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Abstract

Recent empirical studies have highlighted that valuation effects associated with fluctuations of nominal exchange rates are one of the key components that drive the behavior of the net foreign assets position of a country. In this paper, we propose a two-country overlapping-generations model of nominal exchange rate determination with endogenous portfolio choice in line with this evidence. We show that a country runs a current account deficit when its share of world GDP decreases. As the domestic currency depreciates in equilibrium, a positive wealth effect partially offsets the current deficit and therefore has a stabilizing impact on the net external position of the country. The model rationalizes the deterioration of the US external position over the past 20 years as a consequence of the rise of emerging market countries in the world economy, while being consistent with the fact the US have experienced positive valuation effects. Numerical results indicate that valuation effects are quantitatively relevant as they account for more than half of the cumulated US current account deficits, consistently with the data.

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1 Introduction

Cross-border holdings of assets and liabilities have substantially increased since the early 1990s, for both developed and emerging countries (Lane and Milesi-Ferretti, 2001, 2007). It is well known that one of the consequences of the higher degree of financial integration across countries is the increasing importance of the so called "valuation channel" in the dynamics of net foreign assets (e.g. Gourinchas and Rey, 2007, 2015). Traditionally, the net foreign assets position of a country was simply computed by cumulating current account balances over time. While this measure reflects changes in the stocks of foreign assets and liabilities, it is imperfect as it ignores changes in the value of foreign assets and liabilities which can arise due to fluctuations of nominal exchange rates and asset prices. For instance, Figure 1 shows the divergence between the cumulated current accounts and the net foreign assets position of the United States. According to the former measure, the net foreign assets position of the United States amounted to almost -60% of GDP in 2010. However, direct estimates of net foreign assets and liabilities suggest that the net external position was much lower and equal to around -20% of GDP. This shows the significance of the valuation channel in the dynamics of the net foreign assets of the US. In particular, the US have experienced a substantial wealth transfer from the rest of the world over the past 20 years as the value of their foreign assets has risen relatively to the value of their foreign liabilities. The importance of this channel is not specific to the US: it is interesting to observe that emerging countries in East Asia have experienced the opposite situation (Figure 2). While their net external positions have considerably improved over the past decades because of current account surpluses, they have experienced negative valuation effects[1]. For all the above countries, valuation effects seem to have a stabilizing effect on the net foreign assets position.

One of the challenges in international macroeconomics is to "come up with a new generation of portfolio balance models microfounded and embedded in a general equilibrium set up" so as to explain this and other facts in international financial markets (Gourinchas and Rey, 2015). In this paper, we propose a

[1] Gourinchas and Rey (2015) make similar observations for other emerging countries.

two-country overlapping-generations model with endogenous portfolio choice in which the nominal exchange rate is endogenously determined. The main novelty of our work is that it sheds light on the role of nominal exchange rate in countries' portfolio choices and its impact on the dynamics of net external positions through valuation effects[2].

The nominal exchange rate is an important factor as it operates through two different, but related channels. Firstly, it has an impact on the decision of an agent to allocate his savings across a menu of currencies (or assets denominated in different currencies). It is rational for agents to buy assets denominated in currencies that depreciate, as they are relatively cheaper, but also to buy those assets denominated in currencies which are expected to appreciate, as they have a higher purchasing power in the future. Therefore, the nominal exchange rate matters for quantity decisions.

Secondly, fluctuations of nominal exchange rates have an impact on the net foreign assets position of a country, generating positive or negative valuation effects. It is known that the effect of e.g. a currency depreciation will depend on the currency composition of a country's balance sheet. Lane and Shambaugh (2010) have recently shown that the balance sheet of emerging countries is increasingly similar to the balance sheet of the US and other developed countries, i.e. foreign assets are mainly denominated in foreign currencies while foreign liabilities are mainly denominated in the domestic currency. Figure 3 shows the depreciation of the dollar against the currencies of emerging market economies, especially since 2004[3]. As a consequence, a dollar depreciation does imply positive valuation effects for the US and negative valuation effects for emerging economies, as observed in the data.

This paper provides a theoretical framework suitable to analyze the joint behavior of nominal exchange rates and portfolio choices in a general equilibrium setting, and it is also able to rationalize the above stylized facts. As

[2]In other open economy papers with endogenous portfolio choice, money does not play any role and valuation effects are instead driven by capital gains and losses. For instance, see Pavlova and Rigobon (2007), Heathcote and Perri (2013), Devereux and Sutherland (2010), Tille and Van Wincoop (2010). Tille (2008) makes a first step towards analyzing the wealth effects of exchange rate fluctuations, but portfolios are exogenous in his analysis.

[3]While China and Malaysia do not have a fully flexible exchange rate regime, controls on foreign exchange markets are easing over time (see e.g. IMF, 2014) leading to considerable currency appreciations.

the focus of this paper is modeling valuation effects due to nominal exchange rate fluctuations, we abstract from other sources of valuation effects. It is important to stress that exchange-rate driven valuation effects are known to be empirically important. Lane and Shambaugh (2010) have recently documented that the wealth effects associated with nominal exchange rates fluctuations are substantial as they account for a significant fraction of the overall valuation effects. Moreover, Gourinchas and Rey (2007) have shown that a substantial part of the US cyclical external imbalances are eliminated via predictable movements in nominal exchange rates.

Our framework has two important ingredients: incomplete markets and imperfect substitutability of assets.

Models with complete markets generate very strong predictions, as it is always optimal not to adjust portfolios following a new realization of uncertainty[4]. The fact that portfolio rebalancing is instead observed in the data is evidence that there is some degree of market incompleteness in the real world. In our model, markets are incomplete in the sense that the young cannot insure against the realization of output that they receive when they are born. Moreover, the young lack of a complete set of assets to ensure against risk when old. When markets are incomplete and assets are nominal, the equilibrium allocation can be indeterminate (Balasko and Cass, 1989; Geanakoplos and Mas-Colell, 1989; Polemarchakis, 1988) and this poses particular challenges for applied work. However, if an asset in positive net supply such as money is introduced, the price level can be pinned down and the indeterminacy problem can be avoided (Magill and Quinzii, 1992; Gottardi, 1996; Neumeyer, 1998). Our setting does not suffer from the indeterminacy problem as agents transfer wealth across periods using the two national currencies[5]. As the asset structure is simple, we are able to obtain some analytical results and therefore to gain a very good understanding of agents' portfolio choices as well as the behaviour of the nominal exchange rate. Another advantage of our framework is

[4]See Lucas (1982) and Judd et al. (2003) for a more general version of the Lucas asset pricing model.

[5]If we introduced nominal bonds in zero net supply as well as the currencies, currencies and bonds would be perfect substitutes. As a consequence, the exact allocation of savings between money and bonds would not be determined. For instance, see Gottardi (1994). Therefore, we do not introduce nominal bonds as it would not add too much to our analysis.

that we can compute the global solution of the model, which is known to be more accurate[6]. In particular, we solve for the stationary equilibrium of the model, which is defined as a time-invariant distribution (across state of nature) of nominal prices, exchange rates, consumption and portfolio allocations.

The second important feature of our model is that currencies are imperfect substitutes, as old agents can only buy the country-specific good with the local currency. Since the seminal paper of Kareken and Wallace (1981), it is known that the equilibrium exchange rate and portfolios are not determinate in the absence of some form of legal restrictions in currency trading[7]. This restriction, along with the timing structure, guarantees that the nominal exchange rate is determinate.

The timing is structured as follows. When young, agents receive a state-dependent endowment of the domestic good. They spend part of the domestic output for consumption of both goods in the current period and the rest of their income to buy a portfolio of currencies in view of consuming when old. When old, agents are not allowed to readjust their portfolio after uncertainty realizes so that they are restricted to use the domestic (foreign) currency accumulated in the previous period to buy the domestic (foreign) good. In our model, agents face genuine exchange rate uncertainty, as no portfolio adjustments are possible in the old age. The restriction that agents must buy each good with the local currency is also a feature in Lucas (1982). However, money is not used to transfer wealth across periods in the cash-in-advance literature but to carry out exchange within a given period. As a consequence, the nominal exchange rate is simply a function of current state and does not affect agents' intertemporal decisions[8]. In this paper, the nominal exchange rate is a forward-looking variable which depends on the expected purchasing power of the two currencies weighted by the old's marginal utilities.

In sections 2 and 3, we present the model and define net foreign assets

[6]In the open economy literature, recent work has proposed local solution methods to analyze incomplete markets' model (e.g. Devereux et al. (2010) and Tille et al. (2010)). While these methods can deal with any state space, Rabitsch et al. (2014) showed that the global solution does not always coincide with the local one. See also Coeurdacier and Rey (2012) for a critical assessment of local solution methods.

[7]Sargent (1987) showed that the indeterminacy result holds more generally and is not due to the OLG structure in Kareken and Wallace (1981).

[8]See also Svensson (1985) and Alvarez et al. (2009).

as well as valuation effects in the context of our framework. In section 4, we derive our main analytical result. We show that the country that runs a current account surplus in equilibrium is the country whose share of world GDP has increased over time. As the country is wealthier with respect to the past, the young accumulate more foreign assets and hold less foreign liabilities: at country level, there is a positive change in net foreign assets. We also point out that the surplus country can be poorer than the other country in equilibrium. However, as the country's output grows relatively more than the other country's, its share of world GDP increases. Therefore, our model rationalizes the deterioration of the US external position as the result of the rise of emerging market countries in the world economy.

In section 5, we parametrize the model to illustrate the impact of the nominal exchange rate on the net external position of the US and China. Our finding is that the nominal exchange rate stabilizes the net foreign assets position of each country. The intuition is very simple and can be explained as follows. Because there is persistence in the stochastic process for output, the young expect that prices will stay relatively low in the surplus country (China). As the currency of the surplus country has a higher purchasing power in expectation, the demand for the Chinese currency increases. To restore equilibrium, the currency has to appreciate. Therefore, the surplus (deficit) country experiences negative (positive) valuation effects, consistently with the stylized facts presented above. Our result is also quite robust as it requires mild assumptions such as persistence in the stochastic process for output and the elasticity of substitution between traded goods to be bigger than one so that we avoid episodes of "immiserizing growth".

Another important result is the quantitative relevance of valuation effects. While the model can explain more than a third of the US-China trade imbalances, valuation effects reduce the impact of the US current account deficit on the net foreign assets position by more than a half, consistently with the data.

2 The Model

We consider a two-country pure exchange overlapping-generations economy[9]. In each period, an agent h with a two-period lifetime is born in each country. Therefore, two young and two old populate the world economy at each t.

The young are born with an endowment of the country-specific good ℓ, which is also the total output of the country. Output is denoted as $y^{\ell}(s)$ as it depends on the state of nature realized, where $s = \{1, ..., S\}$. We will use the superscript ℓ to indicate goods and currencies, while we will refer to agents with the subscript h. We assume that output follows a first-order stationary Markov process, where $\rho(ss')$ indicates the probability of transiting from state s to s'. Agents gain utility from the consumption of both goods although they are only endowed with the country-specific good, as in Lucas (1982).

At time 0, the two governments issue fiat money and distribute it to the initial old. M^{ℓ} is the stock of money issued in country ℓ. As the old have no endowment, money is valued in equilibrium as agents would not be able to consume in their second period of life otherwise. For simplicity, we assume that monetary authorities are inactive after the first period. As we study the stationary equilibria of the model, prices will not depend on the history of the shocks but only on the current state of nature.

The timing is organized as follows. In the first period of life, young agents consume part of their endowment of the domestic good and sell the rest to buy the foreign good and the two currencies for saving purposes. Therefore, there is both intra-generational and inter-generational trade in this economy. The two young engage in trade in order to consume the foreign good in the present period. Moreover, they sell part of their endowment to the current old in exchange for money to finance future consumption. We now state the key assumptions of our model.

Assumption 1 *The old can buy good ℓ only with currency M^{ℓ}.*

Assumption 2 *The old cannot adjust their portfolio after the realisation of uncertainty.*

[9]This is with no loss of generality. The model can easily be extended to L countries.

The first restriction that we impose is that agents need the local currency to buy the local good. However, Assumption 1 alone is not useful as agents could hold all their savings in the domestic currency and then buy the foreign currency that they need to buy the foreign good when old in the following period, after uncertainty is realized. In this case, there would be no actual portfolio choice to be made when young and therefore this scenario is not interesting for our purposes. The addition of Assumption 2 guarantees that the young hold a portfolio of two currencies at the end of the period. Moreover, Assumption 2 is important as it introduces an element of exchange rate risk in the agents' decision problems, as uncertainty is realized *after* the currencies are chosen.

These Assumptions are a crucial aspect of the model, as they allow to pin down the equilibrium exchange rate and countries' portfolios. Currencies are not perfect substitutes in the sense that each of them has a specific role, that is to allow agents to consume a particular good. On the contrary, in a world of no legal restrictions in which portfolios and exchange rates are indeterminate, only total money holdings matter and not the currency composition (see Kareken and Wallace, 1981). Moreover, these assumptions are also important for the existence of a stationary equilibrium in itself (see Eugeni, 2013).

We assume the following functional form for the utility function:

$$U_h(s) = \sum_{\ell} \frac{c^{\ell}_{1h}(s)^{1-\frac{1}{\sigma_h}}}{1-\frac{1}{\sigma_h}} + \beta_h \sum_{s'} \rho(ss') \sum_{\ell} \frac{c^{\ell}_{2h}(ss')^{1-\frac{1}{\sigma_h}}}{1-\frac{1}{\sigma_h}} \qquad \sigma_h > 0, \sigma_h \neq 1 \quad (1)$$

Taking as given the vector of transition probabilities and the goods' and currencies' prices, agent h born in state s chooses the consumption vectors and the portfolio of currencies that maximise the above utility function subject to the following constraints:

$$p^1(s)c^1_{1h}(s) + p^2(s)e(s)c^2_{1h}(s) - w_h(s) = -m^1_h(s) - e(s)m^2_h(s) \qquad (2)$$

$$p^1(s')c^1_{2h}(ss') = m^1_h(s) \quad \forall\ s' \qquad (3)$$

$$p^2(s')c^2_{2h}(ss') = m^2_h(s) \quad \forall\ s' \qquad (4)$$

The budget constraint of the young is expressed in units of currency 1, which is our numéraire. $p^{\ell}(s)$ is the nominal price in country ℓ expressed in units of the domestic currency. $e(s)$ is the price of currency 2 in units of currency 1 or the nominal exchange rate. Therefore, we say that if $e(s)$ rises then currency 2 (1) appreciates (depreciates). $w_h(s)$ is the wealth of agent h in units of currency 1, which is equal to the value of the domestic output: $w_1(s) = p^1(s)y^1(s)$ and $w_2(s) = p^2(s)e(s)y^2(s)$.

Notice that, when agents are old, they face two constraints in each state of nature as they use the currencies that they bought in the previous period to purchase each good in the local market with the appropriate currency.

Let $\lambda_h(s)$ be the multiplier associated to the young's budget constraint, $\lambda_h^\ell(ss')$ the multiplier of the constraint of the old related to good ℓ in state s'. The necessary and sufficient conditions for a maximum are the following first-order conditions:

$$\begin{aligned}
c_{1h}^1(s) &: \quad c_{1h}^1(s)^{-\frac{1}{\sigma_h}} = \lambda_h(s)p^1(s) && (5)\\
c_{1h}^2(s) &: \quad c_{1h}^2(s)^{-\frac{1}{\sigma_h}} = \lambda_h(s)p^2(s)e(s) && (6)\\
c_{2h}^\ell(ss') &: \quad \beta_h\rho(ss')c_{2h}^\ell(ss')^{-\frac{1}{\sigma_h}} = \lambda_h^\ell(ss')p^\ell(s') \qquad \forall\ \ell, s' && (7)\\
m_h^1(s) &: \quad -\lambda_h(s) + \sum_{s'}\lambda_h^1(ss') = 0 && (8)\\
m_h^2(s) &: \quad -\lambda_h(s)e(s) + \sum_{s'}\lambda_h^2(ss') = 0 && (9)\\
\lambda_h(s) &: \quad p^1(s)c_{1h}^1(s) + p^2(s)e(s)c_{1h}^2(s) - w_h(s) + \\
&+ \quad m_h^1(s) + e(s)m_h^2(s) = 0 && (10)\\
\lambda_h^1(ss') &: \quad p^1(s')c_{2h}^1(ss') - m_h^1(s) = 0 \qquad \forall\ s' && (11)\\
\lambda_h^2(ss') &: \quad p^2(s')c_{2h}^2(ss') - m_h^2(s) = 0 \qquad \forall\ s' && (12)
\end{aligned}$$

In the Appendix B, we show how to find the following closed-form solutions for the agents' portfolios:

$$m_h^1(s) = \frac{\beta_h^{\sigma_h}\left[\sum_{s'}\rho(ss')p^1(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h}}{A_h(s)}w_h(s) \qquad (13)$$

$$m_h^2(s) = \frac{\beta_h^{\sigma_h}e(s)^{1-\sigma_h}\left[\sum_{s'}\rho(ss')p^2(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h}}{A_h(s)}\frac{w_h(s)}{e(s)} \qquad (14)$$

where

$$A_h(s) \equiv p^1(s)^{1-\sigma_h} + [p^2(s)e(s)]^{1-\sigma_h} + \beta_h^{\sigma_h}\left[\sum_{s'}\rho(ss')p^1(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h} + \\ + \beta_h^{\sigma_h}e(s)^{1-\sigma_h}\left[\sum_{s'}\rho(ss')p^2(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h}$$

Agent h's demand functions can be derived using (13), (14) and the budget constraints (calculations of the demand functions when young are provided in the Appendix):

$$c_{1h}^1(s) = \frac{p^1(s)^{-\sigma_h}}{A_h(s)}w_h(s) \qquad \forall\ \ell \tag{15}$$

$$c_{1h}^2(s) = \frac{[p^2(s)e(s)]^{-\sigma_h}}{A_h(s)}w_h(s) \qquad \forall\ \ell \tag{16}$$

$$c_{2h}^1(ss') = \frac{\beta_h^{\sigma_h}\left[\sum_{s'}\rho(ss')p^1(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h}}{A_h(s)}\frac{w_h(s)}{p^1(s')} \qquad \forall\ s' \tag{17}$$

$$c_{2h}^2(ss') = \frac{\beta_h^{\sigma_h}e(s)^{-\sigma_h}\left[\sum_{s'}\rho(ss')p^2(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h}}{A_h(s)}\frac{w_h(s)}{p^2(s')}\ \forall\ s' \tag{18}$$

As preferences are homothetic, the demand for each good is a linear function of wealth as we would expect. Wealth is premultiplied by a complicated non-linear function of current and future prices as well as the current nominal exchange rate.

2.1 The role of the exchange rate: partial equilibrium

Using equations (8, 9) and (7), we can obtain the following expression for the nominal exchange rate:

$$e(s) = \frac{\sum_{s'}\rho(ss')\frac{c_{2h}^2(ss')^{-\frac{1}{\sigma_h}}}{p^2(s')}}{\sum_{s'}\rho(ss')\frac{c_{2h}^1(ss')^{-\frac{1}{\sigma_h}}}{p^1(s')}} \qquad s = 1, ..., S \tag{19}$$

In our model, the nominal exchange rate is a forward-looking variable, as it depends on the expected marginal utilities derived from the consumption of the two goods as well as from the expected purchasing power of the two currencies. In fact, $\frac{1}{p^\ell(s')}$ gives how many units of good ℓ we can afford in state s' per unit of currency ℓ held. In other words, the nominal exchange rate is the ratio of the expected purchasing power of currency 2 over the expected purchasing power of currency 1, weighted by agent h's marginal utilities. The more a currency can buy tomorrow relatively to the other currency, the higher will be its price today. In other words, the nominal exchange rate follows some sort of asset pricing equation, given that the currencies are used to transfer wealth across periods.

In the cash-in-advance literature, the spot exchange rate simply depends on the *current* realization of the stochastic variables and not on expectations of future variables (see e.g. Lucas (1982)). This is due to the transaction role that it is attributed to money, which is only used to carry out exchange in a given period. In the cash-in-advance literature, money is a "veil" and the exchange rate does not ultimately affect the real allocation, which is the same as in the barter economy.

Let us now consider the role of the nominal exchange rate in the portfolio decision of an agent. We combine the demand for the two currencies (13) and (14) to get:

$$\frac{m_h^1(s)}{m_h^2(s)} = e(s)^{\sigma_h} \frac{\left[\sum_{s'} \rho(ss')p^1(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h}}{\left[\sum_{s'} \rho(ss')p^2(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h}} \tag{20}$$

The above equation shows that the higher is the (relative) price of currency 2 (i.e. the nominal exchange rate) the higher is the (relative) demand for currency 1. In a sense, the two currencies are substitutes, although not perfectly. Moreover, the higher is the expected purchasing power of currency 1, the higher is the relative demand for currency 1 as long as the degree of substitutability between the two goods is high enough ($\sigma_h > 1$).

Obviously, our arguments about the role of the nominal exchange rate in the portfolio choice of the agents are of a partial equilibrium nature as we assume that the nominal exchange rate is fixed. Below, we will show the importance of general equilibrium analysis as the nominal exchange rate does act as a "shock absorber" in this model.

3 Equilibrium

Definition 1 *A stationary equilibrium is a system of prices* $(p, e) \in \mathbb{R}^{2S}_{++} \times \mathbb{R}^{S}_{++}$*, consumption allocations and portfolios* $(c_{1h}(s), c_{2h}(ss'), m_h(s)) \in \mathbb{R}^2_{++} \times \mathbb{R}^{2S}_{++} \times \mathbb{R}^2_{++}$ *for every* $h = 1, ..., H$ *and* $s = 1, ..., S$ *such that:*

(i) *agent h maximizes his utility function subject to the budget constraints in every s;*

(ii) $c_1^\ell(s) + c_2^\ell(s's) = y^\ell(s) \qquad \forall\ s, s' \ \ and\ \ \forall\ \ell$

(iii) $\sum_h m_h^\ell(s) = M^\ell \qquad \forall\ s, \ell$

where $c_1^\ell(s) \equiv \sum_h c_{1h}^\ell(s)$ and $c_2^\ell(s's) \equiv \sum_h c_{2h}^\ell(s's)$.

Notice that we have $3S$ endogenous variables, i.e. $2S$ nominal price levels as well as S nominal exchange rates. On the other hand, we have $2S^2 + 2S$ equations. Goods' markets have to clear for any pair of s and s', as the consumption of the old does depend on the previous state as well as on the current state. Moreover, $2S$ monetary equations have to clear.

First, the system can be reduced by applying Walras Law. In particular, S^2 equations can be made redundant. If we sum across agents the budget constraints of the young and the old and combine them, we get:

$$p^1(s)[c_1^1(s) + c_2^1(s's) - y^1(s)] + p^2(s)e(s)[c_1^2(s) + c_2^2(s's) - y^2(s)] = 0 \qquad \forall\ s', s$$

Therefore, if for every pair of (s', s) the market for good 1 clears, the market for good 2 clears automatically.

However, we still have $S^2 - S$ equations more than the number of endogenous variables. This is the issue raised by Spear (1985), who proved that a steady state equilibrium does not generically exist in a stochastic OLG economy with money and multiple goods. Heuristically speaking, the non existence result is due to the fact that there are too many equations with respect to the number of unknowns[10].

[10]It is important to stress that his generic result does not rule out the possibility that a stationary equilibrium may exist under some restrictions. For example, he showed that economies with additively time-separable utility functions and one type of agent per generation do have a stationary equilibrium. In an open economy setting, we have heterogenous agents therefore existence of equilibrium is not guaranteed.

Next, we show that Assumptions 1 and 2 imply that $S^2 - S$ equations can be made redundant. As we end up with a system having the same number of equations and unknowns ($3LS$), we can get around the non-existence problem.

Proposition 1 *Under Assumptions 1 and 2, further $S^2 - S$ equations are redundant.*

Proof. Given Walras Law, suppose that the independent equations in the goods' markets are those for good 1. Sum across agents the budget constraints of the old for good 1 in state s:

$$p^1(s)c_2^1(s's) = M^1$$

It is easy to see that the aggregate consumption of the old does not depend on the previous state (the state realized when born) as aggregate real money balances only depend on the current state:

$$c_2^1(s's) = \frac{M^1}{p^1(s)} \qquad \Rightarrow \qquad c_2^1(s's) = c_2^1(s)$$

Suppose that the S equations for which $s' = s$ clear:

$$c_1^1(s) + c_2^1(ss) = y^1(s)$$

Given that the aggregate consumption of the old does not depend on the past, the other $S^2 - S$ clear automatically. ■

3.1 Definitions

Before we discuss the solution method, we introduce some key definitions and make a couple of useful remarks.

3.1.1 Portfolio rebalancing and trade imbalances: a unified view

To start with, let us define the balance of trade of country 1 in state s[11]:

$$tb_1(s's) \equiv p^1(s)[y^1(s) - c_{11}^1(s) + c_{21}^1(s's)] - p^2(s)e(s)[c_{11}^2(s) + c_{21}^2(s's)]$$

Notice that the sign of the balance of trade does depend on the choices that the young make in the current period, but also on the choices made by the

[11]Obviously, by Walras Law we have that $tb_2(s's) = -tb_1(s's)$.

current old in the previous period. Substituting the budget constraints into the trade balance equation, it should be immediate that the above definition can be rewritten as:

$$tb_1(s's) = m_1^1(s) - m_1^1(s') + e(s)[m_1^2(s) - m_1^2(s')] \tag{21}$$

This leads us to the following two remarks:

Remark 1 *If portfolios are constant across states, then trade is always balanced.*

Remark 2 *If today's realized state is the same as yesterday's, then trade is balanced.*

Equation (21) shows that there is a close relationship between agents' behaviour in the assets' markets and the goods' markets. If, for some reason, there is no portfolio rebalancing in equilibrium, then the balance of trade is always in equilibrium. Our framework is very different from the cash-in-advance literature with complete markets. In Lucas (1982), trade imbalances arise and yet portfolio rebalancing is never a possibility with the implication that the change in the net foreign assets position of a country is always zero.

The second remark is related to Polemarchakis and Salto's result for deterministic OLG economies (2002). In a one-currency economy, they showed that the balance of trade is in equilibrium at the monetary steady state. In this paper, the monetary steady state is stochastic and trade imbalances are possible whenever $s \neq s'$.

It is reasonable to expect that the set of parameters of the economy under which portfolios are state invariant has a very small measure. Constant portfolios implies that the consumption of an old person does not depend on the state in which he is born[12]. If output is a random variable, agents born in different states of nature are likely to have different wealth and therefore different demands for the goods. For the consumption of the old to be independent from the state when born, the demand function must be very special. In Appendix C, we show that this behaviour occurs when utility functions are

[12] In the previous section, we showed that the *aggregate* consumption of the old does not depend on the past, but this does not imply that the *individual* consumption is independent of the past as well.

logarithmic. Under logarithmic utility, the demand functions are extremely simple and the model is fully tractable. However, this comes at the cost that agents' behaviour is too simplistic and therefore uninteresting to our purposes. On the other hand, under isoelastic utility functions, constant portfolios will only occur for degenerate values of the endowments but the solution of the model requires a numerical approach.

Our findings for the log case are related to Cass and Pavlova (2004), who have shown that logarithmic utility yields peculiar results when markets are incomplete. In a two-period economy with N Lucas trees, the matrix of portfolio returns is degenerate and that the equilibrium allocation is Pareto optimal despite the incompleteness of the markets. Pavlova and Rigobon (2007) extended the model to the infinite-horizon but output shocks cannot generate time-varying portfolios. On the other hand, there is portfolio rebalancing with demand shocks. In our logarithmic version of the model, we could achieve a similar result if we allowed for state-dependent discount factors. The innovation of this paper is that we are able to explain portfolio dynamics as a consequence of output innovations rather than demand shocks, which is easier to verify in the data.

We now define our main variables of interest, i.e. net foreign assets and valuation effects.

3.1.2 Net foreign assets and valuation effects

In this section, we explore the relationship between net foreign assets, the balance of trade and valuation effects. Consider the balance of trade of country 1 in state $s's$, as defined in the previous section (equation (21)):

$$tb_1(s's) = m_1^1(s) - m_1^1(s') + e(s)[m_1^2(s) - m_1^2(s')] \tag{22}$$

Using the fact that $m_1^1(s) + m_2^1(s) = M^1$ for every s, we can rewrite the first two terms on the right hand side as follows:

$$tb_1(s's) = \underbrace{m_2^1(s')}_{\text{current value } FL_1(s')} - \underbrace{m_2^1(s)}_{FL_1(s)} + \underbrace{e(s)m_1^2(s)}_{FA_1(s)} - \underbrace{e(s)m_1^2(s')}_{\text{current value } FA_1(s')} \tag{23}$$

$FA(s)$ are holdings of foreign assets in state s and $FL(s)$ are foreign holdings of the domestic currency, i.e. foreign liabilities. Now, define net foreign assets as $NFA(s) \equiv FA(s) - FL(s)$ and rewrite the above as follows:

$$NFA_1(s) = \text{current value } NFA_1(s') + tb_1(s's) \tag{24}$$

Equation (24) states that the end-of-period net foreign assets in country 1 is equal to the current value of the net foreign assets accumulated in the previous period and the balance of trade[13].

The next step is to rewrite equation (23) in order to highlight valuation effects. In the right hand side, sum and subtract the foreign assets of country 1 in the previous state ($e(s')m_1^2(s')$) and use the definition of net foreign assets to obtain:

$$tb_1(s's) = NFA_1(s) - NFA_1(s') + [e(s') - e(s)]m_1^2(s') \tag{25}$$

This equation can be rewritten as:

$$\Delta NFA_1(s's) = tb_1(s's) + \underbrace{r(s's)e(s')m_1^2(s')}_{\text{valuation effects}} \tag{26}$$

where

$$r(s's) = R(s's) - 1 \equiv \frac{e(s)}{e(s')} - 1$$

Therefore, the change in the net foreign assets position of country 1 will be determined by the behaviour of the balance of trade and the valuation effects, where $r(s's)$ is the return on the foreign assets accumulated in the previous period. In this model, valuation effects are entirely determined by exchange rate movements[14]. If foreign currencies have appreciated with respect to the past (i.e. $e(s) > e(s')$), then the return on the foreign assets accumulated in the previous period is positive and therefore we say that the country experiences positive valuation effects[15]. Conversely, a country experiences negative valuation effects if foreign currencies have depreciated.

In this framework, currencies are the only assets available and therefore our setting can capture a scenario in which the majority of domestic assets

[13] This equation is equivalent to equation (1) in Gourinchas and Rey (2007, footnote 2).

[14] Moreover, there is no net income from abroad and therefore the trade balance position is equivalent to the current account position.

[15] As the price of the foreign asset is defined in units of the domestic asset, i.e. the exchange rate, the above rate of return has to be interpreted as the return of foreign assets relatively to the return on foreign liabilities.

are denominated in the foreign currency while domestic liabilities are denominated in the domestic currency. As from the findings of Lane and Shambaugh (2010), this is entirely consistent with the currency denomination of the foreign assets and liabilities of the US while it would be less realistic when applied to developing countries. Lane and Shambaugh (2010) also find that emerging market countries are becoming more similar to advanced economies as they issue less foreign-currency denominated debt than developing countries and are accumulating foreign-currency denominated assets in the form of foreign exchange reserves.

The consensus in the empirical literature is that valuation effects are very important in explaining the dynamics of net foreign assets of the US and other countries (see e.g. Gourinchas and Rey (2015)). Lane and Shambaugh (2010) have also shown that the valuation effects stemming from nominal exchange rate changes are an important driver of the overall valuation effects[16]. While most of the theoretical literature has focused on other sources of valuation effects (e.g. Devereux and Sutherland (2010)), the novelty of this paper is that it provides a theoretical framework in which exchange rates related-valuation effects can arise while countries adjust their portfolios over time because of the market incompleteness. In section 5, we will discuss the interaction between exchange rates and net foreign assets, as well as the quantitative importance of valuation effects.

4 Portfolio holdings, the distribution of world GDP and the role of the nominal exchange rate

From now onwards, we focus on the case in which preferences are identical across countries: $\sigma_h = \sigma$ and $\beta_h = \beta$. Plugging the demand functions for the goods and the currencies into the equilibrium conditions, we get the following

[16] They also observed that valuation effects associated to exchange rate fluctuations tend to move in the same direction as valuation effects associated to capital gains and losses.

system of $3S$ equations, which will be solved numerically:

$$\frac{p^2(s)e(s)}{p^1(s)} = \frac{\omega^1(s)}{\omega^2(s)} \frac{[p^2(s)e(s)]^{1-\sigma} + \beta^\sigma e(s)^{1-\sigma} \left[\sum_{s'} \rho(ss') p^2(s')^{\frac{1-\sigma}{\sigma}}\right]^\sigma}{p^1(s)^{1-\sigma} + \beta^\sigma \left[\sum_{s'} \rho(ss') p^1(s')^{\frac{1-\sigma}{\sigma}}\right]^\sigma} \tag{27}$$

$$M^1 = \frac{\beta^\sigma \left[\sum_{s'} \rho(ss') p^1(s')^{\frac{1-\sigma}{\sigma}}\right]^\sigma}{A(s)} \sum_h w_h(s) \tag{28}$$

$$e(s)M^2 = \frac{\beta^\sigma e(s)^{1-\sigma} \left[\sum_{s'} \rho(ss') p^2(s')^{\frac{1-\sigma}{\sigma}}\right]^\sigma}{A(s)} \sum_h w_h(s) \tag{29}$$

The following proposition establishes that there is a strong relationship between the distribution of world GDP across countries, portfolio holdings and trade imbalances when preferences are identical across countries.

Proposition 2 *If $\sigma_h = \sigma$ and $\beta_h = \beta$: (i) country h's portfolio holdings at the end of the period depends on its current share of world GDP; (ii) if country h has a higher (lower) share of world GDP with respect to the past, it runs a trade surplus (deficit).*

Proof. (i) When $\sigma_h = \sigma$ and $\beta_h = \beta$, the demand of agent h for the two currencies has the following form (see equations (13) and (14)):

$$m_h^\ell(s) = k^\ell(s) w_h(s)$$

where $k^\ell(s)$ is identical across agents. Summing across h, we get the following equation:

$$M^\ell = k^\ell(s) \sum_h w_h(s)$$

Dividing the first equation by the second equation, we obtain the desired result:

$$\frac{m_h^\ell(s)}{M^\ell} = \frac{w_h(s)}{w(s)} \qquad \ell = 1, 2$$

where $w(s) = \sum_h w_h(s)$[17].

(ii) Suppose that today's realized state is s and yesterday's state was s'. By hypothesis, $\frac{w_h(s)}{w(s)} > \frac{w_h(s')}{w(s')}$. The first part of the proof implies that:

$$\frac{m_h^\ell(s)}{M^\ell} > \frac{m_h^\ell(s')}{M^\ell} \qquad \ell = 1, 2$$

[17] World GDP is defined as the sum of countries' nominal GDP expressed in units of the numéraire currency.

Finally, equation (21) implies country h has a trade surplus in state s. The other case can be worked out in a similar way. ■

The end-of-the-period wealth of the young is equal to their total money holdings, as the two currencies are the means by which they can save and therefore finance future consumption. Therefore, Proposition 2 suggests that the distribution of world GDP is the same as the distribution of world wealth at the end of the period[18]. If the distribution of world GDP changes across states of nature, then the distribution of wealth will change as well and portfolio rebalancing occurs over time. As we discussed above, this is a likely outcome of the model as markets are incomplete.

As a matter of fact, portfolios are constant across states of nature in complete markets' models precisely because wealth is identical across agents and therefore agents do not adjust their portfolio holdings following new shocks.

Proposition 2 sheds further light on the behavior of the trade balance. If a country is in surplus, it is because it is relatively wealthier with respect to the past. This does not rule out the possibility that such country is poorer than the other country in all states of nature[19]. Therefore, our model offers a novel explanation of the fact that emerging countries run trade surpluses against the United States: global imbalances simply reflect the rise of emerging countries in the world economy.

If a country is classified as "emerging", then its share of world GDP should have increased over time. Using a sample of 146 countries, we find that the share of world GDP of all emerging countries except Argentina has increased over the past 20 years[20]. As expected, China is the emerging country whose share of world GDP has increased the most, as it has gained 7.81% points over the past 20 years. On the other hand, the share of world GDP of the US has fallen by 3.54%. At this stage, the model does not say whether an increase in

[18]As domestic GDP is equal to domestic income, the distribution of world GDP is equal to the distribution of world income.

[19]On the other hand, the poor country is always in trade deficit in cash-in-advance models under isoelastic utility (see Eugeni (2013) for a derivation). The reason is that the sign of the trade balance does only depend on the current shock, and not on the past.

[20]We calculate the change in the share of world GDP as follows: $\Delta share_h = \frac{GDP_{h,2010}}{\sum_h GDP_{h,2010}} - \frac{GDP_{h,1990}}{\sum_h GDP_{h,1990}}$. GDP is taken from the IMF World Economic Outlook database and it is measured at current national prices converted in US dollars. We use the IMF classification of emerging countries.

wealth is due to either output growth or changes in prices and our calculation does not distinguish between the two accordingly. In the next section, we show that because China's real GDP has grown more than the US', then it is wealthier with respect to the past and therefore it runs a trade surplus in equilibrium[21].

Finally, the allocation of savings across currencies deserves some comment. Agents do not have very sophisticated portfolio strategies according to Proposition 2. In fact, agents hold the same share of both money stocks[22]. This is due to the "shock absorbing" role of the nominal exchange rate. If the elasticities of substitution are allowed to differ across countries, the distribution of money holdings is more difficult to characterize and agents could have a preference for different currencies in different states.

4.1 Exchange rate determination and the role of money

Combining equations (28) and (29), we obtain the following expression for the exchange rate:

$$e(s) = \left(\frac{M^1}{M^2}\right)^{\frac{1}{\sigma}} \frac{\sum_{s'} \rho(ss')p^2(s')^{\frac{1-\sigma}{\sigma}}}{\sum_{s'} \rho(ss')p^1(s')^{\frac{1-\sigma}{\sigma}}} \qquad s = 1, ..., S \tag{30}$$

Although the above expression is not a closed-form solution, we can gain some intuition about the role of the nominal exchange rate and the importance of general equilibrium analysis as opposed to partial equilibrium analysis. Recall the equation that linked the portfolio choice of the agents to the nominal exchange rate (equation (20)). In a partial equilibrium setting, an increase in the nominal price levels in country 2 means that the purchasing power of currency 2 is lower and therefore the relative demand for currency 2 falls (provided that the elasticity of substitution is bigger than 1). As the nominal exchange rate is endogenous, it will behave in such a way to counteract expectations on price movements. In particular, currency 2 appreciates if the price of good 2 increases as equation (30) shows. In fact, if we combine equations (20) and (30), we obtain our previous result that each agent holds the money stocks of

[21]This requires that the elasticity of substitution between traded goods is greater than 1, which is supported by empirical evidence as we explain in the next section.

[22]Notice that we do not impose any "home bias" in the preferences, therefore agents hold the two currencies in the same share as they like the two goods equally.

both countries in equal shares. In the numerical section, these arguments will be further clarified.

It is also interesting to note that, if the stochastic process is i.i.d., the exchange rate is constant as the probabilities that agents attach to future events are independent from the state in which they are born.

We conclude this section with a discussion of the role of money in our economy. Since the old face separate budget constraints for each good, then the aggregate consumption of the old of good ℓ is equal to the real money balances of currency ℓ. As a consequence, we can write the following expressions for the nominal price levels in the two countries using the goods' markets clearing conditions:

$$\begin{aligned} p^1(s) &= \frac{M^1}{\omega^1(s) - c_1^1(s)} \\ p^2(s) &= \frac{M^2}{\omega^2(s) - c_1^2(s)} \end{aligned}$$

This also implies that the *level* of the money stocks does not matter for the real allocation. Suppose that the money stock of country 1 doubles. Given the first of the above two equations, the nominal price level will be doubled as well. In other words, prices are homogenous of degree 1 with respect to the domestic money supply. The nominal exchange rate doubles as well (see equation (30)) as currency 1 becomes cheaper. Therefore, the wealth of both agents is doubled. It can be checked from the demand functions that this change in prices does not affect the consumption of both agents.

Although the level of the money stocks does not matter for the real allocations, money is not neutral in our model. If we removed money from the economy, the equilibrium allocation would be very different. While the young would still be able to engage in barter (intragenerational trade), the old would not be able to consume anything. Even if the old had an endowment, such as a pension, he would not be able to trade it because of the multiple budget constraints, so he would limit his consumption to the domestic good. Money is important in our framework as in standard Samuelsonian OLG economies, but in an even stronger sense because of the multiple budget constraints which prevent barter among the old.

5 The US external position and valuation effects

The aim of this section is to gain further insights on the behaviour of the nominal exchange rate, the balance of trade and the net foreign asset positions. For this purpose, we parametrize our two-country model.

In this section, we will refer to country 1 as the United States and country 2 as China. The reason why we choose the United States and China for our numerical exercise is that China is one of the main creditors of the US and the US deficit against China account for a significant fraction of the overall US current account deficit (e.g. Eugeni, 2015). Moreover, the US-China imbalances are persistent and our two-period OLG model is especially suitable to capture low-frequency trends in international financial markets. In our setting, "foreign assets" is a country's foreign currency holdings, while "foreign liabilities" is the domestic currency held abroad. Therefore, our model is able to capture the currency composition of the US and China's balance sheet, for which foreign assets are denominated in foreign currencies while foreign liabilities are denominated in the domestic currency. According to the Lane and Shambaugh (2010) database, 64% of US foreign assets were denominated in foreign currencies while 93% of US foreign liabilities were denominated in dollars in 2004[23]. As far as China is concerned, 100% of the Chinese foreign assets are denominated in foreign currency, 70% of which are dollar denominated. This is consistent with the fact that China is one of the US main lenders. On the other hand, 63% of Chinese liabilities were issued in renmimbi in 2004. This reflects a general trend which sees emerging market economies increasingly able to borrow in their domestic currency (Lane and Shambaugh, 2010)[24].

Therefore, a depreciation of the dollar in our setting would imply a positive wealth effect for the US and a negative wealth effect for China. Although the Chinese currency has considerably appreciated over the past 10 years (Figure

[23]The first figure reflects the fact that many developing economies do still borrow in US dollars as they are unable to issue debt in domestic currency-denominated assets.

[24]Another signal of the increased ability of emerging countries to borrow in their own currency is that a third of the foreign currency-denominated US foreign assets are denominated in currencies other than the Euro, the Yen, the Pound and the Swiss Franc. Therefore, these are assets held in emerging economies and denominated in local currencies.

3), the Chinese exchange rate is not freely floating therefore it is reasonable to expect that the model will tend to over predict valuation effects. It is also important to stress that our model can only capture low-frequency movements of the exchange rate and the balance of trade and does not aim at explaining high-frequency movements (or lack of) in foreign exchange markets.

Since agents live for two periods in our OLG economy, we assume that a period is 20-years long. As we wish to explain the deterioration of the US external position against emerging economies over the past 20 years, we adopt the following strategy. We consider an economy with two states of nature, where state 1 corresponds to the state of the world economy in 1990 while state 2 is the state of the world economy in 2010. Therefore, we will focus on what happens in the world economy in the transition from state 1 to state 2[25].

We take the real GDP per capita of the United States and China in 1990 and 2010 to parametrize output in the two states[26].

$$\begin{array}{ll} y^1(1) = 31,432 & y^1(2) = 41,627 \\ y^2(1) = 2,005 & y^2(2) = 7,693 \end{array}$$

Notice that while US output has grown by 32% over the 20-years period, China has grown by 384%. Although China has experienced higher growth over time, the real GDP per capita level is still much lower than the US. We choose the rest of the parameter values as follows:

$$\begin{aligned} M^1 = M^2 = M &= 1 \\ \sigma_1 = \sigma_2 = \sigma &= 4 \\ \beta_1 = \beta_2 = \beta &= 1 \\ \rho(ss) &= 0.9 \end{aligned}$$

We normalize both money supplies to 1 since the level of the money supply does not affect the real allocation. The *level* of the trade balance does change in

[25]This is not to argue that the world economy can only be in a state that matches the situation of the world economy either of the 1990 or the 2010. However, a two-states example is enough to illustrate our arguments while adding more states of nature would not provide neither more information nor intuition.

[26]We take the output-side real GDP at chained PPPs and the population from the Penn World Tables 8.0.

the money supplies, as portfolios and the nominal exchange rates are affected (equation (21)). However, the size of the money stocks are irrelevant when we normalize the trade balance as a percentage of domestic GDP. The same is true for valuation effects as a percentage of GDP.

The elasticity of substitution is assumed to be greater than 1 as such parametrization rules out episodes of "immiserizing growth". In fact, when $0 < \sigma < 1$, a country that experiences a positive shock (everything else equal) is poorer in value terms since the price of the domestic good falls too much. In other words, the terms of trade effect dominates changes in output[27]. Empirical work based on low-frequency data found elasticities between 4 and 15, while estimates at higher frequency suggest that the elasticity is much lower and in the range of 0.2 to 3.5 (see Ruhl (2008)). Our parametrization is more in line with the low-frequency literature. Below, we show that our results are robust to different parameter values for the elasticity of substitution.

The discount factor is set equal to 1 and identical across countries. We have also assumed that the Markov process is persistent. In the robustness analysis, we will solve the model for different values of $\rho(ss)$.

5.1 Numerical results

We report the equilibrium prices in Appendix D. We can compute relative prices, expressed in the numéraire currency, are follows: $p(s) \equiv \frac{p^2(s)e(s)}{p^1(s)}$. Therefore:

$$\begin{aligned} p(1) &= 2.0968 \\ p(2) &= 1.4969 \end{aligned}$$

Country 1 (the US) experiences an improvement of the terms of trade in the transition from state 1 to state 2, as the price of imports fall relatively to the price of exports. This is due to a supply effect, as output in country 2 (China) has increased relatively more than output in country 1. At the same time, currency 1 depreciates (see the Appendix). The intuition behind this can be explained as follow. While both nominal prices fall in the transition from state

[27] A similar issue arises in the simplest possible setting, i.e. a static GE model with isoelastic utility and corner endowments. See also Cole and Obstfeld (1991) and Lucas (1982).

1 to state 2, as supply increases in both countries, the nominal price of country 2 falls at a higher rate. Therefore, currency 2 gives the current old a higher real return. Because the shock is persistent, then the young expect the current state to realize tomorrow with a very high probability. As a consequence, agents would have an incentive to buy more of currency 2. But since the money supply is fixed, then currency 2 (yuan) has to appreciate in equilibrium.

On the left (right) column, we report the money holdings of agents born in country 1 (2)[28]:

$$m_1^1(1) = 0.8820 \qquad m_2^1(1) = 0.1180$$
$$m_1^2(1) = 0.8820 \qquad m_2^2(1) = 0.1180$$

$$m_1^1(2) = 0.7833 \qquad m_2^1(2) = 0.2167$$
$$m_1^2(2) = 0.7833 \qquad m_2^2(2) = 0.2167$$

As China is poorer in both states, then the country holds a lower share of both currencies. However, this share increases in the transition from state 1 to state 2. As the country experiences higher growth, its share of world GDP increases and the young accumulate more assets (see Proposition 2).

Using equations (21) and (26), we can compute the balance of trade, the change in the net foreign assets position and valuation effects. We report results for country 1, therefore we express the above variables as a percentage of the country's GDP, where $GDP_1(s) = p^1(s)y^1(s)$. As we explained above,

Table 1: Country 1. Numerical results

$\frac{\Delta NFA_1(12)}{GDP_1(2)}\%$	$\frac{tb_1(12)}{GDP_1(2)}\%$	$\frac{VAL_1(12)}{GDP_1(2)}\%$
−1.87%	−6%	4.13%
$\frac{\Delta NFA_1(21)}{GDP_1(1)}\%$	$\frac{tb_1(21)}{GDP_1(1)}\%$	$\frac{VAL_1(21)}{GDP_1(1)}\%$
2%	5.91%	−3.91%

we are interested in the transition from state 1 to state 2, therefore we focus on the first two lines.

As the US experiences lower growth, it runs a trade deficit of 6% of domestic GDP. The reason behind this is that the country accumulates less foreign assets

[28] Since the money supplies are normalized to 1, we can interpret the numbers below as the proportion of the two money stocks held in the two countries in each state.

while foreign liabilities increase. As China becomes wealthier, they accumulate more foreign assets as well as domestic assets. Another way to interpret this result is that as Chinese goods become relatively cheaper, then the US imports more and exports less than before[29]. The mechanism is different than in Lucas (1982), where a country that is richer in all states of nature is always going to run a trade surplus[30]. In this model, the trade balance is backward-looking so the country that runs a surplus is not the richest country in the current state, but the country whose relative position in the world economy has improved.

The trade deficit is partially offset by the positive valuation effects. As the Chinese currency appreciates, the value of the US foreign assets increases while the value of foreign liabilities fall. Therefore, the US experience a positive wealth effect that mitigates the negative impact of the trade deficit on the external position of the country. On the other hand, China experiences a negative wealth effect as the country runs a trade surplus.

5.2 Discussion

Our numerical results are consistent with the observation that the US have experienced positive valuation effects over the past 20 years despite accumulating a substantial trade deficit, while many emerging economies (including China) have experienced exactly the reverse (see Figure 1 and 2). According to our model, the reason why the United States have run a current account deficit for the past two decades is that emerging countries' share of world GDP has increased. Due to their higher wealth, emerging countries have accumulated more domestic and foreign assets and therefore held less foreign liabilities which have led to a positive change in their net foreign assets' position. As US goods become relatively more expensive, there is a lower demand of dollars as compared to the past. Hence, the dollar depreciates and the US experience a positive wealth effect. In the robustness section, we show that the negative relationship between valuation effects and the trade balance is robust to alternative values of the elasticity of substitution and the persistence parameter.

[29] Plugging the budget constraints into (21), we can write the other definition of the balance of trade as the difference between exports and imports: $tb_1(s's) \equiv p^1(s)[c^1_{12}(s) + c^1_{22}(s's)] - p^2(s)e(s)[c^2_{11}(s) + c^2_{21}(s's)]$

[30] See Eugeni (2013) for a derivation of the Lucas model under isoelastic utility.

The purpose of this paper is not to provide a fully-fledged quantitative assessment of valuation effects, as our model cannot generate valuation effects related to capital gains and losses, which are equally as important. Yet, it has considerable explanatory power. We calculate the change in the net foreign assets positions, the current account and valuation effects of the US and China as accumulated over the past 20 years as a percentage of GDP in 2010:

Table 2: The external positions of the US and China, 1990-2010

	$\frac{NFA_{2010}-NFA_{1990}}{GDP_{2010}}\%$	$\frac{\sum_{t=1990}^{2010} CA_t}{GDP_{2010}}\%$	$\frac{\sum_{t=1990}^{2010} VAL_t}{GDP_{2010}}\%$
United States	-15%	-41%	26%
China	25%	31%	-6%
United States vs. China		-15%	

Notes. The change in the net foreign assets position is calculated using the database of Lane and Milesi-Ferretti (2007). The current account of the US as well as the US position against China is calculated using data from the Bureau of Economic Analysis although the second time-series only starts from 1999. Valuation effects are computed as the difference between the two.

First of all, our table supports the ideas the US current account deficit against China is one of the main driving forces behind the US current account deficit. Our model can explain more than a third of the US-China current account imbalance (6% out of 15%). Moreover, valuation effects have stabilize both the net external positions of China and the United States, consistently with our model. Unfortunately, we do not possess data on US net foreign assets disaggregated by countries, differently from the current account balance. As a consequence, we cannot compute the valuation effects between the US and China as a residual and make a full comparison between the model and the data. However, we are able to observe that the overall valuation effects are large for both countries, but especially in the US. Part of the reason could be that the Chinese exchange rate is not fully flexible. The model overestimates the valuation effects and therefore underestimates the change in the net foreign assets position against China, as it assumes that the Chinese exchange rate regime is freely floating. In fact, country 2's nominal exchange rate appreciates by 61% while the renmbimbi has only appreciated by 25% with respect to the dollar over the period 1994-2010[31]. Therefore, our results suggest that the wealth transfer from China to the US would have been much bigger if the Chinese had a fully floating nominal exchange rate regime.

5.3 Robustness

The aim of this section is to check the robustness of the mechanisms described above to alternative specifications of the elasticity of substitution and the persistence parameter.

Table 3 shows that valuation effects do always act as stabilizer of the net external positions of country 1, independently from the chosen value of the elasticity of substitution. Our exercise is done taking into account the range of elasticities estimated with the low-frequency data, which is between 4 and 15 (see Ruhl (2010)). For completeness, we do also report the case in which $\sigma = 1$, which corresponds to the log case whose analytical solution we derive in Appendix C. Interestingly, the negative relationship between valuation effects and the trade balance is not invalidated if we assume an elasticity of substitution lower than 1. As we explained above, this corresponds to the case of "immiserizing growth", where a country whose output increases is actually poorer in equilibrium because the terms of trade falls too much. All signs are simply reversed as country 1 is poorer in equilibrium.

As the elasticity of substitution increases, goods are increasingly substitutable and agents do not react as much to changes in prices in their demand for the currencies. Therefore, the nominal exchange rate has less of a "shock-absorbing" role and valuation effects become less important. However, they do always account for a significant proportion of the change in net foreign assets of country 1.

Table 4 shows that valuation effects do stabilize the net external positions as long as there is persistence in the Markov processes for output. If not, valuation effects move in the same direction as the country's trade balance ($\rho(ss) = 0.4$ in the Table). The reason is the following. In state 2, the young would not expect that similar conditions occur tomorrow with a high probability. On the contrary, they would expect very high prices in country 2 relatively to

[31] We use the end-of-the-period exchange rate time-series provided by Lane and Milesi-Ferretti (2007).

country 1. As they would demand more of currency 1, so that currency 1 has to appreciate, the exchange rate falls. Therefore, valuation effects would be negative for country 1 in the transition from state 1 to state 2. Notice that if the Markov process is instead i.i.d (i.e. $\rho(ss) = 0.5$), then the exchange rate is constant across states since agents' expectations about future events are not conditional on their state of birth.

To conclude, the results that we have shown in Table 1 can be obtained under very mild assumptions such as persistence in the Markov process and the elasticity of substitution bigger than one.

6 Conclusions and future research

This paper provides a two-country model of nominal exchange rate determination where valuation effects substantially contribute to the dynamics of the net foreign assets' position of a country. If preferences are identical across agents, agents born in a given state of nature hold a fraction of the total money stocks equivalent to their share of world GDP. Since the distribution of world GDP varies across states, trade imbalances among the two countries arise. We have also shown that the exchange rate fluctuates because of the Markov structure of uncertainty. The spot exchange rate between any two currencies is a function of the expected purchasing power of the currencies with respect to the related domestic good. Since the agents' expectations depend on the state in which they are born, the exchange rate is state dependent in equilibrium. In this framework, exchange rate movements are wide enough to generate quantitatively big valuation effects, which stabilize countries' net external positions consistently with the data.

An aspect of the model that could be further investigated is countries' portfolio choices when preferences are heterogeneous. If the elasticity of substitution is allowed to vary across agents, agents might prefer to hold more domestic currency in some state while more foreign currency in some other state, or even have a clear cut "preference" for a particular currency. For instance, it would be interesting to investigate under which conditions agents tend to have a bias towards domestic assets, as observed in the empirical literature[32].

Finally, an important step would be to generalize the model in the direction of introducing further assets in addition to the currencies. For instance, by introducing equity or FDI we would be able to analyze other sources of valuation effects than exchange rate movements, as well as understand more deeply the composition of countries' balance sheets. Our overlapping-generations framework is able to explain some main trends in international financial markets and generate quantitatively important valuation effects, so it is a promising line of research in the direction of answering other important research questions in international macroeconomics.

References

[1] Alvarez F., Atkenson A., Kehoe P.J. (2009), Time-Varying Risk, Interest Rates, and Exchange Rates in General Equilibrium, *Review of Economic Studies*, 76, 851-878.

[2] Balasko Y., Cass D. (1989), The Structure of Financial Equilibrium with Exogenous Yields: The Case of Incomplete Markets, *Econometrica*, 57 (1), 135-162.

[3] Cass D., Pavlova A. (2004), On trees and logs, *Journal of Economic Theory*, 116 (2004) 41-83.

[4] Cole H., Obstfeld M. (1991), Commodity trade and international risk sharing, *Journal of Monetary Economics*, 3-24.

[5] Courdacier N., Rey H. (2012), Home Bias in Open Economy Financial Macroeconomics, *Journal of Economic Literature*, 51(1), 63-115.

[6] Devereux M.B., Sutherland A. (2010), Valuation effects and the dynamics of net external assets, *Journal of International Economics*, 80, 129-143.

[7] Eugeni S. (2013), Essays in International Macroeconomics, University of York.

[32]See Coeurdacier et al. (2012) for a recent review of the literature.

[8] Geanakoplos J., Mas-Colell A. (1989), Real indeterminacy with Financial Assets, *Journal of Economic Theory*, 47, 22-38.

[9] Gottardi P. (1994), On the Non-neutrality of Money with Incomplete Markets, *Journal of Economic Theory*, 62, 209-220.

[10] Gottardi P. (1996), Stationary Monetary Equilibria in Overlapping Generations with Incomplete Markets, *Journal of Economic Theory* 71, 75-89.

[11] Gourinchas P., Rey H. (2007), International Financial Adjustment, *Journal of Political Economy*, 115 (4), 665-703.

[12] Gourinchas P., Rey H. (2015), External Adjustment, Global Imbalances, Valuation Effects, Chapter 10 in Handbook of International Economics, Vol. 4, 585-645.

[13] Heathcote J., Perri F. (2013), The International Diversification Puzzle Is Not as Bad as You Think, *Journal of Political Economy*, 121 (6), 1108-1159.

[14] Kareken J., Wallace N. (1981), On the Indeterminacy of Equilibrium Exchange Rates, *Quarterly Journal of Economics*, 96 (2), 207-222.

[15] International Monetary Fund (2014), Annual Report on Exchange Arrangements and Exchange Restrictions, Washington, October.

[16] Judd K.L., Kubler F., Schmedders K. (2003), Asset Trading Volume with Dynamically Complete Markets and Heterogeneous Agents, *Journal of Finance*, 58 (5), 2203-2217.

[17] Lane P.R., Milesi-Ferretti G.M. (2001), The external wealth of nations: measures of foreign assets and liabilities for industrial and developing countries, *Journal of International Economics*, 55 (2), 263-294.

[18] Lane P.R., Milesi-Ferretti G.M. (2002), External wealth, the trade balance, and the real exchange rate, *European Economic Review*, 46, 1049-1071.

[19] Lane P.R., Milesi-Ferretti G.M. (2007), The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970-2004, *Journal of International Economics*, 73 (2), 223-250.

[20] Lane P.R., Shambaugh J.C. (2010), Financial Exchange Rates and International Currency Exposures, *American Economic Review*, 100 (1), 518-540.

[21] Lucas R.E. (1982), Interest Rates and Currency Prices in a Two-Country World, *Journal of Monetary Economics*, 10 (3), 335-359.

[22] Magill M., Quinzii M. (1992), Real effects of money in general equilibrium, *Journal of Mathematical Economics*, 21, 301-342.

[23] Neumeyer P.A. (1998), Currencies and the Allocation of Risk: The Welfare Effects of a Monetary Union, *American Economic Review*, 88 (1), 246-259.

[24] Pavlova A., Rigobon R. (2007), Asset Prices and Exchange Rates, *Review of Financial Studies*, 20 (4), 1139-1180.

[25] Polemarchakis H.M. (1988), Portfolio Choice, Exchange Rates, and Indeterminacy, *Journal of Economic Theory*, 46, 414-421.

[26] Polemarchakis H.M., Salto M. (2002), Balance-of-Payments Equilibrium and the Determinacy of Interest Rates, *Review of International Economics*, 10 (3), 459-468.

[27] Rabitsch K., Stepanchuk S., Tsyrennikov V. (2014), International Portfolios: A Comparison of Solution Methods, Vienna University Department of Economics Working Paper Series, 159.

[28] Ruhl K.J. (2008), The International Elasticity Puzzle, NYU Stern School of Business.

[29] Sargent T.J. (1987), Dynamic Macroeconomic Theory, Harvard University Press, Cambridge MA.

[30] Spear S.E. (1985), Rational Expectations in the Overlapping Generations Model, *Journal of Economic Theory*, 251-275.

[31] Svensson L.E.O. (1985), A general equilibrium asset-pricing cash-in-advance approach, *Journal of International Economics*, 18, 17-41.

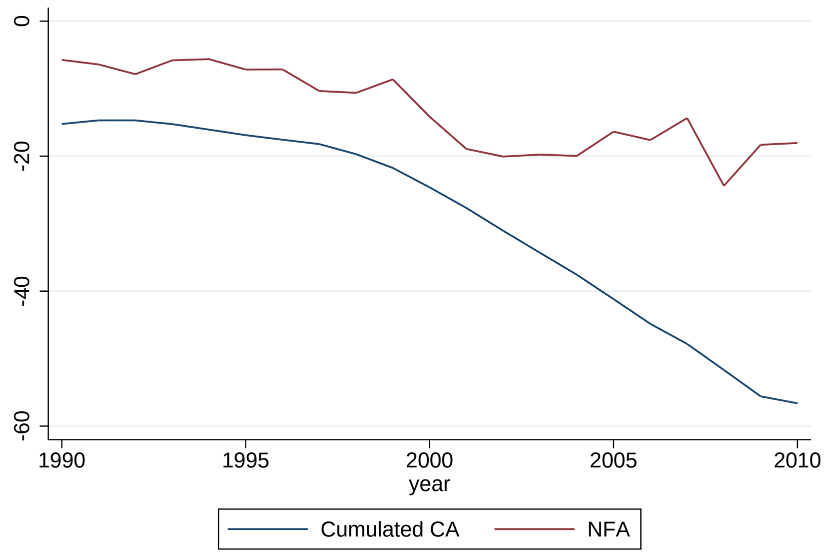
[32] Tille C. (2008), Financial integration and the wealth effects of exchange rate fluctuations, *Journal of International Economics*, 75, 283-294.

[33] Tille C., Van Wincoop E. (2010), International capital flows, *Journal of International Economics*, 80 (2), 157-175.

7 Appendix

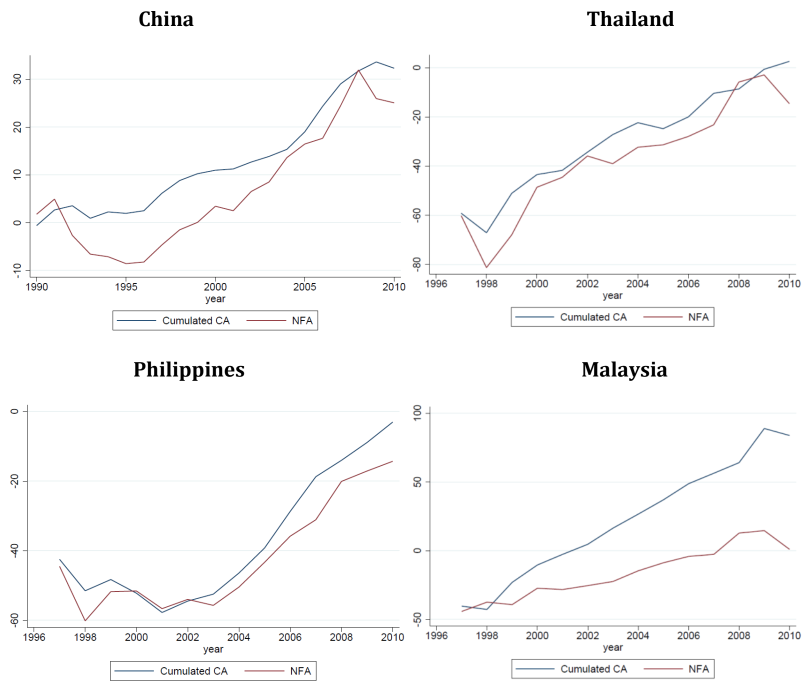
7.1 Appendix A. Figures

Figure 1: Net foreign assets' position and cumulated current accounts of the United States as a percentage of GDP, 1990-2010



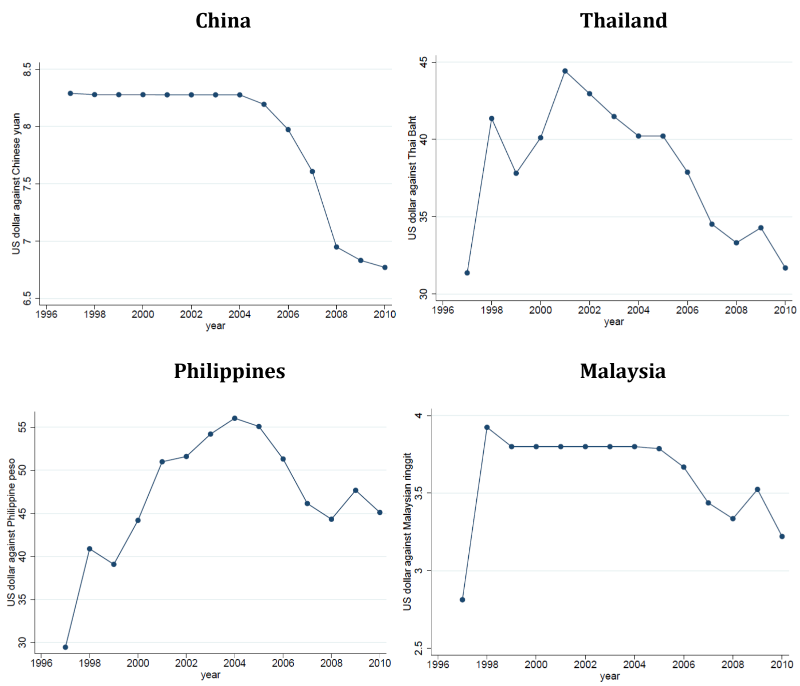
Source: Lane and Milesi-Ferretti database (2007). Updated to 2011.

Figure 2: Net foreign assets' position and cumulated current accounts of selected emerging market countries as a percentage of GDP



Source: Lane and Milesi-Ferretti database (2007). Updated to 2011.

Figure 3: The depreciation of the US dollar against the currencies of selected emerging market economies



Source: Lane and Milesi-Ferretti database (2007). Updated to 2011.

7.2 Appendix B

7.2.1 Derivation of portfolios

In this section, we explain how to derive the demand for the currencies.

First, combine (8), (5) and (7) for $\ell = 1$:

$$\frac{c_{1h}^1(s)^{-\frac{1}{\sigma_h}}}{p^1(s)} = \beta_h \sum_{s'} \frac{\rho(ss')c_{1h}^1(ss')^{-\frac{1}{\sigma_h}}}{p^1(s')} \tag{31}$$

and rewrite it as follows:

$$\frac{p^1(s)^{\frac{1-\sigma_h}{\sigma_h}}}{[p^1(s)c_{1h}^1(s)]^{\frac{1}{\sigma_h}}} = \beta_h \sum_{s'} \frac{\rho(ss')p^1(s')^{\frac{1-\sigma_h}{\sigma_h}}}{[p^1(s')c_{1h}^1(ss')]^{\frac{1}{\sigma_h}}} \tag{32}$$

Plugging $p^1(s')c_{2h}^1(ss') = m_h^1(s)$ for every s', we can sum up the numerators in the right hand side and elevate both sides of the equation to σ_h:

$$\frac{p^1(s)^{1-\sigma_h}}{p^1(s)c_{1h}^1(s)} = \beta_h^{\sigma_h} \frac{\left[\sum_{s'} \rho(ss')p^1(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h}}{m_h^1(s)} \tag{33}$$

Next, we combine the first-order conditions for the goods consumed when young as follows:

$$\frac{c_{1h}^1(s)^{-\frac{1}{\sigma_h}}}{p^1(s)} = \frac{c_{1h}^2(s)^{-\frac{1}{\sigma_h}}}{p^2(s)e(s)} \tag{34}$$

After some manipulations, the above equation can be rewritten as follows:

$$p^2(s)e(s)c_{1h}^2(s) = \frac{[p^2(s)e(s)]^{1-\sigma_h}}{p^1(s)^{1-\sigma_h}} p^1(s)c_{1h}^1(s) \tag{35}$$

Now, plug (35) into the budget constraint when young and obtain:

$$p^1(s)c_{1h}^1(s) = \frac{p^1(s)^{1-\sigma_h}}{p^1(s)^{1-\sigma_h} + [p^2(s)e(s)]^{1-\sigma_h}} [w_h(s) - m_h^1(s) - e(s)m_h^2(s)] \tag{36}$$

Plug it into (33) and rearrange:

$$m_h^1(s) = \frac{\beta_h{}^{\sigma_h} \left[\sum_{s'} \rho(ss')p^1(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h} [w_h(s) - e(s)m_h^2(s)]}{p^1(s)^{1-\sigma_h} + [p^2(s)e(s)]^{1-\sigma_h} + \beta_h^{\sigma_h} \left[\sum_{s'} \rho(ss')p^1(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h}} \tag{37}$$

Now, combine (9) with (7) for $\ell = 2$:

$$\lambda_h(s)e(s) = \beta_h \sum_{s'} \frac{\rho(ss')c_{2h}^2(ss')^{-\frac{1}{\sigma_h}}}{p^2(s')} \tag{38}$$

Multiplying and dividing each term of the right hand side by $p^2(s')^{\frac{1}{\sigma_h}}$ and then substituting $p^2(s')c^2_{2h}(ss') = m^2_h(s)$, we can sum the numerators on the right hand side and get the following equation:

$$\lambda_h(s) = \beta_h \frac{\sum_{s'} \rho(ss')p^2(s')^{\frac{1-\sigma_h}{\sigma_h}}}{m^2_h(s)^{\frac{1}{\sigma_h}} e(s)} \tag{39}$$

Because $\lambda_h(s) = \sum_{s'} \lambda^1_h(ss')$, we can write:

$$\underbrace{\beta_h \frac{\sum_{s'} \rho(ss')p^1(s')^{\frac{1-\sigma_h}{\sigma_h}}}{m^1_h(s)^{\frac{1}{\sigma_h}}}}_{\sum_{s'} \lambda^1_h(ss')} = \beta_h \frac{\sum_{s'} \rho(ss')p^2(s')^{\frac{1-\sigma_h}{\sigma}}}{m^2_h(s)^{\frac{1}{\sigma_h}} e(s)} \tag{40}$$

or

$$\frac{m^1_h(s)}{m^2_h(s)} = e(s)^{\sigma_h} \frac{\left[\sum_{s'} \rho(ss')p^1(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h}}{\left[\sum_{s'} \rho(ss')p^2(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h}} \tag{41}$$

Solving (41) and (37) simultaneously, we obtain the demand for the two currencies:

$$m^1_h(s) = \frac{\beta_h^{\sigma_h} \left[\sum_{s'} \rho(ss')p^1(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h}}{A_h(s)} w_h(s) \tag{42}$$

$$m^2_h(s) = \frac{\beta_h^{\sigma_h} e(s)^{-\sigma_h} \left[\sum_{s'} \rho(ss')p^2(s')^{\frac{1-\sigma_h}{\sigma}}\right]^{\sigma_h}}{A_h(s)} w_h(s) \tag{43}$$

where

$$\begin{aligned} A_h(s) \equiv\ & p^1(s)^{1-\sigma_h} + [p^2(s)e(s)]^{1-\sigma_h} + \beta_h^{\sigma_h} \left[\sum_{s'} \rho(ss')p^1(s')^{\frac{1-\sigma_h}{\sigma}}\right]^{\sigma_h} + \\ +\ & \beta_h^{\sigma_h} e(s)^{1-\sigma_h} \left[\sum_{s'} \rho(ss')p^2(s')^{\frac{1-\sigma_h}{\sigma_h}}\right]^{\sigma_h} \end{aligned}$$

7.2.2 Derivation of the demand functions of the young

Let us recall the budget constraint of agent h born in state s:

$$p^1(s)c^1_{1h}(s) + p^2(s)e(s)c^2_{1h}(s) = w_h(s) - m^1_h(s) - e(s)m^2_h(s)$$

Firstly, we can obtain total expenditure by substituting the demand for the currencies (13) and (14):

$$p^1(s)c^1_{1h}(s) + p^2(s)e(s)c^2_{1h}(s) = \frac{p^1(s)^{1-\sigma_h} + [p^2(s)e(s)]^{1-\sigma_h}}{A_h(s)} w_h(s) \tag{44}$$

Combining the above equation with (35), we can derive the demand functions of the young agents.

7.3 Appendix C. An analytically tractable version: log utility

In this section, we derive analytically the model under logarithmic utility functions ($\sigma_h \to 1$).

Agent h born in state s solves the following maximization problem:

$$\max \sum_{\ell} \log c^{\ell}_{1h}(s) + \beta_h \sum_{s'} \rho(ss') \sum_{\ell} \log c^{\ell}_{2h}(ss')$$

subject to (2), (3) and (4). The first-order conditions are:

$$\begin{aligned}
c^1_{1h}(s) &: \quad \frac{1}{c^1_{1h}(s)} = \lambda_h(s)p^1(s) && (45)\\
c^2_{1h}(s) &: \quad \frac{1}{c^2_{1h}(s)} = \lambda_h(s)p^2(s)e(s) && (46)\\
c^{\ell}_{2h}(ss') &: \quad \frac{\beta_h\rho(ss')}{c^{\ell}_{2h}(ss')} = \lambda^{\ell}_h(ss')p^{\ell}(s') \qquad \forall\ \ell, s' && (47)\\
m^1_h(s) &: \quad -\lambda_h(s) + \sum_{s'} \lambda^1_h(ss') = 0 && (48)\\
m^2_h(s) &: \quad -\lambda_h(s)e(s) + \sum_{s'} \lambda^2_h(ss') = 0 && (49)\\
\lambda_h(s) &: \quad p^1(s)[c^1_{1h}(s) - \omega^1_h(s)] + p^2(s)e(s)[c^2_{1h}(s) - \omega^2_h(s)] + \\
&+ \quad m^1_h(s) + e(s)m^2_h(s) = 0 && (50)\\
\lambda^1_h(ss') &: \quad p^1(s')c^1_{2h}(ss') - m^1_h(s) = 0 \qquad \forall\ s' && (51)\\
\lambda^2_h(ss') &: \quad p^2(s')c^2_{2h}(ss') - m^2_h(s) = 0 \qquad \forall\ s' && (52)
\end{aligned}$$

Solving the maximization problem requires the following steps. First, combine (45) and (46):

$$p^1(s)c^1_{1h}(s) = p^2(s)e(s)c^2_{1h}(s) \tag{53}$$

Plug the above into the young's budget constraint to obtain:

$$p^1(s)c^1_{1h}(s) = \frac{1}{2}[w_h(s) - m^1_h(s) - e(s)m^2_h(s)] \tag{54}$$

Take the first-order conditions for good 1 in all spots and plug them into (48):

$$\frac{1}{p^1(s)c^1_{1h}(s)} = \beta_h \sum_{s'} \frac{\rho(ss')}{p^1(s')c^1_{2h}(ss')} \tag{55}$$

Then, substitute (54) and (51) into (55) and obtain:

$$m_h^1(s)\left(1+\frac{1}{2}\beta_h\right) = \frac{1}{2}\beta_h[w_h(s) - e(s)m_h^2(s)] \tag{56}$$

Now follow the same steps for good 2. First, take (53) and this time rewrite the budget constraint when young getting rid of good 1. Second, combine (49), (46) and (47) for good 2. Finally, plug in the rewritten budget constraint and (52):

$$e(s)m_h^2\left(1+\frac{1}{2}\beta_h\right) = \frac{1}{2}\beta_h[w_h(s) - m_h^1(s)] \tag{57}$$

Solve simultaneously equations (56) and (57) to obtain agent h's demand for the currencies:

$$\begin{aligned} m_h^1(s) &= \frac{1}{2}\frac{\beta_h}{1+\beta_h}w_h(s) \\ m_h^2(s) &= \frac{1}{2}\frac{\beta_h}{1+\beta_h}\frac{w_h(s)}{e(s)} \end{aligned}$$

The demand functions are:

$$\begin{aligned} c_{1h}^1(s) &= \frac{1}{2}\frac{1}{1+\beta_h}\frac{w_h(s)}{p^1(s)} \\ c_{1h}^2(s) &= \frac{1}{2}\frac{1}{1+\beta_h}\frac{w_h(s)}{p^2(s)e(s)} \\ c_{2h}^1(ss') &= \frac{1}{2}\frac{\beta_h}{1+\beta_h}\frac{w_h(s)}{p^1(s')} \qquad \forall\ s' \\ c_{2h}^2(ss') &= \frac{1}{2}\frac{\beta_h}{1+\beta_h}\frac{w_h(s)}{p^2(s')e(s)} \qquad \forall\ s' \end{aligned}$$

As in the more general case, demand is a linear functions of wealth. However, they are not complicated functions of current and future prices.

In the main body of the paper, we have shown that only S equations in the goods' markets are independent. For instance, we can take the market clearing equations for good 1 when the previous state is equal to the current state:

$$\sum_h c_{1h}^1(s) + \sum_h c_{2h}^1(ss) = y^1(s)$$

Now, substitute the demand functions for good 1 into the market clearing equation:

$$\frac{1}{2}\sum_h \frac{1}{1+\beta_h}\frac{w_h(s)}{p^1(s)} + \frac{1}{2}\sum_h \frac{\beta_h}{1+\beta_h}\frac{w_h(s)}{p^1(s)} = y^1(s)$$

Using the fact that $w_1(s) = p^1(s)y^1(s)$ and $w_2(s) = p^2(s)e(s)y^2(s)$, the market clearing equation for good 1 pins down relative prices in each state:

$$\frac{p^2(s)e(s)}{p^1(s)} = \frac{\omega^1(s)}{\omega^2(s)} \qquad s = 1, ..., S \tag{58}$$

Using (58), we can show that the money market clearing equations pin down nominal prices:

$$p^1(s) = \frac{2M^1}{y^1(s)\sum_h \frac{\beta_h}{1+\beta_h}} \qquad s = 1, ..., S \tag{59}$$

$$p^2(s) = \frac{2M^2}{y^2(s)\sum_h \frac{\beta_h}{1+\beta_h}} \qquad s = 1, ..., S \tag{60}$$

Finally, the exchange rate can be computed:

$$e(s) = e = \frac{M^1}{M^2} \tag{61}$$

Under log utility, it is remarkable that the exchange rate is constant even though the stochastic process is Markov and discount factors differ across agents.

We can now compute the solution for the portfolios, which shows that portfolios are constant across states:

$$m_h^\ell(s) = m_h^\ell = \frac{\frac{\beta_h}{1+\beta_h}}{\sum_h \frac{\beta_h}{1+\beta_h}} M^\ell \qquad \forall\ h, \ell$$

Equations (59), (60) and (61) reveal that wealth and the exchange rate are constant in equilibrium, therefore the demand for the currencies cannot be state dependent. As there is no portfolio rebalancing, the balance of trade is always in equilibrium and the change in net foreign assets is equal to zero in all states (see section 4).

In section 4, we emphasized that if portfolio rebalancing does not occur, then the consumption of the old does not depend on the state of birth. In fact, the consumption allocation is the following:

$$\begin{aligned}
c_{11}^1(s) &= \tfrac{1}{2}\tfrac{1}{1+\beta_1}\omega^1(s) & c_{12}^1(s) &= \tfrac{1}{2}\tfrac{1}{1+\beta_2}\omega^1(s) \\
c_{11}^2(s) &= \tfrac{1}{2}\tfrac{1}{1+\beta_1}\omega^2(s) & c_{12}^2(s) &= \tfrac{1}{2}\tfrac{1}{1+\beta_2}\omega^2(s) \\
c_{21}^1(s's) &= \tfrac{1}{2}\tfrac{\beta_1}{1+\beta_1}\omega^1(s) & c_{22}^1(s's) &= \tfrac{1}{2}\tfrac{\beta_2}{1+\beta_2}\omega^1(s) \\
c_{21}^2(s's) &= \tfrac{1}{2}\tfrac{\beta_1}{1+\beta_1}\omega^2(s) & c_{22}^1(s's) &= \tfrac{1}{2}\tfrac{\beta_2}{1+\beta_2}\omega^2(s)
\end{aligned}$$

7.4 Appendix D. Numerical results

7.4.1 Solution of the model

Under the parameter values specified in section 6, the following prices solve our non-linear system of 6 equations:

$$p^1(1) = 6.1486e - 05 \qquad p^1(2) = 4.9539e - 05$$
$$p^2(1) = 8.1947e - 04 \qquad p^2(2) = 2.9220e - 04$$
$$e(1) = 0.1573 \qquad e(2) = 0.2538$$

We solved the system using Matlab.

7.4.2 Robustness

Table 3: Varying the elasticity of substitution parameter

	$m_1(1)$ $m_1(2)$	e(1) e(2)	$\frac{tb_1(21)}{GDP_1(1)}\%$ $\frac{tb_1(12)}{GDP_1(2)}\%$	$\frac{VAL_1(21)}{GDP_1(1)}\%$ $\frac{VAL_1(12)}{GDP_1(2)}\%$	$\frac{\Delta NFA_1(21)}{GDP_1(1)}\%$ $\frac{\Delta NFA_1(12)}{GDP_1(2)}\%$
$\sigma = 0.5$	0.0615	15.0661	-62.73%	59.57%	-3.16%
	0.1401	6.5061	26%	-23.20%	2.8%
$\sigma = 1$	0.5	1	0	0	0
	0.5	1	0	0	0
$\sigma = 2$	0.7926	0.2789	5.81%	-4.6%	1.21%
	0.7029	0.4079	-6.24%	5.05%	-1.19%
$\sigma = 4$	0.8820	0.1573	5.91%	-3.91%	2%
	0.7833	0.2538	-6%	4.13%	-1.87%
$\sigma = 8$	0.9136	0.1220	5.77%	-2.9%	2.87%
	0.8167	0.1889	-5.45%	2.89%	-2.56%
$\sigma = 16$	0.9272	0.1074	5.77%	-1.91%	3.86%
	0.8314	0.1496	-5.06%	1.8%	-3.26%

Table 4: Varying the persistence parameter

	$m_1(1)$	e(1)	$\frac{tb_1(21)}{GDP_1(1)}\%$	$\frac{VAL_1(21)}{GDP_1(1)}\%$	$\frac{\Delta NFA_1(21)}{GDP_1(1)}\%$
	$m_1(2)$	e(2)	$\frac{tb_1(12)}{GDP_1(2)}\%$	$\frac{VAL_1(12)}{GDP_1(2)}\%$	$\frac{\Delta NFA_1(12)}{GDP_1(2)}\%$
$\rho(ss) = 0.4$	0.8747	0.1891	5.48%	0.46%	5.94%
	0.7903	0.1785	-4.53%	-0.93%	5.46%
$\rho(ss) = 0.5$	0.8752	0.1874	5.52%	0	5.52%
	0.7896	0.1874	-4.66%	0	-4.66%
$\rho(ss) = 0.6$	0.8759	0.1846	5.58%	-1.10%	4.48%
	0.7886	0.1986	-4.84%	1.23%	- 3.61%
$\rho(ss) = 0.7$	0.8771	0.1799	5.66%	-2.59%	3.07%
	0.7874	0.2128	-5.09%	2.89%	-2.2%
$\rho(ss) = 0.8$	0.8790	0.1718	5.78%	-2.45%	3.33%
	0.7856	0.2310	-5.46%	2.47%	-2.99%
$\rho(ss) = 0.9$	0.8820	0.1573	5.91%	-3.91%	2%
	0.7833	0.2538	-6%	4.13%	-1.87%