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1. Introduction

A crucial assumption in many statistical models is that of constant variance. Lately, a family of time series models has been developed relaxing the assumption of constant variance through time. This family is called Autoregressive Conditional Heteroskedasticity (ARCH) Models and was introduced by Engle (1982). These are conditional mean zero, serially uncorrelated stochastic processes with non-constant variances conditional on past, but invariant unconditional variances. The variance of the dependent variable is modeled as a function of past values of the dependent variable and independent, or exogenous variables. Autoregressive Conditional Heteroskedasticity (ARCH) models are specifically designed to model and forecast conditional variances. They been successfully applied in macroeconomic and financial time series in order to model and forecast volatility. Some of the areas where the ARCH models are widely used are: i) portfolio risk analysis, ii) option pricing, iii) time-varying confidence intervals forecasting. The aim is to obtain more accurate intervals of conditional mean by modeling the variance of the errors.

In this paper and in section 2 we present the most important regularities that govern asset returns volatility and the incorporation of them in modeling both the conditional mean and conditional variance. In section 3, we examine the dynamic structure of Greek Stock Market. Section 4 contains an empirical application of ARCH processes in Greek Stock Market. Section 5 deals with the conclusions of the paper. The results are based on Margiora and Panaretos (2001).

2. Autoregressive Conditional Heteroskedasticity Processes

Consider a stochastic process of interest $\{y_t(\theta_0)\}$ parametrized by the finite dimensional vector $\theta_0 \in \Theta \subseteq R^m$, where θ_0 denotes the true value, with conditional mean

$$\mu_t(\theta_0) = E(y_t \mid I_{t-1}) = E_{t-1}(y_t) \quad t=1,2,\ldots$$

I_{t-1} denotes any information available at time $t-1$ (Information Set at time $t-1$). Define the $\{u_t(\theta_0)\}$ process by

$$\{u_t(\theta_0)\} \equiv y_t - \{\mu_t(\theta_0)\} \quad \text{t=1,2,…}$$

The $\{u_t(\theta_0)\}$ process is then defined to follow an ARCH model if the conditional mean equals zero,

$$E_{t-1}(u_t(\theta_0)) = 0,$$

but the conditional variance varies through time,

$$h_t(\theta_0) = Var_{t-1}(u_t(\theta_0)) = E_{t-1}(u_t^2(\theta_0)).$$

2.1 Modeling the conditional variance

Numerous parametric specifications for the time varying conditional variance have been proposed in the literature. The first model is the **ARCH(q)** model introduced by Engle (1982). The conditional variance is a linear function of the past q squared innovations[1]:

$$h_t = a_0 + \sum_{i=1}^{q} a_i u_{t-i}^2, \; a_0 > 0, \; a_i \geq 0, \text{ i=1,…,q.}$$

In empirical applications of ARCH(q) models a long lag length and a large number of parameters are often needed. Thus, Bollerslev (1986) generalized the ARCH(q) model and introduced the General Autoregressive Conditional Heteroskedasticity **GARCH(p,q)** model. The conditional variance is a linear function of the past q squared innovations and the past p conditional variances:

$$h_t = a_0 + \sum_{i=1}^{q} a_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j},$$

for a_0>0, $a_i \geq 0$, i=1,…,q, $\beta_j \geq 0$, j=1,…,p. In empirical investigations the estimate of $\sum_{i=1}^{q} a_i + \sum_{j=1}^{p} \beta_j$ is very close to unity. Engle and Bollerslev (1986) referred to a model satisfying $\sum_{i=1}^{q} a_i + \sum_{j=1}^{p} \beta_j = 1$ as an integrated GARCH process, denoted **IGARCH(p,q)**. Under an IGARCH process the unconditional variance of u_t is infinite, so neither u_t not u_t^2 satisfies the definition of a covariance stationary process.

GARCH models are suitable to capture some characteristics of financial markets. They elegantly capture the volatility clustering in asset returns first noted by Mandelbrot (1963): "… large changes tend to be followed by large changes of either sign, and small changes tend to be followed by small changes…". However, the structure of a GARCH model imposes an important limitation. GARCH models assume that only the magnitude and not the positivity or negativity of innovations determines the feature of h_t because h_t is a function of lagged h_t and lagged u_t^2 and so is invariant to changes in the algebraic sign of the u_t^2's.

On the other hand, asset returns tend to be leptokurtic (heavily tailed). Denote as

[1] The term "innovation" is used instead of the "residual" and expresses the unpredictable part of a financial series.

$$z_t(\theta_0) \equiv u_t(\theta_0) / \sqrt{h_t(\theta_0)}$$

the standardized process, it will have conditional mean zero and time invariant conditional variance unity. If the conditional distribution for z_t is furthermore assumed to be time invariant with finite fourth moment, then the unconditional distribution for u_t will have fatter tails than the distribution for z_t. For instance, for the ARCH(1) model with conditionally normally distributed errors, $E(u_t^4) / E^2(u_t^2) = 3(1-a_1^2) / (1-3a_1^2)$, if $3a_1^2 < 1$, and $E(u_t^4) / E^2(u_t^2) = \infty$ otherwise, both of which exceed the normal value of three.

Financial markets are characterized by the "leverage effect", first noted by Black (1976). The "leverage effect" refers to the tendency for the changes in the stock prices to be negatively correlated with changes in stock volatility. I.e. volatility tends to rise in response to "bad news" (returns lower than expected) and fall in response to "good news" (returns higher than expected).

Nelson (1991), proposed the following model for the evolution of the conditional variance of u_t:

$$\log(h_t) = a_0 + \sum_{j=1}^{\infty} \pi_j \left\{ \left| \frac{u_{t-j}}{\sqrt{h_{t-j}}} \right| - E\left| \frac{u_{t-j}}{\sqrt{h_{t-j}}} \right| + \delta \frac{u_{t-j}}{\sqrt{h_{t-j}}} \right\}$$

This model is referred to as exponential GARCH, or EGARCH model. In this model, h_t depends on both the magnitude and the sign of lagged residuals. The δ parameter allows for the asymmetric effect. If $\delta = 0$ then a positive surprise has the same effect on volatility as a negative surprise. If $-1 < \delta < 0$, a positive surprise increases volatility less than a negative surprise. If $\delta < -1$, a positive surprise actually reduces volatility while a negative surprise increases volatility. For $\delta < 0$ the "leverage effect" exists. Since EGARCH describes the log of h_t, the h_t will be positive regardless of whether the π_j coefficients are positive. Thus, in contrast to the GARCH model, no restrictions need to be imposed on the model for estimation. We can express the infinite moving average representation of the model as the ratio of two finite order polynomials. Thus, an ARMA process provides a simpler parameterization of the form. We denote it as **EGARCH(p,q)**:

$$\log(h_t) = a_0 + \sum_{j=1}^{p} \left\{ a_j \left| \frac{u_{t-j}}{\sqrt{h_{t-j}}} \right| - a_j E\left| \frac{u_{t-j}}{\sqrt{h_{t-j}}} \right| + \delta_j \frac{u_{t-j}}{\sqrt{h_{t-j}}} \right\} + \sum_{i=1}^{q} \left(\beta_i \log(h_{t-i}) \right).$$

The family of ARCH models is remarkably rich. Another route for introducing asymmetric effects is to set:

$$\sqrt{h_t} = a_0 + \sum_{i=1}^{q} \left[a_i^+ I(u_{t-i} > 0) |u_{t-1}| + a_i^- I(u_{t-i} \leq 0) |u_{t-1}| \right] + \sum_{j=1}^{p} \beta_j \sqrt{h_{t-j}} ,$$

where $I(\cdot)$ denotes the indicator function[1]. The model introduced by Zakoian (1990) is called Threshold ARCH or **TARCH(p,q)**.

Glosten, Jagannathan and Runkle (1993) introduced the **GJR(p,q)** model with the following form:

[1] $I(u_{t-i} > 0) = 1$ if $u_{t-i} > 0$, otherwise zero. $I(u_{t-i} \leq 0) = 1$ if $u_{t-i} \leq 0$, otherwise zero.

$$h_t = a_0 + \sum_{i=1}^{q} a_i u_{t-i}^2 + \delta_1 I(u_{t-1} < 0) u_{t-1}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \ ,$$

where $I(\cdot)$ denotes the indicator function. The "leverage effect" is supported if $\delta_1 > 0$. "Good news" has got an impact of a_i and "bad news" has got an impact of $a_i + \delta_1$.
Engle (1990), proposed the Asymmetric ARCH or **AARCH(p,q)** model:

$$h_t = a_0 + \sum_{i=1}^{q} \left(a_i u_{t-i}^2 + \delta_i u_{t-i} \right) + \sum_{j=1}^{p} \beta_j h_{t-j} \ ,$$

where a negative value of δ_i means that positive returns increase volatility less than negative returns.
Taylor (1986) modeled the conditional standard deviation function instead of conditional variance. Schwert (1989) modeled the conditional standard deviation as a linear function of lagged absolute residuals.
The **Taylor/Schwert GARCH(p,q)** model is defined as

$$h_t^{1/2} = a_0 + \sum_{i=1}^{q} a_i |u_{t-1}| + \sum_{j=1}^{p} \beta_j h_{t-j}^{1/2} \ .$$

Higgins and Bera (1992) introduced the Non-linear ARCH or **NARCH(p,q)** model:

$$h_t^{\gamma/2} = a_0 + \sum_{i=1}^{q} a_i |u_{t-1}|^{\gamma/2} + \sum_{j=1}^{p} \beta_j h_{t-j}^{\gamma/2} \ .$$

Geweke (1986) and Pantula (1986) introduced the **log-ARCH(p,q)** model:

$$\log(h_t) = a_0 + \sum_{i=1}^{q} a_i \log\left(u_{t-i}^2\right) + \sum_{j=1}^{p} \beta_j \log\left(h_{t-j}\right).$$

Sentana (1995) introduced the Quadratic ARCH or **QARCH(p,q)** model of the form

$$h_t = a_0 + \sum_{i=1}^{q} a_i u_{t-i}^2 + \sum_{i=1}^{q} \delta_i u_{t-i} + 2\sum_{i=1}^{q} \sum_{j=i+1}^{q} \delta_{ij} u_{t-i} u_{t-j} + \sum_{j=1}^{p} \beta_j h_{j-1} \ .$$

Ding, Granger and Engle (1993) introduce the Asymmetric Power ARCH or **APARCH(p,q)** model:

$$h_t^{\gamma/2} = a_0 + \sum_{i=1}^{q} a_i \left(|u_{t-1}| - \delta_i u_{t-i} \right)^{\gamma/2} + \sum_{j=1}^{p} \beta_j h_{t-j}^{\gamma/2} \ ,$$

which includes seven ARCH models as special cases[1]. Ding, Granger and Engle (1993) estimate the Standard & Poor's 500 (hereafter S&P 500) returns by the APARCH(1,1) model and the estimated power $\gamma/2$ for the conditional heteroskedasticity function is 1.43, which is significantly different from 1 (Taylor/Schwert model) or 2 (GARCH model).

Non-trading periods

Information that accumulates when financial markets are closed is reflected in prices after the markets reopen. If, for example, information accumulates at a constant rate over calendar time, then the variance of returns over the period from the Friday close to the Monday close should be three times the variance from the Monday close to the Tuesday close. Fama (1965) and French and Roll (1986) have found, however, that information accumulates more slowly when the markets are closed than when they are open. Variances are higher following weekends and holidays than on other days, but not nearly by as much as would be expected if the news arrival rate were constant.

[1] ARCH, GARCH, Taylor/Schwert GARCH, GJR, TARCH, NARCH and logARCH

2.2 Modeling the conditional mean

The conditional mean $\mu_t(\theta_0) = E_{t-1}(y_t)$ should be modeled in order to incorporate information from empirical regularities of asset returns.

Non-synchronous trading

According to efficient market theory, the stock market returns themselves contain little serial correlation. Moreover, when high frequency data is used, the non-synchronous trading in the stocks making up an index induces positive first order serial correlation in the return series. To control this Scholes and Williams (1977) suggested a first order moving average form, while Lo and Mackinlay (1988) suggested a first order autoregressive form. Nelson (1991) wrote "as a practical matter, there is little difference between an AR(1) and an MA(1) when the AR and MA coefficients are small and the autocorrelations at lag one are equal".

Risk return tradeoff

Many theories in finance are dealt with the tradeoff between the expected returns and variance, or the covariance among the returns. According to the Capital Asset Pricing Model (CAPM) the excess returns[1] on all risky assets are proportional to the non-diversifiable risk as measured by the covariances with the market portfolio. Merton (1973) in Intertemporal Capital Asset Pricing Theory showed that the expected excess return on the market portfolio is linear in its conditional variance.

The ARCH in mean or ARCH-M model, introduced by Engle et al. (1987), was designed to capture such relationships. In the ARCH-M model the conditional mean is an explicit function of the conditional variance:

$$\mu_t(\theta) = g[h_t(\theta), \theta],$$

where the derivative of the $g(.,.)$ function with respect to the first element is non-zero. The most commonly employed specifications of the ARCH-M model postulate a linear relationship in h_t or $h_t^{1/2}$, e.g. $g[h_t(\theta), \theta] = \mu_0 + \mu_1 h_t$. A positive as well as a negative relationship between risk and return could be consistent with the financial theory. We expect a positive relationship if we assume a rational risk averse investor who requires a larger risk premium during times the payoff of the security is riskier. But we expect a negative relationship under the assumption that during relatively riskier periods the investors may want to save more. In applied researches, there is evidence for both relationships.

Volatility and Serial Correlation

LeBaron (1992) found a strong inverse relation between volatility and serial correlation for Standard & Poor, CRSP value weighted index, Dow Jones and IBM returns. He introduced the Exponential Autoregressive GARCH model or EXP-GARCH in which the conditional mean is a non-linear function of conditional variance:

$$\mu_t(\theta) = \mu_2 e^{\frac{-h_t}{\mu_3}} y_{t-1},$$

As LeBaron stated, it is difficult to estimate μ_3 in conjunction with μ_2 when using a gradient type of algorithm. For this reason, μ_2 is set to the sample variance of the series.

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[1] Asset return minus the risk free interest rate. As an approximation to the risk free interest rate we usually use the three month Treasury Bill return.

LeBaron found that μ_2 is significantly negative and remarkably robust to the choice of sample period, market index, measurement interval and volatility measure.

3. Modeling the Dynamic Structures of the Greek Stock Market: Applying an ARCH model

The data set we will analyze is the General Index of Athens Stock Exchange[1] (hereafter GI). There are totally 2982 observations from 31 July 1987 to 30 July 1999. Define $y_t = \log(p_t / p_{t-1})$ as the continuously compounded rate of return for GI at time t $(t = 1,...,2981)$, where p_t is the daily closing price of GI. In the following lines, we estimate a model to examine several issues previously investigated in the economics and financial literature namely a) the relation between the level of market risk and required return, b) the asymmetry between positive and negative returns in their effects on conditional variance, c) fat tails in the conditional distribution of returns d) the contribution of non-trading days to volatility e) the inverse relation between volatility and serial correlation and f) the non-synchronous trading.

We use the model developed by Nelson (1991), assuming an Autoregressive Moving Average representation for $\ln(h_t)$. To allow for the possibility of non-normality in the conditional distribution of returns, we assume that the $z_t \equiv u_t \big/ \sqrt{h_t}$ are i.i.d. draws from the Generalized Error Distribution (GED)[2]. The density of a GED random variable normalized to have a mean of zero and a variance of one is given by

$$f(z_t) = \frac{v e^{-2^{-1}\left|\frac{z_t}{\lambda}\right|^{v}}}{\lambda 2^{v+1/v}\,\Gamma\left(1/v\right)},$$

$-\infty < z < \infty$, $0 < v \le \infty$, where $\Gamma(.)$ denotes the gamma function, and

$$\lambda \equiv \left(\frac{2^{-2v^{-1}}\,\Gamma\left(1/v\right)}{\Gamma\left(3/v\right)}\right)^{1/2}.$$

The v is a tail-thickness parameter. When $v = 2$, z has a standard normal distribution. For $v < 2$, the distribution of z has thicker tails than the normal (for $v = 1$, z_t has a double exponential distribution) and for $v > 2$, the distribution of z has thinner tails than the normal (for $v = \infty$, z_t is uniformly distributed on the interval $\left[-\sqrt{3}, \sqrt{3}\right]$).

Thus, we model the log of the conditional variance as:

$$\ln(h_t) = a + \frac{\left(\Psi_1 L + ... + \Psi_q L^q\right)}{\left(1 - \Delta_1 L - ... - \Delta_p L^p\right)} z_t,$$

where L is the lag operator.

To account for the contribution of non-trading periods to market variance, we assume that each non-trading day contributes as much to variance as some fixed fraction

[1] We are grateful to GrStocks.com for providing the data.

[2] Other distributions that have been employed are the Normal-Poisson mixture distribution of Jorion (1988), the t-distribution of Bollerslev (1987), the Generalized t-distribution of Bollerslev, Engle and Nelson (1994), the Power Exponential distribution of Baillie and Bollerslev (1989), the normal-log normal mixture of Hsieh (1989) and others.

of a trading day does. If, for example, this fraction is one tenth, than h_t on a typical Monday would be 20 per cent higher than on a typical Tuesday.

Thus we replace the constant term a with:

$$a_t = a_0 + \ln(1 + N_t \delta_0),$$

where N_t is the number of non-trading days between trading days $t-1$ and t, and a_0 and δ_0 are parameters. Fama (1965) and French and Roll (1986) have found that non-trading periods contribute much less than do trading periods to market variance, so we expect that $0 < \delta_0 \le 1$.

To accommodate the asymmetric relation between stock returns and volatility changes we should use a function $g(z_t)$ instead of z_t. The $g(z_t)$ must be a function of both the magnitude and the sign of z_t. One choice, that in certain cases turns out to give h_t well behaved moments, is to make $g(z_t)$ a linear combination of z_t and $|z_t|$:

$$g(z_t) = \theta z_t + |z_t| - E|z_t|.$$

Over the range $0 < z_t < \infty$, $g(z_t)$ is linear with slope $\theta + 1$, and over the range $-\infty < z_t \le 0$, $g(z_t)$ is linear with slope $\theta - 1$. Thus, $g(z_t)$ allows the conditional variance process h_t to respond asymmetrically to rises and falls in stock price.

Finally, we model the log of conditional variance as

$$\ln(h_t) = a_t + \frac{(\Psi_1 L + \ldots + \Psi_q L^q)}{(1 - \Delta_1 L - \ldots - \Delta_p L^p)} g(z_t).$$

The returns are modeled as:

$$y_t = \mu_0 + \mu_1 h_t + \left(\mu_2 + \mu_3 e^{-h_t/\mu_4} \right) y_{t-1} + u_t,$$

where the conditional mean and variance of u_t at time t are 0 and h_t respectively and $\mu_0, \mu_1, \mu_2, \mu_3$ and μ_4 are parameters.

The $\mu_2 y_{t-1}$ term allows for the autocorrelation induced by discontinuous trading in the stocks making up an index. The $\mu_1 h_t$ term allows the tradeoff between the expected returns and variance. The $\mu_3 e^{-h_t/\mu_4} y_{t-1}$ term allows for the inverse relation between volatility and serial correlation of returns. As we have already stated, it is difficult to estimate μ_4 in conjunction with μ_3 when using a gradient type of algorithm. For this reason, μ_4 is set to the sample variance of the series,

$$\mu_4 = \frac{\sum_{t=1}^{T} (y_t - \bar{y})^2}{T - 1}.$$

In order to maximize the likelihood functions, we use the Eviews 3.1 object LogL. The maximum likelihood parameter estimates were computed using the Marquardt algorithm as the BHHH algorithm fails to converge.

For a given ARMA(p,q) order, the $\{z_t\}_{t=1,T}$ and $\{h_t\}_{t=1,T}$ sequences can be easily derived recursively given the data $\{y_t\}_{t=1,T}$ and the initial values $h_1, \ldots, h_{1+\max(p.q+1)}$. Also, $\ln(h_1), \ldots, \ln(h_{1+\max(p.q+1)})$ were set equal to their unconditional expectations

$a_0 + \ln(1+N_{1t}\delta_0), \ldots, a_0 + \ln(1+N_{1+\max(p,q+1)}\delta_0)$. This allows us to write the log likelihood as:

$$L_T(\theta) = \sum_{t=1}^{T} \ln(f(y_t \mid I_{t-1};\theta)) =$$

$$= \sum_{t=1}^{T} \ln\left(\frac{v}{\lambda}\right) - \frac{1}{2}\left|\frac{y_t - \mu_0 - \mu_1 h_t - \left(\mu_2 + \mu_3 e^{-h_t/\mu_4}\right) y_{t-1}}{\sqrt{h_t}\,\lambda}\right|^{v} - (1+v^{-1})\ln(2) - \ln\left[\Gamma(v^{-1})\right] - \frac{1}{2}\ln(h_t).$$

To select the order of the ARMA process for $\ln(h_t)$, we use the Schwarz Criterion(SC) (Schwarz (1978)),

$$SC = (-2l + k\ln(n))n^{-1},$$

where k is the number of estimated parameters, n is the number of observations, and l is the value of the log likelihood function using the k estimated parameters. The model with the lowest SC value is chosen as the most appropriate. Hannan (1980) showed that the SC provides consistent order estimation in the context of linear ARMA models. The asymptotic properties of the SC in the context of ARCH models are unknown. We do not use the Akaike Information Criterion (Akaike (1973)) as it tends to choose the model with the higher number of parameters. Table 3.1 lists the SC values for the various ARMA orders of the model.

Table 3.1.
Schwarz criterion for exponential-E-GARCH(p,q) in Mean model.

	MA order(q)				
AR order(p)	0	1	2	3	4
0	-5.33388	-5.40385	-5.47692	-5.5028	-5.51895
1	-5.33227	-5.61135	-5.61126	-5.61382	-5.61454
2	-5.35678	-5.61002	**-5.62066**	-5.60734	-5.61569
3	-5.33149	-5.60973	-5.61867	-5.61603	-5.61303
4	-5.33477	-5.60905	-5.60569	-5.60204	-5.61363

The ARMA(2,2) gives the SC lowest value. Nelson applied a similar model in daily returns for CRSP value weighted market index for July 1962 to December 1987 and selected the ARMA(2,1) order. Table 3.2 gives the parameters estimates, the estimated standard errors and the t-statistics of the Exponential E-GARCH(2,2) in Mean model:

$$y_t = \mu_0 + \mu_1 h_t + \left(\mu_2 + \mu_3 e^{-h_t/\mu_4}\right) y_{t-1} + u_t$$

$$\ln(h_t) = a_0 + \ln(1+N_t\delta_0) + \frac{(\Psi_1 L + \Psi_2 L^2)}{(1-\Delta_1 L - \Delta_2 L^2)}\left(\theta\frac{u_t}{\sqrt{h_t}} + \left|\frac{u_t}{\sqrt{h_t}}\right| - E\left|\frac{u_t}{\sqrt{h_t}}\right|\right)$$

Table 3.2.
Parameters estimates for
exp-EGARCH(2,2) in Mean model

	Coefficient	Std. Error	z-Statistic	Prob.
α_0	-8.0154	0.2537	-31.5996	0.000

δ_0	0.3777	0.0479	7.8887	0.000
θ	0.0289	0.0426	0.6783	0.249
Δ_1	1.7464	0.0490	35.6073	0.000
Δ_2	-0.7488	0.0484	-15.4651	0.000
Ψ_1	0.5387	0.0428	12.5845	0.000
Ψ_2	-0.4979	0.0393	-12.6682	0.000
μ_1	2.2854	0.9667	2.3641	0.009
μ_0	-0.0003	0.0002	-1.3339	0.091
μ_2	0.1105	0.0353	3.1325	0.001
μ_3	0.2121	0.0638	3.3234	0.000
v	1.3908	0.0406	34.2749	0.000

The estimated correlation matrix of the parameter estimates is presented in Table 3.3. These are computed from the inverse of the sum of the outer product of the first derivatives evaluated at the optimum parameter values.

Table 3.3. Estimated correlation matrix for parameter estimates for exp-EGARCH(2,2) in Mean model (only lower triangle reported)

	α_0	δ_0	Θ	Δ_1	Δ_2	Ψ_1	Ψ_2	μ_1	μ_0	μ_2	μ_3	v
α_0	1											
δ_0	-0.1284	1										
θ	-0.0617	-0.1437	1									
Δ_1	-0.0774	0.0778	-0.0911	1								
Δ_2	0.0821	-0.0756	0.0920	-0.9999	1							
Ψ_1	0.2972	0.0002	-0.0494	-0.4498	0.4504	1						
Ψ_2	-0.2803	-0.0490	0.0747	0.2130	-0.2150	-0.9548	1					
μ_1	-0.0807	-0.0183	0.1054	0.0050	-0.0049	-0.0843	0.0841	1				
μ_0	0.0170	0.0512	0.1833	-0.0343	0.0331	0.0245	-0.0001	-0.5411	1			
μ_2	0.0243	-0.0665	-0.0519	-0.0608	0.0606	0.1136	-0.1028	-0.0141	-0.0495	1		
μ_3	-0.0012	0.0538	0.0250	0.0545	-0.0550	-0.0739	0.0678	-0.0142	0.0807	-0.8585	1	
v	0.0388	0.2607	0.0660	-0.0895	0.0876	-0.1568	0.2211	0.0413	-0.0551	0.0125	-0.0397	1

Let now examine the empirical issues raised in the previous Section.

a) Market Risk and Expected Return: The estimated risk premium is positively correlated with conditional variance, with $\mu_1 = 2.2854$ being statistically significant. This agrees with the significant positive relation between returns and conditional variance found by researchers using GARCH-M models (Chou (1987) and French, Schwert and Stambaugh (1987)), but contracts with the findings of Nelson (1991) who used a similar model and of other researchers not using GARCH models (Pagan and Hong (1988)).
b) The asymmetric relation between returns and changes in volatility, as represented by θ is insignificant. According to the leverage effect, θ should be negative, as we should expect the volatility to rise (fall) when returns surprises are negative (positive). Episodes of high volatility should be associated with market drops. But looking at the plots of the daily conditional standard deviation of returns and the log value of the GI (Figure 3.1), we find that high volatility episodes are associated both with market peaks and drops.
c) Fat Tails. It is well known that the distribution of stock returns has more weight in the tails than the normal distribution (much higher kurtosis than 3), and that a stochastic process is thick tailed if it is conditionally normal with a randomly changing conditional variance (like GARCH processes). In our case the model

generates thick tails with both a randomly changing conditional variance h_t and a thick tailed conditional distribution for u_t. The estimated v is approximately 1.39 with a standard error of about 0.04, so the distribution of z_t is significantly thicker tailed than the normal.

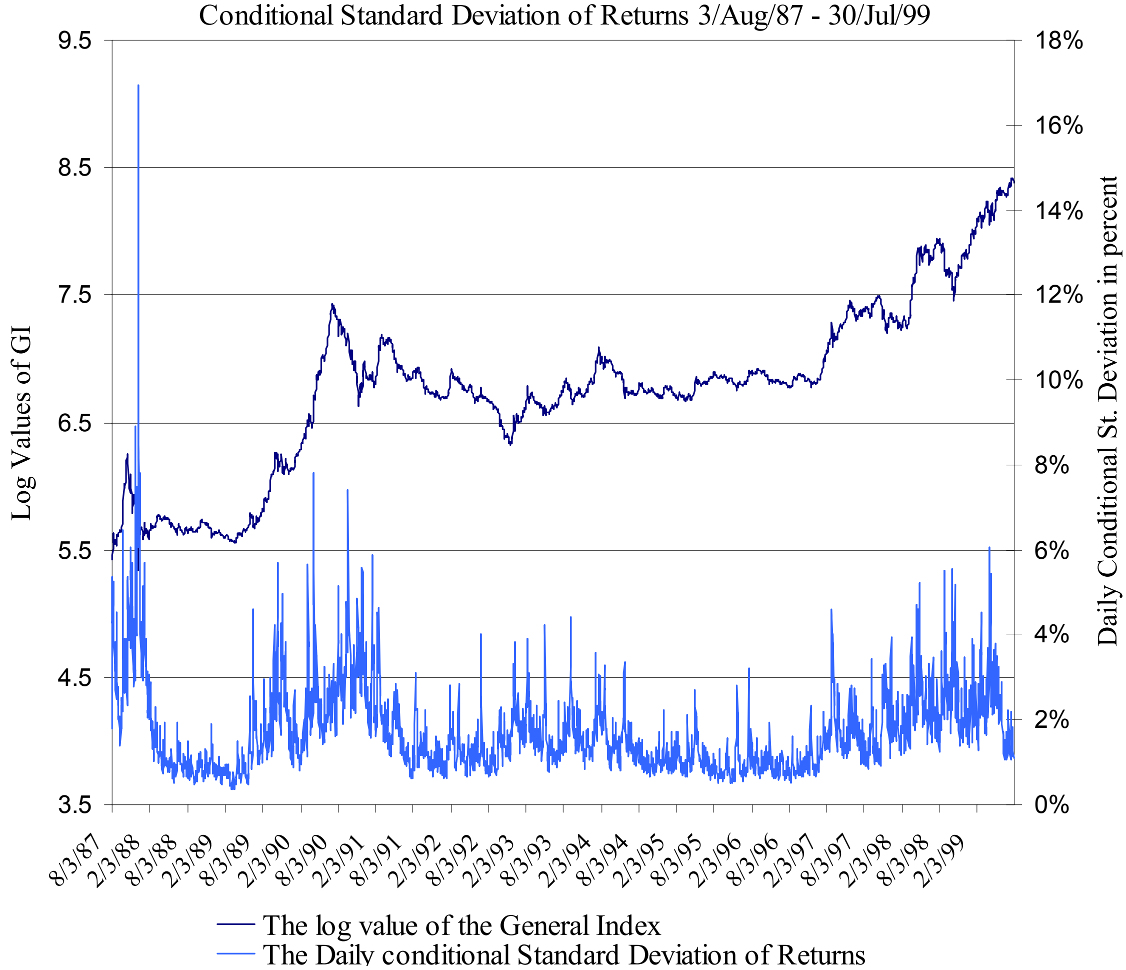
d) The estimated contribution of non-trading days to conditional variance is roughly consistent with the results of French and Roll (1986). The estimated value of δ_0 is about 0.38, which statistically significant, so a non-trading day contributes more than a third as much volatility as a trading day.

e) The inverse relation between volatility and serial correlation for GI daily returns as represented by μ_3 term is statistically significant. Thus the conditional mean is a positive non-linear function of conditional variance.

f) The μ_2 term equals 0.11 shows that the positive non-synchronous trading effect exists in the construction of the GI.

Figure 3.1

The Log Value of the General Index of Athens Stock Exchange and the Daily Conditional Standard Deviation of Returns 3/Aug/87 - 30/Jul/99



5. Conclusion

The paper presented the most important theoretical regularities that govern the dynamic structure of financial time series and tests their validity in Athens Stock Exchange. The Greek Stock Market is examined by applying an ARCH model on the log-returns of the General Index of Athens Stock Exchange from 31 July 1987 to 30 July

1999. The ARCH model fits well to Greek Stock Market data and provides empirical evidence on theoretical regularities. Some of the conclusion are: the existence of a positive (non-linear) trade-off between stock returns and volatility, the absence of leverage effects, the thick tailed stock returns distribution, the information accumulation in a slower rate when the market is closed than when it is open, the existence of positive non-synchronous trading effects and the existence of a long-term memory pattern in stock returns. It would be interesting to develop applications of the model in the fields of portfolio risk management and financial derivatives pricing. Moreover we should test the efficiency gained, if any, in modeling with distributions other than normal. These questions are the scope of further research.

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