisi, 2011).

Let us define a threshold level of entrepreneurial ability $T \in [x_{\min}, x_{\max}]$ such that two entrepreneurs drawn from the two sectors yield equal expected profitability, i.e.:

$$V_r(x = T) = V_s(x = T) - \kappa \cdot T \qquad [6]$$

T can therefore be derived from equations [1], [2], and [6]:

$$T = \frac{(\tau + z + c_r \cdot A)/(A+1) - (z + c_s \cdot B)/(B+1)}{y_r/(A+1) - y_s/(B+1) + r\kappa} \qquad [7]$$

with $A \equiv \dfrac{r + \delta + \beta_r \cdot g(\theta_r)}{(1-\beta_r) \cdot f(\theta_r)}$ and $B \equiv \dfrac{r + \delta + \rho + \beta_s \cdot g(\theta_s)}{(1-\beta_s) \cdot f(\theta_s)}$.

Equation [7] defines *T* as a special *x*, so that the condition $x \geq x_{\min} > 0$ requires that $T > 0$. Sufficient conditions for $T > 0$ are that both the numerator and the denominator of [7] are positive. The numerator is positive if $(\tau + z) > c_s$, $c_r > z$, and $c_r > c_s$, which are realistic conditions.[7] The denominator is positive if y_r (augmented by the moral cost to open an irregular vacancy as captured by κ) is sufficiently greater than y_s, which is a necessary condition for the regular sector to be able to survive, and it qualifies our Assumption 1. A further result can be obtained from these restrictions which characterises the entrepreneurs'

[7] The cost to post a vacancy in the underground sector c_s should be very low, since ease of entry is often one of the criteria used to define the informal sector (Gërxhani, 2004). By contrast, the cost c_r is often very heavy because of regulations, administrative burdens, licence fees and bribery (Bouev, 2005).

choice: the intercept of $V_r(x)$ is lower than the intercept of $V_s(x)$, and the slope of $V_r(x)$ is steeper than the slope of $V_s(x)$ (see Fig. 1).

========== *Fig. 1 about here (now at the end)* ==========

Note that another threshold for R emerges, i.e. $R(x=T)$, which separates the entrepreneurs of the two sectors. Since *R* is a positive function of *x*, this threshold is greater than the other threshold $\tilde{R}$, which separates individuals who search for a job (workers). More precisely, it must be true that: $R(x = x_{\max}) > R(x = T) > R(x = x_{\min}) > \tilde{R} > R(x = 0)$.

From the macroeconomic point of view, the entrepreneurs' indifference condition [6] implies that, given the set of entrepreneurs $1-l$, the share of entrepreneurs who open a vacancy in the regular sector is:

$$1 - F(T) = v_r \qquad [8]$$

while the share

$$F(T) - l = v_s \qquad [9]$$

opens a vacancy in the underground sector. Entrepreneurs may thus post a vacancy and then fill the job, or fail to fill it, in one of the two sectors, so that it can be simply stated that $v_r = 1 - l - v_s$.[8]

Equation [7] can be re-written in a more general form as follows:

$$T = T(v_s) \qquad [10]$$

Equation [10] highlights the relationship between the two variables v_s and *T*, and it can thus be called the *T*-curve. Only the variable v_s is defined in [10] because in this subsection the variable *u* appearing in [7] is taken as exogenous, thus underlining the fact that it is taken by entrepreneurs as given, while in the next subsection *u* will be a function of v_s. The relationship is negative in equation [10] because of the wage cost effect, and the effects due to *search* or *congestion* externalities (see Pissarides, 2000). In fact, if the irregular vacancies increase, wages increase and the probability of filling them is lower. Therefore, it is more difficult to fill an irregular vacancy and fewer entrepreneurs enter the irregular sector. It can be proved that $\partial T / \partial v_s < 0$ under restrictions very similar to those required for $T = T(v_s) > 0$ (see *Appendix A*).

[8] In this model, the number of incumbent entrepreneurs, who run $n_r + n_s$ firms, is exogenous and adds to those who enter the market. Matters thus become simpler without loss of generality.

Equation [10] can be coupled with equation [9], which represents the distribution of ability across (irregular) entrepreneurs. In this equation v_s is monotonically rising in *T* from $x_{\min}$ up to $x_{\max}$. Both equations [9] and [10] can thus be depicted in the diagram with axes [v_s, T], as in Fig. 2. Equation [10] has been built under the following condition:

$$\lim_{v_s \to 1-l-\tilde{v}_r} T = \frac{(\tau + z + c_r \cdot A)/(A+1) - z}{y_r/(A+1) - y_s + r\kappa} \geq x_{\min}$$

so that the available entrepreneurial ability is sufficient to open some vacancies.

Lemma 2. *A unique intersection between the two curves exists, thus determining the partial equilibrium of the model, since u is taken as given.*

========== *Fig. 2 about here (now at the end)* ==========

From this result, and from the previous one represented in Fig. 1, a further result follows, thus substantiating the statement that the minimum level of entrepreneurial ability to profitably open a new vacancy, i.e. $x_{\min}$, strictly regards the underground sector.

Lemma 3. *The less able entrepreneurs open irregular vacancies; the abler entrepreneurs open regular vacancies.*

This result strengthens the previous section's conclusion about wages.

2.3 Unemployment and the steady state general equilibrium

Although the economy has two sectors, we empirically observe a single rate of unemployment, which is defined thus:

$$\begin{cases} u = u_r + u_s \\ u_r + u_s = l - n_r - n_s \end{cases} \Rightarrow u = l - n_r - n_s \qquad [11]$$

where n_r and n_s represent steady-state employment in the regular and underground sector, respectively. Since unemployed workers can search for a job only in one of the two sectors, we assume that the larger the size of the sector, the greater the number of job-seekers in that sector. Without loss of generality, we can thus assume that $u_s = \alpha \cdot u$ and $u_r = (1-\alpha) \cdot u$, where $\alpha = \alpha(v_s)$, $\lim_{v_s \to 0} \alpha = \underline{\alpha} > 0$ and $\lim_{v_s \to (1-l)} \alpha = \overline{\alpha} < 1$, $\partial\alpha / \partial v_s = \dfrac{\overline{\alpha} - \underline{\alpha}}{1-l}$. The parameters $\underline{\alpha}$ and $\overline{\alpha}$ can be very close to 0 and 1 respectively.

Since jobs arrive to unemployed workers at the rate $g(\theta_i)$, with $i \in \{r, s\}$, and regular and irregular filled jobs are destroyed at the rate δ and $(\delta + \rho)$, respectively, then in the steady-state equilibrium it must be that:

$$\delta \cdot n_r = (1-\alpha(v_s)) \cdot u \cdot g(\theta_r) \qquad [12]$$

$$(\delta+\rho) \cdot n_s = \alpha(v_s) \cdot u \cdot g(\theta_s) \qquad [13]$$

Steady-state unemployment is thus given by equations [11], [12] and [13]:

$$u = \frac{l}{\frac{(1-\alpha(v_s)) \cdot g\left(\frac{\theta_r}{1-\alpha(v_s)}\right)}{\delta} + \frac{\alpha(v_s) \cdot g\left(\frac{\theta_s}{\alpha(v_s)}\right)}{\delta+\rho} + 1} = \frac{l}{\frac{M(\theta_r)}{\delta} + \frac{M(\theta_s)}{\delta+\rho} + 1} \qquad [14]$$

where the *M*-function has the same basic properties as the original *g*-function (see *Appendix A*), and where $\theta_r = v_r / u_r$, $v_r = (1-l-v_s)$ and $\theta_s = v_s / u_s$.

Equation [14] can be rewritten in general and explicit form as follows:

$$u = u(v_s) \qquad [15]$$

where steady-state unemployment u is a function of vacancies in the underground sector only. Equation [15] can be depicted as a U-shaped curve in the (v_s, u)-axes over the relevant range of v_s, with perfect symmetry in the case of $\rho = 0$. Equation [15] closes the general equilibrium model formed by the system including the three main equations [7], [9] and [15] in the three unknowns v_s, T, and u. It is intuitive that the equilibrium result obtained in the previous subsection (where u was taken as given), which concerned the intersection between the curves represented in [9] and [10], does not qualitatively change under the condition that u changes only moderately through equation [15]. It can be proved that this condition is $-\frac{1}{\theta_r} < \frac{\partial u(v_s)}{\partial v_s} < \frac{1}{\theta_s}$, which obviously holds for intermediate levels of v_s (see *Appendix A*).

It can also be proved that the equilibrium result does not qualitatively change, even in the complementary conditions, i.e. $\frac{\partial u(v_s)}{\partial v_s} < -\frac{1}{\theta_r}$ and $\frac{\partial u(v_s)}{\partial v_s} > \frac{1}{\theta_s}$, which may hold when v_s takes extreme values. In these two cases the macroeconomic condition of the labour market affects both the regular and the underground sector. In fact, for v_s close to zero, $\partial u(v_s)/\partial v_s$ may be so negative that both θ_s and θ_r rise, but θ_s rises more than θ_r, while for v_s close to ($1-l$), $\partial u(v_s)/\partial v_s$ may be so positive that both θ_s and θ_r diminish, but θ_s diminishes less than θ_r (see *Appendix A*).

Therefore, this concluding proposition can be obtained:

Proposition 1. *The solutions for the four key variables v_s, v_r, T and u are obtained by considering: 1) the present discounted values of the vacancies, i.e. equations [1] and [2]; 2) the entrepreneurs' indifference condition between open vacancies in the two sectors, given* *their entrepreneurial ability distribution, and the threshold level of entrepreneurial ability, i.e. equations [6] and [7]; 3) the unemployment identity [11] and the equilibrium condition of the transition flows on the supply side of the labour market, i.e. equations [12] and [13]. The solutions can be interior solutions for v_s, v_r within their relevant range, and are not necessarily unique solutions.*

2.4 Discussion

The main result of the model of this section is that not only is there an interior solution whereby both the underground sector and the regular sector survive in equilibrium (Boeri and Garibaldi, 2006; Albrecht et. al., 2009), but this equilibrium is determined by allocating heterogeneous entrepreneurial ability between the two sectors (Rauch, 1991; Carillo and Pugno, 2004). This may explain the so-called "shadow puzzle", i.e. the persistence of the underground sector despite advances in detection technologies and greater organisation by public authorities to reduce irregularities (issue (*i*) in the Introduction). This kind of explanation runs counter to the argument that the underground sector is an incubator of infant industries (see also La Porta and Shleifer, 2008; Rauch, 1991; Levenson and Maloney, 1998).

The second result is that the greater the relative size of the regular sector, the higher the wage gap between the regular and the underground sector. The gap is always positive for a sufficiently higher level of productivity in the regular sector.

A number of other important results can be drawn from comparative statics exercises, although described only in dynamic terms for conciseness. A general exercise concerns the effects of the shift of the *T*-curve due to changes in some parameters. Its downward shift decreases both the (partial) equilibrium of v_s in Fig. 2, and the model's (general) equilibrium of v_s, and hence also θ_s. Therefore, this downward shift squeezes the proportion of the underground sector and expands the proportion of the regular sector, as clearly emerges from equations [8] and [9].

The downward shift of the *T*-curve can thus increase overall output, because it increases the proportion of the most productive sector. The regular sector is in fact more productive than the underground sector for two reasons: the regular sector exhibits a greater labour productivity, and the most able entrepreneurs prefer this sector. In fact, for a greater number of regular vacancies made possible by the shift of the abler entrepreneurs from the underground sector, both the number of regular matches, $m_r = m(v_r, u_r)$, and employment in the regular sector, n_r, are greater because of the greater probability of finding a regular job.

The main policy implications can be drawn from the effects of the changes in the policy parameters on T, and hence on the proportion of the underground sector, i.e.: $\frac{\partial T}{\partial \rho} < 0$; $\frac{\partial T}{\partial \tau} > 0$; $\frac{\partial T}{\partial c_r} > 0$.

In other words, closer monitoring, lower taxation and lower costs for posting official vacancies reduce the underground sector. This is in line with the conclusions of other models (see e.g. Friedman et al., 2000; Johnson et al., 2000; Sarte, 2000; Bouev, 2005). Considering the parameter κ in [7] adds the result that a greater moral cost to open an irregular vacancy reduces the underground sector.

A new contribution of this model regards a much more controversial question, i.e. the ambiguous relationship between underground economy and unemployment (issue (*ii*) in the Introduction). This relationship is represented by equation [15], which is U-shaped, thus showing that $\partial u(v_s)/\partial v_s$ <0 when v_s is relatively small, and $\partial u(v_s)/\partial v_s$ >0 when v_s is relatively great. But if ρ increases, then the minimum of $u=u(v_s)$ shifts in the region where v_s is closer to zero. A more precise Proposition can thus be stated:

Proposition 2. *If $v_s \leq v_r$, the relationship between v_s and u is negative if ρ is sufficiently low, it is positive if ρ is sufficiently high. If $v_s > v_r$ the relationship between v_s and u is positive for any ρ (see Appendix B for proof).*

This is an interesting result from a policy implications point of view. In fact, the role of the monitoring parameter is strengthened, since any policy intended to reduce the irregular sector may also reduce the unemployment rate if ρ is sufficiently high.[9]

3. The model with investment in education and endogenous productivity growth

The relationship between economic growth and unemployment has been recently investigated by focusing the analysis on the role of technology. The present paper takes another look, although maintaining the long-run perspective, by recognising that in economies not at the technological frontier, the underground sector may play a major role because a substantial share of the economy produces with this different type of organisation.

[9] Bosch and Esteban-Pretel (2009) focus on the role of the job destruction rate. According to their matching model, policies that reduce the cost of formality (or those that increase the cost of informality) produce an increase in the share of formal employment while also reducing unemployment because the reallocation between formal and informal jobs has non-neutral effects on the unemployment rate, since informal jobs record much higher separation rates (namely, more inflows into unemployment).

This section will show that economic growth and unemployment may be negatively or positively correlated, depending on the level of the monitoring rate on the regularity of firms, which both captures the country's rule of law and affects the size of the underground sector. For this purpose, our matching model will be extended in order to study steady-state growth, and some cross-country evidence will provide empirical plausibility to backup the theoretical results. Possibilities of multiple equilibria of steady-growth will be further explored.

3.1 A steady-growth solution of the model

This paper assumes that human capital accumulation is the primary engine of economic growth. In the growth literature, workers' human capital usually refers to "*the average level of educational attainment*" (Nelson and Phelps, 1966; Benhabib and Spiegel, 1994) or similarly to "*the average total years of schooling*" (Savvides and Stengos, 2009). Specifically, education and schooling enable workers to absorb knowledge and acquire additional human capital once employed (Rosen, 1976; Stokey, 1991; Laing et al., 1995). Therefore, it can be stated that the higher the level of schooling or knowledge (k) and the larger the human capital accumulation (h), the higher the rate of economic growth.

To simplify matters, and without loss of generality, we assume $h = k$, so that education and human capital can be used interchangeably. Then, we specify a simple equation for the rate of productivity growth (γ):

$$\gamma = \gamma(h) \qquad \text{with } \gamma'(h) > 0 \tag{16}$$

with the further property that $r > \gamma(h)\ \ \forall\, h$, in order to keep present values finite.

Since the education level and skill of the workers employed in the regular sector are higher than those in the underground sector (Albrecht et. al., 2009; Cappariello and Zizza, 2009), growth is expected to be faster in the regular sector. This link is assumed in the form of labour-augmenting technological progress *à la* Pissarides (2000), where, specifically, workers' human capital plays two roles, as suggested by Laing et al. (1995). In fact, since human capital is firstly acquired through formal education, workers can be employed with an initial productivity (y_0) that depends on the level of schooling (h). Secondly, workers' productivity increases according to the rate of productivity growth. Let us then state the following assumption:

Assumption 3. *The total discounted value of productivity in the regular sector is given by:*

$$y_r(h)=\int_0^\infty e^{-r\cdot t}\cdot y_0(h)\cdot e^{\gamma(h)\cdot t}\,dt \Rightarrow \frac{y_0(h)}{r-\gamma(h)} \tag{17}$$

where:

$$y_0 = y_0(h) \qquad \text{with } y_0'(h)>0,\ \lim_{h\to 0} y_0>0,\ \lim_{h\to\infty} y_0<\infty \tag{18}$$

Productivity in the underground sector is given by:

$$y_s=\varphi\cdot y_r(h) \qquad \text{with } 0<\varphi<1 \tag{19}$$

According to this assumption, the underground sector partially benefits from this process because of spill-over effects in the diffusion of knowledge. Therefore, both sectors can grow at the same rate $\gamma(h)$, while the level of productivity in the regular sector remains higher than that of productivity in the underground sector. The static model of the previous section should also be re-specified so that the variables c_r, c_s, τ and z are indexed to the productivity levels. In this way, the costs of posting vacancies, taxes and the opportunity cost of employment are commensurate with the changing production level of the economy.

In order to endogenise the rate of productivity growth, let us consider the optimal choice of education for individuals, given that schooling investment is costly (cf. Laing *et al.*, 1995; Decreuse and Granier, 2007), and that only regular firms profitably employ educated workers. Formally:

Assumption 4. *Let the cost function of education be c(k), with $c'(k)>0$, $c''(k)>0$ and $\partial c(0)/\partial k=0$, because of either a direct pecuniary cost or the disutility from scholastic effort. Each job-seeker in the regular sector solves the following program:*[10]

$$\max_{k\geq 0}\{[w_r(y_r(k))-w_s]-c(k)\}$$

where $w_r(y_r(k))-w_s$ is the net gain from investing in education, i.e. the wage differential. The optimal investment in education (k^*) can be thus obtained by the usual condition:

$$\Rightarrow \frac{\partial w_r(y_r(k))}{\partial k}=\frac{\partial c(k)}{\partial k} \tag{20}$$

Condition [20] shows a positive relationship between θ_r and k, i.e. $\frac{\partial k}{\partial \theta_r}>0$, besides the implication that $k^*>0$, since $\frac{\partial w_r}{\partial \theta_r}>0$. In fact, a rise in θ_r increases the regular wages.

[10] Workers invest in education before entering the labour market in Laing et al. (1995). Unlike Laing et al. (1995) and Decreuse and Granier (2007), in this model the optimal choice of education is linked to the wage differential rather than to the value of searching for a job. Furthermore, following Dulleck *et al.* (2006), we can assume that the costs of higher education in the shadow sector tend to infinity.

Hence, in order to search for a job (work) in the regular sector, more workers choose to invest in education. In turn, the higher the optimal investment in education, the greater the human capital and the greater the productivity level of the economy. Therefore, the increase in the size of the regular sector, i.e. θ_r, spurs economic growth by a greater investment in education.

It follows that, from a macroeconomic point of view, the investment in education is on the one hand negatively linked to the size of the underground sector, and on the other, positively linked to the productivity growth of the economy. The following Proposition can thus be stated:

Proposition 3. *The solution of the steady-state model can be extended to include the optimal investment in education* (k^*), *and the rate of productivity growth of the economy* (γ), *thus finding a steady-growth solution.*

These results, together with *Proposition 2* of the previous section regarding the relationship between the underground economy and unemployment, help understand the relationship between economic growth and unemployment (issue (*iii*) in the Introduction). Indeed, under the condition that $v_s \leq v_r$, the relationship between $\gamma(h)$ and u is positive if ρ is low, while this relationship is negative if ρ is high.

Our analysis is thus able to reconcile the conflicting results found in the literature on growth and unemployment. This suggestion is alternative to Aghion and Howitt's approach, nevertheless it refers to the structure of the economy. Since the condition $v_s \leq v_r$ is the usual condition used throughout the world, the monitoring rate becomes a very important parameter. Not only does it affect the size of the underground sector, but it may positively affect both unemployment and economic growth.

The ambiguous relationship between growth and unemployment that has been found in theory is plausible from an empirical point of view. In fact, different authors obtain different results concerning the sign of the correlation between growth and unemployment, both across countries and across long periods of time in the same country (Aghion and Howitt, 1994; Bean and Pissarides, 1993; Caballero, 1993; Hoon and Phelps, 1997; Muscatelli and Tirelli, 2001). This issue has been effectively synthesised by Mortensen (2005), who shows that the correlation between average growth and average unemployment over the past ten years across 29 European countries is essentially zero.

However, Pissarides and Vallanti (2004, 2007) have found that, on the basis of a panel of advanced countries, productivity growth is strongly negatively correlated with unemployment in the long run. They thus conclude that technological progress seems to have mainly taken the form of ‘disembodied technology’.[11]

If less advanced countries are also considered in the study of the relationship between growth and unemployment, such as European transition countries and Latin American countries, interesting results emerge. In fact, simple cross-country econometric estimates show results that are both consistent with those of Pissarides and Vallanti when the sample is restricted to advanced countries, and that support the conclusions of our model when the sample is extended.

Since the key variable of our model is the monitoring rate, we have to find a suitable variable which is negatively correlated with the underground economy but linked to the monitoring rate. A natural candidate is the ‘rule of law’ index, which captures perceptions of the extent to which agents have confidence in and abide by the rules of society, and in particular the quality of contract enforcement, property rights, the police and the courts, as well as the likelihood of crime and violence. The correlation coefficient between shadow (underground) economy and the ‘rule of law’ index is about –0.75.[12] This result does not change by using other proxies for the monitoring rate, such as the ‘government effectiveness’ index and/or ‘the corruption perception’ index, since the correlation coefficients among them are extremely high.[13]

Econometric estimates show clear results (see Tables 3 and 4 and the dataset in Tables 1 and 2). In synthesis, the unemployment rate has been regressed by using the rates of economic growth for the whole sample. The estimate has been firstly controlled for three regional dummies, thus capturing institutional and cultural differences, i.e. $D_{EU\_non\text{-}transition}$, $D_{EU\_transition}$, and $D_{Latin\text{-}American}$. The estimate of the equation:

$$unempl_i = cons + \beta_1 \cdot D_{EU\_transition} + \beta_2 \cdot D_{Latin-American} + \beta_3 \cdot growth_i + \varepsilon_i \qquad \text{(I)}$$

exhibits a non significant correlation, since the null hypothesis that $\beta_3 = 0$ is not rejected at any customary confidence level. Obviously, both β_1 and β_2 exhibit positive signs since the unemployment rate in the EU transition countries and Latin-American countries is higher than in the EU non-transition countries.

[11] More precisely, the “capitalisation” effect seems to dominate, while the “creative destruction” effect seems to play no role in the steady-state unemployment dynamics, since at reasonable parameter values a nontrivial negative impact of growth on unemployment is incompatible with embodied technology.

[12] The shadow (underground) economy is calculated as % of GDP (1996-2007 average) according to Schneider et al. (2010).

[13] Indeed, the correlation between “rule of law” and “government effectiveness” is higher than 0.90.

In the second estimate, the control for the 'rule of law' index is instead introduced. The dummy variable D_{rule} assumes value 1 if the index is high and zero otherwise.[14] The estimate of the equation:

$$unempl_i = cons + \beta_1 \cdot D_{rule} \cdot growth_i + \beta_2 \cdot growth_i + \varepsilon_i \qquad \text{(II)}$$

exhibits significant results.[15] When D_{rule} = 0, higher growth predicts greater unemployment ($\beta_2 = 0.511$), and when D_{rule} = 1, higher growth predicts lower unemployment ($\beta_1 + \beta_2 = -0.373$). In the estimate (II) there is also an increase in the Adjusted-R-square (from 17% to 19%). Both estimates (I) and (II) are statistically correct and satisfy all the main statistical tests.

========== *Tabs. 1,2,3 and 4 about here* =========

3.2 The case of multiple equilibria

The extended model may also be adapted in order to account for a special case: that of regional dualism, i.e. the failure of the more backward region to catch up with the more developed region.

Let us assume that $y_0(h)$ is a logistic function, i.e. it performs increasing returns to human capital before the usual and eventual decreasing returns. This form may be due to thresholds in human capital, i.e. once human capital attains a certain threshold level (*critical mass*) productivity may reach a higher steady-state level (Azariadis and Drazen, 1990). This pattern has also received some empirical evidence (Savvides and Stengos, 2009).[16]

Under this assumption, the relationship between T and v_s may change significantly. Indeed, if the functions [17] and [19] are plugged into [7], then multiple equilibria become possible since the *T*–curve may display an increasing part in the middle, thus crossing the other curve twice, as depicted in Fig. 2 (dotted line).[17]

The two extreme equilibria may be labelled as "good" and "bad" because they define two different conditions where the proportion of the underground sector is small and,

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[14] Precisely, we use as threshold value an index of rule of law equal to 82. This value is calculated as the mean between the maximum and average values (by excluding, however, the too low value of Venezuela).

[15] The results do not differ much if the 'rule of law' index is used as a continuous variable. Indeed, in this case, we have a lower adjusted-R-square.

[16] The models which describe general nonlinearities in the relationship between growth and human capital do not provide specific functional forms (Savvides and Stengos, 2009). Azariadis and Drazen (1990) even study a step functional form, where thresholds are more than one.

[17] As shown by Savvides and Stengos (2009) – adapted from Azariadis and Drazen (1990) – a step functional form may generate the possibility of multiple equilibria, with different balanced growth paths. This growth process comes to an end when *"labour productivity attains the highest possible value and the system settles down on the ultimate stage of growth"* (Azariadis and Drazen, 1990, p. 517).

respectively, large, with the consequent desirable and undesirable characterisations. Specifically, in the "good" equilibrium one region exhibits higher productivity, a more efficient use of entrepreneurial ability, higher investment in education, greater employment of skilled workers, and, finally, a higher rate of economic growth with respect to the region in the "bad" equilibrium.

This result is interesting because it can represent an economy characterised by a uniform institutional set-up, as captured by the parameters of the model, but with two regions that differ in their histories, as captured by the different relative sizes of the underground sector. The region that has inherited from the past a sufficiently greater proportion of the underground sector will tend to the "bad" equilibrium, at least in the case of scarce mobility of labour. The region that has inherited a smaller proportion of the underground sector will tend to the "good" equilibrium. The economy will thus exhibit regional dualism, which may persist even in the long run, because the region in the "bad" equilibrium also performs a relatively lower steady-growth. This case seems to be the best fit with the Italian North-South divide, which is distinctive but not unique in the world.

4. Conclusions

The persistence of the underground sector, albeit different extents, in all countries around the world has been documented by many studies, and has been called the 'shadow puzzle'. How the underground sector relates with unemployment has also been studied, without clarifying, however, whether the relationship is positive or negative. Research on the underground sector encounters a third issue which is related to economic growth, i.e. whether the relationship between unemployment and economic growth is negative or positive. At microeconomic level, several studies have found that underground firms employ relatively backward technology, less educated, less skilled, and less paid workers, as well as less able entrepreneurs, i.e. lower quality inputs for growth. This microeconomic evidence has suggested useful links for building a matching-type model of endogenous growth that is able to account for both the 'shadow puzzle' and the two ambiguous relationships.

The key assumption of the model, which is also rather new in matching models, is that entrepreneurial ability is a heterogeneous input for production. If also workers are not homogeneous because they can become educated and skilled, then two types of firms emerge: the more productive firms where the ablest entrepreneurs match with skilled workers, and the less productive firms where the less able entrepreneurs match with unskilled workers. These latter firms form the underground sector, because they can persistently survive on the market by evading taxes and paying lower wages. Their role in the economic growth is negative, because they discourage human capital accumulation and hence productivity growth.

The model predicts that monitoring firms' regularity determines whether or not unemployment is complementary to underground employment, and, consequently, whether unemployment is positively or negatively related with economic growth. Some econometric evidence supports this role of law enforcement in making the relationship between unemployment and economic growth positive or negative.

The model used is rather general (and the growth equation rather simple), so that it can be applied to a number of cases. In the paper, the case is made for EU non-transition and Latin American countries, which exhibit a high share of underground sector. Another interesting case is regional dualism, e.g. the North-South dualism in Italy, where some backward regions diverge from the others, although both groups share the same institutional set-up. For this case the model has been made more specific by assuming sufficient non-linearities in the human capital accumulation function, so that multiple equilibria emerge in the size of the underground sector.

A number of policy implications ensue from this analysis. Reducing the tax burden becomes especially effective if monitoring is at a high level, because underground firms are discouraged without raising unemployment. In the long run, this may also enhance growth. These same results follow if monitoring is itself increased. In the case of regional dualism, a one-shot change in the policy parameters may trigger an endogenous dynamic of convergence between the two regions. More generally, an effective policy should seek to increase entrepreneurial ability, typically through education, so that overall economic performance improves, both because of the sectoral composition effect, and because of the positive level effect of each firm.

Appendices

Appendix A: Proof that $\partial T/\partial v_s < 0$

It will be firstly proved that $\partial T/\partial v_s < 0$ (with $v_s \in]1-l-\tilde{v}_r, \tilde{v}_s[$ and v_r=1–l–v_s) when u is assumed as exogenous, as in subsection 2.2, and then when u is assumed as endogenous, as in subsection 2.3.

Sufficient conditions for $\partial T/\partial v_s < 0$ are that $\partial N/\partial v_s < 0$ and $\partial D/\partial v_s > 0$, where *N* and *D* are the numerator and the denominator of *T* in [7]. To prove this, let us observe, from the definitions of *A* and *B* in [7], that $\partial A/\partial v_s < 0$ and $\partial B/\partial v_s > 0$, because $\partial A/\partial \theta_r > 0$, $\partial \theta_r/\partial v_s < 0$, and $\partial B/\partial \theta_s > 0$, $\partial \theta_s/\partial v_s > 0$. Therefore, $\partial N/\partial v_s$ is negative if $c_r > (\tau + z)$ and $c_s > z$, as it emerges from the derivative of *N*:

$$\frac{\partial}{\partial v_s}\left(\frac{\tau + z + c_r A}{A+1} - \frac{z + c_s B}{B+1}\right) = \frac{\partial A}{\partial v_s}\left(\frac{c_r(A+1) - (\tau + z + c_r A)}{(A+1)^2}\right) - \frac{\partial B}{\partial v_s}\left(\frac{c_s(B+1) - (z + c_s B)}{(B+1)^2}\right) \qquad \text{[A.1]}$$

while $\partial D/\partial v_s$ is always positive, as it emerges from the derivative of *D*:

$$\frac{\partial}{\partial v_s}\left(\frac{y_r}{A+1} - \frac{y_s}{B+1} + r\kappa\right) = \frac{\partial B}{\partial v_s}\frac{y_s}{(B+1)^2} - \frac{\partial A}{\partial v_s}\frac{y_r}{(A+1)^2} \qquad \text{[A.2]}$$

The restriction set of the parameters for both *T*>0 and $\partial T/\partial v_s < 0$ thus becomes: $c_r > (\tau + z) > c_s > z$, and y_r sufficiently greater than y_s.

Subsection 2.3 assumes that *u* is endogenous through equation [14]. In this equation the *M*-function has the same basic properties as the original *g*-function, i.e.:

$\lim_{v_s \to 0} g(\theta_r) = g\left(\frac{1-l}{u}\right) < \infty$; $\lim_{v_s \to 0} M(\theta_r) = (1-\underline{\alpha})g\left(\frac{1-l}{u(1-\underline{\alpha})}\right) < \infty$;

$\lim_{v_s \to (1-l)} g(\theta_r) = \lim_{v_s \to (1-l)} M(\theta_r) = 0$; $\lim_{v_s \to 0} g(\theta_s) = \lim_{v_s \to 0} M(\theta_s) = 0$;

$\lim_{v_s \to (1-l)} g(\theta_s) = g\left(\frac{1-l}{u}\right) < \infty$; $\lim_{v_s \to (1-l)} M(\theta_s) = \overline{\alpha} g_s\left(\frac{1-l}{u\overline{\alpha}}\right) < \infty$; both the *M*-function and the *g*-function are monotonic and concave.

Equation [14] is U-shaped within the relevant range of v_s. In fact, the derivative of $u(v_s)$ can thus be calculated through some manipulations:

$$\frac{\partial u}{\partial v} = \frac{\dfrac{M'(\theta_r)}{\delta} - \dfrac{M'(\theta_s)}{\delta + \rho}}{\dfrac{M(\theta_s)}{\delta + \rho} - \dfrac{\theta_s M'(\theta_s)}{\delta + \rho} + \dfrac{M(\theta_r)}{\delta} - \dfrac{\theta_r M'(\theta_r)}{\delta} + 1} \qquad \text{[A.3]}$$

While the denominator of [A.3] is always positive because $M(\theta_i)$ is a concave function so that $\frac{M(\theta_i)}{\theta_i} > \frac{\partial M(\theta_i)}{\partial \theta_i}$, the numerator is negative for relatively small v_s, and it is positive for relatively great v_s, because, again, $M(\theta_i)$ is a concave function.

The fact that $u(v_s)$ is U-shaped maintains that $\partial N/\partial v_s < 0$ and $\partial D/\partial v_s > 0$ in [7], so that $\partial T/\partial v_s < 0$. This can be proved by distinguishing the intermediate range of v_s around the minimum of $u(v_s)$, from the extreme ranges, where v_s is either close to zero or close to $(1-l)$. In the former case, $\partial u(v_s)/\partial v_s$ is relatively small, so that it can satisfy these conditions: $-\frac{1}{\theta_r} < \frac{\partial u(v_s)}{\partial v_s} < \frac{1}{\theta_s}$, which guarantee that $\partial\theta_r/\partial v_s < 0$ and $\partial\theta_s/\partial v_s > 0$, and thus also that $\partial A/\partial v_s < 0$ and $\partial B/\partial v_s > 0$, because $\frac{\partial((1-l-v_s)/u(v_s))}{\partial v_s} = -\frac{1}{u}\left(\frac{1}{u}\frac{\partial u(v_s)}{\partial v_s}\cdot(1-l-v_s)+1\right)$ and $\frac{\partial(v_s/u(v_s))}{\partial v_s} = \frac{1}{u}\left(1-\theta_s\frac{\partial u(v_s)}{\partial v_s}\right)$. This case also holds for the extreme ranges of v_s, if $M(\theta_i)$ is not very concave.

In the lower range of v_s, where it is close to zero, the condition $-\frac{1}{\theta_r} > \frac{\partial u(v_s)}{\partial v_s}$ emerges, if $M(\theta_i)$ is very concave, as in the Cobb-Douglas specification of the matching equation. In this case, the derivatives $\partial\theta_r/\partial v_s$ and $\partial A/\partial v_s$ take the "perverse" positive sign, while $\partial\theta_s/\partial v_s$ and $\partial B/\partial v_s$ maintain the positive sign, although increasing in size both because the numerator of θ_s rises and because its denominator diminishes. The limit of [A.1] makes it evident that $\partial N/\partial v_s$ <0:

$\lim_{v_s\to 0} N = \frac{\partial A}{\partial v_s}\left(\frac{c_r - t - z}{(A+1)^2}\right) - \frac{\partial B}{\partial v_s}(c_s - z)$, which would be equal to $-\infty$ if the matching function were Cobb-Douglas. Similar reasoning can be applied to D, which would be equal to ∞ at the limit of the Cobb-Douglas case.

In the upper range of v_s, where it is close to $(1-l)$, the condition $\frac{\partial u(v_s)}{\partial v_s} > \frac{1}{\theta_s}$ emerges, if $M(\theta_i)$ is very concave. In this case, the derivatives $\partial\theta_s/\partial v_s$ and $\partial B/\partial v_s$ take the "perverse" negative sign, while the derivatives $\partial\theta_r/\partial v_s$ and $\partial A/\partial v_s$ maintain the negative sign, although becoming even more negative, both because the numerator of θ_s diminishes and because its denominator rises. The limit of [A.1] makes it evident that, again, $\partial N/\partial v_s$ <0:

$\lim_{v_s\to 1-l} N = \frac{\partial A}{\partial v_s}(c_r - t - z) - \frac{\partial B}{\partial v_s}\frac{c_s - z}{(B+1)^2}$, which would be equal to $-\infty$ if the matching function were Cobb-Douglas. Similar reasoning can be applied again to D, which would be equal to ∞ at the limit of the Cobb-Douglas case.

Appendix B: Proof of Proposition 2

Equation [14] is perfectly symmetric with respect to v_s if ρ=0, so that $u(v_s)$ is at the minimum when $v_s=v_r$. If ρ>0, the minimum lies in the region where $v_s<v_r$. In fact, the condition for the minimum $\partial u(v_s)/\partial v_s=0$ that can be derived from [A.3] is $\frac{\delta+\rho}{\delta}=\frac{M'(\theta_s)}{M'(\theta_r)}$. This condition states that the greater is ρ, the smaller the level of v_s for which $u(v_s)$ is at the minimum. Therefore, for any given v_s such that $\partial u(v_s)/\partial v_s<0$ at some level of ρ, there exists a sufficiently greater level of ρ such that $\partial u(v_s)/\partial v_s>0$. Note that this important result holds even if two different concave matching functions governed the two sectors, although the downward bound of the range of v_s where $\partial u(v_s)/\partial v_s>0$ for any ρ would be different from $v_s=v_r$.

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Figures and Tables

FIGURE 1. Entrepreneurs' indifference condition

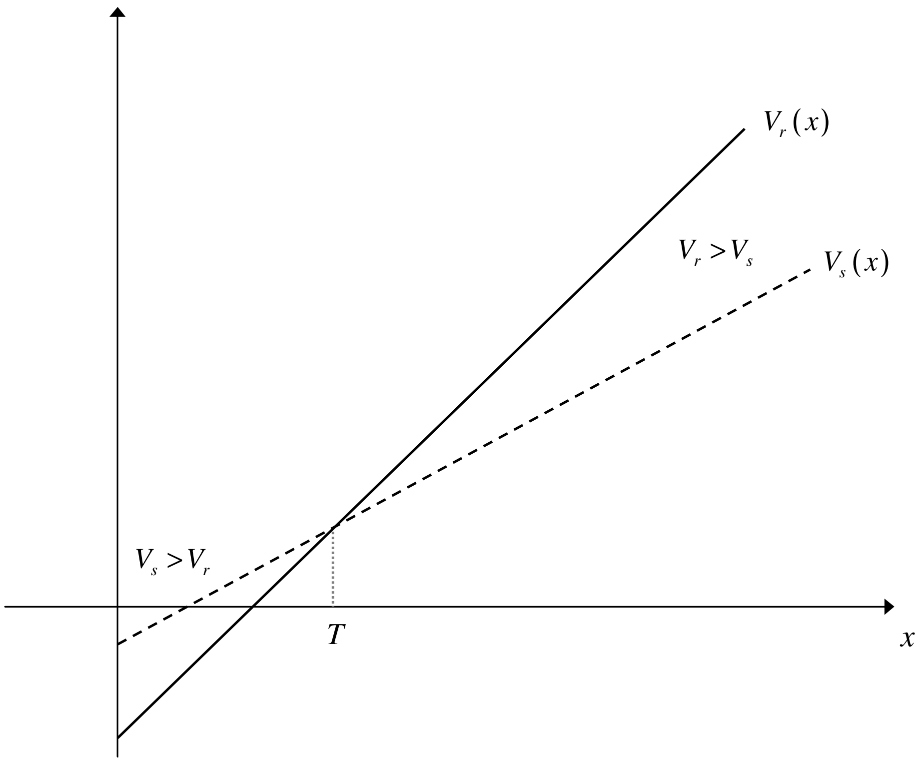

FIGURE 2. Interior equilibrium and multiple equilibria

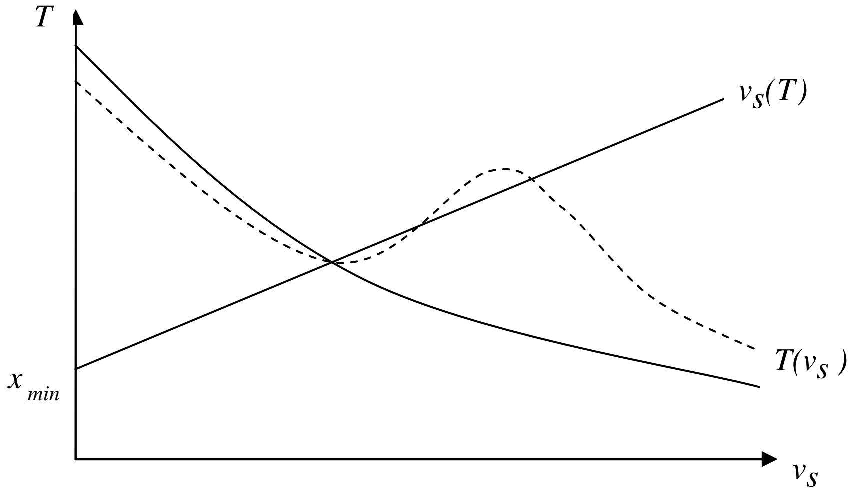

Table 1. Dataset

EU non-transition countries	unemployment rate (%) *	GDP growth rate (%) *	Rule of Law ** (Percentile Rank ***)
Austria	3.73	2.29	99.0
Belgium	6.40	2.04	89.0
Cyprus	3.63	3.77	84.2
Denmark	3.88	1.56	99.5
Finland	6.76	3.21	97.6
France	7.44	1.90	90.0
Germany	8.51	1.47	93.3
Greece	7.98	3.98	73.2
Ireland	3.74	5.02	94.3
Italy	6.43	1.16	62.2
Luxembourg	3.06	4.27	96.2
Malta	4.69	1.80	91.4
Netherlands	2.88	2.16	94.7
Portugal	5.48	5.84	83.7
Spain	8.43	2.80	85.2
Sweden	4.92	4.71	98.1
United Kingdom	3.71	1.70	92.3

EU transition countries	unemployment rate (%) *	GDP growth rate (%) *	Rule of Law ** (Percentile Rank ***)
Bulgaria	10.94	5.59	51.2
Czech Republic	6.18	4.20	77.0
Estonia	7.86	7.02	84.7
Hungary	5.73	3.52	76.1
Latvia	8.91	7.32	71.3
Lithuania	9.47	6.97	67.5
Poland	13.11	1.32	65.1
Romania	5.46	5.70	53.6
Slovakia	13.52	4.32	67.0
Slovenia	4.92	3.31	82.3

** (2000 - 2008) average.*
Source: (http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/themes)
*** Source: (http://info.worldbank.org/governance/wgi/mc_countries.asp)*
**** Percentile rank, from 0 (worst) to 100 (best). More precisely, according to the World Bank, the 'Rule of Law' index measures the quality of contract enforcement, the police and the courts, as well as the likelihood of crime and violence.*

Table 2. Dataset

Latin America countries	Unemployment rate *	GDP growth rate **	Rule of Law index ***
Argentina	12.95	2.51	32.10
Bolivia	5.2	1.47	12.00
Brazil	8.99	2.08	46.40
Chile	7.46	3.51	88.00
Colombia	13.92	2.73	37.80
Costa Rica	6.01	2.74	62.70
Dominican Republic	14.7	3.85	33.00
Ecuador	8.99	3.05	9.10
El Salvador	6.75	1.06	30.60
Guatemala	2.25	1.50	12.90
Honduras	4.48	2.85	20.60
Mexico	3.2	1.72	29.70
Nicaragua	7.64	0.91	21.10
Panama	11.85	3.45	49.80
Paraguay	7.52	-0.20	15.30
Peru	7.94	3.40	25.80
Uruguay	12.77	1.58	65.60
Venezuela, R. B. de	12.28	2.63	2.90

* *(%) of labour force (2000-2008) average. Source:* http://data.worldbank.org/indicator/

** *(2000 - 2007) average.*
Source: Alan Heston, Robert Summers and Bettina Aten, Penn World Table Version 6.3, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, August 2009.
http://pwt.econ.upenn.edu/php_site/pwt_index.php

*** *Source:* (http://info.worldbank.org/governance/wgi/mc_countries.asp).
Percentile rank, from 0 (worst) to 100 (best).

Table 3. Estimate of the equation (I)

Dependent variable: *unemployment rate*	
Independent variables	Coefficients
Growth	0.138 (0.42)
Dummy (EU transition countries)	2.94 (2.12) **
Dummy (Latin-American)	3.30 (3.12) ***
cons	4.99 (4.09) ***
Statistical tests	
Ramsey RESET test *H_0: model has no omitted variables*	F(3, 38) = 0.79 Prob > F = 0.51
Breusch-Pagan / Cook-Weisberg test for heteroskedasticity *H_0: Constant variance*	chi2(1) = 3.92 Prob > chi2 = 0.0478 *Robust standard errors*
Mean VIF	1.48
Skewness/Kurtosis tests for Normality	Prob>chi2 = 0.494
Shapiro-Wilk W test for normal data	Prob>z = 0.351
Full model	Prob > F = 0.0148 **
Adj R-squared	0.167
Obs.	45

Notes**.** *The estimation method is OLS. Numbers in brackets after the coefficients are t-statistics. The symbol *** denotes significance at 1% confidence level; ** denotes significance at 5% confidence level; * denotes significance at 10% confidence level.*

Table 4. Estimate of the equation (II)

Dependent variable: *unemployment rate*	
Independent variables	Coefficients
Growth	0.511 (1.81) *
*Dummy of interaction (Rule of Law * Growth)*	–0.884 (–3.42) ***
cons	6.95 (7.44) ***
Statistical tests	
Ramsey RESET test *Ho: model has no omitted variables*	F(3, 39) = 1.10 Prob > F = 0.361
Breusch-Pagan / Cook-Weisberg test for heteroskedasticity *Ho: Constant variance*	chi2(1) = 2.25 Prob > chi2 = 0.133
Mean VIF	1.15
Skewness/Kurtosis tests for Normality	Prob>chi2 = 0.272
Shapiro-Wilk W test for normal data	Prob>z = 0.151
Full model	Prob > F = 0.0050 ***
Adj R-squared	0.186
Obs.	45

Notes. *The estimation method is OLS. Numbers in brackets after the coefficients are t-statistics. The symbol *** denotes significance at 1% confidence level; ** denotes significance at 5% confidence level; * denotes significance at 10% confidence level.*