Debt and Deficit Fluctuations in a Time-Consistent Setup

Grechyna, Daryna

Middlesex University London

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Daryna Grechyna[‡]

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Abstract

This paper compares the stochastic behavior of fiscal variables under optimal fiscal policy for the cases of full commitment by the government (Ramsey problem) and no commitment by the government (focusing on differentiable Markov perfect equilibrium). It shows that the cyclical properties of fiscal variables are similar for both commitment assumptions. These conclusions are robust to two different specifications of the structure of public bonds (risk-free and state-contingent), and to different sets of the parameters. The cyclical properties of fiscal variables, regardless of commitment assumptions, can be determined by the parameters of the utility function.

Keywords: optimal taxation; time-consistent policy; market incompleteness.

JEL Classification Numbers: E61, E62, H21, H63.

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[‡]Department of Economics, Middlesex University London, Business School, Hendon Campus, The Burroughs, London NW4 4BT, UK. Tel.: +44 (0)2084115000. E-mail: dgrechna@gmail.com.

1 Introduction

A government's commitment to its fiscal plan is a potential determining factor of fiscal policy outcomes. In the ideal world, the government could minimize the economic distortions from exogenous shocks by announcing the fiscal policy once for all future periods and not deviating from this announcement as time goes on. Practical implementation of such a full-commitment policy is difficult; one reason for this is that this type of fiscal policy would be time-inconsistent (see Kydland and Prescott, 1977). In modern economies, governments usually are not fully committed to following a fiscal policy chosen in one period during future periods. Instead, the fiscal plans are revised periodically, with the frequency of these revisions being dependent on the institutions and on external events such as exogenous shocks or political elections. If the economic outcomes from periodic reoptimization by the government are significantly different from those that would occur given a government's full commitment to its fiscal plan, then welfare of the population may be significantly improved by constructing mechanisms that guide the government towards full commitment.

In this paper, we analyze the effect of the absence of government commitment on the cyclical properties of fiscal variables by comparing them to the case of full commitment. To this end, we consider a version of the optimal fiscal policy model developed by Lucas and Stokey (1983) with public bonds of one-period maturity and with endogenous public spending. The government reoptimizes its fiscal plan every period, taking into consideration the current state of the economy and the effect of current policy on the anticipated future policy (thus, we consider the Markov perfect equilibria). The equilibrium transition path of the analogous economy has been characterized by Debortoli and Nunes (2013) and Occhino (2012), who showed that the optimal government policy under no commitment is such that the time-inconsistency problem reduces over time and disappears at the steady state.

The time-inconsistency problem originates from the temptation of the government to manipulate the price of the public bonds which are in inelastic supply. At a given period, the discretionary government chooses to unexpectedly deflate accumulated public debt to increase the revenue side of its budget; at the same time, it anticipates that future governments will have an incentive to follow the same discretionary policy, inefficiently (from today's perspective) manipulating future prices. Therefore, the current government wants to use its discretion but it would like to force future governments to act in a committed way. The way to do that is to decrease the time-inconsistency problem.

For a broad range of parameter values, the only stable (deterministic) steady state is the steady state with zero public debt. The less debt, the lower the temptation to manipulate prices in the future. As a result, in the long run the debt converges to a zero mean independently of the initial level of debt. In the economy with aggregate uncertainty, where the government mission is to mitigate the effect of exogenous shocks, the same discretion mechanism should reduce the ability of the government to smooth taxes. Intuitively, once debt is around zero, and because there is little temptation to manipulate prices, the stochastic properties of the model should resemble those of the full-commitment case, with a lower variance for debt and more variance in taxes, as the government tries to keep the debt around zero to minimize the commitment problem. In this paper we verify this intuition using the numerical simulations of the no-commitment and full-commitment economies.

We discuss two alternative structures for public bonds markets. In the first case, the government has access to risk-free bonds, unconditional on realization of uncertainty in the next period. In the second case, the government has access to state-contingent bonds, conditional on the realizations of the next period exogenous state. Marcet and Scott (2009) compare the stochastic properties of fiscal variables under these alternative market structures when the government is fully committed to its fiscal plan. We show that, without commitment, the statistical nature of fiscal variables changes, but the consequences for their cyclical properties are insignificant. In particular, the public debt is stationary in the no-commitment-incomplete-markets setup and is less persistent than output in the no-commitment-complete-markets setup, differently from the full-commitment cases with respective market structures. The absence of government commitment results in more volatile and less persistent fiscal variables, but does not change the direction of responses of variables to exogenous shocks.

Furthermore, we show that the cyclical properties of fiscal variables, regardless of the commitment assumption, depend on the parameters of the utility function. In particular, in the considered setup with endogenous public expenditures and separable CRRA utility, the responses of fiscal variables to the productivity shock depend on the relative intertemporal elasticities of substitution of private and public consumption. An increase in productivity (a positive supply shock) implies that both types of consumption, private and public, should increase. One of the aims of optimal fiscal policy is to smooth taxes over time. If the public consumption is relatively more elastic, taxes increase to finance higher government expenditures, but the government deficits increase as well; opposite occurs if private consumption is relatively more elastic. A shock to government expenditure (a demand shock) affects the optimal public consumption directly and leads to conventional responses of fiscal variables (a positive shock to government spending increases deficits and taxes). These findings are related to several studies that have discussed the role of utility parameters for economic outcomes under time-consistent optimal fiscal policy. In particular, Martin (2009) has shown that the elasticity of consumption defines the long-run level of public debt in a model without capital in which the discretionary government can issue both public bonds and money. Barseghyan et al. (2013) have considered fluctuations of fiscal variables in a model with political frictions; the authors successfully replicated the pattern of public debt in the U.S., but obtained counterfactual predictions of a counter-cyclical tax rate and pro-cyclical public spending. We confirm the conjecture of those authors that the cyclicality of taxes may depend on the curvature of utility from consumption.

For the case of full-commitment, there are numerous characterizations of the cyclical properties of fiscal variables. For example, Chari et al. (1994) describe the stochastic properties of the business-cycle model; Aiyagari et al. (2002) characterize the stochastic behavior of fiscal variables in the model without capital when financial markets are incomplete; Scott (2007) studies the properties of labor taxes in the stochastic world; and Marcet and Scott (2009) compare the cyclical properties of a variety of models under complete and incomplete financial markets.

Several examples exist of closely related studies that have discussed the Markov perfect equilibrium properties in line with this paper. Klein and Ríos-Rull (2003) compare the cyclical properties of optimal fiscal policy without commitment with those under full commitment in the neoclassical growth model with a balanced government budget. In their model, government cannot commit to a capital income tax. The authors show that capital income tax is positive and high and that labor income tax is highly volatile in the no-commitment case, contrary to the full-commitment case. Klein et al. (2008) describe the Markov perfect equilibrium for a benevolent government's problem with a balanced budget when the government has to choose public expenditures. Ortigueira et al. (2012) and Debortoli and Nunes (2013) discuss optimal fiscal policy with an unbalanced budget and endogenous government spending in a deterministic economy with and without capital, respectively. Krusell et al. (2006) discuss a time-consistent fiscal policy with government debt and exogenous government spending in a deterministic economy; the authors find a Markov perfect equilibrium with discontinuous policy functions and debt time-series similar to those under full commitment. Martin (in press) analyzes welfare consequences of different institutional arrangements of time-consistent government policy.

The paper is organized as follows. Section 2 describes the model and the government problem under different bond-market structures and with no commitment by the government. Section 3 characterizes stability properties of the deterministic steady state under no commitment. Section 4 compares the stochastic properties of fiscal variables under both no commitment by the government and full commitment by the government. Sections 5 and 6 discuss the role of the utility parameters and of the bounds on debt for cyclical properties of fiscal variables. Section 7 concludes.

2 The Model

The model represents a version of Lucas and Stokey's (1983) economy, where a benevolent government decides how to finance government spending through taxes on labor income and through issues of public debt. The only departures from their original model are that government spending is endogenous, the sources of uncertainty are a labor productivity shock and a shock to preferences for public consumption, and bonds traded between the government and households are only of one-period maturity.[1] The structure of the economy is described in detail below.

The exogenous state of the economy is given by $s_t = \{\theta_t, \varphi_t\}$, where θ_t represents a shock to labor productivity and φ_t represents a shock to the marginal value of the public good (government expenditure shock). It is assumed that s_t belongs to a finite set S and follows a Markov process with a stationary transition matrix P; we denote by $P(s_{t+1}|s_t)$ the probability of the exogenous state being s_{t+1} at period $t+1$ conditional on being s_t at period t. We denote by s^t the history of the exogenous state from the initial period until t and by $P_t(s^t|s_0)$ the probability of s^t conditional on s_0, where s_0 is given.

The economy is inhabited by identical households. The time endowment of a representative household is 1 for each period of time. The resource constraint of the economy satisfies:

$$c_t(s^t) + g_t(s^t) = \theta_t(1 - x_t(s^t)), \tag{1}$$

where $c_t(s^t)$, $g_t(s^t)$, and $x_t(s^t)$ denote the period t private consumption, public consumption, and leisure, respectively. At period t, a representative household derives utility from private consumption and leisure, $h(c_t(s^t), x_t(s^t))$, separable in its arguments, as well as utility from public consumption, $\varphi_t v(g_t(s^t))$. Functions h and v are twice differentiable, increasing and concave in their arguments. The total instantaneous utility function of the household can be denoted as $u(c_t(s^t), x_t(s^t), g_t(s^t)) \equiv h(c_t(s^t), x_t(s^t)) + \varphi_t v(g_t(s^t))$. The household consumes and saves in the form of public bonds out of labor income net of tax payments and interest income from its holdings of government bonds.

The benevolent government sets the labor taxes $\tau_t(s^t)$ and chooses the amount of public bonds to issue at time t to finance a stream of government spending and be repaid at time $t+1$, maximizing the welfare of the representative household; initial public debt is given. It is assumed that the government cannot default on its debt; it repays all its obligations outstanding on a particular date.

We consider two different structures for the public bonds, risk-free and state-contingent (as in Aiyagari et al., 2002), and analyze the implications for the cyclical properties of fiscal variables. Risk-free public bonds are unconditional on the realization of uncertainty at period $t+1$. We refer to the case in which the government has access only to risk-free public bonds as the case of incomplete markets and denote by $b_t(s^t)$ the amount of risk-free public bonds issued by the government at time t. State-contingent public bonds represent a vector indexed to possible realizations of uncertainty at $t+1$. We refer to the case in which the government has access to state-contingent bonds as the case of complete markets and denote by $\{b_t(s^{t+1})\}_{s_{t+1}\in S}$ the amount of state-contingent public bonds issued by the government at time t. The labor taxes and public expenditures are chosen within a period and are state-contingent. All debt issued by the government is held by the public.

Given the government policy, which is defined by the level of government expenditures and taxes, a representative household chooses its consumption, leisure, and savings to maximize its expected lifetime utility

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t u(c_t(s^t), x_t(s^t), g_t(s^t)) P_t(s^t|s_0), \tag{2}$$

subject to its budget constraint. We denote by p_t the price of a public bond in units of time t consumption. In the case of risk-free public bonds, the household chooses the sequence $\{c_t(s^t),\ x_t(s^t),\ b_t(s^t)\}_{t=0}^{\infty}$ and its budget constraint is given by:

$$c_t(s^t) + p_t(s^t)b_t(s^t) \leq (1 - \tau_t(s^t))\theta_t(1 - x_t(s^t)) + b_{t-1}(s^{t-1}). \tag{3}$$

In the case of state-contingent public bonds, the household chooses the sequence $\{c_t(s^t),$ $x_t(s^t),\ \{b_t(s^{t+1})\}_{s_{t+1}\in S}\}_{t=0}^{\infty}$ and its budget constraint is given by:

$$c_t(s^t) + \sum_{s_{t+1}\in S} p_t(s^{t+1})b_t(s^{t+1}) \leq (1 - \tau_t(s^t))\theta_t(1 - x_t(s^t)) + b_{t-1}(s^t). \tag{4}$$

It is assumed that the issues of public debt are such that the transversality condition of the household's problem is satisfied. The environment is competitive and the wage in period t is equal to θ_t. The government faces a budget constraint that restricts its spending on public consumption and public debt services from exceeding the tax revenues and income from the newly issued bonds:[2]

$$\begin{aligned} & g_t(s^t) + b_{t-1}(s^{t-1}) \leq \tau_t(s^t)\theta_t(1 - x_t(s^t)) + p_t(s^t)b_t(s^t), \\ & if\ bonds\ are\ risk-free; \\ & g_t(s^t) + b_{t-1}(s^t) \leq \tau_t(s^t)\theta_t(1 - x_t(s^t)) + \sum_{s_{t+1}\in S} p_t(s^{t+1})b_t(s^{t+1}), \\ & if\ bonds\ are\ state-contingent. \end{aligned} \tag{5}$$

A competitive equilibrium in the considered economy, given government policy and the initial public debt, consists of stochastic processes for bond prices and a household's allocations for consumption, leisure, and savings such that, given bond prices, the allocations maximize *the household's expected lifetime utility subject to its budget constraint; the government budget constraint is satisfied; and the resource constraint of the economy is satisfied.*

The optimality conditions of a representative household set its intratemporal choice of consumption and leisure as a function of income taxes,

$$u_{x,t}(x_t(s^t))/u_{c,t}(c_t(s^t)) = (1-\tau_t(s^t))\theta_t, \tag{6}$$

and intertemporal choice of consumption levels as a function of bond price,

$$\begin{aligned}
&p_t(s^t)u_{c,t}(c_t(s^t)) - \beta \sum_{s_{t+1}\in S} P(s_{t+1}|s_t)u_{c,t+1}(c_{t+1}(s^{t+1})) = 0, \\
&if\ bonds\ are\ risk-free; \\
&p_t(s^{t+1})u_{c,t}(c_t(s^t)) - \beta P(s_{t+1}|s_t)u_{c,t+1}(c_{t+1}(s^{t+1})) = 0, \forall s_{t+1} \in S, \\
&if\ bonds\ are\ state-contingent.
\end{aligned} \tag{7}$$

For the case of state-contingent bonds, (7) represents a system of equations for each state s_{t+1}. The optimality conditions (6) and (7) can be used to substitute away taxes and prices from the household budget constraint to obtain the implementability constraint to be taken into account by the government:[3]

$$\begin{aligned}
&c_t(s^t)u_{c,t}(c_t(s^t)) + \beta \sum_{s_{t+1}\in S} P(s_{t+1}|s_t)u_{c,t+1}(c_{t+1}(s^{t+1}))b_t(s^t) = \\
&= u_{x,t}(x_t(s^t))(1-x_t(s^t)) + u_{c,t}(c_t(s^t))b_{t-1}(s^{t-1}),\ if\ bonds\ are\ risk-free; \\
&c_t(s^t)u_{c,t}(c_t(s^t)) + \beta \sum_{s_{t+1}\in S} P(s_{t+1}|s_t)u_{c,t+1}(c_{t+1}(s^{t+1}))b_t(s^{t+1}) = \\
&= u_{x,t}(x_t(s^t))(1-x_t(s^t)) + u_{c,t}(c_t(s^t))b_{t-1}(s^t),\ if\ bonds\ are\ state-contingent.
\end{aligned} \tag{8}$$

The resource and implementability constraints (1) and (8) implicitly define the optimal consumption and leisure of the household as functions of government policy (described next) and the exogenous state.

2.1 Government Policy

The government is unable to commit to a fiscal plan developed for the long term. Instead, it chooses the issues of public debt and labor income taxes (or, equivalently, the levels of public expenditures and labor income taxes) every period, given the inherited debt and the stochastic state of the economy. That is, the government policy functions depend only on fundamentals. The policy chosen by the government in a given period affects the state of the economy (in particular, the amount of debt outstanding) in the next period, which in turn defines the next period policy choices of the government, and so on. This effect of current policy on the anticipated future policy is taken into account in the government maximization problem.

The government decides on its policy before households make their choices about the variables they control. The decisions are made only once for each period of time and after the exogenous shocks have been realized. Under such policy-making timing, the optimal allocations of households in a given period represent the reaction functions on the government policy undertaken in that period; differently from the government, a representative household's choices do not have direct influence on the future government policy.

Given that policy chosen by the government depends only on the state of the economy, we switch to a shortcut notation, useful for recursive formulation of the government problem. First, we change the notation for the amount of risk-free bonds $b_{t-1}(s^{t-1})$ to b and $b_t(s^t)$ to b'. Second, we change the notation for the amount of state-contingent bonds $b_{t-1}(s^t)$ to b_s and $\{b_t(s^{t+1})\}_{s_{t+1}\in S}$ to $\{b'_{s'}\}_{s'\in S}$. Finally, we substitute the time subindex $t+1$ with superindex prime and drop the time subindex t and the arguments s^t or s^{t+1} from the remaining variables. Then, the resource constraint (1) reads as

$$c + g = \theta(1 - x), \tag{9}$$

and implementability constraint (8) reads as:

$$\begin{aligned} cu_c(c) + \beta \sum_{s'\in S} P(s'|s)u'_c(c')b' &= u_x(x)(1-x) + u_c(c)b, \ \textit{or} \\ cu_c(c) + \beta \sum_{s'\in S} P(s'|s)u'_c(c')b'_{s'} &= u_x(x)(1-x) + u_c(c)b_s, \end{aligned} \tag{10}$$

in the case of risk-free public bonds and state-contingent public bonds, respectively.

The problem of the current government can be formulated in terms of household allocations: the government chooses consumption, leisure, and the amount of new public bonds to maximize the expected utility of households, given (9) and (10) and factors including the amount of public bonds to be repaid in current period, the exogenous state, and the fact that anticipated future policy depends on the government's current policy via the amount of public bonds to be repaid in the future. In particular, (10) includes future consumption implemented by the next period government, depending on the inherited level of debt and the realization of uncertainty.[4]

Let $\Gamma \in [\underline{B}; \bar{B}]$ be the set of possible debt levels, $\underline{B} < 0 < \bar{B}$. In the case of risk-free public bonds (incomplete markets), the states of the economy are b and s. Denote as $\mathbf{C}(b,s)$ the policy that the current government operating under incomplete markets anticipates will be followed in the future, so that $c' = \mathbf{C}(b', s')$. Then, given b, s, and the perception that future governments implement $\mathbf{C}(b,s)$ with corresponding net present value utility $V(b,s)$, substituting g from the resource constraint (9), the problem of the current government operating under incomplete markets can be formulated as follows:

$$\max_{c,x,b'} \left\{ u(c, x, \theta(1-x) - c) + \beta \sum_{s' \in \Theta} P(s'|s) V(b', s') \right\}, \tag{11}$$

$$s.t.: u_c(c) + \beta \sum_{s' \in \Theta} P(s'|s) u_c'(\mathbf{C}(b', s'))b' = u_x(x)(1-x) + u_c(c)b. \tag{12}$$

In equilibrium, the anticipated policy function will coincide with the actual policy function.

We can define the equilibrium in the case of risk-free public bonds as follows:

A Markov perfect equilibrium in a given incomplete markets economy consists of functions $V(b,s)$, $\mathbf{C}(b,s)$, $\mathbf{X}(b,s)$, *and* $\mathbf{B}(b,s)$ *such that for all* $(b,s) \in \Gamma \times S$*:*

$$\{\mathbf{C}(b,s), \mathbf{X}(b,s), \mathbf{B}(b,s)\} = \underset{c,x,b'}{\operatorname{argmax}} \{u(c, x, \theta(1-x) - c) + \beta \sum_{s' \in \Theta} P(s'|s) V(b', s')\},$$

subject to (12); and

$$V(b,s) = u(\mathbf{C}(b,s), \mathbf{X}(b,s), \theta(1 - \mathbf{X}(b,s)) - \mathbf{C}(b,s)) + \beta \sum_{s' \in \Theta} P(s'|s) V(\mathbf{B}(b,s), s').$$

In the case of state-contingent public bonds (complete markets), the states of the economy are b_s and s. Denote as $\mathbf{C}(b_s, s)$ the policy that the current government operating under complete markets anticipates will be followed in the future, so that $c' = \mathbf{C}(b'_{s'}, s')$. Then, given

b_s, s, and the perception that future governments implement $\mathbf{C}(b_s, s)$ with corresponding net present value utility $V(b_s, s)$, substituting g from the resource constraint (9), the problem of the current government operating under complete markets can be formulated as follows:

$$\max_{c,x,\{b'_{s'}\}_{s'\in\Theta}} \left\{ u(c, x, \theta(1-x) - c) + \beta \sum_{s'\in\Theta} P(s'|s) V(b'_{s'}, s') \right\}, \tag{13}$$

$$s.t.: u_x(x)(1-x) + u_c(c)(b-c) - \beta \sum_{s'\in\Theta} P(s'|s) u'_c(\mathbf{C}(b'_{s'}, s')) b'_{s'} = 0. \tag{14}$$

A Markov perfect equilibrium in a given complete markets economy consists of functions $V(b_s, s)$*,* $\mathbf{C}(b_s, s)$*,* $\mathbf{X}(b_s, s)$*, and* $\{\mathbf{B}_{s'}(b_s, s)\}_{s'\in\Theta}$*, such that for all* $(b_s, s) \in \Gamma \times S$*:*

$$\{\mathbf{C}(b_s, s), \mathbf{X}(b_s, s), \{\mathbf{B}_{s'}(b_s, s)\}_{s'\in\Theta}\} = \underset{c,x,\{b'_{s'}\}_{s'\in\Theta}}{\text{argmax}} \{u(c, x, \theta(1-x)-c) + \beta \sum_{s'\in\Theta} P(s'|s) V(b'_{s'}, s')\},$$

subject to (14); and

$$V(b_s, s) = u(\mathbf{C}(b_s, s), \mathbf{X}(b_s, s), \theta(1 - \mathbf{X}(b_s, s)) - \mathbf{C}(b_s, s)) + \beta \sum_{s'\in\Theta} P(s'|s) V(\mathbf{B}_{s'}(b_s, s), s').$$

Assume that equilibrium policy functions are differentiable under both complete and incomplete markets.[5]

The necessary conditions for government problems (11)-(12) or (13)-(14) consist of (9), (10), and the following equations (see derivations in appendix):[6]

$$u_x - \theta u_g - \gamma(u_{xx}(1-x) - u_x) = 0, \tag{15}$$

and

$$u_c - u_g - \gamma(u_{cc}(b-c) - u_c) = 0 \textit{ and} \tag{16}$$

$$\beta \sum_{s'\in\Theta} P(s'|s)[\gamma u'_c + \gamma u'_{cc} \mathbf{C}'_b b' - \gamma' u'_c] = 0, \textit{ if bonds are risk} - \textit{free}, \tag{17}$$

or

$$u_c - u_g - \gamma(u_{cc}(b_s - c) - u_c) = 0 \textit{ and} \tag{18}$$

$$\gamma u'_c + \gamma u'_{cc} \mathbf{C}'_{b_s} b'_s - \gamma' u'_c = 0, \forall s' \in S, \textit{ if bonds are state} - \textit{contingent}, \tag{19}$$

where γ is the Lagrange multiplier attached to the implementability constraint (10). Note that the number of equations in (19) is equal to the number of possible states s'.

The respective government problems under full commitment are formulated in the appendix. Equations (15) and (16)-(17) or (18)-(19) are close to their full commitment counterparts (equations (21)-(23) or (25)-(27) in the appendix), but do not include the term that measures the effect on the current government budget constraint of maintaining the price of the bond consistent with the value of the bond in the previous period (this term is described in appendix). Instead, equations (17) and (19) include either the term $\gamma u'_{cc}\mathbf{C}'_b b'$ or $\gamma u'_{cc}\mathbf{C}'_{b_s} b'_s$, each of which measures the effect of current government budget constraints on the policy choices of future governments (that is, the impact of choice of debt in the present on future choices).

In equilibrium, the reaction by households to discretionary government policy should reduce the ability of the government to smooth economic fluctuations in comparison to the full-commitment case under both complete and incomplete bond markets. The importance of government discretion as compared with full commitment by the government is evaluated in the following sections using numerical simulations.

3 Deterministic Steady States

In a deterministic setup, equation (17) or (19) implies that the model economy features three types of steady states: 1) $b = 0$, 2) $\mathbf{C}_b = 0$ with $\gamma \neq 0$, and 3) $\gamma = 0$. In all three cases, the time-inconsistency problem disappears.

The steady state without distortionary taxation, $\gamma = 0$, is not stable and is also characterized by $\mathbf{C}_b = 0$.[7] In equilibrium with a sufficiently large amount of accumulated assets, there is no need to rely on distortionary taxation, and allocations are unaffected by government policy.

The stability properties are not easily characterized for the other two types of steady states: 1) $b = 0$ and 2) $\mathbf{C}_b = 0$ with $\gamma \neq 0$. Only for a restricted set of parameters is it possible to prove analytically that the steady state with $b = 0$ is asymptotically stable (e.g. when utility is linear in leisure). To explore the stability properties of these types of steady states for a general utility function, we use numerical solutions.

For further analysis, we specify the utility function with the general form as follows:

$$u(c_t, x_t, g_t) = \phi_c \frac{c_t^{1-v_c}}{1-v_c} + \phi_x \frac{x_t^{1-v_x}}{1-v_x} + (1-\phi_c-\phi_x)\varphi_t \frac{g_t^{1-v_g}}{1-v_g}, \; v_c, v_x, v_g > 0. \tag{20}$$

In this section, we set $\varphi_t = 1$, $\theta_t = 1$, for all t. The weights in the utility functions ϕ_c and ϕ_x are chosen so that leisure accounts for 66% of the time endowment and the ratio of public to private consumption is 0.36 in the first-best equilibrium (same values as in Debortoli and Nunes, 2013). The discount factor β is set to 0.98 to approximately match the return on public bonds. We consider different combinations of utility parameters: $v_c, v_x, v_g \in [0.5; 2.5]^3$.

We reduce the system of optimality conditions (9), (10), (15), (16), (17) without uncertainty to three equations by substituting away g and γ. For each combination of $\{v_c, v_x, v_g\}$, we solve the resulting system of three equations, together with the first derivatives of these equations with respect to b, at the steady state $\mathbf{C}_b = 0$ and then at the steady state $b = 0$. As a result, we obtain the values of $\mathbf{C}(b)$, $\mathbf{X}(b)$, $\mathbf{B}(b)$, and their derivatives with respect to b, $\mathbf{C}_b$, $\mathbf{X}_b$, $\mathbf{B}_b$, for a given set of parameters at a given steady state. If the solution features $|\mathbf{B}_b| < 1$, we conclude that the corresponding steady state is asymptotically stable. When there are multiple solutions, we choose the solution that delivers maximum utility as the relevant one.

The results of this exercise indicate that for all considered combinations of parameters, the steady state $\mathbf{C}_b = 0$ is not asymptotically stable (it is characterized by $\mathbf{B}_b = 1$), while the steady state $b = 0$ is asymptotically stable. The latter steady state is characterized by $\mathbf{C}_b > 0$.[8]

As is well known from the literature (see, for example, Marcet and Scott, 2009), in a deterministic environment with full commitment on the part of the government, the steady state of the economy is determined by initial public indebtedness and is asymptotically stable. Thus, the types of equilibrium steady states that exist without commitment by the government are quite different from the corresponding steady states under a fully committed government. In the case where initial public debt is zero, the full-commitment and the no-commitment steady states coincide, and there is no time-inconsistency problem to deal with.

In a world with uncertainty, however, restrictions on policy functions, imposed by the presence or absence of government commitment, can affect the cyclical properties of the economy even around the time-consistent steady state. The next sections describe the model economies with uncertainty.

4 Comparison of No- and Full-Commitment Outcomes

This section compares the stochastic properties of fiscal variables under no commitment and full commitment using simulations of the model economies. Solutions for the no-commitment case are based on global approximations of policy functions on the grid of states. Numerical algorithms and accuracy measures are described in the appendix. Corresponding problems under full commitment by the government with state-contingent and risk-free bonds have been extensively characterized in the literature (see Marcet and Scott, 2009) and are summarized in the appendix.

For the purpose of simulations, the model parameters are set as follows. The weights in the utility functions ϕ_c, ϕ_x and the discount factor β are set in the same way as in the previous section. In the literature, v_c and/or v_x are usually set to one or two, while v_g is usually set to one; it can also be slightly lower or greater than one (Debortoli and Nunes, 2013). We report the results of the model simulations for $v_c = v_x = 1$, $v_g = \{0.9, 1.1\}$. The role of different utility parameters is discussed in the next section.

We assume that the labor productivity shock and the public expenditure shock are independent and follow AR(1) processes: $\ln\theta_t = \rho_\theta \ln\theta_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, $\ln\varphi_t = \rho_\varphi \ln\varphi_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. We set $\rho_\theta = \rho_\varphi = 0.85$ and $\sigma_\varepsilon = \sigma_\epsilon = 0.01$.[9] The transition matrix for the exogenous state of the economy $s_t = \{\theta_t, \varphi_t\}$ is constructed using Tauchen's (1986) method.

We report the results of simulations in the form of impulse-response functions for environments including complete and incomplete markets given either no or full commitment by the government for the following variables: public debt, measured as the debt to be repaid (b_t), primary deficits, measured as the deficit net of debt service costs ($g_t - \tau_t\theta_t(1 - x_t)$), labor taxes, and output. The impulse responses are calculated as the difference between the value of the variable in period t and in the period preceding the shock as the percentage of output in the period preceding the shock (e.g., for debt $100(b_t - b_0)/(\theta_0(1 - x_0))$, where the shock occurred in $t = 1$), excepting the labor taxes, for which the responses are calculated as the percentage deviation from their value in the period preceding the shock.

Figures 1 and 2 present the impulse responses to one standard deviation positive productivity shock and positive public expenditure shock, respectively.

[FIGURE 1]

[FIGURE 2]

We can observe the following:

i) The direction of response of fiscal variables to a shock is the same for the full-commitment and no-commitment cases, but depends on the structure of bond markets. Moreover, for labor productivity shock, the direction of response of fiscal variables depends on the parameters of the utility function. In particular, given $v_c = v_x = 1$, a positive productivity shock leads to an increase in public debt, deficits, and taxes when public consumption is elastic, and to a decrease in these variables when public consumption is inelastic, under incomplete markets. The direction of response of public debt is opposite for the complete markets case. A positive shock to government spending leads to higher debt, deficits, and taxes under incomplete markets, for all considered utility parameters, and to a reduction in debt under complete markets.

ii) Public debt and deficits are more volatile and taxes are less volatile under full commitment, as compared to the no-commitment case, both under incomplete and under complete markets. This reflects the role of full commitment by the government in taxation smoothing. Although the time-inconsistency problem is minor around the steady state, the magnitude of fluctuations is affected by the absence of commitment.

iii) Public debt and deficits are less persistent in the no-commitment than in the full-commitment case (though the differences in persistency are not very significant under complete markets). To see that, note that the responses of these variables come back to zero faster under no commitment. Higher persistency of the variables under full commitment is a consequence of the government obligation to keep bond prices consistent with the value of previously accumulated debt, which is absent under the no-commitment policy. In the no-commitment environment, the persistence of the variables is affected by the impact of the state variable, public debt, on policy functions, under both bond market structures. Previous positive indebtedness induces the government to deflate the debt price (as also discussed in Debortoli and Nunes, 2013), reducing the reaction of fiscal policy to the shocks.

As has been shown by Marcet and Scott (2009), under full commitment by the government all of the variables are characterized by the same persistence as output when state-contingent bonds are available, whereas public debt and taxes are more persistent than other variables when only risk-free bonds are available. This is only partially true in the no-commitment case studied here. Public debt and taxes are still the most persistent variables when the government does not commit to fiscal policy and operates risk-free public bonds. However, when the bonds are state-contingent, public debt is less persistent than output. Moreover, under no commitment, the state-contingent public debt and corresponding primary deficit inherit some characteristics of their risk-free counterparts: their response changes sign in the long run and does not vanish over a long period of time. This persistence is because the policy functions depend on the inherited debt in the no-commitment case under complete markets, acting differently from the full-commitment case. Thus, regardless of the market structure, the effect of bad shocks is partially spread over time, leading to a decrease in deficits sufficiently far into the future to service the additional interest, similar to the full-commitment-incomplete-markets case discussed in Marcet and Scott (2009).

Comparing these results to the four facts about the behavior of public debt and deficits in the U.S. described in Marcet and Scott (2009), we can conclude that the first three facts (high persistence of public debt, co-movement of deficit and debt and persistent fluctuations in deficits) hold for the incomplete markets economy with no commitment by the government, while the fourth fact, about pro-cyclicality of deficits and debt, is not robust to different parametrizations of the utility function under both full and no commitment. Differences in the directions of responses of fiscal variables for small changes in the utility parameters impose challenges for successful calibration of the model. Therefore, this paper does not attempt to use the considered model to replicate the moments of corresponding variables in the data.

From Figures 1 and 2 we infer that under no commitment, the government partially loses the ability to smooth taxes in response to exogenous shocks. However, it is unclear whether different market structures have different effects on taxation policy under no commitment as compared with full commitment by the government. The differences in responses of public debt are smaller when the government has access to state-contingent debt. This is partially due to the fact that stationarity of public debt is independent of the commitment assumption under complete markets, distinguishing it from the incomplete markets case. With full commitment by the government when markets are incomplete, the public debt has a unit root component that originates from an extra state variable that resembles a martingale (see more on this in Aiyagari et al., 2002). Under no commitment, the only endogenous state is inherited debt; therefore, the public debt is stationary regardless of the market structure. Moreover, in the full-commitment-incomplete-markets setup, given the unit-root nature of public debt, the bounds $\underline{B}$ and $\bar{B}$ can be binding and influence the variance of public debt series (see more on the role of the bounds on debt in Section 6).

To draw an inference about the importance of market structure, we calculate the ratio of impulse responses for labor taxes under no commitment to the corresponding impulse responses under full commitment. The response of labor taxes is four and ten times greater under no commitment than under full commitment with incomplete and complete markets, respectively. The ratios of the responses of households allocations in the no-commitment and full-commitment cases (not shown in the figures) are also greater under complete markets. This suggests that the absence of government commitment has more severe implications under complete than under incomplete markets. Nevertheless, similar to Aiyagari et al. (2002) we find negligible differences in welfare across different market structures and commitment assumptions.

Overall, the results of simulations imply that removing the government commitment assumption changes the quantitative characteristics, but does not qualitatively affect the cyclical properties of fiscal variables. Next, we discuss the role of different parametrizations of the utility function.

5 The Role of Utility Parameters

The analysis above suggests that the stability of the equilibrium under no commitment and the direction of responses of fiscal variables to the productivity shock under both no and full commitment by the government depend on the relative intertemporal elasticities of substitution of the components of the utility function. To explore the latter issue, we consider a linear approximation of the responses of variables to a productivity shock as a function of the inverse of the elasticity of public consumption, v_g, around the point $v_g = 1$, given $v_c = v_x = 1$. We establish the following result (see details on the numerical procedure in appendix):

In the economy with full commitment and complete markets, given $v_c = v_x = 1$ *and a positive labor productivity shock, the response of private consumption and debt are increasing in* v_g *and the response of leisure, labor taxes, and deficits are decreasing in* v_g*; moreover, the responses of taxes, leisure, and deficit (debt) are negative (positive) for* $v_g > 1$, *positive (negative) for* $v_g < 1$*, and zero for* $v_g = 1$.

Numerical simulations described in the previous section suggest that this result should also hold in the economy with complete markets and no commitment (due to the similarity of full- and no-commitment outcomes) and in the full- and no-commitment economies with incomplete markets (for all the variables, excepting public debt, which has a response opposite to that in complete markets). Moreover, varying the elasticity of substitution of private consumption, given v_g, has the opposite implications for the direction of responses: the higher (lower) v_c is, the more positive (negative) the response of taxes and deficits.

Sensitivity of the cyclical properties of fiscal variables to the elasticities is specific to the economy with endogenous public spending and originates from a trade-off between public and private consumption in a given period. Intuitively, the more inelastic public consumption is, the more the private consumption increases in response to positive productivity shock as compared with the public consumption. This leads to relatively greater increase in the marginal rate of substitution between consumption and leisure and, therefore, to relatively lower response of optimal taxes. All together, this is likely to lead to a decrease in deficits. The effect on public debt is determined by the market structure: a decrease in deficits should lead to a decrease (increase) in debt under incomplete (complete) markets.

A shock to government expenditure affects the optimal public consumption directly and leads to conventional responses of the fiscal variables (a positive shock to government spending increases deficits and taxes) for all combinations of parameters from the set $\{v_c, v_x, v_g\} \in [0.5; 2.5]^3$. At the same time, a positive shock to the value of endogenous government spending increases output, acting differently from the exogenous government spending shock that is usually discussed in the literature. This is because an increase in the relative value of public consumption leads to an optimal increase in the labor supply from households (the relative value of leisure decreases).

In addition to the cyclical properties of the variables, the utility parameters affect the response of bond price to discretionary policy: u_{cc} depends on the elasticity of private consumption. The numerical simulations suggest that the response of consumption to changes in debt, $\mathbf{C}_b$, is also sensitive to the intertemporal elasticities of substitution of private and public consumption.

Thus, the parameters of the utility function may define the cyclical properties of fiscal variables and the magnitude of the difference between the policies under full and no commitment by the government.

6 The Role of Bounds on Debt, $\underline{\mathbf{B}}$ and $\bar{\mathbf{B}}$

In formulating the government problem, we assumed that possible debt levels belong to set $\Gamma \in [\underline{B}, \bar{B}]$, $\underline{B} < 0 < \bar{B}$. The bounds $\underline{B}$ and $\bar{B}$ are not restrictive for the no-commitment economy, which converges to the steady state with zero public debt. In equilibrium, the households' response to the government policy leads to a reduction in debt level until the time-inconsistency problem disappears (in the deterministic setup) or is negligible (in the stochastic setup). Under full commitment, the bounds on debt can be occasionally binding and affect the volatility of variables in the incomplete markets economy (see Aiyagari et al., 2002).

To get more insights on the role of bounds on debt, consider an alternative set of possible debt levels: $\Gamma \in [\underline{B}, \bar{B}]$ with positive debt bounds $0 < \underline{B} < \bar{B}$. These bounds are restrictive for the economy with no commitment by the government: a positive lower bound on public debt fixes the distortions from discretion at the level where public debt is equal to the lower bound.

[FIGURE 3]

Figure 3 plots an example of simulations of public debt series for no-commitment and full-commitment economies with $\underline{B}$ and $\bar{B}$ equal to 50% and 70% of output (to proxy the recent OECD average debt levels). In the full-commitment case, public debt behaves as a random walk; in the no-commitment case it converges to the lower positive bound, with occasional insignificant deviations from the bound after the most severe shocks. Thus, restrictions that prevent convergence of public debt to zero strengthen the differences in cyclical properties between the full- and no-commitment environments. The further the level of public debt is from zero, the greater the gains from the discretion are, and the greater the response to discretion – the derivative $\mathbf{C}_b$ is positive and increasing in b. Given that households' consumption function counteracts the changes in public debt for high public debt levels, it is difficult if not impossible for the government to use public debt to smooth fluctuations far from the steady state. In contrast, in the low level of discretion (around the steady state), households react more to the tax-smoothing policy of the government and less to the discretionary policy, increasing the similarity between no- and full-commitment outcomes.

7 Conclusions

This paper compared the stochastic behavior of fiscal variables under optimal fiscal policy with no commitment by the government and full commitment by the government in an economy with endogenous government spending. It showed that the cyclicality and persistence of the variables are not significantly affected if the government reoptimizes its policy every period instead of committing to a fiscal plan chosen once for an infinite number of time periods in the future. This result suggests that the timing of policymaking is not necessarily the defining force of the model's outcomes.

This paper also showed that the parameters of the utility function can affect the cyclical properties of fiscal variables. In particular, the relative intertemporal elasticities of substitution of private and public consumption define the direction of responses of public debt, deficits, and taxes to a supply (the labor productivity) shock. This finding represents a challenge for the calibration of optimal fiscal policy models.

Notes

[1]Thus, the particular management of the maturity of public debt to make the full-commitment policy time-consistent, as proposed by Lucas and Stokey (1983) and Domínguez (2005), or to complete the market, as proposed by Angeletos (2002), cannot be applied.

[2]There is no need to introduce public transfers to ensure that the government budget constraint will be always satisfied with equality. Whenever the government constraint is loose, welfare may be improved by increasing the level of public spending; it will not distort the household's decisions, given that public spending enters the utility in an additive way.

[3]Thus, the primal approach to solve the government problem is used.

[4]In case of non-separable preferences, (10) also includes future labor; future government spending is defined by the feasibility constraint.

[5]The corresponding equilibrium for a deterministic economy has been analyzed by Debortoli and Nunes (2013). They show that equilibrium with differentiable policy function exists for the particular utility function.

[6]Here and in the appendix we skip the arguments of the functions to simplify the notation.

[7]To see this, note that without distortionary taxation, the first-order conditions corresponding to the control variables become $u_c = u_g = u_x$. These equations define steady-state consumption and leisure as functions only of the utility parameters. Thus, consumption and leisure are constant over time and are independent of public debt. Substituting consumption and leisure into (8) without uncertainty yields, in this steady state, $\mathbf{B}_b = 1/\beta$; that is, this steady state is unstable.

[8]Further numerical experiments suggest that the steady state $b = 0$ is not stable ($|\mathbf{B}_b| > 1$) for some combinations of considered parameters when the weights in the utility functions ϕ_c, ϕ_x are chosen so that the ratio of public to private consumption is less than 0.36 in the first-best equilibrium.

[9]The exact values of these parameters are not crucial given that it is not a quantitative exercise.

References

[1] Aiyagari, S. Rao, Albert Marcet, Thomas J. Sargent, and Juha Seppälä (2002) Optimal taxation without state-contingent debt. *Journal of Political Economy* 110, 1220–1254.

[2] Angeletos, George-Marios (2002) Fiscal policy with noncontingent debt and the optimal maturity structure. *The Quarterly Journal of Economics* 117, 1105–1131.

[3] Barseghyan, Levon, Marco Battaglini, and Stephen Coate (2013) Fiscal policy over the real business cycle: A positive theory. *Journal of Economic Theory* 148, 2223–2265.

[4] Chari, V.V., Lawrence J. Christiano, and Patrick J. Kehoe (1994) Optimal fiscal policy in a business cycle model. *Journal of Political Economy* 102, 617–652.

[5] Debortoli, Davide and Ricardo Nunes (2013) Lack of commitment and the level of debt. *Journal of the European Economic Association* 11, 1053–1078.

[6] Domínguez, Begoña (2005) Reputation in a model with a limited debt structure. *Review of Economic Dynamics* 8, 600–622.

[7] Judd, Kenneth L. (1992) Projection methods for solving aggregate growth models. *Journal of Economic Theory* 58, 410–452.

[8] Klein, Paul, Per Krusell, and José-Víctor Ríos-Rull (2008) Time-consistent public policy. *Review of Economic Studies* 75, 789–808.

[9] Klein, Paul and José-Víctor Ríos-Rull (2003) Time-consistent optimal fiscal policy. *International Economic Review* 44, 1217–1245.

[10] Krusell, Per, Martin F.M., and José-Víctor Ríos-Rull (2006) Time-consistent debt. Manuscript.

[11] Kydland, Finn E. and Edward C. Prescott (1977) Rules rather than discretion: The inconsistency of optimal plans. *Journal of Political Economy* 85, 473–492.

[12] Ljungqvist, Lars and Thomas J. Sargent (2000) *Recursive Macroeconomic Theory.* Cambridge: MIT press.

[13] Lucas Jr, Robert E. and Nancy L. Stokey (1983) Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics* 12, 55–93.

[14] Marcet, Albert and Ramon Marimon (2011) Recursive contracts. Economics Working Papers 552, Barcelona Graduate School of Economics.

[15] Marcet, Albert and Andrew Scott (2009) Debt and deficit fluctuations and the structure of bond markets. *Journal of Economic Theory* 144, 473–501.

[16] Martin, Fernando M. (2009) A positive theory of government debt. *Review of Economic Dynamics* 12, 608–631.

[17] Martin, Fernando M. (2013) Government policy in monetary economies. *International Economic Review* 54, 185–217.

[18] Martin, Fernando M. (in press) Policy and welfare effects of within-period commitment. *Macroeconomic Dynamics*.

[19] Occhino, Filippo (2012) Government debt dynamics under discretion. *The B.E. Journal of Macroeconomics* 12.

[20] Ortigueira, Salvador, Joana Pereira, and Paul Pichler (2012) Markov-perfect optimal fiscal policy: the case of unbalanced budgets. Economics Working Papers we1230, Universidad Carlos III.

[21] Scott, Andrew (2007) Optimal taxation and OECD labor taxes. *Journal of Monetary Economics* 54, 925–944.

[22] Tauchen, George (1986) Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters* 20, 177–181.

Appendix

Derivation of the optimality conditions (15), (16), (17) and (15), (18), (19)

In the case of risk-free bonds, attaching the multiplier γ to the constraint (10) the first-order conditions to the problem (11)-(12) are as follows:

$$\begin{aligned}
[x] &: u_x - \theta u_g - \gamma(u_{xx}(1-x) - u_x) = 0, \\
[c] &: u_c - u_g + \gamma(u_c + cu_{cc} - u_{cc}b) = 0, \\
[b'] &: \beta \sum_{s' \in S} P(s'|s)\left[V_b' + \gamma u_{cc}' \mathbf{C}_b' b' + \gamma u_c'\right] = 0.
\end{aligned}$$

Using the Envelope condition:

$$\begin{aligned}
V_b = u_c \mathbf{C}_b + u_x \mathbf{X}_b + u_g(-\theta \mathbf{X}_b - \mathbf{C}_b) + \\
\gamma(u_c \mathbf{C}_b + cu_{cc}\mathbf{C}_b - u_{cc}\mathbf{C}_b b - u_c - u_{xx}(1-x)\mathbf{X}_b + u_x \mathbf{X}_b) + \\
+\gamma\beta \sum_{s' \in S} P(s'|s)\left[\left(u_{cc}'\mathbf{C}_b' b' + u_c'\right) b_b' + \beta V_b' \mathbf{B}_b'\right].
\end{aligned}$$

Combining the Envelope condition with the first-order conditions and simplifying obtains:

$$V_b = -\gamma u_c.$$

Forwarding the last equation one period and substituting back into the first-order condition yields the optimality conditions (15), (16), (17).

In the case of state-contingent bonds, attaching the multiplier γ to the constraint (10) the first-order conditions to the problem (13)-(14) are as follows:

$$\begin{aligned}
[x] &: u_x - \theta u_g - \gamma(u_{xx}(1-x) - u_x) = 0, \\
[c] &: u_c - u_g + \gamma(u_c + cu_{cc} - u_{cc}b_s) = 0, \\
[b_s'] &: V_{b_s}' + \gamma u_{cc}' \mathbf{C}_{b_s}' b_s' + \gamma u_c' = 0, \forall s' \in S.
\end{aligned}$$

Combining the Envelope conditions which define V_{b_s} for each state $s \in S$ with the first-order conditions and simplifying obtains:

$$V_{b_s} = -\gamma u_c, \forall s' \in S.$$

Forwarding the last equation one period and substituting back into the first-order condition yields the optimality conditions (15), (18), (19).

The optimality conditions for the government problem under full commitment

Under full commitment, the government maximizes (2) subject to (1) and (8), choosing its fiscal plan in period 0 for all the infinite periods of the economy's life. This economy is similar to the economy studied by Aiyagari et al. (2002), except that public spending is endogenous.

In the case of risk-free bonds, the optimality conditions to the government's problem consist of (9), (10), and the following equations:

$$u_c - u_g + \mu u_{cc} b - \gamma(u_{cc}(b-c) - u_c) = 0, \tag{21}$$

$$u_x - \theta u_g - \gamma(u_{xx}(1-x) - u_x) = 0, \tag{22}$$

$$\beta \sum_{s' \in S} P(s'|s)[\mu' u'_c - \gamma' u'_c] - v_1 + v_2 = 0, \tag{23}$$

$$\mu' = \gamma,\ v_1(\underline{B} - b) = 0,\ v_2(b - \bar{B}) = 0, \tag{24}$$

where γ is the Lagrange multiplier attached to the implementability constraint (10), b_0 is given, $\mu_0 = 0$, and v_1 and v_2 are the Kuhn-Tucker multipliers associated with the last two equations, which ensure that bond issues belong to set Γ. As explained in Marcet and Marimon (2011) and Ljungqvist and Sargent (2000), the co-state μ associated with the government implementability constraint represents the marginal cost of commitment; namely, of the government keeping a promise to fulfill in period t the plan announced in the previous period. The term $-\mu u_{cc} B$, which is positive for positive debt, implies that the government must sacrifice current consumption ($u_c - u_g$ is higher) to maintain the price of the bond consistent with the value of the bond in the previous period. This term disappears in the optimality conditions of the government problem without commitment.

In the case of state-contingent bonds, the optimality conditions corresponding to the government problem consist of (9) and the following equations:

$$u_c - u_g + \gamma(u_{cc} c + u_c) = 0, \tag{25}$$

$$u_x - \theta u_g - \gamma(u_{xx}(1-x) - u_x) = 0. \tag{26}$$

$$b_0 = \frac{1}{u_{c_0}} \sum_{j=0}^{\infty} \sum_{s^j} \beta^j \left(c_j u_{c_j} - u_{x_j}(1-x_j) \right) P_j(s^j|s_0), \tag{27}$$

where γ is the Lagrange multiplier attached to constraint (27) and b_0 is given. The sequence of public bond issues, $\{b_t\}_{t=1}^{\infty}$, can be obtained from equation (27) forwarded t-times, after solving the system (25)-(27). In this case, the term that measures the effect of full commitment is given by $\gamma u_{cc} b$.

Numerical Algorithm for No Commitment

The numerical solutions for the no-commitment economies are obtained by the projection method described in Judd (1992) using a similar procedure to that used in Martin (2013) except that we include exogenous states as arguments rather than index the policy functions by exogenous states. Figure 4 presents the policy functions and the derivative $\mathbf{C}_b(b,s)$ for the several particular realizations of the exogenous state s.

The solutions to the government problem in the case of *risk-free bonds* are obtained by solving the systems of equations (9), (10), (15), (16), (17), on a pre-specified three-dimensional grid of states: $\{[\underline{B};\bar{B}] \times [1-3\sigma_\theta^2; 1+3\sigma_\theta^2] \times [1-3\sigma_\varphi^2; 1+3\sigma_\varphi^2]\}$, where $\sigma_\theta^2 = \frac{\sigma_\varepsilon^2}{1-\rho_\theta^2}$, $\sigma_\varphi^2 = \frac{\sigma_\epsilon^2}{1-\rho_\varphi^2}$, $[\underline{B};\bar{B}] = \pm 15\%$ of output, and by iterating on the fixed points of the policy functions. The policy functions $\mathbf{C}(b,s)$, $\mathbf{X}(b,s)$, $\mathbf{B}(b,s)$, where $s = \{\theta, \varphi\}$, are approximated by 3-cubic splines.

The solutions to the government problem in the case of *state-contingent bonds* are obtained by solving the systems of equations (9), (10), (15), (18), (19), on the same pre-specified grid of the states for the functions $\mathbf{C}(b_s, s)$ and $\mathbf{X}(B_s, s)$, which are approximated by 3-cubic splines. The debt policy $\{\mathbf{B}(b_s, s')\}_{s' \in S}$ is conditional on the realization of uncertainty, $s' = \{\theta', \varphi'\}$. Given the stationarity of the shocks, all possible realizations of the next period's exogenous state, conditional on the current period state, are defined by S. Therefore, the debt function in the complete markets case is approximated by a 5-cubic spline on the grid $\{[\underline{B};\bar{B}] \times [1-3\sigma_\theta^2; 1+3\sigma_\theta^2]^2 \times [1-3\sigma_\varphi^2; 1+3\sigma_\varphi^2]^2\}$. Once a new exogenous state $s' = \{\theta', \varphi'\}$ is realized, only the portion of the debt conditional on s', $b'_{s'} = \mathbf{B}(b_s, s')$, becomes relevant as the next-period state.

[FIGURE 4]

For the baseline model solution the grid consists of 11 nodes for b, 5 nodes for θ, and 5 nodes for φ. The accuracy of approximations on the grid is evaluated with rescaled to unit-free generalized Euler equation errors (equations (17) and (19) divided by the marginal utility of consumption). The maximum absolute unit-free generalized Euler equation errors calculated on the grid of 1000 equally distant nodes for b within $[\underline{B}; \bar{B}]$ are: $1.03e-07$ and $1.29e-07$ for the incomplete markets approximation and $1.33e-06$ and $1.88e-06$ for the complete markets approximation, for $v_g = 0.9$ and $v_g = 1.1$, respectively. An alternative accuracy measure (as in Martin, 2013), the sum of squared residuals for each necessary optimality condition of the government problem (reduced to three equations by substituting away g and γ), delivers the following results: $8.92e-12$, $3.38e-11$, $2.22e-14$ and $1.30e-11$, $7.34e-11$, $2.21e-14$ for incomplete markets, $v_g = 0.9$ and $v_g = 1.1$, respectively; and $5.80e-09$, $3.31e-11$, $1.42e-12$ and $1.19e-08$, $7.15e-11$, $3.16e-12$, for complete markets, $v_g = 0.9$ and $v_g = 1.1$, respectively. Increasing the number of nodes leads to moderate improvements in accuracy at the expense of much longer computational time. Increasing the bounds of endogenous state, b, for a given number of nodes reduces accuracy far from the steady state.

Analysis of the direction of responses of fiscal variables to a labor productivity shock

Consider the government that operates under full commitment and issues state-contingent bonds; assume without loss of generality that $\varphi_t = 1$ and $b_0 = 0$.

In the case when $v_g = 1$, the optimality conditions (25)-(27) imply that $c_t = \frac{\theta_t \phi_c^2}{(1-\phi_x)(\phi_x+\phi_c)}$, $x_t = \frac{\phi_x}{\phi_x+\phi_c}$, $\gamma = \frac{\phi_x - \phi_x(\phi_x+\phi_c)}{\phi_c(\phi_x+\phi_c)}$, $b_t = 0$, and equation (6) implies that, over time, τ_t is constant and the primary deficit is zero.

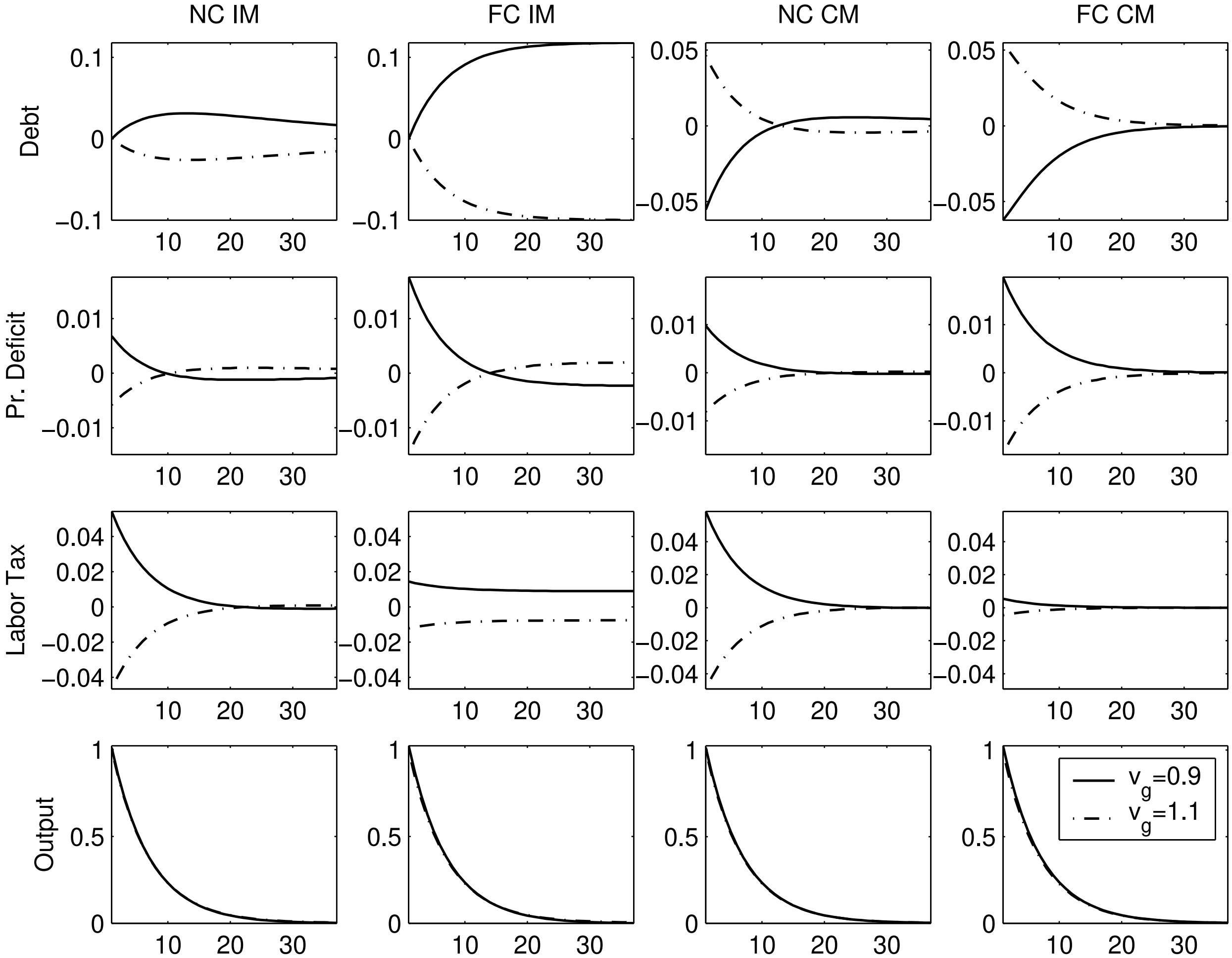
In the case when $v_g \neq 1$, compute the derivatives of consumption and leisure with respect to θ_t, $dx_t/d\theta_t$ and $dc_t/d\theta_t$, from the system (25)-(26) using the Implicit function theorem (these two equations implicitly define c_t and x_t as functions of θ_t; γ is constant over time). These derivatives are functions of v_g. Consider their linear approximation around $v_g = 1$:[10]

$$\frac{dx_t}{d\theta_t}(v_g) \simeq \frac{dx_t}{d\theta_t}(1) + \left.\frac{d(dx_t/d\theta_t)}{dv_g}\right|_{v_g=1} (v_g - 1)\,,$$

$$\frac{dc_t}{d\theta_t}(v_g) \simeq \frac{dc_t}{d\theta_t}(1) + \left.\frac{d(dc_t/d\theta_t)}{dv_g}\right|_{v_g=1} (v_g - 1)\,.$$

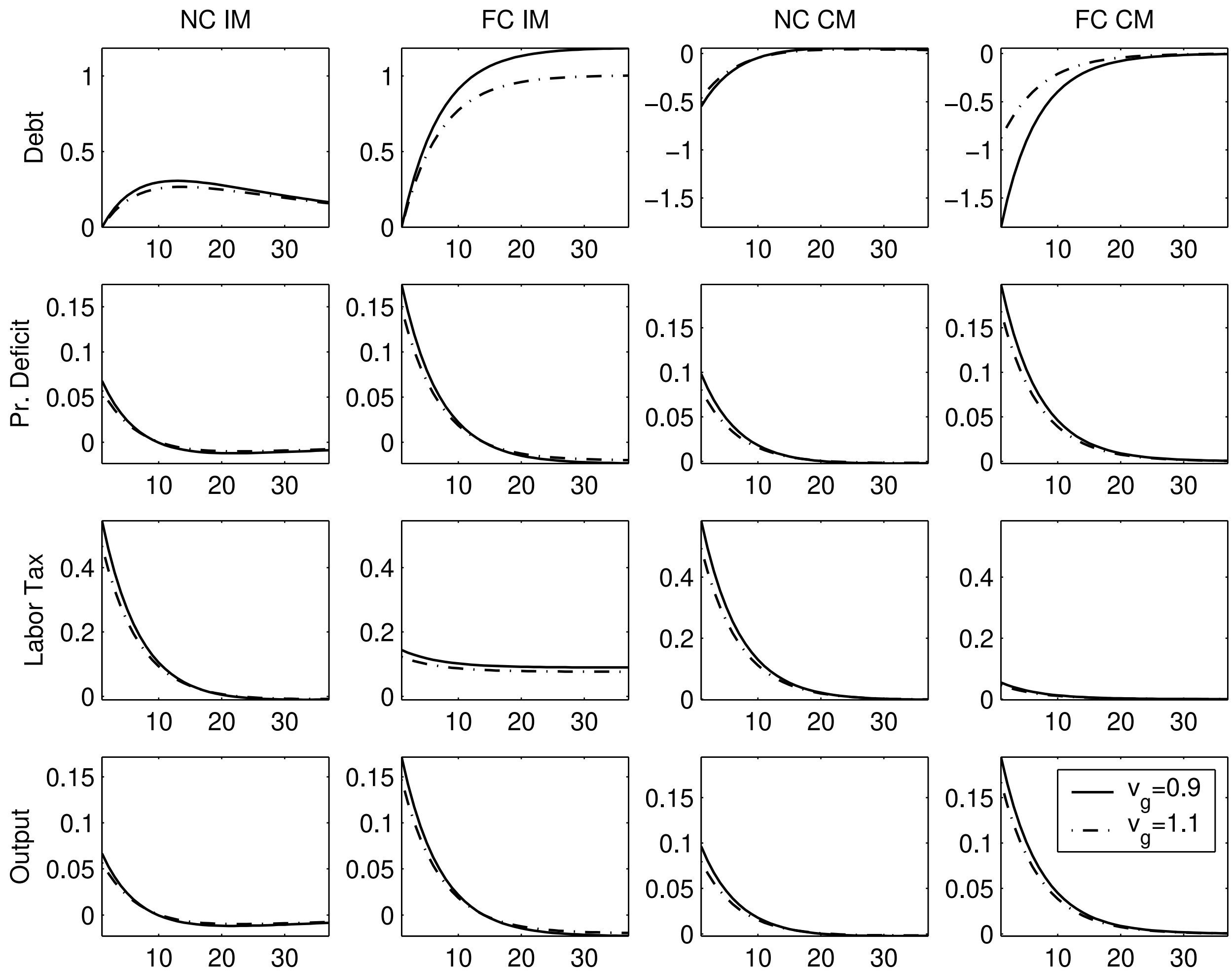
Using a numerical solver, we find that $\left.\frac{d(dx_t/d\theta_t)}{dv_g}\right|_{v_g=1} < 0$ and $\left.\frac{d(dc_t/d\theta_t)}{dv_g}\right|_{v_g=1} > 0$. Thus, the response of private consumption to a positive productivity shock is increasing in v_g and the response of leisure is decreasing in v_g and changing sign from positive to negative at $v_g = 1$. Moreover, the response of consumption is greater than the response of leisure in absolute value, which combined with (6) implies that the response of taxes to a positive productivity shock is decreasing in v_g (and changing sign from positive to negative at $v_g = 1$). The public debt in period t, $b_t = c_t/\phi_c \sum_{j=t}^{\infty} \sum_{s^j} \beta^j \left(\phi_c + \phi_x + \phi_c/x_j\right) P_j(s^j|s_t)$, is increasing in c_t and decreasing in x_t; thus, the response of public debt to a positive productivity shock is increasing in v_g (and changing sign from negative to positive at $v_g = 1$). Finally, we refer to the results from Marcet and Scott (2009) to conclude that the response of the primary deficit to a shock is opposite to the response of public debt.

Figure 1: Impulse responses to 1 standard deviation positive productivity shock.



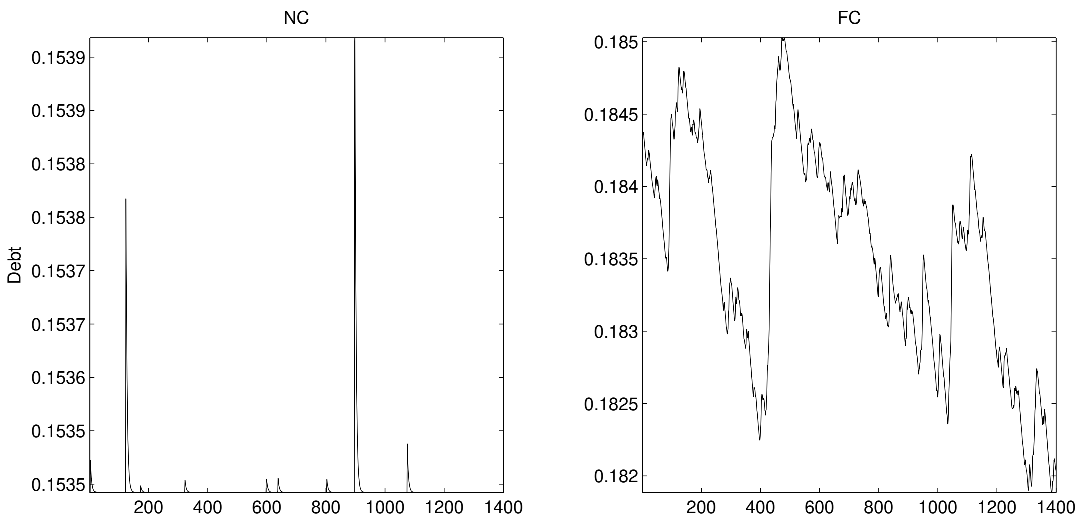
The impulse responses for debt, deficits, and output are calculated as the difference between the value of the variable in period t and in the period preceding the shock as the percentage of output in the period preceding the shock; the responses for labor taxes are calculated as the percentage deviation from their value in the period preceding the shock. Columns "NC IM" and "FC IM" present the responses for no commitment and full commitment with incomplete markets, respectively; columns "NC CM" and "FC CM" present the responses for no commitment and full commitment with complete markets, respectively.

Figure 2: Impulse responses to 1 standard deviation positive government expenditure shock.



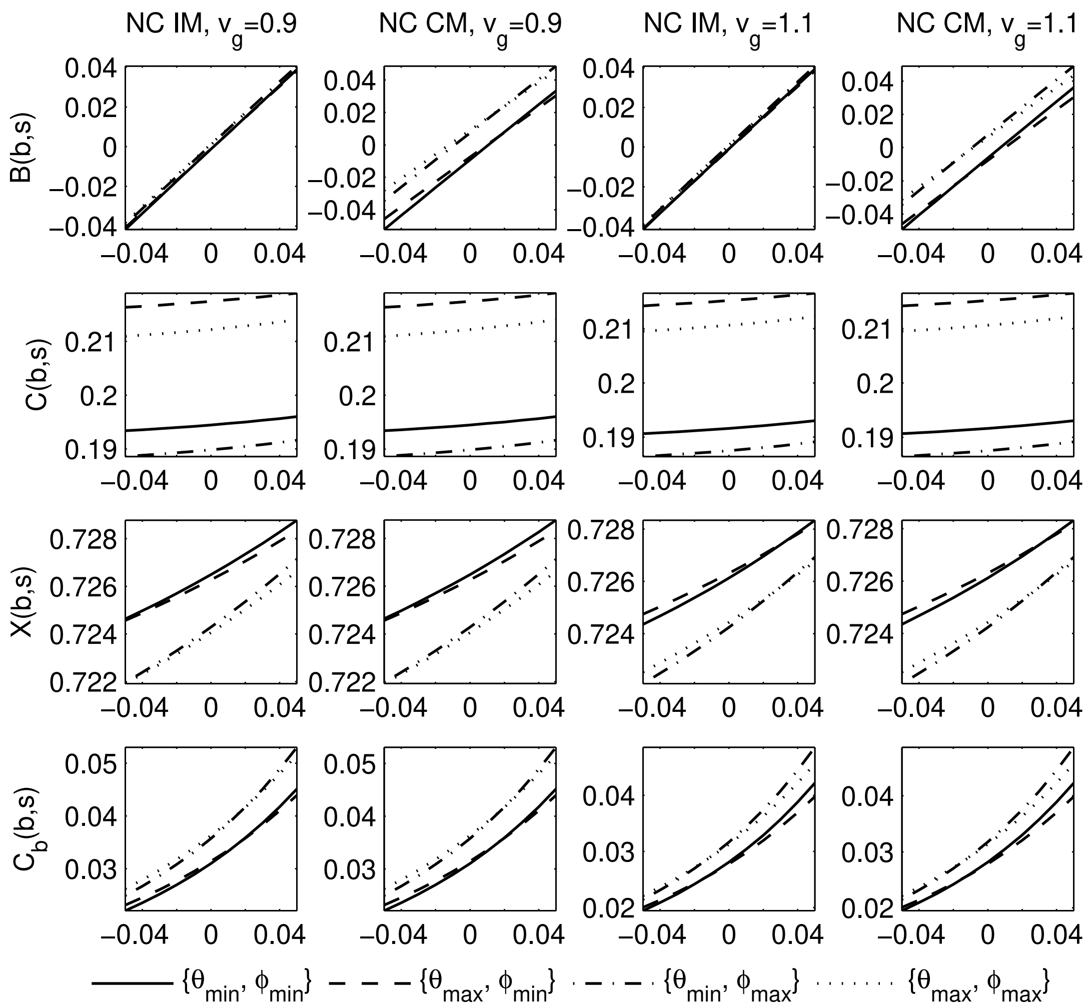
The impulse responses for debt, deficits, and output are calculated as the difference between the value of the variable in period t and in the period preceding the shock as the percentage of output in the period preceding the shock; the responses for labor taxes are calculated as the percentage deviation from their value in the period preceding the shock. Columns "NC IM" and "FC IM" present the responses for no commitment and full commitment with incomplete markets, respectively; columns "NC CM" and "FC CM" present the responses for no commitment and full commitment with complete markets, respectively.

Figure 3: Simulations of the public debt series when $\underline{B}$ and $\bar{B}$ are equal to 50% and 70% of the steady state output.



The time-series of public debt in the case of stochastic labor productivity, $\underline{B}$ and $\bar{B}$ equal to 50% and 70% of the steady state output. Columns “NC” and “FC” present public debt series for the no-commitment and full-commitment cases, respectively.

Figure 4: The policy functions and the derivative $\mathbf{C}_b(b,s)$.



The policy functions for debt, $\mathbf{B}(b,s)$, consumption, $\mathbf{C}(b,s)$, leisure, $\mathbf{X}(b,s)$, and the derivative $\mathbf{C}_b(b,s)$ for $s = \{\{\theta_{min}, \phi_{min}\}; \{\theta_{max}, \phi_{min}\}; \{\theta_{min}, \phi_{max}\}; \{\theta_{max}, \phi_{max}\}\}$. Columns "NC IM, $v_g = 0.9$" and "NC CM, $v_g = 0.9$" present functions for no commitment with incomplete and complete markets, respectively, for $v_g = 0.9$; columns "NC IM, $v_g = 1.1$" and "NC CM, $v_g = 1.1$" present functions for no commitment with incomplete and complete markets, respectively, for $v_g = 1.1$.