Bank's choice of loan portfolio under high regulation – example of Croatia

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Summary

This paper creates a mathematical model in which the banks are faced with two optimization problems. The first optimization problem is how to optimize their behavior in order to maximize profits. The second optimization is how to optimize the structure of liabilities in order to have minimum regulation. The regulatory regime is imposed by the central bank. This paper investigates the behavior of banks when faced with high regulation and provides a theoretical framework for analysis of the impact of high regulation on the choice of the bank's portfolio structure. The model shows the banks have a learning framework in which the banks learn the central bank's true model and adjust their credit policies to existing regulatory regime. However this adjustment also creates changes in the choice of credit.

JEL Classification: E58, C61, C73, E51

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1. Introduction

Banks have always been an interesting research subject. The main reason for this interest are bank's inherent ties to the business cycles and the fact the banks are the transmission mechanism of monetary policy. Banks are interrelated with the monetary policy and are the ones that actually transfer the effects of monetary policy on to other economic participants, the households and the firms. The object of this paper is to look at how does monetary policy and choice of regulatory regime affect the distribution of credit towards three main bank's clients: households, firms, government[1].

This paper will proceed to model the banks as profit maximizing economic individuals that are faced with dynamic constraints imposed by the central bank. The banks are trying to optimize their behavior under uncertainty. The uncertainty in this paper will come from frequent and erratic changes of regulation imposed by the central bank. The regulation imposed by the central bank can be relatively large as a percentage of the bank's balance sheet. The variation in regulation therefore changes the size of the bank's funds available for lending and depending on the regulation the banks has to adjust their behavior. For the purpose of our model the behavior of banks is in fact: credit policy.

For the purposes of this paper we will considers regulation just on bank's assets. The scope and the size of the regulation are dictated by the structure of the bank's liabilities. In small open economies like Croatia the structure of liabilities can be separated between the currency structure (local currency, foreign currency) and origin of the funds (resident or nonresident funding).

The banks have a similar optimization problem like households. While households try to maximize the utility, which comes from consumption, the banks are faced with optimization on two fronts. The first optimization is the optimization of sector distribution of loans with the purpose to maximize profits. This optimization is focused to which sector in the economy to give loans to: corporate, households or government. In terms of the balance sheet this could be termed the optimization of the assets. The second optimization problem for banks comes from regulation imposed by the central bank and how to create the liabilities structure in order to have as little as possible regulatory burden. In terms of the balance sheet this could be termed the optimization of the liabilities. But when it comes to the optimization of the liabilities the banks are in inferior position since they cannot manipulate the monetary policy decisions that central bank is undertaking. The regulation is imposed on them by the central bank. In this case the banks are faced with the Stackelberg problem.

All of the above elements, taken into consideration, present interesting modeling challenge of the banks and their behavior. The strategic choices in dynamic setting combined with the Stackelberg problem of the interplay with the central bank in a small open economy is the objective of research for this paper. The paper proceeds as follows. Part two of the paper gives a brief overview of the major problems in banking history in Croatia, part three creates a model in which the banks optimize their income. Part four gives a model in which the banks optimize their behavior in respect to the central bank. Part five concludes.

[1] The views expressed herein are those of the authors and do not necessarily reflect the views anyone else.

2. Brief history of banks in Croatia

Banking industry in Croatia met many challenges. These challenges can be separated into internal and external ones.

The externals ones are the interest rate risk and the fact that over 90% of the Croatian banking sector in foreign owned[2]. So there is possibility of external crises to be spilled over onto Croatian banks like it was the case with the start of the crisis in 2009. However this is of a minor concern due to high capitalization and high liquidity of the banking sector. Although the banking sector in Croatia is highly developed and is over 110% of GDP, the Croatian banks are relatively small in comparison to their mothers companies. The best example of the relative small size of the Croatian banking sector and the overall lack of importance is the fact that there was not a single problem with a bank in Croatia due to the problem with mother company.

The internal challenges are two fold, the first problem is the legacy of the past and the second problem is the regulation imposed by the central bank. According to Ribnikar (1999, 1999, 2004) the fact the companies did not have strictly determined capital, but what Ribnikar calls "sort of capital" was one of the main source of the propagating inflation in Yugoslavia. The banks were "forced" give loans as needed by the companies, regardless of the credit risk. At the same time for retail customers it was hard to obtain loans. Because of this the banking sector in Yugoslavia was not entirely stable by today's standards. The purpose of banks was to facilitate the real sector of the economy and not to take heed of the capital and profit standards. As for the retail clients the households were allowed to save in foreign currency, but there was no abundance of retail credit.

Legacy of the past is again two fold. The first element is political and the second one is economical. Croatia was part of Yugoslavia, a socialist closed economy whose banking system depended on domestic funding and domestic sources of money. In Yugoslavia the growth of the bank's balance sheet came from households, which had unusually higher rate of savings and business which were owned by the people[3] as presented in Rohatinski et all. (1995).

The households were perceived as economic elements that produced savings and firms the economic elements that used savings funds obtained by the banks. This circle of taking household's savings and lending it to companies was the basis of the socialist economies, although this economic structure was more related to the basic capitalism. It was Keynes who postulated that S=I in a closed economy, this doctrine was enforced by the socialist system where banks served as a financial service for companies. However the credit risk restrictions were not heavily enforced at the time which caused banks to be the least profitable sector of the economy.

After the dissolution of Yugoslavia the banks were privatized one by one, some of them collapsed under the heavy burden of bad loans and the new banks were formed. In Croatia there were two large banking crises. The first crises were caused by the collapse of the Dubrovačka banka and the second one was caused by the collapse of Riječka banka. It is generally considers Dubrovačka banka collapsed because of the political pressure to give bad loans with doubtful collateral, while Riječka banka collapsed because of the lack of proper

[2] The dana can be found in the Croatian National Bank Bulletin.
[3] This is the closes translation of "Opće društveno vlasništvo ili narodno vlasništvo".

supervision and illegal dealings of Head trader. Banking crisis and the process of privatization were explained in Kraft (2002) and Kraft, Faulend, Tepus, (1998).

The failure of banks, need to patch up budgetary deficits and moral hazard problem has led Government owned Privatization fund to privatize banks in Croatia. Most banks in Croatia were privatized by 2000 mostly to large foreign conglomerates. Today only two banks are still owned by the government; Hrvatska poštanska banka and Croatia Banka which have around 7% of market share together. The number of banks in Croatia is also decreasing, from all time high number of 60 in 1998. to 27 at the end of 2014.

Banks in Croatia also have to contend with the legacy of hyperinflation. In the period from 1992 to 1994 Croatia experienced period of spiraling hyperinflation which reached at one point level of 25% per month. Then in late 1994 a stabilization program was undertaken. The effect of the stabilization program was sudden stop in inflation which stabilized the economy and made possible economic restructuring described in Rohatinski et all (1995).

This turbulent banking past brings us to the second internal challenge the banks face in Croatia: high levels of regulation. The lack of supervision and political pressure led the regulator to be very strict with the banks. Large balance sheet burdens were imposed on the banks through the regulation.

The main regulations are the reserve requirement on domestic and foreign funds (currently 12%), minimal foreign currency short term loans (for liabilities received in foreign currency and foreign currency claims the bank had to maintain at least 17% in strictly determined assets which may not have maturity larger than 3 months). For the overview of the current regulation the best source is the Central bank bulletin and for the evolution of the regulation there is a great description in Galac (2010)[4].

However the theoretical level of the percentage of liabilities that are subject to regulation is from 0% (capital and funds received from other banks in Croatia are not subject to regulation) to 25%. However in the time periods where there was a special regulation on funding obtained from nonresidents the percentage of funds blocked could go as high as 72% of incoming liabilities. Large differences in the amount of possible regulation on funds received, ranging from 0 to 72% of the funds received, makes governance of the regulation inside of the bank's balance sheet of major importance for any bank in Croatia[5]. The regulation presented here refers just to the regulation which affects the structure of the balance sheet. As presented in the overview from Galac (2010) there is also regulation limiting the growth to retail and corporate clients to 12% per year imposed in 2007 and continued in 2008. These changes in the regulation are precisely the object of investigation of this paper and how have changes in the regulation affected bank’s credit policy choices.

3. Dynamic model

In this part of the paper we are going to create a dynamic model of bank’s balance sheet. But before we move to explicit mathematical model we have to create some theoretical foundations. First issue is the issue of risk per unit of credit. Here we have two options. The first option is to have absolute risk exposure. In this case the bank is always willing to risk a

[4] Additional descriptions and history of regulation can be found on the website of Croatian national bank.

[5] In Croatia both oversight of the banks and conduct of the monetary policy are duties of the Croatian National Bank.

fixed monetary value per unit of credit. The second option is to have a relative risk exposure where bank is always willing to risk x % per unit of exposure. The first option is not realistic since it is not possible for a bank to risk 100 monetary units per both credit in the mount of 1000 and 1000000 monetary units. Because of this we will use relative risk exposure. This is much more realistic with the respect of the banking business.

The importance of determining the risk exposure lies in the modelling approach in this paper. It is standard practice in economics for households to be modeled as utility maximizing participants in the economy. We are going to use the same approach with the banks. We are going to present them as utility maximizing agents with constant relative risk aversion.

Using the business logic of the previous two paragraphs we are going to set up the utility function for a representative bank. As noted the difference between the bank and a household is that the households try to maximize consumption and the banks try to maximize profits. So instead of c for consumption in utility functions for banks we are going to have π for profits. However we should note that the profit is an accounting variable, not economic variable. The formula for profit can be shown as:

1. $$\pi = rA - \delta L$$

Where π is profit, A are assets and L are liabilities[6] r is the average interest rate on assets and δ is the average interest rate on liabilities. Since by the accounting equality assets have to equal liabilities we get the following equation for banks profits.

2. $$\pi = rA - \delta A = A(r - \delta) = \tau A$$

Where τ is the net interest rate the bank gets or the interest rate spread between the assets and liabilities. In this paper we are using profits, although we could be using τA, the results are the same. We now turn back to the form of the utility function.

In this model we are going to assume that the banks are using CRRA style function as it was explained in the previous section. The utility function we are going to use as the control function for our optimal control problem.

Now we are going to set up the optimal control problem the banks are facing. The problem will be stochastic since the banks have uncertainty in their business and have to create expectation about future. Basic set up of the problem can be also found in Cooper and Adda (2003) and initially developed by Bellman (1957). The optimization problem for the banks is how to maximize present value of expected profits over time.

3. $$\max \ E\left[\sum_{0}^{\infty} (1+\theta)^{-t} u(\pi_t)\right]$$

[6] In this paper we are only going to look at the interest bearing assets and interest bearing liabilities plus capital. We are not going to look at other items of the bank's balance sheet. So when we are referring to assets or liabilities we are referring to the interest bearing parts of the balance sheet plus capital.

The profits are noted as π and are discounted over by the rate θ in order to get the present value of profits; E stated for the expectations. The value function takes the form:

$$4. \quad V_t(x_t) = \max E\left[\sum_0^\infty (1+\theta)^{-t} u(\pi_t)\right]$$

Subject to $x_0 > 0$,

$x_\infty > 0$ is free

$x_t > 0$ for all time periods

Where x is the part of the bank's balance sheet that is interest bearing and is net of regulation imposed by the central bank. The rest of the balance sheet is tied up with regulation and it will be optimized in the next section. We are assuming that x has some initial value of the bank's balance sheet, this is the bank's balance sheet where when the bank starts the control problem, the end value is free. We are also assuming that through time the value of x is greater than 0, since it would be impossible for the bank to have no interest bearing assets.

The transition equation for interest bearing part of the balance sheet is:

$$5. \quad x_t = x_{t-1} + (y_t - c_t)$$

In each period the bank has net interest bearing assets x_{t-1} from the previous period and the current period changes in the values of interest bearing assets. Where y_t represents the incoming funds into the bank, c_t are the outgoing funds in each time period.

The assets in the bank are earning some interest. The bank has two choices of investing. The first choice is to invest in the risk free government bonds that have return r or to lend at some rate z, but with risk[7]. The balance sheet is split into parts that are invested with risk and the parts that are invested without risk.

$$6. \quad x_t = [x_{t-1} + (y_t - c_t)] * [(1 + r_t)\omega_t + (1 + z_t)(1 - \omega_t)]$$

with $0 \le \omega \le 1$

In equation (6) z is the rate of return on risk bearing asset, r is the interest rate obtained by investing in risk bearing assets. Both rates of return are net rates of return. The portion of the balance sheet invested into risk bearing asset is ω.

To obtain the recursive solution to the bank's optimization problem we will use Bellman principle of optimality as described in Adda and Cooper (2003). Now we can set up the bellman equation

[7] We are going to split the profit (business part) of the bank's balance sheet between risk free part which is obviously the government and risk part which include the firm and households. We could make additional split for firms and households but we are not investigating credit distribution of loans, but the risk preference when it comes to regulation.

$$7. \quad V_t(x_t) = \max_{c_t \omega_t} \left\{ u(\pi_t) + (1+\theta)^{-1} E[V_{t+1}(x_{t+1})] \right\}$$

Equation (7) is subject to equations (5) and (6).

The E in bellman equation indicates expectations. Equation (7) is telling how the bank is going to behave. By solving the bellman equation we can find the optimal path for the bank in order to achieve the maximization of profits. First order conditions are: for π

$$8. \quad u'(\pi_t) = E\left[(1+\theta)^{-1}((1+r_t)\omega_t + (1+z_t)(1-\omega_t))V'_{t+1}(x_{t+1})\right]$$

For ω:

$$9. \quad E[V'_{t+1}(x_{t+1})(r_t - z_t)] = 0$$

In this equation we have used the fact that $(1+\theta)^{-1}(x_t + y_t - c_t)$ is known at time t. Using equation (8) and the envelope theorem described in Blanchard and Fisher (1989) we can get:

$$10. \quad V'(x_t) = E\left[(1+\theta)^{-1}((1+r_t)\omega_t + (1+z_t)(1-\omega_t))V'_{t+1}(x_{t+1})\right] = u'(\pi_t)$$

Now we can conclude that marginal utility of profit has to be equal to the marginal increase in x, the interest bearing part of the balance sheet. Using this relationship we can eliminating V'(x+1) from the first order conditions and get equations in (9) and (10).

$$11. \quad u'(\pi_t) = E\left[(1+\theta)^{-1}((1+r_t)\omega_t + (1+z_t)(1-\omega_t))u'(\pi_{t+1})\right]$$

$$12. \quad E[(1+r_t)u'(\pi_{t+1})] = E[u'(\pi_{t+1})(1+z_t)]$$

We can now substitute (11) into (10) and get:

$$13. \quad u'(\pi_t) = (1+\theta)^{-1}(1+r_t)E[u'(\pi_{t+1})]$$

$$14. \quad u'(\pi_t) = (1+\theta)^{-1}E[u'(\pi_{t+1})(1+z_t)]$$

In this we have solved the problem of bank’s optimization of business part of the balance sheet. Bank is trying to optimize profits and therefore it will try to find the optimal distribution of risk and risk free assets in its balance sheet. We will not move to the analysis of the relationship between the central bank and bank.

4. Banks vs. Central bank

In the previous part we have looked at the general solution to the optimal behavior of a bank in economic setting where the bank has to optimize the balance sheet in order to maximize profit. In the previous part we have only looked at the business part of the balance sheet, which we have called the interest bearing part of the balance sheet. In this part we are going to look at the behavior of banks in relationship with the central bank. The relationship of a bank with the central bank is in fact the problem of the optimization of the bank's balance sheet due to regulation.

The banks are under direct supervision of the central bank and have to obey the rules imposed by the central bank. In terms of game theory this set up puts banks directly in inferior position to central bank. The banks have to obey a large set of rules and regulation. The specific regulation we are going to analyze is the regulation which impacts the ability of the bank to perform its business activities. This is the regulation which forces bank to exclude some funds out of the lending process (like reserve requirement); limits banks decision on credit distribution per sector[8], or limit the growth of loans[9].

The restrictions imposed on the banks are done through regulation which specifically states a certain portion of assets has to be invested in a certain way. Usually this is determined through the structure of liabilities. The most common and the most practiced regulation of this kind is the reserve requirement. The reserve requirement requests a portion of funds received from certain liabilities has to be deposited with the central bank, or some other institution. The exact specification of the reserve requirement can vary over many parameters: term, currency, source, and residency. Apart from the reserve requirement the banks might also be regulated with strict ratios imposing certain structure on the bank's balance sheet.

The regulation is usually used by the central bank with some very specific design in mind. The complexity of regulation automatically increases the complexity of the optimization problem for the bank. The smaller the amount of regulation, the less complex is the bank's optimization with respect to the central bank.

The scope of the possible regulation is very broad. If the regulation is large and complex it is very important for banks to optimize the total impact of the regulation, both on the bank's balance sheet and on the bank's profits. Another problem for the banks is the fact the regulation might change over time and the bank also has to make expectations over the future how the regulation will look like and what will be the impact of the regulation. The possibility of changes in regulation and instability of regulation increases complexity of the optimization problem for the banks.

What we will specifically pay attention to is going to be the impact of the regulation on business choices made by the banks and how does the choice of the monetary regime impact the behavior of banks. Since the central bank has power to impose regulation when it sees new regulation necessary the banks are in fact faced with Stackelberg problem where they have to optimize not just their internal behavior, but also the shocks made by the behavior of the central bank.

[8] In Serbia there was regulation which imposed the maximum lending to retail as percentage of capital.
[9] Croatia implemented this limit as 1% loan growth to some sectors.

Since banks in their essence are intermediaries between those who have funds and those who need funds the structure of balance sheet regulation[10] is exceptionally important. First thing about regulation is that the regulation dictates the interest rates and therefore the cost of funding in the economy. The amount of regulation imposes the minimum interest rate on assets placed based on the interest rate of liabilities[11]: higher the regulation, higher the interest rate. The banks do not create regulation; they just try to optimize the existing regulation so the banks can maximize their profits. This is an important point, because it implies **the banks will dance to the tune of the central bank, not the other way around**. The banks adjust to the regulation through a dynamic process.

The basic premise developed here lies on the theoretical assumption the bank is playing a game with the central bank. The object of the bank is to have the maximum benefit from the regulation, or stated differently the bank is trying to minimize the "damage" to the income caused by the regulation. Since the central bank imposes the regulation, forces banks to obey the regulation, the banks are forced to play along. So we have two players, one which is imposing the rules and the second one which is forced to play by the rules.

The model we are going to present here relies on Hansen and Sargent (2005).We shall denote the total balance sheet, regulation plus business part with vector A, let A_t^S denote the history from t so s. If the subscript is omitted we take it to be 0. If the super script is omitted we take it to be $+\infty$. The balance sheet will have two components, two vectors with different investments in them. The first vector is noted to be the endogenous vector x (bank chooses what to do with the loans placed as described in the previous part) and the exogenous vector q (the central bank imposes the regulation). The endogenous one is the component that is under director control of the bank, while the exogenous one is a component not under direct control of the central bank. Using the vector notation these two components can be presented as:

$$15. \quad A_t = \begin{bmatrix} x_t \\ q_t \end{bmatrix}$$

The exogenous part of the assets shall have the transition law,

$$16. \quad q_{t+1} = \begin{bmatrix} 0 \\ q_t \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_t \end{bmatrix}$$

Where ε is identical and independently distributed changes in the regulation with a distribution Φ, this distribution is unknown to the bank and the bank can only made assumptions about it, but it is known to the central bank since the central bank is in charge of the regulation.

The shocks ε in fact are the changes in the regulation. The only way a bank can know in advance what there will be the change in regulation is if the central bank explicitly states when and to what extent the regulation is going to be changed. So the natural question arises:

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[10] By balance sheet regulation I mean all regulation which dictates the structure of assets in any way. Either by imposing structure of assets based on liabilities like reserve requirement or limits on certain assets or capital adequacy ponders.

[11] A simple example: If reserve requirement 10% and is remunerated at 0%, if the interest rate on deposit is 5% the minimum interest rate in assets to cover the cost of interest is 5,55%.

how does the bank obtain expectations about the future regulation changes? In order to answer this we have to use the Bayesian learning approach.

One of the main characteristics of the Bayesian approach is the fact that the prior distribution can be updated at any point in time. Using this property we can expect the bank to have the ability to learn the behavior of the central bank. Any time a change in the regulation occurs the bank can update the prior distribution used in order to determine the future changes in the regulation. With this tool the banks have a powerful learning mechanism which allows them to adopt their expectations and change their forecasts of regulation over time.

Naturally the more levels of regulation (term structure, source, currency) more expectations will have to be created and the probability distribution of expectations is going to be greater, which implies a larger margin of error for the banks. So the more regulation there is the more stochastic elements the banks are faced with.

We have set up the interest bearing part of the balance sheet and we have set up the regulatory part of the balance sheet. Now we have to ask: how does total structure of loans A evolve over time? In order to develop transition equation we have to understand the process. By definition assets equal liabilities. In order to place assets the bank has to obtain liabilities. There are three main sources of funding for any bank. The primary source of funding which is funding received from clients; the funding which comes to the bank. The bank does not go out and try to obtain it. The secondary sources of funding are funds received from financial institutions or specific investors; the bank has obtained them by actively pursuing funding. This is the funding the bank has obtained through borrowing from someone else. The third source of funding, which is also a part of the secondary sources is the capital. In this particular case the owners of the banks or new investors are willing to give funding to the bank in exchange for the part of the ownership of the bank.

The funds enter into the balance sheet and are in terms of accounting booked in liabilities. In assets the funds get separated into two parts. The first part is part we have noted as q and it is the part needed for regulation. The second part is the part used for business activities and that part has been noted as x. We can show changes in the size of the balance sheet mathematically as:

17. $$A_t = \begin{bmatrix} x_{t-1} \\ q_{t-1} \end{bmatrix} + \begin{bmatrix} 1-p_t & 1-s_t & 1 \\ p_t & s_t & 0 \\ & & \end{bmatrix} * \begin{bmatrix} P^{\Lambda}_{t-1} - P^{\lambda}_{t-1} \\ S^{\Lambda}_{t-1} - S^{\lambda}_{t-1} \\ C_{t-1} - d_{t-1} \end{bmatrix}$$

The equation (17) has the following notation: A are assets; P are primary sources of funds (superscript y denotes incoming and c denotes outgoing funds); S are secondary sources of funds (superscript y denotes incoming and c denotes outgoing funds); C is capital; p is regulation requirement on primary sources of funds, has value $0 \leq p \leq 1$; s is regulatory requirement on secondary source of funds $0 \leq s \leq 1$ and d are dividends paid.

As we can see from the equation (17) the funds enter or exit and so does the structure of the balance sheet between the regulatory part and the free part change. Also we have not made any specifications on the regulation, except for the fact that there is no regulation on the capital. No regulation on capital is also assumption since the bank can impose reserve requirement on subordinated debt, which is a part of the capital. As for the values of p and s they have been bounded between 0 and 1. With 0 meaning there is no regulation and 1 meaning that the whole amount of funds received from the source has to go towards the regulation.

Now that we have set up the transition equation we can see that the problem the bank is facing is to minimize the size of q in the balance sheet. That is to minimize the amount of funds used for regulation in the balance sheet.

$$18. \quad \min E\left[\sum_{t=0}^{\infty}(1+\theta)^t f(q_t)\right]$$

subject to $A_{t=\infty} = free, A_t > 0$ with initial values $A_{t=0} = \begin{bmatrix} x_0 \\ q_0 \end{bmatrix}$

Where θ is the discount factor with values 0<θ<1 and E denoted expectations. So the bank wants to minimize the size of the regulation in the bank's balance sheet.

Since we are dealing with a complex environment we are going to make some changes in the model. First of all we are going to reformulate the problem. We are not going to analyze the balance sheet values at all, what we are going to analyze is the percentages of the regulation and interest bearing assets in the balance sheet, this implies that x+q=1. We are also going to assume that at any point in time there is some optimal distribution of interest bearing assets x* and regulation q* which will give the bank the most profit given the current regulation. The bank wants to be as close to that optimal distribution at any point in time.

The bank does not know what the central bank is planning to do with the regulation in the future. So the bank has to assume what the central bank is going to do. Over time the bank has the ability to learn what the central bank is doing. This model is universal for any regulatory regime and has nothing to do with the choice of the monetary policy. The central bank can control the banks regardless of the choice of the regulatory regime.

As one bank can take the regulation and calculate the optimal balance between the x and q in the balance sheet, so can the central bank. Given the objectives of the central bank, the central bank aggregates all of the balance sheets of the banks in the economy and sets up the optimal distribution between regulatory part of the balance sheet and the interest rate bearing part of the aggregated balance sheet. The central bank wants the balance sheet of the banks to evolve over time according to the following model:

$$19. \quad A_t^* = \alpha_t x_{t-1} + \beta_t q_{t-1}$$

Where A* is the size of the balance sheet of all of the banks, x is the interest rate part and q is the regulation part. The parameters α and β represent the rates of change of each part of the balance sheet in each time period. The central bank controls the change in structure of x and q over time.

Given the current state and the future expected states of the regulation the bank wants to be as close to the optimal distribution of x and q as possible. The problem the bank is trying to solve in that case is:

$$20. \quad \min E\left[\sum_{t=0}^{\infty}(1+\theta)^t((x_t - x_t^*)^2 + (q_t - q_t^*)^2)\right]$$

What the bank wants is to minimize the covariance between the optimal structure x* and q* of the balance sheet and the actual structure x and q of the balance sheet. Since q and x are tied together the bank can either try to maximize x or minimize q, we are going to model the problem of the minimization of q part of the balance sheet. The minimization is subject to

$$21. \qquad q_t = \lambda'_{t|t-1}\,\Xi_t + \sigma$$

The bank is trying to minimize the variance between the regulation and the actual structure of the balance sheet. Where λtǀt-1 and Ξ are r×1 vectors which contain the bank's model of prediction of the changes in the regulation. The bank's model is not the "true model" used by the central bank and the bank does not know the true model. The parameter σ is the vector of random i.i.d. disturbances.

In the minimization problem we have E, this E denotes expectations, which are created rationally, but based on the wrong model. The parameter λ also has the subscript t conditional on t-1, this means that the bank every time period collects the current set of regulation and then updates its knowledge about the changes in the regulation. This means the model the bank is using is getting better and better with each time period. The bank will update its "wrong" model in each time period:

$$22. \quad q_t = \lambda'_t\,\Xi_t + \sigma$$

$$23. \quad \lambda_t = \lambda_{t-1} + \Omega$$

Where Ω is not correlated with σ and is an i.i.d. Gaussian random vector with mean zero and some covariance matrix K. The mean estimate of the model is going to be

$$24. \quad \hat{\lambda}_{t|t-1} \equiv E(q_t | \zeta_{t-1})$$

$$25. \quad \zeta_{t-1} \equiv \{q_1, q_2, q_3, q_4, ...q_t\}$$

If we let

$$26. \quad Q_{t|t-1} \equiv Var(\lambda | \zeta_{t-1})$$

Given the bank's model the mean estimates are optimally updated via the Kalman filter. Given the values of $\hat{\lambda}_{1|0}$ and $Q_{t|t-1}$ the Kalman filter algorithm updates $\hat{\lambda}_{t|t-1}$ with the following formula

$$27. \quad \hat{\lambda}_{t+1|t} = \hat{\lambda}_{t|t-1} + \frac{Q_{t|t-1}\Xi_t (q_t - \Xi' \hat{\lambda}_{t|t-1})}{\sigma^2 + \Xi'_t\, Q_{t|t-1}\Xi_t}$$

$$28. \quad Q_{t+1|t} = Q_{t|t-1} - \frac{Q_{t|t-1}\Xi_t}{\sigma^2 + \Xi'_t\, Q_{t|t-1}\Xi_t} + K$$

The learning process from the Kalman filter[12] is eventually going to converge to the correct model of the central bank. The desired distribution of the bank's balance sheet is going to be achieved as the banks learn the true model. The model presented here refers to the structure of the balance sheet given the distribution of the regulation and the non-regulation parts of the balance sheet. We have confronted the bank with the central bank and have not juxtaposed bank and other participants in the economy.

In the model presented here we have allowed banks to learn the process of regulation and optimize their behavior depending on the regulatory regime the central bank is imposing. Since the banks do not know the exact model of the central bank, they have to learn the model over time. The fact the banks do not know the model is the source of uncertainty in the model and in the financial system in general.

5. Conclusion

This paper presents banks as economic participants which are faced with two dynamic problems. The first dynamic problem is how to optimize their balance sheet in order to maximize their profits. The second problem the banks are faced is how to optimize their behavior when faced with the inferior position with the central bank which dictates monetary policy and through regulation and the nature and structure of regulation directly dictates the structure of the bank's balance sheet. The shocks to the balance sheet are stochastic and hard to predict. Since the bank is faced with two exogenous sets of shocks the market interest rates and the regulation imposed by the central bank the bank has a hard problem to optimize the business side.

This paper provides a theoretical framework for the analysis of a dual problem in banking. What this framework provides is the theoretical model for the analysis off banking strategies. The model shows that there is a strong impact of regulation and that the bank transfers the regulatory structure onto the structure of the interest bearing assets. The model in this paper directly implies there is an impact of regulation on bank's credit policy and sector distribution of credit in the economy.

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