A Note on DD Approach

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Abstract

This paper attempts to assess the impact of treatment effect or programme applying difference in difference (DD) approach. This study also identifies that the DD estimators are biased under certain conditions.

***Key* Words**: Difference-in-Difference, Treatment group, Control group.

1. Introduction

How do we measure the impact of a treatment (or programme) on the socio-economic outcomes? This study prepares a note on the impact measurement techniques for such outcomes. There appears a welcome trend in the current literature to consider a broader concept of treatment variance as much as average effects, and considers variation in impacts across the distribution of treatment effect. Here, to address this issue, the key requirement is the methodological issue. Econometricians, generally, apply sophisticated techniques to measure the treatment effects with certain assumptions, but accuracy of the estimators depend on assumptions. Recently, the popular impact estimation is the difference in difference approach. Now, we note on program evaluation and the Difference in Difference (DD) Approach.

2. Methodology

How do we evaluate the impact of a program or treatment? We evaluate the impact of treatment or program on an outcome Y over population individuals. Here, we apply and follow the standard notations used in the literature.

Suppose there are two groups indexed by treatment status T = 0, 1; where 0 and 1 indicate individuals who do not receive treatment (i.e., the control group) and individuals who receive treatment (i.e., treatment group), respectively. Assume that we observe individuals in two time periods, t = 0, 1 where 0 indicates a time period before the treatment group receives treatment (i.e. pre-treatment), and 1 indicates a time period after the treatment group receives treatment (i.e. post-treatment). Every observation is indexed by the letter i = 1,...,N; individuals will typically have two observations each, one pre-treatment and one post-treatment. For the sake of notation let $\overline{Y_0^T}$ and $\overline{Y_1^T}$ be the sample averages of the outcome for the treatment group before and after treatment, respectively, and let $\overline{Y_0^C}$ and $\overline{Y_1^C}$ be the corresponding sample averages of the outcome for the control group. Subscripts correspond to time period and superscripts to the treatment status.

2.1 Modelling the Outcome

The outcome Yi is modelled by the following equation

$$Y_i = \beta_0 + \beta_1\, T_i + \beta_2\, t_i + \beta_3\, (T_i * t_i) + \varepsilon_i \qquad (2.1)$$

where the β_0, β_1, β_2, β_3, coefficients are all unknown parameters and ε_i is a random, unobserved "error" term which contains all determinants of Y_i which the model omits. By inspecting the equation we should observe the coefficients and have the following interpretation: β_0 = constant term, β_1 = treatment group specific effect (to account for average permanent differences between treatment and control), β_2 = time trend common to control and treatment groups, β_3 = true effect of treatment.

The purpose of the program evaluation is to find a “good” estimate of δ, $\hat{\delta}$, given the data that we have available.

2.2 Assumptions for an Unbiased Estimator

A reasonable criterion for a good estimator is that it be unbiased which means that "on average" the estimate will be correct, or mathematically that the expected value of the estimator

$$E\,[\widehat{\beta_3}] = \beta_3$$

The assumptions we need for the difference in difference estimator to be correct are given by the following

1) The model in equation (2.1) is correctly specified. For example, the additive structure imposed is correct.
2) The error term is on average zero: E [ε_i] = 0. Not a hard assumption with the constant term β_0 put in.
3) The error term is uncorrelated with the other variables in the equation:

   Cov (ε_i, T_i) = 0

   Cov (ε_i, t_i) = 0

$Cov\ (\varepsilon_i, T_i{*}\ t_i) = 0$

the last of these assumptions, also known as the parallel-trend assumption, is the most critical.

Under these assumptions we can use equation (2.1) to determine that expected values of the average outcomes are given by

$$E[Y_0^T] = \beta_0 + \beta_1$$
$$E[Y_1^T] = \beta_0 + \beta_1 + \beta_2 + \beta_3$$
$$E[Y_0^C] = \beta_0$$
$$E[Y_1^C] = \beta_0 + \beta_2$$

These equations are helpful to identify the estimated impact of a treatment.

3. The Difference in Difference Estimator

Before explaining the difference in difference estimator it is best to review the two simple difference estimators and understand what can go wrong with these. Understanding what is wrong about as an estimator is as important as understanding what is right about it.

3.1 Simple Pre versus Post Estimator

Consider first an estimator based on comparing the average difference in outcome Y_i before and after treatment in the treatment group alone[1].

$$\widehat{\delta_1} = \bar{Y}_1^T - \bar{Y}_0^T \qquad \text{(D1)}$$

Taking the expectation of this estimator we get

$$\begin{aligned} E[\widehat{\delta_1}] &= E[\bar{Y}_1^T] - E[\bar{Y}_0^T] \\ &= [\beta_0 + \beta_1 + \beta_2 + \beta_3] - [\beta_0 + \beta_1] \\ &= \beta_2 + \beta_3 \end{aligned}$$

which means that this estimator will be biased so long as $\beta_2 \neq 0$, i.e. if a time-trend exists in the outcome Y_i then we will confound the time trend as being part of the treatment effect.

3.2 Simple Treatment versus Control Estimator

Next consider the estimator based on comparing the average difference in outcome Y_i post-treatment, between the treatment and control groups, ignoring pre-treatment outcomes[2].

$$\widehat{\delta_2} = \bar{Y}_1^T - \bar{Y}_1^C \qquad \text{(D2)}$$

Taking the expectation of this estimator we get

$$\begin{aligned} E[\widehat{\delta_2}] &= E[\bar{Y}_1^T] - E[\bar{Y}_1^C] \\ &= [\beta_0 + \beta_1 + \beta_2 + \beta_3] - [\beta_0 + \beta_2] \end{aligned}$$

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[1] This would be the estimate one would get from an OLS estimate on a regression equation of the form $Y_i = \alpha_1 + \delta_1 T_i + \varepsilon_i$ on the sample from the treatment group only.

[2] This would be the estimate one would get from an OLS estimate on a regression equation of the form $Y_i = \alpha_2 + \delta_2 T_i + \varepsilon_i$ on the post-treatment samples only.

$$= \beta_1 + \beta_3$$

So, this estimator is biased so long as $\beta_1 \neq 0$, i.e. there exist permanent average differences in outcome Yi between the treatment groups. The true treatment effect will be confounded by permanent differences in treatment and control groups that existed prior to any treatment. Note that in a randomized experiments, where subjects are randomly selected into treatment and control groups, β_1 should be zero as both groups should be nearly identical: in this case this estimator may perform well in a controlled experimental setting typically unavailable in most program evaluation problems seen in economics.

3.3 The Difference in Difference Estimator

The difference in difference (or "double difference") estimator is defined as the difference in average outcome in the treatment group before and after treatment minus the difference in average outcome in the control group before and after treatment[3] : it is literally a "difference of differences".

$$\widetilde{\delta_{DD}} = (\bar{Y}_1^T - \bar{Y}_0^T) - (\bar{Y}_1^C - \bar{Y}_0^C) \qquad \text{(DD)}$$

Taking the expectation of this estimator we will see that it is unbiased

$$E[\widetilde{\delta_{DD}}] = E[\bar{Y}_1^T] - E[\bar{Y}_0^T] - (E[\bar{Y}_1^C] - E[\bar{Y}_0^C])$$
$$=([\beta_0 + \beta_1 + \beta_2 + \beta_3] - [\beta_0 + \beta_1]) - ([\beta_0 + \beta_2]\text{-}\ \beta_0)$$
$$= [\beta_2 + \beta_3] - (\beta_2)$$
$$= \beta_3$$

This estimator can be seen as taking the difference between two pre-versus-post estimators seen above in (D1), subtracting the control group's estimator, which captures the time trend β_2, from the treatment group's estimator to get β_3. We can also rearrange terms in equation (DD) to get $\widetilde{\delta_{DD}} = (\bar{Y}_1^T - \bar{Y}_1^C) - (\bar{Y}_0^T - \bar{Y}_0^C)$ in which it can be interpreted as taking the difference of two estimators of the simple treatment versus control type seen in equation (D2). The difference estimator for the pre-period is used to estimate the permanent difference β_1, which is then subtracted away from the post-period estimator to get β_3.

Another interpretation of the difference in difference estimator is that is a simple difference estimator between the actual $\overline{Y_1^T}$ and the $\overline{Y_1^T}$ that would occur in the post treatment period to the treatment group had there been no treatment $\overline{Y_{cf}^T} = \overline{Y_0^T} + (\overline{Y_1^C}\text{-}\ \overline{Y_0^C})$, where the subscript "*cf*" refers to the term "*counterfactual*", so that $\widetilde{\delta_{DD}} = (\bar{Y}_1^T - \bar{Y}_{cf}^T)$. This observation $\bar{Y}_{cf}^T$, which has expectation E[$\bar{Y}_{cf}^T$] $= \beta_0 + \beta_1 + \beta_2$, does not exist: it is literally "contrary to fact"

[3] This would be the estimate one would get from an OLS estimate of a regression equation of the form given by (2.1) on the entire sample.

since there actually was a treatment in fact. However if our assumption are correct we can construct legitimate estimate of $\bar{Y}_{cf}^{T}$, taking the pre treatment average $\bar{Y}_{0}^{T}$ and adding the our estimate β_1 using the pre versus post difference for the control group.

3.4 Problems with Difference in Difference Estimators

If any of the assumptions listed above do not hold then we have no guarantee that the estimator $\hat{\delta}_{DD}$ is unbiased. Unfortunately, it is often difficult and sometimes impossible to check the assumptions in the model as they are made about unobservable quantities. Keep in mind that small deviations from the assumptions may not matter much as the biases they introduce may be rather small, biases are a matter of degree. It is also possible, however, that the biases may be so huge that the estimates we get may be completely wrong, even of the opposite sign of the true treatment effect.

One of the most common problems with difference in difference estimates is the failure of the parallel trend assumption. Suppose that Cov (ε_i, T_i * t_i) = E (ε_i (T_i * t_i)) = Δ so that Y follows a different trend for the treatment and control group. The control group has a time trend of $\gamma C = \gamma$, while the treatment group has a trend of $\gamma T = \gamma + \Delta$. In this case the difference in difference estimator will be biased as

$$\begin{aligned} E[\hat{\delta}_{DD}] &= \gamma T + \delta - \gamma C \\ &= \gamma + \Delta + \delta - \gamma \\ &= \delta + \Delta \end{aligned}$$

The failure of the parallel trend assumption may in fact be a relatively common problem in many program evaluation studies, causing many differences in difference estimators to be biased.

One way to help avoid these problems is to get more data on other time periods before and after treatment to see if there are any other pre-existing differences in trends. It may also be possible to find other control groups which can provide additional underlying trends.

Conclusion

This study suggests that difference in difference approach is very simple and easy to estimate the impact assessment of a treatment (or programme). The difference in difference approach provides good estimated results only under certain assumptions; otherwise, DD estimators will be biased especially in case of failure of the parallel trend assumption.

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