Governments Should Not Use Declining Discount Rates in Project Analysis

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A number of governments have already adopted the policy of applying Declining Discount Rates (DDRs) to long lived projects, a move that could affect public sector investment decisions. Arguments for the use of Declining Discount Rates are based on the consideration of uncertainty, both for discount rates derived from social welfare functions, and for those derived from the characteristics of capital markets. The case for the latter is based on Martin L. Weitzman's assertion that certainty equivalent discount rates are a declining function of time and tend to the lowest possible interest rate when interest rates are stochastic but perfectly auto-correlated. This paper finds that this conclusion is the consequence of Weitzman's use of time reversed negative compounding, rather than of discounting, in the definition of net present value. When discounting is used instead, Weitzman's conclusions are reversed, and do not support the use of Declining Discount Rates.

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In "Should Governments Use a Declining Discount Rate in Project Analysis?" Kenneth J. Arrow *et al.* (2014:146-7) presents a case for the adoption of Declining Discount Rates (DDRs) by showing that two separate branches of research on the subject of discounting recommend it. The authors state: "Over the last decade, two branches of the literature have emerged concerning DDRs. The first branch extends the Ramsey formula for discounting benefits and costs to allow for uncertainty in the rate of consumption growth" while "The second branch of the DDR literature is based on the expected net present value (ENPV) approach. This approach was initially developed by Weitzman (1998, 2001, 2007), who argued that the uncertainty about future discount rates justifies using a decreasing term structure (i.e., time pattern) of discount rates today."

The cited second branch is the primary subject of the present paper, which will contend that Martin L. Weitzman's (1998) definition of expected present value (EPV) is not based on discounting, but rather on time reversed negative compounding. The conclusion that certainty equivalent discount rates decline with time is a property of negative compounding and not of discounting. Consequently, when discounting is used instead, as intended, the Weitzman's model yields the conclusion that certainty equivalent discount rates are a growing function of time. This considerably weakens the case for DDRs.

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In addition, the proper role of discount rates solely derived from welfare functions in project analysis will be reviewed. This relates to the question raised by Arrow *et al.* (2014:154) as follows: "What if society is *not* on an optimal consumption path? In this case, theory tells us that we need to calculate the social opportunity cost of capital." It will be argued that if society is not on an optimal consumption path, then, for an investment to be welfare increasing, its ENPV should be positive at both the welfare function derived discount rate, and the one derived from the social opportunity cost of capital. It will be argued that the latter would be the higher of the two, and would therefore constitute the binding hurdle rate that investment projects face. The former would not then be relevant to the investment decision to be made. This argument also weakens the case for DDRs.

This paper is organized as follows. Section 1 analyzes Weitzman's EPV formula; Section 2 explains what negative compounding is and shows that Weitzman's EPV formula does not correspond to discounting, but rather to time reversed negative compounding; Section 3 addresses the issue of correlated interest rates implicit in Weitzman's model; Section 4 discusses the role of discounting in project analysis; and Section 5 concludes.

1. Weitzman's EPV

The essence of the Weitzman (1998) model can be formulated as follows. A risk neutral investor must make an investment choice with consequences in the distant future before knowing what the capital market interest rate will be. Once the investor has made his choice, the constant interest rate to prevail thence until the distant future is revealed. There are j possible scenarios with probabilities p_j and interest rates r_j. If the length of time between the present and the distant future is t years, the conditional discount factor for each scenario j is as follows, as a function of t:

$$a_j(t) = e^{-r_j t} \tag{1}$$

Weitzman defined the certainty equivalent discount factor used to compute the present value of \$1 due at time t as

$$A_W(t) = \sum p_j a_j(t) = \sum p_j e^{-r_j t} \tag{2}$$

The corresponding certainty-equivalent discount rate R_W will fulfill the following condition[2]

$$A_W(t) = e^{-R_W t} \tag{3}$$

from which it follows that

[2] Subscripts W attribute to Weitzman.

$$R_W(t) = -\left(\frac{1}{t}\right)\ln\left(\sum_j p_j e^{-r_j t}\right) \tag{4}$$

which, as Weitzman (1998) shows, tends to the lowest possible interest rate for large *t*. This conclusion is also quoted in Arrow *et al.* (2014:154).

Expression (2) above[3] does not conform to the definition of present value, however, which requires that compounding the present value of a future sum yield the same future value that was discounted[4], because

$$\left(\sum p_j e^{-r_j t}\right)\left(\sum p_j e^{r_j t}\right) \neq 1 \tag{5}$$

where the second term is the expected compound factor derived from the same interest rates and probabilities. The correct EPV of \$1 due in year *t* is

$$A(t) = \frac{1}{\sum p_j e^{r_j t}} \tag{6}$$

which, when multiplied by the expected compound factor $\sum p_j e^{r_j t}$, will clearly result in an expected future value (EFV) equal to \$1.

The correct certainty equivalent rate (CER) can be derived from (6) and is

$$R\ (t) = \left(\frac{1}{t}\right)\ln\left(\sum_j p_j e^{r_j t}\right) \tag{7}$$

which, as Christian Gollier (2003) shows, tends to the highest possible interest rate for large *t*.

Gollier (2003) arrived at conclusions that are diametrically opposed to those of Weitzman (1998) by computing CERs based on EFVs, rather than EPVs. This is the work that gave rise to the Weitzman-Gollier puzzle, the name by which the literature came to know the paradox that the Weitzman's EPV rule results in a different CER than that which Gollier derived from the EFV rule, and which corresponds to expression (7).

The paradox was this: if as a consequence of the perfect auto-correlation of interest rates assumed in the Weitzman (1998) model[5], EFV derived CERs are growing with time, then how can EPV derived CERs, which should measure the opportunity cost of capital, be declining? The formulation of the EFV derived CERs could not be faulted, for their expression follows from a description of how capital markets operate and from the definition of expected value. Cleary the conflicting CER definitions could not both

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3 Equivalent to (3) in Arrow *et al.* (2014:154)

4 "The present value of an asset is obtained by calculating how much money invested today would be needed, at the going interest rate, to generate the asset's future stream of receipts." (Paul A. Samuelson and William D. Nordhaus, 1992:271.)

5 A necessary consequence of a two time period model being used to represent the distant future.

be right. But because the EPV formulation proposed by Weitzman (1998) (expression 2 above) appeared to be so plausible, it occurred to no one to question it. Consequently, the contradiction observed in the risk neutrality context of the Weitzman (1998) model was left unresolved.

Gollier (2003) was the first to suggest the that the timing of risk bearing might be the cause of the discrepancy, even though a discrepancy between alternatively measured risk neutral CERs cannot be solved by introducing risk aversion. A sizable literature ensued that used utility functions to justify Weitzman's DDR conclusions. Many consider that the matter was settled by Gollier and Weitzman (2009), which claimed to have resolved the puzzle. Szekeres (2015) finds, however, that Gollier and Weitzman (2009) does not address the original puzzle, and that in general adding risk aversion to the Weitzman (1998) model to derive risk adjusted CERs cannot resolve the inconsistency between alternative measures of risk neutral CERs, because investors' risk aversion only affects their own valuations, not market yields.

There is no mention of risk aversion in the second part of Arrow *et al.* (2014). The arguments presented there for the use of DDRs are therefore the same as those of Weitzman (1998), and consequently suffer from the same flaw. As will be discussed in Section 2.2 below, using the CER proposed in Arrow *et al.* (2014:154 expression 4) results in the violation of the generally accepted requirement that preferences be transitive.

Expression (5) above shows that Weitzman's EPV measure is not a present value, because it does not comply with the definition of present value. The next Section describes what it is instead.

2. Negative compounding

How can Weitzman's EPV expression be interpreted if it does not compute the correct EPV? Notice that Weitzman's expected discount factor $\sum p_j e^{-r_j t}$ corresponds exactly to the expected compound factor $\sum p_j e^{r_j t}$ when the product $r \cdot t$ is negative. Having negative r would correspond to a capital market in which resources are stored for a fee, rather than being lent to someone willing to pay a positive interest rate. Having t negative would imply reversing the flow of time.

As Weitzman did not postulate negative interest rates (in fact he ruled them out in his model[6]), his expected discount factor must be regarded to be time reversed negative compounding, which is not the same as discounting when interest rates are stochastic.

For simplicity, and to be able to use numerical examples later on, the following analysis assumes that there are only two states of the world, with interest rates r_1 and r_2, and probabilities p_1 and p_2, respectively.

[6] "The only restriction being applied to the $\{r_i\}$ is that they each be nonnegative." (Weitzman, 1998:204).

The difference between discounting and negative compounding will be explained with the help of two Figures. Figure 1 shows the compound and discount factors curves applicable to an investment of \$1 made at time 0, in continuous time, with a deterministic annual interest rate of 5%, between years –200 and 200. The equations being plotted are $e^{0.05\,t}$ for the compound factors curve, and $1/e^{0.05\,t}$ for the discount factors curve. The vertical scale in the figures is logarithmic, which is why both the compound factors and discount factors curves are seen to be linear. The fact that one is the inverse of the other is evidenced by their symmetry with respect to the horizontal line passing through the value of 1. It is important to note that the negative range of the compound factor curve is symmetric to the positive range of the discount factor curve around the vertical axis (year 0), which means that in the deterministic case discounting and negative compounding are equivalent.

Figure 1

COMPOUND AND DISCOUNT FACTORS,
5% INTEREST P.A., LOGARITHMIC SCALE.

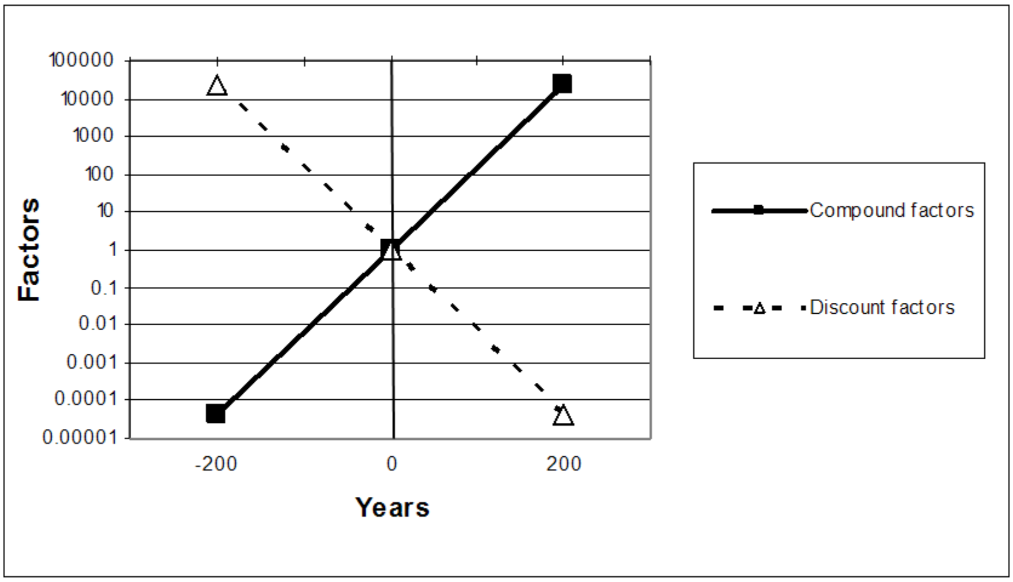
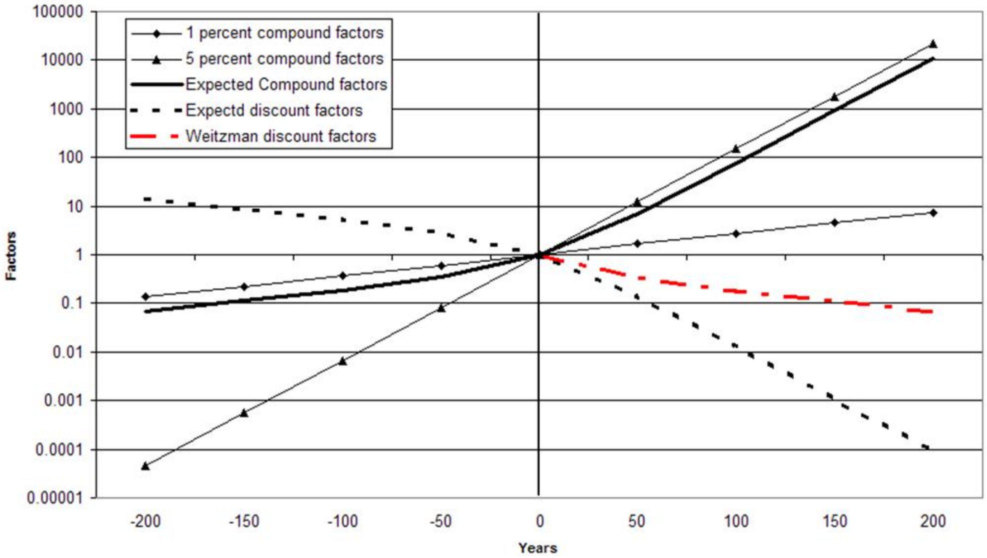


Figure 2 illustrates the stochastic case. It is assumed that interest rates can be either 1% or 5%, with equal probabilities. Figure 2 shows the compound factor curves corresponding to 1% and 5%, both of which are linear in logarithmic terms. Their expectation is no longer linear, however. Moving forward in time (positive range of years), compound factors corresponding to the high interest rate grow comparatively larger relative to those of the low interest rate, thereby pulling their expected value ever closer to the compound factors curve of the high rate. The same happens moving backwards into the past (negative range of years), in which case it is the compound factors corresponding to the low interest rate that grow relatively larger, and it is therefore towards the compound factors curve of the low interest rate that their expected values tend asymptotically. In other words, the higher discount factors pull the expected discount factors upwards over the entire time range, this effect being stronger as the absolute value of time increases.

Figure 2

COMPOUND FACTORS AT 1% AND 5%, THEIR EXPECTED VALUE AND THE CORRESPONDING DISCOUNT FACTORS, LOGARITHMIC SCALE



The immediate consequence of this is that the expected compound factors curve is no longer linear logarithmically. This is also true of the expected discount factors curve, which is the inverse of the expected compound factors curve. Because of this lack of linearity, the negative range of the expected compound factors curve is no longer symmetric, with respect to the vertical axis, to the positive range of the expected discount factors curve, and cannot be used, therefore, to calculate present values correctly. As Figure 2 shows, the negative range of the compound factors curve is significantly higher than the positive range of the discount factors curve, for all absolute values of time.

This is the reason why the probability weighted average of the conditional discount factors of alternative interest rate scenarios (which is what the negative range of the expected compound factors curve is, and which Weitzman used to calculate EPVs) does not yield the correct EPVs of amounts compounded to the future. To facilitate comparison with the correct discount factors, the former are time reversed to the positive range of years and labeled Weitzman discount factors in Figure 2.

It is only when negative compounding is used for discounting that DDRs will result, because it is only in the negative time range that the expected compound factors are pulled towards those of the lowest interest rate. Certainty equivalent discount rates derived from the expected discount factors curve (or from the compound factors curve, since the results are the same) will be an increasing function of time.

2.1 A numerical comparison of discounting and negative compounding

A numerical example will be used to further illustrate the difference between discounting and negative compounding, and how Weitzman discounting relates to these concepts. First, $1 will be compounded for 200 years and then discounted both with the simple equivalent of Weitzman discounting (expression 2) and that of conventional discounting (expression 6). The data of the example are the following:

Table 1

SIMPLE INTEREST RATE UNCERTAINTY

First rate of interest	r_1	1%
Second rate of interest	r_2	5%
Probability of 1st rate	p_1	0.5
Probability of 2nd rate	p_2	0.5
Time elapsed in years between the present and the future in years	t	200
Future value of \$1 at r_1	$L = \mathrm{e}^{r_1 t}$	\$7.39
Conditional discount factor for r_1	$d_1 = 1/L$	0.1353
Future value of \$1 at r_2	$H = \mathrm{e}^{r_2 t}$	\$22,026.47
Conditional discount factor for r_2	$d_2 = 1/H$	4.540E-05
Expected future value of \$1	$EFV = p_1\,\mathrm{e}^{r_1 t} + p_2\,\mathrm{e}^{r_2 t}$	\$11,016.93
Expected discount factor	$D = \dfrac{1}{p_1 \mathrm{e}^{r_1 t} + p_2 \mathrm{e}^{r_2 t}}$	9.077E-05

With discounting, the present value of the expected future value in this simple two scenario case is:

$$EPV = EFV \cdot D = \$11{,}016.93 \cdot 9.077\text{E-}05 = \$1 \tag{8}$$

The result cannot be anything other than \$1, since the discount factor is the inverse of the expected compound factor. Notice that the compound factor itself is an expected value, which reflects the uncertainty of the states of the world. The corresponding CER is 4.65%.[7]

Weitzman discounting, in contrast, discounts each state of the world separately with its own conditional discount factor (conditional to the relevant state of the world occurring). It is these alternative conditional present values that are then probability weighted to compute an expected present value, denoted $\mathrm{EPV_W}$, which yields a markedly different result:

$$\begin{aligned} \mathrm{EPV_W} &= p_1 \cdot \$11{,}016.93 \cdot d_1 + p_2 \cdot \$11{,}016.93 \cdot d_2 = \\ &= \$11{,}016.93\,(0.5 \cdot 0.1353 + 0.5 \cdot 4.54\text{E-}05) = \$745.74 \end{aligned} \tag{9}$$

This is a lot more than the original investment of \$1. The corresponding CER 1.35%.[8]

7 ln(11016.93)/200

8 ln(11016.93/745.74)/200

The reason why discounting yields the correct present value is because it exactly undoes the effects of compounding. The correct result could also be obtained with the approach of using probability weighted conditional discount factors, however, if these were used to multiply the corresponding conditional future values, as follows:

$$\text{EPV} = p_1\, L\, d_1 + p_1\, H\, d_1$$

$$= 0.5 \cdot \$7.39 \cdot 0.1353 + 0.5 \cdot \$22{,}026.47 \cdot 4.540\text{E-}05 = \$1 \qquad (10)$$

But negative compounding applies the conditional discount factors not to the conditional future value of each scenario, but to their expected value. This overstates the amount to be discounted for the lower interest rate scenario ($11,016.93 instead of $7.39), and understates it for the high interest rate scenario ($11,016.93 instead of $22,026.47)[9]. These errors do not cancel out, because the underestimation is proportionally much more diminished by the lower discount factor corresponding to the high interest rate (4.540E-05) than the overestimation is by the high discount factor corresponding to the low interest rate (0.1353). This necessarily results in an overestimation of the computed EPV.

Weitzman assumed that the future value to be discounted would be certain, whereas in the above numerical example the future value is the expected value of two states of the world. This makes no difference for either calculation method, however. Using their respective CERs, each method again obtains its own present value result when probability weighting the two future value scenarios. This is as it should be for risk neutral investors.

The following Table 2 will illustrate the relationship between Weitzman discounting and negative compounding, that is, compounding with negative interest rates. The two states of the world are still assumed to be equally likely, and t = 200. The initial amount compounded and discounted is $1, corresponding to which EPV is the expected discount factor, and EFV is the expected compound factor.

Table 2

COMPARISON OF NEGATIVE COMPOUNDING AND WEITZMAN DISCOUNTING

Compounding		Weitzman discounting	
r_1	-1%	r_1	1%
r_2	-5%	r_2	5%
$EFV = p_1 e^{r_1 t} + p_2 e^{r_2 t}$	0.06769	$EPV = p_1 e^{-r_1 t} + p_2 e^{-r_2 t}$	0.06769
Certainty equivalent compound rate R = ln(*EFV*) / 200	-1.35%	Certainty equivalent discount rate R_W = ln(1/*EPV*) / 200	1.35%

[9] Thereby loosing information.

What the above shows, is that the expected compound factor, when interest rates are negative, is the same as the expected discount factor implicit in Weitzman discounting. The absolute values of the implicit CERs are identical, but their signs differ, due to the fact that the first is computed from an EFV factor, while the second is computed from what is taken to be an EPV.

But note that if for the compounding case EFV = $e^{(-0.0135)(200)}$ we specified that t = -200, reversing the arrow of time, then for the EFV to be invariant we would have to multiply the CER by -1, which will then become the same as R_W. Consequently, Weitzman discounting is the same as time reversed negative compounding. It shifts the origin of time measurement to when the future value is due, which it then negatively compounds backwards to the present, in lieu of discounting.

The inverse of compounding is discounting, and the two will always be consistent and will yield the same CERs. Weitzman discounting also has an inverse, with which it is internally consistent, and with which it shares CERs. The inverse of time reversed negative compounding is time reversed negative discounting, which is the expected value of the discount factors curves in the negative time range, shown in Figure 2. Just as time reversed negative compounding is a kind of pseudo-discounting that exponentially declines while moving backwards in time, negative discounting is a kind of pseudo-compounding that exponentially grows while moving backwards in time. Time reversed negatively discounting \$745.74 (the Weitzman EPV computed with the data of Table 1) will yield \$11,016.93 by year -200. Time reversed negative discounting of \$1 accruing in year 0 will result in \$14.77 in year -200, whereas compounding the same amount yields \$11,016.93 in year 200, using the same interest rate assumptions.

Thus time reversed compounding and discounting are mutually consistent, but yield values that are very different from those derived from compounding and discounting when the arrow of time is not turned around.

2.2 Negative compounding in the capital markets

The use of Weitzman discounting (and hence of DDRs) leads to the violation of the requirement that preferences be transitive. According to Weitzman discounting, the EPV of \$11,016.93 (A) due in year 200 is \$745.74 (B), so the investor should be indifferent between them (A~B). The EFV in year 200 of investing \$745.74 (B) is \$8,215,765.37 (C), so the investor should be indifferent between them (B~C). A~B~C means that the investor should be indifferent between having \$11,016.93 and having \$8,215,765.37, both in year 200. An investor using DDRs would be a money pump, which is only possible because of the implicit violation of the transitivity of his preferences.

As there is no dispute about how to compute EFVs, it is the use negative compounding to measure EPVs that must be wrong.

The inconsistency between the CERs derived from negative compounding and the kind of compounding that operates in real capital markets will ensure that markets will not behave as Weitzman discounting would dictate. Therefore, it is not the CERs derived from Weitzman discounting that will be characteristic of capital markets, but that which is derived from either compounding or discounting, as conventionally defined.

3. Correlation of interest rates

Substituting discounting for negative compounding in Weitzman's model will result in increasing discount rates, rather than declining ones. This conclusion still depends on the key implicit assumption that annual interest rates are perfectly auto-correlated through time, however. This assumption is a necessary consequence of having a two-period model represent the very long run, in which a single interest rate will prevail unaltered between year 0 and year *t*. Gollier (2009:4) concedes that "the decreasing nature of the term structure obtained in this framework depends heavily upon the assumption that shocks on the interest rate are permanent. If they are purely transitory, the term structure of discount rates should be flat."

Arrow *et al.* (2014) review several empirical estimates of possible DDR schedules for the United States. One of the earliest ones was made by Richard Newell and William Pizer (2001:18), which did not find the statistical evidence for the correlation required to observe DDRs to be particularly strong: "[the] inconsistency between the mean-reverting forecasts and the realized interest rate is particularly troubling because we know that the lower range of possible interest rates ultimately determines the future certainty-equivalent rate. Because the random walk model does a better job of predicting this possibility, we find it more compelling for our application, even though evidence based on standard statistical tests is ambiguous."

However, Newell and Pizer (2001:7) also defined (in its expression 3) the certainty equivalent discount factor $E[P_t]$ by using negative compounding:

$$E[P_t] = E\left[\exp\left(-\sum_{s=1}^{t} r_s\right)\right] \tag{11}$$

Were this be replaced by the expression corresponding to discounting, $E[P_t]$ would become:

$$E[P_t] = 1 \Big/ E\left[\exp\left(\sum_{s=1}^{t} r_s\right)\right] \tag{12}$$

on the basis of which their model should show, to the extent justified by the observed correlation, a schedule of increasing, rather than declining discount rates. The same will be true of the other instances of the empirical ENPV literature.

If the misunderstanding about ENPV were corrected, this branch of the literature should suggest the use of growing, rather than declining discount rates. But since the evidence for the requisite correlation of interest rates is not sufficiently strong, it is safer to just conclude that the use of DDRs is not supported by either theory or empirical evidence.

4. The Choice of Discount Rate in Project Analysis

The classic classification of approaches to discounting is due to Arrow *et al.* (1996), according to which the "prescriptive" approach seeks to define a social welfare function, while the "descriptive approach" "focusses on the (risk adjusted) opportunity cost of capital" (page 132).

Arguably, the first part of Arrow *et al.* (2014) could be classified as being prescriptive. Its equation (1) (p. 148), which presents the Ramsey formula that is the starting point of the arguments made in its first part, is the same as Equation 4.1 of Arrow *et al.* (1996:131), which defines the social time preference rate as a function of the pure rate of time preference, the growth rate of consumption and its marginal utility.

The second part could be considered to be descriptive, as its main focus is determining the certainty equivalent of market interest rates, hence of the expected opportunity cost of capital.

Therefore, the results of the first part could be taken to express time preferences, while the second to define consumption reallocation opportunities, thus defining alternative discount rates that are theoretically the same only when the optimal allocation of consumption is reached. In this regard Arrow *et al.* (2014:155-156) states that "In an optimal growth model (e.g., the Ramsey model), the consumption rate of discount will equal the marginal product of capital along an optimal consumption path. [] But what if society is *not* on an optimal consumption path? In this case, theory tells us that we need to calculate the social opportunity cost of capital."

In a dynamic world, society is unlikely to ever be on the optimal consumption path, but it will strive to move towards it. It is the role of Cost Benefit Analysis (CBA) to evaluate public investment projects to determine whether they would be welfare enhancing or not. Opinions diverge of how to account for the social opportunity cost of capital in CBA[10], which also depends on the style of CBA being employed (choice of numeraire). This complex question needs not be settled for the purposes of the arguments to be made here, however, as long as it could be said that the estimated social opportunity cost of capital would be a positive, monotonic function of the monetary opportunity cost of capital.

As the main concern of Arrow *et al.* (2014), in both of its parts, was the impact of uncertainty on discount rates, the discount rates suggested are taken to be certainty equivalents[11]. Let's assume, along with Arrow *et al.* (2014:146, footnote 2), that we can "ignore uncertainty in the stream of benefits and costs associated with a project, effectively assuming that these have been converted to certainty-equivalents," while adding to this that the techniques of CBA will have been used to adjust such flows for any market distortions present.

[10] For the opinion of this author, written before the discovery reported on in this paper, see Szekeres (2011) "Discounting in Cost-Benefit Analysis."

[11] The discussion of certainty equivalents in the second part of Arrow *et al.* (2014) is framed in the context of risk neutrality.

How should project analysis use the alternative welfare discount rate ρ_t and the social opportunity cost of capital R_t? If a given project has investment costs of I in year 0 and benefits B in year t, then, for it to be economically feasible, it must have positive ENPVs at both discount rates:

$$B\,e^{-t\rho_t} - I > 0 \tag{13}$$

$$B\,e^{-tR_t} - I > 0 \tag{14}$$

If a project failed to meet condition (13), then investing in it would be welfare reducing, as evaluated by the welfare function, and should therefore be rejected. This step could be called *valuation*, as it only depends on the welfare function.

If a project failed to meet condition (14), then investing in it would also be welfare reducing, even if it met condition (13), because the opportunity cost of the project would be higher than its benefits, and therefore it should be rejected. For such projects it will be true that

$$\frac{B}{e^{t\rho_t}} < \frac{I\,e^{tR_t}}{e^{t\rho_t}} \tag{15}$$

In such cases, welfare would be enhanced by diverting investment cost I to the capital markets, as amount I would compound to a value higher than B by year t. Checking for compliance with (14) could be called (opportunity cost) *discounting*, and is a required step in CBA, for it attributes the correct opportunity cost of capital to the project, thereby ensuring that the ENPV computed is a true measure of the welfare surplus generated by the project.[12] Notice that (14) and (15) are equivalent, the only difference being that in (14) present values are compared, whereas in (15) the time preference discounted future values are compared. In this comparison, valuation only provides a scaling factor that does not affect the outcome of the comparison, and for this reason it could be omitted, just like in (14).

The decision rule to be adopted in project analysis is that for projects to be accepted, they should have a positive ENPV when discounted at the higher of ρ_t and R_t.

This paper has not addressed the first part of Arrow *et al.* (2014), so its conclusion that ρ_t declines as a function of time is assumed to hold. However R_t does not decline as a function of time, as shown in the preceding Sections. It is generally assumed that that $\rho_t < R_t$.[13] This difference would become ever more pronounced with the passage of time if the former declined and the latter did not. Therefore, R_t will be the higher of the two.

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[12] Many writers on this topic have complained that this step short-changes future generations and have proposed that valuation replace opportunity cost discounting. But valuation is present in the denominators of (15). Even those who argue that ρ_t should equal zero for ethical reasons should prefer that those alive in year t have Ie^{tR}, rather than just B. Rejecting inefficient projects does not necessarily increase present day consumption at the expense of that of future generations. If amount I is not taken for the project at hand, but is left in the capital markets, wealth at time t will increase by Ie^{tR}, rather than just by B. As Gary S. Becker *et al.* (2010:18) stated, "Future generations would not thank us for investing in a low-return project."

[13] Were it not the case, it would signal that society should no longer increase its investments, but should consume more in the present, except for projects with returns sufficiently exceeding R_t.

As project analysis should use the higher of these discount rates, only R_t is relevant in practice. As R_t is not declining, DDRs have no role in the analysis of public projects.

5. Conclusions

The literature supporting DDRs was given impetus by Weitzman's (1998) assertion that, in the presence of stochastic but perfectly auto-correlated interest rates, certainty equivalent discount rates would be a declining function of time and tend to the lowest possible interest rate. This paper has shown that this result was due to the use of time reversed negative compounding instead of discounting to compute EPVs. Using discounting instead, the proper conclusion of the Weitzman (1998) model is that certainty equivalent discount rates are an increasing function of time and tend to the highest possible rate.

This conclusion is contingent on the assumption of perfect auto-correlation of stochastic interest rates. If there is no correlation, then the term structure of interest rates will be flat, meaning that certainty equivalent discount rates will be constant. Because the empirical evidence for the requisite auto-correlation is not sufficiently robust, the conclusion that certainty equivalent discount rates should be growing cannot be asserted with confidence, but that they should not be declining can.

For a public sector project to be welfare enhancing, it is not enough for it to generate future benefits that are valued at least as highly, in welfare terms, as its present investment costs, but must also yield a return that is superior to what its investment costs could earn in the capital markets, provided that the comparison has been made using the adjustments required to take market imperfections and non-market welfare impacts into account. This is so because the option of investing in the capital markets exists, and if that option yielded greater future benefits than the project being analyzed, then accepting the project would reduce welfare from what it could otherwise be.

As the discount rate used to compare welfare values is likely to be below that which measures the opportunity cost of capital (otherwise consumption should be increased at the expense of investments), it is latter that should be used as a hurdle discount rate in project analysis. As this has been shown not to be a declining function of time, it can be concluded that governments should not use DDRs in project analysis.

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