Do positional preferences for wealth and consumption cause inter-temporal distortions?

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Abstract. This paper derives necessary and sufficient conditions under which positional preferences do not induce inter-temporal distortions. When labor supply is exogenous, positional preferences for consumption have been shown to be non-distortionary for a class of models. However, it has not been explored whether the same holds when households also exhibit positional preferences for wealth. The analysis identifies a restricted homogeneity-property which, when not satisfied, induces positional preferences to be distortionary, despite inelastic labor supply. Without positional preferences for wealth, a constant marginal rate of substitution-property is necessary and sufficient for a consumption positionality to be non-distortionary. Once a household also has positional preferences for wealth in addition, the consumption positionality almost always becomes distortionary, as the implied effects of the positional concerns induce opposing effects on a household's saving behavior. Under a constant marginal rates of substitution-property, these opposing effects exactly offset each other.

Keywords and Phrases: Status, keeping up with the Joneses, positional preferences, distortion, endogenous growth, Ak model.

JEL Classification Numbers: D91, O40

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1 Introduction

Within the endogenous growth framework of an Ak model, this paper derives necessary and sufficient conditions for positional preferences to be non-distortionary, when labor supply is inelastic. By positional preferences we mean a situation in which households not only derive utility from own consumption and wealth, but also from own consumption and wealth relative to some consumption- and wealth reference levels.[1] The nature of the distortions we analyze is inter-temporal (in contrast to intra-temporal distortions when labor supply is elastic).

Social distinction or status is an important motivation of human behavior. This was already shown by Darwin (1871), who emphasized *sexual* selection besides *natural* selection. "To spread across the population, genes of sexual species not only need to survive in their natural and social environment, but also need to be or appear a more attractive mating partner than their same sex competitors." (Truyts 2010, p.137) Clearly, Darwin was not the first to think about positional preferences. Philosophers have started to comment on positional preferences more than 2400 years ago. In his *The Republic* (Book II), Plato argues: *Since ... appearance tyrannizes over truth and is lord of happiness, to appearance I must devote myself.* This passage astoundingly resembles Darwin's argument on sexual selection. In more recent times, Easterlin (1995) demonstrated that while national incomes have increased over the decades, happiness levels have not grown. One explanation for this *Easterlin Paradox* is that people have positional preferences, as emphasized by Clark et al. (2008). The recent literature provides abundant significant empirical evidence for positional preferences. Pioneering studies include Johansson-Stenman et al. (2002), Johansson-Stenman and Martinsson (2006), and Solnick and Hemenway (1998, 2005). A recent brief discussion is provided in Wendner (2014).

The prior literature argues that, in a standard framework (with neoclassical

[1] Different authors employ various terms, with slightly varying meanings, to describe positional preferences. These terms include *(negative) consumption externality*, *relative wealth or consumption*, *jealousy*, *envy*, *keeping or catching up with the Joneses*, *external habits*, *positional concerns*, *conspicuous wealth or consumption.*

production), positional preferences with respect to consumption have no impact on the steady state equilibrium, once labor supply is exogenous.[2] However, it has not been generally explored whether the same holds when households also exhibit positional preferences for wealth. Moreover this paper focuses on the *distortionary* impact of positional preferences – not on the question whether or not there is an impact on the equilibrium path at all.

Three related prior papers investigate the conditions under which positional preferences with respect to consumption are non-distortionary. Alonso-Carrera et al. (2006) consider an Ak model in which habit-forming households exhibit positional preferences for consumption. In case of no habits, a constant marginal rate of substitution-property is required for positional preferences to be non-distortionary. Their result coincides with that in this paper – for the case in which households are not positional with respect to wealth. Though, Alonso-Carrera et al. (2006) focus on the interaction between relative consumption and habits, this paper works out conditions when households are concerned with both relative consumption and relative wealth. Arrow and Dasgupta (2009) also demonstrate that a constant marginal rate of substitution-property is required for positional preferences to be non-distortionary. However, while their paper works out the conditions for the case of endogenous labor supply and positional concerns for both consumption and leisure, this paper focuses on positional preferences with respect to consumption and wealth (with exogenous labor supply). In their endogenous growth framework, Liu and Turnovsky (2005) also develop a constant marginal rate of substitution-condition. Though they do not consider wealth-dependent preferences.

Three further related papers consider positional preferences for both consump-

[2]Brekke and Howarth (2002, p.142) argue that "we have established that augmenting a standard neoclassical growth model to incorporate a concern for relative consumption has no impacts on long-run economic behavior." Fisher and Hof (2000, p.249) show that the result that "relative consumption does not affect the long-run steady state...is robust with respect to the specification of the instantaneous utility function." Liu and Turnovsky (2005, p.1106) state that "[w]ith exogenous labor supply, consumption externalities, which impact through the labor-consumption tradeoff, have no channel to affect steady state output" in a framework with neoclassical production. Rauscher (1997, p.38) argues that "conspicuous consumption does not affect the long-run steady state."

tion and wealth. Nakamoto (2009) demonstrates, in a framework with exogenous growth, that even with exogenous labor supply, positional preferences for consumption always introduce a distortion, once preferences are wealth dependent. In a framework with endogenous growth, this result is (only) partially confirmed. Nakamoto (2009) does not consider positional preferences for wealth. Here, we show that once households exhibit *positional* preferences for wealth, positional preferences for consumption need not necessarily introduce a distortion (see below), in contrast to the result of Nakamoto (2009). Tournemaine and Tsoukis (2008) investigate positional preferences with respect to both consumption and wealth. However, they analyze the impact of positional preferences on growth (they do not consider the distortionary effects). A third closely related paper is Ghosh and Wendner (2014), which considers an endogenous growth model with positional concerns with respect to both consumption and wealth. The paper shows that a preference for wealth is critical for a consumption positionality to be distortionary. Their paper, however, focuses on the effects of positional preferences on growth, welfare, and optimal taxation for a functionally specified utility function. In contrast, this paper works out necessary and sufficient conditions for preferences to be non-distortionary for a fully general utility function.

This paper contributes to the prior literature in two ways. First, the paper provides necessary and sufficient conditions for positional concerns for consumption to be non-distortionary, in a framework with exogenous labor supply. Second, the paper explores necessary and sufficient conditions for positional preferences to be non-distortionary when households have positional preferences with respect to both consumption and wealth.

This paper displays three main results. First, if households do not exhibit a preference for wealth, positional preferences with respect to consumption do not cause a distortion if a constant marginal rate of substitution-property is satisfied. Under this property, the marginal utility of consumption of individual households is proportional to that of a social planner, and the market equilibrium path equals that of the social optimum. In contrast, if households are positional with respect to wealth but not with respect to consumption, the market equilibrium path is always inefficient. Second, if households exhibit a preference for wealth – but are not positional with respect for wealth – then positional preferences with respect to consumption always introduce a distortion. Third, if households exhibit *positional* preferences for both consumption and wealth, then, under restrictive assumptions, the market equilibrium path can be socially optimal. The key property is that the marginal rates of substitution of reference consumption for consumption and of reference wealth for wealth must be constant and identical. Under this property, the opposing effects of positional concerns (for consumption and wealth) on savings exactly offset each other.

This paper is structured as follows. Section 2 presents the endogenous growth model (with inelastic labor supply) for both the market economy and the social optimum. Section 3 analyzes the distortionary effects of positional preferences for consumption without and with wealth-dependent preferences. Subsequently, the section analyzes positional preferences for both consumption and wealth. Section 4 concludes the paper.

2 The Model

We consider a dynamic general equilibrium model of a closed economy that allows for fully endogenous growth. Endogenous growth stems from constant returns to capital (Ak model). Time is considered to be continuous. There is a large number of households and firms, the respective number of which we normalize to unity. Households are homogeneous and exhibit positional preferences. They derive utility not only from own consumption but also from own consumption relative to some consumption reference level, and from own wealth relative to some wealth reference level.

2.1 Preferences

The representative household has preferences for consumption c, relative consumption $c/\bar{c}$, wealth k, and relative wealth $k/\bar{k}$. Relative consumption is given by individual consumption relative to some consumption reference level $\bar{c}$. As households are homogeneous in our framework, we consider the economy's average consumption level as a natural choice for a household's consumption reference level. By the same token, relative wealth is given by individual wealth relative to the average wealth in the economy, $\bar{k}$. Individual households consider $\bar{c}$ and $\bar{k}$ as exogenous.

Wealth-dependent preferences have been considered before (cf., among others, Corneo and Jeanne 1997, 2001, Fisher and Hof 2000, Fisher and Hof 2005, Futagami and Shibata 1998, Hof and Wirl 2008, Pham 2005, Rauscher 1997). However, only few papers consider both positional consumption- *and* positional wealth concerns (cf. Nakamoto 2009, Tournemaine and Tsoukis 2008, Ghosh and Wendner 2014). In this paper, both relative consumption and relative wealth enter the utility function. The *reduced-form* instantaneous utility function is given by:[3]

$$u(c, \bar{c}, k, \bar{k})\,. \tag{1}$$

In the standard model, $u_c(c, \bar{c}, k, \bar{k}) > 0$, and $u_i(c, \bar{c}, k, \bar{k}) = 0$ for some $i \in \{\bar{c}, k, \bar{k}\}$, where a subindex refers to the partial derivative: $u_x(.) \equiv \partial\, u(.)/(\partial\, x)$. If $u_{\bar{c}}(c, \bar{c}, k, \bar{k}) < 0$, preferences exhibit positional concerns for consumption. For a given own consumption level, a rise in the others' consumption (the reference consumption level) lowers own utility. If $u_k(c, \bar{c}, k, \bar{k}) > 0$, households derive utility not only from consumption, but also from wealth. Finally, If $u_{\bar{k}}(c, \bar{c}, k, \bar{k}) < 0$, preferences exhibit positional concerns for wealth. For a given own wealth level, a rise in the others' wealths (the reference wealth level) lowers own utility. The time index t is suppressed, unless necessary to avoid ambiguities.

Throughout, we assume that the utility function (1) is strictly concave, twice continuously differentiable, and strictly increasing in c.

[3]Specifically, $u(c, \bar{c}, k, \bar{k})$ captures the special case: $v(c, c/\bar{c}, k, k/\bar{k})$.

The intertemporal utility function, U, as viewed from date $t = 0$, is given by:

$$U = \int_{t=0}^{\infty} u(c, \bar{c}, k, \bar{k})\, e^{-\rho\, t} dt\,, \tag{2}$$

where $\rho > 0$ is the household's constant pure rate of time preference.

Example 1 (Ghosh and Wendner (2014))

An example of such preferences is given by the instantaneous utility function in Ghosh and Wendner (2014). They analyze an economy, in which preferences are specified by

$$u(c, \bar{c}, k, \bar{k}) = \frac{1}{\gamma}\left[c^{1-\eta_c}\, \left(\frac{c}{\bar{c}}\right)^{\eta_c}\, \left[k^{1-\eta_k}\, \left(\frac{k}{\bar{k}}\right)^{\eta_k}\right]^{\xi}\right]^{\gamma}\,, \tag{3}$$

with $-\infty < \gamma < 1$*,* $\xi \geq 0$*,* $0 \leq \eta_c < 1$*,* $0 \leq \eta_k < 1$*. In this example,* $u_{\bar{c}}(c, \bar{c}, k, \bar{k}) < 0$ *refers to* $\eta_c > 0$*;* $u_k(c, \bar{c}, k, \bar{k}) > 0$ *corresponds to* $\xi > 0$*;* $u_{\bar{k}}(c, \bar{c}, k, \bar{k}) < 0$ *is implied by* $\xi\, \eta_k > 0$*. This is the leading example illustrating the theoretical results below.*

2.2 Technology

A homogeneous output, y, is produced by capital according to the linear technology (Rebelo 1991):

$$y = A\, k\,, \quad A > 0 \tag{4}$$

where y is gross production per capita, and k is capital per capita. The depreciation rate of capital is $\delta \in [0, 1]$. We assume $(A - \delta) \geq 0$ to ensure a nonnegative net-productivity. Moreover, there is no population growth.

2.3 Market equilibrium

Let the superscript m indicate a market (decentralized) equilibrium. Households choose a consumption stream so as to maximize intertemporal utility (2) subject to:

$$\dot{k}^m = y^m - c^m - \delta k^m = (A-\delta)k^m - c^m\,, \tag{5}$$

$$k_0^m \;\; \text{given}\,, \tag{6}$$

$$\bar{c}^m\,, \bar{k}^m \quad \text{exogenous}\,, \tag{7}$$

$$\lim_{t\to\infty} \mu_t^m k_t^m e^{-\rho t} = 0\,. \tag{8}$$

Differential equation (5) reflects the flow budget constraint of the representative household. Restriction (6) is obvious; every household is required to base her plans on the initial value of her wealth. Notice that (5) and (6) hold for both the market framework and the social optimum (see below). Restriction (7) reflects the fact that individual households consider the positionality reference levels as exogenous. Finally, (8) is the transversality condition.

For the market economy, the current value Hamiltonian is given by:

$$H^m(c^m, \bar{c}^m, k^m, \bar{k}^m, \mu^m) = u(c^m, \bar{c}^m, k^m, \bar{k}^m) + \mu^m\left[(A-\delta)k^m - c^m\right], \tag{9}$$

where the costate variable μ^m represents the shadow price of capital. An interior solution implies the following first-order conditions:

$$\mu^m = u_c(c^m, c^m, k^m, k^m)\,, \tag{10}$$

$$\frac{\dot{\mu}^m}{\mu^m} = -[(A-\delta)-\rho] - \frac{u_k(c^m, c^m, k^m, k^m)}{u_c(c^m, c^m, k^m, k^m)}\,. \tag{11}$$

Notice that without wealth in the utility function, $u_k(.) = 0$, and (11) simplifies to become

$$\frac{\dot{\mu}^m}{\mu^m} = -[(A-\delta)-\rho]\,. \tag{11'}$$

For the decentralized economy, an equilibrium path is characterized by (5), (6), (10), and (11).

2.4 Social Optimum

Let the superscript o indicate a social planner's optimal equilibrium. By employing a utilitarian social welfare function, the social planner chooses a consumption stream so as to maximize intertemporal utility

$$U = \int_{t=0}^{\infty} u(c^o, c^o, k^o, k^o)\, e^{-\rho\, t} dt\,, \tag{12}$$

subject to

$$\dot{k}^o = (A - \delta)k^o - c^o\,, \tag{13}$$

$$k_0^o \ \text{given}\,, \tag{14}$$

$$\lim_{t\to\infty} \mu_t^o k_t^o e^{-\rho\, t} = 0\,. \tag{15}$$

Restrictions (13) – (15) have the same interpretations as those given for the market economy. The main difference with respect to the decentralized framework is the fact that the social planner takes into account that $\bar{c}^o = c^o$ and $\bar{k}^o = k^o$.

For the social optimum, the current value Hamiltonian is given by:

$$H^o(c^o, c^o, k^o, k^o, \mu^o) = u(c^o, c^o, k^o, k^o) + \mu^o\,[(A - \delta)k^o - c^o], \tag{16}$$

where the costate variable μ^o represents the optimal shadow price of capital. An interior solution implies the following first-order conditions:

$$\mu^o = u_c(c^o, c^o, k^o, k^o) + u_{\bar{c}}(c^o, c^o, k^o, k^o)\,, \tag{17}$$

$$\frac{\dot{\mu}^o}{\mu^o} = -[(A - \delta) - \rho] - \frac{u_k(c^o, c^o, k^o, k^o) + u_{\bar{k}}(c^o, c^o, k^o, k^o)}{u_c(c^o, c^o, k^o, k^o) + u_{\bar{c}}(c^o, c^o, k^o, k^o)}\,. \tag{18}$$

Notice that without wealth in the utility function, $u_k(.) = u_{\bar{k}}(.) = 0$, and (18) simplifies to become

$$\frac{\dot{\mu}^o}{\mu^o} = -[(A - \delta) - \rho]\,. \tag{18'}$$

That is, without wealth in the utility function, it is *always* the case that the shadow price of capital in the decentralized economy grows at the same rate as that in the centralized framework:

$$\frac{\dot{\mu}^m}{\mu^m} = \frac{\dot{\mu}^o}{\mu^o}\,. \tag{19}$$

In the above-mentioned example (Ghosh and Wendner 2014), equation (19) is equivalent to $\xi = 0$.

For the centralized economy framework, an equilibrium path is characterized by (13), (14), (17), and (18).

3 Distortionary effects of positional preferences

In this section, we address two cases. The first case is the special case in which households have positional concerns only with respect to (relative) consumption. We develop a necessary and sufficient condition for positional preferences to be non-distortionary. The results developed here are essential for understanding those of the general second case, in which households have positional concerns with respect to both (relative) consumption and (relative) wealth.

3.1 Positional concerns with respect to consumption

In this subsection, we focus on the case: $u_{\bar{k}}(.) = 0$. This is the case in which households are not concerned about others' wealth levels, i.e., about a wealth reference level. However, households may be concerned about own wealth, in which case $u_k(.) \neq 0$. In order to sharpen our results, we distinguish $u_k(.) = 0$ from $u_k(.) > 0$.

3.1.1 No preference for wealth: $u_k(.) = 0$

We compare the equilibrium path of a market economy with that of the socially optimal one. Clearly, we need to consider $\bar{c} = c$ here. With $u_k(.) = 0$, (11) and (18) become:

$$\frac{\dot{\mu}^m}{\mu^m} = -[(A - \delta) - \rho]\,, \tag{20}$$

$$\frac{\dot{\mu}^o}{\mu^o} = -[(A - \delta) - \rho]\,. \tag{21}$$

As the right hand sides of (20) and (21) are identical, the growth rate of the shadow prices μ^m and μ^o must be identical as well. As a consequence, for the market equilibrium path to be efficient (non-distortionary), the following must hold:

$$\mu^o = \phi\,\mu^m \tag{22}$$

for some constant $\phi > 0$. We assume:

$$0 < \phi < 1\,. \tag{A1}$$

As shown analytically below, the left hand side of Assumption (A1) ensures that in equilibrium (when $\bar{c} = c$): $u_c(c, c) + u_{\bar{c}}(c, c) > 0$. A rise in c for every household raises marginal utility, even though $u_{\bar{c}}(c, c) < 0$. The impact on utility of the rise in the consumption reference level is dominated by the impact of the rise in own consumption. The right hand side of Assumption (A1) implies that $u_{\bar{c}}(c, c, ., .) < 0$, which is the case of positional concerns.

Considering (22) together with (10) and (17), the following condition must be satisfied for the market equilibrium path to be efficient:

$$\frac{\mu^o}{\mu^m} = \frac{u_c(c, c, ., .) + u_{\bar{c}}(c, c, ., .)}{u_c(c, c, ., .)} = 1 + \frac{u_{\bar{c}}(c, c, ., .)}{u_c(c, c, ., .)} = \phi\,, \quad 0 < \phi < 1\,, \tag{23}$$

with ϕ being constant. For any wealth levels, the marginal rate of substitution of $\bar{c}$ for c is constant along the 45-degree line in $(\bar{c}, c)$-space (cf. Arrow and Dasgupta 2009). (23) is implied by a key property that we label *restricted homogeneity with respect to c* (RHc).

Definition 1 (RHc)

Utility function $u(c, \bar{c}, ., .)$ satisfies (RHc) if both functions $[u_c(c, \bar{c}, ., .)+u_{\bar{c}}(c, \bar{c}, ., .)]|_{\bar{c}=c}$ and $[u_c(c, \bar{c}, ., .)]|_{\bar{c}=c}$ are homogeneous in $(c, c, ., .)$ for all $c \in \mathbb{R}_+$.

Property (RHc) restricts the homogeneity requirement to symmetric equilibria, where $\bar{c} = c$. In employing this property, the utility function needs not be homogeneous for $\bar{c} \neq c$.

Lemma 1

Assume (A1). *If and only if the utility function $u(c, c, ., .)$ satisfies (RHc), requirement* (23) *holds. That is, the marginal rate of substitution of $\bar{c}$ for c is constant and negative along an equilibrium path.*

Proof. Sufficiency. If both $[u_c(c, \bar{c}, ., .)]|_{\bar{c}=c}$ and $[u_c(c, \bar{c}, ., .) + u_{\bar{c}}(c, \bar{c}, ., .)]|_{c,c}$ are homogeneous in $(c, c, ., .)$, then both functions have to be homogeneous of the same degree R in $(c, c, ., .)$ (cf. Alonso-Carrera et al. 2006). Consequently,

$$\begin{aligned} \frac{\mu^o}{\mu^m} &= \frac{u_c(c, c, ., .) + u_{\bar{c}}(c, c, ., .)}{u_c(c, c, ., .)} \\ &= \frac{c^R[u_c(1, 1, ., .) + u_{\bar{c}}(1, 1, ., .)]}{c^R[u_c(1, 1, ., .)]} = 1 + \frac{u_{\bar{c}}(1, 1, ., .)}{u_c(1, 1, ., .)} = \phi\,. \end{aligned}$$

Necessity. If

$$\begin{aligned} &\frac{u_c(c, c, ., .) + u_{\bar{c}}(c, c, ., .)}{u_c(c, c, ., .)} = \phi \\ \Rightarrow &\frac{u_c(c, c, ., .) + u_{\bar{c}}(c, c, ., .)}{u_c(c, c, ., .)} = \frac{u_c(\lambda c, \lambda c, ., .) + u_{\bar{c}}(\lambda c, \lambda c, ., .)}{u_c(\lambda c, \lambda c, ., .)} \\ = &\frac{u_c(1, 1, ., .) + u_{\bar{c}}(1, 1, ., .)}{u_c(1, 1, ., .)} = \frac{c^R\,[u_c(1, 1, ., .) + \ u_{\bar{c}}(1, 1, ., .)]}{c^R\, u_c(1, 1, ., .)}\,, \end{aligned}$$

where we set $\lambda = 1/c$. ||

Graphically speaking, (RHc) requires indifference curves for $(\bar{c}, c)$ to have the same slope along the 45-degree line from the origin. Put differently, the trade-off between $\bar{c}$ and c does not depend on the level of consumption for $\bar{c} = c$.

Proposition 1 ($u_{\bar{k}}(.) = u_k(.) = 0$)
Assume (A1) *and let* $k_0^m = k_0^o$. *If and only if (RHc) holds, the market equilibrium path is efficient, that is, it coincides with the socially optimal one.*

Proof. 1. We first note that the market economy and the social optimum face identical restrictions: (i) $k_0^m = k_0^o$; (ii) $\dot{k}_t^i = (A - \delta)k_t^i - c_t^i$, $i \in \{m, o\}$.
2. By Lemma 1, (RHc) implies that the shadow values of capital, μ^o and μ^m are proportional and grow at the same rate. As a consequence, the Euler equations (11') and (18') are identical. As one consequence, consumption grows at the same rate in the two frameworks (see Appendix).
3. Consider $k_t^m = k_t^o$ for all $t > 0$. Then property (RHc) implies that the two transversality conditions (8) and (15) are equivalent. Let $k_t = k_t^m = k_t^o$. As $\phi > 0$,

$$\lim_{t\to\infty} \mu_t^o k_t e^{-\rho t} = 0 \Leftrightarrow \lim_{t\to\infty} \phi\, \mu_t^m k_t e^{-\rho t} = 0 \Leftrightarrow \lim_{t\to\infty} \mu_t^m k_t e^{-\rho t} = 0\,.$$

As an implication, $c_0^m = c_0^o$. As the consumption growth rates coincide, $c_t^m = c_t^o$. ||

Proposition 1 shows that positional preferences need not be distortionary. In order for the market equilibrium path to be efficient (identical to the optimal one), a restriction on preferences must hold: RHc. This restriction ensures that the private marginal utility of consumption is proportional to the social marginal utility of consumption *in a symmetric equilibrium with* $\bar{c} = c$. Specifically, the constant of proportionality, ϕ, is independent of the level of c. In this case individual households and the social planner follow the same Euler equation in determining the consumption growth rate. Intuitively, for the same initial value of the capital stock, thus, the paths of consumption and capital essentially coincide.

A condition in the nature of (RHc) was first introduced by Alonso-Carrera et al. (2006). However, while their paper concentrates on habits, this paper focuses on positional preferences regarding (relative) wealth. Proposition 1, though, is essential for the discussion of preferences for wealth. Interestingly, Arrow and Dasgupta (2009) discuss an equivalent condition to that of Alonso-Carrera et al. (2006) – however without being aware of the latter paper.

Let us turn to the example of Ghosh and Wendner (2014). In the absence of a preference for wealth ($\xi = 0$), their preference specification becomes

$$u(c, \bar{c}, ., .) = \frac{1}{\gamma}\left[c^{1-\eta_c}\left(\frac{c}{\bar{c}}\right)^{\eta_c}\right]^{\gamma} = \frac{1}{\gamma}\left[c\,\bar{c}^{-\eta_c}\right]^{\gamma}. \qquad (24)$$

As can be easily verified, (RHc) is satisfied:

$$\frac{u_c(c, c, ., .) + u_{\bar{c}}(c, c, ., .)}{u_c(c, c, ., .)} = 1 - \eta_c\,, \quad 0 \leq \eta_c < 1\,.$$

Thus, $0 < \phi = 1-\eta_c$ is constant. Consequently, Proposition 1 applies, and positional concerns with respect to consumption do *not* introduce any distortion.

A second interesting example is given in Rauscher (1997). In his endogenous growth framework, Rauscher (1997) argues that relative consumption *does* introduce a distortion, unless the elasticity of marginal utility equals unity (Proposition

2, p. 38). Rauscher specifies instantaneous utility as follows:

$$u(c, \bar{c}) = v(c) + w\left(\frac{c}{\bar{c}}\right) .$$

That is, the marginal rate of substitution of $\bar{c}$ for c becomes:

$$\frac{u_c(c, \bar{c}) + u_{\bar{c}}(c, \bar{c})}{u_c(c, \bar{c})}|_{\bar{c}=c} = \frac{v'(c)}{v'(c) + w'(1)/c} ,$$

which, in general needs not be constant (in c). In fact, condition (RHc) is satisfied – that is, this marginal rate of substitution is constant – only under the condition that $v'(c)$ is homogeneous of degree -1. In this case,

$$\frac{v'(c)}{v'(c) + w'(1)/c} = \frac{c^{-1}v'(1)}{c^{-1}v'(1) + c^{-1}w'(1)/1} = \frac{v'(1)}{v'(1) + w'(1)} , \quad \text{constant.}$$

Homogeneity of degree (-1) of $v'(c)$ implies $v'(c) = c^{-1}v'(1)$, thus $v''(c) = -c^{-2}v'(1)$. Thus, the elasticity of marginal utility (regarding $v(c)$, as defined by Rauscher) becomes:

$$-\frac{v''(c)c}{v'(c)} = -\frac{-c^{-2}v'(1)c}{c^{-1}v'(1)} = 1 ,$$

that is, in this example (RHc) is satisfied only if the elasticity of marginal utility equals one. The latter is exactly the condition, identified by Rauscher (1997) for positional preferences for consumption not to introduce inter-temporal distortions.

In what follows, we show that the result of Proposition 1 is *not* robust with respect to wealth-dependent preferences.

3.1.2 Preference for wealth: $u_k(.) > 0$, $u_{\bar{k}}(.) = 0$

In contrast to the previous subsection, we allow households to have a preference for wealth. Here, households care about own wealth, $u_k(.) > 0$, but they have no positional preference for wealth, $u_{\bar{k}}(.) = 0$. We relax this assumption in the subsequent subsection. To sharpen results, though, we distinguish preferences for wealth from *positional* preferences for wealth (when $u_{\bar{k}}(.) > 0$).

The steps followed here and in the next subsection resemble those of the above discussion — with the right adjustments, though. With $u_k(.) > 0$, (11) and (18) become:

$$\frac{\dot{\mu}^m}{\mu^m} = -[(A-\delta)-\rho] - \frac{u_k(c^m, c^m, k^m, .)}{u_c(c^m, c^m, k^m, .)}\,, \tag{25}$$

$$\frac{\dot{\mu}^o}{\mu^o} = -[(A-\delta)-\rho] - \frac{u_k(c^o, c^o, k^o, .)}{u_c(c^o, c^o, k^o, .) + u_{\bar{c}}(c^o, c^o, k^o, .)}\,. \tag{26}$$

Following the above arguments, for the Euler equations (25) and (26) to be satisfied, it is necessary that the right hand sides coincide. Inspection of the right hand sides of (25) and (26), however, immediately shows the following. If $u_{\bar{c}}(c, c, k, .) \neq 0$, then, if $c^o_t = c^m_t$ and $k^o_t = k^m_t$ for some t, the right hand sides of (25) and (26) cannot be equal.

Proposition 2 ($u_k(.) > 0$, $u_{\bar{k}}(.) = 0$)
Let $k^m_0 = k^o_0$. If households exhibit a preference for wealth, $u_{\bar{k}}(c, c, k, .) > 0$, then positional concerns for consumption always *introduce a distortion. This holds true, regardless of whether or not (RHc) is satisfied.*

Proof. Considering the Euler equations (25) and (26), if $u_{\bar{c}}(c^o, c^o, k^o, .) \neq 0$, the shadow prices μ^o and μ^m grow at differing rates (even if $k^m_0 = k^o_0$). Specifically, if $u_{\bar{c}}(c^o, c^o, k^o, .) < 0$, μ^o declines at a higher rate than μ^m. Consequently, both k^m and c^m grow at a rate different from the growth rate of k^o and c^o. ||

Proposition 2 shows that positional concerns with respect to consumption are always distortionary, once households care for wealth. In this case, the market equilibrium path is *never* efficient. Referring to Proposition 1, we see that positional concerns with respect to consumption need not be distortionary when preferences do not exhibit a desire for wealth. In that context, if (RHc) is satisfied, then positional concerns with respect to consumption are not distortionary. The (RHc) property itself is not "too" restrictive – it is satisfied for many specifications of the utility function (and Ghosh and Wendner (2014) is one of these). However, as soon as households exhibit a preference for wealth, positional concerns with respect to consumption are distortionary *for all* specifications of the utility function.

The intuition for this result stems from the fact that the marginal rate of substitution of wealth for consumption always differs between the market equilibrium and the social optimum. Specifically, the marginal utility of wealth is affected by the reference consumption level. The social planner always considers the marginal disutility from reference consumption, while individual households in the market equilibrium never do. For this reason, the marginal rates of substitution of capital for consumption differ. As a consequence, the Keynes-Ramsey rules differ between the market equilibrium and the social optimum, and so do the consumption- (capital-) growth rates.

In the example of Ghosh and Wendner (2014), the current case corresponds to: $\xi > 0$, $\eta_k = 0$. That is, their preference specification becomes

$$u(c, \bar{c}, ., .) = \frac{1}{\gamma} \left[c^{1-\eta_c} \left(\frac{c}{\bar{c}} \right)^{\eta_c} k^{\xi} \right]^{\gamma} = \frac{1}{\gamma} \left[c \, \bar{c}^{-\eta_c} \, k^{\xi} \right]^{\gamma} . \tag{27}$$

As shown above, (RHc) is satisfied. Here however, once $\eta_c > 0$, the right hand sides of the Euler equations do not coincide. For any date t, consider their difference:

$$\frac{u_k(c, c, k, .)}{u_c(c, c, k, .) + u_{\bar{c}}(c, c, k, .)} - \frac{u_k(c, c, k, .)}{u_c(c, c, k, .)} = \frac{c}{k} \, \frac{\eta_c \, \xi}{1 - \eta_c} .$$

Thus, it is the preference for wealth, $\xi > 0$, that makes the consumption positionality distortionary.

In what follows, we show that positional concerns with respect to wealth introduce a further distortion that is capable of exactly counterbalancing the distortionary effect of relative consumption under wealth dependent preferences.

3.2 Positional concerns with respect to consumption *and* wealth

In contrast to the subsection above, here we consider the case in which households have positional preferences not only with respect to consumption but also with respect to wealth. That is we consider $u_{\bar{c}} < 0$ *and* $u_{\bar{k}} < 0$. Contrary to the subsection above, here households exhibit not only a preference for wealth but also a preference for *relative wealth*.

Following the methodology developed in subsection 3.1.1, we first consider the Euler equations:

$$\frac{\dot{\mu}^m}{\mu^m} = -[(A-\delta)-\rho] - \frac{u_k(c^m, c^m, k^m, k^m)}{u_c(c^m, c^m, k^m, k^m)}\,. \tag{28}$$

$$\frac{\dot{\mu}^o}{\mu^o} = -[(A-\delta)-\rho] - \frac{u_k(c^o, c^o, k^o, k^o) + u_{\bar{k}}(c^o, c^o, k^o, k^o)}{u_c(c^o, c^o, k^o, k^o) + u_{\bar{c}}(c^o, c^o, k^o, k^o)}\,. \tag{29}$$

In contrast to the case without wealth-positionality, here, if $u_{\bar{k}}$ and $u_{\bar{c}}$ are "well behaved" in a specific sense, the right hand sides of (28) and (29) can be equal. Specifically, the right hand sides of (28) and (29) are equal (for $c^m = c^o$ and $k^m = k^o$) if and only if

$$\begin{aligned}
&\frac{u_k(c,c,k,k)}{u_c(c,c,k,k)} = \frac{u_k(c,c,k,k) + u_{\bar{k}}(c,c,k,k)}{u_c(c,c,k,k) + u_{\bar{c}}(c,c,k,k)} \\
&\Leftrightarrow \frac{u_c(c,c,k,k) + u_{\bar{c}}(c,c,k,k)}{u_c(c,c,k,k)} = \frac{u_k(c,c,k,k) + u_{\bar{k}}(c,c,k,k)}{u_k(c,c,k,k)} \\
&\Leftrightarrow \frac{u_{\bar{c}}(c,c,k,k)}{u_c(c,c,k,k)} = \frac{u_{\bar{k}}(c,c,k,k)}{u_k(c,c,k,k)}\,.
\end{aligned} \tag{30}$$

If (RHc) is satisfied, then a parallel homogeneity condition regarding k must be satisfied for (30) to potentially hold.

Definition 2 (RHk)

Utility function $u(.,.,k,\bar{k})$ satisfies (RHk) if both functions $[u_k(.,.,k,\bar{k})+u_{\bar{k}}(.,.,k,\bar{k})]|_{\bar{k}=k}$ and $[u_k(.,.,k,\bar{k})]|_{\bar{k}=k}$ are homogeneous in $(.,.,k,k)$ for all $k \in \mathbb{R}_+$.

Property (RHk) restricts the homogeneity requirement to symmetric equilibria, where $\bar{k} = k$. In employing this property, the utility function needs not be homogeneous for $\bar{k} \neq k$. Graphically speaking, (RHk) requires indifference curves for $(\bar{k}, k)$ to have the same slope along the 45-degree line from the origin. Put differently, the trade-off between $\bar{k}$ and k does not depend on the level of capital for $\bar{k} = k$. Moreover, the degrees of homogeneity for (RHc) and (RHk) are not restricted to be equal.

Proposition 3 ($u_k(.) > 0$, $u_{\bar{k}}(.) < 0$)

Assume $k_0^m = k_0^o$, (RHc) and (RHk). If and only if the utility function is weakly *separable in c and k*

$$u(c, \bar{c}, k, \bar{k}) = U\left(v(c, \bar{c}), w(k, \bar{k})\right)$$

and the constant marginal rates of substitution of $\bar{c}$ *for* c *(of* $\bar{k}$ *for* k*) are identical,*

$$\frac{u_{\bar{c}}(c, c, k, k)}{u_c(c, c, k, k)} = \frac{u_{\bar{k}}(c, c, k, k)}{u_k(c, c, k, k)}, \tag{31}$$

the market equilibrium path is efficient, that is, it coincides with the socially optimal one.

Proof. The proof follows the same steps as that for Proposition 1 and will not be repeated here. However, we need to establish equality of the Euler equations.

1. By (31), the shadow prices of capital, μ^m and μ^o grow (decline) at the same, not necessarily constant rate. Consequently, $\mu^o = \phi\,\mu^m$, for some *constant* $0 < \phi < 1$. The necessary first-order conditions for consumption are given by

$$\mu^m = u_c(c, c, k, k)\,, \tag{32}$$

$$\mu^o = u_c(c, c, k, k) + u_{\bar{c}}(c, c, k, k)\,. \tag{33}$$

(RHc) implies that the marginal rate of substitution of $\bar{c}$ for c ($MRS_{\bar{c}c}$) is independent of c. However, as k grows at a rate different from zero, $MRS_{\bar{c}c}$ also needs to be independent of k – otherwise ϕ is not constant. Weak separability of the utility function in c is equivalent to independence of $MRS_{\bar{c},k}$ from k. We note that $MRS_{\bar{c}c} = (\phi - 1)$ is a constant.

2. Equality of the marginal rates of substitution (31) requires

$$MRS_{\bar{k},k} \equiv \frac{u_{\bar{k}}(c, c, k, k)}{u_k(c, c, k, k)} = \phi - 1\,.$$

That is, $MRS_{\bar{k},k}$ needs to be independent of c and k as well. (RHk) is equivalent to independence of k (see Lemma 1 for the argument). Weak separability of the utility function in k is equivalent to independence of $MRS_{\bar{k},k}$ from c. ||

The proposition specifies the conditions under which positional preferences for consumption and wealth are not distortionary. If either of the conditions is violated, the market equilibrium path differs from the socially optimal one. The key condition is equality (and constancy) of the marginal rates of substitution. Intuitively, while positionality in consumption induces households to lower savings, positionality on wealth induces households to increase savings. Under equality of the marginal rates of substitution (31) these opposing effects exactly cancel each other. Therefore, if preferences depend on wealth but not on relative wealth, positional preferences with respect to consumption are always distortionary – as there is no counteracting positionality with respect to wealth. Here, with positionality with respect to both consumption and wealth, Proposition 3 shows that positional preferences need not be distortionary, though only under restrictive assumptions. This result is in contrast to Nakamoto (2009) who argues that once households exhibit a preference for wealth, concern for relative consumption always introduces a distortion.

Corollary 1 ($u_{\bar{c}}(.) = 0$, $u_{\bar{k}}(.) < 0$)
If preferences are positional with respect to wealth but not with respect to consumption, the market equilibrium path is always inefficient, that is, it never coincides with the socially optimal one.

The corollary follows directly from (30). As preferences depend on relative wealth but not on relative consumption – as there is no counteracting positionality with respect to consumption – the positional preferences are always distortionary. Corollary 1 provides an interesting insight. While positional preferences for consumption alone need not be distortionary (cf. Proposition 1), positional preferences for wealth alone are always distortionary.

To illustrate the proposition, let us turn to the example of Ghosh and Wendner (2014). For the general case, their preference specification is given by

$$u(c, \bar{c}, k, \bar{k}) = \frac{1}{\gamma} \left[c\, \bar{c}^{-\eta_c} \left(k\, \bar{k}^{-\eta_k} \right)^{\xi} \right]^{\gamma} . \tag{34}$$

As can easily be verified, (RHc) and (RHk) are both is satisfied. Moreover, the utility function is weakly separable in c and k. Specifically:

$$\frac{u_{\bar{c}}(c,c,k,k)}{u_c(c,c,k,k)} = -\eta_c\,, \qquad\qquad 0 \le \eta_c < 1\,,$$
$$\frac{u_{\bar{k}}(c,c,k,k)}{u_k(c,c,k,k)} = \begin{cases} -\eta_k & \xi > 0 \\ 0 & \xi = 0 \end{cases}\,, \qquad\qquad 0 \le \eta_k < 1\,.$$

Equality of the marginal rates of substitution (31) requires $\eta_c = \eta_k$, in which case $\phi = 1 - \eta_c = 1 - \eta_k$. Corollary 1 is immediate, as $\eta_k > \eta_c = 0$ – thus, condition $\eta_c = \eta_k$ is not satisfied.

4 Conclusions

In an endogenous growth context with exogenous labor supply, this paper addresses the research question of whether or not positional preferences are distortionary. The paper shows that the answer depends on the type of positionality. Preferences may exhibit positional concerns for consumption or for wealth (or both).

Without positional concerns for consumption, positional preferences for wealth are always distortionary. The reason is that the marginal rate of substitution of wealth for consumption is always smaller in a social optimum compared to the market equilibrium, as the social planner takes the negative wealth externality (coming from a rise in k) into account whereas individual households do not. As a consequence, households over-save compared to the social optimum.

The same result does not hold *in the presence* of positional concerns for consumption. In that case, the consumption positionality exerts an opposing impact on savings. A constancy- and equality of marginal rates of substitution-property ensures that these opposing effects of the wealth- and consumption positionalities exactly offset each other. If this property holds, the market equilibrium path is efficient despite positional preferences.

Without positional concerns for wealth, positional preferences for consumption are always distortionary – except for the case that preferences are not wealth dependent and the marginal rate of substitution of reference consumption for consumption is constant. In this latter case, the shadow prices of capital of the market economy and the social optimum are proportional to each other, and the market economy path is efficient.

The paper derives necessary and sufficient conditions for positional preferences to be non-distortionary. These conditions, though, are shown to be quite restrictive.

Appendix

Proposition 1

By (RHc), considering (10), (17) and (23), u_c grows at the same rate as $u_{\bar{c}}$:

$$\frac{(u_{\bar{c}c}(c,c,.,.)+u_{\bar{c}\bar{c}}(c,c,.,.))c}{u_{\bar{c}}(c,c,.,.)}\left(\frac{\dot{c}}{c}\right)^{o}=\frac{(u_{cc}(c,c,.,.)+u_{c\bar{c}}(c,c,.,.))c}{u_c(c,c,.,.)}\left(\frac{\dot{c}}{c}\right)^{m}.$$

Note that (RHc) implies that the elasticity of marginal utility with respect to c is is identical between the market equilibrium and the social optimum:

$$\begin{aligned}\frac{(u_{\bar{c}c}(c,c,.,.)+u_{\bar{c}\bar{c}}(c,c,.,.))c}{u_{\bar{c}}(c,c,.,.)}&=\frac{((\phi-1)u_{cc}(c,c,.,.)+(\phi-1)u_{c\bar{c}}(c,c,.,.))c}{(\phi-1)u_c(c,c,.,.)}\\&=\frac{(u_{cc}(c,c,.,.)+u_{c\bar{c}}(c,c,.,.))c}{u_c(c,c,.,.)},\end{aligned}$$

where we make use of the fact that $u_{\bar{c}}(c,c,.,.)=(\phi-1)u_c(c,c,.,.)$. Thus,

$$\left(\frac{\dot{c}}{c}\right)^{m}=\left(\frac{\dot{c}}{c}\right)^{o}.\ ||$$

References

[1] Alonso-Carrera, J., J. Caball, X. Raurich (2006), Welfare Implications of the Interaction between Habits and Consumption Externalities, International Economic Review 47, 557-571.

[2] Arrow, K.J., P.S. Dasgupta (2009), Conspicuous Consumption, Inconspicuous Leisure, Economic Journal 119, F497-F516.

[3] Brekke, K. A., R. B. Howarth (2002), Status, Growth and the Environment. Goods as Symbols in Applied Welfare Economics, Cheltenham: Edward Elgar.

[4] Clark, A.E., P. Frijters, M.A. Shields (2008), Relative Income, Happiness, and Utility: An Explanation for the Easterlin Paradox and other Puzzles, Journal of Economic Literature 46, 95–114.

[5] Corneo, G., O. Jeanne (1997), On Relative-wealth Effects and the Optimality of Growth, Economics Letters 54, 87-92.

[6] Corneo, G., O. Jeanne (2001), On Relative Wealth Effects and Long-run Growth, Research in Economics 55, 349358.

[7] Darwin, C. (1871), The Descent of Men and Selection in Relation to Sex, London: John Murray.

[8] Easterlin, R. (1995), Will Raising the Incomes of All Increase the Happiness of All? Journal of Economic Behavior & Organization 47, 35-47.

[9] Fisher, W.H., F.X. Hof (2000), Relative Consumption, Economic Growth, and Taxation, Journal of Economics 72, 241262.

[10] Fisher, W.H., F.X. Hof (2005), Status Seeking in the Small Open Economy, Journal of Macroeconomics 27, 209-232.

[11] Futagami, K., A. Shibata (1998), Keeping One Step Ahead of the Joneses: Status, the Distribution of Wealth, and Long Run Growth, Journal of Economic Behavior & Organization 36, 93111.

[12] Ghosh, S., R. Wendner (2014), Positional Preferences, Endogenous Growth, and Optimal Income- and Consumption Taxation, SSRN: http://ssrn.com/abstract=2532312.

[13] Hof, F.X., F. Wirl (2008), Wealth Induced Multiple Equilibria in Small Open Economy Versions of the Ramsey Model, Homo Oeconomicus 25, 107-128.

[14] Johansson-Stenman, O., F. Carlsson, D. Daruvala (2002), Measuring Future Grandparents' Preferences for Equality and Relative Standing, Economic Journal 112, 362–383.

[15] Johansson-Stenman, O., P. Martinsson (2006), Honestly, Why Are You Driving a BMW?, Journal of Economic Behavior & Organization 60, 129-146.

[16] Liu, W.F., S.J. Turnovsky (2005), Consumption Externalities, Production Externalities, and Long-Run Macroeconomic Efficiency, Journal of Public Economics 89, 1097-1129.

[17] Nakamoto, Y. (2009), Jealousy and Underconsumption in a One-sector Model with Wealth Preference, Journal of Economic Dynamics and Control 33, 2015-2029.

[18] Pham, T.K.C. (2005), Economic Growth and Status-seeking through Personal Wealth, European Journal of Political Economy 21, 404427.

[19] Rauscher, M. (1997), Conspicuous Consumption, Economic Growth, and Taxation, Journal of Economics 66, 3542.

[20] Rebelo, S. (1991), Long Run Policy Analysis and Long Run Growth, Journal of Political Economy 99, 500-521.

[21] Solnik, S.J., D. Hemenway (1998), Is More Always Better? A Survey on Positional Concerns, Journal of Economic Behavior & Organization 37, 373-383.

[22] Solnik, S.J., D. Hemenway (2005), Are Positional Concerns Stronger in Some Domains than in Others?, American Economic Review 95, 147-151.

[23] Truyts, T. (2010), Social Status in Economic Theory, Journal of Economic Surveys 24, 137 - 169.

[24] Tournemaine, F., C. Tsoukis (2008), Relative Consumption, Relative Wealth and Growth, Economics Letters 100, 314-316.

[25] Wendner, R. (2014), Ramsey, Pigou, Heterogeneous Agents, and Non-atmospheric Consumption Externalities, Journal of Public Economic Theory 16, 491521.