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A Nonparametric Option Pricing Model Using Higher Moments

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Abstract

A nonparametric model that includes non-Gaussian characteristics of skewness and kurtosis is proposed based on the cubic market capital asset pricing model. It is an equilibrium pricing model but risk-neutral valuation can be introduced through return data transformation. The model complies with the put-call parity principle of option pricing theory. The properties of the model are studied through simulation methods and compared with the Black-Scholes model. Simulation scenarios include cases on nonnormality in skewness and kurtosis, nonconstant variance, moneyness, contract duration, and interest rate levels. The proposed model can have negative prices in cases of out-of-the-money options and in simulation cases that are different from real-market situations, but the frequency of negative prices is reduced when risk-neutral valuation is implemented. The model is more adaptive and more conservative in pricing options compared to the Black-Scholes model when nonnormalities exist in the returns data.

1 Introduction

The paper proposes a nonparametric option pricing model that accounts for higher-moment features of the underlying asset returns data. This model extends the technology developed by [4] in which the capital asset pricing model [CAPM] was used to derive an option pricing model. The extended version of the model is based on the Cubic Market Model [9, 15]. This model complies with the Four-Moment CAPM model [9, 7] which incorporates non-Gaussian information such as skewness and kurtosis. The derived model also complies with the put-call parity principle of option pricing modeling [18]. The proposed model is based on an equilibrium asset pricing principle similar to [4] but risk-neutral valuation [5] can also be integrated by appropriate adjustment of returns

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data that will preserve other distribution characteristics.

The properties of the proposed model is studied and compared with the Black-Scholes model through simulation methods by changing the assumptions on moneyness, interest rate, duration, and return distribution characteristics in terms of variance, skewness, and kurtosis.

2 Review of Literature

2.1 CAPM for Option Pricing

The capital asset pricing model [17, 8, 4] is specified as follows: at time t , let $R_{it} = 1$ plus unannualized rate of return of asset i , $R_{ft} = 1$ plus unannualized rate of return of a risk-free asset at time t , $R_{mt} = 1$ plus the unannualized rate of return of a market portfolio of assets, and $E_t(\bullet)$ be the expectation operator based on the information set available at time t ; then

$$E_t(R_{it}) = R_{ft} + \beta_{im} [E_t(R_{mt}) - R_{ft}]. \quad (1)$$

The beta in equation (1) is the index of systematic risk for the asset i and is expressed as the following:

$$\beta_{im} = \frac{Cov_t(R_{it}, R_{mt})}{Var_t(R_{mt})} \quad (2)$$

The term $Cov_t(R_{it}, R_{mt})$ is the covariance between the asset i and market portfolio, while $Var_t(R_{mt})$ is the market portfolio variance, both values based on available information at time t .

To derive an option pricing model [4], the terms of the CAPM were replaced as follows: let t and T be fractions of time in a year and $T > t$, $C_{t,T,K}$ the price of a call option at time t with time-to-maturity $T - t$ and strike price K , implying that $C_{T,T,K} = \max\{S_T - K, 0\}$, and $R_{t,T} = S_T/S_t = 1$ plus the unannualized rate of return of the underlying asset with respect to its price held from time t to T , and $R_{f,t,T} = (1 + r_A)^{T-t} = 1$ plus the unannualized rate of return of a risk-free asset from time t to T where r_A is the annual effective rate of the risk-free asset; then

$$E_t\left(\frac{C_{T,T,K}}{C_{t,T,K}}\right) = R_{f,t,T} + \beta_{t,T,K} [E_t(R_{t,T}) - R_{f,t,T}]. \quad (3)$$

and the $\beta_{t,T,K}$ is defined as such below:

$$\beta_{t,T,K} = \frac{Cov_t\left(\frac{C_{T,T,K}}{C_{t,T,K}}, R_{t,T}\right)}{Var_t(R_{t,T})}. \quad (4)$$

By defining $C_{t,T,K}^* = \frac{C_{t,T,K}}{S_t}$, $K_t^* = \frac{K}{S_t}$, and thus making $C_{T,T,K}^* = \frac{C_{T,T,K}}{S_t} = \max\{R_{t,T} - K_t^*, 0\}$, then equations (3) and (4) are restated as follows:

$$E_t \left(\frac{C_{T,T,K}^*}{C_{t,T,K}^*} \right) = R_{f,t,T} + \beta_{t,T,K} [E_t(R_{t,T}) - R_{f,t,T}] \quad (5)$$

$$\beta_{t,T,K} = \frac{Cov_t \left(\frac{C_{T,T,K}^*}{C_{t,T,K}^*}, R_{t,T} \right)}{Var_t(R_{t,T})}. \quad (6)$$

To solve for the adjusted call option price $C_{t,T,K}^*$, the solution given by [4] is:

$$C_{t,T,K}^* = \frac{E_t(C_{T,T,K}^*)}{R_{f,t,T} + \beta_{t,T,K} [E_t(R_{t,T}) - R_{f,t,T}]} \quad (7)$$

The $\beta_{t,T,K}$ of equation (7) is equation (6), which contains $C_{t,T,K}^*$. Iterative methods are used to jointly solve for $\beta_{t,T,K}$ and $C_{t,T,K}^*$. The expectations are solved using the method of moments estimation on the returns data $\{R_{t,T,1}, \dots, R_{t,T,n}\}$ where $R_{t,T,i} = \frac{S_i}{S_{i-N(T-t)}}$ where N is the number of time periods in a year of which the data was disaggregated; for example $N = 252$ for trading days in a year for daily data. From the model, expectation operations are replaced with arithmetic mean summations. In this case, the estimator for the call option price would be:

$$\tilde{C}_{t,T,K}^* = \frac{\frac{1}{n} \sum_{i=1}^n \max\{R_{t,T,i} - K_t^*, 0\}}{R_{f,t,T} + \tilde{\beta}_{t,T,K} \left[\frac{1}{n} \sum_{i=1}^n (R_{t,T,i}) - R_{f,t,T} \right]} \quad (8)$$

with

$$\tilde{\beta}_{t,T,K} = \frac{\frac{1}{n} \sum_{i=1}^n \left(\frac{\max\{R_{t,T,i} - K_t^*, 0\}}{C_{t,T,K}^*} R_{t,T,i} \right) - \left[\frac{1}{n} \sum_{i=1}^n \left(\frac{\max\{R_{t,T,i} - K_t^*, 0\}}{C_{t,T,K}^*} \right) \right] [\bar{R}_{t,T}]}{\frac{1}{n} \sum_{i=1}^n [R_{t,T,i} - \bar{R}_{t,T}]^2} \quad (9)$$

where $\bar{R}_{t,T} = \frac{1}{n} \sum_{i=1}^n R_{t,T,i}$.

The returns data may be scaled in terms of applying assumptions on the variance of returns, since in its current form, the overall historical variance of the returns is assumed for future option price movements. It is suggested by [4] that the variance of previous five years of returns data from point of valuation

may persist in the future, so the data is scaled by multiplying $\tilde{v}_{t,T}$ through the transformation below to produce a new set of returns, $\ddot{R}_{t,T,i}$ to produce a variance equal to the most recent 5 years of data:

$$\ddot{R}_{t,T,i} = \frac{\tilde{v}_{t,T}}{s_{R_{t,T,i}}} [R_{t,T,i} - \bar{R}_{t,T}] + \bar{R}_{t,T} \quad (10)$$

where $s_{R_{t,T,i}}$ = overall historical standard deviation of the data. This transformation does not change the mean of the data.

So the method of [4] involves an iterative method of evaluating $\tilde{C}_{t,T,K}^*$, $\tilde{\beta}_{t,T,K}$, and $\tilde{v}_{t,T}$ from equations (10) and the two equations below:

$$\tilde{C}_{t,T,K}^* = \frac{\frac{1}{n} \sum_{i=1}^n \max \{ \ddot{R}_{t,T,i} - K_t^*, 0 \}}{R_{f,t,T} + \tilde{\beta}_{t,T,K} \left[\frac{1}{n} \sum_{i=1}^n (\ddot{R}_{t,T,i}) - R_{f,t,T} \right]} \quad (11)$$

with

$$\tilde{\beta}_{t,T,K} = \frac{\frac{1}{n} \sum_{i=1}^n \left(\frac{\max \{ \ddot{R}_{t,T,i} - K_t^*, 0 \}}{C_{t,T,K}^*} \ddot{R}_{t,T,i} \right) - \left[\frac{1}{n} \sum_{i=1}^n \left(\frac{\max \{ \ddot{R}_{t,T,i} - K_t^*, 0 \}}{C_{t,T,K}^*} \right) \right] [\bar{R}_{t,T}]}{\frac{1}{n} \sum_{i=1}^n [\ddot{R}_{t,T,i} - \bar{R}_{t,T}]^2} \quad (12)$$

The call option price $C_{t,T,K} = C_{t,T,K}^* \times S_t$.

The method of [4] is nonparametric since it does not assume a specific distribution on the returns of the underlying. However, it uses the information on the mean and variance of the returns being used. Based on their valuation on real data, the method eliminates the volatility smile seen [6] in the Black-Scholes model [2].

One problem of the method is that the CAPM does not account the non-Gaussian characteristics of the stock returns [19] such as skewness and heavy tails as measured by kurtosis. By these means, an extension of the CAPM with respect to using higher moments of the return distributions is sought.

2.2 Higher-Moments Extensions of the CAPM Model

The CAPM has been extended to include higher moments such as skewness and kurtosis. The first of these extensions incorporates the skewness of the market portfolio [14], called the three-moment CAPM. The model is expressed in this

general form:

$$E_t(R_{it}) - R_{ft} = c_1\beta_{im} + c_2\gamma_{im} \quad (13)$$

The terms of the model are: $R_{it} = 1$ plus unannualized rate of return of asset i at time t , $R_{ft} = 1$ plus unannualized rate of return of a risk-free asset at time t , and $E_t(\bullet)$ be the expectation operator based on the information set available at time t , β_{im} is the same as equation (2), and γ_{im} is defined as follows:

$$\gamma_{im} = \frac{Coskew_t [R_{it}, R_{mt}]}{\mu_{3t} [R_{mt}]} \quad (14)$$

The term $Coskew_t [R_{it}, R_{mt}] = E_t \left\{ [R_{it} - E_t(R_{it})] [R_{mt} - E_t(R_{mt})]^2 \right\}$ is defined as the coskewness between the return of asset i and the market portfolio return. The term $\mu_{3t} [R_{mt}] = E_t \left\{ [R_{mt} - E_t(R_{mt})]^3 \right\}$ is the unadjusted skewness of the market portfolio return. Taken together, γ_{im} is the systematic skewness of the return of asset i with respect to the market portfolio. The terms c_1 and c_2 are called the risk premiums due to systematic covariance β_{im} and systematic skewness γ_{im} , respectively.

The three-moment CAPM generates the simple CAPM by setting $c_2 = 0$ and letting the market risk premium c_1 be the portfolio market risk premium $E_t(R_{mt}) - R_{ft}$.

The special case of the three-moment CAPM is the quadratic market model [1], with model specifications as follows:

$$E_t [R_{it}] = R_{ft} + \alpha_{1im} [E_t (R_{mt}) - R_{ft}] + \alpha_{2im} E_t \left[(R_{mt} - R_{ft})^2 \right] \quad (15)$$

The quadratic market model complies with the specifications of the three-moment CAPM [1] as α_{1im} and α_{2im} can be expressed in terms of β_{im} and γ_{im} and is solvable given that the quantities are known [1, 15]:

$$\beta_{im} = \alpha_{1im} + \alpha_{2im} \frac{Cov_t \left[(R_{mt} - R_{ft})^2, R_{mt} \right]}{Var (R_{mt})} \quad (16)$$

$$\gamma_{im} = \alpha_{1im} + \alpha_{2im} \frac{Coskew_t \left[(R_{mt} - R_{ft})^2, R_{mt} \right]}{\mu_{3t} (R_{mt})} \quad (17)$$

If $\alpha_{2im} = 0$, the quadratic market model reduces to the CAPM as $\beta_{im} = \alpha_{1im}$ but also implies that $\beta_{im} = \gamma_{im}$, that is, systematic variance is equal to systematic skewness, of which has been shown [14] to be not evident in its empirical study.

A further extension of the CAPM includes the kurtosis of the market portfolio returns [9, 7], the four-moment CAPM. The specification of the model is shown below:

$$E_t(R_{it}) - R_{ft} = c_1\beta_{im} + c_2\gamma_{im} + c_3\delta_{im} \quad (18)$$

The model terms are similar to equation (13) except for the additional δ_{im} , which is defined as follows:

$$\delta_{im} = \frac{Cokurt_t[R_{it}, R_{mt}]}{\mu_{4t}[R_{mt}]} \quad (19)$$

The term $Cokurt_t[R_{it}, R_{mt}] = E_t\left\{[R_{it} - E_t(R_{it})][R_{mt} - E_t(R_{mt})]^3\right\}$ is defined as the cokurtosis between the return of asset i and the market portfolio return. The term $\mu_{4t}[R_{mt}] = E_t\left\{[R_{mt} - E_t(R_{mt})]^4\right\}$ is the unadjusted kurtosis of the market portfolio return. Taken together, δ_{im} is the systematic kurtosis of the return of asset i with respect to the market portfolio. The term c_3 is the risk premium due to systematic kurtosis.

From the four-moment CAPM, the three-moment CAPM can be generated by setting $c_3 = 0$, and the simple CAPM will be generated by setting $c_2 = c_3 = 0$ and letting $c_1 = E_t(R_{mt}) - R_{ft}$.

A special case of the four-moment CAPM is the cubic market model [9, 15], specified below:

$$\begin{aligned} E_t[R_{it}] = & R_{ft} + \alpha_{1im}[E_t(R_{mt}) - R_{ft}] + \alpha_{2im}E_t\left[(R_{mt} - R_{ft})^2\right] \\ & + \alpha_{3im}E_t\left[(R_{mt} - R_{ft})^3\right] \end{aligned} \quad (20)$$

The cubic market model complies with the specifications of the four-moment CAPM [9] as α_{1im} , α_{2im} , and α_{3im} can be expressed in terms of β_{im} , γ_{im} , and δ_{im} and is solvable given that the quantities are known [15]:

$$\begin{aligned}\beta_{im} &= \alpha_{1im} + \alpha_{2im} \frac{Cov_t \left[(R_{mt} - R_{ft})^2, R_{mt} \right]}{Var_t (R_{mt})} \\ &+ \alpha_{3im} \frac{Cov_t \left[(R_{mt} - R_{ft})^3, R_{mt} \right]}{Var_t (R_{mt})}\end{aligned}\quad (21)$$

$$\begin{aligned}\gamma_{im} &= \alpha_{1im} + \alpha_{2im} \frac{Coskew_t \left[(R_{mt} - R_{ft})^2, R_{mt} \right]}{\mu_{3t} (R_{mt})} \\ &+ \alpha_{3im} \frac{Coskew_t \left[(R_{mt} - R_{ft})^3, R_{mt} \right]}{\mu_{3t} (R_{mt})}\end{aligned}\quad (22)$$

$$\begin{aligned}\delta_{im} &= \alpha_{1im} + \alpha_{2im} \frac{Cokurt_t \left[(R_{mt} - R_{ft})^2, R_{mt} \right]}{\mu_{4t} (R_{mt})} \\ &+ \alpha_{3im} \frac{Cokurt_t \left[(R_{mt} - R_{ft})^3, R_{mt} \right]}{\mu_{4t} (R_{mt})}\end{aligned}\quad (23)$$

Setting $\alpha_{3im} = 0$ will produce the quadratic market model and letting $\alpha_{2im} = \alpha_{3im} = 0$ will produce the simple CAPM since $\beta_{im} = \alpha_{1im}$.

Research on the extensions of the CAPM highlight these main points: first, under the presence of skewness and kurtosis on asset return distributions, the excess expected asset return $E(R_{it}) - R_{ft}$ would be related to the systematic variance, systematic skewness [14], and systematic kurtosis [9, 7]; second, that investors tend to take into account the variance, skewness, and kurtosis of the asset returns, in the sense that they have aversion to variance and prefer positive skewness [14] and have aversion towards kurtosis [7], which in turn investors are compensated with higher returns when they take assets with high systematic variance, i.e., beta, and high systematic kurtosis, and are not concerned with being compensated for higher systematic skewness [9].

From these pointers, an option pricing model that incorporates non-normal features of asset returns via the generalization of the CAPM model [9, 7, 15] with the technology [4] that possesses nonparametric features is derived.

3 Derivation of Option Pricing Model

Using the cubic market model as described in [15], its terms are substituted similar to [4] and the resulting system of equations will be the following:

$$E_t \left[\frac{C_{T,T,K}}{C_{t,T,K}} \right] = R_{f,t,T} + \alpha_{1,t,T,K} [E_t (R_{t,T}) - R_{f,t,T}] + \alpha_{2,t,T,K} E_t \left[(R_{t,T} - R_{f,t,T})^2 \right] + \alpha_{3,t,T,K} E_t \left[(R_{t,T} - R_{f,t,T})^3 \right] \quad (24)$$

with t and T as fractions of time in a year and $T > t$, $C_{t,T,K}$ as the call price at time t with time-to-maturity $T - t$ and strike price K , $C_{T,T,K} = \max\{S_T - K, 0\}$, $R_{t,T} = S_T/S_t = 1$ plus the unannualized rate of return of the underlying asset with respect to its price held from time t to T , $R_{f,t,T} = (1 + r_A)^{T-t} = 1$ plus the unannualized rate of return of a risk-free asset from time t to T where r_A is the annual effective rate of the risk-free asset, and $\alpha_{1,t,T,K}$, $\alpha_{2,t,T,K}$, and $\alpha_{3,t,T,K}$ such that

$$\beta_{t,T,K} = \alpha_{1,t,T,K} + \alpha_{2,t,T,K} \frac{Cov_t \left[(R_{t,T} - R_{f,t,T})^2, R_{t,T} \right]}{Var(R_{t,T})} + \alpha_{3,t,T,K} \frac{Cov_t \left[(R_{t,T} - R_{f,t,T})^3, R_{t,T} \right]}{Var_t(R_{t,T})} \quad (25)$$

$$\gamma_{t,T,K} = \alpha_{1,t,T,K} + \alpha_{2,t,T,K} \frac{Coskew_t \left[(R_{t,T} - R_{f,t,T})^2, R_{t,T} \right]}{\mu_{3t}(R_{t,T})} + \alpha_{3,t,T,K} \frac{Coskew_t \left[(R_{t,T} - R_{f,t,T})^3, R_{t,T} \right]}{\mu_{3t}(R_{t,T})} \quad (26)$$

$$\delta_{t,T,K} = \alpha_{1,t,T,K} + \alpha_{2,t,T,K} \frac{Cokurt_t \left[(R_{t,T} - R_{f,t,T})^2, R_{t,T} \right]}{\mu_{4t}(R_{t,T})} + \alpha_{3,t,T,K} \frac{Cokurt_t \left[(R_{t,T} - R_{f,t,T})^3, R_{t,T} \right]}{\mu_{4t}(R_{t,T})} \quad (27)$$

where $\beta_{t,T,K}$, $\gamma_{t,T,K}$, and $\delta_{t,T,K}$ are

$$\beta_{t,T,K} = \frac{Cov_t \left(\frac{C_{T,T,K}}{C_{t,T,K}}, R_{t,T} \right)}{V_t (R_{t,T})} \quad (28)$$

$$\gamma_{t,T,K} = \frac{Coskew_t \left(\frac{C_{T,T,K}}{C_{t,T,K}}, R_{t,T} \right)}{\mu_{3t} (R_{t,T})} \quad (29)$$

$$\delta_{t,T,K} = \frac{Cokurt_t \left(\frac{C_{T,T,K}}{C_{t,T,K}}, R_{t,T} \right)}{\mu_{4t} (R_{t,T})} \quad (30)$$

Letting $C_{t,T,K}^* = \frac{C_{t,T,K}}{S_t}$, $K_t^* = \frac{K}{S_t}$, and $C_{T,T,K}^* = \frac{C_{T,T,K}}{S_t} = \max \{ R_{t,T} - K_t^*, 0 \}$, equations (24), (28), (29) and (30) are changed to

$$\begin{aligned} E_t \left[\frac{C_{T,T,K}^*}{C_{t,T,K}^*} \right] &= R_{f,t,T} + \alpha_{1,t,T,K} [E_t (R_{t,T}) - R_{f,t,T}] \\ &\quad + \alpha_{2,t,T,K} E_t \left[(R_{t,T} - R_{f,t,T})^2 \right] \\ &\quad + \alpha_{3,t,T,K} E_t \left[(R_{t,T} - R_{f,t,T})^3 \right] \end{aligned} \quad (31)$$

$$\beta_{t,T,K} = \frac{Cov_t \left(\frac{C_{T,T,K}^*}{C_{t,T,K}^*}, R_{t,T} \right)}{V_t (R_{t,T})} \quad (32)$$

$$\gamma_{t,T,K} = \frac{Coskew_t \left(\frac{C_{T,T,K}^*}{C_{t,T,K}^*}, R_{t,T} \right)}{\mu_{3t} (R_{t,T})} \quad (33)$$

$$\delta_{t,T,K} = \frac{Cokurt_t \left(\frac{C_{T,T,K}^*}{C_{t,T,K}^*}, R_{t,T} \right)}{\mu_{4t} (R_{t,T})} \quad (34)$$

the $C_{t,T,K}$ is taken out of the expectation, moment, and comoment operations similar to [4], thus changing equations (31) to (34) to

$$\begin{aligned} \frac{E_t [C_{T,T,K}^*]}{C_{t,T,K}^*} &= R_{f,t,T} + \alpha_{1,t,T,K} [E_t (R_{t,T}) - R_{f,t,T}] \\ &\quad + \alpha_{2,t,T,K} E_t \left[(R_{t,T} - R_{f,t,T})^2 \right] \\ &\quad + \alpha_{3,t,T,K} E_t \left[(R_{t,T} - R_{f,t,T})^3 \right] \end{aligned} \quad (35)$$

$$\beta_{t,T,K} = \frac{Cov_t(C_{T,T,K}^*, R_{t,T})}{C_{t,T,K}^* \times V_t(R_{t,T})} \quad (36)$$

$$\gamma_{t,T,K} = \frac{Coskew_t(C_{T,T,K}^*, R_{t,T})}{C_{t,T,K}^* \times \mu_{3t}(R_{t,T})} \quad (37)$$

$$\delta_{t,T,K} = \frac{Cokurt_t(C_{T,T,K}^*, R_{t,T})}{C_{t,T,K}^* \times \mu_{4t}(R_{t,T})} \quad (38)$$

Multiplying $C_{t,T,K}^*$ in equations (35) to (38) will produce

$$\begin{aligned} E_t [C_{T,T,K}^*] &= C_{t,T,K}^* R_{f,t,T} + C_{t,T,K}^* \alpha_{1,t,T,K} [E_t(R_{t,T}) - R_{f,t,T}] \\ &+ C_{t,T,K}^* \alpha_{2,t,T,K} E_t [(R_{t,T} - R_{f,t,T})^2] \\ &+ C_{t,T,K}^* \alpha_{3,t,T,K} E_t [(R_{t,T} - R_{f,t,T})^3] \end{aligned} \quad (39)$$

$$C_{t,T,K}^* \beta_{t,T,K} = \frac{Cov_t(C_{T,T,K}^*, R_{t,T})}{V_t(R_{t,T})} \quad (40)$$

$$C_{t,T,K}^* \gamma_{t,T,K} = \frac{Coskew_t(C_{T,T,K}^*, R_{t,T})}{\mu_{3t}(R_{t,T})} \quad (41)$$

$$C_{t,T,K}^* \delta_{t,T,K} = \frac{Cokurt_t(C_{T,T,K}^*, R_{t,T})}{\mu_{4t}(R_{t,T})} \quad (42)$$

If the multiplication was also done on equations (25) to (27), then

$$\begin{aligned} C_{t,T,K}^* \beta_{t,T,K} &= C_{t,T,K}^* \alpha_{1,t,T,K} + C_{t,T,K}^* \alpha_{2,t,T,K} \frac{Cov_t[(R_{t,T} - R_{f,t,T})^2, R_{t,T}]}{Var(R_{t,T})} \\ &+ C_{t,T,K}^* \alpha_{3,t,T,K} \frac{Cov_t[(R_{t,T} - R_{f,t,T})^3, R_{t,T}]}{Var_t(R_{t,T})} \end{aligned} \quad (43)$$

$$\begin{aligned} C_{t,T,K}^* \gamma_{t,T,K} &= C_{t,T,K}^* \alpha_{1,t,T,K} + C_{t,T,K}^* \alpha_{2,t,T,K} \frac{Coskew_t[(R_{t,T} - R_{f,t,T})^2, R_{t,T}]}{\mu_{3t}(R_{t,T})} \\ &+ C_{t,T,K}^* \alpha_{3,t,T,K} \frac{Coskew_t[(R_{t,T} - R_{f,t,T})^3, R_{t,T}]}{\mu_{3t}(R_{t,T})} \end{aligned} \quad (44)$$

$$\begin{aligned} C_{t,T,K}^* \delta_{t,T,K} &= C_{t,T,K}^* \alpha_{1,t,T,K} + C_{t,T,K}^* \alpha_{2,t,T,K} \frac{Cokurt_t[(R_{t,T} - R_{f,t,T})^2, R_{t,T}]}{\mu_{4t}(R_{t,T})} \\ &+ C_{t,T,K}^* \alpha_{3,t,T,K} \frac{Cokurt_t[(R_{t,T} - R_{f,t,T})^3, R_{t,T}]}{\mu_{4t}(R_{t,T})} \end{aligned} \quad (45)$$

By re-expressing the parameters $\beta_{t,T,K}^* = C_{t,T,K}^* \beta_{t,T,K}$, $\gamma_{t,T,K}^* = C_{t,T,K}^* \gamma_{t,T,K}$, $\delta_{t,T,K}^* = C_{t,T,K}^* \delta_{t,T,K}$, and $\alpha_{i,t,T,K}^* = C_{t,T,K}^* \alpha_{i,t,T,K}$ for $i = 1, 2, 3$, then equations (39) to (45) are restated as

$$\begin{aligned} E_t [C_{T,T,K}^*] &= C_{t,T,K}^* R_{f,t,T} + \alpha_{1,t,T,K}^* [E_t (R_{t,T}) - R_{f,t,T}] \\ &+ \alpha_{2,t,T,K}^* E_t [(R_{t,T} - R_{f,t,T})^2] \\ &+ \alpha_{3,t,T,K}^* E_t [(R_{t,T} - R_{f,t,T})^3] \end{aligned} \quad (46)$$

$$\beta_{t,T,K}^* = \frac{Cov_t (C_{T,T,K}^*, R_{t,T})}{V_t (R_{t,T})} \quad (47)$$

$$\gamma_{t,T,K}^* = \frac{Coskew_t (C_{T,T,K}^*, R_{t,T})}{\mu_{3t} (R_{t,T})} \quad (48)$$

$$\delta_{t,T,K}^* = \frac{Cokurt_t (C_{T,T,K}^*, R_{t,T})}{\mu_{4t} (R_{t,T})} \quad (49)$$

$$\begin{aligned} \beta_{t,T,K}^* &= \alpha_{1,t,T,K}^* + \alpha_{2,t,T,K}^* \frac{Cov_t [(R_{t,T} - R_{f,t,T})^2, R_{t,T}]}{Var (R_{t,T})} \\ &+ \alpha_{3,t,T,K}^* \frac{Cov_t [(R_{t,T} - R_{f,t,T})^3, R_{t,T}]}{Var_t (R_{t,T})} \end{aligned} \quad (50)$$

$$\begin{aligned} \gamma_{t,T,K}^* &= \alpha_{1,t,T,K}^* + \alpha_{2,t,T,K}^* \frac{Coskew_t [(R_{t,T} - R_{f,t,T})^2, R_{t,T}]}{\mu_{3t} (R_{t,T})} \\ &+ \alpha_{3,t,T,K}^* \frac{Coskew_t [(R_{t,T} - R_{f,t,T})^3, R_{t,T}]}{\mu_{3t} (R_{t,T})} \end{aligned} \quad (51)$$

$$\begin{aligned} \delta_{t,T,K}^* &= \alpha_{1,t,T,K}^* + \alpha_{2,t,T,K}^* \frac{Cokurt_t [(R_{t,T} - R_{f,t,T})^2, R_{t,T}]}{\mu_{4t} (R_{t,T})} \\ &+ \alpha_{3,t,T,K}^* \frac{Cokurt_t [(R_{t,T} - R_{f,t,T})^3, R_{t,T}]}{\mu_{4t} (R_{t,T})} \end{aligned} \quad (52)$$

From equation (46), the solution for $C_{t,T,K}^*$ will be

$$\begin{aligned} C_{t,T,K}^* &= \frac{1}{R_{f,t,T}} \times \{ E_t [C_{T,T,K}^*] - \alpha_{1,t,T,K}^* [E_t (R_{t,T}) - R_{f,t,T}] \\ &- \alpha_{2,t,T,K}^* E_t [(R_{t,T} - R_{f,t,T})^2] - \alpha_{3,t,T,K}^* E_t [(R_{t,T} - R_{f,t,T})^3] \} \end{aligned} \quad (53)$$

The values of $\alpha_{1,t,T,K}^*$, $\alpha_{2,t,T,K}^*$, and $\alpha_{3,t,T,K}^*$ are derived from equations (47) to (52), which has a closed-form solution since equations (50) to (52) is a system of linear equations. The call option price will finally be $C_{t,T,K} = C_{t,T,K}^* \times S_t$.

To solve the estimated call option price $\hat{C}_{t,T,K} = \hat{C}_{t,T,K}^* \times S_t$, the method of moments estimation on the returns data $\{R_{t,T,1}, \dots, R_{t,T,n}\}$ where $R_{t,T,i} = \frac{S_i}{S_{i-N(T-t)}}$ is used, where N is the number of time periods in a year of which the data was disaggregated; for example $N = 252$ for trading days in a year for daily data. The expectation operations are replaced with arithmetic mean summations. The estimators for the quantities in the model are as follows:

$$\hat{E}_t (C_{t,T,K}^*) = \frac{1}{n} \sum_{i=1}^n \max \left\{ R_{t,T,i} - \frac{K}{S_t}, 0 \right\} \quad (54)$$

$$\hat{E}_t (R_{t,T}) = \bar{R}_{t,T} = \frac{1}{n} \sum_{i=1}^n R_{t,T,i} \quad (55)$$

$$\hat{E}_t \left[(R_{t,T} - R_{f,t,T})^2 \right] = \frac{1}{n} \sum_{i=1}^n \left[(R_{t,T,i} - R_{f,t,T})^2 \right] \quad (56)$$

$$\hat{E}_t \left[(R_{t,T} - R_{f,t,T})^3 \right] = \frac{1}{n} \sum_{i=1}^n \left[(R_{t,T,i} - R_{f,t,T})^3 \right] \quad (57)$$

$$\hat{V}ar (R_{t,T}) = \frac{1}{n} \sum_{i=1}^n (R_{t,T,i} - \bar{R}_{t,T})^2 \quad (58)$$

$$\hat{\mu}_{3t} (R_{t,T}) = \frac{1}{n} \sum_{i=1}^n (R_{t,T,i} - \bar{R}_{t,T})^3 \quad (59)$$

$$\hat{\mu}_{4t} (R_{t,T}) = \frac{1}{n} \sum_{i=1}^n (R_{t,T,i} - \bar{R}_{t,T})^4 \quad (60)$$

$$\hat{\beta}_{t,T,K}^* = \frac{\frac{1}{n} \sum_{i=1}^n \left\{ \left[\max \left\{ R_{t,T,i} - \frac{K}{S_t}, 0 \right\} - \hat{E}_t (C_{t,T,K}^*) \right] [R_{t,T,i} - \bar{R}_{t,T}] \right\}}{\frac{1}{n} \sum_{i=1}^n (R_{t,T,i} - \bar{R}_{t,T})^2} \quad (61)$$

$$\hat{\gamma}_{t,T,K}^* = \frac{\frac{1}{n} \sum_{i=1}^n \left\{ \left[\max \left\{ R_{t,T,i} - \frac{K}{S_t}, 0 \right\} - \hat{E}_t (C_{t,T,K}^*) \right] [R_{t,T,i} - \bar{R}_{t,T}]^2 \right\}}{\frac{1}{n} \sum_{i=1}^n (R_{t,T,i} - \bar{R}_{t,T})^3} \quad (62)$$

$$\hat{\delta}_{t,T,K}^* = \frac{\frac{1}{n} \sum_{i=1}^n \left\{ \left[\max \left\{ R_{t,T,i} - \frac{K}{S_t}, 0 \right\} - \hat{E}_t (C_{T,T,K}^*) \right] [R_{t,T,i} - \bar{R}_{t,T}]^3 \right\}}{\frac{1}{n} \sum_{i=1}^n (R_{t,T} - \bar{R}_{t,T})^4} \quad (63)$$

$$\begin{aligned} & \hat{Cov}_t \left[(R_{t,T} - R_{f,t,T})^2, R_{t,T} \right] \\ &= \frac{1}{n} \sum_{i=1}^n \left\{ \left[(R_{t,T,i} - R_{f,t,T})^2 - \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^2 \right] \right] [R_{t,T,i} - \bar{R}_{t,T}] \right\} \quad (64) \end{aligned}$$

$$\begin{aligned} & \hat{Cov}_t \left[(R_{t,T} - R_{f,t,T})^3, R_{t,T} \right] \\ &= \frac{1}{n} \sum_{i=1}^n \left\{ \left[(R_{t,T,i} - R_{f,t,T})^3 - \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^3 \right] \right] [R_{t,T,i} - \bar{R}_{t,T}] \right\} \quad (65) \end{aligned}$$

$$\begin{aligned} & \hat{Coskew}_t \left[(R_{t,T} - R_{f,t,T})^2, R_{t,T} \right] \\ &= \frac{1}{n} \sum_{i=1}^n \left\{ \left[(R_{t,T,i} - R_{f,t,T})^2 - \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^2 \right] \right] [R_{t,T,i} - \bar{R}_{t,T}]^2 \right\} \quad (66) \end{aligned}$$

$$\begin{aligned} & \hat{Coskew}_t \left[(R_{t,T} - R_{f,t,T})^3, R_{t,T} \right] \\ &= \frac{1}{n} \sum_{i=1}^n \left\{ \left[(R_{t,T,i} - R_{f,t,T})^3 - \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^3 \right] \right] [R_{t,T,i} - \bar{R}_{t,T}]^2 \right\} \quad (67) \end{aligned}$$

$$\begin{aligned} & \hat{Cokurt}_t \left[(R_{t,T} - R_{f,t,T})^2, R_{t,T} \right] \\ &= \frac{1}{n} \sum_{i=1}^n \left\{ \left[(R_{t,T,i} - R_{f,t,T})^2 - \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^2 \right] \right] [R_{t,T,i} - \bar{R}_{t,T}]^3 \right\} \quad (68) \end{aligned}$$

$$\begin{aligned} & \hat{Cokurt}_t \left[(R_{t,T} - R_{f,t,T})^3, R_{t,T} \right] \\ &= \frac{1}{n} \sum_{i=1}^n \left\{ \left[(R_{t,T,i} - R_{f,t,T})^3 - \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^3 \right] \right] [R_{t,T,i} - \bar{R}_{t,T}]^3 \right\} \quad (69) \end{aligned}$$

The adjusted call option price estimator $\hat{C}_{t,T,K}^*$ is finally equal to

$$\begin{aligned} \hat{C}_{t,T,K}^* &= \frac{1}{R_{f,t,T}} \times \left\{ \hat{E}_t [C_{T,T,K}^*] - \hat{\alpha}_{1,t,T,K}^* \left[\hat{E}_t (R_{t,T}) - R_{f,t,T} \right] \right. \\ &\quad \left. - \hat{\alpha}_{2,t,T,K}^* \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^2 \right] - \hat{\alpha}_{3,t,T,K}^* \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^3 \right] \right\} \quad (70) \end{aligned}$$

The values of $\hat{\alpha}_{1,t,T,K}^*$, $\hat{\alpha}_{2,t,T,K}^*$, and $\hat{\alpha}_{3,t,T,K}^*$ are solutions to the system of linear equations,

$$\begin{aligned}\hat{\beta}_{t,T,K}^* &= \hat{\alpha}_{1,t,T,K}^* + \hat{\alpha}_{2,t,T,K}^* \frac{\hat{Cov}_t \left[(R_{t,T} - R_{f,t,T})^2, R_{t,T} \right]}{\hat{Var}(R_{t,T})} \\ &+ \hat{\alpha}_{3,t,T,K}^* \frac{\hat{Cov}_t \left[(R_{t,T} - R_{f,t,T})^3, R_{t,T} \right]}{\hat{Var}_t(R_{t,T})}\end{aligned}\quad (71)$$

$$\begin{aligned}\hat{\gamma}_{t,T,K}^* &= \hat{\alpha}_{1,t,T,K}^* + \hat{\alpha}_{2,t,T,K}^* \frac{\hat{Coskew}_t \left[(R_{t,T} - R_{f,t,T})^2, R_{t,T} \right]}{\hat{\mu}_{3t}(R_{t,T})} \\ &+ \hat{\alpha}_{3,t,T,K}^* \frac{\hat{Coskew}_t \left[(R_{t,T} - R_{f,t,T})^3, R_{t,T} \right]}{\hat{\mu}_{3t}(R_{t,T})}\end{aligned}\quad (72)$$

$$\begin{aligned}\hat{\delta}_{t,T,K}^* &= \hat{\alpha}_{1,t,T,K}^* + \hat{\alpha}_{2,t,T,K}^* \frac{\hat{Cokurt}_t \left[(R_{t,T} - R_{f,t,T})^2, R_{t,T} \right]}{\hat{\mu}_{4t}(R_{t,T})} \\ &+ \hat{\alpha}_{3,t,T,K}^* \frac{\hat{Cokurt}_t \left[(R_{t,T} - R_{f,t,T})^3, R_{t,T} \right]}{\hat{\mu}_{4t}(R_{t,T})}\end{aligned}\quad (73)$$

The call option price estimator will finally be $\hat{C}_{t,T,K} = \hat{C}_{t,T,K}^* \times S_t$.

With using the formula in its current state, it assumes that the historical mean and the standard deviation of data will persist in future price movements of the asset. Adjusting the pricing model to account for desired mean and variance assumptions on the returns of the asset, $\check{\mu}_{t,T,i}$ and $\check{\sigma}_{t,T,i}^2$ respectively, the following transformation of the data can be done from $R_{t,T,i}$ to an adjusted $\check{R}_{t,T,i}$ analogous to equation (10):

$$\check{R}_{t,T,i} = \frac{\check{\sigma}_{t,T,i}}{s_{R_{t,T,i}}} \left[R_{t,T,i} - \bar{R}_{t,T} \right] + \check{\mu}_{t,T,i}\quad (74)$$

The transformation changes the mean and standard deviation of the data to desired levels, but maintains the shape of the distribution of returns. This means that the transformation above still assumes that the historical skewness and kurtosis of the data persists to the future.

Using the transformation method above can account for risk-neutral valuation of options, by letting the model fulfill the martingale condition [5], with $r = \ln(1 + r_A)$ as the continuously compounding interest rate:

$$E_t [R_{t,T,i}] = \exp \{r(T - t)\} = (1 + r_A)^{T-t} = R_{f,t,T}\quad (75)$$

which implies setting $\check{\mu}_{t,T,i} = R_{f,t,T}$ to achieve risk-neutral valuation.

Special cases of the pricing model can be generated by setting conditions on some parameters. Letting $\alpha_{2,t,T,K}^* = \alpha_{3,t,T,K}^* = 0$ will produce the call option price estimator similar in concept to Chen and Palmon [4], denoted as $\hat{C}_{t,T,K}^{CP}$; letting $\alpha_{3,t,T,K}^* = 0$ will produce the quadratic market model call option price estimator $\hat{C}_{t,T,K}^{QMM}$; while the general case, the cubic market model estimator, will be denoted as $\hat{C}_{t,T,K}^{CMM}$.

Special notations are used for denoting models described in the paper. If a model facilitates risk-neutral valuation via equations (74) and (75), the *RN* superscript is added. For example, $\hat{C}_{t,T,K}^{CMM.RN}$ implies the cubic pricing model has been adjusted for risk-neutral valuation. The asterisk on the pricing model notation indicates that the underlying-adjusted pricing formula is used, meaning that the pricing formula is divided by the underlying price S_t . As example, the notation $\hat{C}_{t,T,K}^{*CMM}$ means that the cubic pricing model has been divided by the underlying price.

One special case of the proposed pricing model is the $\hat{C}_{t,T,K}^{*CP.RN}$ formula, written in full as

$$\hat{C}_{t,T,K}^{*CP.RN} = \frac{1}{R_{f,t,T}} \times \hat{E}_t(C_{T,T,K}^*) = \frac{1}{R_{f,t,T}} \times \frac{1}{n} \sum_{i=1}^n \max \left\{ \tilde{R}_{t,T,i} - \frac{K}{S_t}, 0 \right\} \quad (76)$$

where $\tilde{R}_{t,T,i} = [R_{t,T,i} - \bar{R}_{t,T}] + R_{f,t,T}$, which shifts the distribution of returns to a risk-neutral distribution having a new mean of $R_{f,t,T}$. This formula has a unique feature compared to the other forms of the proposed pricing model in that it will always be a positive value. It is also the simplest of all the model variations as it does not use the information on skewness and kurtosis and it deals with one summation formula. It also reduces the idea of option pricing as solving for a truncated average value of differences between possible future returns and the return from the strike price, with the truncated average brought to present value through a discount factor.

4 Put-Call Parity Property

The proposed pricing model complies with the put-call parity property [18]

$$\hat{C}_{t,T,K} - \hat{P}_{t,T,K} = S_t - K \exp \{-r(T-t)\} \quad (77)$$

which, by dividing by S_t and letting $\exp \{-r(T-t)\} = R_{f,t,T}^{-1}$, can be restated as

$$\hat{C}_{t,T,K}^* - \hat{P}_{t,T,K}^* = 1 - K_t^* \times R_{f,t,T}^{-1} \quad (78)$$

To solve for $\hat{P}_{t,T,K}^*$, it is similar in respect to the call-option price formula, with the except of replacing $\max \left\{ R_{t,T,i} - \frac{K}{S_t}, 0 \right\}$ to $\max \left\{ \frac{K}{S_t} - R_{t,T,i}, 0 \right\}$ in all the

necessary equations. The proof for the put-call parity property of the pricing model is shown below.

Proof: Define $\hat{C}_{t,T,K}^*$ and $\hat{P}_{t,T,K}^*$ as follows:

$$\begin{aligned} \hat{C}_{t,T,K}^* &= \frac{1}{R_{f,t,T}} \times \left\{ \hat{E}_t [C_{T,T,K}^*] - \hat{\alpha}_{1,t,T,K}^{*C} \left[\hat{E}_t (R_{t,T}) - R_{f,t,T} \right] \right. \\ &\quad \left. - \hat{\alpha}_{2,t,T,K}^{*C} \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^2 \right] - \hat{\alpha}_{3,t,T,K}^{*C} \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^3 \right] \right\} \quad (79) \end{aligned}$$

$$\begin{aligned} \hat{P}_{t,T,K}^* &= \frac{1}{R_{f,t,T}} \times \left\{ \hat{E}_t [P_{T,T,K}^*] - \hat{\alpha}_{1,t,T,K}^{*P} \left[\hat{E}_t (R_{t,T}) - R_{f,t,T} \right] \right. \\ &\quad \left. - \hat{\alpha}_{2,t,T,K}^{*P} \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^2 \right] - \hat{\alpha}_{3,t,T,K}^{*P} \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^3 \right] \right\} \quad (80) \end{aligned}$$

where $\hat{\alpha}_{i,t,T,K}^{*C}$ and $\hat{\alpha}_{i,t,T,K}^{*P}$, $i = 1, 2, 3$ are solutions to the linear equations (71) to (73) and using the appropriate set of quantities $(\hat{\beta}_{t,T,K}^{*C}, \hat{\gamma}_{t,T,K}^{*C}, \hat{\delta}_{t,T,K}^{*C})$ and $(\hat{\beta}_{t,T,K}^{*P}, \hat{\gamma}_{t,T,K}^{*P}, \hat{\delta}_{t,T,K}^{*P})$ for call and put formulas, respectively.

The difference between equation (79) and (80) is

$$\begin{aligned} \hat{C}_{t,T,K}^* - \hat{P}_{t,T,K}^* &= \frac{1}{R_{f,t,T}} \times \left\{ \hat{E}_t [C_{T,T,K}^*] - \hat{E}_t [P_{T,T,K}^*] \right. \\ &\quad - (\hat{\alpha}_{1,t,T,K}^{*C} - \hat{\alpha}_{1,t,T,K}^{*P}) \left[\hat{E}_t (R_{t,T}) - R_{f,t,T} \right] \\ &\quad - (\hat{\alpha}_{2,t,T,K}^{*C} - \hat{\alpha}_{2,t,T,K}^{*P}) \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^2 \right] \\ &\quad \left. - (\hat{\alpha}_{3,t,T,K}^{*C} - \hat{\alpha}_{3,t,T,K}^{*P}) \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^3 \right] \right\} \quad (81) \end{aligned}$$

The term $\hat{E}_t [C_{T,T,K}^*] - \hat{E}_t [P_{T,T,K}^*]$ can be expressed as:

$$\begin{aligned} &\hat{E}_t [C_{T,T,K}^*] - \hat{E}_t [P_{T,T,K}^*] \\ &= \frac{1}{n} \sum_{i=1}^n \max \left\{ R_{t,T,i} - \frac{K}{S_t}, 0 \right\} - \frac{1}{n} \sum_{i=1}^n \max \left\{ \frac{K}{S_t} - R_{t,T,i}, 0 \right\} \\ &= \frac{1}{n} \sum_{i=1}^n \left(R_{t,T,i} - \frac{K}{S_t} \right) = \bar{R}_{t,T} - \frac{K}{S_t} \quad (82) \end{aligned}$$

To consider the differences $\hat{\alpha}_{i,t,T,K}^{*C} - \hat{\alpha}_{i,t,T,K}^{*P}$, for $i = 1, 2, 3$, note that equations (71) to (73) as equations written in matrix form:

$$\hat{\theta}_{t,T,K}^* = \mathbf{M} \times \hat{\underline{\alpha}}_{t,T,K}^* \quad (83)$$

where $\hat{\theta}_{t,T,K}^* = [\hat{\beta}_{t,T,K}^{*C}, \hat{\gamma}_{t,T,K}^{*C}, \hat{\delta}_{t,T,K}^{*C}]^T$, $\hat{\underline{\alpha}}_{t,T,K}^*$ is the vector of $\hat{\alpha}_{i,t,T,K}^*$, and \mathbf{M} is the matrix

$$\mathbf{M} = \begin{bmatrix} 1 & \frac{C\hat{ov}_t[(R_{t,T}-R_{f,t,T})^2, R_{t,T}]}{Var(R_{t,T})} & \frac{C\hat{ov}_t[(R_{t,T}-R_{f,t,T})^3, R_{t,T}]}{Var(R_{t,T})} \\ 1 & \frac{Cos\hat{ke}w_t[(R_{t,T}-R_{f,t,T})^2, R_{t,T}]}{\hat{\mu}_{3t}(R_{t,T})} & \frac{Cos\hat{ke}w_t[(R_{t,T}-R_{f,t,T})^3, R_{t,T}]}{\hat{\mu}_{3t}(R_{t,T})} \\ 1 & \frac{C\hat{o}k\hat{u}r_t[(R_{t,T}-R_{f,t,T})^2, R_{t,T}]}{\hat{\mu}_{4t}(R_{t,T})} & \frac{C\hat{o}k\hat{u}r_t[(R_{t,T}-R_{f,t,T})^3, R_{t,T}]}{\hat{\mu}_{4t}(R_{t,T})} \end{bmatrix}$$

and thus solving for $\hat{\underline{\alpha}}_{t,T,K}^*$ is equal to

$$\hat{\underline{\alpha}}_{t,T,K}^* = \mathbf{M}^{-1} \times \hat{\underline{\theta}}_{t,T,K}^* \quad (84)$$

So, the difference of the vectors $\hat{\underline{\alpha}}_{t,T,K}^{*C} - \hat{\underline{\alpha}}_{t,T,K}^{*P}$ is

$$\hat{\underline{\alpha}}_{t,T,K}^{*C} - \hat{\underline{\alpha}}_{t,T,K}^{*P} = \mathbf{M}^{-1} \times \left(\hat{\underline{\theta}}_{t,T,K}^{*C} - \hat{\underline{\theta}}_{t,T,K}^{*P} \right) \quad (85)$$

The equation above implies that the differences $\hat{\beta}_{t,T,K}^{*C} - \hat{\beta}_{t,T,K}^{*P}$, $\hat{\gamma}_{t,T,K}^{*C} - \hat{\gamma}_{t,T,K}^{*P}$, and $\hat{\delta}_{t,T,K}^{*C} - \hat{\delta}_{t,T,K}^{*P}$ will determine the values of $\hat{\underline{\alpha}}_{t,T,K}^{*C} - \hat{\underline{\alpha}}_{t,T,K}^{*P}$.

The j th element of $\hat{\underline{\theta}}_{t,T,K}^{*C}$ and $\hat{\underline{\theta}}_{t,T,K}^{*P}$, $j = 1, 2, 3$, will be of the form

$$\begin{aligned} & \hat{\theta}_{t,T,K}^{*C(j)} \\ &= \frac{\frac{1}{n} \sum_{i=1}^n \left\{ \left[\max \left\{ R_{t,T,i} - \frac{K}{S_t}, 0 \right\} - \hat{E}_t(C_{T,T,K}^*) \right] [R_{t,T,i} - \bar{R}_{t,T}]^j \right\}}{\frac{1}{n} \sum_{i=1}^n (R_{t,T} - \bar{R}_{t,T})^{j+1}} \end{aligned} \quad (86)$$

$$\begin{aligned} & \hat{\theta}_{t,T,K}^{*P(j)} \\ &= \frac{\frac{1}{n} \sum_{i=1}^n \left\{ \left[\max \left\{ \frac{K}{S_t} - R_{t,T,i}, 0 \right\} - \hat{E}_t(P_{T,T,K}^*) \right] [R_{t,T,i} - \bar{R}_{t,T}]^j \right\}}{\frac{1}{n} \sum_{i=1}^n (R_{t,T} - \bar{R}_{t,T})^{j+1}} \end{aligned} \quad (87)$$

So solving the difference between equations (86) and (87) will be similar to solving the difference of the numerators, which results to

$$\begin{aligned}
& \frac{1}{n} \sum_{i=1}^n \left\{ \left[\max \left\{ R_{t,T,i} - \frac{K}{S_t}, 0 \right\} - \hat{E}_t (C_{T,T,K}^*) \right] [R_{t,T,i} - \bar{R}_{t,T}]^j \right\} \\
& - \frac{1}{n} \sum_{i=1}^n \left\{ \left[\max \left\{ \frac{K}{S_t} - R_{t,T,i}, 0 \right\} - \hat{E}_t (P_{T,T,K}^*) \right] [R_{t,T,i} - \bar{R}_{t,T}]^j \right\} \\
& = \frac{1}{n} \sum_{i=1}^n \left\{ \left[\max \left\{ R_{t,T,i} - \frac{K}{S_t}, 0 \right\} - \max \left\{ \frac{K}{S_t} - R_{t,T,i}, 0 \right\} \right. \right. \\
& \quad \left. \left. - \left[\hat{E}_t (C_{T,T,K}^*) - \hat{E}_t (P_{T,T,K}^*) \right] \right] [R_{t,T,i} - \bar{R}_{t,T}]^j \right\} \\
& = \frac{1}{n} \sum_{i=1}^n \left\{ \left[R_{t,T,i} - \frac{K}{S_t} - \left[\bar{R}_{t,T} - \frac{K}{S_t} \right] \right] [R_{t,T,i} - \bar{R}_{t,T}]^j \right\} \\
& = \frac{1}{n} \sum_{i=1}^n \left\{ [R_{t,T,i} - \bar{R}_{t,T}] [R_{t,T,i} - \bar{R}_{t,T}]^j \right\} \\
& = \frac{1}{n} \sum_{i=1}^n [R_{t,T,i} - \bar{R}_{t,T}]^{j+1}. \tag{88}
\end{aligned}$$

Therefore for any $j = 1, 2, 3$,

$$\hat{\theta}_{t,T,K(j)}^{*C} - \hat{\theta}_{t,T,K(j)}^{*P} = \frac{\frac{1}{n} \sum_{i=1}^n [R_{t,T,i} - \bar{R}_{t,T}]^{j+1}}{\frac{1}{n} \sum_{i=1}^n [R_{t,T,i} - \bar{R}_{t,T}]^{j+1}} = 1, \tag{89}$$

and thus

$$\hat{\underline{\alpha}}_{t,T,K}^{*C} - \hat{\underline{\alpha}}_{t,T,K}^{*P} = \mathbf{M}^{-1} \times [1, 1, 1]^T. \tag{90}$$

Note that $\mathbf{M}^{-1} \times \mathbf{M} = \mathbf{I}$, the 3×3 identity matrix, and the vector $[1, 1, 1]^T$ is the first column of \mathbf{M} so

$$\hat{\underline{\alpha}}_{t,T,K}^{*C} - \hat{\underline{\alpha}}_{t,T,K}^{*P} = [1, 0, 0]^T. \tag{91}$$

From the equations (55), (82), and (91), it can now be shown that

$$\begin{aligned}
\hat{C}_{t,T,K}^* - \hat{P}_{t,T,K}^* &= \frac{1}{R_{f,t,T}} \times \left\{ \hat{E}_t [C_{T,T,K}^*] - \hat{E}_t [P_{T,T,K}^*] \right. \\
&\quad - (\hat{\alpha}_{1,t,T,K}^{*C} - \hat{\alpha}_{1,t,T,K}^{*P}) \left[\hat{E}_t (R_{t,T}) - R_{f,t,T} \right] \\
&\quad - (\hat{\alpha}_{2,t,T,K}^{*C} - \hat{\alpha}_{2,t,T,K}^{*P}) \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^2 \right] \\
&\quad \left. - (\hat{\alpha}_{3,t,T,K}^{*C} - \hat{\alpha}_{3,t,T,K}^{*P}) \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^3 \right] \right\} \\
&= \frac{1}{R_{f,t,T}} \times \left\{ \bar{R}_{t,T} - \frac{K}{S_t} - (1) [\bar{R}_{t,T} - R_{f,t,T}] \right. \\
&\quad \left. - (0) \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^2 \right] - (0) \hat{E}_t \left[(R_{t,T} - R_{f,t,T})^3 \right] \right\} \\
&= \frac{1}{R_{f,t,T}} \times \left\{ \bar{R}_{t,T} - \frac{K}{S_t} - [\bar{R}_{t,T} - R_{f,t,T}] \right\} \\
&= \frac{1}{R_{f,t,T}} \times \left\{ R_{f,t,T} - \frac{K}{S_t} \right\} \\
&= 1 - K_t^* \times R_{f,t,T}^{-1} \tag{92}
\end{aligned}$$

Therefore, the proposed pricing model complies with the put-call parity exactly, whether the returns data were transformed by equation (74) or not. This also means that when the proposed call option pricing model is used for valuation, the put price can be derived by put-call parity equation.

5 Simulation Study

The properties of the new pricing model and its special cases will be studied and compared with the Black Scholes pricing model [2]:

$$\begin{aligned}
\hat{C}_{t,T,K}^{BS} &= S_t \Phi \left[\frac{\ln \left(\frac{S_t}{K} \right) + \left(r + \frac{\hat{\sigma}^2}{2} \right) (T - t)}{\hat{\sigma} \sqrt{T - t}} \right] \\
&\quad - K \exp \{ -r(T - t) \} \Phi \left[\frac{\ln \left(\frac{S_t}{K} \right) + \left(r - \frac{\hat{\sigma}^2}{2} \right) (T - t)}{\hat{\sigma} \sqrt{T - t}} \right] \tag{93}
\end{aligned}$$

In the formula, $r = \ln(1 + r_A)$ is the continuous compounding interest rate of the risk-free asset, where r_A is the annual effective interest rate of the asset.

The term $\Phi(\bullet)$ is the cumulative density function of the standard normal distribution, given below:

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{x^2}{2}\right\} dx \quad (94)$$

The formula for the estimator of σ in the Black-Scholes model will be based on [10]:

$$\hat{\sigma} = \frac{\sqrt{\frac{1}{n-1} \sum_{i=1}^n (u_{t,T,i} - \bar{u}_{t,T})^2}}{\sqrt{T-t}} \quad (95)$$

The terms $u_{t,T,i} = \ln\left(\frac{S_i}{S_{i-N(T-t)}}\right) = \ln R_{t,T,i}$ is the log-returns at time i with duration of accumulation $T-t$. N is the number of time periods in a year of which the data was disaggregated, for example $N = 252$ for trading days in a year for daily data, and $\bar{u}_{t,T}$ is the mean of these log-returns.

The underlying-adjusted Black-Scholes pricing model $\hat{C}_{t,T,K}^{*BS}$ is derived by dividing equation (93) with respect to S_t , giving the formula

$$\begin{aligned} \hat{C}_{t,T,K}^{*BS} = & \Phi \left[\frac{-\ln K_t^* + \left(r + \frac{\hat{\sigma}^2}{2}\right)(T-t)}{\hat{\sigma}\sqrt{T-t}} \right] \\ & - K_t^* \exp\{-r(T-t)\} \Phi \left[\frac{-\ln K_t^* + \left(r - \frac{\hat{\sigma}^2}{2}\right)(T-t)}{\hat{\sigma}\sqrt{T-t}} \right] \end{aligned} \quad (96)$$

The pricing models that will be compared through simulation studies are $\hat{C}_{t,T,K}^{*BS}$, $\hat{C}_{t,T,K}^{*CP}$, $\hat{C}_{t,T,K}^{*CP,RN}$, $\hat{C}_{t,T,K}^{*QMM}$, $\hat{C}_{t,T,K}^{*QMM,RN}$, $\hat{C}_{t,T,K}^{*CMM}$, and $\hat{C}_{t,T,K}^{*CMM,RN}$.

The scenarios for the simulation will be a combination of cases from different elements of the pricing model formulas: (a) interest rate of the risk-free asset r_A , (b) moneyness K_t^* , (c) duration of the contract $T-t$, (d) variance structure of the log-returns, (e) skewness of log-returns, and (f) kurtosis of log-returns. Each scenario will consist of 100 simulated return series data, with each series data having 1260 return periods, equivalent to five-year's worth of data.

For the risk-free interest rate, three cases are assumed: the 4-week US Treasury Bills secondary market interest rate for the low rate case, the 3-year US Treasury Bond interest rate for the middle rate case, and the 20-year Treasury Bond interest rate for the high rate case. The rates used are based on [16] for the date of 31 March 2015. These rates are, respectively, 0.05% p.a., 0.89% p.a. compounded semiannually, and 2.31% p.a. compounded semiannually.

For moneyness, the term K_t^* is varied to five values as cases: $K_t^* = 0.90$ represents the case of 11.11% in-the-money for the call option, 0.95 represents

5.26% in-the-money, 1.00 represents at-the-money, 1.05 represents 4.76% out-the-money, and 1.10 represents 9.09% out-the-money.

On contract duration, three cases are assumed: 21 trading days equivalent to one month for the short case, 63 trading days as 3 months for the middle case, and 126 trading days as 6 months for the long case. As fractions of a year, the lengths in days are divided with respect to 252 days. Thus, $T - t$ for each case respectively will be $1/12$, $1/4$, and $1/2$. The duration in days is used as a basis for real-data based values for the distributional assumptions of the log-returns, of which the S&P 500 index data of five year's worth, which is 1260 return periods, ending at 13 February 2015 is used as the data from which values of cases are derived.

With respect to variance, two structures are assumed: (a) constant variance over the whole span of the series, and (b) a GARCH(1,1) model for variance as determined from real data. The GARCH(1,1) model is defined as [3]:

$$\begin{aligned} u_{t,T,i} &= \mu_{t,T} + \epsilon_{t,T,i}, & \epsilon_{t,T,i} &\sim (0, \sigma_{t,T,i}^2) \\ \sigma_{t,T,i}^2 &= \omega_{t,T} + \phi_{1,t,T} \epsilon_{t,T,i-1}^2 + \phi_{2,t,T} \sigma_{t,T,i-1}^2 \end{aligned} \quad (97)$$

The GARCH(1,1) model makes the individual return variances $\sigma_{t,T,i}^2$ of log-returns fluctuate through time given that the unconditional historical variance of the data still is a finite constant value.

The value $\mu_{t,T}$ for simulations will be the mean as estimated from the S&P 500 data and will only change with respect to the contract duration and the variance model of the case assumed. The statement $\epsilon_{t,T,i} \sim (0, \sigma_{t,T,i}^2)$ means that the distribution will be generated from a standardized distribution that will have zero mean. The standardized distribution depends on the value of skewness and kurtosis. For the constant variance case, $\phi_{1,t,T} = \phi_{2,t,T} = 0$ is set.

Duration	Constant Variance		GARCH(1,1)			
	$\mu_{t,T}$	$\omega_{t,T}$	$\mu_{t,T}$	$\omega_{t,T}$	$\phi_{1,t,T}$	$\phi_{2,t,T}$
21-day	0.01034198	0.001467624	0.021169	0.000118	0.893725	0.029669
63-day	0.03072180	0.003206283	0.043351	0.000069	0.776531	0.222469
126-day	0.06242610	0.005116157	0.073590	0.000088	0.972670	0.026330

Table 1: Parameter Values for Variance Cases

Table 1 shows the values of the parameters for different cases. The estimates of $\mu_{t,T}$ and $\omega_{t,T}$ for the constant variance are the corresponding method-of-moments estimates, while the GARCH(1,1) model estimates are based on quasi-maximum likelihood estimation on the S&P 500 data ending at 13 February 2015 with a span of 5 years.

Duration	S&P 500		Skewness Cases		Kurtosis Case
	Skewness	Kurtosis	Negative	Positive	Leptokurtic
21-day	-1.034952	5.463981	-1.034952	1.034952	5.463981
63-day	-0.944574	4.170992	-0.944574	0.944574	4.170992
126-day	-0.663999	3.705460	-0.663999	0.663999	3.705460

Table 2: Parameter Values for Skewness and Kurtosis Cases

About skewness, three cases are assumed: negative or skewed to the left, zero or symmetric, and positive or skewed to the right. The value of skewness is based on the S&P 500 data and would be different for every contract duration considered. Table 2 shows the value of skewness per case and duration, based on the skewness of the S&P 500 data. The formula used for the skewness of the data is the following, based on equations (58) and (59):

$$Skew(u_{t,T}) = \frac{\hat{\mu}_{3t}(u_{t,T})}{\left[\hat{Var}_t(u_{t,T})\right]^{3/2}} \quad (98)$$

With respect to kurtosis, two cases are considered: the mesokurtic case, where kurtosis is equal to 3, and the leptokurtic or heavy-tails case, where the kurtosis is based on the S&P 500 data and returns based on contract duration. Table 2 contains the values of kurtosis that will be used for the simulation studies. The formula for kurtosis used to derive the values from data are the following, based on equations (58) and (60):

$$Kurt(u_{t,T}) = \frac{\hat{\mu}_{4t}(u_{t,T})}{\left[\hat{Var}_t(u_{t,T})\right]^2} \quad (99)$$

Since nonnormal features will be part of the cases of the simulations, when nonzero skewness or nonnormal kurtosis is the case, then the Johnson family of distributions [13] is used to generate the simulated returns data. The Johnson family of distributions has the following cumulative distribution formula $F(x)$:

$$F(x) = \Phi \left[\eta + \zeta \times g \left(\frac{x - \xi}{\lambda} \right) \right] \quad (100)$$

The function $g(\bullet)$ is a function that determines the type of distribution used for generating returns. If $g(z) = z$, then the Johnson distribution type is the normal or Gaussian distribution, denoted as SN. If $g(z) = \ln(z)$, then the Johnson distribution type is the lognormal distribution, marked as SL. The Johnson SU or unbounded distribution type is generated by letting $g(z) = \sinh^{-1} z$, the inverse hyperbolic sine function, and the bounded distribution or SB type is by letting $g(z) = \ln \left(\frac{z}{1-z} \right) = \text{logit}(z)$, the logit function.

The parameters ξ , λ , η , and ζ describe the location, scale, skewness, and kurtosis of the data, respectively; that is, changing the corresponding parameter changes the specific feature of the distribution. They are not the exact values of the mean, variance, skewness, nor kurtosis. However, the distribution parameters can be derived by moment-matching [11], of which estimated values of the parameters and the type of distribution to be used are found through solving a system of nonlinear equations that matches the desired moment values to the functions that describe the corresponding moments through the parameters.

Table 3 shows the different parameters and types of the Johnson family used to generate each combination of cases for skewness and kurtosis. For each case, it is assumed that the mean is zero and the variance is one since these can be included over the simulated returns. An asterisk beside the SL indicates that SB estimation would not converge, so the lognormal distribution was used to approximate the nonnormal features.

Duration	Combination of Cases		Johnson Family				
	Skewness	Kurtosis	Type	ξ	λ	η	ζ
21-day	Negative	Mesokurtic	SB	-2.852114	3.876390	-0.856400	0.534582
21-day	Negative	Leptokurtic	SU	1.609838	1.484898	2.050504	2.345450
21-day	Zero	Mesokurtic	SN	0.000000	1.000000	0.000000	1.000000
21-day	Zero	Leptokurtic	SU	0.000000	1.422798	0.000000	1.706085
21-day	Positive	Mesokurtic	SB	-1.024276	3.876390	0.856400	0.534582
21-day	Positive	Leptokurtic	SU	-1.609837	1.484898	-2.050504	2.345450
63-day	Negative	Mesokurtic	SB	-3.155855	4.353278	-0.908420	0.673890
63-day	Negative	Leptokurtic	SB	-10.935399	13.237225	-3.086450	1.873181
63-day	Zero	Mesokurtic	SN	0.000000	1.000000	0.000000	1.000000
63-day	Zero	Leptokurtic	SU	0.000000	1.964320	0.000000	2.188413
63-day	Positive	Mesokurtic	SB	-1.197423	4.353278	0.908420	0.673890
63-day	Positive	Leptokurtic	SB	-2.301826	13.237225	3.086450	1.873181
126-day	Negative	Mesokurtic	SB	-4.559566	6.540898	-1.183885	1.251032
126-day	Negative	Leptokurtic	SL*	4.589575	-1.000000	-6.967738	4.643311
126-day	Zero	Mesokurtic	SN	0.000000	1.000000	0.000000	1.000000
126-day	Zero	Leptokurtic	SU	0.000000	2.476362	0.000000	2.661838
126-day	Positive	Mesokurtic	SB	-1.981333	6.540898	1.183885	1.251032
126-day	Positive	Leptokurtic	SL*	-4.589575	1.000000	-6.967738	4.643311

Table 3: Johnson Distribution Parameter Values and Types for Simulations

Overall, there are 540 scenarios in the simulation experiments. These simulation results will be evaluated based comparisons of valuations from different cases. The average price value will be reported in tables. From the simulations, negative option prices were observed from some cases in the proposed pricing model. To investigate this, the percentage of negative option prices per case in each model is computed.

6 Results and Discussion

Table 4 contains the summary of simulation results considering the moneyness and interest rate, ignoring other simulation scenarios. From the table, negative prices may tend to occur for the CMM, CMM.RN, QMM, QMM.RN, and CP models, and these occurrences increase as the K^* increases. Risk-neutral valuated models are less likely to have negative prices, and the CP.RN and BS models, which are risk-neutral models, cannot have negative prices.

For CMM and CMM.RN, the overall percentage of negative price occurrence in the simulation study was 21.20% and 16.69%, respectively. Negative prices tend to occur for the CMM, CMM.RN models as K^* increases. The average call option price tends to be negative for the CMM starting at $K^* = 1.05$ for a low interest rate level and at $K^* = 1.10$ for other interest levels. CMM.RN has less negative price occurrences compared to CMM, but these occurrences increase when $K^* = 1.10$. For QMM, QMM.RN, and CP, negative prices occurred in the simulation study 29.48%, 26.13%, and 26.26% of the time, respectively. Negative option prices occur starting at $K^* = 1.05$ for the three models. QMM.RN tend to have the lowest frequency of negative prices of the three models at $K^* = 1.05$.

With respect to average call option prices, CMM, CMM.RN, and QMM.RN tend to have lower prices compared to the BS model for in-the-money cases. QMM, CP, and CP.RN tend to have higher price valuations compared to the BS for in-the-money cases. For at-the-money cases, all proposed models tend to have lower valuations compared to the BS model, with the CP.RN model being the closest model to the BS valuation. It is notable that the CP.RN model tends to have higher or equal valuations to the BS model, except for at-the-money cases.

Table 5 contains the summary of simulation results considering the duration, variance structure, skewness, and kurtosis, ignoring other simulation scenarios. Generally, the BS model gives higher valuations compared to the CMM, CMM.RN, QMM, QMM.RN, and CP. The RN models tend to have lower frequency of negative call option prices compared to the non-RN counterparts. For all proposed models except CP.RN, the occurrence of negative option prices tends to be spread to almost all cases but of differing frequency, so analysis will point out on which cases were higher-than-average frequency of price occurrences are observed.

Noting that the overall percent of negative prices for the CMM is 21.20%, the cases when higher-than-average occurrences were observed are the 1-month GARCH cases, 3-month GARCH cases, 6-month GARCH cases when there is nonnegative skewness, and leptokurtic cases under the constant variance and nonpositive skewness with 1 month duration. When risk-neutral valuation is done, i.e., the CMM.RN model, the frequency is reduced overall for GARCH cases, yet tend to worsen for cases with constant variances and nonnegative

K^*	r_A	BS		CMM		CMM.RN		QMM		QMM.RN		CP		CP.RN	
		Aver.	% Neg.	Aver.	% Neg.	Aver.	% Neg.	Aver.	% Neg.	Aver.	% Neg.	Aver.	% Neg.	Aver.	% Neg.
0.90	Low	10.11%	0.00%	9.09%	0.22%	9.96%	0.06%	10.28%	0.00%	9.92%	0.00%	10.24%	0.00%	10.19%	0.00%
	Mid	10.31%	0.00%	10.28%	0.44%	10.16%	0.00%	10.46%	0.00%	10.13%	0.00%	10.44%	0.00%	10.39%	0.00%
	Hi	10.64%	0.00%	10.66%	0.22%	10.48%	0.08%	10.77%	0.00%	10.48%	0.00%	10.77%	0.00%	10.72%	0.00%
0.95	Low	5.59%	0.00%	5.60%	1.08%	5.19%	0.08%	5.63%	0.00%	5.04%	0.00%	5.58%	0.00%	5.59%	0.00%
	Mid	5.76%	0.00%	5.29%	1.22%	5.32%	0.06%	5.79%	0.00%	5.24%	0.00%	5.78%	0.00%	5.78%	0.00%
	Hi	6.06%	0.00%	5.66%	0.92%	3.76%	0.08%	6.07%	0.00%	5.58%	0.00%	6.11%	0.00%	6.08%	0.00%
1.00	Low	2.25%	0.00%	0.10%	7.81%	1.42%	0.31%	1.80%	0.00%	1.25%	0.00%	1.69%	0.00%	2.07%	0.00%
	Mid	2.36%	0.00%	2.66%	6.86%	1.52%	0.39%	1.90%	0.00%	1.36%	0.00%	1.87%	0.00%	2.18%	0.00%
	Hi	2.57%	0.00%	1.93%	6.89%	1.66%	0.31%	2.09%	0.00%	1.58%	0.00%	2.17%	0.00%	2.38%	0.00%
1.05	Low	0.66%	0.00%	-3.09%	52.25%	0.17%	22.75%	0.01%	64.03%	0.03%	51.61%	-0.34%	61.81%	0.69%	0.00%
	Mid	0.71%	0.00%	0.22%	50.14%	0.22%	20.11%	0.03%	63.33%	0.06%	46.03%	-0.22%	53.78%	0.73%	0.00%
	Hi	0.80%	0.00%	0.43%	49.22%	0.21%	16.83%	0.08%	61.03%	0.11%	36.47%	0.00%	43.72%	0.80%	0.00%
1.10	Low	0.16%	0.00%	-0.04%	44.78%	-0.06%	65.36%	-0.22%	80.64%	-0.14%	86.67%	-0.73%	82.53%	0.27%	0.00%
	Mid	0.17%	0.00%	-0.01%	47.53%	-0.04%	62.78%	-0.24%	84.47%	-0.14%	86.92%	-0.65%	80.44%	0.28%	0.00%
	Hi	0.21%	0.00%	-2.67%	48.53%	0.00%	61.22%	-0.26%	88.72%	-0.13%	84.25%	-0.51%	71.64%	0.31%	0.00%

Table 4: Average Call Prices in Percent of Underlying Asset Price and Percentage of Negative Call Prices by Model, Interest Rate, and Moneyness Cases

skewness.

With the QMM model which has 29.48% overall occurrence of negative prices, 6-month cases tend to have negative prices occur less often than its overall percentage. This can be improved by risk-neutral valuation, meaning the use of QMM.RN, especially for GARCH cases.

The CP model tends to be worse than its overall percentage of 26.26% in cases of GARCH variances, 6-month durations, or both, but gets improved by risk-neutral valuation which eliminates all negative occurrences.

With respect to average call price values, the proposed models except CP.RN tend to be more conservative than BS in the sense that they tend value options with lower prices compared to the BS over most cases. The CP.RN model tends to give higher valuations in cases of constant variance compared to the BS model, and would be more conservative for the GARCH variances cases compared to the BS.

Highlighting the real-data cases of GARCH variance, negative skewness, and leptokurtosis for all durations, the BS model tends to value call options higher than the proposed models. The longer the duration, the higher the valuations tend to be except for the CMM and CMM.RN which tend to exhibit nonlinearities on the pattern of the averages.

It is notable that the whether nonnormal features are evident or not, the BS model tends to have similar values up to the second decimal of the percentage, and would only differ with respect to duration and variance structure. For the proposed models, skewness and kurtosis tend to change the option price at differing magnitudes.

Overall, for five of the proposed models, negative prices are possible because: (1) the models are not based on a no-arbitrage pricing principle, but on equilibrium asset pricing models such as the CAPM [4] and its extensions, where negative prices tend to be possible, indicating possible arbitrage gains [12], and (2) the nature of their formula, which involves differences between quantities. Negative quantities would imply large $\alpha_{i,t,T,K}^*$, which may mean large systematic variance, skewness, or kurtosis. This implies that these large risks cannot be eliminated or reduced.

7 Conclusion

Based on the simulation studies, the proposed models for option pricing tend to be more conservative than the Black-Scholes model in the sense that they set a lower value for call options especially in cases of nonconstant variance and existing nonnormalities such as skewness and kurtosis, which are more evident

Dur	Var	Skew	Kurt	BS		CMM		CMM.RN		QMM		QMM.RN		CP		CP.RN	
				Aver.	% Neg.	Aver.	% Neg.	Aver.	% Neg.	Aver.	% Neg.	Aver.	% Neg.	Aver.	% Neg.	Aver.	% Neg.
1 month	Constant	Neg	Meso	3.42%	0.00%	3.05%	20.00%	3.26%	0.00%	3.16%	20.00%	3.13%	0.00%	3.46%	0.00%	3.42%	0.00%
			Lepto	3.42%	0.00%	3.14%	36.33%	3.26%	20.60%	3.16%	36.60%	3.14%	26.67%	3.46%	0.20%	3.41%	0.00%
		Zero	Meso	3.42%	0.00%	3.21%	12.80%	3.13%	40.07%	3.15%	40.00%	3.13%	40.00%	3.39%	0.33%	3.43%	0.00%
			Lepto	3.42%	0.00%	3.14%	22.40%	3.17%	37.47%	3.17%	40.00%	3.15%	39.80%	3.38%	10.40%	3.42%	0.00%
		Pos	Meso	3.42%	0.00%	3.33%	0.00%	3.27%	22.13%	3.15%	40.00%	3.13%	40.00%	3.34%	0.00%	3.45%	0.00%
			Lepto	3.42%	0.00%	3.32%	0.13%	3.29%	19.80%	3.18%	39.87%	3.16%	39.73%	3.32%	11.93%	3.43%	0.00%
	GARCH	Neg	Meso	3.44%	0.00%	3.03%	38.60%	3.24%	31.53%	3.24%	40.00%	3.17%	38.60%	3.55%	34.47%	3.42%	0.00%
			Lepto	3.44%	0.00%	3.18%	30.60%	3.25%	26.87%	3.26%	40.00%	3.17%	33.73%	3.46%	39.33%	3.41%	0.00%
		Zero	Meso	3.44%	0.00%	3.39%	28.67%	3.19%	25.53%	3.27%	40.00%	3.18%	35.53%	3.29%	40.00%	3.43%	0.00%
			Lepto	3.44%	0.00%	2.80%	33.13%	3.22%	19.40%	3.27%	40.00%	3.20%	30.33%	3.26%	40.00%	3.42%	0.00%
		Pos	Meso	3.44%	0.00%	3.26%	23.00%	3.31%	5.67%	3.21%	39.93%	3.22%	28.00%	3.05%	40.00%	3.45%	0.00%
			Lepto	3.44%	0.00%	3.30%	27.07%	3.28%	8.40%	3.24%	40.00%	3.22%	26.53%	3.09%	40.00%	3.43%	0.00%
3 months	Constant	Neg	Meso	3.85%	0.00%	1.86%	59.73%	3.49%	0.00%	3.40%	40.00%	3.21%	18.33%	3.97%	0.00%	3.85%	0.00%
			Lepto	3.85%	0.00%	2.71%	41.07%	3.52%	20.00%	3.40%	40.00%	3.25%	30.53%	3.93%	0.73%	3.85%	0.00%
		Zero	Meso	3.85%	0.00%	6.03%	15.40%	3.22%	39.27%	3.42%	38.67%	3.23%	40.00%	3.66%	15.00%	3.89%	0.00%
			Lepto	3.85%	0.00%	0.52%	19.40%	3.26%	32.93%	3.46%	36.93%	3.28%	38.47%	3.64%	19.13%	3.88%	0.00%
		Pos	Meso	3.85%	0.00%	3.68%	0.00%	3.53%	20.00%	3.41%	13.60%	3.24%	40.00%	3.39%	18.53%	3.92%	0.00%
			Lepto	3.85%	0.00%	3.61%	0.00%	3.58%	10.27%	3.43%	19.93%	3.32%	36.73%	3.39%	27.80%	3.90%	0.00%
	GARCH	Neg	Meso	3.90%	0.00%	2.90%	34.00%	3.46%	21.60%	3.62%	40.00%	3.32%	29.20%	3.96%	27.80%	3.83%	0.00%
			Lepto	3.90%	0.00%	3.87%	23.60%	3.61%	18.80%	3.64%	40.00%	3.35%	22.73%	3.82%	36.87%	3.82%	0.00%
		Zero	Meso	3.90%	0.00%	-2.50%	35.20%	3.33%	16.40%	3.67%	40.00%	3.40%	20.07%	3.40%	39.93%	3.86%	0.00%
			Lepto	3.90%	0.00%	2.00%	36.40%	3.44%	14.73%	3.66%	40.00%	3.43%	18.80%	3.36%	40.00%	3.84%	0.00%
		Pos	Meso	3.90%	0.00%	3.46%	35.13%	3.61%	0.13%	3.44%	40.00%	3.48%	17.53%	2.94%	40.00%	3.89%	0.00%
			Lepto	3.90%	0.00%	3.52%	34.93%	3.57%	2.00%	3.48%	40.00%	3.50%	14.73%	2.99%	40.00%	3.87%	0.00%
6 months	Constant	Neg	Meso	4.35%	0.00%	5.15%	0.00%	3.77%	20.00%	4.12%	12.60%	3.39%	24.20%	4.16%	11.40%	4.43%	0.00%
			Lepto	4.35%	0.00%	4.74%	0.40%	3.75%	19.67%	4.12%	16.93%	3.43%	22.13%	4.10%	16.87%	4.42%	0.00%
		Zero	Meso	4.35%	0.00%	4.65%	15.20%	3.41%	25.33%	4.08%	2.80%	3.43%	30.13%	3.67%	27.53%	4.47%	0.00%
			Lepto	4.35%	0.00%	-3.52%	18.27%	3.47%	21.33%	4.08%	6.33%	3.50%	23.53%	3.67%	30.20%	4.45%	0.00%
		Pos	Meso	4.35%	0.00%	3.98%	0.00%	3.86%	0.87%	3.87%	0.00%	3.49%	29.60%	3.24%	37.07%	4.49%	0.00%
			Lepto	4.35%	0.00%	3.92%	0.67%	3.88%	1.00%	3.85%	0.47%	3.60%	21.73%	3.27%	37.33%	4.47%	0.00%
	GARCH	Neg	Meso	4.39%	0.00%	5.71%	2.87%	3.60%	16.13%	4.47%	20.00%	3.56%	19.80%	4.05%	29.93%	4.32%	0.00%
			Lepto	4.39%	0.00%	2.26%	6.07%	-0.60%	16.53%	4.46%	20.13%	3.59%	18.93%	3.93%	33.67%	4.30%	0.00%
		Zero	Meso	4.39%	0.00%	5.05%	29.67%	3.26%	11.93%	4.33%	20.80%	3.67%	15.53%	3.44%	39.47%	4.34%	0.00%
			Lepto	4.39%	0.00%	1.00%	29.60%	3.47%	11.87%	4.30%	21.60%	3.70%	12.73%	3.41%	39.47%	4.32%	0.00%
		Pos	Meso	4.39%	0.00%	3.92%	26.13%	3.78%	0.80%	3.92%	26.53%	3.76%	9.13%	2.91%	40.00%	4.38%	0.00%
			Lepto	4.39%	0.00%	4.06%	27.00%	3.79%	1.93%	3.95%	27.60%	3.79%	7.13%	2.96%	40.00%	4.35%	0.00%

Table 5: Average Call Price and Percentage of Negative Call Prices by Model, Duration, Variance Structure, Skewness, and Kurtosis Cases

in price movements and returns of underlying asset prices.

There are possibilities that the proposed models may have negative prices but these may only occur at out-of-the-money options and their likelihood can be reduced by using risk-neutral valuation methods. These models were generated not through no-arbitrage pricing theory, but through the equilibrium asset pricing philosophy, of which negative prices can imply arbitrage gains. Negative prices can also mean that there are large systematic non-Gaussian risks that cannot be reduced.

In using these models, risk-neutral valuations offer better results. It should be noted that there is more potential for the model to improve via data transformation using equation (74), as seen by using risk-neutral valuations.

The research opens a new approach to assessing options that includes non-Gaussian features and is under a nonparametric framework. Further extensions of pricing model can be done, as other features of options transactions have not yet been included, such as including transaction costs into the model.

References

- [1] Giovanni Barone-Adesi. Arbitrage equilibrium with skewed asset returns. *The Journal of Financial and Quantitative Analysis*, 20(3):299–313, Sep. 1985.
- [2] Fischer Black and Myron Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3):637–654, May-Jun 1973.
- [3] Tim Bollerslev. Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics*, 31:307–327, 1986.
- [4] Ren-Raw Chen and Oded Palmon. A non-parametric option pricing model: Theory and empirical evidence. *Review of Quantitative Finance and Accounting*, 24:115–134, 2005.
- [5] John C. Cox and Stephen A. Ross. The valuation of options for alternative stochastic processes. *Journal of Financial Economics*, 3:145–166, 1976.
- [6] Emanuel Derman, Iraj Kani, and Joseph Z. Zou. The local volatility surface: Unlocking the information in index option prices. *Financial Analysts Journal*, 52(4):25–36, Jul-Aug 1996.
- [7] Robert F. Dittmar. Nonlinear pricing kernels, kurtosis preference, and evidence from the cross section of equity returns. *The Journal of Finance*, 57(1):369–403, February 2002.

- [8] Eugene F. Fama. Risk, return and equilibrium: Some clarifying comments. *The Journal of Finance*, 23(1):29–40, March 1968.
- [9] Hsing Fang and Tsong-Yue Lai. Co-kurtosis and capital asset pricing. *The Financial Review*, 32(2):293–307, May 1997.
- [10] Peter Gross. Parameter estimation for black-scholes equation. Technical report, Department of Mathematics, The University of Arizona, 2006.
- [11] I. D. Hill, R. Hill, and R. L. Holder. Algorithm as 99: Fitting johnson curves by moments. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 25(2):180–189, 1976.
- [12] Robert A. Jarrow and Dilip B. Madan. Is mean-variance analysis vacuous: Or was beta still born? *European Finance Review*, 1:15–30, 1997.
- [13] N. L. Johnson. Systems of frequency curves generated by methods of translation. *Biometrika*, 36(1/2):149–176, Jun. 1949.
- [14] Alan Kraus and Robert H. Litzenberger. Skewness preference and the valuation of risk assets. *The Journal of Finance*, 31(1):1085–1100, Sep. 1976.
- [15] Serdar Neslihanoglu. *Validating and Extending the Two-Moment Capital Asset Pricing Model for Financial Time Series*. PhD thesis, School of Mathematics and Statistics, College of Science and Engineering, University of Glasgow, 2014.
- [16] Board of Governors of the Federal Reserve System. Selected interest rates (daily) - h.15. <http://www.federalreserve.gov/releases/h15/data.htm>, April 2015. Website accessed last 7 April 2015.
- [17] William F. Sharpe. Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance*, 19(3):425–442, September 1964.
- [18] Hans R. Stoll. The relationship between put and call option prices. *The Journal of Finance*, 24(5):801–824, Dec. 1969.
- [19] Ruey S. Tsay. *Analysis of Financial Time Series*. John Wiley & Sons, 2002.

8 Syntax for R Implementation

```
1 #Pricing Models Program
3 #Preliminary Functions
#Generating Covariance, symmetric function covar(X,Y)=covar(Y,X)
5 covar<-function(x=c(),y=c()) {
  mean((x-mean(x))*(y-mean(y)))
7 }
#Generating Coskewness, not symmetric, since coskew(X,Y)≠coskew(Y
,X)
9 coskew<-function(x=c(),y=c()) {
  mean((x-mean(x))*(y-mean(y))^2)
11 }
#Generating Cokurtosis, not symmetric, since cokurt(X,Y)≠cokurt(Y
,X)
13 cokurt<-function(x=c(),y=c()) {
15   mean((x-mean(x))*(y-mean(y))^3)
  }
17
#Underlying-Adjusted Black-Scholes Model.
#Arguments
19 # x=log-returns data
# K.star=Moneyiness
21 # ra= risk-free asset annual effective interest rate
# Dur = contract duration
23 BS.Price.Adj<-function(x=c(),K.star=c(),ra=c(),Dur=c()) {
25   r<-log(1+ra)
  sigma<-sqrt(sum((x-mean(x))^2)/(length(x)-1))/sqrt(Dur)
27   price<-pnorm((-log(K.star)+(r+sigma^2/2)*Dur)/(sigma*sqrt(Dur)))-
    K.star*exp(-r*Dur)*pnorm((-log(K.star)+(r-sigma^2/2)*Dur)/(
    sigma*sqrt(Dur)))
  return(price)
29 }
31 #Underlying-Adjusted Cubic Market Pricing Model.
#Arguments
33 # x=log-returns data
# K.star=Moneyiness
35 # ra= risk-free asset annual effective interest rate
# Dur = contract duration
37 CMM.Price.Adj<-function(x=c(),K.star=c(),ra=c(),Dur=c()) {
39   rft<-(1+ra)^Dur
  Ret<-exp(x)
41   v1<-covar((Ret-K.star)*((Ret-K.star)>0),Ret)/covar((Ret),(Ret))
  v2<-coskew((Ret-K.star)*((Ret-K.star)>0),Ret)/coskew((Ret),(Ret))
43   v3<-cokurt((Ret-K.star)*((Ret-K.star)>0),Ret)/cokurt(Ret,Ret)
45   bvec<-c(v1,v2,v3)
47   k12<-covar((Ret-rft)^2,Ret)/covar(Ret,Ret)
  k22<-coskew((Ret-rft)^2,Ret)/coskew(Ret,Ret)
49   k23<-cokurt((Ret-rft)^2,Ret)/cokurt(Ret,Ret)
51   k13<-covar((Ret-rft)^3,Ret)/covar(Ret,Ret)
```

```

53 k23<-coskew((Ret-rft)^3,Ret)/coskew(Ret,Ret)
k33<-cokurt((Ret-rft)^3,Ret)/cokurt(Ret,Ret)

55 kmat<-matrix(c(1,1,1,k12,k22,k23,k13,k23,k33),nrow=3,ncol=3)

57 alpha.vec<-solve(kmat,bvec)
rmom.vec<-c(mean(Ret-rft),covar((Ret-rft),(Ret-rft)),coskew((Ret-
rft),(Ret-rft)))

59 cpnew<-(mean((Ret-K.star)*((Ret-K.star)>0))-sum(alpha.vec*rmom.
vec))/(rft)
61 return(cpnew)
}

63 #Underlying-Adjusted Cubic Market Pricing Model, Risk-Neutral
Valuation.
65 #Arguments
# x=log-returns data
67 # K.star=Moneyiness
# ra= risk-free asset annual effective interest rate
69 # Dur = contract duration
CMM.RN.Price.Adj<-function(x=c(),K.star=c(),ra=c(),Dur=c()) {
71 rft<-(1+ra)^Dur
Pret<-exp(x)
73 Ret<-Pret-mean(Pret)+rft

75 v1<-covar((Ret-K.star)*((Ret-K.star)>0),Ret)/covar((Ret),(Ret))
v2<-coskew((Ret-K.star)*((Ret-K.star)>0),Ret)/coskew((Ret),(Ret))
77 v3<-cokurt((Ret-K.star)*((Ret-K.star)>0),Ret)/cokurt(Ret,Ret)

79 bvec<-c(v1,v2,v3)

81 k12<-covar((Ret-rft)^2,Ret)/covar(Ret,Ret)
k22<-coskew((Ret-rft)^2,Ret)/coskew(Ret,Ret)
83 k23<-cokurt((Ret-rft)^2,Ret)/cokurt(Ret,Ret)

85 k13<-covar((Ret-rft)^3,Ret)/covar(Ret,Ret)
k23<-coskew((Ret-rft)^3,Ret)/coskew(Ret,Ret)
87 k33<-cokurt((Ret-rft)^3,Ret)/cokurt(Ret,Ret)

89 kmat<-matrix(c(1,1,1,k12,k22,k23,k13,k23,k33),nrow=3,ncol=3)

91 alpha.vec<-solve(kmat,bvec)
rmom.vec<-c(mean(Ret-rft),covar((Ret-rft),(Ret-rft)),coskew((Ret-
rft),(Ret-rft)))

93 cpnew<-(mean((Ret-K.star)*((Ret-K.star)>0))-sum(alpha.vec*rmom.
vec))/(rft)
95 return(cpnew)
}

97 #Underlying-Adjusted Quadratic Market Pricing Model.
99 #Arguments
# x=log-returns data
101 # K.star=Moneyiness
# ra= risk-free asset annual effective interest rate
103 # Dur = contract duration

```



```

105 QMM.Price.Adj<-function(x=c(),K.star=c(),ra=c(),Dur=c()) {
106   rft<-(1+ra)^Dur
107   Ret<-exp(x)
108
109   v1<-covar((Ret-K.star)*((Ret-K.star)>0),Ret)/covar((Ret),(Ret))
110   v2<-coskew((Ret-K.star)*((Ret-K.star)>0),Ret)/coskew((Ret),(Ret))
111
112   bvec<-c(v1,v2)
113
114   k12<-covar((Ret-rft)^2,Ret)/covar(Ret,Ret)
115   k22<-coskew((Ret-rft)^2,Ret)/coskew(Ret,Ret)
116
117   kmat<-matrix(c(1,1,k12,k22),nrow=2, ncol=2)
118
119   alpha.vec<-solve(kmat,bvec)
120   rmom.vec<-c(mean(Ret-rft),covar((Ret-rft),(Ret-rft)))
121
122   cpnew<-(mean((Ret-K.star)*((Ret-K.star)>0))-sum(alpha.vec * rmom.
123     vec))/(rft)
124   return(cpnew)
125 }
126
127 #Underlying-Adjusted Quadratic Market Pricing Model, Risk-Neutral
128 #Valuation.
129 #Arguments
130 # x=log-returns data
131 # K.star=Moneyness
132 # ra= risk-free asset annual effective interest rate
133 # Dur = contract duration
134 QMM.RN.Price.Adj<-function(x=c(),K.star=c(),ra=c(),Dur=c()) {
135   rft<-(1+ra)^Dur
136   Pret<-exp(x)
137   Ret<-Pret-mean(Pret)+rft
138
139   v1<-covar((Ret-K.star)*((Ret-K.star)>0),Ret)/covar((Ret),(Ret))
140   v2<-coskew((Ret-K.star)*((Ret-K.star)>0),Ret)/coskew((Ret),(Ret))
141
142   bvec<-c(v1,v2)
143
144   k12<-covar((Ret-rft)^2,Ret)/covar(Ret,Ret)
145   k22<-coskew((Ret-rft)^2,Ret)/coskew(Ret,Ret)
146
147   kmat<-matrix(c(1,1,k12,k22),nrow=2, ncol=2)
148
149   alpha.vec<-solve(kmat,bvec)
150   rmom.vec<-c(mean(Ret-rft),covar((Ret-rft),(Ret-rft)))
151
152   cpnew<-(mean((Ret-K.star)*((Ret-K.star)>0))-sum(alpha.vec * rmom.
153     vec))/(rft)
154   return(cpnew)
155 }
156
157 #Underlying-Adjusted Chen-Palmon CAPM Pricing Model.
158 #Arguments
159 # x=log-returns data
160 # K.star=Moneyness
161 # ra= risk-free asset annual effective interest rate

```

```

# Dur = contract duration
159 CP.Price.Adj<-function(x=c(),K.star=c(),ra=c(),Dur=c()) {
  rft<-(1+ra)^Dur
161   Ret<-exp(x)

163   beta<-covar((Ret-K.star)*((Ret-K.star)>0),Ret)/covar((Ret),(Ret))

165   cpnew<-(mean((Ret-K.star)*((Ret-K.star)>0))-beta*mean(Ret-rft))/(
     rft)
167   return(cpnew)
}

169 #Underlying-Adjusted Chen-Palmon CAPM Pricing Model, Risk-Neutral
     Valuation.
#Arguments
171 # x=log-returns data
# K.star=Moneyiness
173 # ra= risk-free asset annual effective interest rate
# Dur = contract duration
175 CP.RN.Price.Adj<-function(x=c(),K.star=c(),ra=c(),Dur=c()) {
  rft<-(1+ra)^Dur
177   Pret<-exp(x)
  Ret<-Pret-mean(Pret)+rft

179   beta<-covar((Ret-K.star)*((Ret-K.star)>0),Ret)/covar((Ret),(Ret))

181   cpnew<-(mean((Ret-K.star)*((Ret-K.star)>0))-beta*mean(Ret-rft))/(
     rft)
183   return(cpnew)
}
185

187 #Extract S&P 500 Data
SandP.500.data <- read.csv("SandP 500 data.csv")
189

#Generate 1-month Log Returns Data from S&P 500 Data
191 n<-length(SandP.500.data$S.Close)
SPDat<-SandP.500.data$S.Close
193 SPDatH.21<-SPDat[-1:-21]
SPDatT.21<-SPDat[-(n-20):-n]
195 logret.21<-log(SPDatH.21)-log(SPDatT.21)

#Generate 3-month Log Returns Data from S&P 500 Data
197 SPDatH.63<-SPDat[-1:-63]
SPDatT.63<-SPDat[-(n-62):-n]
199 logret.63<-log(SPDatH.63)-log(SPDatT.63)

201 #Generate 6-month Log Returns Data from S&P 500 Data
203 SPDatH.126<-SPDat[-1:-126]
SPDatT.126<-SPDat[-(n-125):-n]
205 logret.126<-log(SPDatH.126)-log(SPDatT.126)

207 #Moneyiness Cases Vector
K.star.vec<-c(0.9,0.95,1.0,1.05,1.1)
209

#Annual Effective Interest Data As of 31 March, 2015
211 ra.vec<-c(0.0005, (1+0.0089/2)^2-1, (1+0.0231/2)^2-1)

```

```

213 #Duration Vector
Dur.vec<-c(1/12,1/4,1/2)
215
216 #Standard Deviation Vector
217 SD.vec<-c()
218
219 #Skewness vector
Skew.vec<-c()
221
222 #Kurtosis Vector
223 Kurt.vec<-c()
224
225 #Mean Vector
Mean.vec<-c()
227
228 #Generate Stats on 1-month Log Returns Data from S&P 500 Data
229 n.21<-length(logret.21)
SD.vec[1]<-sqrt(covar(logret.21[(n.21-5*252+1):n.21], logret.21[(n
.21-5*252+1):n.21]))
231 Skew.vec[1]<-coskew(logret.21[(n.21-5*252+1):n.21], logret.21[(n
.21-5*252+1):n.21])/(covar(logret.21[(n.21-5*252+1):n.21],
logret.21[(n.21-5*252+1):n.21]))^(3/2)
Kurt.vec[1]<-cokurt(logret.21[(n.21-5*252+1):n.21], logret.21[(n
.21-5*252+1):n.21])/(covar(logret.21[(n.21-5*252+1):n.21],
logret.21[(n.21-5*252+1):n.21]))^(2)
233
234 #Generate Stats on 3-month Log Returns Data from S&P 500 Data
235 n.63<-length(logret.63)
SD.vec[2]<-sqrt(covar(logret.63[(n.63-5*252+1):n.63], logret.63[(n
.63-5*252+1):n.63]))
237 Skew.vec[2]<-coskew(logret.63[(n.63-5*252+1):n.63], logret.63[(n
.63-5*252+1):n.63])/(covar(logret.63[(n.63-5*252+1):n.63],
logret.63[(n.63-5*252+1):n.63]))^(3/2)
Kurt.vec[2]<-cokurt(logret.63[(n.63-5*252+1):n.63], logret.63[(n
.63-5*252+1):n.63])/(covar(logret.63[(n.63-5*252+1):n.63],
logret.63[(n.63-5*252+1):n.63]))^(2)
239
240 #Generate Stats on 6-month Log Returns Data from S&P 500 Data
241 n.126<-length(logret.126)
SD.vec[3]<-sqrt(covar(logret.126[(n.126-5*252+1):n.126], logret
.126[(n.126-5*252+1):n.126]))
243 Skew.vec[3]<-coskew(logret.126[(n.126-5*252+1):n.126], logret.126[(n
.126-5*252+1):n.126])/(covar(logret.126[(n.126-5*252+1):n.126],
logret.126[(n.126-5*252+1):n.126]))^(3/2)
Kurt.vec[3]<-cokurt(logret.126[(n.126-5*252+1):n.126], logret.126[(n
.126-5*252+1):n.126])/(covar(logret.126[(n.126-5*252+1):n.126],
logret.126[(n.126-5*252+1):n.126]))^(2)
245
246 #Generate Means on all Durations of Log Returns Data from S&P 500
Data
247 Mean.vec<-c(mean(logret.21[(n.21-5*252+1):n.21]), mean(logret.63[(n
.63-5*252+1):n.63]), mean(logret.126[(n.126-5*252+1):n.126]))
249
n.vec<-c(n.21, n.63, n.126)
251 #GARCH (1,1) Model Estimation

```

```

require(rmgarch)
253 garch11.spec <- ugarchspec(mean.model = list(armaOrder = c(0,0)),
                             variance.model = list(garchOrder = c
(1,1),
255                                     model = "sGARCH"),
                             distribution.model = "norm")

257 # Fit the model for 21-trading-day duration
garch.fit.21 <- ugarchfit(garch11.spec, data = logret.21[(n.21-5*
252+1):n.21], fit.control=list(scale=TRUE))

259 # Fit the model for 63-trading-day duration
261 garch.fit.63 <- ugarchfit(garch11.spec, data = logret.63[(n.63-5*
252+1):n.63], fit.control=list(scale=TRUE))

263 # Fit the model for 126-trading-day duration
garch.fit.126 <- ugarchfit(garch11.spec, data = logret.126[(n.126-5
*252+1):n.126], fit.control=list(scale=TRUE))

265 #Johnson Fitting by Moment Matching
267 require(JohnsonDistribution)

269 #Set Seed Number
n.seed<-1000
271 #Matrix for Changing Duration, Constant Variance [1], Negative
Skewness[1], Mesokurtic [1]
Dur.1m.111<-c(Dur.vec[1], Mean.vec[1], SD.vec[1],
FitJohnsonDistribution(0,1,Skew.vec[1],3))
273 Dur.3m.111<-c(Dur.vec[2], Mean.vec[2], SD.vec[2],
FitJohnsonDistribution(0,1,Skew.vec[2],3))
Dur.6m.111<-c(Dur.vec[3], Mean.vec[3], SD.vec[3],
FitJohnsonDistribution(0,1,Skew.vec[3],3))
275 Dur.Mat.111<-matrix(0,nrow=3,ncol=9)
Dur.Mat.111[1,]<-Dur.1m.111
277 Dur.Mat.111[2,]<-Dur.3m.111
Dur.Mat.111[3,]<-Dur.6m.111
279 colnames(Dur.Mat.111)<-c("Dur", "mu", "sigma", "ITYPE", "GAMMA", "DELTA",
"XLAM", "XI", "IFAULT")
##Simulation Skeleton
281 ###K.star.vec[j] uses "j" as index
###ra.vec[k] uses "k" as index
283 ###Dur.Mat[l,] uses "l" as index
set.seed(n.seed)
285 price.mat.111<-matrix(0,nrow=(4500),ncol=13)
colnames(price.mat.111)<-c("CMM", "CMM.RN", "QMM", "QMM.RN", "CP", "CP.
RN", "BS", "K*", "R.A", "Dur", "Var", "Skew", "Kurt")
287 for (l in 1:3) {
for (k in 1:3) {
289 for (j in 1:5) {
for (i in 1:100) {
291 z<-rnorm(1260)
u<-Dur.Mat.111[1,"mu"]+Dur.Mat.111[1,"sigma"]*
yJohnsonDistribution(z,Dur.Mat.111[1,"ITYPE"],Dur.Mat.111[1,"
GAMMA"],Dur.Mat.111[1,"DELTA"],Dur.Mat.111[1,"XLAM"],Dur.Mat
.111[1,"XI"])
293 price.CMM<-CMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat.111[1,
"Dur"])

```

```

price.CMM.RN<-CMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.111[1,"Dur"])
295 price.QMM<-QMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat.111[1,
"Dur"])
price.QMM.RN<-QMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.111[1,"Dur"])
297 price.CP<-CP.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat.111[1,"
Dur"])
price.CP.RN<-CP.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.111[1,"Dur"])
299 price.BS<-BS.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat.111[1,"
Dur"])
price.mat.111[(i+(j-1)*100+(k-1)*500+(1-1)*1500),]<-c(price.CMM,
price.CMM.RN,price.QMM,price.QMM.RN,price.CP,price.CP.RN,price.
BS,j,k,1,1,1,1)
301 }
}
303 }
}
305
#Matrix for Changing Duration, Constant Variance [1], Negative
Skewness [1], Leptokurtic [2]
307 Dur.1m.112<-c(Dur.vec[1],Mean.vec[1],SD.vec[1],
FitJohnsonDistribution(0,1,Skew.vec[1],Kurt.vec[1]))
Dur.3m.112<-c(Dur.vec[2],Mean.vec[2],SD.vec[2],
FitJohnsonDistribution(0,1,Skew.vec[2],Kurt.vec[2]))
309 Dur.6m.112<-c(Dur.vec[3],Mean.vec[3],SD.vec[3],
FitJohnsonDistribution(0,1,Skew.vec[3],Kurt.vec[3]))
Dur.Mat.112<-matrix(0,nrow=3,ncol=9)
311 Dur.Mat.112[1,]<-Dur.1m.112
Dur.Mat.112[2,]<-Dur.3m.112
313 Dur.Mat.112[3,]<-Dur.6m.112
colnames(Dur.Mat.112)<-c("Dur","mu","sigma","ITYPE","GAMMA","DELTA"
,"XLAM","XI","IFAULT")
315 ##Simulation Skeleton
###K.star.vec[j] uses "j" as index
317 ###ra.vec[k] uses "k" as index
###Dur.Mat[1,] uses "1" as index
319 set.seed(n.seed)
price.mat.112<-matrix(0,nrow=(4500),ncol=13)
321 colnames(price.mat.112)<-c("CMM","CMM.RN","QMM","QMM.RN","CP","CP.
RN","BS","K*","R_A","Dur","Var","Skew","Kurt")
for (l in 1:3) {
323 for (k in 1:3) {
for (j in 1:5) {
325 for (i in 1:100) {
z<-rnorm(1260)
327 u<-Dur.Mat.112[1,"mu"]+Dur.Mat.112[1,"sigma"]*
yJohnsonDistribution(z,Dur.Mat.112[1,"ITYPE"],Dur.Mat.112[1,"
GAMMA"],Dur.Mat.112[1,"DELTA"],Dur.Mat.112[1,"XLAM"],Dur.Mat
.112[1,"XI"])
price.CMM<-CMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.112[1,"Dur"])
329 price.CMM.RN<-CMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
Dur.Mat.112[1,"Dur"])
price.QMM<-QMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.112[1,"Dur"])

```

```

331     price.QMM.RN<-QMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
Dur.Mat.112[1,"Dur"])
     price.CP<-CP.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.112[1,"Dur"])
333     price.CP.RN<-CP.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.
Mat.112[1,"Dur"])
     price.BS<-BS.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.112[1,"Dur"])
335     price.mat.112[(i+(j-1)*100+(k-1)*500+(l-1)*1500),]<-c(price
.CMM,price.CMM.RN,price.QMM,price.QMM.RN,price.CP,price.CP.RN,
price.BS,j,k,l,1,1,2)
     }
337   }
339 }

341 #Matrix for Changing Duration, Constant Variance [1], Zero Skewness
[2], Mesokurtic [1]
343 Dur.1m.121<-c(Dur.vec[1],Mean.vec[1],SD.vec[1],
FitJohnsonDistribution(0,1,0,3))
Dur.3m.121<-c(Dur.vec[2],Mean.vec[2],SD.vec[2],
FitJohnsonDistribution(0,1,0,3))
345 Dur.6m.121<-c(Dur.vec[3],Mean.vec[3],SD.vec[3],
FitJohnsonDistribution(0,1,0,3))
Dur.Mat.121<-matrix(0,nrow=3,ncol=9)
347 Dur.Mat.121[1,]<-Dur.1m.121
Dur.Mat.121[2,]<-Dur.3m.121
349 Dur.Mat.121[3,]<-Dur.6m.121
colnames(Dur.Mat.121)<-c("Dur","mu","sigma","ITYPE","GAMMA","DELTA",
"XLAM","XI","IFAULT")
351 ##Simulation Skeleton
###K.star.vec[j] uses "j" as index
353 ###ra.vec[k] uses "k" as index
###Dur.Mat[1,] uses "l" as index
355 set.seed(n.seed)
price.mat.121<-matrix(0,nrow=(4500),ncol=13)
357 colnames(price.mat.121)<-c("CMM","CMM.RN","QMM","QMM.RN","CP","CP.
RN","BS","K*","R_A","Dur","Var","Skew","Kurt")
for (l in 1:3) {
359   for (k in 1:3) {
     for (j in 1:5) {
361       for(i in 1:100) {
         z<-rnorm(1260)
363         u<-Dur.Mat.121[1,"mu"]+Dur.Mat.121[1,"sigma"]*
yJohnsonDistribution(z,Dur.Mat.121[1,"ITYPE"],Dur.Mat.121[1,"
GAMMA"],Dur.Mat.121[1,"DELTA"],Dur.Mat.121[1,"XLAM"],Dur.Mat
.121[1,"XI"])
         price.CMM<-CMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.121[1,"Dur"])
365         price.CMM.RN<-CMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
Dur.Mat.121[1,"Dur"])
         price.QMM<-QMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.121[1,"Dur"])
367         price.QMM.RN<-QMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
Dur.Mat.121[1,"Dur"])

```

```

    price.CP<-CP.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.121[1,"Dur"])
369 price.CP.RN<-CP.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.
Mat.121[1,"Dur"])
    price.BS<-BS.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.121[1,"Dur"])
371 price.mat.121[(i+(j-1)*100+(k-1)*500+(1-1)*1500),]<-c(price
.CMM,price.CMM.RN,price.QMM,price.QMM.RN,price.CP,price.CP.RN,
price.BS,j,k,1,1,2,1)
  }
373 }
}
375 }
##Matrix for Changing Duration, Constant Variance [1], Zero Skewness
[2], Leptokurtic [2]
377 Dur.1m.122<-c(Dur.vec[1],Mean.vec[1],SD.vec[1],
FitJohnsonDistribution(0,1,0,Kurt.vec[1]))
Dur.3m.122<-c(Dur.vec[2],Mean.vec[2],SD.vec[2],
FitJohnsonDistribution(0,1,0,Kurt.vec[2]))
379 Dur.6m.122<-c(Dur.vec[3],Mean.vec[3],SD.vec[3],
FitJohnsonDistribution(0,1,0,Kurt.vec[3]))
Dur.Mat.122<-matrix(0,nrow=3,ncol=9)
381 Dur.Mat.122[1,]<-Dur.1m.122
Dur.Mat.122[2,]<-Dur.3m.122
383 Dur.Mat.122[3,]<-Dur.6m.122
colnames(Dur.Mat.122)<-c("Dur","mu","sigma","ITYPE","GAMMA","DELTA"
,"XLAM","XI","IFAULT")
385 ##Simulation Skeleton
###K.star.vec[j] uses "j" as index
387 ###ra.vec[k] uses "k" as index
###Dur.Mat[1,] uses "1" as index
389 set.seed(n.seed)
price.mat.122<-matrix(0,nrow=(4500),ncol=13)
391 colnames(price.mat.122)<-c("CMM","CMM.RN","QMM","QMM.RN","CP","CP.
RN","BS","K*","R_A","Dur","Var","Skew","Kurt")
for (l in 1:3) {
393 for (k in 1:3) {
for (j in 1:5) {
395 for (i in 1:100) {
z<-rnorm(1260)
397 u<-Dur.Mat.122[1,"mu"]+Dur.Mat.122[1,"sigma"]*
yJohnsonDistribution(z,Dur.Mat.122[1,"ITYPE"],Dur.Mat.122[1,"
GAMMA"],Dur.Mat.122[1,"DELTA"],Dur.Mat.122[1,"XLAM"],Dur.Mat
.122[1,"XI"])
price.CMM<-CMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.122[1,"Dur"])
399 price.CMM.RN<-CMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
Dur.Mat.122[1,"Dur"])
price.QMM<-QMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.122[1,"Dur"])
401 price.QMM.RN<-QMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
Dur.Mat.122[1,"Dur"])
price.CP<-CP.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.122[1,"Dur"])
403 price.CP.RN<-CP.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.
Mat.122[1,"Dur"])

```

```

    price.BS<-BS.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.122[1,"Dur"])
405   price.mat.122[(i+(j-1)*100+(k-1)*500+(l-1)*1500),]<-c(price
.CMM,price.CMM.RN,price.QMM,price.QMM.RN,price.CP,price.CP.RN,
price.BS,j,k,l,1,2,2)
    }
407 }
409 }

411 #Matrix for Changing Duration, Constant Variance [1], Positive
Skewness[3], Mesokurtic[1]
413 Dur.1m.131<-c(Dur.vec[1],Mean.vec[1],SD.vec[1],
FitJohnsonDistribution(0,1,-Skew.vec[1],3))
Dur.3m.131<-c(Dur.vec[2],Mean.vec[2],SD.vec[2],
FitJohnsonDistribution(0,1,-Skew.vec[2],3))
415 Dur.6m.131<-c(Dur.vec[3],Mean.vec[3],SD.vec[3],
FitJohnsonDistribution(0,1,-Skew.vec[3],3))
Dur.Mat.131<-matrix(0,nrow=3,ncol=9)
417 Dur.Mat.131[1,]<-Dur.1m.131
Dur.Mat.131[2,]<-Dur.3m.131
419 Dur.Mat.131[3,]<-Dur.6m.131
colnames(Dur.Mat.131)<-c("Dur","mu","sigma","ITYPE","GAMMA","DELTA",
"XLAM","XI","IFAULT")
421 ##Simulation Skeleton
###K.star.vec[j] uses "j" as index
423 ###ra.vec[k] uses "k" as index
###Dur.Mat[l,] uses "l" as index
425 set.seed(n.seed)
price.mat.131<-matrix(0,nrow=(4500),ncol=13)
427 colnames(price.mat.131)<-c("CMM","CMM.RN","QMM","QMM.RN","CP","CP.
RN","BS","K*","R.A","Dur","Var","Skew","Kurt")
for (l in 1:3) {
429   for (k in 1:3) {
     for (j in 1:5) {
431       for (i in 1:100) {
         z<-rnorm(1260)
433         u<-Dur.Mat.131[1,"mu"]+Dur.Mat.131[1,"sigma"]*
yJohnsonDistribution(z,Dur.Mat.131[1,"ITYPE"],Dur.Mat.131[1,"
GAMMA"],Dur.Mat.131[1,"DELTA"],Dur.Mat.131[1,"XLAM"],Dur.Mat
.131[1,"XI"])
         price.CMM<-CMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.131[1,"Dur"])
435         price.CMM.RN<-CMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
Dur.Mat.131[1,"Dur"])
         price.QMM<-QMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.131[1,"Dur"])
437         price.QMM.RN<-QMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
Dur.Mat.131[1,"Dur"])
         price.CP<-CP.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.131[1,"Dur"])
439         price.CP.RN<-CP.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.
Mat.131[1,"Dur"])
         price.BS<-BS.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.131[1,"Dur"])

```



```

441     price.mat.131[(i+(j-1)*100+(k-1)*500+(1-1)*1500),]<-c(price
      .CMM,price.CMM.RN,price.QMM,price.QMM.RN,price.CP,price.CP.RN,
      price.BS,j,k,1,1,3,1)
    }
443  }
445 }

447 #Matrix for Changing Duration, Constant Variance [1], Positive
      Skewness [3], Leptokurtic [2]
Dur.1m.132<-c(Dur.vec[1],Mean.vec[1],SD.vec[1],
  FitJohnsonDistribution(0,1,-Skew.vec[1],Kurt.vec[1]))
449 Dur.3m.132<-c(Dur.vec[2],Mean.vec[2],SD.vec[2],
  FitJohnsonDistribution(0,1,-Skew.vec[2],Kurt.vec[2]))
Dur.6m.132<-c(Dur.vec[3],Mean.vec[3],SD.vec[3],
  FitJohnsonDistribution(0,1,-Skew.vec[3],Kurt.vec[3]))
451 Dur.Mat.132<-matrix(0,nrow=3,ncol=9)
Dur.Mat.132[1,]<-Dur.1m.132
453 Dur.Mat.132[2,]<-Dur.3m.132
Dur.Mat.132[3,]<-Dur.6m.132
455 colnames(Dur.Mat.132)<-c("Dur","mu","sigma","ITYPE","GAMMA","DELTA",
  "XLAM","XI","IFAULT")
##Simulation Skeleton
457 ###K.star.vec[j] uses "j" as index
###ra.vec[k] uses "k" as index
459 ###Dur.Mat[1,] uses "1" as index
set.seed(n.seed)
461 price.mat.132<-matrix(0,nrow=(4500),ncol=13)
colnames(price.mat.132)<-c("CMM","CMM.RN","QMM","QMM.RN","CP","CP.
  RN","BS","K*","R.A","Dur","Var","Skew","Kurt")
463 for (l in 1:3) {
  for (k in 1:3) {
465     for (j in 1:5) {
      for (i in 1:100) {
467         z<-rnorm(1260)
         u<-Dur.Mat.132[1,"mu"]+Dur.Mat.132[1,"sigma"]*
         yJohnsonDistribution(z,Dur.Mat.132[1,"ITYPE"],Dur.Mat.132[1,"
         GAMMA"],Dur.Mat.132[1,"DELTA"],Dur.Mat.132[1,"XLAM"],Dur.Mat
         .132[1,"XI"])
469         price.CMM<-CMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
         .132[1,"Dur"])
         price.CMM.RN<-CMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
         Dur.Mat.132[1,"Dur"])
471         price.QMM<-QMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
         .132[1,"Dur"])
         price.QMM.RN<-QMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
         Dur.Mat.132[1,"Dur"])
473         price.CP<-CP.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
         .132[1,"Dur"])
         price.CP.RN<-CP.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.
         Mat.132[1,"Dur"])
475         price.BS<-BS.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
         .132[1,"Dur"])
         price.mat.132[(i+(j-1)*100+(k-1)*500+(1-1)*1500),]<-c(price
         .CMM,price.CMM.RN,price.QMM,price.QMM.RN,price.CP,price.CP.RN,
         price.BS,j,k,1,1,3,2)
477     }
  }
}

```

```

479   }
480 }
481
482 #Mus for Nonconstant Variance Cases
483 Mu.mat<-matrix(0,nrow=3,ncol=1260)
484 Mu.mat[1,]<-fitted(garch.fit.21)
485 Mu.mat[2,]<-fitted(garch.fit.63)
486 Mu.mat[3,]<-fitted(garch.fit.126)
487
488 #Sigmas for Nonconstant Variance Cases
489 Sigma.mat<-matrix(0,nrow=3,ncol=1260)
490 Sigma.mat[1,]<-sigma(garch.fit.21)
491 Sigma.mat[2,]<-sigma(garch.fit.63)
492 Sigma.mat[3,]<-sigma(garch.fit.126)
493
494 #Matrix for Changing Duration, GARCH Variance [2], Negative
495   Skewness[1], Mesokurtic[1]
496 Dur.1m.211<-c(Dur.vec[1], FitJohnsonDistribution(0,1,Skew.vec[1],3)
497 )
498 Dur.3m.211<-c(Dur.vec[2], FitJohnsonDistribution(0,1,Skew.vec[2],3)
499 )
500 Dur.6m.211<-c(Dur.vec[3], FitJohnsonDistribution(0,1,Skew.vec[3],3)
501 )
502 Dur.Mat.211<- matrix(0,nrow=3,ncol=7)
503 Dur.Mat.211[1,]<-Dur.1m.211
504 Dur.Mat.211[2,]<-Dur.3m.211
505 Dur.Mat.211[3,]<-Dur.6m.211
506 colnames(Dur.Mat.211)<-c("Dur", "ITYPE", "GAMMA", "DELTA", "XLAM", "XI",
507 "IFAULT")
508
509 ##Simulation Skeleton
510 ###K.star.vec[j] uses "j" as index
511 ###ra.vec[k] uses "k" as index
512 ###Dur.Mat[1,] uses "1" as index
513 set.seed(n.seed)
514 price.mat.211<-matrix(0,nrow=(4500),ncol=13)
515 colnames(price.mat.211)<-c("CMM", "CMM.RN", "QMM", "QMM.RN", "CP", "CP.
516 RN", "BS", "K*", "R_A", "Dur", "Var", "Skew", "Kurt")
517 for (l in 1:3) {
518   for (k in 1:3) {
519     for (j in 1:5) {
520       for (i in 1:100) {
521         z<-rnorm(1260)
522         u<-Mu.mat[1,]+Sigma.mat[1,]*yJohnsonDistribution(z,Dur.Mat
523 .211[1, "ITYPE"], Dur.Mat.211[1, "GAMMA"], Dur.Mat.211[1, "DELTA"],
524 Dur.Mat.211[1, "XLAM"], Dur.Mat.211[1, "XI"])
525         price.CMM<-CMM.Price.Adj(u,K.star.vec[j], ra.vec[k], Dur.Mat
526 .211[1, "Dur"])
527         price.CMM.RN<-CMM.RN.Price.Adj(u,K.star.vec[j], ra.vec[k],
528 Dur.Mat.211[1, "Dur"])
529         price.QMM<-QMM.Price.Adj(u,K.star.vec[j], ra.vec[k], Dur.Mat
530 .211[1, "Dur"])
531         price.QMM.RN<-QMM.RN.Price.Adj(u,K.star.vec[j], ra.vec[k],
532 Dur.Mat.211[1, "Dur"])
533         price.CP<-CP.Price.Adj(u,K.star.vec[j], ra.vec[k], Dur.Mat
534 .211[1, "Dur"])

```

```

    price.CP.RN<-CP.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.
523 Mat.211[1,"Dur"])
    price.BS<-BS.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
    .211[1,"Dur"])
    price.mat.211[(i+(j-1)*100+(k-1)*500+(l-1)*1500),]<-c(price
    .CMM,price.CMM.RN,price.QMM,price.QMM.RN,price.CP,price.CP.RN,
525 price.BS,j,k,l,2,1,1)
  }
}
527 }
529 }
##Matrix for Changing Duration, GARCH Variance [2], Negative
Skewness[1], Leptokurtic [2]
531 Dur.1m.212<-c(Dur.vec[1], FitJohnsonDistribution(0,1,Skew.vec[1],
Kurt.vec[1]))
Dur.3m.212<-c(Dur.vec[2], FitJohnsonDistribution(0,1,Skew.vec[2],
Kurt.vec[2]))
533 Dur.6m.212<-c(Dur.vec[3], FitJohnsonDistribution(0,1,Skew.vec[3],
Kurt.vec[3]))
Dur.Mat.212<- matrix(0,nrow=3,ncol=7)
535 Dur.Mat.212[1,]<-Dur.1m.212
Dur.Mat.212[2,]<-Dur.3m.212
537 Dur.Mat.212[3,]<-Dur.6m.212
colnames(Dur.Mat.212)<-c("Dur","ITYPE","GAMMA","DELTA","XLAM","XI",
"IFAUULT")
539 ##Simulation Skeleton
###K.star.vec[j] uses "j" as index
541 ###ra.vec[k] uses "k" as index
###Dur.Mat[1,] uses "l" as index
543 set.seed(n.seed)
price.mat.212<-matrix(0,nrow=(4500),ncol=13)
545 colnames(price.mat.212)<-c("CMM","CMM.RN","QMM","QMM.RN","CP","CP.
RN","BS","K*","R.A","Dur","Var","Skew","Kurt")
for (l in 1:3) {
547   for (k in 1:3) {
     for (j in 1:5) {
549       for (i in 1:100) {
         z<-rnorm(1260)
551         u<-Mu.mat[1,]+Sigma.mat[1,]*yJohnsonDistribution(z,Dur.Mat
.212[1,"ITYPE"],Dur.Mat.212[1,"GAMMA"],Dur.Mat.212[1,"DELTA"],
Dur.Mat.212[1,"XLAM"],Dur.Mat.212[1,"XI"])
         price.CMM<-CMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.212[1,"Dur"])
553         price.CMM.RN<-CMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
Dur.Mat.212[1,"Dur"])
         price.QMM<-QMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.212[1,"Dur"])
555         price.QMM.RN<-QMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
Dur.Mat.212[1,"Dur"])
         price.CP<-CP.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.212[1,"Dur"])
557         price.CP.RN<-CP.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.
Mat.212[1,"Dur"])
         price.BS<-BS.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.212[1,"Dur"])

```

```

559     price.mat.212[(i+(j-1)*100+(k-1)*500+(1-1)*1500),]<-c(price
      .CMM,price.CMM.RN,price.QMM,price.QMM.RN,price.CP,price.CP.RN,
      price.BS,j,k,1,2,1,2)
    }
561  }
563 }
565 #Matrix for Changing Duration, GARCH Variance [2], Zero Skewness
      [2], Mesokurtic [1]
567 Dur.1m.221<-c(Dur.vec[1], FitJohnsonDistribution(0,1,0,3))
568 Dur.3m.221<-c(Dur.vec[2], FitJohnsonDistribution(0,1,0,3))
569 Dur.6m.221<-c(Dur.vec[3], FitJohnsonDistribution(0,1,0,3))
570 Dur.Mat.221<- matrix(0,nrow=3,ncol=7)
571 Dur.Mat.221[1,]<-Dur.1m.221
572 Dur.Mat.221[2,]<-Dur.3m.221
573 Dur.Mat.221[3,]<-Dur.6m.221
574 colnames(Dur.Mat.221)<-c("Dur","ITYPE","GAMMA","DELTA","XLAM","XI",
      "IFAULT")
575 ##Simulation Skeleton
576 ###K.star.vec[j] uses "j" as index
577 ###ra.vec[k] uses "k" as index
578 ###Dur.Mat[1,] uses "1" as index
579 set.seed(n.seed)
580 price.mat.221<-matrix(0,nrow=(4500),ncol=13)
581 colnames(price.mat.221)<-c("CMM","CMM.RN","QMM","QMM.RN","CP","CP.
      RN","BS","K*","R.A","Dur","Var","Skew","Kurt")
582 for (l in 1:3) {
583   for (k in 1:3) {
584     for (j in 1:5) {
585       for (i in 1:100) {
586         z<-rnorm(1260)
587         u<-Mu.mat[1,]+Sigma.mat[1,]*yJohnsonDistribution(z,Dur.Mat
          .221[1,"ITYPE"],Dur.Mat.221[1,"GAMMA"],Dur.Mat.221[1,"DELTA"],
          Dur.Mat.221[1,"XLAM"],Dur.Mat.221[1,"XI"])
588         price.CMM<-CMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
          .221[1,"Dur"])
589         price.CMM.RN<-CMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
          Dur.Mat.221[1,"Dur"])
590         price.QMM<-QMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
          .221[1,"Dur"])
591         price.QMM.RN<-QMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
          Dur.Mat.221[1,"Dur"])
592         price.CP<-CP.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
          .221[1,"Dur"])
593         price.CP.RN<-CP.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.
          Mat.221[1,"Dur"])
594         price.BS<-BS.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
          .221[1,"Dur"])
595         price.mat.221[(i+(j-1)*100+(k-1)*500+(1-1)*1500),]<-c(price
          .CMM,price.CMM.RN,price.QMM,price.QMM.RN,price.CP,price.CP.RN,
          price.BS,j,k,1,2,2,1)
596       }
597     }
598   }
599 }

```

```

#Matrix for Changing Duration, GARCH Variance [2], Zero Skewness
[2], Leptokurtic[2]
601 Dur.1m.222<-c(Dur.vec[1], FitJohnsonDistribution(0,1,0,Kurt.vec[1])
)
Dur.3m.222<-c(Dur.vec[2], FitJohnsonDistribution(0,1,0,Kurt.vec[2])
)
603 Dur.6m.222<-c(Dur.vec[3], FitJohnsonDistribution(0,1,0,Kurt.vec[3])
)
Dur.Mat.222<- matrix(0,nrow=3,ncol=7)
605 Dur.Mat.222[1,]<-Dur.1m.222
Dur.Mat.222[2,]<-Dur.3m.222
607 Dur.Mat.222[3,]<-Dur.6m.222
colnames(Dur.Mat.222)<-c("Dur","ITYPE","GAMMA","DELTA","XLAM","XI",
"IFault")
609 ##Simulation Skeleton
###K.star.vec[j] uses "j" as index
611 ###ra.vec[k] uses "k" as index
###Dur.Mat[l,] uses "l" as index
613 set.seed(n.seed)
price.mat.222<-matrix(0,nrow=(4500),ncol=13)
615 colnames(price.mat.222)<-c("CMM","CMM.RN","QMM","QMM.RN","CP","CP.
RN","BS","K*","R.A","Dur","Var","Skew","Kurt")
for (l in 1:3) {
617 for (k in 1:3) {
for (j in 1:5) {
619 for (i in 1:100) {
z<-rnorm(1260)
621 u<-Mu.mat[1,]+Sigma.mat[1,]*yJohnsonDistribution(z,Dur.Mat
.222[1,"ITYPE"],Dur.Mat.222[1,"GAMMA"],Dur.Mat.222[1,"DELTA"],
Dur.Mat.222[1,"XLAM"],Dur.Mat.222[1,"XI"])
price.CMM<-CMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.222[1,"Dur"])
623 price.CMM.RN<-CMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
Dur.Mat.222[1,"Dur"])
price.QMM<-QMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.222[1,"Dur"])
625 price.QMM.RN<-QMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
Dur.Mat.222[1,"Dur"])
price.CP<-CP.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.222[1,"Dur"])
627 price.CP.RN<-CP.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.
Mat.222[1,"Dur"])
price.BS<-BS.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.222[1,"Dur"])
629 price.mat.222[(i+(j-1)*100+(k-1)*500+(l-1)*1500),]<-c(price
.CMM,price.CMM.RN,price.QMM,price.QMM.RN,price.CP,price.CP.RN,
price.BS,j,k,l,2,2,2)
}
}
631 }
}
633 }

635 #Matrix for Changing Duration, GARCH Variance [2], Positive
Skewness[3], Mesokurtic[1]
637 Dur.1m.231<-c(Dur.vec[1], FitJohnsonDistribution(0,1,-Skew.vec
[1],3))

```

```

Dur.3m.231<-c(Dur.vec[2], FitJohnsonDistribution(0,1,-Skew.vec
[2],3))
639 Dur.6m.231<-c(Dur.vec[3], FitJohnsonDistribution(0,1,-Skew.vec
[3],3))
Dur.Mat.231<- matrix(0,nrow=3,ncol=7)
641 Dur.Mat.231[1,]<-Dur.1m.231
Dur.Mat.231[2,]<-Dur.3m.231
643 Dur.Mat.231[3,]<-Dur.6m.231
colnames(Dur.Mat.231)<-c("Dur","ITYPE","GAMMA","DELTA","XLAM","XI",
"IFault")
645 ##Simulation Skeleton
###K.star.vec[j] uses "j" as index
647 ###ra.vec[k] uses "k" as index
###Dur.Mat[1,] uses "1" as index
649 set.seed(n.seed)
price.mat.231<-matrix(0,nrow=(4500),ncol=13)
651 colnames(price.mat.231)<-c("CMM","CMM.RN","QMM","QMM.RN","CP","CP.
RN","BS","K*","R_A","Dur","Var","Skew","Kurt")
for (l in 1:3) {
653   for (k in 1:3) {
     for (j in 1:5) {
655       for (i in 1:100) {
         z<-rnorm(1260)
657         u<-Mu.mat[1,]+Sigma.mat[1,]*yJohnsonDistribution(z,Dur.Mat
.231[1,"ITYPE"],Dur.Mat.231[1,"GAMMA"],Dur.Mat.231[1,"DELTA"],
Dur.Mat.231[1,"XLAM"],Dur.Mat.231[1,"XI"])
         price.CMM<-CMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.231[1,"Dur"])
659         price.CMM.RN<-CMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
Dur.Mat.231[1,"Dur"])
         price.QMM<-QMM.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.231[1,"Dur"])
661         price.QMM.RN<-QMM.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],
Dur.Mat.231[1,"Dur"])
         price.CP<-CP.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.231[1,"Dur"])
663         price.CP.RN<-CP.RN.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.
Mat.231[1,"Dur"])
         price.BS<-BS.Price.Adj(u,K.star.vec[j],ra.vec[k],Dur.Mat
.231[1,"Dur"])
665         price.mat.231[(i+(j-1)*100+(k-1)*500+(l-1)*1500),]<-c(price
.CMM,price.CMM.RN,price.QMM,price.QMM.RN,price.CP,price.CP.RN,
price.BS,j,k,l,2,3,1)
       }
     }
667   }
669 }

671 #Matrix for Changing Duration, GARCH Variance [2], Positive
Skewness[3], Leptokurtic [2]
Dur.1m.232<-c(Dur.vec[1], FitJohnsonDistribution(0,1,-Skew.vec[1],
Kurt.vec[1]))
673 Dur.3m.232<-c(Dur.vec[2], FitJohnsonDistribution(0,1,-Skew.vec[2],
Kurt.vec[2]))
Dur.6m.232<-c(Dur.vec[3], FitJohnsonDistribution(0,1,-Skew.vec[3],
Kurt.vec[3]))
675 Dur.Mat.232<- matrix(0,nrow=3,ncol=7)

```

```

Dur.Mat.232[1,]<-Dur.1m.232
677 Dur.Mat.232[2,]<-Dur.3m.232
Dur.Mat.232[3,]<-Dur.6m.232
679 colnames(Dur.Mat.232)<-c("Dur", "ITYPE", "GAMMA", "DELTA", "XLAM", "XI",
"IFAULT")
##Simulation Skeleton
681 ###K.star.vec[j] uses "j" as index
###ra.vec[k] uses "k" as index
683 ###Dur.Mat[1,] uses "1" as index
set.seed(n.seed)
685 price.mat.232<-matrix(0,nrow=(4500),ncol=13)
colnames(price.mat.232)<-c("CMM", "CMM.RN", "QMM", "QMM.RN", "CP", "CP.
RN", "BS", "K*", "R.A", "Dur", "Var", "Skew", "Kurt")
687 for (l in 1:3) {
for (k in 1:3) {
689 for (j in 1:5) {
for (i in 1:100) {
691 z<-rnorm(1260)
u<-Mu.mat[1,]+Sigma.mat[1,]*y.JohnsonDistribution(z,Dur.Mat
.232[1, "ITYPE"], Dur.Mat.232[1, "GAMMA"], Dur.Mat.232[1, "DELTA"],
Dur.Mat.232[1, "XLAM"], Dur.Mat.232[1, "XI"])
693 price.CMM<-CMM.Price.Adj(u,K.star.vec[j], ra.vec[k], Dur.Mat
.232[1, "Dur"])
price.CMM.RN<-CMM.RN.Price.Adj(u,K.star.vec[j], ra.vec[k],
Dur.Mat.232[1, "Dur"])
695 price.QMM<-QMM.Price.Adj(u,K.star.vec[j], ra.vec[k], Dur.Mat
.232[1, "Dur"])
price.QMM.RN<-QMM.RN.Price.Adj(u,K.star.vec[j], ra.vec[k],
Dur.Mat.232[1, "Dur"])
697 price.CP<-CP.Price.Adj(u,K.star.vec[j], ra.vec[k], Dur.Mat
.232[1, "Dur"])
price.CP.RN<-CP.RN.Price.Adj(u,K.star.vec[j], ra.vec[k], Dur.
Mat.232[1, "Dur"])
699 price.BS<-BS.Price.Adj(u,K.star.vec[j], ra.vec[k], Dur.Mat
.232[1, "Dur"])
price.mat.232[(i+(j-1)*100+(k-1)*500+(l-1)*1500),]<-c(price
.CMM, price.CMM.RN, price.QMM, price.QMM.RN, price.CP, price.CP.RN,
price.BS, j, k, l, 2, 3, 2)
701 }
}
703 }
}
705
price.mat<-matrix(0,nrow=4500*12,ncol=13)
707 colnames(price.mat)<-c("CMM", "CMM.RN", "QMM", "QMM.RN", "CP", "CP.RN", "
BS", "K*", "R.A", "Dur", "Var", "Skew", "Kurt")
price.mat[1:4500,]<-price.mat.111
709 price.mat[4501:9000,]<-price.mat.112
price.mat[9001:13500,]<-price.mat.121
711 price.mat[13501:18000,]<-price.mat.122
price.mat[18001:22500,]<-price.mat.131
713 price.mat[22501:27000,]<-price.mat.132
price.mat[27001:31500,]<-price.mat.211
715 price.mat[31501:36000,]<-price.mat.212
price.mat[36001:40500,]<-price.mat.221
717 price.mat[40501:45000,]<-price.mat.222
price.mat[45001:49500,]<-price.mat.231

```

```
719 price.mat[49501:54000,]<-price.mat.232  
721 write.table(price.mat,"Price_Mat.csv")
```

SimulationProg.R