Problem of Reduction of the Quantum State's Vector

Zayko, Yuriy

1Russian Presidential Academy of National Economy and Public Administration Stolypin Volga Region Institute,

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Zayko Yu.N.

Russian Presidential Academy of National Economy and Public Administration, Stolypin Volga Region Institute

ABSTRACT

This article presents an investigation of problem of quantum system state's measurement by using an example of particles registered by a measuring device (screen). Some variants of R-procedure which is responsible for measurements are discussed. New variant of R-procedure is suggested. It is based on quantum description of measuring device (screen). In frame of this model R-procedure can be described as part of unitary evolution of the whole system "particle + screen"

Keywords: U – evolution, R- procedure, quantum system, measuring, reduction of state's vector.

1. INTRODUCTION

The behavior of any quantum system according to today's point of view is characterized [1, 2] by smooth evolution which is described with the help of *U*-operator and which is supplemented by abrupt deviations caused by observation (measuring) of the system which is ascribed to action of some operator denoted by *R*. Operator *U* – is a unitary one which is expressed through the system's Hamiltonian H

$$\Psi(t) = U(t)\Psi(0), U(t) = \exp\left(-\frac{itH}{\hbar}\right) \quad (1)$$

Ψ – is a state's vector (wave function, obeying Schrödinger equation), *t* – is a time, *ħ*- Planck constant. There is no such expression for the *R* – operator. Moreover, at present time, any commonly adopted view about the mechanism of the *R*-procedure action is absent. In brief *R* – operator action consists in that under its influence quantum superposition of possible states of the system presented by *Ψ*, is tightened to one state which is fixed by measuring, i.e. so called reduction of state happens. There exist a number of points of view on this process. Its diapason is too much spread. The extremes on them [1, 2] suggest including of consciousness of the observer (E.P. Wigner) or whole neglecting of the *R*-procedure and considering *U*-evolution only with character superposition at classical level too (like Schrödinger cat) but in the different worlds which number is infinitely growing in the process of evolution of the system and its surrounding (H. Everett).
In any case discussion about physical meaning of *R* – procedure concerns the very basic groundings of quantum mechanics enforcing to search new interpretations which are often lie outside the frames of traditional quantum theory. For example in [2] R. Penrose takes an attempt to explain *R* – procedure as a physical process taking into account gravitational

interaction of alternative states of the observing system. According to this point of view he introduces a time of reduction $\Delta t \geq \hbar/\Delta E$. During that time superposition is conserved. Here ΔE - is energy (or indetermination of energy) of the abovementioned gravitational interaction. Estimations which are made in frames of the Newtonian theory of gravitation show that for the microscopic particles (nucleons) time of reduction is greater than 10^7 years what is large enough for the observation particles in superposition (interference experiments). On the other hand for macroscopic particles (couples of water) reduction time in dependence of radius of couples from 10^{-5} to 10^{-3} sm lies in the diapason from several hours to less than 10^{-6} Sec. This shows that with transition from micro- to macroscopic level of description possibility to find a system in a state of superposition is lost[1].

This article concerns the possibility of the physical description of *R* - procedure on the base of quantum description of measuring. It should be noted that present approach differs from the existing ones, using some physical phenomena both real and hypothetical (X-factor [2], zero-point fields [3], quantum Boltzmann entropy [4] and other) at least in two aspects. First, it doesn't involve any well- or unknown physical phenomena for the description of *R* – procedure but concerns on the problem of information handling during the process of measurement, especially on the process of device's preparation to measurement. Second, this approach seems to be simplest than others, but it may be own opinion of the author.

2. REDUCTION OF WAVE PACKET.

A simple experiment which will help to understand the essence of problem looks as follows (see fig. 1). Particles which are emitted by the source *S* through collimator *K* reach the screen *P* (photoplate), where they make a traces – black regions which are revealed after developing the photoplate. Particle with momentum *p*, which is perpendicular to the screen (indeterminacy of *x*-component of momentum $\Delta p_x = 0$) is described by the wave function Ψ which has a form of plane wave whose front is parallel to the screen.

The probability of particle distribution along the screen doesn't depend on co-ordinate *x*, so the indeterminacy of *x*-coordinate of particle $\Delta x = \infty$, but it spoils the screen only at one point (if we neglect the size of spoil spot). Just that reduction of wave packet is ascribed to the action of *R* – procedure. For better understanding the essence of problem one can imagine a case when source is sending and screen is registering particles one by one what isn't a problem taking into account contemporary level of experimental technic.

Traditional description of measuring problem is based on observation of quantum system with the help of classical device. As we will show below quantum description of device can lead to physical interpretation of *R* – procedure.

[1] That is, Schrodinger cat is most likely either dead or alive, than dead and live simultaneously.

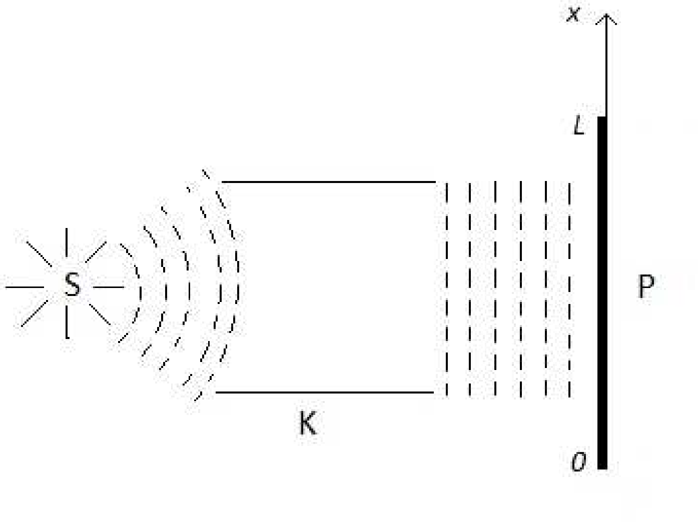

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Fig. 1. Scheme of the experiment for particle's registration by screen. S – source of particles, K – collimator, P – screen of length L; Dashed line shows fronts of wave function Ψ before and after the collimator.

3. QUANTUM SCREEN AND MEASURING.

A screen consists of separate atoms which are interacting with particle under consideration. If we do not take into account an atomic structure of the screen for a time, so as interaction between screen's atoms we may consider a screen as a system which is described by sole wavefunction Φ. If one denotes wavefunction of particle as Ψ, then amplitude of probability of finding a particle in definite point of the screen looks like as $\Phi\Psi$. In order to extremely simplify a problem we consider the screen as one-dimensional one along x, $0 < x < L$, with its longitude L. We neglect dependence of Ψ from all co-ordinates beside x. It is obviously that for $x < 0$ and for $x > L$ $\Phi = 0$. Under this conditions Φ obeys Schrödinger equation in potential $V(x)$ which looks like one-dimensional box with infinite depth. Registration of particle by screen means that particle has been captured by screen. Precision of registration depends on what eigenstates of a screen take part in formation of particle's wave packet.
The fact that particle hits (or doesn't hit) the screen brings one bit of information. Registration of particle in the right or left side of the screen needs one bit of information too. Generally, registration of particle within screen with precision L/N, where $N=2^s$, s is integer, needs $s + 1$ bits of information[2]. Handling of arbitrary amount of information is connected with energy expenditures [6]. Particle itself can't bring this energy, in other case observation of its collision with the screen will violate the law of energy conservation [6]. Thus, measuring device, i.e. the screen, must deliver energy which is needed for information handling from its own stocks. For the purpose of provisioning desirable precision of measuring Δx, it is needed to prepare initial state of the screen, i.e., Φ in a form of wave packet whose size doesn't exceed Δx. It can be done with the help of superposition of screen's eigenstates Φ_n, which corresponds to n – th quantum level for particle with mass m in given potential $V(x)$ ($1 \le n \le N$, $N \sim L/\Delta x$ – number of eigenstates in superposition)[3]. Later this wave packet will

[2] It is so due to definition of a bit: "A bit is an amount of information which is contained in the answer on question which allows only two answers, "yes" or "no" with equal probability" [5].
[3] Further reasoning reminds preparing of squeezed states in given potential [7].

evolve changing its shape. Size of character domains of its amplitude will be of the order of size of the region of initial packet's localization Δx. In other words, evolution of the wave packet has week influence on precision of place of particle's registration.

One can prove that final result doesn't depend on initial shape of the packet $\Phi(x, t = 0)$, t – time. Thus for simplicity of calculation we choose it looks as $\Phi(x, 0) = (N/L)^{1/2}$ for $0 \le x \le L/N$ and $\Phi(x, 0) = 0$ at $x < 0$ and $x > L/N$. So, representing $\Phi(x, t)$ as a sum of first N screen's eigenstates we receive, taking into account an explicit expressions for eigenstates $\Phi_n(x)$ and corresponding eigenvalues E_n [8]

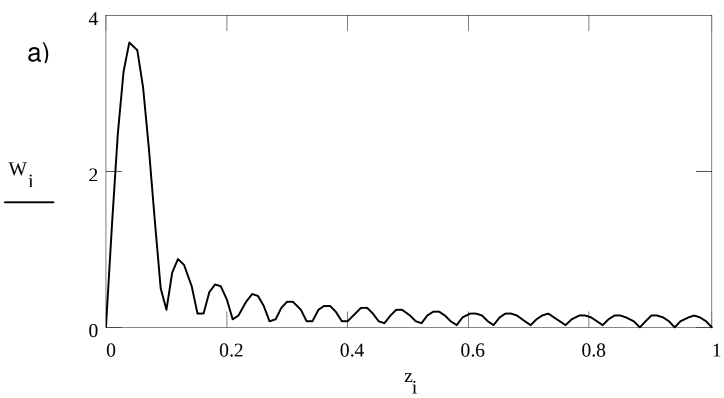
$$\begin{aligned}
&\Phi(x,t) = \sum_{n=1}^{N} c_n \exp(-itE_n/\hbar)\Phi_n(x) \ , \\
&\Phi(x,0) = \sum_{n=1}^{N} c_n \Phi_n(x) \ , \\
&\Phi_n(x) = \sqrt{\frac{2}{L}} Sin \frac{\pi n}{L} x, E_n = \frac{\pi^2 \hbar^2}{2mL^2} n^2 \\
&c_n = \int_0^{L/N} \Phi(x,0)\Phi_n^*(x)dx = \frac{\sqrt{2N}}{n\pi}\left(1 - Cos\frac{n\pi}{N}\right)
\end{aligned} \qquad (2)$$

Or, in dimensionless form

$$\begin{aligned}
&\Phi(z,t) \approx \frac{1}{\pi}\sqrt{\frac{2N}{L}} \sum_{n=1}^{N} \frac{1 - Cos\frac{\pi n}{N}}{n\sqrt{L}} \exp(-in^2\tau) \cdot Sin\pi nz \\
&\tau = \frac{t}{T}, T = \frac{2mL^2}{\pi^2 \hbar}, z = \frac{x}{L}
\end{aligned} \qquad (3)$$

m – is a mass whose sense will be clarified later.

Decomposition of $\Phi(x, t)$ on $\Phi_n(x)$ in (2) is approximate. It becomes precise when upper limit of the sum $N \to \infty$, but this needs infinite amount of energy. State which is prepared in this manner corresponds to needed precision of particle's registration ~ L/N ~ Δx. Particle hits a screen at time t in the point x with probability $W(x) = |\Psi(x,t)\Phi(x,t)|^2 = |\Phi(x,t)|^2$, which can be calculated according to formulas (2). The result of calculation is presented in Fig. 2.



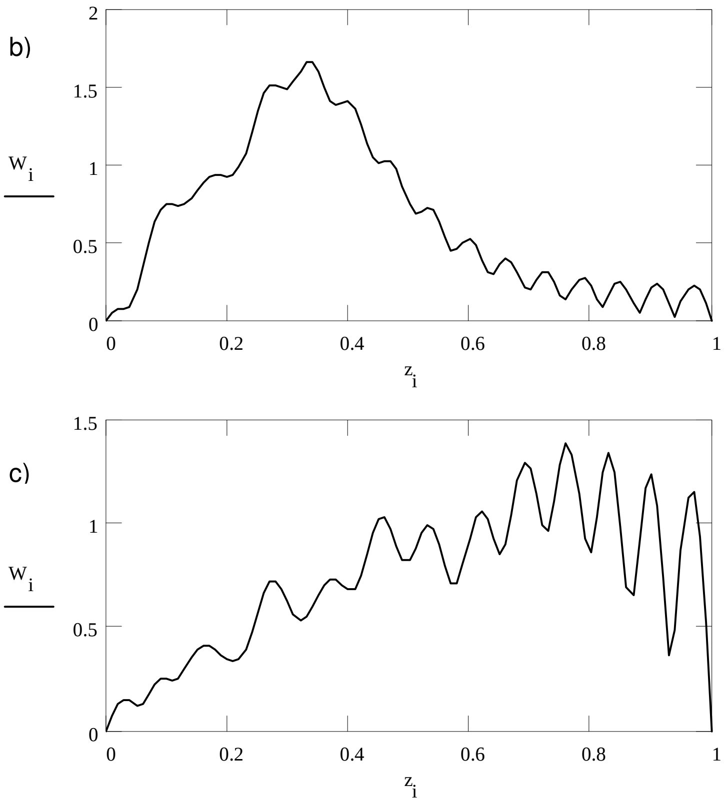


Fig.2. Dependence of probability W(x) for different values of τ = t/T. z = x/L; N=16. a) τ = 0, b) τ =0.05, c) τ = 0.1

As it leads from above, at the moment of the screen's preparation to registration of particle (*t = 0*), the size of region of wave packet's localization is determined by desirable precision, which in turn depends on number of bits of information which is supposed to be spent. Localization of this region could be arbitrary. We choose it at the left side of the screen. Later the region of wave packet's localization will be spreading in the limits of the screen. Nevertheless, particle will be registered most probably in some points of the screen than in others with the given precision.

4. AN EXPERIMENT WITH PARTICLE'S INTERFERENCE.

Above discussion can be implemented for the explanation of well-known experiment with particle's interference. In this experiment, particles hit a screen after going through the wall which has two slots. Results of that experiment prove the wave properties of particles. Besides that this experiment demonstrates the role which plays its conditions. If one knows, at least in principle, which slot particle went through, then superposition will be destroyed, and interference picture will be vanished. In order to avoid mysticism, one must tractate this result not in the sense that Nature can withstand to all our contrivances but in the sense that not all principles of Nature are known.
In order to explain this experiment in frame of our model we, as before, will tractate the screen as quantum object and two slots in the wall – as independent one from another. A preparation of the screen for registration of particles with needed precision looks like as before, with some difference, which consists in that wave function of the screen has now two maximums instead one. More precision we wish to obtain, most narrow these maximums

have to be. In other respects our method stays the same as earlier. Let us consider the screen as a harmonic oscillator with frequency *ω*, which is described by orthonormal system of eigenstates

$$\Psi_n(z) = \frac{1}{\pi^{1.4}} \frac{1}{\sqrt{2^n n!}} e^{-z^2/2} H_n(z), z = x\sqrt{\frac{m\omega}{\hbar}},$$
$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \tag{4}$$

m – is mass of particle, x – is its co-ordinate, E_n – are energy levels, $n = 0, 1, 2, \ldots$ - integer, H_n – Hermit polinoms [8]. An initial state of the screen we take as follows

$$\Phi(z, t=0) = \sqrt[4]{\frac{\pi}{a}} \left\{ \exp\left[-\frac{a(z-b)^2}{2}\right] + \exp\left[-\frac{a(z+b)^2}{2}\right] \right\} \tag{5}$$

Here values $a^{-1/2}$ and $b >> a^{-1/2}$ characterize precision and place of particle's registration. This corresponds to the wall before screen with two slots separated one from another at distance $2b$. Let us represent $\Phi(z, t)$ in the form of superposition

$$\Phi(z,t) \approx \sum_{n=0}^{N} A_n \Psi_n(z) e^{-iE_n t/\hbar}$$
$$A_n = \int_{-\infty}^{\infty} \Phi(z, t=0) \Psi_n(z) dz \tag{6}$$

Value $N=2^s-1$, where s represents amount of bits of information which is needed for providing given precision. As before decomposition (6) is approximate one. In converts to explicit expression if $N \to \infty$. At Fig. 3 the results of calculating of $|\Phi(z, t)|$ are presented for different values of time t (in units of $1/\omega$). Optimal value for $N = 7$ was chosen experimentally.

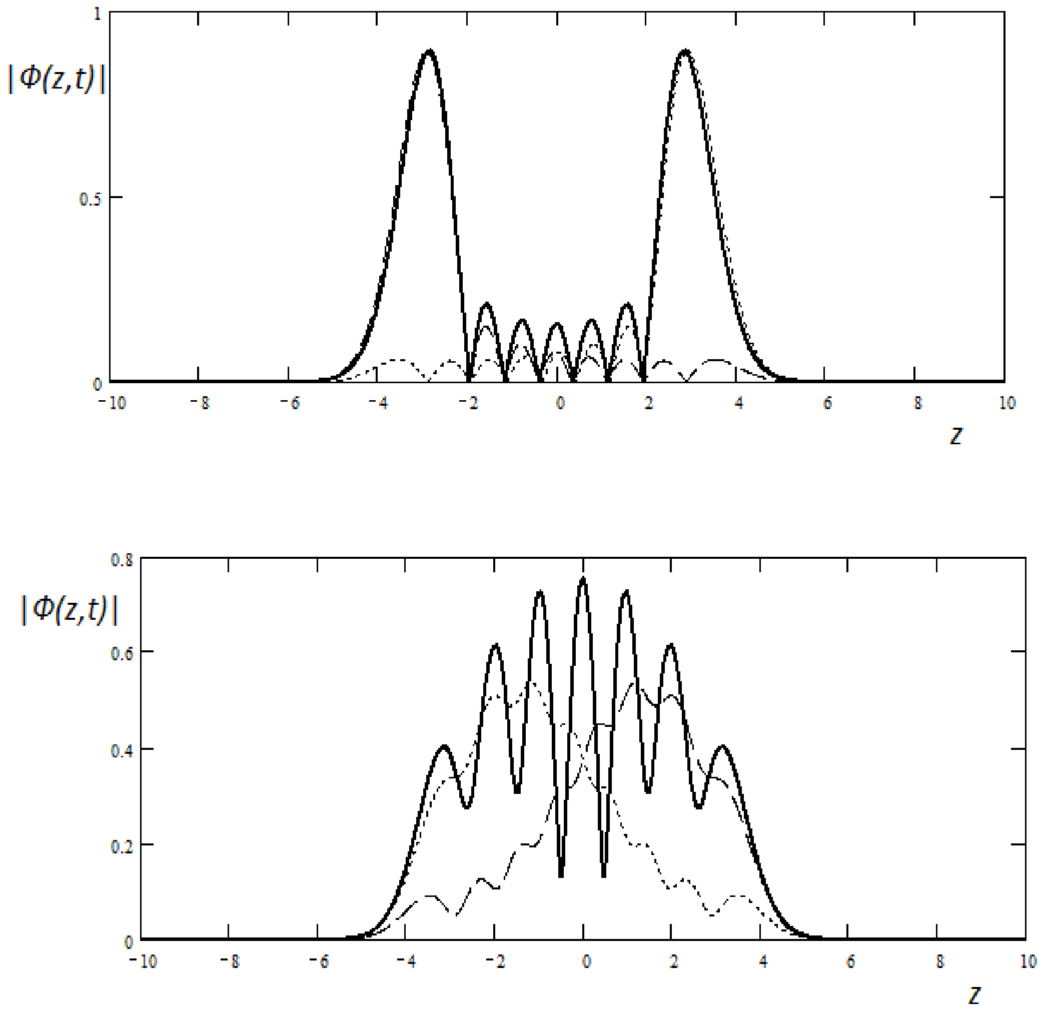


Fig. 3. Dependence of |*Φ(z,t)*| from z = x/L at *t = 0* (upper case) and at *t = 2* (low case). Bold solid line corresponds to |Φ(z,t)|, dashed line and points are corresponding to two additions in formulas (5) separately; a = 8, b = 3

Fig. 3 explicitly demonstrates interference picture for the waves of information

5. DISCUSSION

It was shown in present article that some progress could be achieved in interpretation of quantum measurements if registration device (screen) is assumed as quantum object. In addition to this preliminary stage of measurements is introduced, which is connected with setting needed precision of measurements.

Preparation of the device (screen in our case) for measurements is an important stage of the same measurement which is omitted in earlier discussions cited in [1, 2], for some reasons. It is known, that any device, or more generally, any receiver of information, will not be able to fulfill their task if it will not be in a state of readiness [4] to receive information which was sent to it. Preliminary setting of device, which is concerned with establishing of needed precision, could be fulfilled, if lowest eigenstates of quantum model of device, which may be excited by registered particle, is used. So, usage of quantum model of device is essential and neediness.

More detailed picture of the screen's preparation process looks as follows. Despite that atomic structure of the screen was neglected earlier its whole neglecting is impossible. So, proposed model needs clarification. Firstly, not all the screen's atoms take part in the process of registration in the equal manner. Only those atoms which are in the non-excited state and could be excited by the particle to be registered may initialize the chemical process which will be revealed as darkness of o photo plate. All other atoms could not interact with the particle in a proper way (see footnote 4). Secondly, the wave function of the screen Φ in the form (4) corresponds to superposition of Φ_n which are the eigenfunctions of screen atoms with mass *m* considered as non-interacting particles putting in square box with infinite depth. This is very crude model of the screen and its application may be approved only as first approximation to the problem.

Besides that, as was shown in the last paragraph, this approach can be used for two-slots interference experiment.

It should be stressed that process of the reduction of wave packet considered here in frame of the theory of quantum mechanical measuring has common nature. It is intrinsic to all situations in which evolution connects two principally different pictures of events: probabilistic and deterministic ones. While event did not happen we have set of probabilities for different possible events. When event has become we definitely speak about it and "forget" all other ones, which could but didn't happen. The process that take place in the moment of happening of that event could be named as reduction of probabilities' set to one value corresponding to the event which was happened. If one does neglect that process' duration he will receive complete analogy with quantum mechanical reduction. If we will use just the same methods of description (probabilistic in present article) before so as after happening of the event problem of reduction is vanished.

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[4] Or in a state of waiting. This fact is well-known in the theory of operating systems [9]. When one process is sending a signal to other, this second process can receive it if it is in the state of waiting. In our case particle will not be registered by screen, if no one atom of screen will be exited by falling particle and will affect on it as repulsing center. Such a phenomena are well-known in nuclear physics and find practical implementation, for example, for creating traps for ultra-cold neutrons (Zeldovich Y. B., Sov Phys JETP. 1959; **36,** 1952.Russian)

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