A Liquidity-Based Resolution of the Uncovered Interest Parity Puzzle

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ABSTRACT

A new monetary theory is set out to resolve the "Uncovered Interest Parity Puzzle (UIP Puzzle)". It explores the possibility that liquidity properties of money and nominal bonds can account for the puzzle. A key concept in our model is that nominal bonds carry *liquidity premium* due to their medium of exchange role as either collateral or means of payment. In this framework no-arbitrage condition ensures a positive comovement of real return on money and nominal bonds. Thus, when inflation in one country becomes relatively lower, i.e., real return on this currency is relatively higher, its nominal bonds should also yield higher real return. We show that their *nominal* returns can also become higher under the economic environment where collateral pledgeability and/or liquidity of nominal bonds and/or collateralized credit based transactions are relatively bigger. Since a currency with lower inflation is expected to appreciate, the high interest currency does indeed appreciate in this case, i.e., the UIP puzzle is no longer an anomaly in our model. Our liquidity based theory in fact has interesting implications on many empirical observations that risk based explanations find difficult to reconcile with.

JEL Classification: E4, E31, E52, F31

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1 Introduction

In a seminal paper, Fama (1984) presented that relatively high interest rate currencies tend to appreciate on average. Interestingly, regardless of the increasing sophistication of the econometric techniques employed and of the increasing quality of the data sets utilized, researchers generally keep documenting similar results.[1] What is more important is that this empirical finding is an anomaly in a sense that high interest rate currencies have predictably positive excess returns, which contradicts the very foundation of the UIP condition. Accordingly, this anomaly has been widely referred to as the UIP or "forward premium" puzzle in the literature.

Yet, as Burnside, Eichenbaum, and Rebelo (2009) and Backus, Gavazzoni, Telmer, and Zin (2010) put it, a vast literature is empirically oriented, and relatively little theoretical attempts has been made to tackle the puzzle. Even among the theoretical literature, no consensus seems to be reached. For instance, most prevailing theories revolve around the idea that the failure of the UIP has a close connection with the way the risk premium behaves.[2] Nevertheless, many recent studies have become critical on these risk-based explanations.[3] Deficiency of prevailing explanations does not end here. The UIP evidence is primarily about return on assets, i,e., bonds and currencies. However, the literature, focusing too much on risk aspects of the asset return, has overlooked equally important aspects of the latter, *liquidity*, which could potentially solve for the puzzle.[4]

In this paper we set out a new liquidity-based monetary model of international asset pricing, and study under what conditions the model rationalizes the UIP puzzle. Our approach is novel for the following reasons. First, we model the economy where nominal bonds and currencies explicitly play a liquidity role during the exchange process, and macro fundamentals, i.e., monetary policy, endogenously determine their liquidity values. Second, we take a radical departure from the conventional approach by entirely abstracting from risk considerations. In other words, asset pricing in this framework is only driven by changes in liquidity values of assets. This approach allows one to isolate the role of asset liquidity, if any, in solving the UIP puzzle.

[1]See Bacchetta and Wincoop (2010), Backus, Gavazzoni, Telmer, and Zin (2010), and the references therein for a recent survey of empirical findings and the literature

[2]Some recent studies offer non-risk-based explanations. For instance, see Corsetti, Dasgupta, Morris, and Shin (2004), Burnside, Eichenbaum, and Rebelo (2009), Bacchetta and Wincoop (2010), and Ilut (2014).

[3]For instance, Burnside, Eichenbaum, and Rebelo (2009) point out, "It has been extremely difficult to tie deviations from uncovered interest parity to economically meaningful measures of risk". Also, see Burnside (2007) and Burnside, Eichenbaum, Kleshchelski, and Rebelo (2008) for a critical review of recent risk-based explanations of the puzzle.

[4]For instance, Atkeson, Alvarez, and Kehoe (2007) argue that standard monetary policy models are not suitable for studying exchange rate and therefore, call for a new monetary model of exchange rates in which time-varying liquidity drives fluctuations in the excess return on foreign bonds. In fact, Brunnermeier, Nagel, and Pedersen (2008) is the first one that introduces a liquidity channel through which the UIP puzzle is rationalized. We come back to this study and discuss similarities and differences with ours in detail later.

To concertize these novel features, our model adopts a microfounded monetary framework developed by Lagos and Wright (2005). The basic structure of the model goes as follows. There are two countries, and each country issues its own currency and nominal bond. For each period, trade in goods first takes place separately for each country. Agents in goods markets trade in a pairwise meeting. Furthermore, the trading process in goods market exhibit frictions such as anonymity and limited commitment. These trading frictions are precisely what makes assets endogenously emerge as a medium of exchange (MOE), thereby creating asset liquidity values. Specifically, we allow bonds to serve as collateral in some pairwise meetings, while they compete with money as a direct means of payment in some other meetings. Once the goods trade ends, all agents move into a perfectly integrated Walrasian financial market where they frictionlessly reshuffle their portfolio of assets in the anticipation of the next period's goods trade.

The key feature of this framework lies in a transmission mechanism of monetary policy, i.e., change in money supply, into nominal bond return. Intuitively, monetary policy not only affects real value of currency but its relative scarcity as a medium of exchange. This in turn has an effect on the liquidity value or return of other competing MOEs, i.e., nominal bonds. What is crucial is that this liquidity-based transmission mechanism opens up new possibilities for a richer set of joint dynamics between currency and nominal bond return in a way that the traditional Lucas (1982) model of international asset pricing could never generate.[5]

To understand the rich joint dynamics of our liquidity-based story, it's useful to first review how the conventional Lucas (1982) model poses the UIP puzzle. In the latter, a sudden increase in interest rate, for example, would lead to an instantaneous appreciation of the currency, to be followed by an expected depreciation. Technically speaking, the *nominal* intertemporal marginal rate of substitution (i.e., *nominal* bond price) of a country is negatively correlated with its inflation rate. Thus, the *Fisher effect* holds true in equilibrium, thereby implying the UIP condition.[6] The bottom line here is that the classical dichotomy, i.e., the separation between money supply and intertemporal marginal rate of substitution, *effectively* forces the nominal bond price to move in the opposite direction of inflation rate, which always ensures the UIP condition in equilibrium.

Our story is also built upon similar premises of the conventional monetary approach such as fully flexible prices and complete FX market. But, we break down the classical dichotomy since the intertemporal marginal rate of substitution is augmented by exchange value of assets, which in turn depends upon money supply in our model economy. What is of utmost importance is that our framework allows the correlation between anticipated inflation rate and nominal bond yield to critically hinge upon market microstructure of pairwise meetings. For instance, sup-

[5]See Hu and Rocheteau (2015) for an extensive review on monetary search models where correlation between the currency and bond return could go either way.

[6]For more detailed explanation on the rigorous relationship between monetary policy, pricing kernel, and the UIP puzzle, one can refer to Bekaert (1994), Bekaert and Hodrick (2001), Backus, Foresi, and Telmer (2001), and Alvarez, Atkeson, and Kehoe (2009).

pose that nominal bonds are perfectly illiquid, meaning that no one in our model economy accepts bonds as any kind of MOE in pairwise meetings. Then, our model is effectively equivalent to Lucas (1982), and the *Fisher effect* always holds true in equilibrium. Likewise, one can also design our model economy such that bonds and money are perfect substitutes for each other. For instance, no collateralized credit transactions take place in any pairwise meeting, and no exogenous liquidity differential between money and bonds exist as a direct means of payment. In this case, the no-arbitrage principle always guarantees a constant zero nominal interest rate, i.e., open market operations become irrelevant.

Yet, suppose now that money cannot perfectly substitute nominal bonds. In line with our initial model setup, let us restrict our model economy in a way that some portion of pairwise meetings must use collateralized credit as a MOE. This effectively creates somewhat extra exchange value of bonds compared to money. If this extra exchange value of bonds is high enough, the inflation elasticity of real bond price can be sufficiently higher than that of money. Since a higher anticipated inflation leads to a higher real money price, the former can induce the real bond price to increase even further to the extent that the *nominal* rate of return on bonds actually decreases. Moreover, the higher anticipated inflation always means the expected currency depreciation relative to its partner currency. In all, a low interest currency is expected to depreciate in this scenario and therefore, the UIP in fact ends up being violated in equilibrium.

The main message of this paper is well reflected upon the aforementioned examples. In our microfounded monetary model of international asset pricing, the UIP does not have to hold uniformly. In particular, the negative relationship between anticipated inflation and nominal bond yield is shown to be sufficient for the UIP deviation. Crucially, our framework implies that nominal bonds must somehow exhibit extra exchange value in order to guarantee the sufficient condition. We show in the model that the sufficiently higher exchange value of bonds can be indeed achieved when the portion of collaterlized-credit-transaction-based pairwise meetings is large and/or the pledgeability of bonds as collateral is high and/or exogenous illiquidity discount on bonds as a direct means of payment is low.

One may question if our framework where bonds exhibit the higher exchange value is empirically substantive. Yet, we argue that it is by no means a pure theoretical abstraction based on a recent empirical work by Krishnamurthy and Vissing-Jorgensen (2012). They basically show that U.S. Treasury bonds exhibit superb liquidity properties just like money. One can then address another potential concerns. First, not every nominal bonds, especially those issued by emerging economies, are same as the U.S. Treasury bonds. Second, the bond liquidity is surely time-varying, e.g., extreme dry-up of bond liquidity during the recent liquidity crunch episode.

Very interestingly, these two issues are what precisely leads to the non-uniform UIP deviation in our framework. Put it another way, our model implies that the sufficient condition for the UIP deviation cannot be met whenever bonds are illiquid enough. This bond illiquidity is one of the defining characteristics of emerging market bonds and the liquidity crisis. Thus, our model predicts that the UIP should be confined to emerging economies and the liquidity crunch period. These two predictions are well supported by prominent empirical studies as well as our own empirical evidence reported in Table 1. Bansal and Dahlquist (2000) empirically confirm that the UIP deviation is pervasive only among developed currency pairs. In addition, Brunnermeier, Nagel, and Pedersen (2008) demonstrate that interbank liquidity crunch has a strongly negative correlation with carry trade returns, i.e., the UIP tends to hold true when measures of market liquidity shrink. In short, our model can provide a microfoundation for what Bansal and Dahlquist (2000) and Brunnermeier, Nagel, and Pedersen (2008) have found, which many risk-based explanations find hard to justify.

As for the related literature, we do not intend to thoroughly review a vast number of theories that have been proposed to make sense of the UIP evidence.[7] Broadly speaking, the theories can be assigned into two big categories, *non-rational expectation based models* and *rational expectation based models*. The former is relatively scarce and based on the idea that expectational errors or behavioral biases of investors drive the UIP deviation, e.g., behavioral biases based explanation by Froot and Thaler (1990) and *peso problems* based explanation by Lewis (1995). Yet, most theoretical attempts to solve the UIP puzzle have maintained the assumption of rational expectations. Our explanation also fits into this category. As mentioned already, most conventional rational expectation based theories argue that the failure of UIP is attributed to the behavior of the risk premium, e.g., Lustig and Verdelhan (2007) and Lustig, Roussanov, and Verdelhan (2011) among others. Non-risk based models include, but are not limited to OTC FX market based explanation of Burnside, Eichenbaum, and Rebelo (2009), the rational inattention model of Bacchetta and Wincoop (2010), and the long run risk based explanation of Bansal and Shaliastovich (2013).

Among the rational expectation based models, Backus, Gavazzoni, Telmer, and Zin (2010) have in common with our story to the extent that monetary policy jointly determines exchange and interest rates. The difference is they explicitly abandon the model of money in favor of the Taylor rule. They show that asymmetries in foreign and domestic Taylor rules can account for the UIP deviation. They admit, however, that their approach is partial where the consumption process and asymmetric Taylor rule coefficients are *exogenously* specified, while ours is a fully-fledged general equilibrium approach. Brunnermeier, Nagel, and Pedersen (2008) also have a close connection to our theory in terms of the role that liquidity plays. They focus on *liquidity frictions* that carry traders face as a driving force behind why the UIP deviation cannot be eliminated quickly in the short run, and how marketwide liquidity crunch affects carry trade speculations and eventually a sudden shift in the UIP relation. Our story differs in that aggregate liquidity needs of the country on top of the behavior of particular carry trade speculators can also drive the UIP relation. In this respect, our theory is complementary to Brunnermeier,

[7] One can refer to many excellent papers for the extensive review, e.g., Engel (2015) and Verdelhan (2010) among others.

Nagel, and Pedersen (2008).

In terms of methodology, our paper is also related to a growing body of money-search literature that studies how monetary policy affects asset prices through the liquidity of assets. Many find that higher nominal interest rates raise asset prices by fueling liquidity premia: for instance Geromichalos, Licari, and Suarez-Lledo (2007), Jacquet and Tan (2012), Lester, Postlewaite, and Wright (2012), and Nosal and Rocheteau (2012). Recently, others such as Hu and Rocheteau (2015) and Lagos and Zhang (2014) found the opposite. Our framework can in fact nest both of these cases by allowing the correlation between nominal interest rates and inflation to go either way depending on the microstructure of decentralized market. This paper also contributes to the money-search literature that tackles traditional asset pricing puzzle through the notion of asset liquidity. Lagos (2010) explains the equity premium and risk-free rate puzzle, while Geromichalos and Simonovska (2014) studies how the home asset bias puzzle can be solved under the explicit modeling of assets as facilitators of trade.

The rest of the paper is organized as follows. In Section 2, we describe the physical environment. Section 3 studies the agents' optimal behavior. In Section 4, we define a stationary, symmetric, and two-country equilibrium, and study how the UIP condition is related to monetary policy and market microstructure of pairwise meetings. Section 5 concludes.

2 Physical Environment

Time is discrete and infinite. Each period is divided into two subperiods. There are two counties, A and B. Each country has two types of agents, buyers and sellers, both of which are populated with a continuum of 1. The identity of buyers and sellers is fixed over time. All agents live infinitely and consider dynamics with a discount factor equal to $\beta \in (0,1)$. They discount future only between periods, but not between subperiods. We will often refer to a buyer (seller) from country i as buyer i (seller i) for notational simplicity. There are three kinds of nonstorable and perfectly divisible goods: a general good produced by all agents and a special good i produced only by sellers in each country $i \in \{A, B\}$.

There are also two different types of (financial) assets in this model. First, a perfectly divisible and storable fiat currency is issued by each country's monetary authority. We denote this asset as $money_i$, $i \in \{A, B\}$. The $money_i$ supply is stochastically determined by each country's monetary authority who injects or withdraws $money_i$ via lump-sum transfers or taxes to buyers of country i at the end of every period. Specifically the $money_i$ stock is initially given by $M_{i,0} \in \mathbb{R}_{++}$, and thereafter it grows at a stochastic rate $\gamma_{i,t}$ (i.e., $M_{i,t+1} = \gamma_{i,t} M_{i,t}$), which is assumed to follow a Markov process defined by its transition function $F(\gamma\prime, \gamma) = \Pr(\gamma_{i,t+1} \leq \gamma\prime \mid \gamma_{i,t} = \gamma)$ where $F : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ is continuous. Assume that the process defined by F has a stationary distribution $\Phi(\cdot)$ as well as a unique solution to $\Phi(\gamma') = \int F(\gamma', \gamma) d\Phi(\gamma)$, and that F has the

Feller property. The second type is nominal bond. In each country, a new set of Lucas (1978) trees are born every period. Each unit of the tree in country i delivers d_i units of a general good in the next period, and dies immediately afterwards. We assume that d_i always equals the real value of $money_i$ in terms of a general good in every period as if each unit of the tree in country i delivered one unit of $money_i$ in the next period. This crucial assumption makes the share of these trees in country i equivalent to one-period nominal bonds of that country. For this reason we will hereinafter refer to shares of trees from country i as $(nominal)\, bond_i$. The supply of $bond_i$ is fixed over time and denoted by B_i.

We now proceed to a detailed description of the subperiods characterized by different economic activities. We start with the second subperiod, and move backward. In the second subperiod, all agents have a linear technology that transforms a unit of labor into a unit of general good. All agents can then trade the general good and all types of financial assets, i.e., $money_i$ and $bond_i$, $\forall i$, within one single Walrasian or centralized market (henceforth, FM). $\varphi_{i,t}$ and $\psi_{i,t}$ respectively denotes the FM price of $money_i$ and $bond_i$ in terms of the general good at period t. Further, the nominal exchange rate at time t is defined as the FM price of $money_B$ in terms of $money_A$: $E_t = \varphi_{B,t}/\varphi_{A,t}$. Notice that the perfectly competitive FM assumption allows agents to trade two monies at the market clearing exchange rate. Thus the law of one price holds every period.

In the first subperiod, a decentralized goods market opens separately for each country (henceforth GM). We assume that agents can only trade in their 'domestic' market during the first subperiod. As a result, only buyer i and seller i can trade special good i in GM of country i.[8] Within any GM, trade is bilateral and anonymous. In addition, agents cannot make binding commitments, and trading histories are private in a way that precludes any borrowing and lending. This premise necessitates a medium of exchange (MOE) in any GM trade. What is crucial here is that we differentiate GM in each country into two types of sub-markets, depending on methods of payment: Goods Market 1 (henceforth $GM1$) and Goods Market 2 (henceforth $GM2$).

In $GM1$, when seller i meets buyer i, the latter can pay the former with a combination of any asset, i.e., $money_i$ and $bond_i$, $\forall i$, in exchange for a special good i. For simplicity, we assume that the seller i is endowed with a linear technology, i.e., a unit of labor is transformed to produce a unit of special good i. A key point here is that we adopt the mechanism design approach pioneered by Wallace (2001) and Zhu and Wallace (2007) for the terms of trade determination. We propose a trading mechanism in the $GM1$ that induces allocations to be pairwise Pareto

[8] This assumption precludes our model from considering international trade in goods, and studying its implications on the UIP puzzle. One could surely relax this assumption to make our model empirically more relevant. However doing so would greatly complicate the analysis, particularly trading protocols in Section 3.2, without providing any critical insight to the model. Furthermore, most studies that offered explanations for the UIP puzzle have stressed investment behavior in financial markets rather than trade-related factors. Given this emphasis, we also think that the no-international-trade assumption here is not a major caveat of our model.

optimal, but treats assets asymmetrically depending on their types as well as nationality. This trading mechanism is meant to capture two intuitive notions. First one is that agents receive better terms of trade in a country by using domestic assets rather than foreign ones. The second is that bonds can be accepted as payment by sellers but for less output than what the buyer could have obtained with money, i.e., the illiquidity of bonds. A main advantage of the proposed trading mechanism is that despite asymmetric treatment of the assets, it leaves no gains from trade unexploited, i.e., allocations are socially efficient, which is not generally the case for other mechanisms (e.g., Nash Bargaining). On top of that, it yields nominal exchange rate determinacy without imposing any *ad-hoc* cash-in-advance type restrictions.[9]

Unlike the $GM1$, $GM2$ only allows credit as a method of payment. Notice that the $GM2$ intuitively stands for a fraction of GM where transactions involve some form of credit, following Williamson (2012).[10] Specifically the credit in the $GM2$ means a promise that buyer i will pay back to seller i a certain amount of general good in the coming FM in exchange for special good i. Due to anonymity and limited commitment, the buyer i cannot pay with unsecured credit (e.g., an IOU). Hence she needs to offer the seller i bonds held in a form of collateral to back the credits. The credit limit is determined by the real value of bonds the buyer i places as collateral and the pledgeability parameter $h \in [0, 1)$ - the extent to which they can be used to secure loans. Similar to the $GM1$, we also adopt a trading mechanism that has good efficiency properties, i.e., pairwise Pareto optimality, but treats domestic and foreign bonds asymmetrically for the terms of trade determination. For instance, this proposed mechanism does not impose any constraint on the use of domestic or foreign bonds as collateral. However, it does lead to a better terms of trade for a buyer i when placing $bond_i$ rather than $bond_{-i}$ as collateral. Detailed descriptions of the pricing mechanism in the $GM1$ and $GM2$ will be provided in Section 3.2. Lastly, agents from country i visit $GM1$ ($GM2$) with probability θ $(1-\theta)$ where $\theta \in (0, 1)$, and therefore all buyers and sellers match each other within each county.

Following Lagos and Wright (2005) (LW henceforth), the utility of buyer i and seller i is respectively given by

$$\mathbb{E}_0 \textstyle\sum_{t=0}^{\infty} \beta^t \{U(x_t) - h_t + u(q_t)\},$$
$$\mathbb{E}_0 \textstyle\sum_{t=0}^{\infty} \beta^t \{U(x_t) - h_t - q_t\},$$

[9] Kareken and Wallace (1981) showed that the nominal exchange rate indeterminacy is pervasive in monetary models unless *ad-hoc* frictions such as the cash-in-advance constraint, i.e., agents trade only with their domestic currency in their home market, are imposed. Yet, as Nosal and Rocheteau (2011) have argued, the cash-in-advance constraint seems particularly odd when the two currencies have different rates of return, i.e, inflation. Also see Wallace (2010) for various disadvantages of cash-in-advance type models from a monetary theorist's point of view.

[10] Introduction of the $GM2$ in an empirically relevant way within this model is not an end itself. As will be analyzed later, it actually boosts liquidity properties of bonds to the extent that the comovement of exchange rate and nominal interest rate can violate the UIP condition in equilibrium. Therefore modeling decentralized markets without credit secured by assets as collateral in this framework is *not* without loss of generality for studying the UIP puzzle.

where x_t and h_t stand for the consumption of general good and labor inputs to produce that good in the second subperiod of period t, respectively. q_t represents the amount of special good i produced by the seller i and consumed by the buyer i in the first subperiod of period t. Without loss of generality, we assume the disutility from producing q_t for the seller i is linear. In addition, we denote the utility function for the general good (the special good i) by $U : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ ($u : \mathbb{R}^+ \rightarrow \mathbb{R}^+$). We also assume that both are twice continuously differentiable, increasing, strictly concave and bounded by B on support $\Xi \subseteq (-\infty, \infty)$ with $u(0) = U(0) = 0$, $u'(0) = U'(0) = \infty$ and $u'(\infty) = U'(\infty) = 0$. $\mathbb{E}_0$ denotes the expectation with respect to the probability measure induced by the random trading process in the GM_1 and GM_2. Figure 1 illustrates the timing of events.

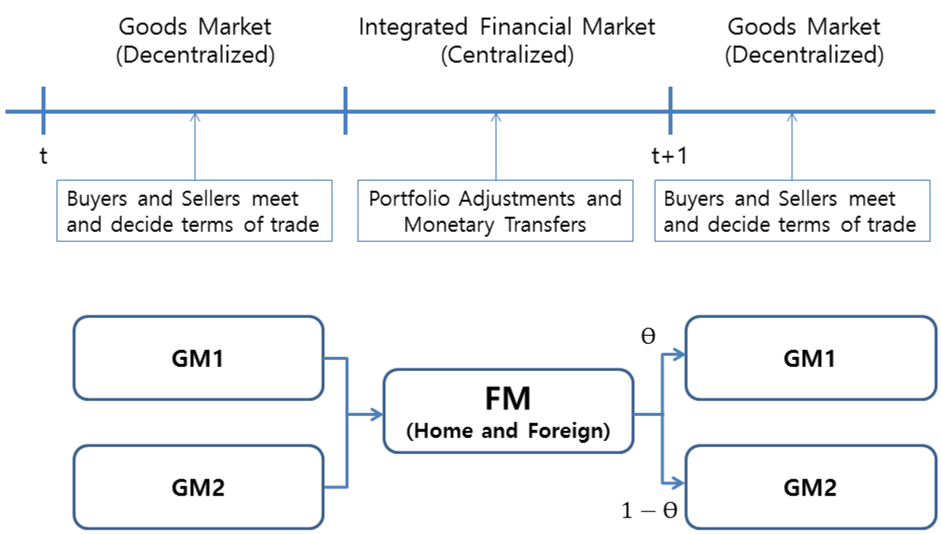

Figure 1: Timing of Events

3 Value Functions and Optimal Behavior

3.1 Value Functions in the Financial Market and Goods Market

First, let $\mathbf{w}_t = (m_{A,t}, a_{A,t}, m_{B,t}, a_{B,t})$ denote the portfolio of any agent at period t. Note that $m_{i,t}$ and $a_{i,t}$ represents units of $money_i$ and $bond_i$ at period t respectively. Define $\boldsymbol{\ell}_t = (\ell_{A,t}, \ell_{B,t})$ as a portfolio of credit (or loan) in terms of the general good which the buyer i borrowed from seller i she met in $GM2$ of previous subperiod, and should pay back at the beginning of the second subperiod of t. Note that $\ell_{i,t}$ means the credit borrowed against $bond_i$ in the $GM2$ of period t. In addition, let $\mathbf{s}_t = (\gamma_{A,t}, M_{A,t}, \gamma_{B,t}, M_{B,t})$ denote the aggregate state of the economy at period t. Then the Bellman's equation for buyer i, who enters FM with a portfolio $\mathbf{w}_t$ and $\boldsymbol{\ell}_t$ is given by[11]

$$
\begin{aligned}
W_i^B(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) &= \max_{x_t, h_t, \mathbf{w}_{t+1}} \left\{ U(x_t) - h_t + \beta \mathbb{E}_t \left[V_i^B(\mathbf{w}_{t+1}, \mathbf{s}_{t+1}) \right] \right\} \\
\text{s.t. } x_t + \boldsymbol{\phi}_t' \mathbf{w}_{t+1} &= h_t + \boldsymbol{\phi}_t \mathbf{w}_t - \boldsymbol{\ell}_t + \varphi_{A,t} T(\gamma_{A,t}) \mathbb{I}_{\{i=A\}} + \varphi_{B,t} T(\gamma_{B,t}) \mathbb{I}_{\{i=B\}},
\end{aligned}
$$

where $\boldsymbol{\phi}_t' = (\varphi_{A,t}, \psi_{A,t}, \varphi_{B,t}, \psi_{B,t})$, $\boldsymbol{\phi}_t = (\varphi_{A,t}, \varphi_{A,t}, \varphi_{B,t}, \varphi_{B,t})$, and $\boldsymbol{\phi}_t' \mathbf{w}_{t+1}$ ($\boldsymbol{\phi}_t \mathbf{w}_t$) denotes the dot product of $\boldsymbol{\phi}_t'$ ($\boldsymbol{\phi}_t$) and $\mathbf{w}_{t+1}$ ($\mathbf{w}_t$). $T(\gamma_{i,t})$ denotes the monetary transfers in country i, and equals to $(\gamma_{i,t} - 1) M_{i,t}$. $\mathbb{I}_{\{i=n\}}, n \in \{A, B\}$, is an indicator function that equals 1 if $i = n$. The function $V_i^B(\mathbf{w}_{t+1}, \mathbf{s}_{t+1})$ represents the GM value function of the buyer i next period. We can easily verify that $x_t = \tilde{x}$, $\forall t$ at the optimum where $\tilde{x}$ is such that $U'(\tilde{x}) = 1$. Based on this fact, we can plug h_t in the budget constraint into W_i^B. It leads to

$$
W_i^B(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) = \boldsymbol{\phi}_t \mathbf{w}_t - \boldsymbol{\ell}_t + \Lambda_{i,t}^B, \tag{1}
$$

where $\Lambda_{i,t}^B \equiv U(\tilde{x}) - \tilde{x} + \mathbf{T}_{i,t} + \max_{\mathbf{w}_{t+1}} \left\{ -\boldsymbol{\phi}_t' \mathbf{w}_{t+1} + \beta \mathbb{E}_t \left[V_i^B(\mathbf{w}_{t+1}, \mathbf{s}_{t+1}) \right] \right\}$.[12] In line with models based on LW, the buyer i's FM value function becomes linear in asset holdings as well as credit owing to quasi-linearity of the preference. This implies that there exists no wealth effects on the choice of $\mathbf{w}_{t+1}$.

Now we consider the FM value function of a seller i. She will never leave the FM with any money or bond holdings because she does not need any liquidity service from those assets in the forthcoming GM simply due to her fixed identity as a seller of the special good (see Rocheteau and Wright (2005) for a rigorous proof). Nevertheless, when she enters the FM, she will generally hold a portfolio of money, bonds, and credit received as payment in either $GM1$ or $GM2$. The Bellman's equation for her is then given by

$$
\begin{aligned}
W_i^S(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) &= \max_{x_t, h_t} \left\{ U(x_t) - h_t + \beta \mathbb{E}_t \left[V_i^S(\mathbf{0}, \mathbf{s}_{t+1}) \right] \right\} \\
\text{s.t. } x_t &= h_t + \boldsymbol{\phi}_t \mathbf{w}_t + \boldsymbol{\ell}_t.
\end{aligned}
$$

[11] The budget constraint implies that the buyer i always pays back the credit borrowed in a previous subperiod. This is in fact not an *assumption* but an equilibrium outcome. In principle, she could instead default and let the seller i whom she met in a previous $GM2$ take collateral she placed, i.e., bonds. However, as will be seen in Section 3.2, this type of strategy becomes always inferior to paying back the credit due to the pledgeability parameter h, less than one. In other words, she would always lose $1 - h$ portion of her real bond balances by defaulting on the seller i. Hence, allowing for less than perfect pledgeability of bonds as collateral is not only empirically relevant, but prevents our model from considering more complex equilibrium default cases.

[12] $\mathbf{T}_{i,t}$ is a short expression for $\varphi_{A,t} T(\gamma_{A,t}) \mathbb{I}_{\{i=A\}} + \varphi_{B,t} T(\gamma_{B,t}) \mathbb{I}_{\{i=B\}}$

Similar to the buyer i, the seller i will also choose $x_t = \tilde{x}$, $\forall t$. Replacing h_t from the budget constraint into W_i^S yields

$$W_i^S(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) = \boldsymbol{\phi}_t \mathbf{w}_t + \boldsymbol{\ell}_t + \Lambda_{i,t}^S, \tag{2}$$

where $\Lambda_{i,t}^S \equiv U(\tilde{x}) - \tilde{x} + \beta \mathbb{E}_t \left[V_i^S(\mathbf{0}, \mathbf{s}_{t+1}) \right]$.

Next we consider the value functions in the GM. First, consider a value of the typical buyer i who enters the GM with a portfolio $\mathbf{w}_t$. Let $q_{i,t}^{GMj}$, $j \in \{1, 2\}$ denote the consumption of special good i that the buyer i obtained from GMj at period t. $\boldsymbol{p}_t = (p_t^{m_A}, p_t^{a_A}, p_t^{m_B}, p_t^{a_B})$ represents a portfolio of assets exchanged in a meeting with a seller in $GM1$ at period t. For instance, $p_t^{m_i}$ ($p_t^{a_i}$) denotes the units of $money_i$ ($bond_i$) handed over to the seller in $GM1$ at period t. All these terms will be determined in Section 3.2. Since the buyer i visits the $GM1$ ($GM2$) with the probability of θ $(1-\theta)$, her GM value function is given by

$$V_i^B(\mathbf{w}_t, \mathbf{s}_t) = \theta \left[u(q_{i,t}^{GM1}) + W_i^B(\mathbf{w}_t - \boldsymbol{p}_t, \mathbf{0}, \mathbf{s}_t) \right] + (1-\theta) \left[u(q_{i,t}^{GM2}) + W_i^B(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) \right]. \tag{3}$$

The typical seller i visits the $GM1$ or $GM2$ with the same probabilities as the buyer i. The GM value function of the seller i with no money and bonds carried over from a previous period is given by

$$V_i^S(\mathbf{0}, \mathbf{s}_t) = \theta \left[-q_{i,t}^{GM1} + W_i^S(\boldsymbol{p}_t, \mathbf{0}, \mathbf{s}_t) \right] + (1-\theta) \left[-q_{i,t}^{GM2} + W_i^S(\mathbf{0}, \boldsymbol{\ell}_t, \mathbf{s}_t) \right].$$

3.2 The Trading Mechanism in Goods Market

In this section, we study the trading mechanism and the associated terms of trade for each GMj in detail. First, consider a meeting in $GM1$ between seller i and buyer i who carries a portfolio $\mathbf{w}_t$. As explained in Section 2, we propose a mechanism that maximizes social welfare given frictions in the economic environment, i.e., limited commitment and anonymity, and social conventions, i.e., buyers receive better terms of trade when using domestic assets rather than foreign assets, and bonds are generally less liquid than money as means of payment. Given this intuition, one can conceptually understand the proposed mechanism in two stages.

In the first stage, the buyer i makes a take-it-or-leave-it offer to the seller i under restrictions that the former can only use $money_i$ and $bond_i$ for payment, and is able to transfer at most a fraction $g \in [0, 1]$ of her $bond_i$ holdings, i.e., if $g = 0$, then bonds are completely illiquid, and if $g = 1$, then they are perfectly liquid.[13] Then, they move together to the second stage where

[13] In fact, the usefulness of g in this framework is not limited to its empirical relevance. This type of restriction on the illiquidity of bonds has been suggested as a most basic solution for the *rate-of-return dominance puzzle* (see Hu and Rocheteau (2013) for an extensive literature review). We adopt the g for a similar reason. Given the introduction of $GM2$, our model induces somewhat higher liquidity properties of nominal bonds than money to

no restrictions on the use of any asset exist. The final terms of trade, which are actually implemented, is determined by maximizing the surplus of the seller i, while maintaining the buyer i's surplus at the first stage payoff level. As a result, the final allocation will be pairwise Pareto efficient. Nevertheless, the buyer i can obtain additional gain neither from using foreign assets nor from paying with more $bond_i$ and less $money_i$. Thus, our mechanism can in fact allows asset-specific transaction restrictions i.e., the cash-in-advance constraint (only domestic assets used in domestic markets) and the exogenous illiquidity of bonds, to be fully endogenized in equilibrium.

Let us now look at the buyer i's surplus at the first stage of the mechanism, $\mathbb{U}^b_{11}(\mathbf{w}_t)$. Following intuitive descriptions above, it can be expressed as

$$\mathbb{U}^b_{11}(\mathbf{w}_t) = \max_{q_{i,t},\boldsymbol{p}_t} \left\{ u(q_{i,t}) + W^B_i(\mathbf{w}_t - \boldsymbol{p}_t, 0, \mathbf{s}_t) - W^B_i(\mathbf{w}_t, 0, \mathbf{s}_t) \right\}$$
$$\text{s.t.} - q_{i,t} + W^S_i(\boldsymbol{p}_t, \mathbf{0}, \mathbf{s}_t) - W^S_i(\mathbf{0}, \mathbf{0}, \mathbf{s}_t) = 0,$$

with the liquidity augmented feasibility constraint $\boldsymbol{p}_t \leq \widetilde{\mathbf{w}}_t$ where $\widetilde{\mathbf{w}}_t = (m_{A,t}, ga_{A,t}, m_{B,t}, ga_{B,t})$ and the cash-in-advance constraint $p^{m_{-i}}_t = p^{a_{-i}}_t = 0$. Note that the subscript 'kj' of $\mathbb{U}^b_{kj}$ denotes the $k\ th$ stage in the GMj, $k, j \in (1, 2)$. Given the linearity of W^B_i and W^S_i, $\mathbb{U}^b_{11}(\mathbf{w}_t)$ simplifies to

$$\mathbb{U}^b_{11}(\mathbf{w}_t) = \max_{q_{i,t},\, p^{m_i}_t,\, p^{a_i}_t} \{u(q_{i,t}) - \varphi_{i,t}(p^{m_i}_t + p^{a_i}_t)\}$$
$$\text{s.t.} - q_{i,t} + \varphi_{i,t}(p^{m_i}_t + p^{a_i}_t) = 0,$$

and $p^{m_i}_t \leq m_{i,t},\ p^{a_i}_t \leq ga_{i,t}$.

Interpretation of the problem above is standard. The buyer i's payoff is obtained by choosing her consumption and and the transfer of her domestic money and bonds in order to maximize her surplus. It is important to note that while she can transfer $money_i$ up to her entire $money_i$ holdings, an upper bound for her $bond_i$ transfers is a fraction of g of her $bond_i$ holdings. Furthermore, the aim of the first stage is to pin down a payoff level for the buyer i. It is worth emphasizing that the terms of trade chosen in this stage are not necessarily the ones that are finally implemented.

Next, we move to the second stage where the buyer i is allowed to use any of her assets to pay without any exogenous liquidity restrictions. The actual terms of trade for $GM1$ are determined such that the seller i maximizes her surplus, taking the predetermined surplus level of the buyer i from the first stage as given. The seller i's surplus at the second stage of the

prevail in equilibrium, i.e., the zero nominal interest rate bound will be violated. However, as will be analyzed in Section 4, the g can potentially offset this effect by making money more liquid than nominal bonds in $GM1$, and therefore the nominal interest rate does not necessarily goes below zero in equilibrium.

mechanism, $\mathbb{U}_{21}^s(\mathbf{w}_t)$ where $\mathbf{w}_t$ denotes the buyer i's portfolio holdings, is then expressed as

$$\mathbb{U}_{21}^s(\mathbf{w}_t) = \max_{q_{i,t},\boldsymbol{p}_t} \left\{-q_{i,t} + W_i^S(\boldsymbol{p}_t, \mathbf{0}, \mathbf{s}_t) - W_i^S(\mathbf{0}, \mathbf{0}, \mathbf{s}_t)\right\}$$
$$\text{s.t. } u(q_{i,t}) + W_i^B(\mathbf{w}_t - \boldsymbol{p}_t, 0, \mathbf{s}_t) - W_i^B(\mathbf{w}_t, 0, \mathbf{s}_t) = \mathbb{U}_{11}^b(\mathbf{w}_t),$$

and the feasibility constraint $\boldsymbol{p}_t \leq \mathbf{w}_t$. Given the linearity of W_i^B and W_i^S, $\mathbb{U}_{21}^s(\mathbf{w}_t)$ again simplifies to

$$\mathbb{U}_{21}^s(\mathbf{w}_t) = \max_{q_{i,t},\boldsymbol{p}_t} \{-q_{i,t} + \boldsymbol{\phi}_t \boldsymbol{p}_t\}$$
$$\text{s.t. } u(q_{i,t}) - \boldsymbol{\phi}_t \boldsymbol{p}_t = \mathbb{U}_{11}^b(\mathbf{w}_t),$$

and $\boldsymbol{p}_t \leq \mathbf{w}_t$.

Notice that the buyer i is never restricted to use any of her assets as means of payment. Furthermore, the feasibility constraint does not impose any asymmetric liquidity restrictions (e.g., bonds are now fully liquid, i.e., $g = 1$). Further, the constraint that the buyer i's surplus must equal to $\mathbb{U}_{11}^b$ guarantees that the final allocation is pairwise Pareto efficient. The next lemma describes the results of the proposed pricing mechanism in $GM1$.

Lemma 1. *Define $q^* = \{q : u'(q) = 1\}$. The total real balances of the buyer i are denoted as $z(\mathbf{w}_t) \equiv \boldsymbol{\phi_t}\mathbf{w}_t$. Finally, define $z^* = u(q^*) - \mathbb{U}_{11}^b$ and $\tilde{\boldsymbol{p}}(\mathbf{w}_t)$ as the set of $(m_{A,t}, a_{A,t}, m_{B,t}, a_{B,t})$ such that $\boldsymbol{\phi_t} \cdot \tilde{\boldsymbol{p}}(\mathbf{w}_t) = z^*$. When the buyer i with a portfolio $\mathbf{w}_t$ meets seller i in $GM1$, the proposed pricing mechanism yields the following results. The terms of trade in the first stage are given by $q_{i,t} \equiv q_i(\mathbf{w}_t)$, $p_t^{m_i} \equiv p^{m_i}(\mathbf{w}_t)$, and $p_t^{a_i} \equiv p^{a_i}(\mathbf{w}_t)$ such that $q_i(\mathbf{w}_t) = \varphi_{i,t}\left(p^{m_i}(\mathbf{w}_t) + p^{a_i}(\mathbf{w}_t)\right) = \min\{q^*, \varphi_{i,t}(m_{i,t} + ga_{i,t})\}$. The actual terms of trade determined in the second stage are given by*

$$q_i(\mathbf{w}_t) = \begin{cases} q^*, & \text{if } z(\mathbf{w}_t) \geq z^*, \\ b_t, & \text{if } z(\mathbf{w}_t) < z^*. \end{cases} \qquad \boldsymbol{p}(\mathbf{w}_t) = \begin{cases} \tilde{\boldsymbol{p}}(\mathbf{w}_t), & \text{if } z(\mathbf{w}_t) \geq z^*, \\ \mathbf{w}_t, & \text{if } z(\mathbf{w}_t) < z^*, \end{cases}$$

where $b_t = u^{-1}\left[z(\mathbf{w}_t) + \mathbb{U}_{11}^b(\mathbf{w}_t)\right]$. The surplus for the buyer i and the seller i is respectively given by

$$\mathbf{U}_{11}^b(\mathbf{w}_t) = \begin{cases} u(q^*) - q^*, & \text{if } \varphi_{i,t}(m_{i,t} + ga_{i,t}) \geq q^*, \\ u(\varphi_{i,t}(m_{i,t} + ga_{i,t})) - \varphi_{i,t}(m_{i,t} + ga_{i,t}), & \text{if } \varphi_{i,t}(m_{i,t} + ga_{i,t}) < q^*, \end{cases}$$
$$\mathbf{U}_{21}^s(\mathbf{w}_t) = \begin{cases} 0, & \text{if } \varphi_{i,t}(m_{i,t} + ga_{i,t}) \geq q^*, \\ u(q^*) - q^* - \mathbf{U}_{11}^b(\mathbf{w}_t), & \text{if } \varphi_{i,t}(m_{i,t} + ga_{i,t}) < q^* \textit{ and } z(\mathbf{w}_t) \geq z^*, \\ u(b_t) - b_t - \mathbf{U}_{11}^b(\mathbf{w}_t), & \text{if } \varphi_{i,t}(m_{i,t} + ga_{i,t}) < q^* \textit{ and } z(\mathbf{w}_t) < z^*. \end{cases}$$

Proof. See the appendix. □

It is straightforward to interpret the first stage outcome. Only the buyer i's domestic asset holdings determine the terms of trade (q_i, p^{m_i}, p^{a_i}). More importantly, when her bond-illiquidity augmented domestic real balances, $\varphi_{i,t}(m_{i,t}+ga_{i,t})$ exceeds the first best quantity, q^*, she receives the latter, and hands over any combination of $money_i$ and $bond_i$ whose total real value exactly equals to q^*. On the other hand, if $\varphi_{i,t}(m_{i,t} + ga_{i,t})$ falls short of q^* then, she is liquidity constrained and therefore, gives up all her $money_i$ and $bond_i$. In return, she receives as much q_i as her $\varphi_{i,t}(m_{i,t}+ga_{i,t})$ allows. Then, her first stage surplus $\mathbf{U}^b_{11}(\mathbf{w}_t)$ equals to total gains from trade due to a take-it-or-leave-it offer by her, and is non-decreasing in $\varphi_{i,t}(m_{i,t} + ga_{i,t})$.

For the second stage outcome notice again that only the buyer i's total real balances determine the actual terms of trade. When $z(\mathbf{w}_t)$ exceeds z^* that guarantees the first best outcome for the seller i, the buyer i also receives the first best, q^* in return for any combination of her asset holdings whose real value equals to z^*, i.e., $\tilde{\boldsymbol{p}}(\mathbf{w}_t)$. Otherwise, the buyer i is liquidity constrained. Hence, she gives up her entire portfolio in order to obtain as much $q_{i,t}$ as possible, subject to the constraint that her net consumption utility of $q_{i,t}$ in $GM1$ equals to $\mathbf{U}^b_{11}(\mathbf{w}_t)$. The seller i's second stage payoff $\mathbf{U}^s_{21}(\mathbf{w}_t)$ in Lemma 1 then immediately follows by replacing the buyer i's participation constraint into her objective function.

There are three key observations worth emphasizing here. First, the fact that $\mathbf{U}^s_{21}(\mathbf{w}_t)$ is non-negative given the same level of buyer surplus as in the first stage indicates that the proposed mechanism yields a pairwise Pareto efficient outcome.[14] Second, $\mathbf{U}^b_{11}(\mathbf{w}_t)$ is never affected by foreign asset holdings of the buyer. Thus, it is immediate that she will never choose to hold any foreign asset in equilibrium with positive asset holding costs, i,e., the cash-in-advance constraint rises endogenously. Lastly, although no restrictions on the illiquidity of bonds are imposed, i.e., $g = 1$, in the second stage, the buyer i gets exactly the same payoff that she would have obtained in a model with the exogenous liquidity constraint, i.e., g affects the level of $\mathbf{U}^b_{11}(\mathbf{w}_t)$. Therefore, our mechanism also endogenously derive, rather than impose, the bond illiquidity constraint.

Now, let us look at the details in the $GM2$ where credit is only accepted in payments. We take the same steps as in the $GM1$. In the first stage, the buyer i makes a take-it-or-leave-it offer to the seller i under restrictions that only $bond_i$ can be used as collateral to obtain credit, and the buyer i can acquire that credit only up to a fraction h of her real $bond_i$ balance. In the second stage, we remove the restriction that only domestic bonds should be used as collateral. Then, we let the seller i choose the actual terms of trade by maximizing her surplus subject to the credit constraint that now applies to both $\ell_{i,t}$ and $\ell_{-i,t}$. Importantly, she also has to make sure that the buyer i's surplus remains at the first stage payoff level.

[14] See Nosal and Rocheteau (2011) for detailed graphical illustration of the Pareto improvement from the first stage to the second, as well as the shapes of Pareto frontiers for the two stages.

The first stage surplus for the buyer i is then given by

$$\mathbb{U}_{12}^{b}(\mathbf{w}_t) = \max_{q_{i,t},\boldsymbol{\ell}_t} \left\{ u(q_{i,t}) + W_i^B(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) - W_i^B(\mathbf{w}_t, 0, \mathbf{s}_t) \right\}$$
$$\text{s.t.} - q_{i,t} + W_i^S(\mathbf{0}, \boldsymbol{\ell}_t, \mathbf{s}_t) - W_i^S(\mathbf{0}, \mathbf{0}, \mathbf{s}_t) = 0,$$

and $\ell_{-i,t} = 0$ with the credit limit constraint $\ell_{i,t} \leq h\varphi_{i,t}a_{i,t}$. The linearity of W_i^B and W_i^S simplifies $\mathbb{U}_{12}^{b}(\mathbf{w}_t)$ to

$$\mathbb{U}_{12}^{b}(\mathbf{w}_t) = \max_{q_{i,t},\,\ell_{i,t}} \left\{ u(q_{i,t}) - \ell_{i,t} \right\}$$
$$\text{s.t.} - q_{i,t} + \ell_{i,t} = 0,$$

with the same constraints above.

In the second stage, the restriction on the use of foreign bonds as collateral, i.e., $\ell_{-i,t} = 0$, is removed. Thus the pricing mechanism is given by

$$\mathbb{U}_{22}^{s}(\mathbf{w}_t) = \max_{q_{i,t},\boldsymbol{\ell}_t} \left\{ -q_{i,t} + W_i^S(\mathbf{0}, \boldsymbol{\ell}_t, \mathbf{s}_t) - W_i^S(\mathbf{0}, \mathbf{0}, \mathbf{s}_t) \right\}$$
$$\text{s.t. } u(q_{i,t}) + W_i^B(\mathbf{w}_t, \boldsymbol{\ell}_t, \mathbf{s}_t) - W_i^B(\mathbf{w}_t, 0, \mathbf{s}_t) = \mathbb{U}_{12}^{b}(\mathbf{w}_t),$$

and the credit limit constraints $\ell_{i,t} \leq h\varphi_{i,t}a_{i,t},\ \forall i$. Using the linear value functions, $\mathbb{U}_{22}^{s}(\mathbf{w}_t)$ is again simplified to

$$\mathbb{U}_{22}^{s}(\mathbf{w}_t) = \max_{q_{i,t},\,\ell_{i,t},\,\ell_{-i,t}} \left\{ -q_{i,t} + \ell_{i,t} + \ell_{-i,t} \right\}$$
$$\text{s.t. } u(q_{i,t}) - \ell_{i,t} - \ell_{-i,t} = \mathbb{U}_{12}^{b}(\mathbf{w}_t),$$

and $\ell_{i,t} \leq h\varphi_{i,t}a_{i,t},\ \forall i$.

The following Lemma 2 summarizes the solutions to the proposed mechanism in $GM2$.

Lemma 2. *Define the total real value of the buyer i's bond holdings as $z_a(\mathbf{w}_t) \equiv \varphi_{A,t}a_{A,t} + \varphi_{B,t}a_{B,t}$, and $\boldsymbol{a}_t^h$ as a set of $(h\varphi_{A,t}a_{A,t},\ h\varphi_{b,t}a_{B,t})$. Finally, let $\tilde{\boldsymbol{\ell}}$ denote the set of $(\ell_{A,t}, \ell_{B,t})$ such that $\ell_{A,t} + \ell_{B,t} = u(q^*) - \mathbb{U}_{12}^{b}(\mathbf{w}_t)$, and define $z_a^* = u(q^*) - \mathbb{U}_{12}^{b}$. When the buyer i with a portfolio $\mathbf{w}_t$ meets seller i in $GM2$, the proposed pricing mechanism yields the following results. The terms of trade in the first stage are given by $q_{i,t} \equiv q_i(\mathbf{w}_t)$ and $\ell_{i,t} \equiv \ell_i(\mathbf{w}_t)$ such that $q_i(\mathbf{w}_t) = \ell_i(\mathbf{w}_t) = \min\{q^*, h\varphi_{i,t}a_{i,t}\}$. The actual terms of trade determined in the second stage are given by*

$$q_i(\mathbf{w}_t) = \begin{cases} q^*, & \text{if } z_a(\mathbf{w}_t) \geq z_a^*, \\ c_t, & \text{if } z_a(\mathbf{w}_t) < z_a^*. \end{cases} \qquad \boldsymbol{\ell}(\mathbf{w}_t) = \begin{cases} \tilde{\boldsymbol{\ell}}, & \text{if } z_a(\mathbf{w}_t) \geq z_a^*, \\ \boldsymbol{a}_t^h, & \text{if } z_a(\mathbf{w}_t) < z_a^*, \end{cases}$$

where $c_t = u^{-1}\left[z_a(\mathbf{w}_t) + \mathbb{U}^b_{12}(\mathbf{w}_t)\right]$. *The surplus for the buyer* i *and the seller* i *is respectively given by*

$$\mathbf{U}^b_{12}(\mathbf{w}_t) = \begin{cases} u(q^*) - q^*, & \text{if } h\varphi_{i,t}a_{i,t} \geq q^*, \\ u(\ h\varphi_{i,t}a_{i,t}) -\ h\varphi_{i,t}a_{i,t}, & \text{if } h\varphi_{i,t}a_{i,t} < q^*, \end{cases}$$

$$\mathbf{U}^s_{22}(\mathbf{w}_t) = \begin{cases} 0, & \text{if } h\varphi_{i,t}a_{i,t} \geq q^*, \\ u(q^*) - q^* - \mathbf{U}^b_{12}(\mathbf{w}_t), & \text{if } h\varphi_{i,t}a_{i,t} < q^* \text{ and } z_a(\mathbf{w}_t) \geq z^*_a, \\ u(c_t) - c_t - \mathbf{U}^b_{12}(\mathbf{w}_t), & \text{if } h\varphi_{i,t}a_{i,t} < q^* \text{ and } z_a(\mathbf{w}_t) < z^*_a. \end{cases}$$

Proof. The proof follows similar steps as in Lemma 1, and it is, therefore, omitted. □

These results are intuitive, and admit almost identical interpretation as in Lemma 1. A key difference is that in $GM2$ money holdings are irrelevant, and the liquidity restriction on bonds as collateral is now reflected by the pledgeability parameter h. Following these differences, outcomes in each stage are straightforward to understand. The terms of trade in the first stage now depend on the buyer i's credit pledgeability augmented real $bond_i$ balance, i.e., $h\varphi_{i,t}a_{i,t}$. If this is less than the first best amount, q^* then, she becomes liquidity constrained, and therefore place all her $bond_i$ holdings as collateral to get as much credit as possible. Otherwise, she just borrows q^* by placing whatever amounts of $bond_i$ as collateral needed to obtain that q^*. In the second stage no restrictions on the use of $bond_{-i}$ exist any more. Therefore, whether the buyer i is liquidity constrained or not depends on the relative value of her total real bond balances, $z_a(\mathbf{w}_t)$, to the first best amount which is now z^*_a. Then, the solution to the second stage terms of trade again follows trivially. Similar to the $GM1$ mechanism, the actual allocation in $GM2$ is also pairwise Pareto efficient, i.e., $\mathbf{U}^s_{22}(\mathbf{w}_t)$ is non-negative. Likewise, it implies endogenously driven cash-in-advance constraint in equilibrium, i.e., $bond_{-i}$ has no effect on $\mathbf{U}^b_{12}(\mathbf{w}_t)$.

3.3 Euler Equations

This section describes the optimal portfolio choice of buyers. The optimal behavior can be derived by solving the maximization problem in (1). To that end, lead eq.(3) by one period and substitute the emerging expression into (1). Notice that the buyer i's portfolio choice, i,e., $\mathbf{w}_{t+1}$, does not depend on her private trading history. Furthermore, the fact that $V^B_i(\mathbf{w}_t, \mathbf{s}_t)$ is a concave function of $z(\mathbf{w}_t)$ implies that total real balances held by buyer i will be degenerate in equilibrium (see Lagos and Wright (2005) for a rigorous proof). The necessary and sufficient first-order conditions for the buyer i's choices of $\mathbf{w}_{t+1} = (m_{A,t+1}, a_{A,t+1}, m_{B,t+1}, a_{B,t+1})$ are given by

$$\varphi_{i,t} \geq \beta \mathbb{E}_t \frac{\partial V_i^B(\mathbf{w}_{t+1}, \mathbf{s}_{t+1})}{\partial m_{i,t+1}} \text{ with equality if } m_{i,t+1} > 0,\ \forall i \in \{A, B\},$$
$$\psi_{i,t} \geq \beta \mathbb{E}_t \frac{\partial V_i^B(\mathbf{w}_{t+1}, \mathbf{s}_{t+1})}{\partial a_{i,t+1}} \text{ with equality if } a_{i,t+1} > 0,\ \forall i \in \{A, B\}.$$

Substitute solutions from Lemma 1 and 2 into eq.(3), and lead the emerging function by one period again. Finally, by taking this function's first derivative with respect to $m_{i,t+1}$ and $a_{i,t+1}$, $\forall i \in \{A, B\}$, one could achieve the following Euler equations for the buyer i.

$$\varphi_{i,t} = \beta \int \left\{(1-\theta) + \theta u'\left(q_i^{GM1}(\cdot)\right)\right\} \varphi_{i,t+1} dF(\gamma_{-i,t+1}, \gamma_{-i,t}) dF(\gamma_{i,t+1}, \gamma_{i,t}), \tag{4}$$

$$\psi_{i,t} = \beta \int \left\{\tau_1 + \tau_2 u'\left(q_i^{GM1}(\cdot)\right) + \tau_3 u'\left(q_i^{GM2}(\cdot)\right)\right\} \varphi_{i,t+1} dF(\gamma_{-i,t+1}, \gamma_{-i,t}) dF(\gamma_{i,t+1}, \gamma_{i,t}), \tag{5}$$

$$\varphi_{-i,t} \geq \beta \int \varphi_{-i,t+1} dF(\gamma_{-i,t+1}, \gamma_{-i,t}) \tag{6}$$
$$\text{“}=\text{”if } m_{-i,t+1} > 0,$$

$$\psi_{-i,t} \geq \beta \int \varphi_{-i,t+1} dF(\gamma_{-i,t+1}, \gamma_{-i,t}) \tag{7}$$
$$\text{“}=\text{”if } a_{-i,t+1} > 0.$$

where $q_{i,t+1}^{GM1}(\cdot) \equiv q_i^{GM1}(\boldsymbol{\phi}_{t+1}\mathbf{w}_{t+1})$, $q_{i,t+1}^{GM2}(\cdot) \equiv q_i^{GM2}(h \cdot \varphi_{i,t+1} a_{i,t+1})$, $\tau_1 = (1-\theta)(1-h) + \theta(1-g)$, $\tau_2 = \theta g$, and $\tau_3 = (1-\theta)h$.

Interpretation of these Euler equations above is standard. The left side of each condition refers to a marginal cost of purchasing $money_i$ or $bond_i$, $\forall i$, while the right side represents the expected marginal benefit from carrying that asset into GM. For instance, condition (4) is the buyer i's Euler equation for $money_i$. The left side simply means the real cost of purchasing a unit of $money_i$. On the other hand, the right side represents the weighted average of the discounted gain from this marginal $money_i$ in the following period, i.e., she carries the $money_i$ until the second subperiod of $t+1$ to consume $\varphi_{i,t+1}$ units of general goods if she happens to visit $GM2$ with probability $1-\theta$, while she can instead gain consumption utility in $GM1$ from using that $money_i$ to purchase special good i, i.e., $u'\left(q_i^{GM1}(\cdot)\right)\varphi_{i,t+1}$, with probability θ.

Condition (5) is the buyer i's Euler equation for $bond_i$. A key difference here is that the discounted expected benefit from carrying additional unit of $bond_i$ into the following period has three components. First, if she happens to visit $GM1$ in the next period then, she enjoys consumption utility from placing the $bond_i$ as means of payment, i.e., $\tau_2 u'\left(q_i^{GM1}(\cdot)\right)\varphi_{i,t+1}$. On the other hand, if she visits $GM2$ instead, she can gain consumption utility by using that $bond_i$ as collateral, i.e., $\tau_3 u'\left(q_i^{GM2}(\cdot)\right)\varphi_{i,t+1}$. Lastly, regardless of which GM she enters, she will *effectively* face a certain restriction on the use of the $bond_i$ as a consequence of the pricing mechanism explained earlier, i.e., there always exists illiquid portion of the $bond_i$ that can not be liquidated. Then, she *effectively* carries that portion of the $bond_i$ into the second subperiod, and consume whatever amounts of general goods it allows her to purchase, i.e., $\tau_1 \varphi_{i,t+1}$.

Condition (6) and (7) is respectively the buyer i's Euler equation for $money_{-i}$ and $bond_{-i}$. Notice that she never gains any additional benefit from carrying foreign assets into GM, and only values them as the claim to the next period's general goods, i.e., the right side of (6) and (7) is only the discounted expected value of $\varphi_{i,t+1}$. To interpret this, it is useful to rewrite the terms within the max operator in the buyer i's portfolio choice problem (1). Substitute (3) into (1) and rearrange terms within the max operator using solutions to $\mathbb{U}_{1j}^b$ from Lemma 1 and 2. Then, one can arrive the following expression.

$$\max_{\mathbf{w}_{t+1}} (-\boldsymbol{\phi}_t + \beta \mathbb{E}_t \boldsymbol{\phi}_{t+1}) \mathbf{w}_{t+1} + \beta \mathbb{E}_t \left[\theta \mathbb{U}_{11}^b(\mathbf{w}_{t+1}) + (1-\theta) \mathbb{U}_{12}^b(\mathbf{w}_{t+1}) \right].$$

Notice that her discounted expected benefit from GM trade in the next period is pinned down by $\beta \mathbb{E}_t \left[\theta \mathbb{U}_{11}^b(\mathbf{w}_{t+1}) + (1-\theta) \mathbb{U}_{12}^b(\mathbf{w}_{t+1}) \right]$ which is never affected by her $money_{-i}$ or $bond_{-i}$ holdings according to Lemma 1 and 2. Thus, the buyer i never appreciates the liquidity value of foreign assets in the forthcoming GM. One can also relate this intuition directly to the pricing mechanism proposed in the GM trade. Our mechanism induces the buyer i to obtain worse terms of trade if she chooses to purchase special good i with foreign assets than domestic ones in both $GM1$ and $GM2$. For instance, without loss of generality, consider buyer i who enters $GM1$. From the constraint for $\mathbb{U}_{11}^b(\mathbf{w}_t)$ in Section 3.2, one can show that she can obtain a unit of special good i in return for an additional unit of real $money_i$ ($bond_i$), i,.e., $1/\varphi_{i,t}$ ($1/(g\varphi_{i,t})$). Yet, according to the constraint for $\mathbb{U}_{21}^s(\mathbf{w}_t)$, she gets less than or equal to a unit of the good, i.e., $1/u'(q_{i,t}^{GM1}) \leq 1$, with an additional real $money_{-i}$ ($bond_{-i}$). This implies that domestic assets are always superior to foreign ones in terms of the marginal surplus generated in the $GM1$. Same results follow trivially for the $GM2$. Thus agents in each country will hold only domestic assets in all states, i.e., condition (6) and (7) hold with strict inequality, even though no restrictions on the use of any asset exist.

4 Equilibrium and Characterization

In this section, we describe the definition of a recursive equilibrium and then, derive expressions for functions of the equilibrium prices such as nominal interest rate for each country and exchange rate. Finally, we will discuss how these variables interrelate with each other, and specify conditions under which the UIP puzzle is resolved.

4.1 Definition of Equilibrium

Before we proceed for the definition of equilibrium, let us first define a few more variables. The transition function F along with the stochastic process for $\gamma_{i,t}$, $\forall i$ also yield a transition function for the aggregate state of the economy, s_t. Specifically, if $\mathrm{s} = (\gamma_A, M_A, \gamma_B, M_B)$ and $\mathrm{s}' = (\gamma'_A, M'_A, \gamma'_B, M'_B)$ then, $\Pr(\mathrm{s}_{t+1} \leq \mathrm{s}' \mid \mathrm{s}_t = \mathrm{s}) = \prod_i \mathbb{I}_{\{\gamma_i M_i \leq M'_i\}} F(\gamma'_i, \gamma_i) \equiv \boldsymbol{F}(\mathrm{s}', \mathrm{s})$. Also let $\boldsymbol{\Psi}$ be the associated stationary distribution, i.e., let $\boldsymbol{\Psi}$ be the unique solution to $\boldsymbol{\Psi}(\mathrm{s}') = \int \boldsymbol{F}(\mathrm{s}', \mathrm{s}) d\boldsymbol{\Psi}(\mathrm{s})$. We define a recursive equilibrium where all prices are time-invariant functions of the aggregate state s_t: $\boldsymbol{\phi}'_t = \boldsymbol{\phi}'(\mathrm{s}_t) = [\psi_A(\mathrm{s}_t), \varphi_A(\mathrm{s}_t), \psi_B(\mathrm{s}_t), \varphi_B(\mathrm{s}_t)]$ and $E_t = E(\mathrm{s}_t)$.

Definition 1. *A recursive equilibrium is a list of individual decision rules for buyer i, $\forall i \in \{A, B\}$, $\mathbf{w}_{t+1}$=$\mathbf{w}(\mathbf{s_t})$ =$[m_A(\mathrm{s}_t), a_A(\mathrm{s}_t), m_B(\mathrm{s}_t), a_B(\mathrm{s}_t)]$, pricing functions $\boldsymbol{\phi}'_t$=$\boldsymbol{\phi}'(\mathrm{s}_t)$ and $E_t = E(\mathrm{s}_t)$, bilateral terms of trade in $GM1$: $Q_i(\mathrm{s}_t) = q_i(\mathbf{w}(\mathrm{s}_t))$ and $\boldsymbol{P}(\mathrm{s}_t) = \boldsymbol{p}(\mathbf{w}(\mathrm{s}_t))$, $\forall i \in \{A, B\}$ where $q_i(\cdot)$ and $\boldsymbol{p}(\cdot)$ are given by Lemma 1, and bilateral terms of trade in $GM2$: $\widetilde{Q}_i(\mathrm{s}_t) = q_i(\mathbf{w}(\mathrm{s}_t))$ and $\boldsymbol{C}(\mathrm{s}_t) = \boldsymbol{\ell}(\mathbf{w}(\mathrm{s}_t))$, $\forall i \in \{A, B\}$ where $q_i(\cdot)$ and $\boldsymbol{\ell}(\cdot)$ are given by Lemma 2 such that:*

(i) the decision rule $\mathbf{w}(\cdot)$ solves the individual optimization problem (1), taking prices as given;

(ii) prices are such that the FM clears, i.e., $\mathbf{w}_{t+1} = [\gamma_{A,t} M_{A,t}, B_A, 0, 0]$ for buyer A and $\mathbf{w}_{t+1} = [0, 0, \gamma_{B,t} M_{B,t}, B_B]$ for buyer B;

(iii) the law of one price holds, i.e., $\varphi_A(\mathrm{s}_t) E(\mathrm{s}_t) = \varphi_B(\mathrm{s}_t)$.

In the remainder of the paper we only focus on a symmetric-recursive equilibrium case where all exogenously given parameters in this model are same across countries. That is θ, g, h are identical across the two countries, and $B_A = B_B = \bar{B}$. This fact implies that the list of equilibrium objects does not depend on the agent's citizenship but only on the aggregate state of the economy, s_t.

Definition 1 reveals some important properties of equilibrium. The fact that $m_i(\mathrm{s}_t) = \gamma_{i,t} M_{i,t}$ for all i in all states implies that the equilibrium is *always* monetary, meaning $\varphi_i(\mathrm{s}_t) > 0$ for all i and s_t. Intuition is straightforward. By construction, a unit of $bond_i$ yields $\varphi_i(\mathrm{s}_t)$ units of general good. Therefore, if the non-monetary equilibrium prevails then, the $bond_i$ must yield no general goods in any states. This means that both $money_i$ and $bond_i$ are never valued so that no GM trade takes place in the equilibrium which would surely be inferior to any monetary equilibrium outcome, i.e., $money_i$, $\forall i$ is always essential in this economy. Secondly, as explained earlier, the competitive nature of the FM does not allow any arbitrage in currency trade to arise in equilibrium, i.e., $\varphi_A(\mathrm{s}_t) E(\mathrm{s}_t) = \varphi_B(\mathrm{s}_t)$ for all states. Lastly, the fact that $m_{-i}(\mathrm{s}_t) = 0$ and $a_{-i}(\mathrm{s}_t) = 0$, i.e., no international diversification in asset holdings occurs, follows naturally from the optimality in Section 3.3.

In order to study equilibrium prices in the next section, one needs to consider "general equilibrium" counterparts of Euler equations in Section 3.3. To that end, let $Z_i(\mathrm{s}_t)$ denote "bond-illiquidity-augmented" equilibrium total real balances held by country i in state s_t. Likewise, define $Z_i^a(\mathrm{s}_t)$ as "pledgeability-augmented" equilibrium real $bond_i$ balances held by the country i in state s_t as follows.

$$Z_i(\mathbf{s}_t) \equiv \varphi_i(\mathbf{s_t})[M_{i,t} + g\bar{B}], \text{ and } Z_i^a(\mathbf{s}_t) \equiv h \cdot \varphi_i(\mathbf{s}_t)\bar{B}.$$

In equilibrium, the Euler equations for $money_i$ and $bond_i$ holdings are then given by

$$\varphi_i(\mathbf{s}_t) = \beta \int \boldsymbol{L}[Z_i(\mathbf{s}_{t+1})]\varphi_i(\mathbf{s}_{t+1})d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t), \tag{8}$$

$$\psi_i(\mathbf{s}_t) = \beta \int \boldsymbol{N}[Z_i(\mathbf{s}_{t+1}), Z_i^a(\mathbf{s}_{t+1})]\varphi_i(\mathbf{s}_{t+1})d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t), \tag{9}$$

where the *stochastic liquidity factors* for $money_i$ and $bond_i$ are respectively given by $\boldsymbol{L}[Z_i(\mathbf{s}_{t+1})]$ and $\boldsymbol{N}[Z_i(\mathbf{s}_{t+1}), Z_i^a(\mathbf{s}_{t+1})]$ as below.

$$\boldsymbol{L}[Z_i(\mathbf{s}_{t+1})] \equiv (1-\theta) + \theta u'\left(\min\{Z_i(\mathbf{s}_{t+1}), q^*\}\right), \tag{10}$$

$$\begin{aligned}\boldsymbol{N}[Z_i(\mathbf{s}_{t+1}), Z_i^a(\mathbf{s}_{t+1})] \equiv{}& \boldsymbol{L}[Z_i(\mathbf{s}_{t+1})] - \theta(1-g)\left[u'\left(\min\{Z_i(\mathbf{s}_{t+1}), q^*\}\right) - 1\right] \\ &+ (1-\theta)h\left[u'\left(\min\{Z_i^a(\mathbf{s}_{t+1}), q^*\}\right) - 1\right].\end{aligned} \tag{11}$$

Note that $\boldsymbol{L}\left[Z_i(\mathbf{s}_{t+1})\right] \geq 1$ for all $money_i$ growth rate realizations $\gamma_{i,t+1} \in \Xi_i$, with strict inequality if $\gamma_{i,t+1} \in \Gamma_{m_i}(\mathbf{s_t})$, where

$$\Gamma_{m_i}(\mathbf{s}_t) = \{\gamma_{i,t+1} \in \Xi_i : \varphi_i(\gamma_{i,t+1}, \gamma_{i,t}M_{i,t})[\gamma_{i,t}M_{i,t} + g\bar{B}] < q^*\},$$

Interpretation of $\boldsymbol{L}[Z_i(\mathbf{s}_{t+1})]$ is standard. It is stochastic and endogenously driven by the aggregate state of the economy. Most importantly, it captures the extra exchange value of $money_i$ in addition to its store of value. Thus, it is always bounded below by 1. It becomes unity, i,.e., the extra exchange value disappears, only when buyers already achieve the first best in the $GM1$, i.e., $Z_i(\mathbf{s}_{t+1}) \geq q^*$ in (10), or no opportunity to visit $GM1$ exists, i.e., $\theta = 0$ in (10).

On the other hand, the stochastic liquidity factor for $bond_i$, $\boldsymbol{N}[Z_i(\mathbf{s}_{t+1}), Z_i^a(\mathbf{s}_{t+1})]$ is richer and more interesting. If $money_i$ and $bond_i$ are perfect substitutes (e.g., $h = 0$ and $g = 1$) then, it should equal to $\boldsymbol{L}[Z_i(\mathbf{s}_{t+1})]$ for all s_t. This result complies with the rate-of-return dominance puzzle literature. When no illiquidity constaints on the use of bonds *endogenously* arise in our framework due to social conventions, money and bonds must be valued equally to prevent arbitrage. Thus, the nominal bond must yield zero nominal return, i.e., the right side of eq.(8) and (9) must be equalized.

However, our proposed GM trade mechanism endogenously induces the equilibrium to nest imperfect substitutability cases as well. If social conventions dictate that $h > 0$ and $g < 1$

then, the stochastic liquidity value of $bond_i$ exhibits two offsetting components that potentially make $\boldsymbol{N}[Z_i(\mathrm{s}_{t+1}), Z_i^a(\mathrm{s}_{t+1})]$ deviate from $\boldsymbol{L}[Z_i(\mathrm{s}_{t+1})]$. First, since only up to g portion of $bond_i$ holdings can be fully liquidated in $GM1$, the net liquidity value of the $bond_i$ has to be proportionally discounted relative to that of $money_i$, i.e., $-\theta(1-g)\left[u'\left(\min\{Z_i(\mathrm{s}_{t+1}), q^*\}\right)-1\right]$ in eq.(11). On the contrary, a unit of $bond_i$, serving as collateral in $GM2$, generates extra liquidity value relative to $money_i$, i.e., $(1-\theta)h\left[u'\left(\min\{Z_i^a(\mathrm{s}_{t+1}), q^*\}\right)-1\right]$. Which of these two offsetting forces dominates critically determines nominal bond yields relative to the zero nominal money return. In fact, structural parameters of the economy such as g, h, and θ turn out to be significant in this context. Suppose no $GM2$ exists, i,e., $\theta = 1$, or bonds are completely useless as collateral, i.e., $h = 0$, then, the second effect vanishes and therefore, the $bond_i$ is always traded at a discount compared to $money_i$, i.e., nominal bonds always dominate money in terms of the rate of return. Instead, let us imagine a economy where bonds are almost as liquid as money in $GM1$, i.e., $g \approx 1$. In this case, the $bond_i$ becomes an almost perfect substitute for $money_i$ in the $GM1$. Additionally, the former can still exhibit extra liquidity properties in $GM2$. This means nominal bonds must be sold at a premium in relation to money, meaning non-positive nominal interest rate prevails in this economy.

4.2 Inflation Rate

The price of $money_i$ in terms of general goods is $\varphi_i(\mathrm{s}_t)$. The nominal price of a general good is $1/\varphi_i(\mathrm{s}_t)$ in country i whose actual (gross) inflation rate between t and $t+1$ is then given by

$$\pi_i(\mathrm{s}_{t+1} = \mathrm{s}', \mathrm{s}_t = \mathrm{s}) \equiv \frac{\varphi_i(\mathrm{s})}{\varphi_i(\mathrm{s}')}.$$

One can also devise expected (gross) inflation in country i in terms of change in the $money_i$ price of a general good, conditional on the information available at s_t. Let us denote the expected (gross) inflation rate for country i at state s_t as $\tilde{\pi}_i(\mathrm{s}_t)$. Without loss of generality, we define the latter as the reciprocal of the harmonic mean of $\pi_i(\mathrm{s}_{t+1} = \mathrm{s}', \mathrm{s}_t = \mathrm{s})$ in the following way.

$$\frac{1}{\tilde{\pi}_i(\mathrm{s}_t)} \equiv \int \frac{1}{\pi_i(\mathrm{s}_{t+1} = \mathrm{s}', \mathrm{s}_t = \mathrm{s})} d\boldsymbol{F}(\mathrm{s}', \mathrm{s}).$$

4.3 Nominal Exchange Rate

Due to the law of one price as an equilibrium condition, $E(\mathrm{s}_t)$ can be rewritten as $\varphi_B(\mathrm{s}_t)/\varphi_A(\mathrm{s}_t)$. The expected exchange rate is then given by $E^e(\mathrm{s}_{t+1}, \mathrm{s}_t) = \mathbb{E}_t[\varphi_B(\mathrm{s}_{t+1}, \mathrm{s}_t)]/\mathbb{E}_t[\varphi_A(\mathrm{s}_{t+1}, \mathrm{s}_t)]$ owing to independent Markov process of $\gamma_{i,t}$ for each country. The expected depreciation (appreciation) of the $money_A$ ($money_B$) between t and $t+1$ is therefore given by

$$\frac{E^e(\mathbf{s}_{t+1},\mathbf{s}_t)}{E(\mathbf{s}_t)} = \frac{\mathbb{E}_t[\varphi_B(\mathbf{s}_{t+1},\mathbf{s}_t)]/\mathbb{E}_t[\varphi_A(\mathbf{s}_{t+1},\mathbf{s}_t)]}{\varphi_B(\mathbf{s}_t)/\varphi_A(\mathbf{s}_t)} = \frac{\int \frac{1}{\pi_B(\mathbf{s}_{t+1},\mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1},\mathbf{s}_t)}{\int \frac{1}{\pi_A(\mathbf{s}_{t+1},\mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1},\mathbf{s}_t)} = \frac{\tilde{\pi}_A(\mathbf{s}_t)}{\tilde{\pi}_B(\mathbf{s}_t)}. \tag{12}$$

Proposition 1. *The expected depreciation of a currency is positively correlated with its relative expected inflation rate to the partner country's, i.e., if $\tilde{\pi}_i(\mathbf{s}_t) > \tilde{\pi}_{-i}(\mathbf{s}_t)$ then, $E^e(\mathbf{s}_{t+1},\mathbf{s}_t)$ is higher (lower) than $E(\mathbf{s}_t)$ for $i = A$ ($i = B$).*

Proof. The proof is trivial, it it, therefore, omitted. □

The expected nominal exchange rate here behaves exactly same as in DSGE models with complete and Walrasian foreign exchange market.[15] Under such conventional models, a key equation that characterizes a joint stochastic process for nominal exchange rate and inflation is given by

$$\frac{E_{t+1}}{E_t} = \frac{m^*_{t+1}}{m_{t+1}}\frac{\pi_{t+1}}{\pi^*_{t+1}}, \tag{13}$$

where E_t denotes the nominal exchange rate (price of foreign currency in units of domestic), m_{t+1} denotes the inter-temporal marginal rate of substitution (IMRS) of the domestic representative agent (m^*_{t+1} for the foreign counterpart), and lastly, π_{t+1} (π^*_{t+1}) is the domestic (foreign) inflation rate. Notice that eq.(13) is identical to eq.(12) given that in our model, the IMRS for buyer i, $\forall i \in \{A, B\}$, equals to a constant β. This constant IMRS is directly attributed to the fact that equilibrium general good consumption for buyers is fixed at $\tilde{x}$ for every period, which again is an artifact of the quasilinear preference in the FM.[16]

4.4 Nominal Interest Rate

Next, we characterize equilibrium nominal interest rate for each country. $\psi_i(\mathbf{s}_t)$ denotes the state $\mathbf{s}_t$ price of $bond_i$ in terms of general goods, i.e., real price of nominal $bond_i$ in state $\mathbf{s}_t$. Then, $\psi_i(\mathbf{s}_t)/\varphi_i(\mathbf{s}_t)$ is its $money_i$ denominated price, i.e., nominal price of the nominal $bond_i$ in state $\mathbf{s}_t$. Hence, we can define country i's (gross) nominal interest rate in state $\mathbf{s}_t$ as its reciprocal,

[15] A seminal paper in this line of research is Lucas (1982) who pioneered an international asset pricing model in a two-country DSGE setup. See Backus, Foresi, and Telmer (2001) for an extensive literature review.

[16] One could instead choose to work with stochastic IMRS by introducing a stochastic shock process for the general good as in Lagos (2011). However, this would be redundant in our model. As will be seen in 4.5, doing so would not affect the equilibrium relationship between currency and nominal bond prices, i.e., stochastic IMRSs can affect the level of equilibrium currency and nominal bond prices but not a joint dynamics between those two.

$\varphi_i(\mathbf{s}_t)/\psi_i(\mathbf{s}_t)$. Using e.q.(8) and (9), it is given by

$$R_i(\mathbf{s}_t) \equiv \frac{\varphi_i(\mathbf{s}_t)}{\psi_i(\mathbf{s}_t)} = \frac{\int L\left[Z_i(\mathbf{s}_{t+1})\right]\varphi_i(\mathbf{s}_{t+1})d\boldsymbol{F}(\mathbf{s}_{t+1},\mathbf{s}_t)}{\int N\left[Z_i(\mathbf{s}_{t+1}),Z_i^a(\mathbf{s}_{t+1})\right]\varphi_i(\mathbf{s}_{t+1})d\boldsymbol{F}(\mathbf{s}_{t+1},\mathbf{s}_t)}. \tag{14}$$

Before we proceed, we restrict our model economy in a way that zero nominal interest rate bound is never violated. It is important to note that this constraint will not *per se* affect equilibrium relationship between various variables, nominal interest rates and expected inflation in particular. This will be discussed more with examples in the following Proposition 2. However, what makes the zero nominal interest rate bound matter here is that it does put a limit on monetary policy. Specifically, we will hereinafter only consider the set of $money_i$ growth rate realizations, ensuring the greater numerator than the denominator in eq.(14), i.e., $L\left[Z_i(\mathbf{s}_{t+1})\right] \geq N\left[Z_i(\mathbf{s}_{t+1}),Z_i^a(\mathbf{s}_{t+1})\right], \forall i,\mathbf{s}_t$. Technically speaking, we only consider $\gamma_{i,t+1} \in \Gamma_{ZLB_i}(\mathbf{s_t})$, where

$$\Gamma_{ZLB_i}(\mathbf{s}_t) = \left\{\gamma_{i,t+1} \in \Xi_i : \frac{u'\left(\min\{h\varphi_i(\gamma_{i,t+1},\gamma_{i,t}M_{i,t})\bar{B},q^*\}\right)-1}{u'\left(\min\{\varphi_i(\gamma_{i,t+1},\gamma_{i,t}M_{i,t})[\gamma_{i,t}M_{i,t}+g\bar{B}],q^*\}\right)-1} \leq \frac{\theta(1-g)}{(1-\theta)h}\right\}. \tag{15}$$

Next proposition reveals important equilibrium properties regarding the relationship between nominal interest rate and expected inflation in each country.

Proposition 2. *Consider an economy with the zero nominal interest rate bound, i.e.,* $\gamma_{i,t+1} \in \Gamma_{ZLB_i}(\mathbf{s_t})$ *for all* i *and* $\mathbf{s}_t$. *Then,* $R_i(\mathbf{s}_t)$ *and* $\tilde{\pi}_i(\mathbf{s}_t)$ *are related in the following way.*

a) If $\theta = 1$ *or* $h = 0$ *then,* $\Gamma_{ZLB_i}(\mathbf{s_t}) = \Xi_i$ *and* $\partial R_i(\mathbf{s}_t)/\partial\tilde{\pi}_i(\mathbf{s}_t) \geq 0$.

b) Otherwise, $\Gamma_{ZLB_i}(\mathbf{s_t}) \subset \Xi_i$*, and a sufficient condition for* $\partial R_i(\mathbf{s}_t)/\partial\tilde{\pi}_i(\mathbf{s}_t) < 0$ *is* $(1-\theta)h \geq (1-g)$.

Proof. See the appendix. □

In order to interpret these results, it is useful to rewrite the nominal price of $bond_i$ at $\mathbf{s}_t$, $\psi_i(\mathbf{s}_t)/\varphi_i(\mathbf{s}_t)$, in terms of three different values. Appendix shows the following.

$$\begin{aligned}\frac{\psi_i(\mathbf{s}_t)}{\varphi_i(\mathbf{s}_t)} = 1 &- \beta\theta(1-g)\int\frac{\left[u'\left(\min\{Z_i(\mathbf{s}_{t+1}),q^*\}\right)-1\right]}{\pi_i(\mathbf{s}_{t+1},\mathbf{s}_t)}d\boldsymbol{F}(\mathbf{s}_{t+1},\mathbf{s}_t) \\ &+ \beta(1-\theta)h\int\frac{\left[u'\left(\min\{Z_i^a(\mathbf{s}_{t+1}),q^*\}\right)-1\right]}{\pi_i(\mathbf{s}_{t+1},\mathbf{s}_t)}d\boldsymbol{F}(\mathbf{s}_{t+1},\mathbf{s}_t).\end{aligned} \tag{16}$$

The nominal value of a unit $bond_i$ at $\mathbf{s}_t$ can be thought of a sum of three different components as in (16). The first component which always equals to one refers to the nominal value of a unit of $money_i$ at $\mathbf{s}_t$. The negative component in the first line of eq.(16) represents the expected nominal value of illiquidity discount on the $bond_i$ due to g in $GM1$. The third component in the second line of (16), on the other hand, captures the expected nominal premium value of the $bond_i$ as a result of its collateral role in $GM2$.

Part $a)$ of Proposition 2 characterizes the equilibrium relationship between $R_i(\mathbf{s}_t)$ and $\tilde{\pi}_i(\mathbf{s}_t)$ when the nominal $bond_i$ can not serve as collateral. A monetary policy in this case is never restricted since the $bond_i$ always has lower liquidity properties than the $money_i$, i.e., $\psi_i(\mathbf{s}_t)/\varphi_i(\mathbf{s}_t)$ always becomes less than or equal to unity regardless of $\gamma_{i,t+1}$. More importantly, it is straightforward to understand why $\partial R_i(\mathbf{s}_t)/\partial\tilde{\pi}_i(\mathbf{s}_t) \geq 0$. Since anticipated inflation acts as a tax on holding real balances, a higher $\tilde{\pi}_i(\mathbf{s}_t)$ reduces the $Z_i(\mathbf{s}_{t+1})$ and $Z_i^a(\mathbf{s}_{t+1})$. This in turn induces their expected nominal liquidity value to rise in $GM1$ and $GM2$ respectively.[17] Yet, the fact that the $GM2$ is now irrelevant makes the premium value of the $bond_i$ disappear. It is therefore the case that the higher expected nominal liquidity value of $Z_i(\mathbf{s}_{t+1})$ only amplifies the illiquidity discount of the $bond_i$, i.e., the second negative term in the first line of (16) gets bigger. Thus, the nominal $bond_i$ price falls, i.e., $R_i(\mathbf{s}_t)$ increases.

This Fisher effect no longer prevails universally as soon as the $bond_i$ has some liquidity properties as collateral in our model. Part $b)$ of Proposition 2 implies that relatively high pledgeability of $bond_i$ both as means of payment and collateral, and a relatively high portion of credit based transactions in goods trade, i.e., $(1-\theta)h \geq (1-g)$, guarantee a negative correlation between $R_i(\mathbf{s}_t)$ and $\tilde{\pi}_i(\mathbf{s}_t)$. Intuition follows clearly from a previous paragraph. Since a higher $\tilde{\pi}_i(\mathbf{s}_t)$ now amplifies both the illiquidity discount and the premium value of the $bond_i$, what matters is their relative size of effects. As the eq.(16) reveals, the sufficient condition, $(1-\theta)h \geq (1-g)$ secures a higher weight on the premium value of the $bond_i$ than the illiquidity discount value. This intuitively explains why the higher $\tilde{\pi}_i(\mathbf{s}_t)$ brings about increase in the former overweighing that in the latter. To see intuition from a different angle, consider *real* price change of $money_i$ and $bond_i$. When $\tilde{\pi}_i(\mathbf{s}_t)$ rises, the real price of both assets surely goes up due to higher marginal utilities in GM associated with a fall in real balances. Yet, the condition $(1-\theta)h \geq (1-g)$ induces the $bond_i$ to exhibit somewhat higher exchange value as a facilitator of special good i trade. Thus, no arbitrage condition makes sure that the real $bond_i$ price increases more than that of $money_i$, meaning the nominal $bond_i$ price should increase.

Notice that allowing for no illiquidity constraint in $GM1$ $(g = 1)$ would make the sufficient condition become redundant for $\partial R_i(\mathbf{s}_t)/\partial\tilde{\pi}_i(\mathbf{s}_t) < 0$. However, as seen in (15), this would in fact make the zero-nominal-interest-bound (ZLB) inducing monetary policy in country i highly restrictive. Basically such policy should result in the first best outcome in $GM2$ at all states, but strictly less than the first best in $GM1$. A set of $\gamma_{i,t+1}$ that satisfies this condition would be

[17] A higher $Z_i(\mathbf{s}_{t+1})$ or $Z_i^a(\mathbf{s}_{t+1})$ surely causes a higher expected *real* liquidity value of itself due to the concavity of $u(\cdot)$. The fact that the nominal liquidity value also goes up can be understood intuitively as follows. The Euler equation for money holdings requires that (real) cost of carrying them must equal to the (real) net benefit from doing so. This condition must hold true in nominal terms because one would divide both side of the Euler equation by the same real price of money to arrive the nominal Euler equation. Then, the higher anticipated inflation should raise both nominal cost and nominal benefit (nominal liquidity value) to ensure the optimality.

particularly narrow when the supply of $bond_i$ is relatively small. In fact, it would be interesting to study what should be a family of optimal stochastic monetary policies in our framework. This, however, would be beyond the scope of this study. Thus, we leave this task for future research.[18]

Before we go on studying implications of these effects on the UIP condition, it is worth emphasizing that the mechanism in which a negative comovement between nominal interest rates and money supply (anticipated inflation) arises differs from the New Keynesian framework. This inverse relationship, often called as *liquidity effect* of monetary policy, is one major characteristics of traditional sticky-price New Keynesian models. Their key idea is that money supply has a direct positive effect on equilibrium real money balances in the short run due to some form of nominal rigidity. Since bonds only serve as a store of value in their framework, financial market equilibrium requires a positive movement between real bond price and real money balances. Finally, the price stickiness ensures that nominal bond price moves in the same direction as money supply.

On the contrary, our proposed model abandons the price stickiness assumption, and cause the liquidity effect through a different channel. In our model, a higher money growth rate boosts real bond price not because of price stickiness but due to bonds' role as a medium of exchange. Since the fully flexible price movement tends to depress the nominal value of bonds in the event of the higher money growth rate, the nominal bond price finally depends on the relative size of increase in the real bond price. When bonds play an extra facilitator role compared to money then, the real bond price effect tends to dominate, and the nominal bond price rises, i.e., the liquidity effect, otherwise the latter vanishes.[19]

4.5 Uncovered Interest Parity Puzzle

We are finally ready to state main results of our paper. To ease presentation, let us rephrase the UIP puzzle in terms of predictable excess returns on currencies. In our framework, one can define a one-period excess $money_A$ return on $money_B$ at state $\mathbf{s}_t$ as

$$\mathcal{Q}(\mathbf{s}_t) = \ln E^e(\mathbf{s}_{t+1}, \mathbf{s}_t) - \ln E(\mathbf{s}_t) + \ln R_B(\mathbf{s}_t) - \ln R_A(\mathbf{s}_t). \tag{17}$$

The UIP puzzle means that $\mathcal{Q}(\mathbf{s}_t)$ is actually predictable because data supports a positive effect of interest rate differential, i.e., $\ln R_B(\mathbf{s}_t) - \ln R_A(\mathbf{s}_t)$ on $\mathcal{Q}(\mathbf{s}_t)$. So the task here is to find

[18] Lagos (2011) studies a set of optimal stochastic monetary policies that implement the Friedman rule basically in a nested version of our economy, i.e., $g = 0$ and $\theta = 1$, with an inclusion of stochastic dividend paying equity.

[19] This difference at least points to the possibility that open market operations could potentially generate richer implications on monetary policy and asset prices than what nominal friction based New-Keynesian models imply. See Lagos, Rocheteau, and Wright (2014) for more extensive literature review on how micro-founded monetary economics based on search theory offers different monetary policy implications.

conditions under which $\partial \mathcal{Q}(\mathbf{s}_t)/\partial [\ln R_B(\mathbf{s}_t) - \ln R_A(\mathbf{s}_t)] > 0$ for every $\mathbf{s}_t$ in our model. As an intermediate step, it is useful to define the *real* liquidity adjusted stochastic discount factor $\mathcal{M}_{i,t+1}$. Following the conventional international asset pricing model, inversing real (gross) return on $bond_i$ should generate the expected $\mathcal{M}_{i,t+1}$ in our model as well.[20] Therefore,

$$\mathbb{E}_t(\mathcal{M}_{i,t+1}) \equiv \mathcal{M}_i(\mathbf{s}_t) = \frac{\tilde{\pi}_i(\mathbf{s}_t)}{R_i(\mathbf{s}_t)}, \ \forall i \text{ and } \mathbf{s}_t. \tag{18}$$

Substituting (12) and (18) into (17) simplifies $\mathcal{Q}(\mathbf{s}_t)$ as follows.

$$\mathcal{Q}(\mathbf{s}_t) = \ln \mathcal{M}_A(\mathbf{s}_t) - \ln \mathcal{M}_B(\mathbf{s}_t). \tag{19}$$

Equation (19) reveals an important characteristic of excess returns on $money_B$. The latter is now completely driven by liquidity property differential. Notice that any IMRS differential between two countries could have never affected $\mathcal{Q}(\mathbf{s}_t)$ even if it existed. The reason is that its effects on interest rate differential and the expected appreciation of $money_B$ will be completely canceled out. In fact, this would be the same equilibrium property of conventional models where the IMRS simply equals the stochastic discount factor. What makes our model differ is that asset liquidity factors asymmetrically augment the stochastic discount factor for the interest and exchange rate. This mechanically gives rise to the liquidity-differential dependent excess returns on $money_B$ in (17). Next proposition finally states a sufficient condition for the UIP violation in equilibrium.

Proposition 3. *Under $\Gamma_{ZLB_i}(\mathbf{s_t}) \subset \Xi_i$ a sufficient condition for $\partial \mathcal{Q}(\mathbf{s}_t)/\partial [\ln R_B(\mathbf{s}_t) - \ln R_A(\mathbf{s}_t)] > 0$ for every $\mathbf{s}_t$ is $(1-\theta)h \geq (1-g)$.*

Proof. The proof is trivial given Proposition 2, and it is intuitively explained in the following paragraph, therefore it is omitted. □

Proposition 2 states that the condition $(1-\theta)h \geq (1-g)$ under the zero nominal interest rate bound always guarantees a negative effect of $\tilde{\pi}_i(\mathbf{s}_t)$ on $R_i(\mathbf{s}_t)$ for all i and $\mathbf{s}_t$. Combining this result with (18) brings about a positive effect of anticipated inflation on the real liquidity adjusted stochastic discount factor. This implies that a relatively higher anticipated inflation in country A than B leads to higher excess returns on $money_B$. In the meanwhile, the country A's higher anticipated inflation induces the country B's nominal interest rate to become relatively higher than its counterpart since $\partial R_i(\mathbf{s}_t)/\partial \tilde{\pi}_i(\mathbf{s}_t) < 0$. Therefore, $\mathcal{Q}(\mathbf{s}_t)$, regardless of $\mathbf{s}_t$, is always increasing in $\ln R_B(\mathbf{s}_t) - \ln R_A(\mathbf{s}_t)$ under the sufficient condition.

[20] As explained in footnote 16, this simplification is based on the fact that the standard IMRS, i.e., $\beta U'(X_{t+1})/U'(X_t)$, is a constant in our framework due to the fixed amount of general good consumption over time.

This proposition implies that the UIP violation critically hinges upon the extent to which nominal bonds play a liquidity role. Under the conventional international asset pricing model, a sudden increase in one country's interest rate would lead to an expected depreciation of the currency, thereby holding the UIP condition. Again, this is because nominal bonds, playing no liquidity role, always yield inflation-only-dependent return. However, this mechanism is no longer pervasive when bonds play a liquidity role. For instance, if the bonds exhibit somewhat higher liquidity properties than money in a precise sense that $(1-\theta)h \geq (1-g)$ within our model, an increase in real return on money (i.e., fall in inflation rate) leads to a relatively bigger increase in real return on bonds (i.e., rise in nominal interest rate). Therefore, unlike the conventional model, a sudden increase in one country's interest rate would lead to an expected appreciation of the currency, thereby causing the UIP violation. This intuition naturally brings about the following corollary.

Corollary 1. *Under* $\Gamma_{ZLB_i}(\mathbf{s_t}) \subset \Xi_i \; \partial \mathcal{Q}(\mathbf{s}_t)/\partial \left[\ln R_B(\mathbf{s}_t) - \ln R_A(\mathbf{s}_t)\right]$ *depends on* $\mathbf{s}_t$*, and the sign is ambiguous if* $(1-\theta)h < (1-g)$*.*

Corollary 1 states that the UIP violation becomes no longer pervasive when liquidity properties of nominal bonds are relatively lower. That is, the UIP no longer violates uniformly when the portion of credit based transactions are lower ($1-\theta$ is lower) and/or the bond pledgeability and liquidity are lower (h and g are lower). This prediction is consistent with a couple of empirical facts regarding the UIP puzzle. First, it is consistent with Bansal and Dahlquist (2000)'s evidence and our empirical findings (Table 1). Caballero, Farhi, and Gourinchas (2008) argue that pledgeability and/or liquidity of assets for emerging economies are generally lower than developed economies. Furthermore, various measures for cross-country credit market and/or financial market development can confirm that credit based transactions are relatively scarce for emerging economies. Our model can capture a similar notion by assuming that g, h, and $1-\theta$ are lower for emerging economies. Consequently, the model implies that the UIP no longer violates uniformly for emerging economies.

Table 1 reports the results of the time-series cross-sectional regressions to present the so-called UIP coefficients. In particular, the coefficients in regressions (1) and (2) support the theoretic result we mentioned above. The coefficient for the developed countries over the whole sample period (regression (1)) is negative (-0.521), while for the emerging markets (regression (2)) positive (+0.537). This observation suggests that there exists the forward premium puzzle among developed economies, while it does not exist among emerging markets.

Corollary 1 also aligns with an empirical fact that the UIP does not violate when the effective liquidity of the economy suddenly shrinks. Among others, Brunnermeier, Nagel, and Pedersen (2008) show that tightening interbank liquidity predicts carry trade losses (i.e., the UIP suddenly holds). One can capture the sudden "drying up" of economy-wide liquidity by a shock that reduces g, h, and $1-\theta$ in our model. For instance, suppose that the liquidity shock changes $(1-\theta)h \geq (1-g)$ to $(1-\theta)h < (1-g)$. Proposition 3 and Corollary 1 then suggest that such a sudden liquidity shock can lead to a sudden carry trade return reversal in our model. The regressions (3) and (4) in Table 1 reports an empirical evidence for this suggestion. We divide the sample period of the developed countries into "during the period of 2008 and 2009" and "during the period except for 2008 and 2009" to investigate how the UIP coefficient changes during the period when financial assets become illiquid. The coefficient during years 2008 and 2009 presents a great positive number (+11.30). However, during the period when the two years are excluded from the sample period, the coefficient turns into a negative number (-1.017), which implies the strong negative relationship between the expected exchange rates and the forward premiums, relative to -0.521, which is exactly consistent with Brunnermeier, Nagel, and Pedersen (2008).

Table 1: Forward Premium Regression

$$\text{Regression Equation}: \left[\frac{E_{i,t+1}-E_{i,t}}{E_{i,t}}\right] = \alpha_0 + \alpha_1\left[\frac{F_{i,t}-E_{i,t}}{E_{i,t}}\right] + u_{i,t+1}$$

	(1) Developed Economies	(2) Emerging Economies	(3) Developed Economies during 2008-9	(4) Developed Economies during the period except for 2008-9
α_1	-0.521	0.537***	11.30***	-1.017
	(0.414)	(0.0741)	(3.390)	(0.611)
α_0	8.52e-05	-0.000972	0.0139**	-0.000778
	(0.000934)	(0.00146)	(0.00473)	(0.00144)
Period	1/1996-2/2015	1/1996-2/2015	1/2008-12/2009	excl. 08-09
Observations	2,339	1,667	238	2,101
R-squared	0.002	0.192	0.102	0.008
Number of state	11	9	11	11
State FE	Yes	Yes	Yes	Yes

Note: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Monthly data on spot exchange rates, and 3-month forward rates for 20 countries from *Bloomberg* are used in the regressions. Fixed effects are included in all of the regressions, i.e., a country-specific intercept is added to each regression. The developed countries include Switzerland, Hong Kong, Singapore, Japan, Belgium, Austria, Denmark, Canada, UK, Australia, Sweden, and the emerging countries include Czech Republic, Malaysia, Argentina, Mexico, Thailand, Philippines, Indonesia, India, Turkey, Korea according to the *International Finance Corporation (IFC)* of the *World Bank*. The exchange rates for the developed countries are the US Dollar prices per unit and for the emerging countries the Malaysian Ringgit prices per unit.

5 Conclusion

Recent monetary and finance theories tell that liquidity properties of assets can play a significant role for asset pricing. Furthermore, they also show that such liquidity aspects of assets interact with monetary policy. This insight is a point of departure for our liquidity-based explanation of the UIP puzzle. We have shown that monetary policy determines the liquidity premium on nominal bonds, which can account for non-uniform deviations from the UIP condition. Intuitively, the conventional wisdom says a high interest rate currency appreciates because it is riskier. We, on the other hand, argue that the high interest rate currency might be appreciating because it is less liquid when the economy is confined to an environment where bonds serve as a sole MOE in some goods transactions. This property of the model turns out to be consistent with some UIP evidence that many conventional risk-based models find hard to justify.

Last but not least, we admit that this liquidity-based explanation is certainly no *panacea* for all those decades long discussions on the UIP puzzle. Yet, we hope that our approach can shed new light on the debate by offering a new liquidity-based perspective. For instance, Backus, Gavazzoni, Telmer, and Zin (2010) have speculated that carry trade returns are in some sense a mirror image of monetary policy implementation costs. We offer a complementary view that the arbitrage carry trade profits might reflect upon the cost of aggregate liquidity management.

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A Appendix

Proof. Proof of Lemma 1.
First, consider the second stage in the trading mechanism. We substitute the real balances term in the buyer's participation constraint (3.2) into the maximization objective function (3.2). Then, bargaining problem in the second stage where the implemented terms of trade is determined is given by

$$\begin{aligned}\mathbb{U}^s_{21}(\mathbf{w}_t) &= \max_{q^1_t, \boldsymbol{p}_t} u(q^1_t) - q^1_t - \mathbb{U}^b_{11}(\mathbf{w}_t)\\ &\text{s.t. } u(q^1_t) - \boldsymbol{\phi_t} \cdot \boldsymbol{p}_t = \mathbb{U}^b_{11}(\mathbf{w}_t),\\ &\quad \boldsymbol{p}_t \leq \mathbf{w}_t\end{aligned}$$

If $u^{-1}\big[\boldsymbol{\phi_t} \cdot \mathbf{w}_t + \mathbb{U}^b_{11}(\mathbf{w}_t)\big] \geq \tilde{q}$, it is obvious that q^1_t will always be equal to $\tilde{q}$, and $\tilde{\boldsymbol{p}}_t(\mathbf{w}_t)$ can be any combination of money and assets such that $\boldsymbol{\phi}_t \cdot \tilde{\boldsymbol{p}}_t(\mathbf{w}_t) = u(\tilde{q}) - \mathbb{U}^b_{11}(\mathbf{w}_t)$. In addition, $\mathbb{U}^s_{21}(\mathbf{w}_t)$ will be equal to $u(\tilde{q}) - \tilde{q} - \mathbb{U}^b_{11}(\mathbf{w}_t)$, which is zero in the case where $\varphi_t(m_t + a_t) \geq \tilde{q}$, because $\mathbb{U}^b_{11}(\mathbf{w}_t) = u(\tilde{q}) - \tilde{q}$. If $u^{-1}\big[\boldsymbol{\phi_t} \cdot \mathbf{w}_t + \mathbb{U}^b_{11}(\mathbf{w}_t)\big] < \tilde{q}$, the first best choice $\tilde{q}$ cannot be achieved, and so the seller will make an offer to have the buyer hand over all of her real balances in order to sell the special goods as many as possible. In this case, $\boldsymbol{p}_t(\mathbf{w}_t) = \mathbf{w}_t$ and $q^1_t = u^{-1}\big[\boldsymbol{\phi_t} \cdot \mathbf{w}_t + \mathbb{U}^b_{11}(\mathbf{w}_t)\big]$. Lastly, $\mathbb{U}^s_{21}(\mathbf{w}_t)$ will be equal to $u(q^1_t) - q^1_t - \mathbb{U}^b_{11}(\mathbf{w}_t)$.

Likewise, the bargaining problem in the first stage is given by plugging the seller's participation constraint (3.2) into the objective function (3.2) as follows.

$$\begin{aligned}\mathbb{U}^b_{11}(\mathbf{w}_t) = &\max_{q^f_t,\, p^m_t,\, p^a_t} u(q^f_t) - q^f_t\\ &\text{s.t. } -q^f_t + \varphi_t(\,p^m_t + \,p^a_t) = 0\\ &\quad p^m_t \leq m_t, \;\; p^a_t \leq a_t\end{aligned}$$

If $\varphi_t(m_t + a_t) \geq \tilde{q}$, q^f_t will be equal to $\tilde{q}$ and $\mathbb{U}^b_{11}(\mathbf{w}_t)$ is the same as $u(\tilde{q}) - \tilde{q}$. If $\varphi_t(m_t + a_t) < \tilde{q}$, then q^f_t will be equal to $\varphi_t(m_t + a_t)$, because the buyer will give up all of her local money and assets to purchase as many as she can. In this case, $\mathbb{U}^b_{11}(\mathbf{w}_t)$ is $u(q^f_t) - \varphi_t(m_t + a_t)$. $\square$

Proof. Proof of Proposition 2.
Dividing equation (8) by $\varphi(\mathbf{s}_t)$ leads to

$$\begin{aligned}1 &= \beta \int \frac{\boldsymbol{L}[Z_i(\mathbf{s}_{t+1})]}{\pi_i(\mathbf{s}_{t+1}, \mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t)\\ &= \frac{\beta}{\tilde{\pi}_i(\mathbf{s}_t)} + \beta\theta \int \frac{[u'(min\{Z_i(\mathbf{s}_{t+1}), q^*\}) - 1]}{\pi_i(\mathbf{s}_{t+1}, \mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t)\\ &= \frac{\beta}{\tilde{\pi}_i(\mathbf{s}_t)} + \beta\theta \mathbf{X}_i(\mathbf{s}_t)\end{aligned}$$

where $\mathbf{X}_i(\mathrm{s}_t) = \int \frac{[u'(min\{Z_i(\mathbf{s}_{t+1}),q^*\})-1]}{\pi_i(\mathbf{s}_{t+1},\mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t)$. Then,

$$\beta\theta\mathbf{X}_i(\mathbf{s}_t) = 1 - \frac{\beta}{\tilde{\pi}_i(\mathbf{s}_t)}.$$

By the implicit function theorem,

$$\frac{\partial \mathbf{X}_i(\mathbf{s}_t)}{\partial \tilde{\pi}_i(\mathbf{s}_t)} = \frac{1}{\theta[\tilde{\pi}_i(\mathbf{s}_t)]^2} > 0.$$

On the other hand, we rewrite the nominal bond price at s_t in country i (equation (16)) as follows.

$$\begin{aligned}\frac{\psi_i(\mathbf{s}_t)}{\varphi_i(\mathbf{s}_t)} &= 1 - \beta\theta(1-g)\mathbf{X}_i(\mathbf{s}_t) \\ &\quad + \beta(1-\theta)h \int \frac{[u'\left(\min\{Z_i^a(\mathbf{s}_{t+1}), q^*\}\right) - 1]}{\pi_i(\mathbf{s}_{t+1}, \mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t).\end{aligned}$$

Then, the partial derivative of the nominal bond price is

$$\begin{aligned}\frac{\partial[\psi_i(\mathbf{s}_t)/\varphi_i(\mathbf{s}_t)]}{\partial\tilde{\pi}_i(\mathbf{s}_t)} &= \beta\theta(1-g)\frac{\partial \mathbf{X}_i(\mathbf{s}_t)}{\partial\tilde{\pi}_i(\mathbf{s}_t)} + \beta(1-\theta)h\frac{\partial \mathbf{Y}_i(\mathbf{s}_t)}{\partial\tilde{\pi}_i(\mathbf{s}_t)} + \frac{\beta(1-\theta)h}{[\tilde{\pi}_i(\mathbf{s}_t)]^2} \\ &= \frac{\beta[(1-\theta)h - (1-g)]}{[\tilde{\pi}_i(\mathbf{s}_t)]^2} + \beta(1-\theta)h\frac{\partial \mathbf{Y}_i(\mathbf{s}_t)}{\partial\tilde{\pi}_i(\mathbf{s}_t)}\end{aligned}$$

where $\mathbf{Y}_i(\mathbf{s}_t) = \int \frac{\left[u'\left(\min\{Z_i^a(\mathbf{s}_{t+1}),q^*\}\right)-1\right]}{\pi_i(\mathbf{s}_{t+1},\mathbf{s}_t)} d\boldsymbol{F}(\mathbf{s}_{t+1}, \mathbf{s}_t)$. Now, it is obvious from the definitions of $Z_i(\mathbf{s}_t)$ and $Z_i^a(\mathbf{s}_t)$ that $\frac{\partial \mathbf{Y}_i(\mathbf{s}_t)}{\partial\tilde{\pi}_i(\mathbf{s}_t)} > 0$, because $\frac{\partial \mathbf{X}_i(\mathbf{s}_t)}{\partial\tilde{\pi}_i(\mathbf{s}_t)} > 0$. Consequently, the sufficient condition for $\frac{\partial[\psi_i(\mathbf{s}_t)/\varphi_i(\mathbf{s}_t)]}{\partial\tilde{\pi}_i(\mathbf{s}_t)} > 0$ is $(1-\theta)h \geq (1-g)$. □