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Socially optimal Nash equilibrium locations and privatization in a model of spatial duopoly with price discrimination

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Abstract

We generalize Beladi et al. (2014) for any non-negative, increasing, continuous function of distance as transportation costs function. By doing so, we show that in a duopoly, partial privatization does not change the socially optimal character of the Nash equilibrium location. Our results call for further research on testing their robustness under the existence of more than two competing firms.

JEL classification: L13; L32; R32
Keywords: Privatization; Spatial competition; Transportation costs

1 Introduction

Beladi et al. (2014) showed that in a duopoly with spatial price discrimination and linear transportation costs where one of the firms is partly publicly owned, firms' Nash equilibrium locations are socially optimal. Moreover, they conclude that the degree of privatization does not affect the equilibrium locations of the firms. In this note, we generalize Beladi et al. (2014) results for any non-negative, increasing, continuous transportation costs function and we reveal the driving force behind these results.

2 Model and results

Our setting follows that of Braid (2008). We consider a duopoly, with a continuum of consumers uniformly distributed over the interval $[0, 1]$ of a linear city. Three products are

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offered to consumers; J and K from firm D_1 and K and L from firm D_2. Let the fraction of consumers buying only good J equal that of those buying only good L equal to c. Product K is bought by a fraction b of consumers. The above assumptions imply that the two firms have monopoly power over the goods J and L. Let k denote the maximum reservation price that the consumers are willing to pay for a good. Evidently, D_1 and D_2 will charge a uniform price infinitesimally below k for J and L. Spatial price discrimination à la Lerner and Singer (1937) is assumed regarding product K. The location of D_1 and D_2 over the interval $[0, 1]$ is x and y, respectively. D_1 is privately owned whereas D_2 is partly privately owned and partly publicly owned in proportions a and $1-a$, respectively with $a \in [0, 1]$. Transportation costs are equal to $tf(d)$, where t is a positive constant, d is the distance shipped and f any non-negative, increasing, continuous function. D_1 and D_2 simultaneously choose their location in the market.

The aggregate shipping distance is equal to

$$\begin{aligned} T(x,y) = {} & c\left(\int_0^x f(x-z)dz + \int_x^1 f(z-x)dz\right) + b\left(\int_0^x f(x-z)dz + \int_x^{\left(\frac{x+y}{2}\right)} f(z-x)dz\right) \\ & + b\left(\int_{\left(\frac{x+y}{2}\right)}^y f(y-z)dz + \int_y^1 f(z-y)dz\right) \\ & + c\left(\int_0^y f(y-z)dz + \int_y^1 f(z-y)dz\right) \end{aligned} \tag{1}$$

In order to find the socially optimal Nash equilibrium locations we have to minimize (1) with respect to x and y. Hence, the socially optimal locations satisfy the first order conditions:

$$\frac{\partial T(x,y)}{\partial x} = cf(x) - cf(1-x) + bf(x) - bf(\frac{y-x}{2}) = 0 \tag{2}$$

$$\frac{\partial T(x,y)}{\partial y} = bf(\frac{y-x}{2}) - bf(1-y) + cf(y) - cf(1-y) = 0 \tag{3}$$

Following Braid (2008), the profit functions of D_1 and D_2 when both firms are privately owned (i.e. when $a = 1$) are:

$$\begin{aligned}\Pi_{D_1}(x,y) &= ck - ct\left(\int_0^x f(x-z)dz + \int_x^1 f(z-x)dz\right) \\ &\quad + bt\int_0^x [f(y-z)-f(x-z)]dz + bt\int_x^{\left(\frac{x+y}{2}\right)} [f(y-z)-f(z-x)]dz \quad (4)\end{aligned}$$

$$\begin{aligned}\Pi_{D_2}(x,y) &= ck - ct\left(\int_0^y f(y-z)dz + \int_y^1 f(z-y)dz\right) \\ &\quad + bt\int_{\left(\frac{x+y}{2}\right)}^y [f(z-x)-f(y-z)]dz + bt\int_y^1 [f(z-x)-f(z-y)]dz \quad (5)\end{aligned}$$

Therefore, the Nash equilibrium locations when both firms are privately owned is given by the solution of the following system of equations:

$$\frac{\partial \Pi_{D_1}(x,y)}{\partial x} = -ct[f(x)-f(1-x)] + bt[f(y-x)-f(x)] + bt[f(\frac{y-x}{2}) - f(y-x)] = 0 \quad (6)$$

$$\frac{\partial \Pi_{D_2}(x,y)}{\partial y} = -ct[f(y)-f(1-y)] - btf(\frac{y-x}{2}) + btf(1-y) = 0 \quad (7)$$

Following Beladi et al. (2014), when $a \in [0,1)$, the profit function of D_2 is

$$\begin{aligned}\hat{\Pi}_{D_2}(x,y) &= ck - ct\left(\int_0^y f(y-z)dz + \int_y^1 f(z-y)dz\right) \\ &\quad + bt\int_{\left(\frac{x+y}{2}\right)}^y [f(z-x)-f(y-z)]dz \\ &\quad + bt\int_y^1 [f(z-x)-f(z-y)]dz + (1-a)g(x,y) \quad (8)\end{aligned}$$

where

$$
\begin{aligned}
g(x,y) &= ck - ct\left(\int_0^x f(x-z)dz + \int_x^1 f(z-x)dz\right) \\
&\quad +bt\int_0^x [f(y-z)-f(x-z)]dz \\
&\quad +bt\int_x^{\left(\frac{x+y}{2}\right)} [f(y-z)-f(z-x)]dz + b\int_0^{\left(\frac{x+y}{2}\right)} [k-tf(y-z)]dz \\
&\quad +b\int_{\left(\frac{x+y}{2}\right)}^1 [k-tf(z-x)]dz \\
&= ck - ct\left(\int_0^x f(x-z)dz + \int_x^1 f(z-x)dz\right) \\
&\quad -bt\int_0^x f(x-z)dz - bt\int_x^1 f(z-x)dz + b\int_0^1 kdz \qquad (9)
\end{aligned}
$$

The Nash equilibrium locations under $a \in [0,1)$ satisfy (6) and

$$\frac{\partial \hat{\Pi}_{D_2}(x,y)}{\partial y} = 0 \qquad (10)$$

However, since (9) does not depend on y, $\frac{\partial \hat{\Pi}_{D_2}(x,y)}{\partial y} = \frac{\partial \Pi_{D_2}(x,y)}{\partial y}$.

It can be easily noted that the systems of (2) and (3), (6) and (7) and (6) and (10) are equivalent and therefore have the same solution.

The above analysis leads to the following propositions:

Proposition 1 *The Nash equilibrium locations for $a \in [0,1]$ are socially optimal under any non-negative, increasing, continuous transportation costs function.*

Proposition 2 *The degree of privatization does not affect the socially optimal Nash equilibrium locations under any non-negative, increasing, continuous transportation costs function.*

Proposition 3 *The Nash equilibrium locations for $a \in [0,1)$ are equal to those for $a = 1$ under any non-negative, increasing, continuous transportation costs function.*

Propositions 1, 2 and 3 apart from proving the results obtained by Braid (2008) and Beladi et al. (2014) at the same time in complete generality, they establish, most importantly, their independence from the linear nature of the original model.

The key observation behind the invariance of the socially optimal Nash equilibrium locations when firm D_2 is partly privatized is that the summand accounting for the welfare in its profit function, $\hat{\Pi}_{D_2}(x, y)$, is, in fact, independent of its location y regardless of the degree of privatization a.

Putting together the above results with the findings by Cremer et al. (1991), it emerges that these are duopoly results having nothing to do with the quadratic transportation costs considered in Cremer et al. (1991).[1]

3 Conclusion

We show that Beladi et al. (2014) conclusions are robust for any non-negative, increasing, continuous transportation costs function. Examining the robustness of our findings under a two-dimensional spatial framework with more than two competing firms constitutes a topic for future research.

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[1] Cremer et al. (1991) do not assume a duopoly.