# Explaining the Smile in Currency Options: Is it Anchoring?

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**Abstract**

An anchoring adjusted currency option pricing formula is developed in which the risk of the underlying currency is used as a starting point which gets adjusted upwards to arrive at the currency call risk. Anchoring bias implies that such adjustments are insufficient. The new formula converges to the Garman-Kohlhagen formula in the absence of anchoring bias. Anchoring bias generates the implied volatility smile if investors hold heterogeneous exchange rate expectations.

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[1] An earlier version was circulated with the title, "Analogy based valuation of currency options".

# Explaining the Smile in Currency Options: Is it Anchoring?

Arguably, the most popular currency option pricing model among traders is the Garman-Kohlhagen model, which is the Black Scholes model (see Black and Scholes (1973)) suitably modified for currency options (See Garman and Kohlhagen (1983)). The existence of the implied volatility smile in currency options is inconsistent with the model. If the model is correct then implied volatility should not vary with strike, however, at-the-money currency options typically have lower implied volatility than in-the-money and out-of-the-money options (see Derman (2003)).

The Black-Scholes/Garman-Kohlhagen model assumes that markets are informationally efficient in the sense that risk adjusted return from a currency option is equal to the risk adjusted return from the underlying currency itself. A pre-requisite for informational efficiency is that all risks are correctly perceived. However, risk is a highly subjective notion, and the argument that risks are often misperceived in significant ways even when stakes are high is frequently made. See Rotheli (2010), Bhattacharya, Goldman, & Sood (2009), Akerlof and Shiller (2009), and Barberis & Thaler (2002) among many others.

How does one form risk judgments about currency call options? A currency call option's payoffs move in the same direction as the movement in the underlying currency; however, as it is a leveraged position in the underlying currency, it is clearly riskier. Hence, it is reasonable to start with the risk of the underlying currency and add to it to arrive at a risk judgment regarding a currency call option. However, using the risk of the underlying currency as a starting point to form risk judgments regarding currency call options exposes one to the anchoring bias. Kahneman and Tversky (1974) in their experiments noted that adjustments from an initial value are typically insufficient as people tend to stop after they reach a plausible value starting from some initial value. Hence, estimates remain at the boundary of plausible values near the initial value known as the anchor. For a survey of recent research on anchoring, see Furnham and Boo (2011).

As argued in Shiller (1999), anchoring appears to be an important concept for financial markets. Hirshelifer (2001) considers anchoring to be an *"important part of psychology based dynamic asset pricing theory in its infancy"* (p. 1535). Empirical evidence suggests that experienced professionals show economically and statistically significant anchoring bias in forming stock return estimates (see Kausta, Alho, and Pultanen (2008)). Anchoring has been found to matter in the bank loan market as the current spread paid by a firm seems to be anchored to the credit spread the firm had paid earlier (see Douglas, Engelberg, Parsons, and Van Wesep (2015)). The role of anchoring bias has been found to be important in equity markets in how analysts forecast firms' earnings (see Cen, Hillary, and Wei (2013)). Also, Campbell and Sharpe (2009) find that expert consensus forecasts of monthly economic releases are systematically biased toward the value of previous months' releases. Johnson and Schnytzer (2009) show that investors in a particular financial market (horse-race betting) are prone to the anchoring bias.

Given the importance of this bias for financial markets in general, and the presence of this bias in experienced professionals in particular when they form stock return estimates (see Kausta et al (2008)), it is logical to expect that such a bias also exists in call return estimates. This becomes even more compelling when one realizes that a clear starting point exists for call options, which is the underlying stock. In fact, considerable field and experimental evidence suggests that such a bias matters for call options. Market professionals with decades of experience often argue that a call option is a surrogate for the underlying stock.[2] Such opinions are surely indicative of the importance of the underlying stock risk as a starting point for thinking about a call option risk, and point to insufficient adjustment to perceived risk, which creates room for the surrogacy argument. Furthermore, a series of laboratory experiments (see Rockenbach (2004), Siddiqi (2012), and Siddiqi (2011) show that the hypothesis that a call option is priced by equating its expected return to the expected return from the underlying stock outperforms other pricing hypotheses. The results are consistent with the idea that risk of the underlying stock is used as a starting point with the anchoring heuristic ensuring that adjustments to the stock risk to arrive at call risk are insufficient. Hence, expected call returns do not deviate from expected underlying stock returns as much as they should.

Siddiqi (2015) puts forward an anchoring adjusted option pricing formula for equity options and shows that it provides a unified explanation for a number of option pricing puzzles. In this article, I extend the argument in Siddiqi (2015) to currency options. In this article, I show that if the risk of the underlying currency is used as a starting point to form a risk judgment about the

[2] As illustrative examples, see the following:
http://finance.yahoo.com/news/stock-replacement-strategy-reduce-risk-142949569.html
http://ezinearticles.com/?Call-Options-As-an-Alternative-to-Buying-the-Underlying-Security&id=4274772,
http://www.investingblog.org/archives/194/deep-in-the-money-options/
http://www.triplescreenmethod.com/TradersCorner/TC052705.asp,
http://daytrading.about.com/od/stocks/a/OptionsInvest.htm

corresponding call option and anchoring bias leads to insufficient adjustment, then a new option pricing formula arises. The new formula converges to the Black-Scholes/Garman-Kohlhagen formula if there is no anchoring bias. If the prices are determined in accordance with the new formula, and the Black-Scholes/Garman-Kohlhagen formula is used to back out implied volatility, then the smile may arise if exchange rate expectations are heterogeneous. Hence, anchoring provides a potential explanation for the smile in currency options.

The central prediction of asset pricing theory is:

$$E[R_i] = R_F - \frac{1}{E[u'(c_{t+1})]} Cov[u'(c_{t+1}), R_i] \tag{0.1}$$

Where $R_i$ and $R_F$ denote the (gross) return on a risky asset and the return on the risk free asset respectively. Equation (0.1) shows that the return that a subjective expected utility maximizer expects from a risky asset depends on his belief about the covariance of the asset's return with his marginal utility of consumption.

According to (0.1), an investor is required to form a judgment about the covariance of an asset's return with his marginal utility of consumption. It is reasonable to think that instead of forming such judgments in isolation for each asset, such a judgment is formed for a familiar asset and then extrapolated to another similar asset. A currency call option derives its existence from the underlying currency, and their payoffs are strongly related and move together.

With the above in mind, an analogy maker is defined as *a subjective expected utility maximizer who uses the risk of the underlying currency as a reference point for forming risk judgments about the corresponding currency call option.* That is, an analogy maker assesses call risk in comparison with the underlying currency risk. If one forms a judgment about call option risk in comparison with his judgment about the underlying currency risk, then one may write:

$$Cov\left(\frac{u'(c_{t+1})}{E[u'(c_{t+1})]}, R_c\right) = Cov\left(\frac{u'(c_{t+1})}{E[u'(c_{t+1})]}, R_s\right) + \epsilon \tag{0.2}$$

Where $R_c$ and $R_s$ are call and currency returns respectively, and $\epsilon$ is the adjustment used to arrive at call option risk from the underlying stock risk. Almost always, assets pay more (less) when consumption is high (less), hence, the covariance between an asset's return and marginal utility of consumption is negative. That is, I assume that $Cov(u'(c_{t+1}), R_s) < 0$. So, in order to make a call option at least as risky as the underlying currency, I assume $\epsilon \leq 0$. An analogy maker understands that *a call option is a leveraged position in the underlying currency, hence is riskier. However, starting from the risk of the underlying stock, he does not fully adjust for the risk, so he underestimates the risk of a call option.* That is, $|\epsilon|$, which is the absolute value of risk adjustment is not large as it should be.

In contrast, option pricing theory predicts that:

$$Cov\left(\frac{u'(c_{t+1})}{E[u'(c_{t+1})]}, R_c\right) = \pi . Cov\left(\frac{u'(c_{t+1})}{E[u'(c_{t+1})]}, R_s\right) \tag{0.3}$$

Where $\pi > 1$ and typically takes very large values. That is, typically $\pi \gg 1$. To appreciate, the difference between (0.3) and (0.2), note that under the Black Scholes/Garman-Kohlhagen assumptions, $\pi = \Omega$, which is call price elasticity w.r.t the underlying stock price. $\Omega$ takes very large values, especially for out-of-the-money call options. That is $Cov(u'(c_{t+1}), R_c)$ is likely to be a far bigger negative number with correct risk judgment than with analogy making. Hence, a comparison of (0.2) and (0.3) indicates that, with analogy making, one likely remains anchored to the risk of the underlying currency while forming risk judgments about the call option leading to underestimation of its risk.

Substituting (0.2) in (0.1) leads to:

$$E[R_C] = E[R_S] + |\epsilon| \tag{0.4}$$

In contrast, substituting (0.3) in (0.1) under the Black Scholes/Garman-Kohlhagen assumptions yields:

$$E[R_C] = R_F + \Omega \cdot (E[R_S] - R_F) \tag{0.5}$$

$\Omega > 1$ and typically takes very large values. Hence, expected call return is likely to be smaller under analogy making when compared with the Black-Scholes/Garman-Kohlhagen predictions.

Clearly, an analogy maker over-prices a call option. Can rational investors make arbitrage profits against such analogy makers? There are three reasons which make such arbitraging very difficult if not impossible. Firstly, the presence of large transaction costs (currency options are mostly OTC instruments with large bid-ask spreads) in currency options create a significant hurdle for anyone interested in profiting from selling over-priced options and buying replicating portfolios.

Secondly, to a credit constrained seller, selling an over-priced option creates bankruptcy risk whereas buying an option does not create such a risk. The reason is that buying an option confers a right to the buyer without creating an obligation, whereas selling an option creates an obligation. In case of a strong adverse movement, a credit constrained seller may get bankrupt. Finally, a replicating portfolio may not even exist, for example, when the underlying currency follows jump diffusion or stochastic volatility. In this article, I assume geometric Brownian motion for simplicity; however, it is easy to extend the argument to more general processes such as jump diffusion and/or stochastic volatility.

**1. Analogy Making and the Value of Currency Options**

Suppose the marginal investor in a currency call option with a strike of $K$ is an analogy maker. If $C$ denotes the domestic currency price of a call option on one unit of foreign currency, $\mu$ is the expected percentage appreciation in the domestic currency price of foreign currency, and $r_F$ is the foreign interest rate on a risk free bond, then over a time interval $dt$:

$$\frac{E[dC]}{C} = \frac{E[dS]}{S} + |\epsilon_K| = \mu + r_F + |\epsilon_K| \qquad (1.1)$$

That is, the expected return from holding the foreign currency is $\mu$, the drift of the exchange rate (domestic units per foreign unit), plus risk free interest rate earned from holding the foreign currency in a risk free asset such as foreign treasury notes. An analogy maker adds the adjustment term $|\epsilon_K|$ to the return from holding the underlying currency to arrive at the return he expects from a currency call option. As a higher strike call is riskier than a lower strike call, $|\epsilon_K|$ rises with strike.

All the assumptions of the Black-Scholes/Garman-Kohlhagen model are maintained except for two assumptions. Firstly, I allow for (proportional and symmetric) transaction costs, whereas the Black- Scholes/Garman-Kohlhagen model requires that transaction costs must be zero. Secondly, I assume that there is anchoring bias implying that the adjustment to the underlying currency return (risk) to arrive at the currency call return (risk) is insufficient. If $\theta$ is the percentage transaction cost, then (1.1) can be re-written as:

$$\frac{E[dC]}{(1+\theta)C} = \frac{E[dS]}{(1+\theta)S} + \epsilon_K$$

$$=> \frac{E[dC]}{C} = \mu + r_F + \epsilon_K(1+\theta) \tag{1.2}$$

As in the Black-Scholes/Garman Kohlhagen model, I assume that the spot exchange rate (domestic units per foreign unit) follows geometric Brownian motion with drift $\mu$ and standard deviation equal to $\sigma$:

$$dS = \mu S dt + \sigma S dW \tag{1.3}$$

Equations (1.2) and (1.3) are sufficient to arrive at a partial differential equation (PDE) that a currency call option must satisfy under analogy making. The associated PDE is described in proposition 1.

**Proposition 1** ***If analogy making determines the price of a European currency call option in the presence of transaction costs, then its price must satisfy the following PDE:***

$$\boldsymbol{\frac{\partial C}{\partial t} + \mu S \frac{\partial C}{\partial S} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 C}{\partial S^2} = \left(\mu + r_F + \epsilon_K(1+\theta)\right)C} \tag{1.4}$$

**With the boundary condition that at expiry, which is at time $\boldsymbol{T}$, the value of a European call option with a strike of $\boldsymbol{K}$ is given by: $\boldsymbol{C = max(S-K, 0)}$**

**Proof.**

See Appendix A.

■

It is interesting to note that (1.4) is equal to the Garman-Kohlhagen PDE/Black Scholes PDE for currency options if the adjustment is correct, and there are no transaction costs. To see this clearly, note that the correct adjustment to underlying currency return (risk) for estimating currency call return (risk) when $(\theta = 0)$ is:

$$|\epsilon_K| = (\Omega_K - 1)(\mu + r_F - r_D) \tag{1.5}$$

Where $\Omega_K$ is call price elasticity w.r.t the underlying currency exchange rate. That is, $\Omega_K = \frac{S}{C}\frac{\partial C}{\partial S}$.

Substituting (1.5) in (1.4) when $\theta = 0$ leads to:

$$\frac{\partial C}{\partial t} + (r_D - r_F)S\frac{\partial C}{\partial S} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 C}{\partial S^2} = r_D C \qquad (1.6)$$

(1.6) is the Garman-Kohlhagen/Black Scholes PDE for currency call options.

As can be seen from (1.6), correct adjustment leads to the Garman-Kohlhagen/Black-Scholes PDE for currency call options. However, the presence of anchoring bias $\left(\epsilon_K < (\Omega_K - 1)(\mu + r_F - r_D)\right)$ and transaction costs imply that the PDE to solve is given in (1.4).

A special case is the situation where the marginal investor is risk neutral with zero transaction costs. With risk neutrality, it follows that $\mu = r_D - r_F$. It is easy to see that risk neutrality also makes analogy PDE equal to Garman-Kohlhagen PDE (1.4 is equal to 1.6).

It is clear that 1.4 becomes equal to 1.6 in two cases: 1) Risk aversion with correct adjustment, and 2) Risk neutrality. Hence, risk aversion with incorrect adjustment, that is, risk aversion with the anchoring bias creates a divergence between the analogy PDE and the Garman-Kohlhagen PDE.

By using a similar method to what is used in solving the Black-Scholes/Garman-Kohlhagen PDE, the analogy based PDE given in (1.4) can be solved to recover an option pricing formula for a currency call option under analogy making. Proposition 2 presents the formula. By using put-call parity, the price of a currency put option is also obtained.

**Proposition 2** ***If analogy making determines the price of a currency call option in the presence of transaction costs, then the corresponding European call option pricing formula is:***

$$\boldsymbol{C = e^{-(|\epsilon_K|(1+\theta)+r_F)(T-t)}\left\{SN(d_1) - Ke^{-\mu(T-t)}N(d_2)\right\}} \qquad \textbf{(1.7)}$$

**Where** $\boldsymbol{d_1 = \frac{ln\left(\frac{S}{K}\right)+\left(\mu+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}}$ **and** $\boldsymbol{d_2 = \frac{ln\left(\frac{S}{K}\right)+\left(\mu-\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}}$

**Proof.**

See Appendix B.

■

*Corollary 2.1* **The price of a corresponding European put option under analogy making is given by:**

$$\boldsymbol{P} = \boldsymbol{K}\left\{e^{-(r_D)(T-t)} - e^{-(r_F+|\epsilon_K|(1+\theta)+\mu)(T-t)}N(d_2)\right\} - \boldsymbol{S}\left\{e^{-r_F(T-t)} - e^{-\left(r_F+|\epsilon_K|(1+\theta)\right)(T-t)}N(d_1)\right\} \quad \textbf{(1.8)}$$

**Proof.**

Follows from put-call parity for exchange rate options.

■

Next, I show that if option prices are determined in accordance with the analogy formula, and the Garman-Kohlhagen/Black-Scholes formula is used to back-out implied volatility, the smile may arise with heterogeneous expectations.

## 2. The Implied Volatility Smile

An analogy maker uses the return (risk) of the underlying currency as a starting point to form his judgment about the return (risk) of the currency call option, with the anchoring bias leaving him short of the correct adjustment.

There are two broad cases to consider: 1) Homogeneous expectations: marginal investors in all call options use the same starting point. It is the simplest case to analyze, however, not very realistic. 2) Heterogeneous expectations: marginal investors in higher strike calls use higher starting points. That is, marginal investors in higher strike calls are more optimistic about the return from the underlying currency than marginal investors in lower strike calls. This is more realistic.

For the purpose of illustration, the following parameter values are assumed: $S = 100, T - t = 1\ year, \sigma = 20\%, r_D = 2\%, and\ r_F = 3\%$. These values are sufficient to price European currency calls with various strikes by Garman-Kohlhagen (GK) formula. The prices are shown in Table 1 under the heading GK. To use the analogy formula, one also needs to know $\mu$ and $|\epsilon_K|$. For a given value of $\mu$, there must exist a value of $|\epsilon_K|$, which makes the analogy price equal to the GK price. By definition, that value of $|\epsilon_K|$ is the correct adjustment needed to arrive at the correct call return starting from the underlying currency return. Table 1 lists the correct adjustments corresponding to various values of $\mu$. For example, when $\mu = 0\%, and\ \frac{K}{S} = 0.90$, then the correct adjustment is 4.47%.

**Table 1**

| **K/S** | **GK** | **Correct $\|\epsilon_K\|$ when $\mu = 0\%$** | **Correct $\|\epsilon_K\|$ when $\mu = 1\%$** | **Correct $\|\epsilon_K\|$ when $\mu = 2\%$** | **Correct $\|\epsilon_K\|$ when $\mu = 3\%$** |
|---|---|---|---|---|---|
| 0.9 | 12.6116 | 4.47% | 8.81% | 13.04% | 17.16% |
| 0.95 | 9.6969 | 5.14% | 10.15% | 15.04% | 19.80% |
| 1.0 | 7.2910 | 5.85% | 11.55% | 17.12% | 22.55% |
| 1.05 | 5.3664 | 6.57% | 13% | 19.27% | 25.39% |
| 1.1 | 3.8715 | 7.31% | 14.46% | 21.45% | 28.28% |

As Table 1 shows, if investors use correct adjustments, then it does not matter which formula they use as the analogy formula and the GK formula give the same answer. That is, if the analogy formula determines real world prices, and the GK formula is used to infer implied volatility, then the implied volatility would be 20% (correct value of $\sigma$) as long as the adjustment is correct. However, as the large literature on anchoring bias argues, the adjustments from a starting point are usually insufficient. What happens with insufficient adjustment? For the purpose of illustration, I assume that adjustments are always 50% of the correct adjustment. That is, from a given starting point, investors only reach half-way to the correct adjustment. Table 2 shows the values of implied volatility with the assumed half-way adjustment.

| Table 2<br>Implied Volatility with Anchoring Bias | | | | |
|---|---|---|---|---|
| **K/S** | $\boldsymbol{Actual\ \lvert\epsilon_K\rvert = \frac{Correct\ \lvert\epsilon_K\rvert}{2}}$<br>$\boldsymbol{\mu = 0\%}$ | $\boldsymbol{Actual\ \lvert\epsilon_K\rvert = \frac{Correct\ \lvert\epsilon_K\rvert}{2}}$<br>$\boldsymbol{\mu = 1\%}$ | $\boldsymbol{Actual\ \lvert\epsilon_K\rvert = \frac{Correct\ \lvert\epsilon_K\rvert}{2}}$<br>$\boldsymbol{\mu = 2\%}$ | $\boldsymbol{Actual\ \lvert\epsilon_K\rvert = \frac{Correct\ \lvert\epsilon_K\rvert}{2}}$<br>$\boldsymbol{\mu = 3\%}$ |
| 0.9 | 20.86367% | 21.71874% | 22.56226% | 23.39497% |
| 0.95 | 20.68371% | 21.36648% | 22.04672% | 22.72570% |
| 1.0 | 20.55946% | 21.12219% | 21.68592% | 22.25136% |
| 1.05 | 20.47190% | 20.94630% | 21.42449% | 21.90536% |
| 1.1 | 20.40684% | 20.81708% | 21.23064% | 21.64701% |

Table 2 shows that the implied volatility falls monotonically with strike if expectations are homogeneous.

As discussed earlier, assuming that expectations are homogeneous is not very realistic. It is likely that more optimistic investors self-select into higher strike calls. Also, note that it does not make much sense for an investor with a pessimistic outlook to buy a call option. After all, a call option is a leveraged position in the underlying currency. Buying a currency call with a pessimistic outlook is equivalent to intentionally magnifying one's losses. That is, one must at least have a neutral outlook about the underlying currency if he buys a currency call option. So, it makes sense to argue that investors self-select with neutral to moderately bullish investors selecting low strike calls, and strongly bullish buying higher strike calls. As an illustration of this point, continuing with the example presented in Table 2, I assume that $\mu = 0\%$ when $\frac{K}{S}$ is 0.90, 0.95, and 1.0 (neutral), $\mu = 1\%$ when $\frac{K}{S}$ is 1.05 (moderately bullish), and $\mu = 2\%$ when $\frac{K}{S}$ is 1.1 (strongly bullish). Figure 1 is a graphical illustration of the resulting smile.

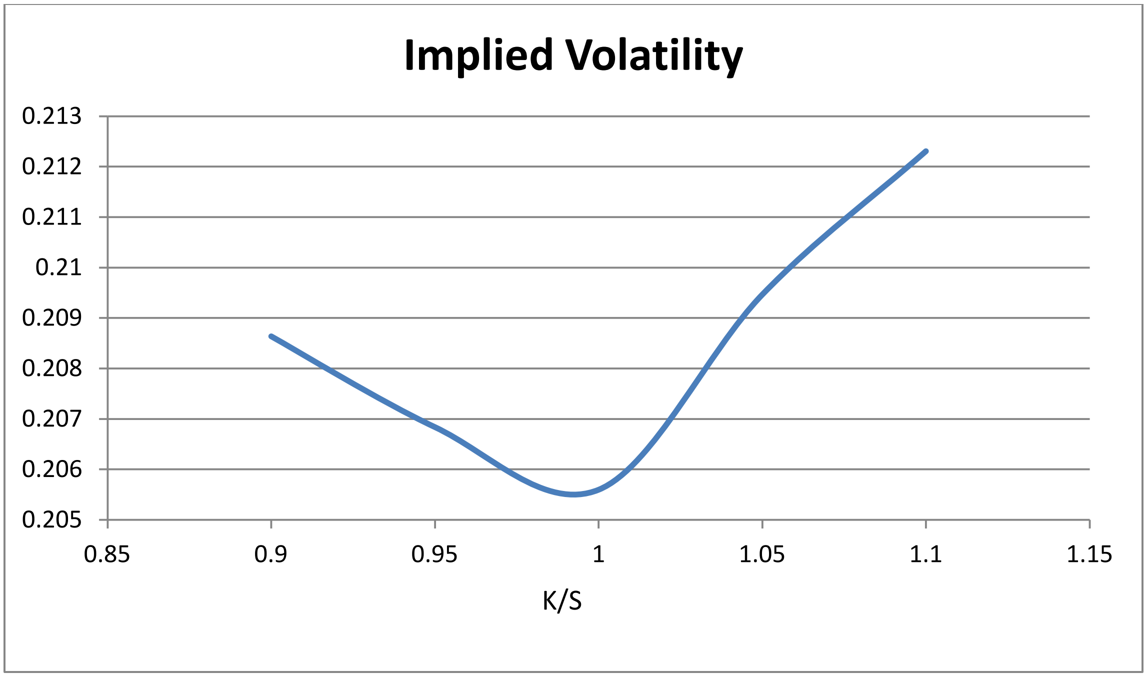


**Figure 1**

As Figure 1 shows, the anchoring bias with heterogeneous expectations may give rise to implied volatility smile. If the anchoring approach is correct, then the degree of heterogeneity or belief dispersion must have a direct impact on the structure of the smile. One would expect greater belief dispersion to increase the curvature of the smile. This is consistent with empirical evidence that heterogeneous exchange rate beliefs are an important determinant of the shape of the implied volatility smile, and greater belief dispersion increases the curvature of the smile (see Beber, A., Buraschi, A., and Breedon, F. (2010)).

## 3. Comparative Statics

For the purpose of hedging risks arising from currency option exposure, partial derivatives of (1.7) and (1.8) are of special interest with 'hedge ratio' being of prime importance:

$$\frac{\partial C}{\partial S} = e^{-(r_F+|\epsilon_K|(1+\theta))(T-t)}N(d_1) > 0 \tag{3.1}$$

Getting the hedge ratio right is of key importance to currency option traders, and they are potentially exposed to large losses without it.

Some other partial derivatives are:

$$\frac{\partial C}{\partial K} = -e^{-(\mu+r_F+|\epsilon_K|(1+\theta))(T-t)}N(d_2) < 0 \tag{3.2}$$

$$\frac{\partial C}{\partial \mu} = (T-t)e^{-(\mu+r_F+|\epsilon_K|(1+\theta))(T-t)}KN(d_1) > 0 \tag{3.3}$$

$$\frac{\partial C}{\partial r_D} = 0 \tag{3.4}$$

$$\frac{\partial C}{\partial r_F} = -(T-t)e^{-(r_F+|\epsilon_K|(1+\theta))(T-t)}SN(d_1) + (T-t)e^{-(\mu+r_F+|\epsilon_K|(1+\theta))(T-t)}KN(d_2) \tag{3.5}$$

To a speculator interested in taking directional bets on exchange rates, these partial derivatives are important as they constitute the risk constraints in a linear programming problem.[3] If prices are determined in accordance with the analogy formula, and Garman-Kohlhagen formula is used to estimate these partial derivatives, quite a few additional problems arise apart from getting the hedge ratio wrong as mentioned earlier.

Firstly, exchange rate expectations have no direct impact in Garman-Kohlhagen setting beyond their impact on spot rate, whereas under analogy making, an increase in expected appreciation directly increases the value of currency call options as (3.3) shows, over and above any indirect impact on the spot rate.

Secondly, in sharp contrast with Garman-Kohlhagen formula, an increase in domestic interest rate does not directly affect the call price. Of course, there are indirect effects due to impacts on exchange rate expectations and the spot rate. An increase in foreign interest rates has a negative impact on call price in Garman-Kohlhagen formula, whereas the impact is ambiguous under analogy making, and can be positive for out-of-the-money call options.

Last but not the least, the presence of transaction costs dampens the magnitude of nearly all partial derivatives.

[3] See Rendlemen (2002) (Chapter 7) for details on setting up linear programming problems associated with directional bets.

The partial derivatives of put option are similarly obtained from (1.8) and they have opposite signs.

## 4. Conclusions

People choose which options to buy based on subjective reasons. This fact compels one to think that investor expectations matter for option demands and prices. Indeed, Bollen and Whaley (2004) find that demand pressures associated with various option series matter for implied volatility. However, theoretical option pricing models typically do not have any role for investor expectations, even though they provide an elegant framework of analysis. It seems that the challenge is to incorporate investor expectations in the elegant option pricing framework with the goal of increasing its explanatory power. The idea of analogy making provides a tool to do just that.

As discussed in the introduction, analogy making approach is inspired by experienced market professionals, who consider the underlying instrument risk to be a starting point for forming risk judgments about corresponding call options. An analogy maker is a subjective expected utility maximize who uses the risk of the underlying instrument as a starting point to which he adds to, with an objective of forming risk judgments regarding corresponding call options. Anchoring bias implies that such adjustments are insufficient.

In this article, the approach is extended to currency options. The approach leads to an analogy based currency option pricing formula which contains the Garman-Kohlhagen formula as a special case corresponding to the absence of the anchoring bias. With anchoring bias, the implied volatility smile may arise.

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## Appendix A

$$\frac{E[dC]}{C} = \mu + r_F + |\epsilon_K|(1+\theta) \tag{A1}$$

From Ito's Lemma:

$$E[dC] = \left(\frac{\partial C}{\partial t} + \mu S\frac{\partial C}{\partial S} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 C}{\partial S^2}\right)dC \tag{A2}$$

Substituting (A2) in (A1) and simplifying leads to:

$$\frac{\partial C}{\partial t} + \mu S\frac{\partial C}{\partial S} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 C}{\partial S^2} = \big(\mu + r_F + |\epsilon_K|(1+\theta)\big)C \tag{A3}$$

(A3) is the analogy based PDE for a currency call option with the boundary condition:

$$C(S,T) = max(S-K,0)$$

## Appendix B

Start by making the following transformations in (A3):

$$\tau = \frac{\sigma^2}{2}(T - t)$$

$$x = ln\frac{S}{K} => S = Ke^x$$

$$C(S,t) = K \cdot\ c(x,\tau) = K \cdot c\left(ln\left(\frac{S}{K}\right), \frac{\sigma^2}{2}(T-t)\right)$$

It follows,

$$\frac{\partial C}{\partial t} = K \cdot \frac{\partial c}{\partial \tau} \cdot \frac{\partial \tau}{\partial t} = K \cdot \frac{\partial c}{\partial \tau} \cdot \left(-\frac{\sigma^2}{2}\right)$$

$$\frac{\partial C}{\partial S} = K \cdot \frac{\partial c}{\partial x} \cdot \frac{\partial x}{\partial S} = K \cdot \frac{\partial c}{\partial x} \cdot \frac{1}{S}$$

$$\frac{\partial^2 C}{\partial S^2} = K \cdot \frac{1}{S^2} \cdot \frac{\partial^2 C}{\partial x^2} - \ K \cdot \frac{1}{S^2}\frac{\partial C}{\partial x}$$

Plugging the above transformations into (A3) and writing $\tilde{r} = \frac{2(\mu)}{\sigma^2}$, $\widetilde{r_F} = \frac{2r_F}{\sigma^2}$, and $\widetilde{\epsilon_K} = \frac{2|\epsilon_K|(1+\theta)}{\sigma^2}$we get:

$$\frac{\partial c}{\partial \tau} = \frac{\partial^2 c}{\partial x^2} + (\tilde{r} - 1)\frac{\partial c}{\partial x} - (\tilde{r} + \widetilde{r_F} + \widetilde{\epsilon_K})c \qquad (B1)$$

With the boundary condition/initial condition:

$$C(S,T) = max\{S - K, 0\}\ becomes\ c(x,0) = max\{e^x - 1,0\}$$

To eliminate the last two terms in (B1), an additional transformation is made:

$$c(x,\tau) = e^{\alpha x + \beta \tau}u(x,\tau)$$

It follows,

$$\frac{\partial c}{\partial x} = \alpha e^{\alpha x+\beta\tau}u + e^{\alpha x+\beta\tau}\frac{\partial u}{\partial x}$$

$$\frac{\partial^2 c}{\partial x^2} = \alpha^2 e^{\alpha x+\beta\tau}u + 2\alpha e^{\alpha x+\beta\tau}\frac{\partial u}{\partial x} + e^{\alpha x+\beta\tau}\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial c}{\partial \tau} = \beta e^{\alpha x+\beta\tau}u + e^{\alpha x+\beta\tau}\frac{\partial u}{\partial \tau}$$

Substituting the above transformations in (B1), we get:

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2} + (\alpha^2 + \alpha(\tilde{r}-1) - (\tilde{r} + \widetilde{r_F} + \widetilde{\epsilon_K}) - \beta)u + \left(2\alpha + (\tilde{r}-1)\right)\frac{\partial u}{\partial x} \tag{B2}$$

Choose $\alpha = -\frac{(\tilde{r}-1)}{2}$ and $\beta = -\frac{(\tilde{r}+1)^2}{4} - (\widetilde{r_F} + \widetilde{\epsilon_K})$. (B2) simplifies to the Heat equation:

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2} \tag{$B3$}$$

With the initial condition:

$$u(x_0, 0) = max\{(e^{(1-\alpha)x_0} - e^{-\alpha x_0}), 0\} = max\left\{\left(e^{\left(\frac{\tilde{r}+1}{2}\right)x_0} - e^{\left(\frac{\tilde{r}-1}{2}\right)x_0}\right), 0\right\}$$

The solution to the Heat equation in our case is:

$$u(x,\tau) = \frac{1}{2\sqrt{\pi\tau}}\int_{-\infty}^{\infty} e^{-\frac{(x-x_0)^2}{4\tau}}u(x_0,0)dx_0$$

Change variables: $= \frac{x_0 - x}{\sqrt{2\tau}}$ , which means: $dz = \frac{dx_0}{\sqrt{2\tau}}$. Also, from the boundary condition, we know that $u > 0 \; iff \; x_0 > 0$. Hence, we can restrict the integration range to $z > \frac{-x}{\sqrt{2\tau}}$

$$u(x,\tau) = \frac{1}{\sqrt{2\pi}}\int_{-\frac{x}{\sqrt{2\pi}}}^{\infty} e^{-\frac{z^2}{2}} \cdot e^{\left(\frac{\tilde{r}+1}{2}\right)(x+z\sqrt{2\tau})}dz - \frac{1}{\sqrt{2\pi}}\int_{-\frac{x}{\sqrt{2\tau}}}^{\infty} e^{-\frac{z^2}{2}} \cdot e^{\left(\frac{\tilde{r}-1}{2}\right)(x+z\sqrt{2\tau})}dz$$

$$=: H_1 - H_2$$

Complete the squares for the exponent in $H_1$:

$$\frac{\tilde{r}+1}{2}\left(x+z\sqrt{2\tau}\right)-\frac{z^2}{2}=-\frac{1}{2}\left(z-\frac{\sqrt{2\tau}(\tilde{r}+1)}{2}\right)^2+\frac{\tilde{r}+1}{2}x+\tau\frac{(\tilde{r}+1)^2}{4}$$

$$=:-\frac{1}{2}y^2+c$$

We can see that $dy = dz$ and $c$ does not depend on $z$. Hence, we can write:

$$H_1=\frac{e^c}{\sqrt{2\pi}}\int_{-x/\sqrt{2\pi}-\sqrt{\tau/2}(\tilde{r}+1)}^{\infty} e^{-\frac{y^2}{2}}dy$$

A normally distributed random variable has the following cumulative distribution function:

$$N(d)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d} e^{-\frac{y^2}{2}}dy$$

Hence, $H_1 = e^c N(d_1)$ where $d_1 = x/\sqrt{2\pi} + \sqrt{\tau/2}\,(\tilde{r}+1)$

Similarly, $H_2 = e^f N(d_2)$ where $d_2 = x/\sqrt{2\pi} + \sqrt{\tau/2}\,(\tilde{r}-1)$ and $f = \frac{\tilde{r}-1}{2}x + \tau\frac{(\tilde{r}-1)^2}{4}$

The analogy based European currency call pricing formula is obtained by recovering original variables:

$$C = e^{-(r_F+|\epsilon_K|(1+\theta))}\left\{SN(d_1) - Ke^{-(\mu)(T-t)}N(d_2)\right\}$$

Where $\boldsymbol{d_1} = \frac{\boldsymbol{ln(S/K)+(\mu+\frac{\sigma^2}{2})(T-t)}}{\boldsymbol{\sigma\sqrt{T-t}}}$ $and$ $\boldsymbol{d_2} = \frac{\boldsymbol{ln\left(\frac{S}{K}\right)+\left(\mu-\frac{\sigma^2}{2}\right)(T-t)}}{\boldsymbol{\sigma\sqrt{T-t}}}$