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Abstract. When procuring multiple products from competing firms, a buyer may choose separate purchase, pure bundling, or mixed bundling. We show that pure bundling will generate higher buyer surplus than both separate purchase and mixed bundling, provided that trade for each good is likely to be efficient. Pure bundling is superior because it intensifies the competition between firms by reducing their cost asymmetry. Mixed bundling is inferior because it allows firms to coordinate to the high prices associated with separate purchase. (Pure) bundling is more likely to be selected as a procurement strategy when: (i) the products' values are higher relative to their possible costs, (ii) costs for different goods are more negatively or less positively dependent, or (iii) the cost distribution of each product is more dispersed.

JEL: D21, D44, L24

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1. Introduction

When purchasing multiple products from competing suppliers, what procurement strategy will maximize a buyer's (expected) surplus? This question arises in many economic situations. For example, the buyer could be an individual who desires to have a kitchen and a bathroom renovated, a company that desires to purchase some computers and printers, an airport in need of an elevator and an escalator, or a government agency procuring a group of military equipment. The buyer may solicit competitive bids for each product separately (separate purchase), procure the multiple products as a package through competitive bidding (pure bundling), or solicit competitive bids simultaneously for individual products and for the package (mixed bundling). This paper examines the buyer's choice among these alternative procurement strategies.

Commodity bundling has been studied extensively from the perspective of a multiproduct monopoly seller. Because consumer values are less dispersed for a bundle than for individual goods, pure bundling potentially allows the seller to extract more consumer surplus than separate selling (e.g., Stigler, 1963; Schmalensee, 1984; Fang and Norman, 2006). Mixed bundling, the practice of selling the products both separately and as a bundle, further endows the seller with the ability to price discriminate; consequently, it always weakly—and sometimes strictly—dominates pure bundling and separate selling (e.g., Adam and Yellen, 1976; Long, 1984; McAfee, McMillian, and Whinston, 1989; Chu, Leslie and Sorensen, 2011; Armstrong, 2013; Chen and Riordan, 2013). Surprisingly, there has been no parallel analysis on the desirability of bundling from a monopsony buyer's perspective. Such an analysis could naturally connect the economics of bundled sales and bundled purchases, despite the apparent difference that in the aforementioned literature on bundled sales there is usually no competition among buyers,[1] whereas in procurement the buyer can typically solicit competitive bids from potential suppliers.[2] As we shall demonstrate, while sharing some common intuition, bundling achieves superiority through different mechanisms in these two different environments, and

[1]A seller with multiple objects may also auction the goods to competing buyers, possibly with bundling. Jehiel, Meyer-ter-Vehn, and Moldovanu (2007) shows that the seller will receive higher revenues from mixed bundling auctions than pure bundling and separate auctions.

[2]There have been studies of procurement that involve bundling, such as the analysis of split-award auctions (e.g., Anton and Yao, 1989; 1992; Gong, Li and McAfee, 2012), and the comparison of separate tasks and bundled tasks in a sequential procurement setting (Li et al., forthcoming). But the models and the interests in these studies are very different from those in the aforementioned commodity bundling literature.

stronger results can be obtained, under general conditions, in the procurement context. Strikingly, unlike for a monopoly seller, for a monopsony buyer mixed bundling is strictly dominated by pure bundling whenever separate purchase is.

We consider a setting where a buyer has unit demand for each of two products, for which her values are known to be v_x and v_y, respectively. Both products can be produced by two competing firms. Each firm's production costs for the two goods are random draws from some joint probability distribution on support $[0, \bar{c}]^2$. We allow the two costs to have any dependence relations except perfect positive dependence, and the joint cost distribution function can take general forms. Firms know the cost realizations but the buyer does not. As in the literature on bundled sales, we assume that the cost of producing two products jointly is equal to the sum of their individual costs, so that there is no complementarity or economies of scope. For ease of exposition, our main model will compare two procurement strategies: separate purchase vs. pure bundling. The model is then extended to include the analysis of mixed bundling. When it causes no confusion, we shall refer to pure bundling simply as bundling.

Our analysis of the main model starts with the base case where the buyer's value for each good is above its highest possible cost (i.e., $v_x, v_y \geq \bar{c}$), so that trade is always efficient. We show in this case that buyer surplus is always higher under bundling than under separate purchase. The reason for this result is closely related to the "dispersion reduction" idea under bundled sales, but due to a different mechanism that we shall term as the "competition effect": firms' costs are less dispersed for the package than for individual products, motivating them to bid more aggressively for the two goods under bundling than under separate purchase, resulting in lower prices.[3] Notice that this result, invariant with the functional form of the cost distribution, is stronger than its counterpart under bundled sales, where (pure) bundling is sometimes less profitable than separate sales even when trade is always efficient (e.g., Fang and Norman, 2006).

When trade for a good may not be efficient (i.e., at least one of v_x and v_y is lower than $\bar{c}$), bundling can reduce buyer surplus for two possible reasons that we shall jointly term as

[3]This is closely related to Dana (2012), where heterogenous consumers with different preferences towards competing firms may form a buyer group that is indifferent between the firms, which eliminates product differentiation and reduces equilibrium prices. By comparison, our model has no consumer heterogeneity, and bundling boosts suppliers' competition by reducing their cost asymmetry for the two goods.

the "adverse tying" effect. First, both firms' costs to supply the bundle may exceed the total value of the bundle, even when their costs for one of the goods are lower than its value. This is analogous to the inefficient tying that may occur under bundled sales. Second, both goods will be purchased, but their total price could be lower under separate purchase, because when the value of a good is lower than the maximum of two firms' costs, it can force a lower price bid on this good under separate purchase but not under bundling. This second adverse tying effect is more subtle and arises for bundled procurement but not for bundled sales. We show that the competition effect dominates the adverse tying effect, so that buyer surplus is higher under bundling than under separate purchase, if trade for each good is likely to be efficient (i.e., if each product value is likely to be higher than its cost); and the reverse is true if at least one of the values is sufficiently low.

We further investigate how the bundling advantage, which we define as the change in buyer surplus from separate to bundled purchase (and can thus be negative), may vary with product values and properties of the cost distribution. The literature on bundled sales has focused on the question of when bundling is more profitable than separate sales, leaving it largely unanswered how large the bundling advantage is under general conditions. For bundled procurement, the different structure of the problem enables us to gain more insight on this issue under general cost distributions.

When a product value increases, buyer surplus under both separate and bundled purchases will become higher, and hence the impact of the value increase on the bundling advantage is *a priori* unclear. We show that when $v_x + v_y < \bar{c}$, the bundling advantage *decreases* in the value of one product if the other value is sufficiently low. This is because in this situation there is likely to be adverse tying, the potential loss of which becomes higher as the product value increases. On the other hand, when $v_x + v_y \geq \bar{c}$, the bundling advantage *increases* in the value of one product if the other value is relatively high, because in this case as the value increases the competition effect of bundling becomes more pronounced while the adverse tying effect is either reduced or absent. (When $v_x, v_y \geq \bar{c}$, the bundling advantage is not affected by a marginal increase in either of the product values.)

On the cost distribution, we consider its properties in two different dimensions: the dependence relations between the two costs and the variance of each cost. For general cost distributions satisfying certain conditions, we prove that the buyer's advantage from bundled purchase is higher when the costs for two goods are more negatively or less positively dependent.[4] The effect of the cost variance is more subtle, because a higher variance of one cost will cause more dispersion in the sum of the costs for two goods, and hence soften price competition under both separate and bundled purchases. For classes of joint cost distributions that are formed by the Farlie-Gumbel-Morgenstern (FGM) copula, we find that higher cost variance for each product increases the bundling advantage.

We finally extend our analysis to include the procurement strategy of mixed bundling, where the buyer solicits supply prices both for individual products and for the two goods as a package. In contrast to bundled sales, for bundled purchase we find that mixed bundling can actually generate lower buyer surplus than pure bundling. This is because the procurement prices are determined through competitive bidding by the sellers. When sellers are invited to bid on the price for the two goods as a package, the option for them to also bid on the prices of individual goods changes their strategic interactions, enabling them to coordinate to higher bids for the bundle so that the separate purchase equilibrium becomes an equilibrium outcome, which would make the buyer worse off if she prefers the (pure) bundling outcome to that of separate purchase.[5] In fact, we find that the equilibrium outcome under separate purchase can always be supported as an equilibrium outcome under mixed bundling, and it is also the sellers' Pareto-dominating outcome if there are multiple equilibria under mixed bundling. Hence, mixed bundling is equivalent to separate purchase, provided that suppliers will play their Pareto-dominating equilibrium in the presence of multiple equilibria. This justifies our focus on the comparison of separate purchase with pure bundling.

In the rest of the paper, we formulate our model in section 2. In section 3, we compare buyer surplus under bundling and separate purchase, and explore how the bundling advantage varies with product values. Section 4 investigates how the bundling advantage changes with cost dependence and variance. Mixed bundling is analyzed in section 5, and section 6 concludes.

[4]The literature on bundled sales has studied the parallel issue of how the profitability of bundling may vary with the dependence of consumer values, but primarily under specific functional forms of consumer value distributions (e.g., Schmalensee, 1984; Chen and Riordan, 2013).

[5]When competing sellers offer bundles to consumers, they may also collectively prefer pure bundling to mixed bundling, but for strategic reasons different from ours. See, for example, Chen (1997), Armstrong and Vickers (2010), and Zhou (2014).

2. The Model

A buyer demands two products, X and Y, both of which can be produced by two competing firms, $i = 1, 2$. The demanded quantity for each product is normalized to 1. A firm's production costs for X and Y are respectively c^x and c^y, which are realizations of two random variables with joint distribution $H(c^x, c^y)$ on $[0, \bar{c}]^2$, where $0 < \bar{c} \leq \infty$.[6] The marginal distributions of c^x and c^y are $F(\cdot)$ and $G(\cdot)$, respectively. We assume that a firm's cost of producing the two products together is $c^x + c^y$, which rules out complementarity or economies of scope as explanations for the potential advantage of bundled procurement. The cost realizations are independent across firms.

The buyer values X and Y at $v_x > 0$ and $v_y > 0$, both of which are known constants and will be treated as parameters of the model. The buyer's objective is to maximize her expected surplus, by choosing from the following two procurement strategies:

- Separate purchase (S). The buyer solicits simultaneous bids of separate supply prices for X and for Y.
- Bundling (B). The buyer solicits bids of prices to supply X and Y as a package.[7]

Bids submitted by the firms are the prices at which they are willing to supply the products. The buyer commits to choose the lower bid, and when two bids are the same, the bidder with a lower cost is assumed to be selected (which can be justified by assuming that the lower-cost supplier bids slightly lower), and each bidder has an equal chance to be selected if they have the same cost.

The procurement game proceeds as follows: First, the buyer announces and commits to the procurement strategy. Second, firm i learns its cost realization, $(c_i^x,\ c_i^y)$, for $i = 1, 2$. Each firm's cost realization is known to both firms, but is unknown to the buyer.[8] Third, firms

[6] We allow $H(c^x, c^y)$ to be discontinuous, and thus the assumption that the two costs have common support $[0,\ \bar{c}]$ is made without loss of generality.

[7] In section 5, we further allow the buyer to choose the procurement strategy of Mixed bundling (M), where the buyer solicits supply prices both for individual products and for X and Y as a package.

[8] If v_x and v_y are above $\bar{c}$, our results would be the same if we alternatively assume that a firm's cost realization is known only to itself. Then, the buyer could run the bidding as a second-price auction: the firm that bids a lower price to supply a good or a package will win the bidding but be paid the highest bid price, instead of its own bid; and bidding one's true cost is a weakly dominant strategy for the firms.

simultaneously submit bids corresponding to the procurement strategy chosen by the buyer. Fourth, the buyer selects the winning bidder(s), and payments are made in exchange for the delivery of goods. Following the convention in the literature (e.g., Anton and Yao, 1989), we assume that a supplier will not bid a price that is below its production cost for the item.

We note that since $(c_1^x,\ c_1^y)$ and $(c_2^x,\ c_2^y)$ are independent and random draws, they follow joint distribution $H\left(c_1^x, c_1^y\right) H\left(c_2^x, c_2^y\right)$.

The relationship between the costs will play an important role in our analysis. Following Nelsen (2006), we have:

Definition 1 *(i) c^x and c^y have perfect positive dependence when, for any two random draws $\left(c_1^x, c_1^y\right)$ and $\left(c_2^x, c_2^y\right)$, $c_1^x \geq c_2^x$ if and only if $c_1^y \geq c_2^y$. (ii) c^x and c^y are not perfectly positively dependent if, for any two random draws $\left(c_1^x, c_1^y\right)$ and $\left(c_2^x, c_2^y\right)$, there is a positive probability that $c_1^x > c_2^x$ but $c_1^y < c_2^y$.*

Unless otherwise stated, we assume that c^x and c^y are not perfectly positively dependent.[9]

3. Analysis: Separate Purchase vs. Bundling

This section compares buyer surplus under separate purchase (S) and bundling (B). In many situations, the buyer may consider both goods as "must–haves", so that $v_x \geq \bar{c}$ and $v_y \geq \bar{c}$. The analysis of this case is especially simple, and it serves as a useful benchmark. We thus start with this base case, followed by a more general analysis with any product values.

3.1 Base Case: $v_x, v_y \geq \bar{c}$

First, under S, firms simultaneously submit bids for X and for Y. Given a pair of realized costs $\left(c_1^x, c_1^y\right)$ and $\left(c_2^x, c_2^y\right)$, the standard logic of Bertrand competition implies that the equilibrium bids for good X and good Y by firm i, $i = 1, 2$, will be:

$$b_i^x = \max\left\{c_1^x, c_2^x\right\}, \quad b_i^y = \max\left\{c_1^y, c_2^y\right\}. \tag{1}$$

[9]If c^x and c^y were perfectly positively dependent, bundling would have no strategic advantage relative to separate purchase.

Thus, the equilibrium total price for the two products under the specific cost realization is

$$t^S = \max\left\{c_1^x, c_2^x\right\} + \max\left\{c_1^y, c_2^y\right\}. \tag{2}$$

The expected total procurement price for the buyer is:

$$T^S = E[t^S] = \int_{[0,\bar{c}]^2} \left[\max\left\{c_1^x, c_2^x\right\} + \max\left\{c_1^y, c_2^y\right\}\right] d\left[H\left(c_1^x, c_1^y\right) H\left(c_2^x, c_2^y\right)\right]. \tag{3}$$

Next, under B, the suppliers simultaneously submit bids for X and Y as a package. Given a pair of realized costs $\left(c_1^x, c_1^y\right)$ and $\left(c_2^x, c_2^y\right)$, the standard logic of Bertrand competition implies that the equilibrium bid for the package by firm i, $i = 1, 2$, will be:

$$b_i^{xy} = \max\left\{c_1^x + c_1^y, c_2^x + c_2^y\right\}. \tag{4}$$

Hence, the equilibrium price for the package given the cost realization is

$$t^B = \max\left\{c_1^x + c_2^y, c_2^x + c_2^y\right\}. \tag{5}$$

The expected equilibrium price for the package is

$$T^B = E[t^B] = \int_{[0,\bar{c}]^2} \max\left\{c_1^x + c_2^y, c_2^x + c_2^y\right\} d\left[H\left(c_1^x, c_1^y\right) H\left(c_2^x, c_2^y\right)\right]. \tag{6}$$

Denoting the buyer's expected surplus under S and under B by W^S and W^B, respectively. Since $v_x \geq \bar{c}$ and $v_y \geq \bar{c}$, we have

$$W^B - W^S = \left[v_x + v_y - T^B\right] - \left[v_x + v_y - T^S\right] = T^S - T^B. \tag{7}$$

Hence, the buyer's (expected) surplus is higher under B than under S if and only if the (expected) total procurement price for the two goods is lower under B. We have:

Proposition 1 *Assume $v_x \geq \bar{c}$ and $v_y \geq \bar{c}$. Then $W^B > W^S$. That is, the buyer achieves higher expected surplus from bundling than from separate purchase.*

Proof. From (7), it suffices to show $T^S > T^B$. For every pair of cost realizations (c_1^x, c_1^y) and (c_2^x, c_2^y), we have

$$t^S = \max\{c_1^x, c_2^x\} + \max\{c_1^y, c_2^y\}$$
$$\begin{cases} > \max\{c_1^x + c_1^y, c_2^x + c_2^y\} = t^B & if \quad c_1^x \geq c_2^x \text{ but } c_1^y < c_2^y \text{ or if } c_1^x < c_2^x \text{ but } c_1^y \geq c_2^y \\ = \max\{c_1^x + c_1^y, c_2^x + c_2^y\} = t^B & otherwise \end{cases}.$$

Thus, $t^S \geq t^B$, and

$$\begin{aligned} & T^S - T^B \\ = & \int_{\substack{c_1^x \geq c_2^x, c_1^y < c_2^y; \\ c_1^x < c_2^x, c_1^y \geq c_2^y}} \left[\max\left\{c_1^x, c_2^x\right\} + \max\left\{c_1^y, c_2^y\right\} - \max\left\{c_1^x + c_1^y, c_2^x + c_2^y\right\}\right] d\left[H\left(c_1^x, c_1^y\right) H\left(c_2^x, c_2^y\right)\right] \\ > & \ 0, \end{aligned}$$

where the inequality holds because, by assumption, c^x and c^y are not perfectly positively dependent. ■

Bundling reduces cost dispersion, making the firms' costs for the two goods less asymmetric. As a result, firms compete more aggressively to supply the two goods under bundling than under separate purchase. When $v_x, v_y \geq \bar{c}$, the realized costs will always be lower than product values, and hence the intensified competition under bundling will lead to lower expected total price for the two goods, and hence also to higher buyer surplus.

If v_x or v_y is lower than $\bar{c}$, it's possible that the price for two goods is lower under separate purchase, because a product value lower than the realized cost can force the firm to lower its price under separate purchase. For example, suppose $v_x = 8$, $v_y = 13$, $c_1^x = 8 < c_2^x = 12$, and $c_1^y = 12 > c_2^y = 10$. Then the equilibrium price for two goods is 21 under bundling but is 20 under separate purchase. The next subsection analyses this general case.

3.2 General Analysis with Any Product Values

We now consider the general case where v_x and v_y may be lower than $\bar{c}$, to compare broadly the buyer's surplus under S and B, and to explore how the buyer's potential advantage from bundling may vary with her valuations for the products. For convenience, this section assumes the costs admit joint density $h(c^x, c^y) > 0$ on $[0, \bar{c}]^2$. Define

$$z_x \equiv \max\{c_1^x, c_2^x\}; \qquad z_y \equiv \max\{c_1^y, c_2^y\}. \tag{8}$$

Then, (z_x, z_y) is the first order statistic of the sample $\{(c_1^x, c_1^y), (c_2^x, c_2^y)\}$, and the joint distribution of (z_x, z_y) is

$$\begin{aligned} H_{(1)}(z_x, z_y) &= \Pr(c_1^x \leq z_x, c_1^y \leq z_y) \Pr(c_2^x \leq z_x, c_2^y \leq z_y) \\ &= [H(z_x, z_y)]^2, \text{ for } (z_x, z_y) \in [0, \bar{c}]. \end{aligned}$$

The marginal distributions of z_x and z_y are

$$F_{(1)}(z_x) = [F(z_x)]^2, \qquad G_{(1)}(z_y) = [G(z_y)]^2. \tag{9}$$

Define $\sigma \equiv c^x + c^y$, which has cdf

$$L(\sigma) = \Pr(c^x + c^y \leq \sigma) = \int_0^\sigma \int_0^{\bar{c}} h(c^x, t - c^x)\, dc^x dt \tag{10}$$

on $[0, 2\bar{c}]$. Then,

$$z \equiv \max\{c_1^x + c_1^y, c_2^x + c_2^y\} \tag{11}$$

has cdf $L^2(z)$ on $[0, 2\bar{c}]$.

Under S, given a pair of realized costs (c_1^x, c_1^y) and (c_2^x, c_2^y), for $i = 1, 2$; $j \neq i$, since v_x and v_y may be lower than c_i now, the equilibrium bids of firm i become:

$$b_i^x = \max\left\{c_i^x, \min\left\{v_x, c_j^x\right\}\right\}, \quad b_i^y = \max\left\{c_i^y, \min\left\{v_y, c_j^y\right\}\right\}. \tag{12}$$

It follows that the equilibrium prices for X and Y are respectively:

$$b^x = \min\{\max\{v_x, \min\{c_1^x, c_2^x\}\},\ z_x\}, \quad b^y = \min\{\max\{v_y, \min\{c_1^y, c_2^y\}\}, z_y\}, \tag{13}$$

and hence we have the result below:

Lemma 1 (Separate Purchase Outcome.) *For cost realizations* (c_1^x, c_1^y) *and* (c_2^x, c_2^y)*, the equilibrium outcome under separate purchase is: (i) if* $v_k \geq \min\{c_1^k, c_2^k\}$ *for* $k = x, y$*, the firm with the lower production cost for* k *supplies product* k *at price equal to* $\min\{v_k, z_k\}$*; (ii) if* $v_k < \min\{c_1^k, c_2^k\}$ *for* $k = x, y$*, the buyer does not purchase product* k*.*

If the realized $z_x \leq v_x$, then z_x is the equilibrium bid price for supplying good X, in which case the buyer's surplus from purchasing X is $v_x - z_x$. If $z_x > v_x$, then the equilibrium bid price will be $\max\{v_x, \min\{c_1^x, c_2^x\}\}$, in which case the buyer's surplus from purchasing X is zero. Therefore, the buyer can have a positive surplus from purchasing X only if $v_x > z_x$. Similarly, the buyer can have a positive surplus from purchasing Y only if $v_y > z_y$. It follows that the buyer's expected surplus under S is

$$\begin{aligned} W^S &= \int_0^{v_x} (v_x - z_x)\, dF^2(z_x) + \int_0^{v_y} (v_y - z_y)\, dG^2(z_y) \\ &= \int_0^{v_x} F^2(z_x)\, dz_x + \int_0^{v_y} G^2(z_y)\, dz_y. \end{aligned} \tag{14}$$

Under B, given a pair of realized costs (c_1^x, c_1^y) and (c_2^x, c_2^y), for $i = 1, 2$; $j \neq i$, the equilibrium bid of firm i now becomes:

$$b_i^{xy} = \max\left\{c_i^x + c_i^y, \min\left\{v_x + v_y, c_j^x + c_j^y\right\}\right\}. \tag{15}$$

We thus immediately have the following:

Lemma 2 (Bundling Outcome) *For cost realizations* (c_1^x, c_1^y) *and* (c_2^x, c_2^y)*, the equilibrium outcome under bundling is: (i) if* $v_x + v_y \geq \min\{c_1^x + c_1^y,\ c_2^x + c_2^y\}$*, the firm with the lower production cost for the package supplies both products at price equal to* $\min\{v_x + v_y,\ z\}$*; (ii) if* $v_x + v_y < \min\{c_1^x + c_1^y,\ c_2^x + c_2^y\}$*, the buyer does not purchase the package.*

By the same logic as under S, the buyer's expected surplus under B is

$$W^B = \int_0^{v_x + v_y} (v_x + v_y - z)\, dL^2(z) = \int_0^{v_x + v_y} L^2(z)\, dz. \tag{16}$$

Therefore:

$$\frac{\partial W^S}{\partial v_x} = \begin{cases} F^2(v_x) & if \quad v_x \leq \bar{c} \\ 1 & if \quad v_x > \bar{c} \end{cases} ; \ \frac{\partial W^S}{\partial v_y} = \begin{cases} G^2(v_y) & if \quad v_y \leq \bar{c} \\ 1 & if \quad v_y > \bar{c} \end{cases} , \tag{17}$$

$$\frac{\partial W^B}{\partial v_x} = \frac{\partial W^B}{\partial v_x} = \begin{cases} L^2(v_x + v_y) & if \quad v_x + v_y \leq 2\bar{c} \\ 1 & if \quad v_x + v_y > 2\bar{c} \end{cases} . \tag{18}$$

Define the bundling advantage as the change in the buyer's expected surplus from S to B:

$$\Delta W = W^B - W^S. \tag{19}$$

We next state our general result concerning when bundled procurement may yield higher or lower surplus to the buyer than separate purchase, and how the bundling advantage may vary with her valuations for the products.

Theorem 1 *(i) There exist* μ_1 *and* μ_2*, with* $0 < \mu_1 \leq \mu_2 < \bar{c}$*, such that* $W^B > W^S$ *if* $v_x, v_y > \mu_2$ *but* $W^B < W^S$ *if* $v_x < \mu_1$ *or* $v_y < \mu_1$*. (ii) For* $k = x, y \neq l$*, when* $v_x + v_y < \bar{c}$*,* ΔW *decreases in* v_k *if* $v_l \leq \delta_1$ *for some sufficiently small* $\delta_1 > 0$*; when* $v_x + v_y \geq \bar{c}$*,* ΔW *increases in* v_k *if* $v_k < \bar{c}$ *and* $v_l \geq \delta_2$ *for some* δ_2 $(< \bar{c})$ *sufficiently close to* $\bar{c}$*.*

Part (i) of Theorem 1 states that, as long as both product values are not much below $\bar{c}$, the buyer's surplus is higher under bundling than under separate purchase; however, if at least one of the values is low enough, then the buyer is better off with separate purchase. This generalizes the result in Proposition 1, which covers the case of both product values being above $\bar{c}$ (so that the buyer always purchases both products). In that base case, the buyer always prefers B to S, because bundling reduces the cost asymmetry between sellers and intensifies their competition, which leads to lower price. When product values are smaller than $\bar{c}$, bundling no longer necessarily leads to lower prices than separate purchase, and the buyer may be better off to purchase only one product when the other product's realized cost exceeds its value. Thus, bundling involves a trade off between the gain from intensified competition and the loss from potential adverse tying. We find that the competition effect dominates when values are relatively high (but both can be lower than $\bar{c}$), while the adverse tying effect dominates when at least one value is sufficiently low.

Part (ii) of Theorem 1 shows how the bundling advantage, ΔW, may vary with product values. When $v_x + v_y < \bar{c}$, if one value is sufficiently low, then ΔW *decreases* as the other value rises, because the potential loss from adverse tying becomes more severe. When $v_x + v_y \geq \bar{c}$, ΔW *increases* in a value when the other value is relatively high, because in this case as the value increases, the adverse tying effect is either reduced or absent, whereas the competition advantage under bundling increases. Notice that when both $v_x \geq \bar{c}$ and $v_y \geq \bar{c}$, a marginal increase in v_x or v_y will have no impact on ΔW.

The proof strategy for Theorem 1 is as follows. While W^B and W^S are difficult to compare directly, it turns out that their rates of change (i.e., partial derivatives) with respect to v_x or v_y can be compared relatively easily. We thus first examine how $\Delta W \equiv W^B - W^S$ may vary with v_x and v_y. We then compare W^B and W^S by using the facts that $\Delta W|_{v_x=v_y=0} = 0$ and, from Proposition 1, $\Delta W|_{v_x,v_y \geq \bar{c}} > 0$.

Specifically, we prove Theorem 1 by establishing two claims below. Claim 1 considers the case of $v_x + v_y < \bar{c}$, where we show that ΔW decreases in v_k when v_l is sufficiently small, and hence, starting from $v_x = v_y = 0$, ΔW is initially negative. The case of $v_x + v_y \geq \bar{c}$ is considered in Claim 2.

Claim 1 *Suppose that $v_x + v_y < \bar{c}$. Then, for $k = x, y \neq l$ and for any given v_k, $\frac{\partial \Delta W}{\partial v_k} < 0$ if $v_l \leq \delta_1$ for some sufficiently small $\delta_1 > 0$. Furthermore, there exists some $\mu_1 \in (0, \bar{c})$ such that $\Delta W < 0$ when $v_x, v_y \leq \mu_1$.*

Proof. When $v_x + v_y < \bar{c}$,

$$L(v_x + v_y) = \Pr(c^x + c^y \leq v_x + v_y) = F(v_x) - \int_0^{v_x} \int_{v_x+v_y-c^x}^{\bar{c}} dH(c^x, c^y) + \int_0^{v_y} \int_{v_x}^{v_x+v_y-c^y} dH(c^x, c^y).$$

Thus, from (17) and (18),

$$\frac{\partial \Delta W}{\partial v_x} = L^2(v_x + v_y) - F^2(v_x) = [L(v_x + v_y) + F(v_x)][L(v_x + v_y) - F(v_x)].$$

For any given $v_x < \bar{c}$, since

$$\begin{aligned} & L(v_x + v_y) - F(v_x) \\ = & \int_0^{v_y} \int_{v_x}^{v_x+v_y-c^y} h(c^x, c^y)\, dc^x dc^y - \int_0^{v_x} \int_{v_x+v_y-c^x}^{\bar{c}} h(c^x, c^y)\, dc^x dc^y \\ \to & -\int_0^{v_x} \int_{v_x-c^x}^{\bar{c}} h(c^x, c^y)\, dc^x dc^y < 0 \text{ as } v_y \to 0, \end{aligned}$$

$\frac{\partial \Delta W}{\partial v_x} < 0$ when v_y is sufficiently small.

Similarly, for any given $v_y < \bar{c}$, when v_x is sufficiently small,

$$\frac{\partial \Delta W}{\partial v_y} = [L(v_x + v_y) + G(v_y)]\,[L(v_x + v_y) - G(v_y)] < 0.$$

Thus, for $k = x, y \neq l$ and for any given $v_k < \bar{c}$, there exists some small $\delta_1 > 0$ such that $\frac{\partial \Delta W}{\partial v_k} < 0$ if $v_l \leq \delta_1$. It follows that there is some small $\varepsilon > 0$ such that when $v_x, v_y \leq \varepsilon$, $\frac{\partial \Delta W}{\partial v_x} < 0$ and $\frac{\partial \Delta W}{\partial v_y} < 0$. Furthermore, since $\Delta W = 0$ if $v_x = v_y = 0$, for $\Delta v_x \leq \varepsilon$ and $\Delta v_y \leq \varepsilon$:

$$\Delta W = \frac{\partial \Delta W}{\partial v_x} \Delta v_x + \frac{\partial \Delta W}{\partial v_y} \Delta v_y < 0.$$

Therefore, there exists some μ_1, with $\bar{c} > \mu_1 > \delta_1 > 0$, such that $\Delta W = W^B - W^S < 0$ when $v_x, v_y \leq \mu_1$. ■

Next, suppose that $v_x + v_y \geq \bar{c}$. We show that if v_l is not much below $\bar{c}$, then $\partial \Delta W / \partial v_k > 0$ for any $v_k < \bar{c}$. Furthermore, since $\Delta W > 0$ when $v_x, v_y \geq \bar{c}$ and since ΔW is continuos in v_x and v_y, we have $\Delta W > 0$ when $v_x, v_y \geq \mu_2$ for some $\mu_2 < \bar{c}$.

Claim 2 *Suppose that $v_x + v_y \geq \bar{c}$. For $k = x, y \neq l$ and for any given $v_k < \bar{c}$, $\frac{\partial \Delta W}{\partial v_k} > 0$ if $v_l \geq \delta_2$ for some δ_2 $(< \bar{c})$ sufficiently close to $\bar{c}$. Furthermore, there exists some $\mu_2 \in [\mu_1, \bar{c})$ such that $\Delta W > 0$ when $v_x, v_y > \mu_2$.*

Proof. First, at any given $v_x < \bar{c}$, if $v_x + v_y \geq 2\bar{c}$, then

$$\frac{\partial \Delta W}{\partial v_x} = 1 - F^2(v_x) > 0.$$

Next, suppose $v_x + v_y < 2\bar{c}$. Then

$$L(v_x+v_y) = F(v_x) - \int_{v_x+\min\{v_y,\bar{c}\}-\bar{c}}^{v_x} \int_{v_x+v_y-c^x}^{\bar{c}} h(c^x,c^y)\,dc^y dc^x + \int_{v_x}^{\bar{c}} \int_0^{v_x+v_y-c_x} h(c^x,c^y)\,dc^y dc^x.$$

Thus, if $v_y \geq \bar{c}$ or if $v_y < \bar{c}$ but $v_y \to \bar{c}$,

$$L(v_x+v_y) - F(v_x) = \int_{v_x}^{\bar{c}} \int_0^{v_x+v_y-c_x} h(c^x,c^y)\,dc^y dc^x - \int_{v_x+\min\{v_y,\bar{c}\}-\bar{c}}^{v_x} \int_{v_x+v_y-c^x}^{\bar{c}} h(c^x,c^y)\,dc^y dc^x > 0.$$

Hence, at any given $v_x < \bar{c}$, if $v_y \geq \delta_2$ for some $\delta_2\,(< \bar{c})$ sufficiently close to $\bar{c}$,

$$\frac{\partial \Delta W}{\partial v_x} = [L(v_x+v_y) + F(v_x)]\,[L(v_x+v_y) - F(v_x)] > 0.$$

Similarly, for any given $v_y < \bar{c}$, if $v_x \geq \delta_2$ for some $\delta_2\,(< \bar{c})$ sufficiently close to $\bar{c}$,

$$\frac{\partial \Delta W}{\partial v_y} = [L(v_x+v_y) + G(v_y)]\,[L(v_x+v_y) - G(v_y)] > 0.$$

Finally, from Proposition 1, when $v_x, v_y \geq \bar{c}$, ΔW is strictly positive for given $H(\cdot,\cdot)$. Therefore, since ΔW is continuous in v_x and v_y, there exists some $\mu_2 \in [\mu_1, \bar{c})$ such that $\Delta W > 0$ when $v_x, v_y > \mu_2$. ■

We note that Theorem 1 follows immediately from Claim 1 and Claim 2.

From Theorem 1, the bundling advantage (ΔW) is negative when product values are below μ_1 and positive when product values are above μ_2. Furthermore, there will be some $\delta_1 \leq \delta_2$ such that for $k = x, y$, $\frac{\partial \Delta W}{\partial v_k} < 0$ when $v_x, v_y < \delta_1$ and $\frac{\partial \Delta W}{\partial v_k} > 0$ when $v_x, v_y > \delta_2$. When $v_x = v_y \equiv v$, it is possible that $\mu_1 = \mu_2$ and $\delta_1 = \delta_2$ so that ΔW first monotonically decreases and then monotonically increases in v (until $v = \bar{c}$), as we illustrate in the two examples below where the two costs are independently distributed with the same marginal distribution.

Example 1 *Suppose that* $h(c^x,c^y) = f(c^x)\,g(c^y) = \frac{1}{a^2}$ *for* $(c^x,c^y) \in [0,a]^2$, *and* $v_x = v_y \equiv v \leq a$. *Then,*

$$L(\sigma) = \begin{cases} \int_0^{\sigma}\left(\int_0^{\sigma-x}\frac{1}{a^2}dy\right)dx = \frac{1}{2}\frac{\sigma^2}{a^2} & if \quad 0 \leq \sigma \leq a \\ \int_0^{\sigma-a}\int_0^a \frac{1}{a^2}dydx + \int_{\sigma-a}^{a}\left(\int_0^{\sigma-x}\frac{1}{a^2}dy\right)dx = \frac{1}{2}\frac{4a\sigma-2a^2-\sigma^2}{a^2} & if \quad a \leq \sigma \leq 2a \end{cases}.$$

$$W^S = 2\int_0^v \frac{z_x^2}{a^2} dz_x = \frac{2}{3}\frac{v^3}{a^2},$$

$$\begin{aligned} W^B &= \int_0^{2v} (2v - z)\, dL^2(z) \\ &= \begin{cases} \int_0^{2v} (2v-z) \frac{z^3}{a^4} dz & if \quad 0 \le 2v \le a \\ \int_0^a (2v-z) \frac{z^3}{a^4} dz + \int_a^{2v} (2v-z) \left(-(2a-z)\frac{-4az+2a^2+z^2}{a^4}\right) dz & if \quad a \le 2v \le 2a \end{cases} \\ &= \begin{cases} \frac{8}{5}\frac{v^5}{a^4} & if \quad 0 \le 2v \le a \\ -\frac{1}{30}\frac{240av^4 - 60a^4 v - 400a^2v^3 + 240a^3v^2 + 5a^5 - 48v^5}{a^4} & if \quad a \le 2v \le 2a \end{cases}. \end{aligned}$$

Thus

$$\Delta W = W^B - W^S = \begin{cases} -\frac{2}{15}v^3 \frac{5a^2 - 12v^2}{a^4} & if \quad 0 \le 2v \le a \\ -\frac{1}{30}\frac{240av^4 - 60a^4v - 380a^2v^3 + 240a^3v^2 + 5a^5 - 48v^5}{a^4} & if \quad a \le 2v \le 2a \end{cases},$$

where $-\frac{2}{15}v^3 \frac{5a^2-12v^2}{a^4} < 0$ *for* $v \le a/2$; *and for* $v \ge a/2$, *it is apparent from numerical analysis that*

$$-\frac{1}{30}\frac{240av^4 - 60a^4v - 380a^2v^3 + 240a^3v^2 + 5a^5 - 48v^5}{a^4} \lesseqgtr 0 \text{ if } v \lesseqgtr 0.6828a.$$

Therefore, $\Delta W \lesseqgtr 0$ *when* $v \lesseqgtr 0.6828a = \mu_1 = \mu_2$. *We also note that* $\frac{\partial \Delta W}{\partial v} \lesseqgtr 0$ *when* $v \lesseqgtr a/2 = \delta_1 = \delta_2$, *because*

$$\frac{\partial \Delta W}{\partial v} = \begin{cases} -2v^2(a-2v)\frac{a+2v}{a^4} < 0 & if \quad 0 \le 2v \le a \\ 2(a-v)(a-2v)\frac{-5av+a^2+2v^2}{a^4} > 0 & if \quad a \le 2v \le 2a \end{cases}.$$

In Example 1, since $\bar{c} = a$, $\delta_1 = \delta_2 = 0.5\bar{c}$, $\mu_1 = \mu_2 = 0.6828\bar{c}$, and ΔW first monotonically decreases and then monotonically increases in v until $v = \bar{c}$.

Example 2 *Suppose that* $F(c) = G(c) = 1 - e^{-\lambda c}$ *and* $h(c^x, c^y) = \left(\lambda e^{-\lambda c^x}\right)\left(\lambda e^{-\lambda c^y}\right)$, *for* $(c^x, c^y) \in [0,\infty)^2$, $\lambda > 0$, *and* $v_x, v_y > 0$. *Then,*

$$W^S = 2\int_0^v \left(1 - e^{-\lambda z_x}\right)^2 dz_x = \frac{1}{\lambda}\left[-e^{2(-\lambda v)} + 4e^{-\lambda v} + 2\lambda v - 3\right],$$

$$l(\sigma) = \int_0^\sigma f(t)g(\sigma - t)dt = \lambda\left(\sigma\lambda e^{-\sigma\lambda}\right),$$
$$L(\sigma;\theta) = \int_0^\sigma \lambda\left(t\lambda e^{-t\lambda}\right)dt = 1 - e^{-\sigma\lambda} - \sigma\lambda e^{-\sigma\lambda},$$

$$\begin{aligned} W^B &= \int_0^{2v}\left(1 - e^{-z\lambda} - z\lambda e^{-z\lambda}\right)^2 dz \\ &= \frac{16e^{-2v\lambda} - 5e^{-4v\lambda} + 8v\lambda - 8v^2\lambda^2 e^{-4v\lambda} + 16v\lambda e^{-2v\lambda} - 12v\lambda e^{-4v\lambda} - 11}{4\lambda}. \end{aligned}$$

$$\Delta W = \frac{-16e^{-v\lambda} + 20e^{-2v\lambda} - 5e^{-4v\lambda} - 8v^2\lambda^2 e^{-4v\lambda} + 16v\lambda e^{-2v\lambda} - 12v\lambda e^{-4v\lambda} + 1}{4\lambda} \lesseqgtr 0 \text{ if } v \lesseqgtr \frac{2.3631}{\lambda}.$$

$$\frac{\partial(\Delta W)}{\partial v} = 2e^{-v\lambda}\left(e^{-v\lambda} + e^{2(-v\lambda)} + 2v\lambda e^{2(-v\lambda)} - 2\right)\left(e^{-v\lambda} + 2v\lambda e^{-v\lambda} - 1\right) \lesseqgtr 0 \text{ if } v \lesseqgtr \frac{1.2564}{\lambda}.$$

In Example 2, the expected cost for each product is $E(c^x) = E(c^y) = \frac{1}{\lambda}$, $\delta_1 = \delta_2 = \frac{1.2564}{\lambda}$, and $\mu_1 = \mu_2 = \frac{2.3631}{\lambda}$. As v rises, ΔW monotonically decreases for $v < \delta_1 = \delta_2$ and then monotonically increases, and $\Delta W < 0$ for $v < \mu_1 = \mu_2$ but $\Delta W > 0$ for $v > \frac{2.3631}{\lambda}$.

We summarize how the bundling advantage ΔW varies with product value when the buyer has the same value v for the two goods in the following:

Corollary 2 *Suppose the buyer has the same value v for the two goods. Then ΔW initially decreases and eventually increases in v until it becomes independent of v for $v > \bar{c}$.*

4. Cost Distribution and Bundling Advantage

In this section, we turn our attention to the role of costs. We are interested in how two key properties of the cost distribution, the variance of c^x or c^y and their dependence relation, affect the buyer's advantage from bundled procurement over separate purchase. To isolate the cost effect, in this section we shall assume that $v_x \geq \bar{c}$ and $v_y \geq \bar{c}$. The comparison of W^B and W^S will then be determined by the comparison of the expected procurement prices, T^B and T^S. Thus, in this section we shall write $\Delta W = W^B - W^S = T^S - T^B$, which is positive from Proposition 1.

We first establish a general result about how cost dependence impacts ΔW under an assumption connecting cost dependence with cost dispersion. We then consider specific classes of cost distributions to illustrate the result and to also explore how cost variance impacts ΔW.

4.1 The Impact of Cost Dependence

Following the standard definition in the literature (e.g., Nelsen, 2006), the two costs are positively (quadrant) dependent if $H(c^x, c^y) > F(c^x) G(c^y)$, and negatively (quadrant) dependent if $H(c^x, c^y) < F(c^x) G(c^y)$. Roughly speaking, the costs are positively dependent if a higher (or lower) c^x is more likely to occur with a higher (or lower) c^y, whereas they are negatively dependent if a higher (or lower) c^x is more likely to occur with a lower (or higher) c^y.[10] Moreover, for two joint distributions $\tilde{H}(c^x, c^y)$ and $H(c^x, c^y)$ with common marginal distributions F and G, if $\tilde{H}(c^x, c^y) \geq H(c^x, c^y)$ for all $(c^x, c^y) \in [0, \bar{c}]^2$, then $\tilde{H}(c^x, c^y)$ is less negatively or more positively dependent than $H(c^x, c^y)$, in which case we shall simply say that $\tilde{H}(c^x, c^y)$ is more dependent than $H(c^x, c^y)$.

Recall from (3) and (6) that the expected procurement prices for the two goods under S and B are respectively

$$T^S = \int_0^{\bar{c}} z_x d\left[F^2(z_x)\right] + \int_0^{\bar{c}} z_y d\left[G^2(z_y)\right]; \qquad T^B = \int_0^{2\bar{c}} z d\left[L^2(z)\right],$$

where $z_x = \max\{c_1^x, c_2^x\}$, $z_y = \max\{c_1^y, c_2^y\}$, and $z = \max\{c_1^x + c_1^y, c_2^x + c_2^y\}$.

When c^x and c^y are more dependent, low values of c^x are more likely to occur together with low values of c^y, and high values of c^x are more likely to occur together with high values of c^y. This suggests that, as the two costs become more dependent, $L(\sigma) = \Pr(c^x + c^y \leq \sigma)$ is likely to increase below some $\sigma = \hat{\sigma} \in (0, 2\bar{c})$ but decrease above $\hat{\sigma}$. Thus, two distribution functions

$$\tilde{L}(\sigma) = \int\int_{c^x + c^y \leq \sigma} d\tilde{H}(c^x, c^y); \qquad L(\sigma) = \int\int_{c^x + c^y \leq \sigma} dH(c^x, c^y), \tag{20}$$

[10] There are plausible situations where costs for the two goods are negatively or positively dependent. For example, when a supplier faces a rigid constraint on some critical resource, devoting more of it to reducing the cost of one product is likely to raise the other product's cost, in which case the costs would be negatively dependent. On the other hand, when two products utilize some common input, their costs can be positively dependent, rising or falling together with the input price.

where $\tilde{H}(c^x, c^y)$ and $H(c^x, c^y)$ have the common marginal distributions F and G, are likely to satisfy the following assumption:

A1. $\tilde{L}(\sigma)$ is a rotation of $L(\sigma)$ with rotation point $\hat{\sigma}$, in the sense that $\tilde{L}(\sigma) \gtreqless L(\sigma)$ if $\sigma \lesseqgtr \hat{\sigma}$, if $\tilde{H}(c^x, c^y)$ is more dependent than $H(c^x, c^y)$.

As discussed in the literature (e.g., Johnson and Myatt, 2006; Chen and Zhang, 2014), when $\tilde{L}(\sigma)$ is a rotation of $L(\sigma)$, $\tilde{L}(\sigma)$ is more dispersed than $L(\sigma)$.[11] Thus, when assumption **A1** holds, the distribution of the sum of the two costs is more dispersed if they are more dependent.

Proposition 2 *Assume that* ***A1*** *holds and $\hat{\sigma}$ is sufficiently small. Then the bundling advantage, ΔW, is lower if c^x and c^y are more dependent.*

Proof. Consider any two joint distributions $\tilde{H}(c^x, c^y)$ and $H(c^x, c^y)$ under common margins F and G, with $\tilde{H}(c^x, c^y)$ being more dependent than $H(c^x, c^y)$. Denote the corresponding bundling advantages by $\Delta\tilde{W} = \tilde{T}^S - \tilde{T}^B$ and $\Delta W = T^S - T^B$. We show that $\Delta\tilde{W} - \Delta W = \left(\tilde{T}^S - T^s\right) + \left(T^B - \tilde{T}^B\right) < 0$.

Since the marginal distributions are the same under the two joint distributions, we have $\tilde{T}^S = T^S$. Therefore,

$$\begin{aligned} \Delta\tilde{W} - \Delta W &= T^B - \tilde{T}^B = \int_0^{2\bar{c}} zd\left[L^2(z) - \tilde{L}^2(z)\right] \\ &= z\left[L^2(z) - \tilde{L}^2(z)\right]\Big|_0^{2\bar{c}} - \int_0^{2\bar{c}}\left[L^2(z) - \tilde{L}^2(z)\right]dz \\ &= \int_0^{\hat{\sigma}}\left[\tilde{L}^2(z) - L^2(z)\right]dz - \int_{\hat{\sigma}}^{2\bar{c}}\left[L^2(z) - \tilde{L}^2(z)\right]dz < 0, \end{aligned}$$

because $\int_0^{\hat{\sigma}}\left[\tilde{L}^2(z) - L^2(z)\right]dz$ is close to zero when $\hat{\sigma}$ is sufficiently small and $0 < \tilde{L}(z) < L(z)$ for $\hat{\sigma} < z < 2\bar{c}$. ■

Therefore, under the conditions of Proposition 2, the bundling advantage is higher if the costs of the two products are more negatively or less positively dependent.

[11] Rotation is a less stringent way of ranking the disperson of two distributions than some other concepts such as the second-order stochastic dominance. As we shall illustrate in the next subsection, the rotation concept will enable us to compare the dispersion of $\tilde{L}(\sigma)$ and $L(\sigma)$ associted with some familiar cost distributions that cannot be ranked, for example, by stochasitic dominance.

To illustrate Proposition 2 and to further gain insights on the effects of cost variance, we next consider classes of joint cost distributions formed by the FGM copula.

4.2 Effects of Cost Dependence and Variance under FGM Copulas

A copula is a bivariate uniform distribution that "couples" arbitrary marginal distributions to form a new joint distribution. By Sklar's Theorem, it is without loss of generality to represent the joint distribution of two variables by a copula and the marginal distributions (Nelsen, 2006). Specifically, we consider classes of joint distributions formed by the FGM copula family, with the marginal distribution functions being either uniform or exponential. In addition to illustrating how the dependence of costs may matter for the bundling advantage, we are also interested in the role played by the dispersion of each cost's marginal distribution, measured by its variance. This copula approach to representing the joint cost distributions enables us to disentangle the effect of each individual cost from the effect of the dependence relation between the two costs.[12]

The joint cost distributions formed by the family of FGM copulas under marginal distribution functions F and G can be written as

$$H\left(c^x, c^y; \theta\right) = F(c^x)G(c^y)\left[1 + \theta(1 - F(c^x))(1 - G(c^y))\right], \tag{21}$$

with joint density

$$h(c^x, c^y; \theta) = f(c^x)g(c^y)\left[1 + \theta\left(2G\left(c^y\right) - 1\right)\left(2F\left(c^x\right) - 1\right)\right]. \tag{22}$$

Here, parameter $\theta \in [-1, 1]$ is a measure of the dependence relationship between c^x and c^y. The two costs are negatively dependent if $\theta < 0$, independent if $\theta = 0$, and positively dependent if $\theta > 0$. Furthermore, as θ increases, the two costs become more dependent.

Suppose first that each cost, c^x or c^y, is uniformly distributed on $[0, a]$. Then, since $\mathrm{Var}(c^x) = \mathrm{Var}(c^y) = \frac{1}{12}a^2$, a is a measure of the dispersion of the marginal distribution.

[12]For an introduction to copulas in statistic analysis, see Nelsen (2006). Copulas have been a useful tool to model consumer preferences for multiple products. See, for example, Chen and Riordan (2013) for a discussion of some recent applications. The FGM copula is frequently used in applications.

The joint density of (c^x, c^y) then becomes

$$h(c^x, c^y; \theta) = \frac{1}{a^2}\left[1 + \theta\left(\frac{2c^y}{a} - 1\right)\left(\frac{2c^x}{a} - 1\right)\right]. \tag{23}$$

With $0 \leq z_x, z_y \leq a$, we have

$$T^S = \int z_x d\left[F(z_x)\right]^2 + \int z_y d\left[G(z_y)\right]^2 = \int_0^a z_x 2\frac{z_x}{a^2} dz_x + \int_0^a z_y 2\frac{z_y}{a^2} dz_y = \frac{4}{3}a.$$

Next, the distribution of $\sigma = c^x + c^y$ is:

$$\begin{aligned} L(\sigma;\theta) &= \begin{cases} \int_0^\sigma \left(\int_0^{\sigma-x} \frac{1}{a^2}\left(1+\theta\left(\frac{2y}{a}-1\right)\left(\frac{2x}{a}-1\right)\right)dy\right)dx \\ \int_0^{\sigma-a}\int_0^a \frac{\left(1+\theta\left(\frac{2y}{a}-1\right)\left(\frac{2x}{a}-1\right)\right)}{a^2}dydx + \int_{\sigma-a}^a \left(\int_0^{\sigma-x} \frac{\left(1+\theta\left(\frac{2y}{a}-1\right)\left(\frac{2x}{a}-1\right)\right)}{a^2}dy\right)dx \end{cases} \\ &= \begin{cases} \frac{1}{6}\sigma^2 \frac{\theta(a-\sigma)(3a-\sigma)+3a^2}{a^4} & if \quad 0 \leq \sigma \leq a \\ \frac{1}{6}\frac{12a^3\sigma - 3a^2\sigma^2 - 6a^4 + \theta(a-\sigma)(a+\sigma)(2a-\sigma)^2}{a^4} & if \quad a \leq \sigma \leq 2a \end{cases}. \end{aligned}$$

Notice that **A1** is satisfied here because, when $\theta_2 > \theta_1$ so that $H(\cdot;\theta_2)$ is more dependent than $H(\cdot;\theta_1)$, $L(\sigma;\theta_2)$ is a rotation of $L(\sigma;\theta_1)$ with rotation point $\hat{\sigma} = a$. Notice also that $\hat{\sigma}$ need not be small for Proposition 2 to hold.

Thus, $\frac{d\left(L(z)^2\right)}{dz} =$

$$\begin{cases} \frac{1}{9}z^3\left(3a^2\theta + z^2\theta + 3a^2 - 4az\theta\right)\frac{3a^2\theta+2z^2\theta+3a^2-6az\theta}{a^8} & if \quad 0 \leq z \leq a \\ \frac{1}{9}(z-2a)\left(a^2\theta - 2z^2\theta - 3a^2 + 2az\theta\right)\frac{12a^3z+4a^4\theta-z^4\theta-3a^2z^2-6a^4-3a^2z^2\theta+4az^3\theta-4a^3z\theta}{a^8} & if \quad a \leq z \leq 2a \end{cases},$$

and

$$T^B = \int_0^{2a} zd\left(L(z)^2\right) = \frac{1}{5670}a\left(234\theta - 17\theta^2 + 6993\right).$$

Therefore,

$$T^S - T^B = \frac{1}{5670}a\left(-234\theta + 17\theta^2 + 567\right),$$

which monotonically increases in a and monotonically decreases in θ for $\theta \in [-1, 1]$. That is, $\Delta W = T^S - T^B$ is larger when the two costs are less dependent or when each cost has a higher variance.

Suppose next that c^x and c^y both follow the exponential distribution $F(c) = G(c) = 1 - e^{-\lambda c}$. Then, since $\mathrm{Var}(c^x) = \mathrm{Var}(c^y) = \frac{1}{\lambda^2}$, a higher $\frac{1}{\lambda}$ implies that each cost is more dispersed. We have:

$$T^S = 2\int z_x d\left[F(z_x)\right]^2 = 2\lambda \int_0^\infty z2\left(1 - e^{-\lambda z}\right) e^{-\lambda z} dz = \frac{3}{\lambda},$$

$$\begin{aligned} l(\sigma;\theta) &= \int_{-\infty}^{\infty} h(c^x, \sigma - c^x)\, dc^x = \int_0^\sigma f(t) g(\sigma - t)\left[1 + \theta\left(2G(\sigma - t) - 1\right)\left(2F(t) - 1\right)\right] dt \\ &= \int_0^\sigma \lambda e^{-\lambda t} \lambda e^{-\lambda(\sigma - t)} \left(1 + \theta\left(2\left(1 - e^{-\lambda(\sigma - t)}\right) - 1\right)\left(2\left(1 - e^{-\lambda t}\right) - 1\right)\right) dt \\ &= \lambda\left(-4\theta e^{-\sigma\lambda} + 4\theta e^{-2\sigma\lambda} + \sigma\lambda e^{-\sigma\lambda} + \sigma\theta\lambda e^{-\sigma\lambda} + 4\sigma\theta\lambda e^{-2\sigma\lambda}\right), \end{aligned}$$

$$\begin{aligned} L(\sigma;\theta) &= \int_0^\sigma \lambda\left(-4\theta e^{-t\lambda} + 4\theta e^{-2t\lambda} + t\lambda e^{-t\lambda} + t\theta\lambda e^{-t\lambda} + 4t\theta\lambda e^{-2t\lambda}\right) dt \\ &= 1 - e^{-\sigma\lambda} - 3\theta e^{2(-\sigma\lambda)} + 3\theta e^{-\sigma\lambda} - \sigma\lambda e^{-\sigma\lambda} - 2\sigma\theta\lambda e^{2(-\sigma\lambda)} - \sigma\theta\lambda e^{-\sigma\lambda}. \end{aligned}$$

It follows that

$$\frac{\partial L(\sigma;\theta)}{\partial \theta} = -e^{-\sigma\lambda}\left(3e^{-\sigma\lambda} + \sigma\lambda + 2\sigma\lambda e^{-\sigma\lambda} - 3\right),$$

which is positive for $\sigma < \frac{2.1491}{\lambda}$ but negative for $\sigma > \frac{2.1491}{\lambda}$. Hence, for $\theta_2 > \theta_1$, $L(\sigma;\theta_2)$ is a rotation of $L(\sigma;\theta_1)$ with rotation point $\hat{\sigma}(\lambda) = \frac{2.1491}{\lambda}$. Thus assumption **A1** is satisfied, and the rotation point $\hat{\sigma}$ can be quite large. Moreover,

$$\begin{aligned} T^B &= \int_0^\infty zd\left(L(z)^2\right) dz \\ &= \int_0^\infty z \begin{pmatrix} -2e^{-z\lambda}\lambda\left(e^{-z\lambda} + 3\theta e^{2(-z\lambda)} - 3\theta e^{-z\lambda} + z\lambda e^{-z\lambda} + 2z\theta\lambda e^{2(-z\lambda)} + z\theta\lambda e^{-z\lambda} - 1\right) \\ \cdot\left(-4\theta + z\lambda + 4\theta e^{-z\lambda} + z\theta\lambda + 4z\theta\lambda e^{-z\lambda}\right) \end{pmatrix} dz \\ &= \frac{1}{216}\frac{20\theta - \theta^2 + 594}{\lambda}. \end{aligned}$$

Therefore

$$T^S - T^B = \frac{3}{\lambda} - \frac{1}{216}\frac{20\theta - \theta^2 + 594}{\lambda} = \frac{1}{216}\frac{-20\theta + \theta^2 + 54}{\lambda},$$

which decreases in θ but increases in $1/\lambda$. That is, $\Delta W = T^S - T^B$ is smaller when the two costs are more dependent but larger when each cost has a higher variance.[13]

Summarizing the discussion above, we have:

Proposition 3 *Assume that the joint distribution of (c^x, c^y) is formed by the class of FGM copulas defined in (21), with the marginal distributions being either (i) uniform on $[0, a]$, with variance $\frac{1}{12}a^2$, or (ii) exponential, with mean $\frac{1}{\lambda}$ and variance $\frac{1}{\lambda^2}$. Then, ΔW always decreases in θ, whereas in case (i) it increases in a and in case (ii) it increases in $\frac{1}{\lambda}$. That is, the bundling advantage is higher when the costs of the two products are less dependent or when the variance of each cost is higher.*

When c^x and c^y are more dependent, the distribution of $c^x + c^y$ is more dispersed, and hence the costs of the bundle are likely to be more asymmetric for the two firms, which softens their competition under bundled procurement. On the other hand, the competitive bidding under separate purchase is not affected by the dependence between the two costs. Therefore more dependence between the two costs reduces the bundling advantage, even though from Proposition 1 the buyer always prefers bundling to separate purchase when $v_x, v_y \geq \bar{c}$. The classes of distributions considered in this subsection also illustrate the plausibility of the conditions for Proposition 2.

When the marginal distributions of costs have a higher variance, the cost of each good is more dispersed, but the cost dispersion of the bundle may also be higher, so that the competitive bidding under separate and bundled purchases may both be less intense. Thus, *a priori*, it is not clear how an increase in the variance of each cost may impact the bundling advantage. For the classes of distributions formed by the FGM copula family with uniform and with exponential margins, we find that the bundling advantage increases in the variance of each cost, suggesting that the dispersion of costs is less affected under bundling than under separate purchase by the higher variance.

[13] Notice that in this case, when $v_x = v_y \geq 10/\lambda$, $F(v_x) = G(v_y) > 0.999$ and $L(v_x + v_y) > 0.999$. Hence, even if v_x and v_y are finite, we can consider the sign and the change in $W^B - W^S$ the same as those in $T^S - T^B$ if $v_x = v_y \geq 10/\lambda$.

5. Allowing Mixed Bundling

We finally extend our analysis to introduce the procurement strategy of mixed bundling (M), where the buyer solicits prices both for individual products and for X and Y as a package. The buyer has the option to buy neither product, only one product, or both of the products. Let firm $i's$ bids for X, Y, and package XY be a_i^x, a_i^y, and a_i^{xy}, respectively. The buyer selects the outcome that maximizes her surplus from the alternatives:

$$\max\{0, v_x - \min\{a_1^x, a_2^x\}, \quad v_y - \min\{a_1^y, a_2^y\}, \quad v_x + v_y - \min\{a_1^{xy}, a_2^{xy}, a_1^x + a_2^y, a_1^y + a_2^x\}\}$$

Lemma 3-5 characterize the equilibrium under mixed bundling for specific cost realizations. We start with the case where it is efficient for one firm, say firm 1, to supply both products.

Lemma 3 *Under cost realizations* (c_1^x, c_1^y) *and* (c_2^x, c_2^y), *suppose that*

$$c_1^x \leq \min\{v_x, c_2^x\} \text{ and } c_1^y \leq \min\{v_y, c_2^y\}. \tag{24}$$

Then the following strategies form an equilibrium:

$$\begin{aligned}
&\hat{a}_1^x = \min\{v_x, c_2^x\}, \quad \hat{a}_1^y = \min\{v_y, c_2^y\}, \quad \hat{a}_1^{xy} = \min\{v_x, c_2^x\} + \min\{v_y, c_2^y\}; \\
&\hat{a}_2^x = c_2^x, \quad \hat{a}_2^y = c_2^y, \quad \hat{a}_2^{xy} = c_2^{xy} \equiv c_2^x + c_2^y,
\end{aligned}$$

with the buyer purchasing both products from firm 1 at a total price equal to

$$\hat{t} = \hat{a}_1^{xy} = \min\{v_x, c_2^x\} + \min\{v_y, c_2^y\},$$

and this is the unique equilibrium outcome of the game if the inequalities in (24) hold strictly.[14]

Proof. [Equilibrium.] Given the firms' strategies, awarding the bundle to firm 1 maximizes the buyer's surplus and thus is the chosen contract outcome. Given firm 1's strategy, firm 2 has no profitable unilateral deviation.

[14] When the inequalities in (24) do not hold strictly, if $c_1^x = v_x < c_2^x$ or $c_1^y = v_y < c_2^y$, the equilibrium outcome is exactly the same as that stated in Lemma 3. If $c_1^x = c_2^x$ (or $c_1^y = c_2^y$), firm 2 has an equal probability of supplying X (or Y) at price $\min\{v_x, c_2^x\}$ (or $\min\{v_y, c_2^y\}$), but the total buyer price remains the same.

Given firm 2's strategy, firm 1 chooses the strategy that maximizes its profit without being undercut by firm 2 in the supply of either product or the bundle. Then, $\hat{a}_1^x = \min\{v_x, c_2^x\}$ and $\hat{a}_1^y = \min\{v_y, c_2^y\}$ are respectively the maximal prices that firm 1 could receive for the supply of product X and Y, and $\hat{a}_1^{xy} = \min\{v_x, c_2^x\} + \min\{v_y, c_2^y\}$ is the maximal price that firm 1 could receive for supplying the bundle. Moreover, under $\hat{a}_1^x$, $\hat{a}_1^y$, and $\hat{a}_1^{xy}$, firm 1 has the same profit supplying the two products separately or as a package, and hence it cannot benefit from a deviation that raises its bid for the package to supply the two products separately.

Therefore, the proposed is indeed an equilibrium. The outcome of this equilibrium is exactly the same as that generated by separate purchase.

[**Uniqueness of the equilibrium outcome.**] By the standard logic for Bertrand competition, the high-cost supplier, firm 2, must bid its cost in equilibrium: $a_2^k = c_2^k$ for $k = x, y, xy$. Thus, firm 2's equilibrium strategy is uniquely determined. Since $\hat{a}_1^k$ for $k = x, y, xy$ is the optimal response by firm 1 given firm 2's strategy and the buyer's purchase strategy, and since any deviation by firm 1 that changes the bidding outcome will reduce its profit, the equilibrium outcome is unique.[15] ■

The next lemma deals with the case where it is efficient for each firm to supply one different good. Without loss of generality, suppose it is efficient for firm 1 to supply good X and firm 2 to supply good Y.

Lemma 4 *Under cost realizations* (c_1^x, c_1^y) *and* (c_2^x, c_2^y), *suppose that*

$$c_1^x \leq \min\{v_x, c_2^x\} \text{ and } c_2^y \leq \min\{v_y, c_1^y\}. \tag{25}$$

The following strategies form an equilibrium under mixed bundling

$$\hat{a}_1^x = \min\{v_x, c_2^x\}, \quad \hat{a}_1^y = c_1^y, \quad \hat{a}_1^{xy} = \min\{v_x, c_2^x\} + \min\{v_y, c_1^y\};$$
$$\hat{a}_2^x = c_2^x, \quad \hat{a}_2^y = \min\{v_y, c_1^y\}, \quad \hat{a}_2^{xy} = \min\{v_x, c_2^x\} + \min\{v_y, c_1^y\},$$

with firm 1 supplying X *at price* $\hat{a}_1^x$ *and firm 2 supplying* Y *at price* $\hat{a}_2^y$. *The total buyer price*

[15] Firm 1's equilibrium strategy is not unique. For instance, $a_1^x = \min\{v^x, c_2^x\}$, $a_1^y = \min\{v^y, c_2^y\}$, $a_1^{xy} = \hat{a}_1^{xy} + \varepsilon$ for any $\varepsilon > 0$ will still be an equilibrium strategy, but this does not change the equilibrium outcome.

is

$$\hat{t} = \hat{a}_1^x + \hat{a}_2^y = \min\{v_x, c_2^x\} + \min\{v_y, c_1^y\}.$$

This equilibrium generates the separate purchase outcome. When the inequalities in (25) hold strictly, any other equilibrium outcome is Pareto dominated by the separate purchase outcome for the firms.[16]

Proof. We proceed in three steps. Step 1, we show that the stated strategies indeed form an equilibrium and the equilibrium generates the separate purchase outcome. Step 2, any possible equilibrium where each firm supplies one product is Pareto dominated by the separate purchase outcome for the firms. Step 3, any bundling equilibrium outcome is Pareto dominated by the separate purchase outcome for the firms.

[**Step 1.**] Given the strategies of the firms, the buyer optimally picks firm 1 to supply X and firm 2 to supply Y. Given firm 2's strategy, it is optimal for firm 1 to bid $\hat{a}_1^x$, $\hat{a}_1^y$, and $\hat{a}_1^{xy}$, resulting in winning the production of X. Similarly, given firm 1's strategy, the proposed strategy of firm 2 is also its best response. The firms' equilibrium profits are $\hat{\pi}_1 = \min\{v_x, c_2^x\} - c_1^x$ and $\hat{\pi}_2 = \min\{v_y, c_1^y\} - c_2^2$. The proposed equilibrium generates exactly the same outcome as separate purchase.

[**Step 2.**] Any equilibrium where each firm supplies one product is Pareto dominated by the separate purchase outcome for the firms. First note that an outcome where firm 1 supplies Y and firm 2 supplies X cannot occur in equilibrium if the inequalities in (25) hold strictly so that $c_1^x < c_2^x$ and $c_1^y > c_2^y$.

Suppose there is an alternative equilibrium where firm 1 supplies product X, firm 2 supplies product Y and $a_1^x \neq \min\{v^x, c_2^x\}$. Then $a_1^x < \min\{v^x, c_2^x\}$ must hold because if $a_1^x > \min\{v^x, c_2^x\}$, the buyer either does not buy product X or procures product X from firm 2 and in these cases firm 1 could lower its bid and wins product X for a profit. In the same logic, for firm 2 to win product Y, $a_2^y \leq \min\{v_y, c_1^y\}$ must hold. Thus, if such an alternative equilibrium indeed exists, the two firms's profits must be such that $\bar{\pi}_1 < \min\{v^x, c_2^x\} - c_1^x = \hat{\pi}_1$ and $\bar{\pi}_2 \leq \min\{v^y, c_1^y\} - c_2^y = \hat{\pi}_2$. Thus such an equilibrium outcome is Pareto dominated by the

[16] If the inequalities in (25) do not hold strict, if $c_1^x < c_2^x$ and $c_2^y < c_1^y$, the equilibrium outcome remains the same; if $c_1^x = c_2^x$ (or $c_2^y = c_1^y$), it is equally possible that firm 2 supplies product X (or firm 1 supplies product Y), but the equilibrium buyer price remains the same as the separate purchase outcome.

separate purchase outcome for the firms. The same holds true for an equilibrium where firm 1 supplies product X, firm 2 supplies product Y and $a_2^y \neq \min\{v^y, c_1^y\}$.

[**Step 3.**] Finally, we show that for the firms, any bundling equilibrium outcome is Pareto dominated by the equilibrium with separate purchase outcome.[17] Suppose without loss of generality that $c_1^{xy} \leq c_2^{xy}$. Then, at the bundling equilibrium, we must have $a_1^{xy} \leq a_2^{xy}$, firm 1 supplies the package with a profit $\tilde{\pi}_1 = a_1^{xy} - c_1^{xy}$ and firm 2's profit is $\tilde{\pi}_2 = 0$. Moreover, the buyer's surplus from the bundle must not be lower than that from purchasing the two goods separately from firm 2 at c_2^x and c_2^y, which implies

$$v_x + v_y - a_1^{xy} \geq \max\{v_x - c_2^x, 0\} + v_y - c_2^y.$$

Thus, if $v_x \geq c_2^x$, then $v_x + v_y - a_1^{xy} \geq v_x - c_2^x + v_y - c_1^y$, or $a_1^{xy} \leq c_2^x + c_1^y$, and hence

$$\tilde{\pi}_1 = a_1^{xy} - c_1^{xy} \leq c_2^x - c_1^x = \min\{v_x, c_2^x\} - c_1^x = \hat{\pi}_1.$$

If $v_x < c_2^x$, then $v_x + v_y - a_1^{xy} \geq \max\{v_x - c_2^x, 0\} + v_y - c_2^y = v_y - c_2^y \geq v_y - c_1^y$, or $v_x + c_1^y \geq a_1^{xy}$, and hence

$$\tilde{\pi}_1 = a_1^{xy} - c_1^x - c_1^y \leq v_x - c_1^x = \min\{v_x, c_2^x\} - c_1^x = \hat{\pi}_1.$$

It follows that

$$\tilde{\pi}_1 + \tilde{\pi}_2 \leq \hat{\pi}_1 + \hat{\pi}_2,$$

where the inequality holds strictly if $\hat{\pi}_2 = \min\{v_y, c_1^y\} - c_2^y > 0$. ■ The next lemma deals with the case where it is efficient to trade only one good. Without loss of generality, suppose it is efficient for firm 1 to supply product X.

Lemma 5 *Under cost realizations* (c_1^x, c_1^y) *and* (c_2^x, c_2^y), *suppose that*

$$c_1^x \leq \min\{v_x, c_2^x\} \textit{ and } v_y < \min\{c_1^y, c_2^y\}. \tag{26}$$

[17] Suppose the inequalities in (25) hold and in addition $v_x \geq \max\{c_1^x, c_2^x\}$ and $v_y \geq \max\{c_1^y, c_2^y\}$, a bundling equilibrium indeed exists, with

$$\tilde{a}_1^x = c_2^x = \tilde{a}_2^x; \quad \tilde{a}_1^y = c_1^y = \tilde{a}_2^y; \quad \tilde{a}_1^{xy} = \max\{c_1^{xy}, c_2^{xy}\} = \tilde{a}_2^{xy},$$

and the firm with the lower total cost for X and Y will supply both goods.

Then the following strategies form an equilibrium

$$\hat{a}_1^x = \min\{v_x, c_2^x\}, \quad \hat{a}_2^x = c_2^x; \quad \hat{a}_1^y = \max\{c_1^y, c_2^y\} = \hat{a}_2^y; \quad \hat{a}_1^{xy} = \max\{c_1^{xy}, c_2^{xy}\} = \hat{a}_2^{xy},$$

where firm 1 supplies product X at price $\min\{v_x, c_2^x\}$, *and product Y is not purchased. This is the unique equilibrium outcome if in addition $c_1^x < c_2^x$.*[18]

Proof. [Equilibrium.] First, given the strategies of the two firms, the contract outcome is that the buyer only procures product X from firm 1. This is optimal for the buyer, because it cannot obtain positive surplus by separately purchasing Y, and its surplus from purchasing the bundle would be

$$v_x + v_y - \max\{c_2^{xy}, c_1^{xy}\} \leq v_x + v_y - c_2^{xy} = v_x - c_2^x + v_y - c_2^y < v_x - c_2^x,$$

lower than its surplus from purchasing X alone.

Next, obviously neither firm can benefit from unilaterally raising any of its bids, and neither firm can benefit from unilaterally lowering its bid for X or for Y.

Finally, if $\max\{c_2^{xy}, c_1^{xy}\} > \min\{c_2^{xy}, c_1^{xy}\}$, we need to consider potential deviating bids for the bundle by the firm with the lower total cost. If $c_2^{xy} < c_1^{xy}$, then in order for firm 2 to profit from a deviation that enables it to supply the bundle, its bid a_2^{xy} must satisfy

$$v_x + v_y - a_2^{xy} > v_x - \min\{v_x, c_2^x\},$$

or

$$a_2^{xy} < v_y + \min\{v_x, c_2^x\} = v_y - c_2^y + \min\{v_x, c_2^x\} + c_2^y < v_y - c_2^y + c_2^x + c_2^y < c_2^x + c_2^y,$$

which is not profitable. On the other hand, if $c_1^{xy} < c_2^{xy}$, then in order for firm 1 to profit from a deviation that enables it to supply the bundle, its bid a_1^{xy} must satisfy

$$v_x + v_y - a_1^{xy} > v_x - \min\{v_x, c_2^x\},$$

[18] If $c_1^x = c_2^x$, each firm has an equal probability of supply product X but the equilibrium price for the buyer remains the same.

or

$$a_1^{xy} < v_y + \min\{v_x, c_2^x\} = v_y - c_1^y + \min\{v_x, c_2^x\} + c_1^y,$$

or

$$a_1^{xy} - (c_1^x + c_1^y) < v_y - c_1^y + \min\{v_x, c_2^x\} - c_1^x < \min\{v_x, c_2^x\} - c_1^x = \hat{a}_1^x - c_1^x,$$

which is not profitable.

Thus, the proposed strategies indeed form an equilibrium. Its outcome is exactly the same as that from separate purchase.

[Uniqueness of the Equilibrium Outcome.] Suppose $c_1^x < c_2^x$. Clearly, there can be no other equilibrium in which only X is supplied, and there can be no equilibrium where only Y is supplied. At a possible equilibrium where the bundle is supplied, it must be supplied by the firm with the lower total cost for the two goods. Suppose $c_1^{xy} \leq c_2^{xy}$. If there exists an equilibrium with a bundling outcome, the winning bundle price must be $a_1^{xy} = \min\{v_x + v_y, c_2^{xy}\}$. Then, firm 1's profit from supplying the bundle is $a_1^{xy} - c_1^{xy}$, whereas if it deviates to supplying X alone at price $\min\{v_x, c_2^x\}$, its profit is $\min\{v_x, c_2^x\} - c_1^x$. The deviation is acceptable to the buyer because

$$v_x + v_y - a_1^{xy} = v_x + v_y - \min\{v_x + v_y, c_2^{xy}\} \leq v_x - c_2^x \leq v_x - \min\{v_x, c_2^x\},$$

and the deviation is profitable to firm 1 because

$$\begin{aligned} a_1^{xy} - c_1^{xy} &= \min\{v_x + v_y, c_2^x + c_2^y\} - c_1^x - c_1^y < \min\{v_x, c_2^x\} + v_y - c_1^x - c_1^y \\ &< \min\{v_x, c_2^x\} - c_1^x. \end{aligned}$$

Suppose next $c_1^{xy} > c_2^{xy}$, then at the possible equilibrium with bundled outcome, to prevent the buyer from purchasing X alone from firm 1, we must have

$$v_x + v_y - a_2^{xy} \geq v_x - c_1^x,$$

which can be true only if

$$v_x + v_y - c_2^{xy} \geq v_x - c_1^x \geq 0,$$

or

$$v_y + c_1^x \geq c_2^x + c_2^y,$$

which is not possible because $v_y < c_2^y$ and $c_1^x < c_2^2$. ■

Notice that Lemmas 3-5 continue to hold if the identities of the firms in (24)-(26) are switched. Moreover, mixed bundling delivers the same equilibrium outcome as separate purchase, unless the cost realizations (c_1^x, c_1^y) and (c_2^x, c_2^y) are such that it is efficient for each firm to supply one different good, in which case there can be other equilibrium outcome under mixed bundling, but Pareto dominated by the separate purchase outcome for the firms. Recall from Theorem 1 that when $v_x, v_y > \mu_2$, buyer surplus is higher under pure bundling than under separate purchase. Hence, when $v_x, v_y > \mu_2$, the expected procurement price is higher under mixed bundling than under pure bundling if, in the presence of multiple equilibria, suppliers choose to play their Pareto dominating equilibrium with separate purchase outcome. We therefore arrive at the following conclusion.

Proposition 4 *The equilibrium outcome under separate purchase is always an equilibrium outcome under mixed bundling. Furthermore, if $v_x, v_y > \mu_2$, then the expected buyer surplus is lower under mixed bundling than under pure bundling if, in the presence of multiple equilibria, sellers choose to play the Pareto dominating equilibrium associated with separate purchase outcome.*

The desirability of mixed bundling is thus very different for a buyer engaged in procurement and for a monopoly seller. In a typical model of bundled sales by a monopoly seller, making the individual goods available together with the bundle gives the seller more options to extract consumer surplus. The seller can often choose the prices for the individual goods and the bundle in such a way that the profit under mixed bundling is higher than that under both pure bundling and separate selling. For a buyer conducting procurement auctions, however, the procurement prices are determined through competitive bidding by the sellers. When sellers are invited to bid on the price for the two goods as a package, the option for them to also bid on the prices of individual goods changes their strategic interactions, enabling them to coordinate to higher bids for the bundle so that the equilibrium outcome becomes that of separate procurement. In particular, when the firms can also bid to supply individual products, each firm has the incentive to raise its bid for the bundle because if he loses the bidding for the bundle, he can still win the bidding for a single product, and this relaxes the competition for the bundle. Mixed bundling can thus result in lower buyer surplus than pure bundling if the latter generates higher expected buyer surplus than separate purchase, as when $v_x, v_y \geq \mu_2$. In fact, provided that firms will play the Pareto dominating equilibrium in the presence of multiple equilibria, mixed bundling is strictly dominated by pure bundling whenever $W^S < W^B$.

6. Concluding Remarks

This paper has derived several results on bundled procurement, in an environment where a buyer procures multiple products from competing firms: (1) Pure bundling generates higher buyer surplus than separate purchase, so long as trade for each good is sufficiently likely. (2) The bundling advantage changes non-monotonically with product values, possibly first decreasing and then increasing. It tends to be larger when costs for two goods are less dependent or when the variance of each cost is higher. (3) Mixed bundling enables firms to coordinate to higher prices and sustain the equilibrium outcome under separate purchase. Consequently, pure bundling will dominate mixed bundling for the buyer when product values are high.

For convenience, we have developed our analysis in a model with two products and two suppliers. But the numbers of products and of suppliers are not essential for the key idea behind our results, namely that firms will compete more aggressively due to more symmetric costs under bundling, and this competition effect dominates the potential adverse tying effect when trade for each good is likely to be efficient. We thus expect that the basic insights from our analysis will continue to be valid in settings with more products and firms.

There are other ways in which our model can be extended or modified. One of our important assumptions is that the buyer has limited commitment ability, in the sense that it can commit to one of the three procurement strategies considered, but not to a specific reserve price. It could be interesting for future research to relax this assumption. Also, instead of assuming that the buyer's product valuations are publicly known, future research might also explore the possibility that product values are realizations of random variables learned privately by the buyer.[19]

The buyer in our analysis has the market power to choose a procurement strategy that maximizes her surplus. As one might expect, this market power can lead to distortions that reduce total welfare. In fact, because the supplier for a good is sometimes not the lowest-cost firm under pure bundling but will always be under separate purchase, pure bundling has lower expected cost efficiency than separate purchase. Thus, the buyer's optimal choice of procurement is generally not socially optimal, in the sense that expected total welfare is lower under pure bundling than under separate purchase.[20]

In practice, there are other factors that could influence the choice of procurement strategies, such as (dis)economies of scope in production or in transaction. Controlling for these other factors, our theory predicts that pure bundling is more likely to be selected as a procurement strategy relative to separate purchase when: (i) the values of products are higher relative to their possible costs, (ii) costs for different goods are more negatively dependent or less positively dependent, and (iii) the cost dispersion of each good is higher. It would be interesting for future research to empirically evaluate these predictions.

[19] Notice that our analysis covers the case where v_x and v_y are realizations of random variables learned privately by the buyer but are known publicly to be above $\bar{c}$.

[20] However, if product values were realizations of random variables and were the buyer's private information, then pure bundling could potentially have higher total welfare than separate purchase, possibly due to higher expected total output associated with the lower total prices under bundling.

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