Growth with Endogenous Direction of Technical Change

LI, Defu and Bental, Benjamin

School of Economics and Management, Tongji University, Department of Economics, University of Haifa

29 January 2015

Growth with Endogenous Direction of Technical Change

Defu Li[†]
School of Economics and Management, Tongji University
Benjamin Bental[‡]
Department of Economics, University of Haifa

Abstract: By extending the range of admissible factor accumulation and innovation investment elasticities, this paper expands the Acemoglu (2003) model and obtains several results. First, it identifies conditions for the existence of a steady-state equilibrium and shows that Uzawa's theorem is obtained as a special case of these conditions. Second, it demonstrates that along a steady-state equilibrium path, technological progress can include both labor-augmenting and capital-augmenting elements. Third, it shows that the direction of technological progress is determined by the relative size of price elasticities of material factors, and is biased towards the factor with the relatively smaller price elasticity. Finally, the paper finds that technical change has two effects on factor income shares. On one hand, factor shares change when the direction of technical progress changes. On the other hand, when the direction of technical change remains unchanged, in general the speed of technical progress also affects factor shares, unless technical progress is Hicks neutral.

Key Words: steady-state, technical change, Uzawa's theorem, investment elasticities, price elasiticities, factor income shares

JEL: E13; O33; O11; Q01

This version: January 29,2015

[†] Corresponding Author. Address: University of Tongji, Economics and Finance, 1500 Siping Road, Shanghai ,P.R. China, tjldf@tongji.edu.cn.
[‡] University of Haifa, Department of Economics, 199 Aba Khoushy Ave.Mount Carmel, Haifa, Israel, bbental@econ.haifa.ac.il.

Growth with Endogenous Direction of Technical Change

1. Introduction

While the direction of technical change and its speed are equally important, the former is not as deeply researched as the latter. Some of the available conclusions are quite unclear. For example, what factors determine the directions of technical change in the long run? Can technical change take a form that is different from the Harrod-neutral one along a steady-state equilibrium path? How does technological progress affect factor income shares? This paper clarifies some of these issues and shows that steady-state growth paths may be consistent with a broad class of technical progress specifications.

The study of the direction of technological change seems to have preceded that of its speed. As early as in 1932, Hicks (Hicks, 1932) pointed out that changes in relative prices could affect the direction of technical change. In the 1960s, the induced innovation literature (Fellner, 1960; Kennedy, 1964; Sanmelson, 1965; Drandakis and Phelps, 1965) introduced an "innovation possibility frontier". Based on Hicks' work it provided the first systematic study of the determinants of technical change, focusing on the role of factor prices in this context. However, this literature was criticized for its lack of micro-foundations. Consequently, for almost 30 years there was little research on the direction of technological progress. Only the work of Acemoglu (1998, 2002, 2003, 2007, and 2009) which studied the issue using the framework of endogenous technological change (as developed by Romer, 1990, and Aghion and Howitt, 1992) has renewed interest in this question. Acemoglu has not only put forward a new framework to analyze the direction of technological progress, but also pointed out that market size may be an important factor affecting that direction beside relative prices. However, Acemoglu's analysis does not consider how investment elasticities affect factor accumulation and technical innovation. These omissions leave the determinants of the direction of technological progress, and the centrality of Harrod-neutrality as open questions.

The centrality of Harrod-neutrality in Uzawa's (1961) "steady state theorem" is likely to be the most important conclusion related to the direction of technical change. It states that in a neoclassical growth model, the only technical change which is compatible with a steady-state growth path must be Harrod-neutral (unless the production function is Cobb-Douglas). Acemoglu (2003), Jones (2005), Jones and Scrimgeour (2008) and Schlicht (2006) prove that the Uzawa steady-state theorem holds under seemingly very general conditions. Clearly, such findings place an obvious constraint on the research of the direction of technical change. Perhaps just due to this, some scholars (Aghion and Howitt, 1996, p16) suspected the validity of the Uzawa theorem, and others (e.g. Sato, Ramachandran, and Lian, 1999; Sato and Ramachandran, 2000; Li and Huang, 2012, 2014; Irmen, 2013) have tried to prove that the conclusion is actually not quite general. These studies also have some deficiencies. Most importantly from the point of view of the current paper, they failed to acknowledge the crucial role played by the investment elasticities of cumulable factors. Furthermore, they did not provide micro mechanisms determining the direction of technical change. Consequently, while it is possible to prove that technical change need not be Harrod-neutral to generate a steady-state equilibrium, it is important to indicate the crucial factors determining the direction of technical change, and reconcile non-Harrod-neutral technical change with consumer inter-temporal optimization and enterprise profit maximization.

The current paper focuses mainly on the investment elasticities of cumulable factors. It turns out that Harrod-neutrality is crucial for the existence of a steady-state growth path only if the investment elasticity of capital accumulation equals 1, as is implicitly assumed in the traditional literature. Otherwise, non-Harrod-neutral technical change can be consistent with a steady-state growth path. While factor accumulation and technological innovation processes differ from those of Acemoglu (2003), other aspects in this paper are identical to his, making the conclusions about the direction of technical change consistent with inter-temporal optimization and profit maximization.

The rest of the paper is organized as follows: the second section describes the economic environment, and analyses the behavior of households and firms; the third section defines the equilibrium and obtains the main conclusions; the fourth section uses numerical simulations to study the influence of the direction of technical change on economic growth, consumption and factor income shares; the fifth section concludes.

2. Model

2.1Economic Environment

The economic environment of the model is an extension of Acemoglu (2003). The economy consists of two kinds of material factors, denoted by K and L,[3]and three sectors of production; a final goods sector, an intermediate goods sector and a research and development (R&D) sector. The preference structure and production functions are identical to Acemoglu's. The current analysis differs from that of Acemoglu's in the factor accumulation functions and the innovation possibilities frontier.

2.1.1 Final good production

The aggregate production function is given by

$$Y = [\gamma Y_L{}^{\eta} + (1-\gamma) Y_K{}^{\eta}]^{1/\eta}, \quad 0 < \gamma < 1 \tag{1}$$

where Y is an aggregate output produced from inputs produced by labor-intensive and capital-intensive processes, respectively Y_L and Y_K, and the factor-elasticity of substitution is given by $\varepsilon=1/(1-\eta)$, with $0 < \varepsilon < +\infty$.

The labor-intensive and capital-intensive inputs are produced competitively using identical constant elasticity of substitution (CES) production functions with corresponding intermediate inputs, X(i) and Z(i):

$$Y_L = \left[\int_0^N X(i)^{\beta} di\right]^{1/\beta} \text{ and } Y_K = \left[\int_0^M Z(i)^{\beta} di\right]^{1/\beta}, \quad 0 < \beta < 1 \tag{2}$$

where the elasticity of substitution is given by $v = 1/(1-\beta)$. Here N and M represent the measure of different types of labor- and capital-intensive intermediate inputs, respectively. As will be seen below, an increase in N or in M corresponds to a labor- or capital-augmenting technical change.

[3]According to the context of any application they can be respectively capital and labor, skilled and unskilled labor, physical capital and human capital, etc.

2.1.2 Intermediate input production

Intermediate inputs are supplied by monopolists who hold the right to use the relevant patent, and are produced linearly from their respective factors:

$$X(i) = L(i) \text{ and } Z(i) = K(i) \tag{3}$$

2.1.3 Accumulation of material factors

While the above follows precisely the Acemoglu (2003) formulation, the following provides an extension. Specifically, we assume:

$$\begin{cases} \dot{K} = b_K I_K^{\alpha_K}, & b_K > 0,\ 0 \le \alpha_K \le 1 \\ \dot{L} = b_L I_L^{\alpha_L}, & b_L > 0,\ 0 \le \alpha_L \le 1, \end{cases} \tag{4}$$

Here I_K and I_L are respectively resource investments needed to accumulate K and L. With $0 < \alpha_K < 1$ and $0 < \alpha_L < 1$, the marginal returns of the investment processes are diminishing. This may reflect, among other things, that investment in either factor is associated with adjustment costs (Irmen, 2013).[4]

2.1.4 The innovation possibilities frontier

The technology innovation functions are also allowed to be characterized by decreasing returns, as given by:[5]

$$\begin{cases} \dot{M} = b_M I_M{}^{\alpha_M},\ b_M > 0, 0 \le \alpha_M \le 1 \\ \dot{N} = b_N I_N{}^{\alpha_N}, b_N > 0, 0 \le \alpha_N \le 1 \end{cases} \tag{5}$$

In the remainder of the paper it is shown that the four parameters, α_K, α_L, α_M and α_N, are the key determinants of the direction of technological progress which is consistent with steady-state growth. Specifically, it turns out that the "standard" linearity assumptions are not required to achieve such a steady-state.

2.1.5 The representative household

The representative household owns material factors such as capital and labor, as well as the indefinite rights over the use of patents of the production of intermediate goods. The household's goal is to maximize the discounted flow of utility, given by:

$$U = \int_0^{+\infty} \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \tag{6}$$

where *C(t)* is consumption at time t, $\rho > 0$ is the discount rate, and $\theta > 0$ is a utility curvatute coefficient of the household.

2.1.6 Budget constraint

[4]In Acemoglu (2002) labor supply is endogenous, depending on wages but not on investment. In Acemoglu (2003) capital accumulates according to $\dot{K} = I_K$, so that $b_K = 1$, and $\alpha_K = 1$. Labor supply is fixed at some level L.

[5]These functions are used in the Rivera-Batiz and Romer (1991) model of experimental equipment and also by Acemoglu (2002, 2003). The latter assumes $\alpha_M = 1$ and $\alpha_N = 1$.

The representative household's income can be used either for consumption or for investment. The latter consists of four options: it can be used to increase the material factors, K and L, or the "number" of intermediate goods of either type. While the material factors K and L are rented in competitive factor markets, the representative household is a monopoly producer of the intermediate goods. Accordingly, the household faces the following budget constraint:

$$C + I_K + I_L + I_N + I_M = wL + rK + \int_0^N \pi_{X(i)} di + \int_0^M \pi_{Z(i)} di = Y \tag{7}$$

Where $I = I_K + I_L + I_N + I_M$ is total investment, w and r are market prices of L and K, $\pi_{X(i)}$ and $\pi_{Z(i)}$ are monopoly profits of the respective intermediate inputs. For the sake of simplicity, this paper will ignore corner solutions and assume that consumption and all investments are strictly positive, that is, C > 0, $I_K > 0$, $I_L > 0$, $I_N > 0$ and $I_M > 0$.

2.2 Enterprise behavior

The analysis of enterprise behavior is similar to that of Acemoglu (2003), and this paper only reports the main results. Through that analysis, one can obtain the prices of the material factors K and L.

2.2.1 Demand for intermediate goods.

The goods Y, Y_L and Y_K are traded in perfectly competitive markets. The final good Y serves as the numeraire, and p_L and p_K are respectively market prices of Y_L and Y_K. The demand for Y_L and Y_K are derived from profit maximization of the final good producers.

$$\begin{cases} p_K = (1-\gamma)[\gamma + (1-\gamma)(Y_K / Y_L)^{(\varepsilon-1)/\varepsilon}]^{1/(\varepsilon-1)} (Y_K / Y_L)^{-1/\varepsilon} \\ p_L = \gamma[\gamma + (1-\gamma)(Y_K / Y_L)^{(\varepsilon-1)/\varepsilon}]^{1/(\varepsilon-1)} \end{cases} \tag{8}$$

Taking the prices, $p_{Z(i)}$ and $p_{X(i)}$, of the generic inputs, X(i) and Z(i), as given, demand for these inputs is obtained from profit maximization:

$$\begin{cases} Z(i) = Y_K (p_K / p_{Z(i)})^{1/(1-\beta)} \\ X(i) = Y_L (p_L / p_{X(i)})^{1/(1-\beta)} \end{cases} \tag{9}$$

2.2.2 Factor market clearing.

Because intermediate goods are supplied by monopolists who hold the relevant patents, and are produced linearly from their respective factors (see equation 3), we can obtain the price of intermediate inputs from the profit maximization conditions of the monopolies:

$$\begin{cases} p_{Z(i)} = r / \beta \\ p_{X(i)} = w / \beta \end{cases} \tag{10}$$

indicating that each of the intermediate inputs has the same mark-up over marginal cost. Substituting (10) into (9), we find that all capital-intensive and all labor-intensive intermediate goods are produced in equal (respective) quantities.

$$\begin{cases} Z(i) = Z = Y_K\left(\beta\, p_K / r\right)^{1/(1-\beta)} \\ X(i) = X = Y_L\left(\beta\, p_L / w\right)^{1/(1-\beta)} \end{cases} \tag{11}$$

By the production functions of the intermediate inputs (3), the monopolists' demand for labor and capital are respectively equal. The material factor market clearing condition implies:

$$\begin{cases} Z = K / M \\ X = L / N \end{cases} \tag{12}$$

Substituting equations (12) into (2), we obtain the equilibrium quantities of labor-intensive and capital-intensive goods:

$$\begin{cases} Y_L = \left[\int_0^N X^\beta di\right]^{1/\beta} = N^{(1-\beta)/\beta} L \\ Y_K = \left[\int_0^M Z^\beta di\right]^{1/\beta} = M^{(1-\beta)/\beta} K \end{cases} \tag{13}$$

Finally, substituting equations (13) into (1), we obtain the amount of the final good produced:

$$Y = [\gamma (N^{(1-\beta)/\beta} L)^\eta + (1-\gamma)(M^{(1-\beta)/\beta} K)^\eta]^{1/\eta} \tag{14}$$

In order to simplify notation, we follow Acemoglu (2003) letting $A \equiv N^{(1-\beta)/\beta}$ and $B \equiv M^{(1-\beta)/\beta}$, to obtain:

$$Y = \left[\gamma (AL)^\eta + (1-\gamma)(BK)^\eta\right]^{1/\eta} \tag{15}$$

Therefore, increasing the variety of capital-intensive or labor-intensive intermediate goods, M and N, implies progress of the capital-augmenting or labor-augmenting technologies B and A.

Let $k \equiv BK/AL$ be the ratio of effective capital to effective labor, then

$$k = (M^{(1-\beta)/\beta} K) / (N^{(1-\beta)/\beta} L) \tag{16}$$

and (15) can be rewritten as:

$$f(k) \equiv Y / AL = \left[\gamma + (1-\gamma) k^\eta\right]^{1/\eta} \tag{17}$$

Using equation (17), we transform the market prices of the capital-intensive and labor-intensive goods (8) into the following forms:

$$\begin{cases} p_K = f'(k) \\ p_L = f(k) - k f'(k) \end{cases} \tag{18}$$

Substituting equation (18), (13), and (12) into (11), we have

$$\begin{cases} r = \beta M^{(1-\beta)/\beta} f'(k) \\ w = \beta N^{(1-\beta)/\beta} [f(k) - k f'(k)] \end{cases} \tag{19}$$

Equations (19) indicate that the prices of the material factors are positively related to the respective "number"of the intermediate goods.

By equations (19), (13) and (10), we find the monopoly profits of the intermediate goods producers:

$$\begin{cases} \pi_Z = (p_Z - r)Z = (1-\beta)M^{(1-2\beta)/\beta}Kf'(k) \\ \pi_X = (p_X - w)X = (1-\beta)N^{(1-2\beta)/\beta}L[f(k) - kf'(k)] \end{cases} \tag{20}$$

Equations (20) show that as long as $\beta < ½$ there is a positive relationship between the monopoly profits of the intermediate inputs and the quantity of material factors. This is just the market scale effect emphasized by Acemoglu (2002, 2003).

2.3 Consumer behavior

Households maximize their objective (6) subject to the budget constraint (7), taking as given the factor accumulation and technological change processes (4) and (5).

The corresponding Euler conditions are derived in Appendix A:

$$\begin{cases} \dot{C}/C = \left\{\alpha_K b_K I_K^{\alpha_K - 1} r - (\alpha_K - 1)\dot{I}_K / I_K - \rho\right\}/\theta \\ \dot{C}/C = \left\{\alpha_L b_L I_L^{\alpha_L - 1} w - (\alpha_L - 1)\dot{I}_L / I_L - \rho\right\}/\theta \\ \dot{C}/C = \left\{\alpha_M b_M I_M^{\alpha_M - 1} \pi_Z - (\alpha_M - 1)\dot{I}_M / I_M - \rho\right\}/\theta \\ \dot{C}/C = \left\{\alpha_N b_N I_N^{\alpha_N - 1} \pi_X - (\alpha_N - 1)\dot{I}_N / I_N - \rho\right\}/\theta \end{cases} \tag{21}$$

These conditions reflect the optimal allocation of income among consumption and the four kinds of investment.

<u>Remark 1</u>: When $\alpha_K = b_K = 1$, the capital accumulation function simplifies to the familiar $\dot{C}/C = (r - \rho)/\theta$. Therefore, in a constant $\dot{C}/C$ implies that *r* must be constant. However, if $0 < \alpha_K < 1$, the rate *r* cannot be constant when $\dot{C}/C$ and $\dot{I}_K/I_K$ are constant, unless $\dot{r}/r = (1 - \alpha_K)\dot{I}_K/I_K$. Thus steady-state growth does not necessarily imply a constant market price of capital.

Finally, the transversality condition is

$$\lim_{t\to\infty} K(t)\exp\left[-\int_0^t r(v)dv\right] = 0 \tag{22}$$

3. Equilibrium

3.1. Market Equilibrium

Definition 1: A <u>market equilibrium</u> is obtained when producers maximize profits, the factor and product markets clear and households meet the Euler equations.

Substituting (18), (19) into the family of Euler equations (21), we obtain the market equilibrium Euler equations:

$$
\begin{cases}
\dot{C}/C = \left\{\alpha_K b_K I_K^{\alpha_K - 1}\beta M^{(1-\beta)/\beta} f'(k) - (\alpha_K - 1)\dot{I}_K / I_K - \rho\right\}/\theta \\
\dot{C}/C = \left\{\alpha_L b_L I_L^{\alpha_L - 1}\beta N^{(1-\beta)/\beta} [f(k) - kf'(k)] - (\alpha_L - 1)\dot{I}_L / I_L - \rho\right\}/\theta \\
\dot{C}/C = \left\{\alpha_M b_M I_M{}^{\alpha_M - 1}(1-\beta) M^{(1-2\beta)/\beta} K f'(k) - (\alpha_M - 1)\dot{I}_M / I_M - \rho\right\}/\theta \\
\dot{C}/C = \left\{\alpha_N b_N I_N{}^{\alpha_N - 1}(1-\beta) N^{(1-2\beta)/\beta} L[f(k) - kf'(k)] - (\alpha_N - 1)\dot{I}_N / I_N - \rho\right\}/\theta
\end{cases}
\quad (23)
$$

3.2. Definition and Existence of a Steady-State Equilibrium

Definition 2: A steady-state growth equilibrium (hereafter SSGE) is a market equilibrium in which the growth rates of the endogenous variables (Y, C, I, I_K, I_L, I_M, I_N, K, L, M, N) are nonnegative constants.

The definition is identical to that of Barro and Sala-i-Martin (2004), but slightly different from the definition of a balanced growth path in Acemoglu (2003). Specifically, Definition 2 does not require that the growth rate of K be equal to that of Y, C, I and does not require *r* to be constant.

Definition 3: The investment elasticity of any variable X is given by

$$E_{X,I} = (\dot{X}/X)/(\dot{I}_X/I_X) \quad (24)$$

Lemma 1: In an SSGE

$$\begin{cases} E_{B,I} = [(1-\beta)/\beta]\alpha_M \\ E_{A,I} = [(1-\beta)/\beta]\alpha_N \end{cases} \text{ and } \begin{cases} E_{K,I} = \alpha_K \\ E_{L,I} = \alpha_L \end{cases} \quad (25)$$

Proof: See Appendix B.

Proposition 1: If $\dot{Y}/Y > 0$, in an SSGE:

1) A necessary condition for $\dot{K}/K > 0$ is $E_{B,I} < 1$, otherwise, if $E_{B,I} = 1$ then $\dot{K}/K$ must be 0. The relationship between $\dot{L}/L$ and $E_{A,I}$ is analogous to that of $\dot{K}/K$ and $E_{B,I}$.

2) A necessary condition for $\dot{B}/B > 0$ is $E_{K,I} < 1$, otherwise, if $E_{K,I} = 1$ then $\dot{B}/B$ must be 0. The relationship between $\dot{A}/A$ and $E_{L,I}$ is analogous to that of $\dot{B}/B$ and $E_{K,I}$.

Proof: See Appendix C.

Remark: The Uzawa (1961) theorem is a special case of the second part of proposition 1. Under the standard assumption that $\dot{K} = I - \delta K$, $E_{K,I} = 1$, so that $\dot{B}/B$ must be 0 which is Uzawa's theorem.

Proposition 2: An SSGE exists only if

$$\begin{cases} E_{K,I} + E_{B,I} = 1 \\ E_{L,I} + E_{A,I} = 1 \end{cases} \text{ or } \begin{cases} \alpha_K + [(1-\beta)/\beta]\alpha_M = 1 \\ \alpha_L + [(1-\beta)/\beta]\alpha_N = 1 \end{cases} \quad (26)$$

Proof: See Appendix D.

Remark: Given Lemma 1, since α_K, α_L, α_M, α_N and β are exogenously given, conditions (26) imply that the existence of an SSGE is a "knife-edge".[6]

[6]Jones (1995), Christiaans (2004) and Growiec (2010) also get similar conclusions.

3.3. The results of steady-state growth equilibrium

In the following part of this paper, we assume that (26) is satisfied. Define $s_N \equiv I_N/Y$, $s_M \equiv I_M/Y$, $s_K \equiv I_K/Y$, $s_L \equiv I_L/Y$, and $s_C \equiv C/Y$. The budget constraint becomes:

$$s_C + s_N + s_M + s_K + s_L = 1 \tag{27}$$

Using (4), (5), (16), (17) and (27), the Euler equations (23) can be re-written as：[7]

$$\begin{cases} \dot{C}/C = \rho/\{\beta\alpha_K{}^2 kf'(k)/[s_K f(k)] + 1 - \alpha_K - \theta\} \\ \dot{C}/C = \rho/\{\beta\alpha_L{}^2 [f(k) - kf'(k)]/[s_L f(k)] + 1 - \alpha_L - \theta\} \\ \dot{C}/C = \rho/\{(1-\beta)\alpha_M{}^2 kf'(k)/[s_M f(k)] + 1 - \alpha_M - \theta\} \\ \dot{C}/C = \rho/\{(1-\beta)\alpha_N{}^2 [f(k) - kf'(k)]/[s_N f(k)] + 1 - \alpha_N - \theta\} \end{cases} \tag{28}$$

On the other hand, we also obtain[8]

$$\begin{cases} \dot{C}/C = \left(\dfrac{b_M s_M{}^{\alpha_M}}{\alpha_M}\right)^{(1-\beta)} \left(\dfrac{b_K s_K{}^{\alpha_K}}{\alpha_K}\right)^{\beta} \left(\dfrac{f(k)}{k}\right)^{\beta} \\ \dot{C}/C = \left(\dfrac{b_N s_N{}^{\alpha_N}}{\alpha_N}\right)^{(1-\beta)} \left(\dfrac{b_L s_L{}^{\alpha_L}}{\alpha_L}\right)^{\beta} [f(k)]^{\beta} \end{cases} \tag{29}$$

The seven equations in (27), (28) and (29) can be solved for the seven steady-state equilibrium variables $(\dot{C}/C)^*, k^*, s_C^*, s_K^*, s_L^*, s_M^*, s_N^*$. These variables are determined by the parameters $(\rho, \theta, \beta, \gamma, \eta, \alpha_L, \alpha_K, \alpha_N, \alpha_M, b_L, b_K, b_N, b_M)$. The steady state equilibrium growth path, including technological progress and the factor accumulation processes are totally endogenous, determined by equations (27), (28), and (29).

Definition 4: The direction of technological progress is $DT \equiv (\dot{B}/B)/(\dot{A}/A)$.

When $\dot{A}/A>0$ and $\dot{B}/B=0$, then $DT=0$, which means a purely labor-augmenting technological progress (i.e. Harrod-neutral); when $\dot{A}/A=0$ and $\dot{B}/B>0$, then $DT\to+\infty$, and technological progress is purely capital augmenting (i.e. Solow-neutral); when $\dot{A}/A=\dot{B}/B>0$, $DT=1$, and technological progress is Hicks-neutral.

Figure 1 shows different directions of technological progress:

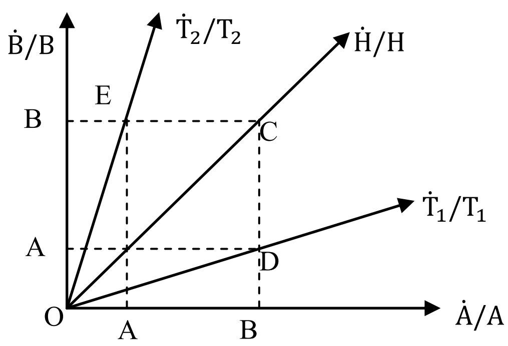

Figure 1: Direction of technological progress

[7]See appendix E.
[8]See appendix F.

Clearly, the axes represent Harrod-neutral (horizontal) and Solow-neutral (vertical) technical change. The diagonal $\dot{H}/H$ represents the location of Hicks-neutral technical changes. The ray $\dot{T}_1/T_1$ indicates technical progress which is close to Harrod-neutrality, while $\dot{T}_2/T_2$ is close to Solow-neutrality. Different types of technical changes maybe be associated with the same growth rates but different directions. They may also have the same direction but different growth rates.

In steady-state equilibrium, we obtain：

$$DT = \frac{\dot{B}/B}{\dot{A}/A} = \frac{\dot{M}/M}{\dot{N}/N} = \frac{\alpha_M}{\alpha_N} = \frac{E_{B,I}}{E_{A,I}} \tag{30}$$

And from (26) we also get:

$$DT = \frac{\dot{B}/B}{\dot{A}/A} = \frac{\dot{M}/M}{\dot{N}/N} = \frac{1-\alpha_K}{1-\alpha_L} = \frac{1-E_{K,I}}{1-E_{L,I}} \tag{31}$$

Using equations (30) and (31) we get proposition 3：

Proposition 3: Along a steady-state growth equilibrium path, technological progress can include both labor-augmenting and capital-augmenting elements. The direction of technological progress is determined by the relative investment elasticities of the technological innovation or factors accumulation processes.

Notice that although this model is an extension of Acemoglu (2003), its conclusions are quite different. In Acemoglu's model, technological progress can only be purely labor-augmenting, whereas here as long as $\alpha_K < 1 (\text{or} \, \alpha_M > 0)$, technological progress can consists of both labor-augmenting and capital-augmenting elements along a steady-state equilibrium path. In fact, the direction of technical progress can take any value between zero and infinity. Clearly, the restricted conclusion obtained by Acemoglu (and others) stems from the assumption that the investment elasticity of capital accumulation equals 1, thereby missing the key factors which affect the direction of technological progress. In addition, along the steady-state growth path, the value of DT depends only on the relative size of the investment elasticities of the technology innovation and factors accumulation processes, and has nothing to do with the parameters of the production and utility functions.

Definition 5: The price elasticity of any variable X is given by

$$\varepsilon_{X,p} = (\dot{X}/X)/(\dot{p}_X/p_X) \tag{32}$$

Lemma 2: In an SSGE the price elasticity of capital and labor are given by:

$$\begin{cases} \varepsilon_{K,r} = \alpha_K/(1-\alpha_K) \\ \varepsilon_{L,w} = \alpha_L/(1-\alpha_L) \end{cases} \tag{33}$$

Proof: See Appendix G.

Remark: When $\alpha_K = 1$ ($\alpha_L = 1$), then in an SSGE $\varepsilon_{K,r} = \infty (\varepsilon_{L,r} = \infty)$. That is, if the investment elasticity of capital (labor) is 1, then the price elasticity of capital (labor) is infinite. Therefore, the existing neoclassical growth models implicitly assume that capital accumulates with an unbounded price elasticity in an SSGE owing to the assumption that $\alpha_K = 1$.

Substituting equations (33) into (31), we obtain

$$DT = \frac{\dot{B}/B}{\dot{A}/A} = \frac{\dot{M}/M}{\dot{N}/A} = \frac{(1+\varepsilon_{L,w})}{(1+\varepsilon_{K,r})} \tag{34}$$

Proposition 4: Along a steady-state growth equilibrium path, the direction of technological progress is determined by the relative price elasticities of the factors accumulation processes and is biased towards the factor with relatively smaller price elasticity.

Remark: In an SSGE, when one factor is characterized by infinite price elasticity, technological progress must not include any augmentation of this factor. For example, if capital accumulates with infinite price elasticity, then the technological progress must be purely labor-augmenting. This is just the conclusion of the Uzawa theorem in a neoclassical growth model. Analogously, Li and Huang (2014b) prove that technological progress cannot include labor-augmenting element in the steady state equilibrium of a Malthusian model owing to its assumption of unlimited labor supply, while Tang, Lin and Li (2014) prove that no technical change can exist in the steady-state equilibrium of the Lewis (1954) model owing to its assumption that both labor and capital have infinite price elasiticities.

Based on the result of steady state equilibrium of this model we can provide Proposition 5.

Proposition 5:

i) Along an SSGE factor income shares are constant.

ii) Factor income shares are independent of the rate of technological progress only if technological progress is Hicks-neutral, i.e. α_M/α_N=1.

Proof: See Appendix H.

Remark: This proposition shows that factor income shares may not change even subject to a capital-augmenting technological progress. On the other hand, when technological progress is Harrod neutral and its direction is unchanged, factor income shares will change when the speed of technological progress changes.

4. A numerical analysis

As the steady state equilibrium is determined by a group of non-linear equations, we analyze the impact of DT on other endogenous variables (such as economic growth, consumption rate and the ratio of factor income shares) by numerical methods.

4.1. Parameter setting and the range of technological progress directions

The relevant exogenous parameters of the model are set as follows: ρ=0.05, θ=0.6, b_K=b_L=b_M=b_N=0.07, γ=0.5, β=0.5. The parameters $\alpha_L$and α_K are determined along an SSGE by α_L=1-[(1-β)/β]α_N and α_K=1-[(1-β)/β]α_M.

Since DT=α_M/α_N, in order to observe the influence of different directions of technological progress on the economy, we change α_M and α_N from 0.3 to 0.7.[9] Figure 2 summarizes the result, where $\dot{T}_1/T_1$ represents the case of α_M= 0.3 (close to the purely labor-augmenting change) and $\dot{T}_2/T_2$ corresponds to α_M= 0.7 (close to the purely capital-augmenting change). The blue arrows represent the resulting route of the direction of technological progress.

[9] Although α_M and α_N can change from 0 to 1, when they less than 0.3 or greater than 0.7, holding other parameters constant, Matlab fails to converge to a solution.

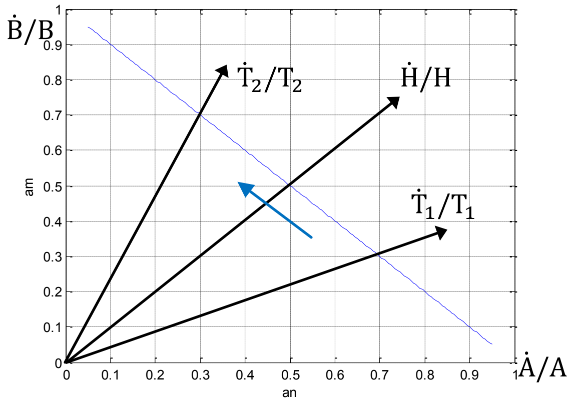

Figure 2: The simulated range of direction of technological progress

4.2 The simulation results

The impact of the above changes of α_M and α_N (through their impact on the direction of technological change) is analyzed for factor substitution elasticities (ε) of 0.6, and 1.2 holding other parameters constant.

First, the changes in α_M and α_N affect the growth rate as shown in Figure 3. As the results are symmetric around (0.5,0.5), we restrict the figures to go from 0.3 to 0.7. For both values of ε, economic growth rates form a spherical surface, where $\alpha_M = \alpha_N = 0.5$ (a Hicks-neutral technological progress) generates the lowest growth rate, as shown in figure 3 (drawn for ε=0.6).

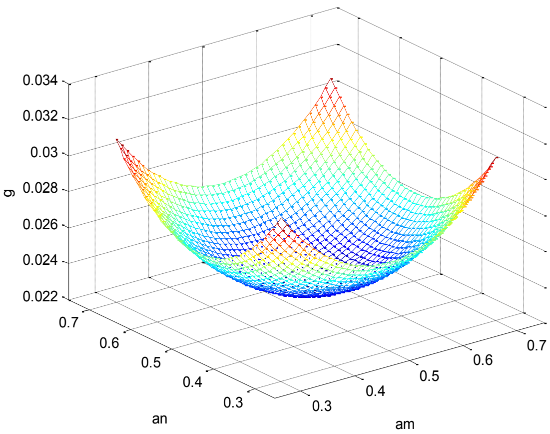

Figure 3 change of economic growth rate (ε=0.6)

Along a path which holds $\alpha_M(\text{or } \alpha_N)$ constant while $\alpha_N(\text{or } \alpha_M)$ changes, economic growth rates form a U-shape contour, where $\alpha_N(\text{or } \alpha_N) = 0.5$ generates the lowest growth rate.

The second effect we investigate is the relationship between the direction of technological progress and the consumption rate, an issue that has received little attention so far. The simulation results below show that the effect is very important, where on the whole, a comparison of Figures 3 and 4 reveals that the consumption and output growth rates are almost mirror-images of one another.

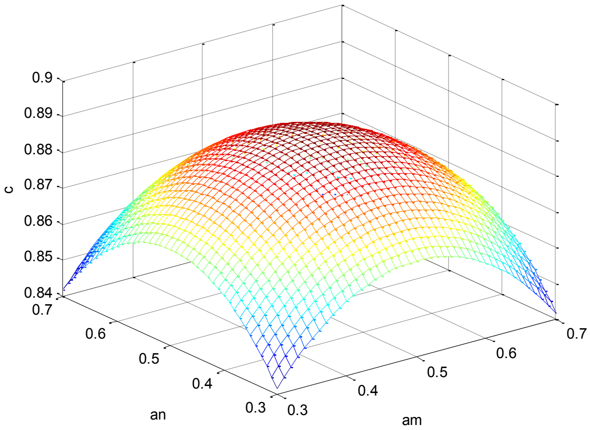

Figure 4: change of consumption rate (ε=0.6)

Thirdly, the direction of technical progress has an important effect on relative factor income shares, as shown in Figure 5.

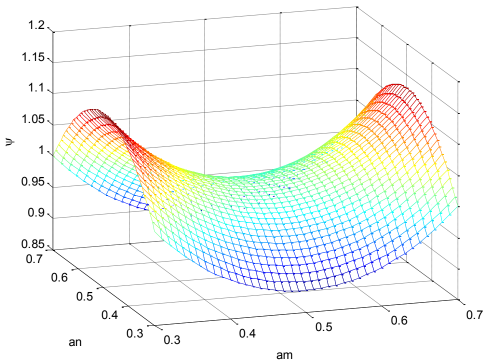

Figure 5a ratio change of factor income share (ε=0.6)

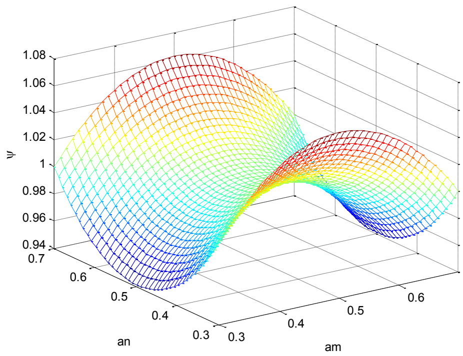

Figure 5b ratio change of factor income share (ε=1.2)

When ε < 1, the contour of the relative factor income share is U-shaped when changing α_M but holding α_N constant and Λ-shaped when changing α_N but holding α_M constant. It is the opposite for ε > 1.

Finally, when we change α_M and α_N keeping $\alpha_M = \alpha_N$, i.e. DT=1, the economic growth rate and consumption rate are still either U-shaped or Λ-shaped, respectively, (see Figures 6 and 7), but the ratio of factor income shares is a constant, see in figure 8.

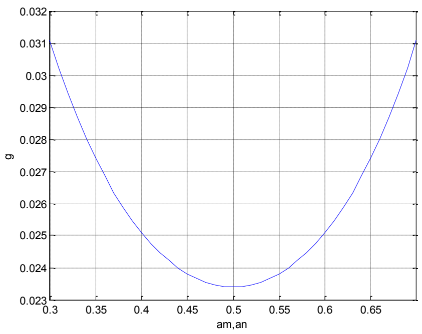

Figure 6 change of economic growth rate (ε=0.6)

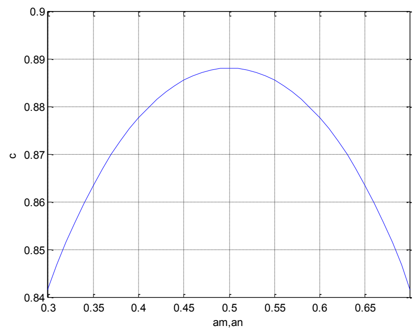

Figure 7 change of consumption rate (ε=0.6)

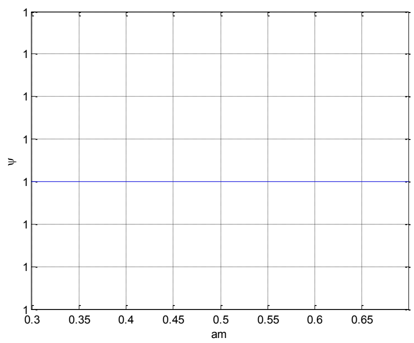

Figure 8 ratio change of factor income share (ε=0.6, $\alpha_N = \alpha_M$)

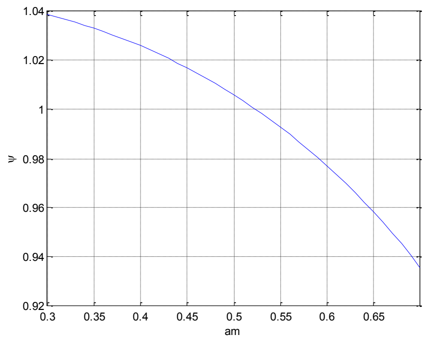

Figure 9 ratio change of factor income share (ε=0.6, $\alpha_N = \alpha_M/1.1$)

However, when keeping $DT = \alpha_M/\alpha_N$ constant (different from 1) while changing α_M and α_N, the ratio of factor income shares changes. For example, figure 9 shows the impact of changing α_M from 0.3 to 0.7, while keeping DT= 1.1.

5. Conclusions

This paper focused on the following questions: what factors determine the long-run direction of technical change? Is a steady-state equilibrium consistent with technical change that is not Harrod-neutral? How does technical change affect factor income shares?

By extending the range of factor accumulation and innovation investment elasticities, this paper expands the Acemoglu (2003) model and shows that the direction of technical change is endogenous and that steady-state equilibrium is compatible with non-Harrod-neutral technical change. Furthermore, the paper shows that factor accumulation and innovation investment elasticities are crucial determinants of the direction of technical change. In particular, Uzawa's steady-state theorem only holds when the investment elasticity of capital accumulation is 1. Finally, numerical simulations reveal that the direction of technical change has an important and quite complicated relationship with economic growth, consumption and the distribution of factor income.

The empirical implications of the above analysis are quite far reaching. In particular, it is crucial to find ways to identify the investment elasticities. Most important is the investment elasticity of capital: if it turns out to be smaller than 1, then capital stock assessed by the usual perpetual inventory method (implicitly assuming that the investment elasticity of capital equals 1) is likely to be wrong. This may affect the conclusions of a significant part of the economic growth literature.

Appendix A： Euler equations

Let the Hamilton associated with the optimization problem be:

$$\begin{aligned} H = U(C)e^{-\rho t} + \lambda_K b_K I_K^{\alpha_K} + \lambda_L b_L I_L^{\alpha_L} + \lambda_M b_M I_M{}^{\alpha_M} + \lambda_N b_N I_N{}^{\alpha_N} \\ + \mu[wL + rK + \pi_X N + \pi_Z M - C - (I_K + I_L + I_N + I_M)] \end{aligned} \tag{A1}$$

The first-order conditions are:

$$
\begin{cases}
C(t)^{-\theta} e^{-\rho t} = \lambda_M \alpha_M b_M I_M{}^{\alpha_M - 1} \\
C(t)^{-\theta} e^{-\rho t} = \lambda_K \alpha_K b_K I_K^{\alpha_K - 1} \\
C(t)^{-\theta} e^{-\rho t} = \lambda_N \alpha_N b_N I_N{}^{\alpha_N - 1} \\
C(t)^{-\theta} e^{-\rho t} = \lambda_L \alpha_L b_L I_L^{\alpha_L - 1} \\
C(t)^{-\theta} e^{-\rho t} = \mu
\end{cases}
\tag{A2}
$$

Taking log-derivatives of both sides of (A2) over time, we obtain

$$
\begin{cases}
-\theta \dot{C}/C - \rho = \dot{\lambda}_M/\lambda_M + (\alpha_M - 1)\dot{I}_M/I_M \\
-\theta \dot{C}/C - \rho = \dot{\lambda}_K/\lambda_K + (\alpha_K - 1)\dot{I}_K/I_K \\
-\theta \dot{C}/C - \rho = \dot{\lambda}_N/\lambda_N + (\alpha_N - 1)\dot{I}_N/I_N \\
-\theta \dot{C}/C - \rho = \dot{\lambda}_L/\lambda_L + (\alpha_L - 1)\dot{I}_L/I_L \\
-\theta \dot{C}/C - \rho = \dot{\mu}/\mu
\end{cases}
\tag{A3}
$$

The motion equations of λ are:

$$
\begin{cases}
\dot{\lambda}_M = -\partial H/\partial M = -\mu \pi_Z \\
\dot{\lambda}_K = -\partial H/\partial K = -\mu r \\
\dot{\lambda}_N = -\partial H/\partial N = -\mu \pi_X \\
\dot{\lambda}_L = -\partial H/\partial L = -\mu w
\end{cases}
\tag{A4}
$$

Based on (A2) and (A4),

$$
\begin{cases}
\dot{\lambda}_M/\lambda_M = -\pi_Z \alpha_M b_M I_M{}^{\alpha_M - 1} \\
\dot{\lambda}_K/\lambda_K = -r \alpha_K b_K I_K^{\alpha_K - 1} \\
\dot{\lambda}_N/\lambda_N = -\pi_X \alpha_N b_N I_N{}^{\alpha_N - 1} \\
\dot{\lambda}_L/\lambda_L = -w \alpha_L b_L I_L^{\alpha_L - 1}
\end{cases}
\tag{A5}
$$

Using (A5) in (A3), we obtain the Euler equations (21).

Appendix B:

Proof: First, from the budget constraint (7) and the definition of a steady-state growth equilibrium, we obtain

$$
\frac{\dot{Y}}{Y} = \frac{\dot{I}}{I} = \frac{\dot{I}_K}{I_K} = \frac{\dot{I}_L}{I_L} = \frac{\dot{I}_M}{I_M} = \frac{\dot{I}_N}{I_N} = \frac{\dot{C}}{C}
\tag{B1}
$$

Then, according to the factor accumulation functions (4) and the innovation possibilities frontier (5), the following must hold in steady-state;

$$\begin{cases} \dot{K}/K = \alpha_K \dot{I}_K / I_K \\ \dot{L}/L = \alpha_L \dot{I}_L / I_L \end{cases} \tag{B2}$$

$$\begin{cases} \dot{M}/M = \alpha_M \dot{I}_M / I_M \\ \dot{N}/N = \alpha_N \dot{I}_N / I_N \end{cases} \tag{B3}$$

By definition, the investment elasticities of technological progress are $E_{B,I} \equiv (\dot{B}/B)/(\dot{I}_M/I_M)$ and $E_{A,I} \equiv (\dot{A}/A)/(\dot{I}_N/I_N)$, and the investment elasticities of factor accumulation are $E_{K,I} \equiv (\dot{K}/K)/(\dot{I}_K/I_K)$ and $E_{L,I} \equiv (\dot{L}/L)/(\dot{I}_L/I_L)$. By the definition of A and B, we get

$$\begin{cases} \dot{B}/B = [(1-\beta)/\beta]\dot{M}/M \\ \dot{A}/A = [(1-\beta)/\beta]\dot{N}/N \end{cases} \tag{B4}$$

Using (B2), (B3) and (B4), we obtain (25).

Appendix C.

<u>Proof:</u> Using the intensive form of the production function (17), we obtain：

$$Y = N^{(1-\beta)/\beta} Lf(k) = M^{(1-\beta)/\beta} Kf(k)/k \tag{C1}$$

In a steady-state growth equilibrium, due to the fact that *k* is constant, we obtain:

$$\begin{cases} \dot{K}/K + [(1-\beta)/\beta]\dot{M}/M = \dot{Y}/Y \\ \dot{L}/L + [(1-\beta)/\beta]\dot{N}/N = \dot{Y}/Y \end{cases} \tag{C2}$$

When technical change is determined by equation (5), using (B1) and (B3) in (C2) yields

$$\begin{cases} \dot{K}/K = (1-E_{B,I})\dot{Y}/Y \\ \dot{L}/L = (1-E_{A,I})\dot{Y}/Y \end{cases} \tag{C3}$$

The above equations show that when technological innovation is endogenously determined by investment, factors get accumulated in a steady state growth equilibrium only if the corresponding investment elasticities of technological innovation are smaller than 1.

On the other hand, if material factors are endogenously determined by equation (4), similarly, using (B1) and (B3) in (C2), we obtain:

$$\begin{cases} \dot{B}/B = (1-E_{K,I})\dot{Y}/Y \\ \dot{A}/A = (1-E_{L,I})\dot{Y}/Y \end{cases} \tag{C4}$$

This result shows that technological progress can happen in a steady state growth equilibrium only if the corresponding investment elasticities of factor accumulation are smaller than 1.

Appendix D.

If material factors are endogenously determined by equation (4), technical change is endogenously determined by equation (5). Combining (B1), (B2), (B3) in (C2), when $\dot{Y}/Y > 0$ we obtain (26) as follow:

$$\begin{cases} E_{K,I} + E_{B,I} = 1 \\ E_{L,I} + E_{A,I} = 1 \end{cases} \text{or} \begin{cases} \alpha_K + [(1-\beta)/\beta]\alpha_M = 1 \\ \alpha_L + [(1-\beta)/\beta]\alpha_N = 1 \end{cases}$$

Accordingly, the sum of the respective investment elasticities must be 1.

Appendix E: Detailed derivation of equation (28)

By arithmetical operation on the Euler equations (23) we get

$$\begin{cases} \dfrac{\dot{C}}{C} = \left\{ \alpha_K \beta \dfrac{b_K I_K^{\alpha_K}}{K} \dfrac{Y}{I_K} \dfrac{M^{(1-\beta)/\beta} K}{Y} f'(k) - (\alpha_K - 1) \dfrac{\dot{I}_K}{I_K} - \rho \right\} / \theta \\ \dfrac{\dot{C}}{C} = \left\{ \alpha_L \beta \dfrac{b_L I_L^{\alpha_L}}{L} \dfrac{Y}{I_L} \dfrac{N^{(1-\beta)/\beta} L}{Y} [f(k) - kf'(k)] - (\alpha_L - 1) \dfrac{\dot{I}_L}{I_L} - \rho \right\} / \theta \\ \dfrac{\dot{C}}{C} = \left\{ \alpha_M (1-\beta) \dfrac{b_M I_M{}^{\alpha_M}}{M} \dfrac{Y}{I_M} \dfrac{M^{(1-\beta)/\beta} K}{Y} f'(k) - (\alpha_M - 1) \dfrac{\dot{I}_M}{I_M} - \rho \right\} / \theta \\ \dfrac{\dot{C}}{C} = \left\{ \alpha_N (1-\beta) \dfrac{b_N I_N{}^{\alpha_N}}{N} \dfrac{Y}{I_N} \dfrac{N^{(1-\beta)/\beta} L}{Y} [f(k) - kf'(k)] - (\alpha_N - 1) \dfrac{\dot{I}_N}{I_N} - \rho \right\} / \theta \end{cases} \tag{E1}$$

Using the factor accumulation and technical innovation processes (4) and (5) we get

$$\begin{cases} \dot{K}/K = b_K I_K^{\alpha_K} / K \\ \dot{L}/L = b_L I_L^{\alpha_L} / L \\ \dot{M}/M = b_M I_M{}^{\alpha_M} / M \\ \dot{N}/N = b_N I_N{}^{\alpha_N} / N \end{cases} \tag{E2}$$

Substituting (E2), the investment rates $s_N \equiv I_N/Y$, $s_M \equiv I_M/Y$, $s_K \equiv I_K/Y$, $s_L \equiv I_L/Y$, $k \equiv (M^{(1-\beta)/\beta}K)/(N^{(1-\beta)/\beta}L)$ and $f(k) \equiv Y/(N^{(1-\beta)/\beta}L)$ into (E1), we obtain

$$\begin{cases} \frac{\dot{C}}{C} = \left\{ \alpha_K \beta \frac{kf'(k)}{s_K f(k)} \frac{\dot{K}}{K} - (\alpha_K - 1) \frac{\dot{I}_K}{I_K} - \rho \right\} / \theta \\ \frac{\dot{C}}{C} = \left\{ \alpha_L \beta \frac{f(k) - kf'(k)}{s_L f(k)} \frac{\dot{L}}{L} - (\alpha_L - 1) \frac{\dot{I}_L}{I_L} - \rho \right\} / \theta \\ \frac{\dot{C}}{C} = \left\{ \alpha_M (1-\beta) \frac{kf'(k)}{s_M f(k)} \frac{\dot{M}}{M} - (\alpha_M - 1) \frac{\dot{I}_M}{I_M} - \rho \right\} / \theta \\ \frac{\dot{C}}{C} = \left\{ \alpha_N (1-\beta) \frac{f(k) - kf'(k)}{s_N f(k)} \frac{\dot{N}}{N} - (\alpha_N - 1) \frac{\dot{I}_N}{I_N} - \rho \right\} / \theta \end{cases} \quad \text{(E3)}$$

Using (B1), (B2) and (B3), we get

$$\begin{cases} \frac{\dot{C}}{C} = \left\{ \alpha_K{}^2 \beta \frac{kf'(k)}{s_K f(k)} \frac{\dot{C}}{C} - (\alpha_K - 1) \frac{\dot{C}}{C} - \rho \right\} / \theta \\ \frac{\dot{C}}{C} = \left\{ \alpha_L{}^2 \beta \frac{f(k) - kf'(k)}{s_L f(k)} \frac{\dot{C}}{C} - (\alpha_L - 1) \frac{\dot{C}}{C} - \rho \right\} / \theta \\ \frac{\dot{C}}{C} = \left\{ \alpha_M{}^2 (1-\beta) \frac{kf'(k)}{s_M f(k)} \frac{\dot{C}}{C} - (\alpha_M - 1) \frac{\dot{C}}{C} - \rho \right\} / \theta \\ \frac{\dot{C}}{C} = \left\{ \alpha_N{}^2 (1-\beta) \frac{f(k) - kf'(k)}{s_N f(k)} \frac{\dot{C}}{C} - (\alpha_N - 1) \frac{\dot{C}}{C} - \rho \right\} / \theta \end{cases} \quad \text{(E4)}$$

Equation (28) is directly obtained from (E4).

Appendix F：Detailed derivation of equation (29)

By the definition of the investment rates, we get

$$\begin{cases} I_K = s_K Y = s_K M^{(1-\beta)/\beta} K f(k) / k \\ I_L = s_L Y = s_L N^{(1-\beta)/\beta} L f(k) \\ I_M = s_M Y = s_M M^{(1-\beta)/\beta} K f(k) / k \\ I_N = s_N Y = s_N N^{(1-\beta)/\beta} L f(k) \end{cases} \quad \text{(F1)}$$

Substituting (F1) into (E2), we obtain

$$
\begin{cases}
\dfrac{\dot{K}}{K} = \dfrac{b_K I_K^{\alpha_K}}{K} = b_K [s_K f(k)/k]^{\alpha_K} \dfrac{M^{\alpha_K (1-\beta)/\beta}}{K^{1-\alpha_K}} \\
\dfrac{\dot{L}}{L} = \dfrac{b_L I_L^{\alpha_L}}{L} = b_L [s_L f(k)]^{\alpha_L} \dfrac{N^{\alpha_L (1-\beta)/\beta}}{L^{1-\alpha_L}} \\
\dfrac{\dot{M}}{M} = \dfrac{b_M I_M{}^{\alpha_M}}{M} = b_M [s_M f(k)/k]^{\alpha_M} \dfrac{K^{\alpha_M}}{M^{1-\alpha_M (1-\beta)/\beta}} \\
\dfrac{\dot{N}}{N} = \dfrac{b_N I_N{}^{\alpha_N}}{N} = b_N [s_N f(k)]^{\alpha_N} \dfrac{L^{\alpha_N}}{N^{1-\alpha_N (1-\beta)/\beta}}
\end{cases}
\tag{F2}
$$

Along a steady-state equilibrium growth path, equation (26) holds. Accordingly we get

$$
\begin{cases}
\dfrac{\dot{K}}{K} = b_K [s_K f(k)/k]^{\alpha_K} \left(\dfrac{M^{\alpha_K}}{K^{\alpha_M}} \right)^{(1-\beta)/\beta} \\
\dfrac{\dot{L}}{L} = b_L [s_L f(k)]^{\alpha_L} \left(\dfrac{N^{\alpha_L}}{L^{\alpha_N}} \right)^{(1-\beta)/\beta} \\
\dfrac{\dot{M}}{M} = b_M [s_M f(k)/k]^{\alpha_M} \dfrac{K^{\alpha_M}}{M^{\alpha_K}} \\
\dfrac{\dot{N}}{N} = b_N [s_N f(k)]^{\alpha_N} \dfrac{L^{\alpha_N}}{N^{\alpha_L}}
\end{cases}
\tag{F3}
$$

Using (B1), (B2), (B3) and (F3), we get

$$
\begin{cases}
b_K (s_K f(k)/k)^{\alpha_K} \left[\dfrac{M^{\alpha_K}}{K^{\alpha_M}} \right]^{(1-\beta)/\beta} = \alpha_K \dfrac{\dot{C}}{C} \\
b_L (s_L f(k))^{\alpha_L} \left[\dfrac{N^{\alpha_L}}{L^{\alpha_N}} \right]^{(1-\beta)/\beta} = \alpha_L \dfrac{\dot{C}}{C} \\
b_M (s_M f(k)/k)^{\alpha_M} \dfrac{K^{\alpha_M}}{M^{\alpha_K}} = \alpha_M \dfrac{\dot{C}}{C} \\
b_N (s_N f(k))^{\alpha_N} \dfrac{L^{\alpha_N}}{N^{\alpha_L}} = \alpha_N \dfrac{\dot{C}}{C}
\end{cases}
\tag{F4}
$$

or:

$$
\begin{cases}
\dfrac{M^{\alpha_K}}{K^{\alpha_M}} = \left[\dfrac{\alpha_K \dot{C}/C}{b_K (s_K f(k)/k)^{\alpha_K}} \right]^{\beta/(1-\beta)} \\
\dfrac{N^{\alpha_L}}{L^{\alpha_N}} = \left[\dfrac{\alpha_L \dot{C}/C}{b_L (s_L f(k))^{\alpha_L}} \right]^{\beta/(1-\beta)} \\
\dfrac{K^{\alpha_M}}{M^{\alpha_K}} = \dfrac{\alpha_M \dot{C}/C}{b_M (s_M f(k)/k)^{\alpha_M}} \\
\dfrac{L^{\alpha_N}}{N^{\alpha_L}} = \dfrac{\alpha_N \dot{C}/C}{b_N (s_N f(k))^{\alpha_N}}
\end{cases} \tag{F5}
$$

Equation (29) is directly obtained from (F5).

Appendix G:

Proof: First, from equation (19) $\begin{cases} r = \beta M^{(1-\beta)/\beta} f'(k) \\ w = \beta N^{(1-\beta)/\beta} [f(k) - kf'(k)] \end{cases}$.

In the SSGE, because k is a constant, we obtain

$$
\begin{cases} \dot{r}/r = [(1-\beta)/\beta]\dot{M}/M \\ \dot{w}/w = [(1-\beta)/\beta]\dot{N}/N \end{cases} \tag{G1}
$$

Substituting (G1) into equation (32), we obtain

$$
\begin{cases} \varepsilon_{K,r} = (\dot{K}/K)/\{[(1-\beta)/\beta]\dot{M}/M\} \\ \varepsilon_{L,w} = (\dot{L}/L)/\{[(1-\beta)/\beta]\dot{N}/N\} \end{cases} \tag{G2}
$$

Substituting (B2) and (B3) into (G2), we obtain

$$
\begin{cases} \varepsilon_{K,r} = \alpha_K/\{[(1-\beta)/\beta]\alpha_M\} \\ \varepsilon_{L,w} = \alpha_L/\{[(1-\beta)/\beta]\alpha_N\} \end{cases} \tag{G3}
$$

Using equations (26) and (G3), we obtain equations (33).

.

Appendix H:

i) Proof:

Define the relative factor income shares as $\varphi \equiv rK/wL$. Using factor returns (19) we obtain,

$$
\varphi = kf'(k)/[f(k) - kf'(k)] \tag{H1}
$$

From the production function (17), we obtain,

$$
f'(k) = \left[\gamma + (1-\gamma)k^{\eta}\right]^{1/\eta - 1}(1-\gamma)k^{\eta-1} \tag{H2}
$$

Therefore, along the steady-state equilibrium growth path we have:

$$
\varphi = [(1-\gamma)/\gamma]k^{\eta} \tag{H3}
$$

Along the SSGE k is determined by the underlying parameters $(\rho, \theta, \beta, \gamma, \eta, \alpha_L, \alpha_K, \alpha_N, \alpha_M, b_L, b_K, b_N, b_M)$ which are given. Accordingly, k and φ remain constant.

ii) Proof:

From equation (28), we can get

$$\begin{cases} \dfrac{\beta\alpha_K{}^2 kf'(k)}{s_K f(k)} - \alpha_K = \dfrac{\beta\alpha_L{}^2[f(k)-kf'(k)]}{s_L f(k)} - \alpha_L \\ \dfrac{(1-\beta)\alpha_M{}^2 kf'(k)}{s_M f(k)} - \alpha_M = \dfrac{(1-\beta)\alpha_N{}^2[f(k)-kf'(k)]}{s_N f(k)} - \alpha_N \end{cases} \tag{H4}$$

Suppose $\alpha_M = \alpha_N$. To guarantee a SSGE we require $\alpha_K = \alpha_L$. Using this in (H4) we get

$$\begin{cases} \dfrac{kf'(k)}{f(k)-kf'(k)} = \dfrac{s_K}{s_L} \\ \dfrac{kf'(k)}{f(k)-kf'(k)} = \dfrac{s_M}{s_N} \end{cases} \tag{H5}$$

Implying:

$$\frac{s_M}{s_N} = \frac{s_K}{s_L} \tag{H6}$$

By equation (29), we have:

$$k = \left(\frac{\alpha_N b_M s_M{}^{\alpha_M}}{\alpha_M b_N s_N{}^{\alpha_N}}\right)^{(1-\beta)/\beta} \left(\frac{\alpha_L b_K s_K{}^{\alpha_K}}{\alpha_K b_L s_L{}^{\alpha_L}}\right) \tag{H7}$$

Since $\alpha_M = \alpha_N$ and $\alpha_K = \alpha_L$, (H7) can be rewritten as:

$$k = \left(\frac{b_K}{b_L}\right)\left(\frac{b_M}{b_N}\right)^{(1-\beta)/\beta} \left(\frac{s_M}{s_N} / \frac{s_K}{s_L}\right)^{\alpha_M(1-\beta)/\beta} \left(\frac{s_K}{s_L}\right) \tag{H8}$$

By (H6), (H8) simplifies to:

$$k = \left(\frac{b_K}{b_L}\right)\left(\frac{b_M}{b_N}\right)^{(1-\beta)/\beta} \frac{s_K}{s_L} \tag{H9}$$

Substituting for k in (H5) we obtain

$$\frac{f(k)-kf'(k)}{f'(k)} = \left(\frac{b_K}{b_L}\right)\left(\frac{b_M}{b_N}\right)^{(1-\beta)/\beta} \tag{H10}$$

Equation (H10) shows that when $\alpha_M = \alpha_N = \alpha$, k is independent of α. As a result, changing α_M and α_N while keeping DT=α_M/α_N=1 keeps factor income shares constant. Otherwise, changes in α_M and α_N will change k even when DT=α_M/α_N remains a constant (that is not equal to 1).

References：

1. Acemoglu, Daron, 1998, "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality," *Quarterly Journal of Economics*, 113, 1055–1090.

2. ______________, 2002, "Directed Technical Change." *Review of Economic Studies* 69: 781–809.

3. ______________, 2003, "Labor- and Capital-Augmenting Technical Change," *Journal of European Economic Association,* Vol.1 (1), pp. 1-37.

4. ______________, 2007, "Equilibrium Bias of Technology." *Econometrica* 75(5): 1371–1410.

5. ______________, 2009, *Introduction to Modern Economic Growth*, Princeton University Press.

6. Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, and David Hemous. 2012. "The Environment and Directed Technical Change." *American Economic Review*, 102(1): 131-66.

7. Aghion, Philippe, and Peter Howitt, 1992, "A Model of Growth through Creative Destruction." *Econometrica,* 60: 323–351.

8. Barro, Robert and Xavier Sala-i-Martin, 2004, *Economic Growth*. MIT Press, Cambridge, MA.

9. Christiaans, Thomas, 2004. Types of balanced growth. *Economics Letters* 82 (2), 253–258.

10. Drandakis, E. M., and Edmund S. Phelps, 1966, "A Model of Induced Invention, Growth, and Distribution," *Economic Journal*, Vol. 76 (304), pp. 823-840.

11. Fellner, William, 1961, "Two Propositions in the Theory of Induced Innovations," *Economic Journal*, Vol. 71(282), pp. 305-308.

12. Growiec, J., 2010, "Knife-edge Conditions in the Modeling of Long-run Growth Regularities", *Journal of Macroeconomics* 32 (2010) 1143-1154.

13. Hicks, John, 1932, *The Theory of Wages*. London: Macmillan.

14. Irmen, Andreas, 2013, "Adjustment Costs in a Variant of Uzawa's Steady-state Growth Theorem", *Economics Bulletin*, Vol. 33 No.4, pp. 2860-2873.

15. Jones, C. I., 1995, "R&D-based Models of Economic Growth". *Journal of Political Economy* 103,759–784.

16. Jones, Charles I., 2005, "The Shape of Production Functions and the Direction of Technical Change." *Quarterly Journal of Economics* 2: 517–549.

17. Jones, Charles I., and Dean Scrimgeour, 2008, "A New Proof of Uzawa's Steady-State Growth Theorem," *Review of Economics and Statistics*, Vol. 90(1), pp. 180-182.

18. Kaldor, Nicholas, 1963, "Capital Accumulation and Economic Growth." In *Proceedings of a Conference Held by the International Economics Association*, Friedrich A. Lutz and Douglas C. Hague (editors). London: Macmillan.

19. Kennedy, Charles M., 1964, "Induced Bias in Innovation and the Theory of Distribution," *Economic Journal*, Vol. 74 (295), pp. 541-547.

20. Li, Defu and Jiuli Huang, 2012, "Can the Steady-State Path of Neoclassical Growth Model Embrace Capital-Augmenting Technological Progress?", *South China Journal of Economics,* October, pp.43-50.

21. ____________________, 2014a, "Is Harrod-Neutrality Indispensable for Steady-State Growth: Uzawa Theorem Revisited", working paper.

22. ____________________, 2014b, "Uzawa (1961)'s Steady-State Theorem in a Malthusian Model", working paper.

23. Rivera-Batiz, L. A. and P. M. Romer, 1991, "Economic Integration and Endogenous Growth", *Quarterly Journal of Economics*, 106, 531-555.

24. Romer, P. M., 1990, "Endogenous Technological Change," *Journal of Political Economy*, 98(5) (October), Part II, S71-S102.

25. Samuelson, Paul A., 1965, "A Theory of Induced Innovation along Kennedy-Weisäcker Lines," *Review of Economics and Statistics*, Vol. 47(4), pp. 343-356.

26. Sato, R., R. V. Ramachandran and C. Lian, 1999, "A Model of Optimal Economic Growth with Endogenous Bias," *Macroeconomic Dynamics*, Vol. 3, pp. 293-310.

27. Sato, R. and R. V. Ramachandran , 2000, "Optimal Growth with Endogenous Technical Progress: Hicksian Bias in a Macro Model," *The Japanese Economic Review*, Vol. 51, No. 2, June 2000.

28. Schlicht, E., 2006, "A Variant of Uzawa's Theorem," *Economics Bulletin*, Vol. 5 (6), pp. 1-5.

29. Uzawa, H., 1961, "Neutral Inventions and the Stability of Growth Equilibrium," *Review of Economic Studies*, Vol. 28, February, pp. 117-124.

30. Xuemei, Tang , Lin Shanlang, and Defu Li, 2014, "Transformation of Economic Growth Mode in the Development Process of Labor Surplus Economy", *Shanghai Journal of Economics*, May.