Information Advantage in Stackelberg Duopoly under Demand Uncertainty

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INFORMATION ADVANTAGE IN STACKELBERG DUOPOLY UNDER DEMAND UNCERTAINTY

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We consider a Stackelberg model under demand slope uncertainty in an environment where the follower owns information advantage. Specifically, we show that the second mover obtains higher expected profit than the first mover when the leader only knows the prior beliefs and the follower gains the posterior probabilities. This result tells us that the leadership advantage is dominated by the information advantage when demand fluctuation is important.

1 INTRODUCTION

The Stackelberg model is one of the most widely used models in industrial organization for analyzing firms' behavior in a competitive environment. In regular perfect information, the Stackelberg leader preempts his follower by investing in a larger capacity, which guarantees him higher profits compared to the follower. However, in Gal-or (1985) it is shown that when two identical firms move sequentially in a game the leader earns higher profits than the follower if reaction functions are downwards sloping and lower profits if reaction functions are upwards sloping. Especially, the market uncertainty always influences the firm's strategies and profits (Ponssard, 1976; Gal-or, 1987; Raju and Roy, 2000; Liu, 2005; Lu and Poddar, 2006). Gal-or (1987) presents a leader-follower game where both the Stackelberg leader and follower have private information on the random demand, and the quantity choice of the leading firm reveals its private information to the follower, thus providing the second mover with an information advantage. Raju and Roy (2000) find that, except under some conditions, more precise market information has a greater impact on profits in a Stackelberg mode of conduct than in a Bertrand mode. In Liu (2005) where the demand

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uncertainty is only for the first mover, however, it is demonstrated that when the realized demand is far from its expected value, the second mover obtains higher profit than the leading firm and when the realized demand is in an intermediate zone does the first mover preserve its advantage.

But most of the previous literature models uncertainty in the demand as an uncertain intercept. The assumption of an unknown intercept often eliminates any interaction between a firm's expectations about its rivals' outputs and the market uncertainty itself. Recently, some authors model an alternative industry in which the slope of the demand function is uncertain under Cournot competition (Malueg and Tsutsui, 1996; Raju and Roy, 2000; Chokler *et al.*, 2006). Slope uncertainty could arise in a setting in which consumers are identical and firms are uncertain about the number of consumers in the market.

We analyze a linear Stackelberg model in which the slope of the demand function is uncertain. Specifically, we focus on the strategic consequence of asymmetric demand information owned by first and second movers. Usually, the followers in markets gain more market information than first movers before sinking their investments. Therefore, we assume that the leader only knows the prior beliefs and the follower gains the posterior probabilities to update its distributions of the demand slope using the signals more accurately. Thus, on the one hand, our model is a natural complement to the intercept uncertainty studied by previous authors. On the other hand, as an alterative representation of demand uncertainty, our model checks the robustness of previous models' predictions about the relationship between leadership and information advantage. We show that if the following firm updates the distribution functions of demand slope parameter by receiving signals, the follower always earns greater profit than the leader with *ex ante* choice. Consequently, under random demand slope and the second mover's information advantage, firms will have the incentive to move second, because the first mover's leadership advantage is dominated by the second mover's information advantage. Thus, the previous models of information advantage and demand uncertainty, while proving tractable, understate the scope for profitable information advantage.

Our work is organized as follows: in the next section we describe the model, in section 3 we derive the equilibrium, and in section 4 we conclude.

2 THE MODEL

We consider a Stakelberg game where two firms compete in quantities sequentially and produce identical goods, the inverse demand for which is given by

$$p_i = a - \beta(q^1 + q^2), \quad i = 1,2;$$

where q^i is firm i quantity and $a > 0$ and $\beta > 0$ are parameters. The value of a is known to both firms. The slope parameter β, however, is random, which takes on one of two values, β_l or β_h, where $\beta_h > \beta_l > 0$. It is assumed that their fixed costs are zero and the firms have equal and constant marginal costs. Therefore, inverse demand is intercepted as net of marginal cost.

Without of loss generality, we assume the firm 1 is the Stakelberg leader and the firm 2 is the Stakelberg follower. Two firms possess common prior beliefs about β, with $\Pr(\beta_l) = \Pr(\beta_h) = 1/2$. The firms are asymmetrically informed about the real state of demand. Before making quantity decisions, the second mover receives private information about the value of β. Firm 2 observes the signal s^2, which supposes one of two values, s_l^2 or s_h^2. The following firm's private signals are equally accurate, with the conditional distribution of signal s^2, given the real demand slope, being as follows: $\Pr(s^2 = s_x^2 \mid \beta = \beta_x) = \sigma$, for $x \in \{l,h\}$. Generally, suppose that $\sigma \geq 1/2$.[1] Given ignorance regarding the true state of nature, firm 1 can commit to a fixed nonnegative quantity $q^1 \in \mathbb{R}_+$ in all states of nature. Outputs for firm 2 are chosen conditional on the observed signals.

Finally, the above assumptions of the environment are common knowledge among the firms.

[1] Here, symmetry of distributions is assumed only for simplicity.

3 DERIVATION OF THE EQUILIBRIUM

We start out by considering the maximization solved by the follower. Let I^2 denote the information available to firm 2 when it chooses its output. His objective is to:

$$\underset{q^2}{Max}\, U^2(q^1, q^2, I^2) = E[(a - \beta(q^1 + q^2))q^2 \mid I^2].$$

The first-order condition is:

$$E[a \mid I^2] = 2E[\beta \mid I^2]q^2(I^2) + E[\beta q^1 \mid I^2]. \qquad (1)$$

Obviously, payoff of firm 1 under uncertain demand is given by

$$\underset{q^1}{Max}\, U^1(q^1, q^2) = E[(a - \beta(q^1 + q^2))q^1].$$

The first-order condition for the leader is:

$$E[a] = 2E[\beta]q^1 + E[\beta q^2]. \qquad (2)$$

As noted above, previous authors modeled demand uncertainty through randomness in the demand intercept, a. Eq. (1) reveals one reason for this focus. When the demand intercept, but not its slope, is uncertain, β can be factored out of the rightmost term in (1). Then, in the equilibrium with strategies that are linear in the information, the system of linear equations described by (1) and (2) can be solved for firm 1's and 2's equilibrium outputs. However, suppose, as we do, that a is known, but the slope parameter β is random. Then (1) and (2) become

$$a = 2E[\beta \mid I^2]q^2(I^2) + E[\beta q^1 \mid I^2] \qquad (1')$$

and

$$a = 2E[\beta]q^1 + E[\beta q^2]. \qquad (2')$$

In Eq. (1') the interaction between the unknown parameter and firm 1's output simply cannot be dealt with by the earlier models that assumed linearity of expectations between signals and the underlying state variables and then considered output strategies that were linear in the signals. This interaction can be analyzed in our model by explicitly considering the (finitely many) possible values of βq_1 for a given output strategy of firm 1.

Proposition: In the two-stage game in which the leader only knows the prior beliefs and the follower gains the posterior probabilities to update its distributions of the demand slope using the signals more accurately, we show that the second mover obtains higher expected profit than the first mover. In other words, the preemptive capabilities of a Stackelberg leader are reduced when the demand slope is random.

Proof. Let q_l^2 and q_h^2 denote the equilibrium output strategy for firm 2, given it has observed the signal $s^2 = s_x^2,\ x \in \{l,h\}$. Firm 2's first-order condition (1') for q_l^2, at the equilibrium, is now given as

$$\begin{aligned} a &= 2E[\beta \mid s_l^2]q_l^2 + E[\beta q^1 \mid s_l^2] \\ &= [\Pr(\beta_l \mid s_l^2)\beta_l + \Pr(\beta_h \mid s_l^2)\beta_h](2q_l^2 + q^1) \\ &= [\sigma\beta_l + (1-\sigma)\beta_h](2q_l^2 + q^1). \end{aligned} \quad (3)$$

Similarly, the first-order condition for q_h^2 is given as

$$a = [\sigma\beta_h + (1-\sigma)\beta_l](2q_h^2 + q^1). \quad (4)$$

Let q_*^1 denote the equilibrium output strategy for firm 1, and rearranging Eq. (2') yields:

$$a = \frac{\beta_l + \beta_h}{2}(Eq^2 + 2q_*^1) = \frac{\beta_l + \beta_h}{2}\left(\frac{q_l^2 + q_h^2}{2} + 2q_*^1\right). \quad (5)$$

Let $y = \sigma(1-\sigma)$, $M = \sigma\beta_l + (1-\sigma)\beta_h$ and $N = \sigma\beta_h + (1-\sigma)\beta_l$, then the solution of Eqs. (3), (4) and (5) provides the equilibrium outputs

$$q_l^2 = \frac{a}{12}\frac{6(\beta_l + \beta_h)N - 8MN + (\beta_l + \beta_h)^2}{(\beta_l + \beta_h)MN}, \quad (6)$$

$$q_h^2 = \frac{a}{12}\frac{6(\beta_l + \beta_h)M - 8MN + (\beta_l + \beta_h)^2}{(\beta_l + \beta_h)MN} \quad (7)$$

and

$$q_*^1 = \frac{a}{6}\frac{8MN - (\beta_l + \beta_h)^2}{(\beta_l + \beta_h)MN}. \quad (8)$$

Let $P_{ij} = \Pr(\beta = \beta_i, s^2 = s_j^2)$,[2] where $i, j \in \{l,h\}$. From (6)-(8), we have the expected equilibrium profits for the leader and the follower:

$$EU^1 = E[\beta(q_*^1)^2] = (q_*^1)^2(\beta_l + \beta_h)/2$$

and

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[2] It is easily verified that $P_{ll} = P_{hh} = \sigma/2$, $P_{lh} = P_{hl} = (1-\sigma)/2$.

$$EU^2 = P_{ll}\beta_l(q_l^2)^2 + P_{lh}\beta_l(q_h^2)^2 + P_{hl}\beta_h(q_l^2)^2 + P_{hh}\beta_h(q_h^2)^2$$
$$= [M(q_l^2)^2 + N(q_h^2)^2]/2 .$$

Hence,

$$EU^2 - EU^1 = \frac{1}{2}[\frac{a}{6}\frac{1}{(\beta_l+\beta_h)MN}]^2\{\frac{1}{4}M[6(\beta_l+\beta_h)N - 8MN + (\beta_l+\beta_h)^2]^2 + \frac{1}{4}N$$
$$[6(\beta_l+\beta_h)M - 8MN + (\beta_l+\beta_h)^2]^2 - (\beta_l+\beta_h)[8MN - (\beta_l+\beta_h)^2]^2\} .$$

Since $\frac{1}{2}[\frac{a}{6}\frac{1}{(\beta_l+\beta_h)MN}]^2 > 0$, we consider

$$F = \frac{1}{4}M[6(\beta_l+\beta_h)N - 8MN + (\beta_l+\beta_h)^2]^2 + \frac{1}{4}N[6(\beta_l+\beta_h)M$$
$$-8MN + (\beta_l+\beta_h)^2]^2 - (\beta_l+\beta_h)[8MN - (\beta_l+\beta_h)^2]^2$$
$$= 9(M+N)^3MN - 6(M+N)MN[8MN - (\beta_l+\beta_h)^2]$$
$$-\frac{3}{4}(M+N)[8MN - (\beta_l+\beta_h)^2]^2$$
$$= (M+N)[27(M+N)^2MN - 96M^2N^2 - \frac{3}{4}(M+N)^4]$$
$$= (M+N)[-4MN + (M+N)^2][24MN - \frac{3}{4}(M+N)^2] .$$

Regarding $(M+N)[-4MN+(M+N)^2] = (M+N)(M-N)^2 \geq 0$, we have

$$V = 24MN - \frac{3}{4}(M+N)^2 = -\frac{3}{4}(M^2+N^2-30MN)$$
$$= -\frac{3}{4}\{[1-32\sigma(1-\sigma)][(\beta_l)^2+(\beta_h)^2] + [64\sigma(1-\sigma)-30]\beta_l\beta_h\}$$
$$= \frac{3}{4}\{(32y-1)[(\beta_l)^2+(\beta_h)^2] + (30-64y)\beta_l\beta_h\} .$$

Let $\mu = \beta_h/\beta_l$,[3] then

$$f(\mu) = (32y-1)\mu^2 + (30-64y)\mu + (32y-1) .$$

On the one hand, for $y \geq 1/32$, $f(\mu)$ is the convex function and $f(1) = 28 > 0$. When $\mu = (64y-30)/(64y-2) < 1$, we get the minimum of the function $f(\mu)$. Thus, if $1 < \mu \leq 4$, $f(\mu) > 0$, then $V > 0$.

On the other hand, for $y < 1/32$, $f(\mu)$ is the concave function and $f(1) = 28 > 0$. When $\mu = (64y-30)/(64y-2) > 4$, the function $f(\mu)$ gets the maximum. Thus, if $1 < \mu \leq 4$, then $V > 0$.

Therefore, if $\sigma = 1/2$, $F = 0$ and if $\sigma > 1/2$, $F > 0$. Then, we have

[3] Generally, fluctuation of variable μ is not too large, so we suppose $1 < \mu \leq 4$. See McGuiqan, J., Moyer, R. and Harris, F. (2001).

when $\sigma = 1/2$, $EU^1 = EU^2$ and when $\sigma > 1/2$, $EU^1 > EU^2$. □

As a result, the leader gains the same profit as the follower provided that the second mover only knows the indifferent probability distribution of the slope. However, if the follower gains the posterior probabilities to update its distributions of the demand slope using the signals more accurately, the preemptive capabilities of a Stackelberg leader are reduced and the follower obtains strictly higher expected profit than the leader. Moreover, the second firm has an incentive to choose Stackelberg rather than Cournot competition.

4 CONCLUSION

In a regular perfect information environment, the Stackelberg leader preempts his follower by investing in a large capacity, which guarantees him higher profits compared to the follower. In contrast, with private information about stochastic demand, the second mover obtains higher expected profit than the first mover, which means that the first mover's leadership advantage is dominated by the second mover's information advantage. In fact, we can find many examples in accord with the conclusion. It was recently reported that Boston Scientific claimed that it had captured about 70 percent of new orders of drug-coated device just one year after Johnson & Johnson first introduced this product (Abelson, 2004). This is because in a market with a high degree of uncertainty, the followers can wait and see the customers' response to the new product introduced by the first movers, as well as move along the 'learning curve' of innovation. By persistently modeling demand uncertainty about the intercept of demand, earlier papers on random demand among Stackelberg competition have been unnecessarily restrictive and to the extend, misleading. We have shown, in a simple linear model with demand of uncertain slope, that the follower may have greater payoff than the leader when it owns more accurate information. This prediction is similar to that of previous models (Ponssard, 1976; Gal-or, 1987; Raju and Roy, 2000; Liu, 2005).

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