Credit expansion and contraction: a simplified model

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ABSTRACT

Presented is a mathematical model of single-product economy where credit expansion is used to increase the demand for product. Explored is the dynamics of affected product's price, supply and demand. Shown is that expansion of the demand carries a temporal character.

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1   Introduction

Concepts of credit and debt carry an enormous role in modern economics. I develop here a model tool, which I would like to use later to explore the impact of these phenomena on economic life.

Below I describe a mathematical model of the market of single-product economy. Economic forces acting on the market represent inherent market forces of demand and supply complemented with the forces caused by credit expansion and corresponding debt servicing. The market actions are expressed through the system of ordinary differential equations.

The model somewhat continues previous research for nominal economic growth and decline performed in Krouglov, 2014.

The presented model carries mostly theoretical character. However, it can be further expanded to deal with more sophisticated tasks.

2   Single-Product Economy at Undisturbed State

Concepts and methodology presented in this section are based on the framework of mathematical dynamics of economic systems developed in Krouglov, 2006; 2009.

When there are no disturbing economic forces, the market is in equilibrium position, i.e., the supply of and demand for product are equal, they are developing with a constant rate and a price of the product is fixed.

I assume the market had been in an equilibrium until time $t = t_0$, volumes of the product supply $V_S(t)$ and demand $V_D(t)$ on market were equal, and they both were developing with a constant rate r_D^0. The product price $P(t)$ at that time was fixed,

$$V_D(t) = r_D^0(t - t_0) + V_D^0 \tag{1}$$

$$V_S(t) = V_D(t) \tag{2}$$

$$P(t) = P^0 \tag{3}$$

where $V_D(t_0) = V_D^0$.

When the balance between the volumes of the product supply and demand is broken, the market is experiencing economic forces, which act to bring the market to a new equilibrium position.

3   Constant-Rate Credit Expansion in a Single-Product Economy

I present model of a single-product economy where the credit is increasing with a constant-rate in order to advance the demand for the product.

According to this scenario, the credit expansion causes a debt growth where the amount of debt $S_D(t)$ on the market rises since time $t = t_0$ according to the following formula,

$$S_D(t) = \begin{cases} 0, & t < t_0 \\ \delta_D(t - t_0), & t \geq t_0 \end{cases} \tag{4}$$

where $S_D(t) = 0$ for $t < t_0$ and $\delta_D > 0$.

Correspondingly, the debt growth causes an increase of debt servicing cost $s_S(t) = \delta_S S_D(t)$, $\delta_S > 0$, where the accumulated amount of debt servicing cost $S_S(t)$ on the market rises according to following formula since time $t = t_0$,

$$S_S(t) = \begin{cases} 0, & t < t_0 \\ \dfrac{\delta_S \delta_D}{2}(t - t_0)^2, & t \geq t_0 \end{cases} \tag{5}$$

where $S_S(t) = 0$ for $t < t_0$ and $\delta_S > 0$.

Economic forces trying to bring the market into a new equilibrium position are described by the following ordinary differential equations with regard to the volumes of product supply $V_S(t)$, demand $V_D(t)$, and price $P(t)$ given the accumulated amounts of debt $S_D(t)$ and of debt servicing cost $S_S(t)$ on the market (see Krouglov, 2006; 2009),

$$\frac{dP(t)}{dt} = -\lambda_P \left(V_S(t) - V_D(t) - S_D(t) + S_S(t) \right) \tag{6}$$

$$\frac{d^2 V_S(t)}{dt^2} = \lambda_S \frac{dP(t)}{dt} \tag{7}$$

$$\frac{d^2 V_D(t)}{dt^2} = -\lambda_D \frac{d^2 P(t)}{dt^2} \tag{8}$$

In Eqs. (6) – (8) above the values $\lambda_P, \lambda_S, \lambda_D \geq 0$ are constants and they characterize price inertness, supply inducement, and demand amortization correspondingly.

One may say if the accumulated amount of debt $S_D(t)$ exceeds the accumulated amount of debt servicing cost $S_S(t)$: $S_D(t) \geq S_S(t)$, credit expansion takes place in the sense $V_D(t) + \left(S_D(t) - S_S(t) \right) \geq V_D(t)$.

On the other hand, when the accumulated amount of debt $S_D(t)$ goes below the amount of debt servicing cost $S_S(t)$: $S_D(t) < S_S(t)$, credit contraction takes place in the sense $V_D(t) + \left(S_D(t) - S_S(t) \right) < V_D(t)$.

Thus, credit expansion takes place in the time interval $t_0 \leq t \leq t_0 + \dfrac{2}{\delta_S}$, and credit contraction happens when $t_0 + \dfrac{2}{\delta_S} < t < +\infty$.

Let me introduce a new variable $D(t) \equiv \left(V_S(t) - V_D(t) - S_D(t) + S_S(t)\right)$ representing the volume of product surplus (or shortage) on the market. Therefore, behavior of $D(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2 D(t)}{dt^2} + \lambda_P \lambda_D \frac{dD(t)}{dt} + \lambda_P \lambda_S D(t) - \delta_S \delta_D = 0 \tag{9}$$

with the initial conditions, $D(t_0) = 0$, $\dfrac{dD(t_0)}{dt} = -\delta_D$.

If one uses another variable $D_1(t) \equiv D(t) - \dfrac{\delta_S \delta_D}{\lambda_P \lambda_S}$, then Eq. (9) becomes,

$$\frac{d^2 D_1(t)}{dt^2} + \lambda_P \lambda_D \frac{dD_1(t)}{dt} + \lambda_P \lambda_S D_1(t) = 0 \tag{10}$$

with the initial conditions, $D_1(t_0) = -\dfrac{\delta_S \delta_D}{\lambda_P \lambda_S}$, $\dfrac{dD_1(t_0)}{dt} = -\delta_D$.

Similar to Eq. (9), the product price $P(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2 P(t)}{dt^2} + \lambda_P \lambda_D \frac{dP(t)}{dt} + \lambda_P \lambda_S \left(P(t) - P^0 - \frac{\delta_D}{\lambda_S} + \frac{\delta_S \delta_D}{\lambda_S}(t - t_0) \right) = 0 \tag{11}$$

with the initial conditions, $P(t_0) = P^0$, $\dfrac{dP(t_0)}{dt} = 0$.

Let me introduce variable $P_1(t) \equiv P(t) - P^0 - \dfrac{\delta_D}{\lambda_S} + \dfrac{\delta_S \delta_D}{\lambda_S}(t - t_0) - \dfrac{\lambda_D}{\lambda_S^2} \delta_S \delta_D$ to simplify an analysis of the product price behavior. The behavior of variable $P_1(t)$ is described by the equation for $t > t_0$,

$$\frac{d^2 P_1(t)}{dt^2} + \lambda_P \lambda_D \frac{dP_1(t)}{dt} + \lambda_P \lambda_S P_1(t) = 0 \tag{12}$$

with the initial conditions, $P_1(t_0) = -\dfrac{\delta_D}{\lambda_S} - \dfrac{\lambda_D}{\lambda_S^2} \delta_S \delta_D$, $\dfrac{dP_1(t_0)}{dt} = \dfrac{\delta_S \delta_D}{\lambda_S}$.

The behavior of solutions for $D_1(t)$ and $P_1(t)$ described by Eqs. (10) and (12) depends on the roots of the corresponding characteristic equations (Piskunov, 1965; Petrovski, 1966). Note that Eqs. (10) and (12) have the same characteristic equations.

When the roots of characteristic equation are complex-valued (i.e., $\dfrac{\lambda_P^2 \lambda_D^2}{4} < \lambda_P \lambda_S$) both the variable $D_1(t)$ and variable $P_1(t)$ experience damped oscillations for time $t \geq t_0$. When the roots of characteristic equation are real and different (i.e., $\dfrac{\lambda_P^2 \lambda_D^2}{4} > \lambda_P \lambda_S$) both the variable $D_1(t)$ and variable $P_1(t)$ don't oscillate for time $t \geq t_0$. When the roots of characteristic equation are real and equal (i.e., $\dfrac{\lambda_P^2 \lambda_D^2}{4} = \lambda_P \lambda_S$) both the variable $D_1(t)$ and variable $P_1(t)$ don't oscillate for time $t \geq t_0$ as well.

It takes place $D_1(t) \to 0$ and $P_1(t) \to 0$ for $t \to +\infty$ if roots of characteristic equations are complex-valued ($\dfrac{\lambda_P^2 \lambda_D^2}{4} < \lambda_P \lambda_S$), real and different ($\dfrac{\lambda_P^2 \lambda_D^2}{4} > \lambda_P \lambda_S$), or real and equal ($\dfrac{\lambda_P^2 \lambda_D^2}{4} = \lambda_P \lambda_S$).

It takes place for the product surplus (shortage) $D(t)$, for the product price $P(t)$, for the product demand $V_D(t)$, for the product supply $V_S(t)$, for the amount of debt $S_D(t)$, and for the amount of debt servicing cost $S_S(t)$ if $t \to +\infty$,

$$D(t) \to \frac{\delta_S \delta_D}{\lambda_P \lambda_S} \tag{13}$$

$$P(t) \to -\frac{\delta_S \delta_D}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_D}{\lambda_S} + \frac{\lambda_D}{\lambda_S^2} \delta_S \delta_D \tag{14}$$

$$V_D(t) \to \left(r_D^0 + \frac{\lambda_D}{\lambda_S} \delta_S \delta_D \right)(t - t_0) + V_D^0 - \frac{\lambda_D}{\lambda_S} \delta_D - \frac{\lambda_D^2}{\lambda_S^2} \delta_S \delta_D \tag{15}$$

$$V_S(t) \to \left(r_D^0 + \delta_D + \frac{\lambda_D}{\lambda_S} \delta_S \delta_D \right)(t - t_0) - \frac{\delta_S \delta_D}{2}(t - t_0)^2 + V_D^0 - \frac{\lambda_D}{\lambda_S} \delta_D + \frac{\delta_S \delta_D}{\lambda_P \lambda_S} - \frac{\lambda_D^2}{\lambda_S^2} \delta_S \delta_D \tag{16}$$

$$S_D(t) = \delta_D(t - t_0) \tag{17}$$

$$S_S(t) = \frac{\delta_S \delta_D}{2}(t - t_0)^2 \tag{18}$$

To analyze an economic growth I use the variable $E_D(t) \equiv P(t) \times r_D(t)$ where $r_D(t) \equiv \frac{dV_D(t)}{dt}$, i.e., a rate of nominal demand for the product, which roughly represents the product earning on the market.

I compare the variable $E_D(t)$, rate of nominal demand changed by the amount of debt $S_D(t)$ and of debt servicing cost $S_S(t)$, with the variable $\tilde{E}_D(t)$, original rate unchanged by the amounts of debt and debt servicing cost, for $t \to +\infty$, $E_D(t) \to \left(-\frac{\delta_S \delta_D}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_D}{\lambda_S} + \frac{\lambda_D}{\lambda_S^2} \delta_S \delta_D \right)\left(r_D^0 + \frac{\lambda_D}{\lambda_S} \delta_S \delta_D \right)$

and $\tilde{E}_D(t) \to P^0 r_D^0$.

Thus, if the amount of debt $S_D(t)$ is increasing with a constant-rate $\delta_D > 0$ to advance the demand for product on the market then the amount of debt servicing cost $S_S(t)$ is increasing with acceleration $\delta_S \delta_D > 0$ and ultimately causing an unrestricted decrease of the rate of nominal demand $E_D(t)$ with the passage of time.

We can estimate a decrease $e_D(t)$ of the rate of nominal demand $E_D(t)$ where $e_D(t) \equiv \dfrac{dE_D(t)}{dt}$, i.e., the decrease of the rate of nominal demand for product, which roughly represents the decrease of the product earning on market.

It takes place, for $t \rightarrow +\infty$, $e_D(t) \rightarrow -\dfrac{\delta_S \, \delta_D}{\lambda_S}\left(r_D^0 + \dfrac{\lambda_D}{\lambda_S}\delta_S \, \delta_D \right) < 0$ when $\delta_S \, \delta_D > 0$.

The limitary value of variable $e_D(t)$ doesn't have extremal points in the region $\delta_S \, \delta_D > 0$. In fact, the variable $e_D(t)$ has maximal limitary value when $\delta_S \, \delta_D = -\dfrac{\lambda_S}{2\lambda_D} r_D^0$. Then, respectively for $t \rightarrow +\infty$,

$$\max\{e_D(t)\} \rightarrow \dfrac{1}{4\lambda_D}\left(r_D^0 \right)^2 > 0.$$

Therefore, the variable $e_D(t)$ is always negative in the region $\delta_S \, \delta_D > 0$. Changes of the rate of nominal demand in the region $\delta_S \, \delta_D > 0$ have negative limitary values. Note, the maximal increase of the rate of nominal demand $\max\{e_D(t)\}$ for product on the market happens outside of the region $\delta_S \, \delta_D > 0$ and is

equal, for time $t \rightarrow +\infty$, $\max\{e_D(t)\} \rightarrow \dfrac{1}{4\lambda_D}\left(r_D^0 \right)^2 > 0$.

I will talk about economic implications of some results in the next section.

4 Implicative Economic Discussion

A single-product economy model of the credit expansion presented here can be briefly described as following. At first, the demand for product and supply of it were equal, and the market was undisturbed.

Then the demand for product in economy was increased by assuming a constant-rate growing debt. The credit expansion in turn caused an increase of the debt servicing cost. Since the debt servicing cost was proportional to the accumulated amount of debt, i.e., an integral of the assumed debt over the time, the amount of debt servicing cost was growing with an accelerated rate and eventually exceeded the amount of accumulated debt. Afterwards, the credit expansion had been transformed into the credit contraction in the sense that the demand for product was reduced by superposition of the amounts of debt and debt servicing cost since that moment.

During the credit expansion period of a limited length the amended demand exceeds the supply and that creates a product shortage on the market, which interrupts supply-demand equilibrium and drives the product price up. On the other hand, an increase of the product price decreases the product demand. As a general rule, the dual effect of price increase and demand decrease caused by the debt rising with a constant rate induces a restricted short-term nominal economic growth (see Krouglov, 2014). However, depending on the model characteristics, the said dual effect can cause either a nominal economic growth or a nominal economic decline of finite value (e.g., short-time fluctuations can distort a growth pattern).

During the credit contraction period, which has an unlimited extent, the amended demand goes below the supply and it creates a product surplus on the market, which interrupts supply-demand equilibrium and drives the product price down. On the other hand, a decrease of the product price increases the product demand. The dual effect of the price decrease and the demand increase can theoretically cause either a nominal economic growth or a nominal economic decline. Though, the amount of debt servicing cost in the model grows with an accelerated rate, which creates an effect of the unlimited long-term nominal economic decline.

It is important to note the structure of debt used in the model is a rolling over debt, which excludes the principal's repayment. It allows working with limitary values in the model and discounting the distortions caused by slight short-time effects.

The model produced here is rather a model tool, which is used to explore an economic impact caused to the market by an accumulated debt. A model example was the scenario when the amount of debt was growing with a constant rate (i.e., as a linear function), which caused the amount of debt servicing cost growing with a constant acceleration (i.e., as a quadratic function). In these circumstances, the amount of debt servicing cost exceeded the amount of debt at some point in time. Afterwards, a credit expansion would be transformed into a credit contraction and induce an unlimited long-term nominal economic decline. The model might be further expanded if one wants to explore a more complex and practical economic task.

5   Conclusions

Presented here is a simplified mathematical model that investigates economic effects caused by a credit expansion. Initially, the demand for a product was increased by assumption of a constant-rate growing debt. The accumulating debt caused an increase of debt servicing cost, where the amount of accumulated cost eventually exceeded the amount of accumulated debt. Accordingly, the credit expansion was transformed into the credit contraction in the sense that the amended demand for product was diminished by the debt servicing cost afterwards.

The model uses a rolling over debt, which has allowed ignoring distortions caused by the minor short-time effects.

At this stage, I rather view the model as a tool to explore an economic impact of the debt to the market. Explored was the scenario when amount of debt was growing with a constant rate and amount of debt servicing cost was growing with an accelerated rate, so the latter in time exceeded the former. Afterwards, a credit expansion would be converted into a credit contraction and produce an unlimited long-term nominal economic decline. The model can be expanded to deal with more complex and practical economic problems further.

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