Team Production and the Allocation of Creativity across Global and Local Sectors

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1 April 2015

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April 3, 2015

Abstract

Team production is introduced into a two-sector Ricardian comparative advantage model in order to investigate its role in shifting high-skilled agents from a sector in which they have comparative advantage to a sector in which they have comparative disadvantage especially focusing on a case where environments of team production in the latter sector are improving. The first result is that team production changes the nature of comparative advantage, possibly leading to reallocation of creativity. The second result is that the likelihood of the shift is limited, and even in a case of success, policy targets (improving the environments of team production) should be selected carefully since those targets are different in the likelihood of shifting creativity, and the most likely case is associated with non-monotonic dynamics of the allocation of creativity.

Keywords: team production, Ricardian comparative advantage, local advantage

[1]I would like to thank Marcus Berliant, Tomoya Mori, and Se-il Mun for their comments and suggestions. All remaining errors are mine. This research is financially supported by the Project "Spatial Economic Analysis on Regional Growth" undertaken at Research Institute of Economy, Trade and Industry (RIETI). E-mail: nagamachi@gsm.kagawa-u.ac.jp

1 Introduction

Transformation of a system of cities from sectoral to functional specialization (Duranton and Puga, 2005) and the increasing importance of interactive tasks in economic activity (Michaels et al., 2013) pose a challenge for local policy makers in attracting creativity since skill-intensive, non-routine economic activity concentrates in larger cities which have a comparative advantage in such activity.[2]

Given the above concern, this paper investigates the role of team production in affecting the allocation of creativity by developing a two-sector Ricardian comparative advantage model with team production and two types of agents, high-skilled and low-skilled. The two sectors consist of global and local sectors, the former (latter) of which is defined as the one in which the high-skilled (low-skilled) have a comparative advantage. In both sectors, team production is allowed, connecting one high-skilled agent as a manager to some low-skilled agents as workers and allowing for managers to leverage their knowledge effectively. Under an interpretation that global and local sectors correspond to larger and smaller cities, respectively, I introduce an additional type of team production in the local sector, in which managers can learn about local advantages through communications with workers.

The implications are two-fold: First, team production changes the nature of comparative advantage, i.e., team production could be a tool of shifting creativity from global to local sectors. Second, the likelihood of the shift is limited, and even in a case of success, policy targets should be selected carefully since those targets are different in the likelihood of attracting creativity, and the most likely case is associated with non-monotonic dynamics of the allocation of creativity.

This paper is related to two literatures: First, the model is an extension of Garicano and Rossi-Hansberg (2006) to multiple sectors and additional form of team production. Second, the literature of knowledge creation such as Berliant and Fujita (2012) is related. However, the focus is on collaborations between high-skilled and low-skilled agents not those between creative people.

The rest of this paper is organized as follows: In Section 2, the structure of the model is described. Then, in Section 3, policy implications are obtained focusing on equilibria resembling functional specialization of cities. The final section, Section 4, concludes this paper.

2 The Model

2.1 Environment

I consider a Ricardian closed economy with two competitive sectors, global g and local ℓ, and two types of agents, high-skilled h and low-skilled l. ℓ agents have a comparative advantage in ℓ sector, the relative price of a good in which is denoted by $p > 0$. Production is specified as problem solving: Given one unit of time endowment, each agent draws one problem per unit of time, each associated

[2] Functional specialization is a system of cities, in which larger cities specialize in skill-intensive, non-routine economic activity such as research and development, while smaller cities in less skill-intensive, more routine ones such as line production which based on technologies developed by the former cities.

with some level in $(0,1)$ of knowledge required to solve. i-agent level of knowledge in k sector is denoted by $k_i \in (0,1)$, and the law of large numbers implies sector-g (-ℓ) income of i agents is given by g_i ($p\ell_i$) under self-employment. The following absolute and comparative advantages are assumed: $g < l < g_h$; $\ell_l < \ell_h$; and $g_l/\ell_l < g_h/\ell_h$. The relative supply of l agents is fixed to $\rho > 1$.

2.2 Team Production

In both sectors, team production á la Garicano and Rossi-Hansberg (2006) is allowed in addition to self-employment. More precisely, there are one common and one ℓ-specific forms of team production.

2.2.1 Common Form

In a team of the common form in sector k, one h-agent manger and n_k l-agent workers constitute a team. First, workers draw and try to solve problems by themselves, implying $(1-k_l)n_k$ problems are left unsolved. Second, workers pass $(1-k_l)n_k$ unsolved problems to their manager with communication cost c_k per unit of problems. Finally, the manager suggests how to fix those problems to her workers if she knows, and the suggested workers solve the problems. In total, the team as a whole can solve $k_h n_k$ problems. The manager's time constraint determines the team size $n_k = 1/[c_k(1-k_l)]$.

In this paper, I focus on a case where communication cost c_k satisfies

$$\frac{1}{\rho(1-g_l)} < c_g < \frac{g_h - g_l}{g_h(1-g_l)}; \quad \frac{1}{\rho(1-\ell_l)} < c_\ell < \frac{\ell_h - \ell_l}{\ell_h(1-\ell_l)}, \tag{1}$$

implying that team production is productive compared with self-employment, and l agents have never bargaining power in wage determination. Given this assumption, h managers exploit the rents of their teams, which are given by the zero-profit condition.

2.2.2 ℓ-specific Form

With ℓ-specific team production, an h agent can invest her time (in addition to communication cost) to raise her productivity from ℓ_h to $a\ell_h$, where $a \in (1, \ell_h^{-1})$. An interpretation is as follows: h manager can apply their knowledge well-suitable to activities in g sector to those in ℓ sector and can still earn income more than l agents do. However, with learning about local advantages such as scenery, culture, history and etc., the quality of output increases further. Rather than simply designing a conventional building in a beautiful scenery, designing a building in accord with such nature makes the place more valuable. The time cost of learning is specifically given by iceberg-type cost $\tau > 1$, i.e., with learning, passing $(1-\ell_l)n_\ell$ unsolved problems in a team requires the manager $\tau c_\ell(1-\ell_l)n_\ell$ units of time.

2.3 l-agent Choice

Since l agents cannot become a manager and have no bargaining power,[3] income levels are equalized across self-employment and workers of teams in both sectors. The resulting environment is exactly the same as in a simple Ricardian model, i.e., letting $p_l^* \equiv g_l/\ell_l$, l agents choose g sector if $p < p_l^*$ and ℓ sector otherwise. This optimal choice implies that the wage rate w_l of l agents is given as follows: $w_l = g_l$ if $p < p_l^*$, and $w_l = p\ell_l$ otherwise.

2.4 h-agent Choice

In addition to self-employment, h agents can become the manager of a team in either g or ℓ sector. In addition, if an h agent chose to form a team in ℓ sector, then she must also choose which type of team she forms. For notational convenience, let g_s and ℓ_s denote self-employment in g sector and that in ℓ sector, respectively. Also let g, $\ell_{w/}$, and $\ell_{w/o}$ denote teams in g sector, those with learning in ℓ sector, and those without learning in ℓ sector, respectively. Therefore, h agents choose one of $\{g_s, \ell_s, g, \ell_{w/}, \ell_{w/o}\}$. Note that for a chosen form $f \in \{g, \ell_{w/}, \ell_{w/o}\}$ of a team, productivity z_f and team size n_f are determined, implying that the wage rate $w_{h,f}$ of h manager is given by $w_{h,f} = (z_f - w_l)n_f$.

Since the wage rate $w_{h,f}$ of managers depends on the wage rate w_l of l workers which in turn depends on the relative price p, h-agent choice should be discussed conditional on p.

2.4.1 Indifference Curves: $p < p_l^*$

If $p < p_l^*$, then the wage rate w_l of l workers is given by $w_l = g_l$. Due to comparative advantage, h agents also choose g_s if they chose self-employment. However, note that g_s is never chosen by h agents when $p < p_l^*$. This is simply because $w_{h,g} > g_s$, which holds under (1).

Therefore, h agents are effectively faced with three options: g, $\ell_{w/}$, or $\ell_{w/o}$. It is convenient to provide equations associated with indifference curves:

$$I_{\ell_{w/}\sim\ell_{w/o}} \quad : \quad \frac{pa\ell_h - g_l}{\tau c_\ell(1-\ell_l)} = \frac{p\ell_h - g_l}{c_\ell(1-\ell_l)} \quad \Longrightarrow \quad p = \frac{g_l}{\ell_h}\frac{\tau - 1}{\tau - a}(\tau \neq a) \tag{2}$$

$$I_{g\sim\ell_{w/}} \quad : \quad \frac{g_h - g_l}{c_g(1-g_l)} = \frac{pa\ell_h - g_l}{\tau c_\ell(1-\ell_l)} \quad \Longrightarrow \quad p = \frac{g_l}{a\ell_h}\left[1 + \tau\frac{c_\ell(1-\ell_l)}{c_g(1-g_l)}\frac{g_h - g_l}{g_l}\right] \tag{3}$$

$$I_{g\sim\ell_{w/o}} \quad : \quad \frac{g_h - g_l}{c_g(1-g_l)} = \frac{p\ell_h - g_l}{c_\ell(1-\ell_l)} \quad \Longrightarrow \quad p = \frac{g_l}{\ell_h}\left[1 + \frac{c_\ell(1-\ell_l)}{c_g(1-g_l)}\frac{g_h - g_l}{g_l}\right]. \tag{4}$$

For notational convenience, let $p_{\ell_{w/}\sim\ell_{w/o}}$ denote the relative price corresponding to the indifference curve associated with $\ell_{w/} \sim \ell_{w/o}$. In a similar manner, I use similar notations for the other cases. When emphasizing that the relative price is a function of some parameter θ, I use the expression like $p_{\ell_{w/}\sim\ell_{w/o}}(\theta)$.

[3] The former is not an exogenous assumption. That is, although l agents can choose to try to form a team, there is no productivity gain, making no agents willing to participate in such a team.

2.4.2 Indifference Curves: $p > p_l^*$

If $p > p_l^*$, then the wage rate w_l of l workers is given by $w_l = p\ell_l$. Due to comparative advantage, h agents also choose g_s if they chose self-employment. In this case, preferring g to g_s is not necessarily the case since the choice is dependent on the relative price p.

Therefore, h agents are effectively faced with four options: g_s, g, $\ell_{w/}$, or $\ell_{w/o}$. It is convenient to provide equations associated with indifference curves:

$$I_{\ell_{w/}\sim\ell_{w/o}} \quad : \quad \frac{pa\ell_h - p\ell_l}{\tau c_\ell(1-\ell_l)} = \frac{p\ell_h - p\ell_l}{c_\ell(1-\ell_l)} \quad \Longrightarrow \quad \tau = \frac{a\ell_h - \ell_l}{\ell_h - \ell_l}, \tag{5}$$

$$I_{g\sim\ell_{w/}} \quad : \quad \frac{g_h - p\ell_l}{c_g(1-g_l)} = \frac{pa\ell_h - p\ell_l}{\tau c_\ell(1-\ell_l)} \quad \Longrightarrow \quad p = \frac{g_h}{\ell_l + (a\ell_h - \ell_l)\frac{c_g(1-g_l)}{\tau c_\ell(1-\ell_l)}}, \tag{6}$$

$$I_{g\sim\ell_{w/o}} \quad : \quad \frac{g_h - p\ell_l}{c_g(1-g_l)} = \frac{p\ell_h - p\ell_l}{c_\ell(1-\ell_l)} \quad \Longrightarrow \quad p = \frac{g_h}{\ell_l + (\ell_h - \ell_l)\frac{c_g(1-g_l)}{c_\ell(1-\ell_l)}}, \tag{7}$$

$$I_{g_s\sim\ell_{w/}} \quad : \quad g_h = \frac{pa\ell_h - p\ell_l}{\tau c_\ell(1-\ell_l)} \quad \Longrightarrow \quad p = g_h \frac{\tau c_\ell(1-\ell_l)}{a\ell_h - \ell_l}, \tag{8}$$

$$I_{g_s\sim\ell_{w/o}} \quad : \quad g_h = \frac{p\ell_h - p\ell_l}{c_\ell(1-\ell_l)} \quad \Longrightarrow \quad p = g_h \frac{c_\ell(1-\ell_l)}{\ell_h - \ell_l}, \tag{9}$$

$$I_{g_s\sim g} \quad : \quad g_h = \frac{g_h - p\ell_l}{c_g(1-g_l)} \quad \Longrightarrow \quad p = \frac{g_h}{\ell_l}[1 - c_g(1-g_l)] \tag{10}$$

3 Results

3.1 h-agent Choice in (θ, p) Coordinates

Assuming that initial parameters satisfy the following

$$1 < c_g(1-g_l) + c_\ell(1-\ell_l)\frac{\ell_l}{\ell_h - \ell_l}, \tag{11}$$

h-agent choice is summarized in (θ, p) coordinate (Figure 1-6),[4] where θ is either one of three parameters of interest: learning cost τ, productivity gain a, and ℓ-sector communication cost c_ℓ. Under this assumption, there exists a range of g_s of some positive measure in (θ, p) coordinate. The condition is rewritten as $p_{g\sim g_s} < p_{g_s\sim\ell_{w/o}}$, where $p_{g\sim g_s}$ and $p_{g_s\sim\ell_{w/o}}$ are given by (10) and (9), respectively. By imposing the above assumption, I focus on a situation "severe" for ℓ sector in that shifts in h-agent choice from g sector to ℓ sector is not smooth.

[4] The definitions of thresholds are given in Appendix A.1.1-A.1.3.

3.2 Relative Supply Curves

Given h-agent choice in Subsection 3.1, the shifts of the relative supply curve of ℓ good for decreasing τ, increasing a, and decreasing c_ℓ are obtained as depicted in Figure 7-10, Figure 11-14, and Figure 15-21, respectively.[5] For ease of exposition, the lower and upper bounds for (τ, a, c_ℓ) are omitted.

3.3 Numerical Experiment

3.3.1 Equilibrium of Interest

In this paper, I focus on the simplest case where agents' preference is specified by a Cobb-Douglas function with expenditure shares, α_g and α_ℓ, of g and ℓ goods, i.e., $\alpha_g + \alpha_\ell = 1$, implying that the relative demand for ℓ good is given by αp^{-1}, where $\alpha \equiv \alpha_\ell/\alpha_g > 0$. Depending on the ratio α, various equilibria are possible, and thus there are many "dynamics" of ℓ-sector share λ_h in h agents when τ or c_ℓ decreases or when a increases.

Therefore, I focus on equilibria with the following properties: First, α must satisfy

$$\frac{g_l}{g_h}[\rho c_g(1-g_l)-1] < \alpha < [\rho c_g(1-g_l)-1][1-c_g(1-g_l)]. \tag{12}$$

Second, it must hold that $\max\{\hat{\tau}_2, 1\} < \tau$, which is equivalent to $1 < a < \hat{a}_2$ given that $\tau > 1$; or $\hat{c}_{\ell,1} < c_\ell$ if $\hat{\tau}_1 < \tau$ and $\tilde{c}_{\ell,1} < c_\ell$ otherwise.

In this type of equilibrium, all h agents are initially g-team managers, while l agents are either employed by those managers or self-employment in ℓ sector. To some extent, this captures functional specialization of cities reported by the literature (Duranton and Puga, 2005) with an interpretation that this system of production corresponds to team production in g sector. Possible scenarios of the dynamics of λ_h are illustrated in Figure 24-36. For decreasing τ, increasing a, and decreasing c_ℓ, Scenario 0-4, Scenario 0-1 and A1-A2, and Scenario 0-4 and C1-C6 apply, respectively.[6]

3.3.2 Results of Monte Carlo Simulation

Compared with the other two parameters, a decrease in ℓ-sector communication cost c_ℓ is most effective in shifting h agents from global to local sectors in that the measure of Scenario 0, the share of Scenario 0 in the samples, is lowest for most of the pair $(\bar{\rho}, \bar{\tau})$ of the upper bounds for the relative supply of l agents and learning cost (Figure 22).

As for $\ell_{w/}$-specific parameters (τ, a), there is no clear ranking in that the measure of Scenario 0 tends to be higher in τ-λ_h dynamics when $\bar{\tau}$ is low, while the contrary holds when $\bar{\tau}$ is high (Figure 22). This suggests effective policy targets depend on cases. When learning cost τ is high ($\bar{\tau}$ is high), team production with learning is costly, making the effect of increasing productivity gain a limited.

[5] The definitions of the relative quantities are given in Appendix A.1.4.
[6] These possible scenarios of comparative statics of λ_h are identified analytically.

I also report the measure of each scenario other than Scenario 0 for each dynamics in Figure 23. The left, the center, and the right panels show measures of scenarios in τ-λ_h, a-λ_h, and c_ℓ-λ_h dynamics, respectively, while the lower, the middle, and the upper panels show measures of scenarios for low, middle, and high $\bar{\tau}$, respectively.

There are at least two properties common across all dynamics. First, as the measure of Scenario 0 decreases due to changes in $\bar{\tau}$, then measures of other scenarios shift upward.

Second, for lower $\bar{\rho}$, an equilibrium with $\lambda_h \in [0,1)$ is likely to happen, while an equilibrium with $\lambda_h = 1$ does for higher $\bar{\rho}$. More specifically, for low $\bar{\rho}$, Scenario 2 (or Scenario A1 in the case of a-λ_h dynamics) and Scenario 1 are likely to happen, while scenarios with $\lambda_h = 1$ happen with zero probability for sufficiently low $\bar{\rho}$. However, for high $\bar{\rho}$, scenarios with $\lambda_h = 1$ become more likely to happen, and among those scenarios, Scenario 4 in τ-λ_h dynamics, Scenario A2 in a-λ_h dynamics, and Scenario 3 in c_ℓ-λ_h dynamics are of measures comparable with those of scenarios with $\lambda_h \in [0,1)$.

The second property is a result of general equilibrium effects of the relative supply ρ of l agents. When ρ is relative large, the relative demand for ℓ good is more likely to be higher than the relative supply, resulting in h agents all engaging in team production in ℓ sector, because the supply for g good including that from self-employment of l agents, those not employed by h agents, increases as the relative supply ρ of l agents increases.

An important policy implication observed in Figure 23 is that when encouraging team production in ℓ sector, i.e., improvements in cost and benefit (τ, a, c_ℓ), effects on λ_h are likely to be non-monotonic, necessitating a careful policy management. Specifically, Scenario 2 (or Scenario A1) has the highest measure than those except for Scenario 0.[7] The mechanism is as follows: When λ_h is increasing, the equilibrium relative price p is higher than p_l^*, implying that l agents not employed by h managers in g sector all engage in ℓ-sector self-employment. Given that those l agents do not benefit from the improvements, a lower communication cost c_ℓ, say, by increasing the relative demand for ℓ good due to a decrease in the price p, requires some h agents to fix the excess demand for ℓ good. The contrasting result holds in a phase where λ_h is decreasing (or constant in a-λ_h dynamics).

4 Conclusion

This paper introduces team production á la Garicano and Rossi-Hansberg (2006) into a two-sector Ricardian comparative advantage model to derive implications for policies encouraging team production in local sectors to attract creativity. The first implication is that team production could be a tool of shifting creativity from global to local sectors. However, policy targets should be selected carefully since those targets are different in the likelihood of attracting creativity, and the most likely case is associated with non-monotonic dynamics of the allocation of creativity, the second implication.

[7] Note that Scenario A1 has no decreasing phase in a-λ_h because the relative demand and supply have the same elasticity with respect to a. However, this phase can be interpreted as an ineffectiveness of policies increasing productivity gain a.

References

BERLIANT, M. AND M. FUJITA, “Culture and diversity in knowledge creation,” *Regional Science and Urban Economics* 42 (2012), 648–662.

DURANTON, G. AND D. PUGA, “From sectoral to functional urban specialisation,” *Journal of Urban Economics* 57 (March 2005), 343–370.

GARICANO, L. AND E. ROSSI-HANSBERG, “The Knowledge Economy at the Turn of the Twentieth Century: The Emergence of Hierarchies,” *Journal of the European Economic Association* 4 (04-05 2006), 396–403.

MICHAELS, G., F. RAUCH AND S. J. REDDING, “Task Specialization in U.S. Cities from 1880-2000,” CEP Discussion Papers dp1186, Centre for Economic Performance, LSE, February 2013.

A Appendix

A.1 Definitions

A.1.1 Thresholds in (τ, p) Coordinate

The thresholds $\hat{\tau}_1$, $\hat{\tau}_2$, and $\hat{\tau}_3$ are given by (5), substituting $p_{g\sim g_s}$ into (6), and substituting p_l^* into (6), respectively:

$$\begin{aligned}
\hat{\tau}_1 &\equiv \frac{a\ell_h - \ell_l}{\ell_h - \ell_l}, \\
\hat{\tau}_2 &\equiv \frac{a\ell_h - \ell_l}{\ell_l} \frac{1 - c_g(1 - g_l)}{c_\ell(1 - \ell_l)}, \\
\hat{\tau}_3 &\equiv \frac{g_l}{\ell_l} \frac{a\ell_h - \ell_l}{g_h - g_l} \frac{1 - g_l}{1 - \ell_l} \frac{c_g}{c_\ell}.
\end{aligned}$$

Given (11), $\hat{\tau}_3 < \hat{\tau}_2 < \hat{\tau}_1$.

The threshold c_1^* is given by

$$c_1^* \equiv \frac{g_l}{\ell_l} \frac{a\ell_h - \ell_l}{g_h - g_l} \frac{1 - g_l}{1 - \ell_l}.$$

A.1.2 Thresholds in (a, p) Coordinate

The thresholds $\hat{a}_1$, $\hat{a}_2$, and $\hat{a}_3$ are given by solving (5) for a, substituting $p_{g\sim g_s}$ into (6), and substituting p_l^* into (6), respectively:

$$
\begin{aligned}
\hat{a}_1 &\equiv \left(1-\frac{\ell_l}{\ell_h}\right)\tau+\frac{\ell_l}{\ell_h}, \\
\hat{a}_2 &\equiv \frac{\ell_l}{\ell_h}\left[1+\tau\frac{c_\ell(1-\ell_l)}{1-c_g(1-g_l)}\right], \\
\hat{a}_3 &\equiv \frac{\ell_l}{\ell_h}\left[1+\tau\frac{c_\ell(1-\ell_l)}{c_g(1-g_l)}\frac{g_h-g_l}{g_l}\right],
\end{aligned}
$$

and the ranking $1 < \hat{a}_1 < \hat{a}_2 < \hat{a}_3$ holds.

A.1.3 Thresholds in (c_ℓ, p) Coordinate

The thresholds $\hat{c}_{\ell,1}$, $\hat{c}_{\ell,2}$, and $\hat{c}_{\ell,3}$ in Pattern 1 are given by substituting $p_{g\sim g_s}$ into (7), substituting p_l^* into (7), and substituting $p_{\ell_{w/}\sim\ell_{w/o}}$ given by (5) into (7), respectively:

$$
\begin{aligned}
\hat{c}_{\ell,1} &\equiv \frac{\ell_h-\ell_l}{\ell_l(1-\ell_l)}[1-c_g(1-g_l)], \\
\hat{c}_{\ell,2} &\equiv \frac{g_l}{\ell_l}\frac{\ell_h-\ell_l}{g_h-g_l}\frac{1-g_l}{1-\ell_l}c_g, \\
\hat{c}_{\ell,3} &\equiv \frac{a-1}{\tau-a}\frac{g_l}{g_h-g_l}\frac{1-g_l}{1-\ell_l}c_g,
\end{aligned}
$$

and the ranking $0 < \hat{c}_{\ell,3} < \hat{c}_{\ell,2} < \hat{c}_{\ell,1}$ holds. The thresholds $\tilde{c}_{\ell,1}$ and $\tilde{c}_{\ell,2}$ in Pattern 2 are given by substituting $p_{g\sim g_s}$ into (6) and substituting p_l^* into (6), respectively:

$$
\begin{aligned}
\tilde{c}_{\ell,1} &\equiv \frac{a\ell_h-\ell_l}{\ell_l}\frac{1-c_g(1-g_l)}{\tau(1-\ell_l)}, \\
\tilde{c}_{\ell,2} &\equiv \frac{g_l}{\ell_l}\frac{a\ell_h-\ell_l}{g_h-g_l}\frac{1-g_l}{1-\ell_l}\frac{c_g}{\tau},
\end{aligned}
$$

and the ranking $0 < \tilde{c}_{\ell,2} < \tilde{c}_{\ell,1}$ holds.

A.1.4 Relative Quantities

The relative quantities in Figure 7-21 are defined as follows:

$$
s_g \equiv \frac{\ell_l}{g_h}[\rho c_g(1-g_l)-1], \quad s_{g_s} \equiv \frac{\ell_l}{g_h}\rho, \quad s_{\ell_{w/}} \equiv \frac{a\ell_h}{g_l}\frac{1}{\rho\tau c_\ell(1-\ell_l)-1}, \quad s_{\ell_{w/o}} \equiv \frac{\ell_h}{g_l}\frac{1}{\rho\tau c_\ell(1-\ell_l)-1} > s_{\ell_{w/}}.
$$

A.2 Numerical Experiment

A.2.1 Algorithm

For each fixed set of upper bounds, $\bar{\rho}$ and $\bar{\tau}$, the algorithm below is used to generate random samples and conduct comparative statics.

Step 1: Generate samples of parameters $(\ell_l, \ell_h, g_l, g_h, \rho, c_g, c_\ell, a, \tau)$ of size of one million from the uniform distribution over the subset of parameters satisfying the stated conditions:

(a) Generate ℓ_l at random such that $0 < \ell_l < (1-\bar{\rho}^{-1})$.

(b) Generate ℓ_h at random such that $[\bar{\rho}/(\bar{\rho}-1)]\ell_l < \ell_h < 1$.

(c) Generate g_h at random such that $0 < g_h < 1$.

(d) Generate g_l at random such that $0 < g_l < (\ell_l/\ell_h)g_h$.

(e) Generate ρ at random such that $\ell_h/(\ell_h - \ell_l) < \rho \le \bar{\rho}$.

(f) Generate a at random such that

$$1 < a < \min\left\{\ell_h^{-1}, \frac{\ell_l}{\ell_h}\left(1+\bar{\tau}\frac{g_h}{g_l}\frac{\ell_h-\ell_l}{\ell_h}\right)\right\}.$$

(g) Generate c_g at random such that

$$\max\left\{\frac{1}{\rho(1-g_l)}, \frac{\ell_h-\ell_l}{\ell_h(1-g_l)}, \frac{1}{1-g_l} - \frac{\bar{\tau}}{1-g_l}\frac{\ell_l}{\ell_h}\frac{\ell_h-\ell_l}{a\ell_h-\ell_l}\right\} < c_g < \frac{g_h-g_l}{g_h(1-g_l)}.$$

(h) Generate c_ℓ at random such that

$$\max\left\{\frac{1}{\rho(1-\ell_l)}, \frac{\ell_h-\ell_l}{\ell_l(1-\ell_l)}[1-c_g(1-g_l)], \frac{a\ell_h-\ell_l}{\bar{\tau}\ell_l(1-\ell_l)}[1-c_g(1-g_l)]\right\} < c_\ell < \frac{\ell_h-\ell_l}{\ell_h(1-\ell_l)}.$$

(i) Generate τ at random such that $\max\{\hat{\tau}_2, 1\} < \tau \le \bar{\tau}$.

Step 2: For each sample, construct equidistant grid points on the following closed interval of α under which an equilibrium relative price p satisfies $p_l^* \le p \le p_{g\sim g_s}$:

$$\frac{g_l}{g_h}[\rho c_g(1-g_l)-1] \le \alpha \le [\rho c_g(1-g_l)-1][1-c_g(1-g_l)],$$

and compute the share of each scenario in the grid points.

Step 3: Compute the sample average of the share of each scenario.

Step 1 in the algorithm ensures that parameters satisfy the required conditions. Possible scenarios are illustrated in Figure 24-36, the conditions of which are omitted due to limitations of space. For $\bar{\tau}$, I consider three values: low (1.1), middle (1.5), and high (2.0).[8] $\bar{\rho}$ ranges from 2 to 20.

[8] Learning within ℓ team increases time cost of passing unsolved problems by $100 \times (\tau - 1)\%$. $\bar{\tau}$ gives the upper bound for this increase, and "low," "middle," and "high" correspond to the maximal increase of 10%, 50%, and 100%, respectively.

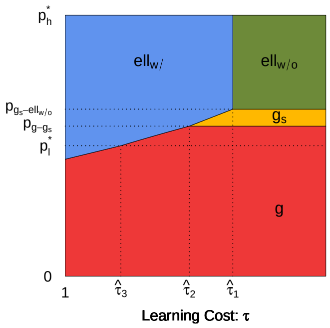

Figure 1: h-agent Choice in (τ, p) Coordinate, Pattern 1, $\frac{c_\ell}{c_g} < c_1^*$

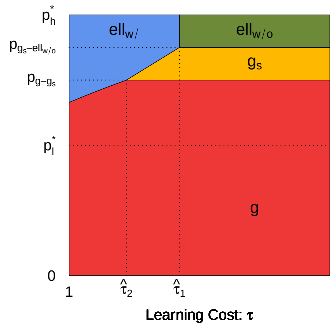

Figure 2: h-agent Choice in (τ, p) Coordinate, Pattern 2, $\frac{c_\ell}{c_g} > c_1^*$, $\hat{\tau}_2 > 1$

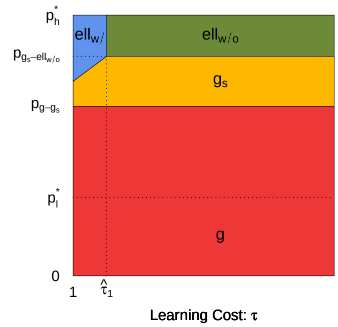

Figure 3: h-agent Choice in (τ, p) Coordinate, Pattern 3, $\frac{c_\ell}{c_g} > c_1^*$, $\hat{\tau}_2 < 1$

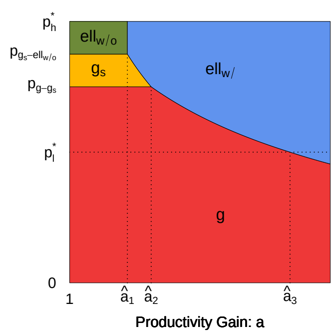

Figure 4: h-agent Choice in (a, p) Coordinate

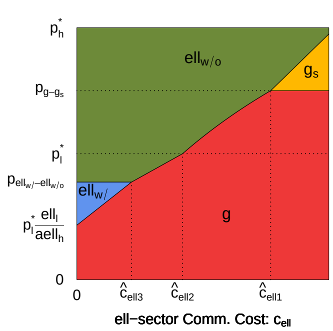

Figure 5: h-agent Choice in (c_ℓ, p) Coordinate, Pattern 1, $\hat{\tau}_1 < \tau$

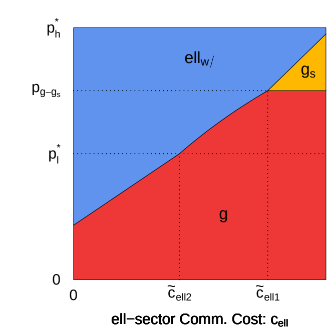

Figure 6: h-agent Choice in (c_ℓ, p) Coordinate, Pattern 2, $\hat{\tau}_1 > \tau$

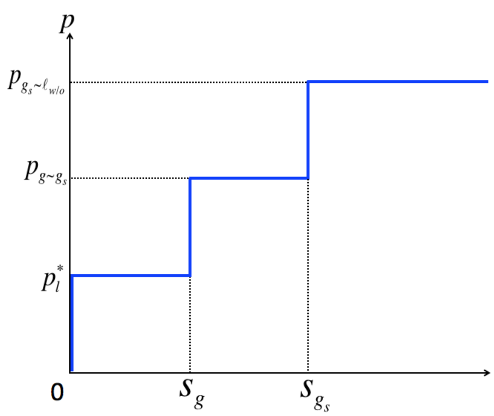

Figure 7: $\hat{\tau}_1 < \tau$

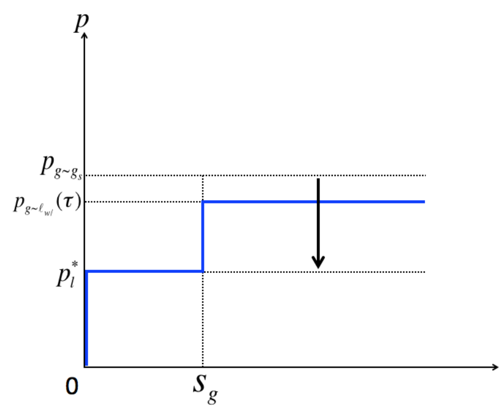

Figure 9: $\hat{\tau}_3 < \tau < \hat{\tau}_2$

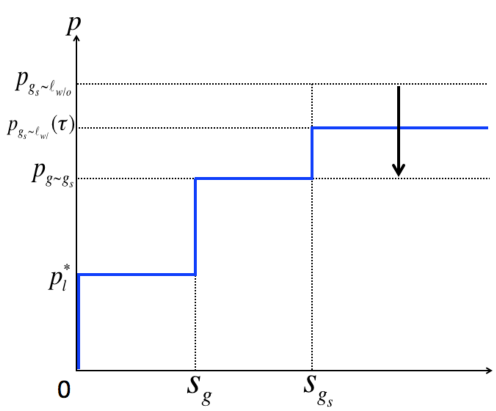

Figure 8: $\hat{\tau}_2 < \tau < \hat{\tau}_1$

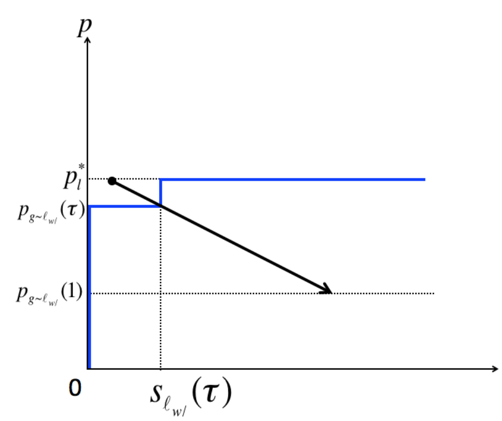

Figure 10: $1 < \tau < \hat{\tau}_3$

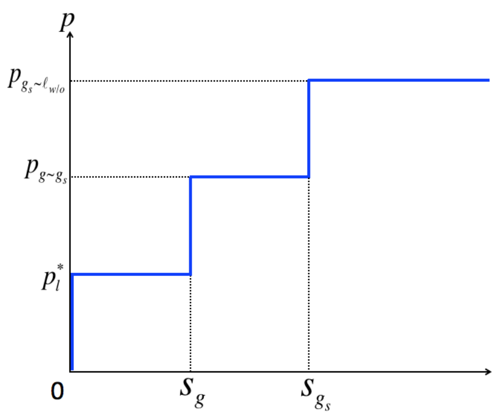

Figure 11: $1 < a < \hat{a}_1$

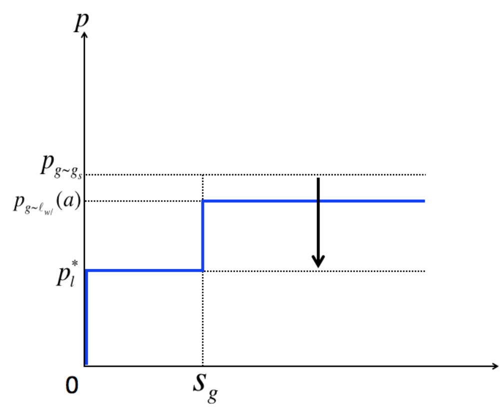

Figure 13: $\hat{a}_2 < a < \hat{a}_3$

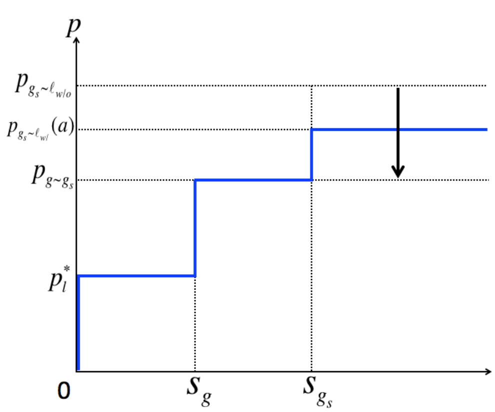

Figure 12: $\hat{a}_1 < a < \hat{a}_2$

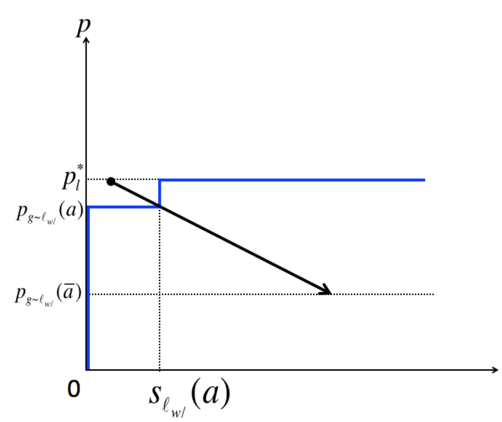

Figure 14: $\hat{a}_3 < a$

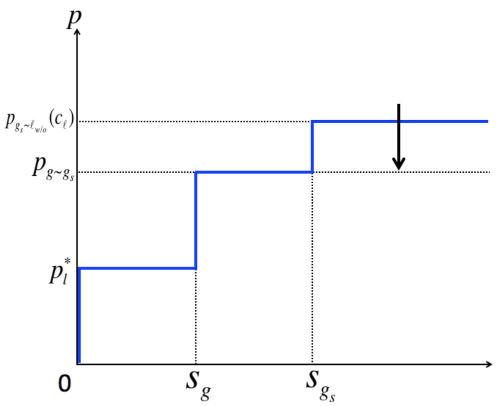

Figure 15: $\hat{c}_{\ell,1} < c_\ell$

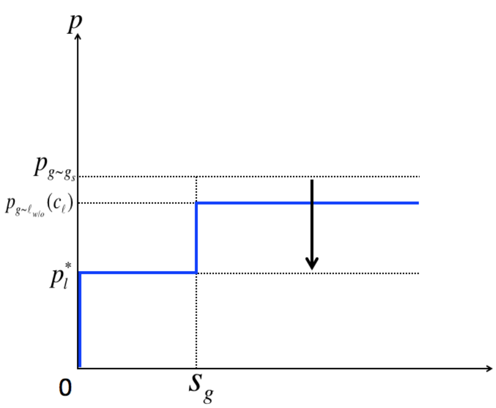

Figure 16: $\hat{c}_{\ell,2} < c_\ell < \hat{c}_{\ell,1}$

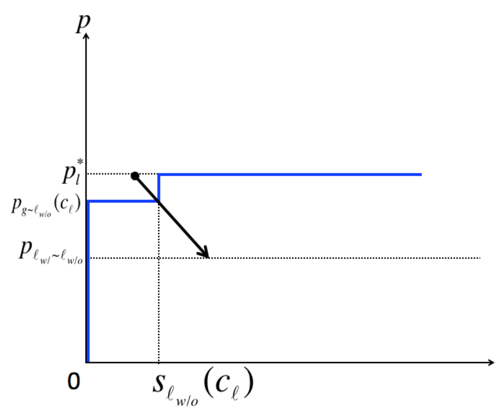

Figure 17: $\hat{c}_{\ell,3} < c_\ell < \hat{c}_{\ell,2}$

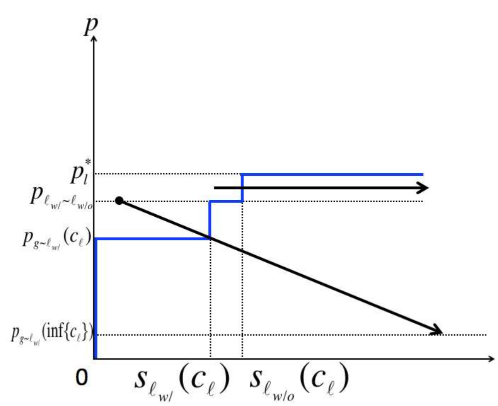

Figure 18: $(\inf\{c_\ell\} =)\underline{c}_\ell < c_\ell < \hat{c}_{\ell,3}$

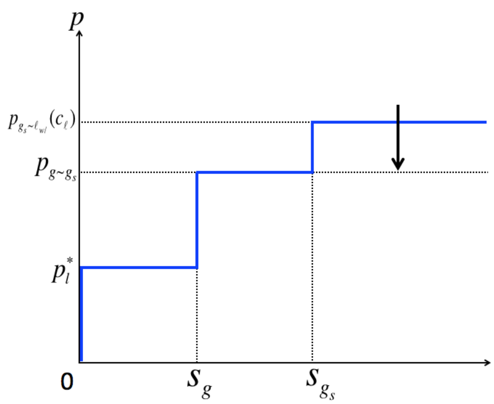

Figure 19: $\tilde{c}_{\ell,1} < c_\ell$

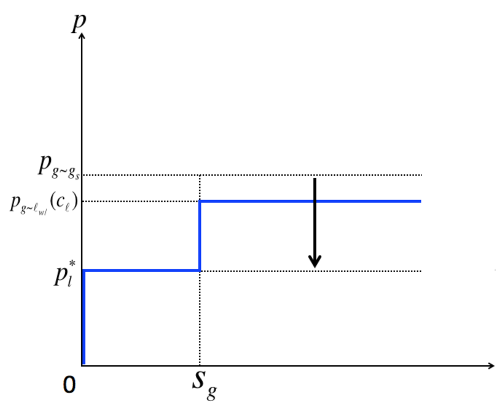

Figure 20: $\tilde{c}_{\ell,2} < c_\ell < \tilde{c}_{\ell,1}$

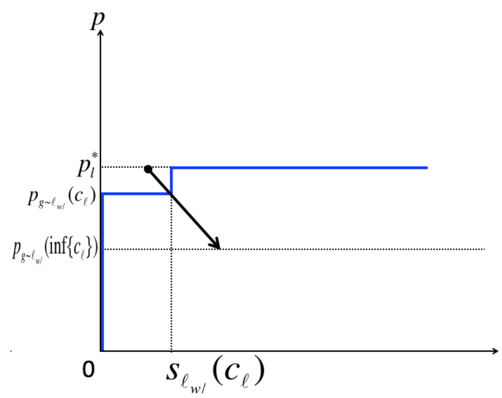

Figure 21: $(\inf\{c_\ell\} =)\underline{c}_\ell < c_\ell < \tilde{c}_{\ell,2}$

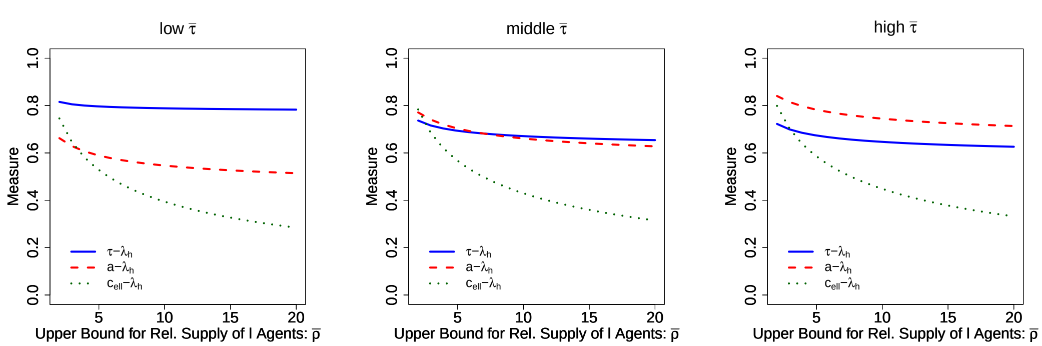

Figure 22: Measure of Scenario 0

Note: Low, middle and high $\bar{\tau}$ correspond to values of 1.1, 1.5, and 2.0, respectively.

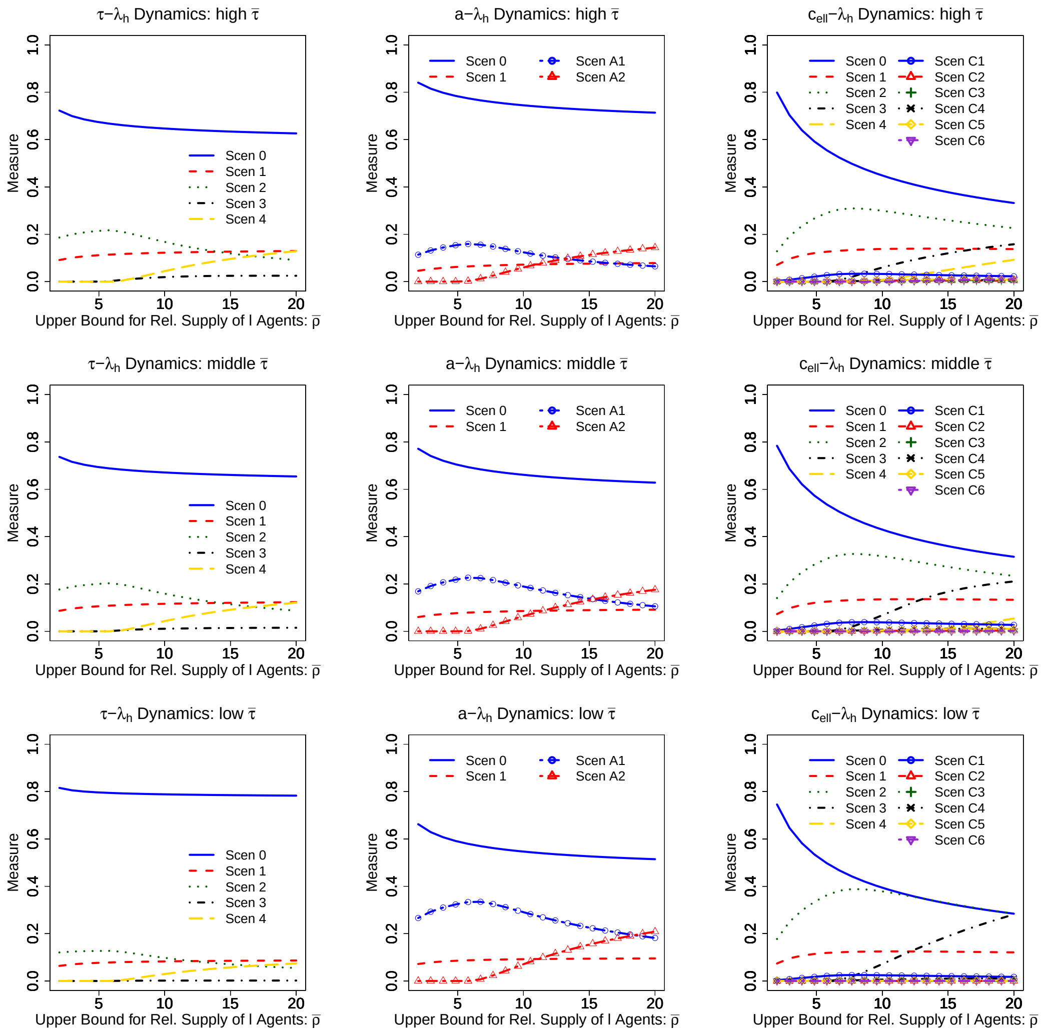

Figure 23: Measures of Scenarios in Dynamics

Note: Low, middle and high $\bar{\tau}$ correspond to values of 1.1, 1.5, and 2.0, respectively.

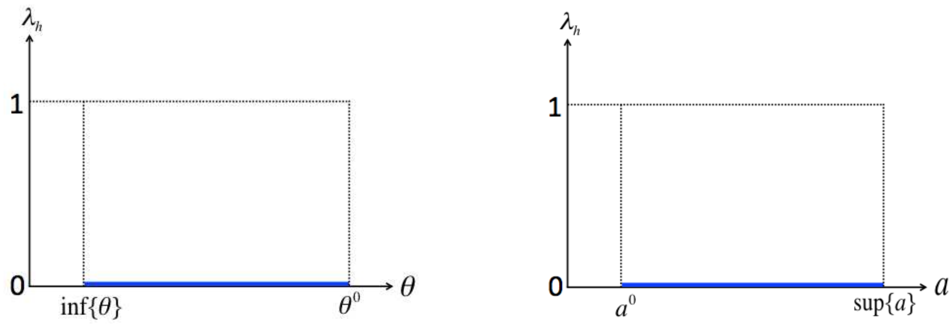

Figure 24: Scenario 0

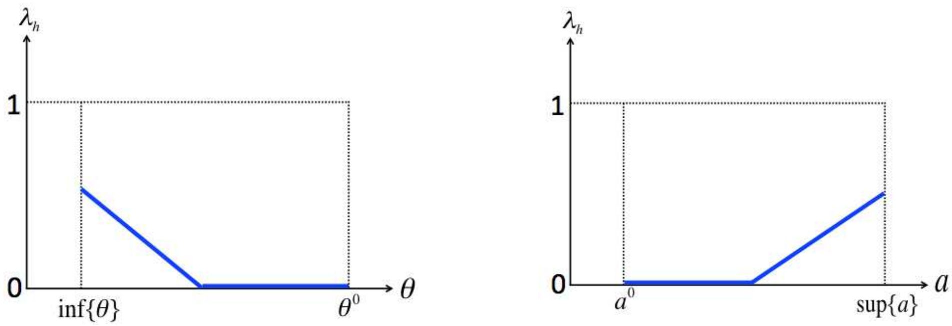

Figure 25: Scenario 1

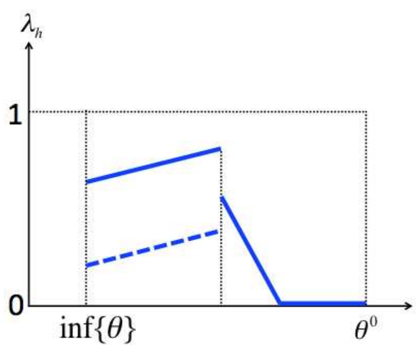

Figure 26: Scenario 2

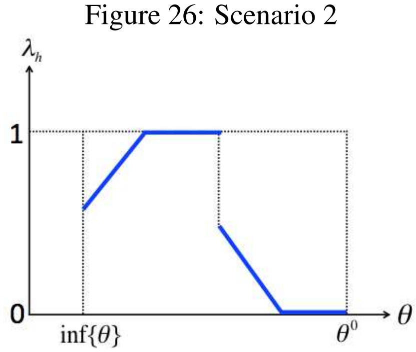

Figure 27: Scenario 3

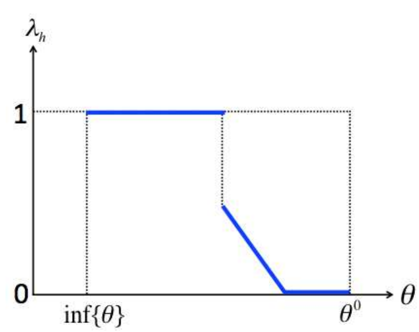

Figure 28: Scenario 4

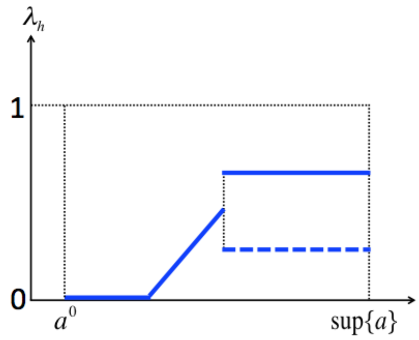

Figure 29: Scenario A1

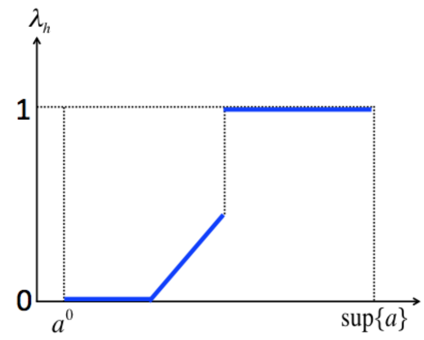

Figure 30: Scenario A2

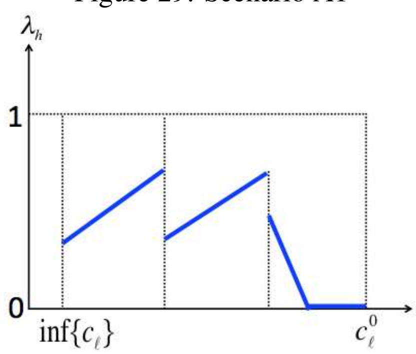

Figure 31: Scenario C1

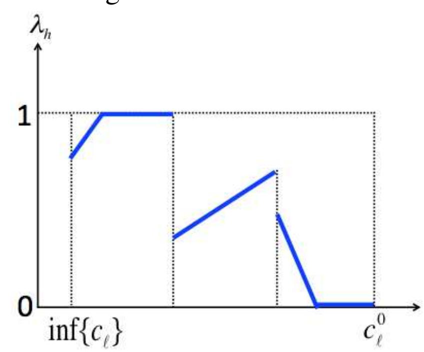

Figure 32: Scenario C2

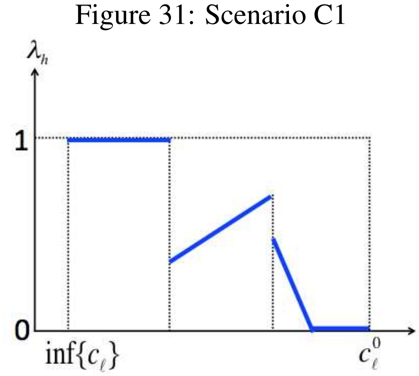

Figure 33: Scenario C3

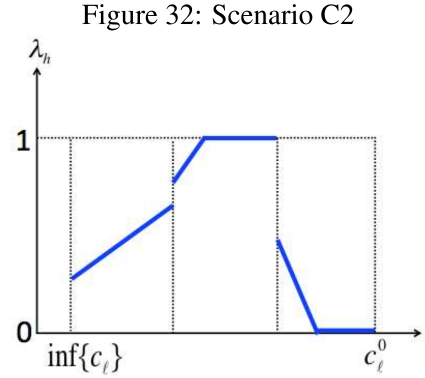

Figure 34: Scenario C4

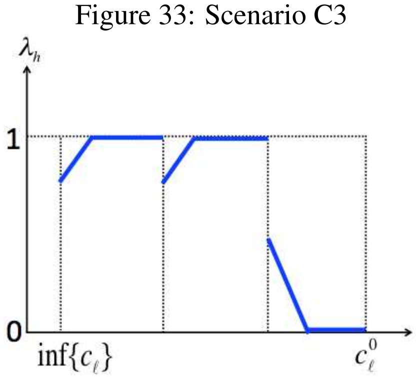

Figure 35: Scenario C5

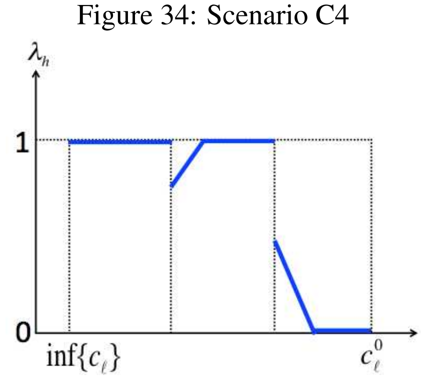

Figure 36: Scenario C6