Intergenerational mobility, composition of human capital and distance to frontier

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Intergenerational Mobility, Human Capital Composition and Distance to Technological Frontier

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Abstract

An economy can improve its technology level through two channels – imitating from the world technology frontier and innovating on its own technology level – innovation being more skilled-intensive than imitation. An endogenous growth model is considered where education decision depends on the cognitive ability of an individual as well as on the parental income. Child of a rich parent has higher probability to become educated than the child of a poor parent. It is shown that more equal society leads to higher upward and downward mobilities. Growth enhancing education policy leads to absolute convergence of all the economies to the world technology frontier. In the imitation-innovation regime, life time utility gap within skilled and unskilled human capital rise due to parental income differences. Moreover, life time utility gap within skilled human capital rises due to cognitive ability differences.

Journal of Economic Literature Classifications: E02, I24, I25, JHO, O11, O31, O32, O40.

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1 Introduction

Aghion et al. [2009], Vandenbussche et al. [2006] and Basu and Mehra [2014] bring out the contribution of the different composition of human capital to economy-wide technological improvements through the twin channels of imitation and innovation. Technological progress is a dualistic phenomenon which uses human capital inputs differently at different stages of development (in terms of its distance to the world technology frontier). Theoretically, it is shown that when an economy is far away from the world technology frontier, imitation of technologies is the main engine of total factor productivity (TFP) growth. In comparison, as an economy bridges its gap from the world technology leader, the scope of imitation falls and the dependence on innovation activity rises. Technologically sufficiently advanced economies rely on innovation activity (constitute **innovation-only** regime) alone whereas technologically sufficiently backward economies perform only imitation activity (form the **imitation-only** regime). Intermediate economies perform both the activities (represent **imitation-innovation** regime). This work aims to extend this line of research to address the relation between the intergenerational mobilities (upward and downward mobilities) of individuals and between (skilled and unskilled) group wage inequality of an economy depending on the level of development when **individuals are credit constrained**.[1] On the consumption side, income of an individual depends not only on its own talent and income level but also on the education decision of his/ her parent and grandparent and so on. Intergenerational income of individuals is related through the level of bequest that parents left for their children. Rich parent can leave higher bequest than a poor parent. This may give a higher opportunity of becoming rich to a child of a rich parent and lesser opportunity to a child of a poor parent. This implies that, different generations are connected on the production side through the endogenous evolution of technology over time, and on the consumption side through income distributions of previous generations. Thus, in a dynamic setting, growth rate, aggregate income and

[1] Upward mobility captures the probability of becoming educated given that parent did not has formal education and downward mobility implies probability of not getting a formal education given that parent was educated.

intergenerational mobilities of an economy are determined by the interaction of these two interrelated components. This study focuses on the convergence possibility of an economy to the world technology frontier. Additionally, life time utility gaps within skilled human capital due to difference in cognitive ability at various stages of technological development have been examined in this part of research. Life time utility gaps within skilled as well as within unskilled human capital due to parental income differences have also been investigated. Moreover, the impact of the different components of human capital on economic growth depending on its distance to frontier have been studied.

By looking at the literature on intergenerational mobility and inequality, it is found that Galor and Tsiddon [1997], with perfect capital market, shows that a major technological invention initially raises inequality and consequently mobility. Over time, as technology becomes more accessible, both mobility and inequality decrease. There exists a positive relation between inequality and intergenerational mobility in the short run. Becker and Tomes [1986] also shows that intergenerational mobility is smaller when endowments are transferred from parents to children. Without considering that the return to education changes over time, under imperfect capital market Becker and Tomes [1979] and Becker and Tomes [1986] and Loury [1981] show that a more equal society leads to higher mobility and economic development. But, with the assumption of capital market imperfection, Owen and Weil [1998] and Maoz and Moav [1999] show that inequality and mobility move in opposite directions.[2] While Owen and Weil [1998] characterizes only the steady state condition, Maoz and Moav [1999] charts out the transitional dynamics of inequality, mobility and allocation of education along the growth path and shows that both downward and upward mobilities increase as an economy progresses technologically. But none of these studies address the issue of technological progress over time and interaction of it with inequality and mobility. Hassler et al. [2007] shows that differences in skill-biased technology or wage compression exhibits a positive relation between inequality and mobility.[3] An economy

[2] The empirical support for this can be found in Andrews and Leigh [2009].

[3] Wage compression implies that differences in wages between skilled and unskilled human capital is

with higher skill-biased technology or with lower wage compression leads to a higher inequality and higher mobility and vise versa. This holds under the assumption that poor parents not only have less ability to spend on children's education but also have a lower willingness to pay for it. Das [2007] shows that initial income differences persist even under convex technology and convex preferences. Empirical findings of Solon [1992], Solon [2002] and Lee and Solon [2009] show that intergenerational correlation in the long run income is relatively high for both son's and daughter's income. By considering macroeconomic dynamics, Borjas [1992] empirically shows that the skills of the current generation depend not only on parental income but also on the average skills of the ethnic group in the parent's generation. However, none of these studies have considered the possibility of endogenous Research and Development (R & D) based approach in the analysis of intergenerational mobilities and inequality.

This research also sheds light on the existing literatures on the relation between the composition of human capital and growth. Vandenbussche et al. [2006] finds that, when economies are in the **imitation-innovation** regime, skilled human capital is growth enhancing for a relatively advanced economy while unskilled human capital does the same for a relatively backward economy. Their study assigned different degrees of importance to skilled and unskilled human capital, based on the distance of the economy to the world technology frontier. This is able to solve the puzzle posed by Krueger and Lindahl [2001] that education has positive and significant impact on growth only for the technologically backward economy but has negative and insignificant impact for the rich countries. This nonlinear relation between human capital and economic growth is also supported by Durlauf and Johnson [1995] and Kalaitzidakis et al. [2001]. Vandenbussche et al. [2006] provides the explanation that unskilled human capital is more efficient in imitation activity and the scope of imitation is higher in a technologically backward economy. This line of argument is similar to Nelson and Phelps [1966]. On the contrary, Romer [1990] and Grossman and Helpman [1991] show that education favors the innovation of new technology (technologically advanced

lower than the productivity level of them. A state with higher labor regulations or stronger labor union exhibits higher wage compression, that is, it leads to a more equal society.

economy rely more on innovation activity) and skilled human capital is more efficient in it. By allowing inter-state migration, Aghion et al. [2009] also supports the theory postulated by Vandenbussche et al. [2006]. But both the earlier papers, that is, Vandenbussche et al. [2006] and Aghion et al. [2009] have assumed that there exists an exogenously given composition of human capital and it remains fixed irrespective of the distance of the economy to the frontier. Basu and Mehra [2014] endogenize the skill composition of an economy, based on an individual's decision to acquire education depending on the heterogeneous cognitive ability among individual's and shows the importance of *skilled human capital in the diversified regime irrespective of its distance to frontier*. Moreover, their study shows that skilled human capital is also growth enhancing in the innovation-only regime while unskilled human capital does the same job in the imitation-only regime.

This work also interfaces with the existing literatures on convergence theory. In growth theory, there exist two types of convergence hypothesis. First, β convergence implies that a poor country grows at a rate faster than the richer one. This induces that a catch up process is underway. Second, σ convergence measures the cross-sectional dispersion of the logarithm of per capita income or output across different countries. It is found that *β convergence is a necessary but not a sufficient condition for σ convergence*. Now, conditional β convergence implies that there exists β convergence among the countries if they are structurally identical/ similar. Using the classical approach Barro et al. [1991], Barro et al. [1992] and Sala-i Martin [1994], Sala-i Martin [1996] show that there exists cross-sectional conditional β and σ convergence among the US states and for across different countries for different time periods. Convergence rate is roughly at 2 percent per year. But by using cointegration, Bernard and Durlauf [1995] shows very little evidence of the convergence of output. By criticizing the earlier methodology, Quah [1996a], Quah [1996b], Quah [1996c] Quah [1997], Quah [1999], Howitt and Mayer-Foulkes [2005] and Maasoumi et al. [2007] empirically show that income distribution is polarizing into twin peaks of rich and poor; that is, poor are becoming poorer, rich are getting richer and the proportion of the middle income group countries are reducing. With the assumption that an economy improves its technology level only through innovation, Howitt [2000] theoretically demonstrates the possibility of club convergence. With nonparametric analysis Mayer-Foulkes [2002] shows that, in the long run, the world income converges in three steady states – semi-stagnation, semi-development and development, depending on whether countries have overcome barriers to human capital and technological innovation. By making the assumption that as technology level increases, innovation becomes more difficult, Howitt and Mayer-Foulkes [2005] theoretically shows club convergence. Aghion and Howitt [2006], Di Maria and Stryszowski [2009] and Basu and Mehra [2014] show that without any distortion, such as migration, technology transfer from advanced to backward economy leads to absolute convergence in the long run.

Next, the key findings of this analysis for imperfect credit market have been elaborated:

1. A technologically sufficiently backward economy with low relative composition of skilled-unskilled human capital specializes in the imitation activity. This regime has been known as *imitation-only* regime. As the distance from the world technology frontier falls or the relative skill composition of the economy rises, it moves from the *imitation-only* regime to *imitation-innovation* regime.[4] In the diversified regime, the economy performs both the imitation and the innovation activities for technology enhancement. After that, the economy shifts from the *imitation-innovation* regime to *innovation-only* regime (performs only innovation activity) if either the economy becomes sufficiently technologically advanced or relative composition of skilled-unskilled human capital is significantly high.

2. The features of the labor market equilibrium conditions for the diversified regime as well as for the specialized regimes have been emphasized. The assumption that innovation is relatively skilled human capital intensive implies that reliance on unskilled human capital is relatively more for the imitation activity (under (contant returns to scale (CRS)) production structure). In equilibrium this translates into a *lower proportion of skilled human capital than unskilled human capital in the imitation-only regime.* Moreover, the importance of the imitation

[4] This regime is also known as diversified regime.

falls as an economy progresses. Given that diminishing effect of the imitation activity has similar effect to both the factors, *there exists a constant proportion of equilibrium composition of skilled and unskilled human capital in the imitation-only regime as an economy progresses to the world technology leader.*

As an economy progresses further, the scope of imitation falls. However, the opportunity for innovation rises. The assumption that innovation is skilled human capital intensive implies that the *equilibrium proportion of skilled human capital rises and unskilled human capital falls as an economy progresses technologically in the diversified regime.* Consequently, *both skilled and unskilled human capital shift from the imitation to the innovation activity in the diversified regime as an economy bridges the gap from the frontier.*

Finally, at the other extreme, an economy may depend on innovation activity only for future technology improvement. The assumption that innovation is more skilled intensive entails that the *equilibrium proportion of skilled human capital is higher than unskilled human capital in the innovation-only regime.* Further, *there exists constant composition of human capital in the innovation-only regime* (since productivity of both the factors change at the same rate) as an economy moves toward the world technology frontier.

3. This analysis illustrates the growth path of an economy depending on its distance from the frontier. In the imitation-only regime, as an economy progresses, the scope of imitation falls and as an outcome of which growth rate falls. *There exists a declining trend displayed by the growth path of an economy in the imitation-only regime. In the diversified regime, there exists a U-shaped growth curve as the economy progresses.* That is, growth rate initially falls only to rise later. However, *the innovation-only regime exhibits a constant growth rate.*

4. The long run dynamics of an economy is such that by implementing a growth enhancing education policy, the economy moves from the imitation-only regime to the imitation-innovation regime and finally to the innovation-only regime. In the long run, all the economies will converge to the world technology frontier and will

grow at the same rate. *There exists absolute convergence among the economies.*

5. The analysis also focuses on the wage rate of skilled and unskilled workers and the wage inequality between skilled and unskilled workers in the three regimes. In the imitation-only (resp. innovation-only) regime, the marginal productivity of both skilled and unskilled human capital fall (resp. rise) that leads to a *reduction (resp. increment) in the wage rate of skilled and unskilled workers* as an economy moves toward the world technology frontier. Moreover, *a constant level of wage inequality is found to prevail between skilled and unskilled workers in both the specialized regimes.*

   In the imitation-innovation regime, as the economy progresses, the importance of innovation rises and that of imitation falls. Consequently, the marginal productivity of skilled human capital rises and unskilled human capital falls which leads to an *increment (resp. decrement) in the wage rate of skilled (resp. unskilled) workers in the diversified regime*. Additionally, *between skilled and unskilled groups, wage inequality rises as an economy moves to the frontier.*

6. There exists *a constant proportion of upward and downward mobilities in the imitation-only and innovation-only regimes* (since a constant wage inequality prevails between skilled and unskilled workers). In the diversified regime, education becomes more correlated with the parental income and less related with the cognitive ability (as between group wage inequality rises) as the economy progresses. *The proportion of both upward and downward mobilities decrease in the imitation-innovation regime.*

7. In the imitation-innovation regime as an economy progresses, the wage inequality between skilled and unskilled groups rises. This implies that the gap between the levels of bequest that an individual receives from his/ her parent rises given the difference in the parental education status. Some of the individuals who are working as skilled (resp. unskilled) today, had their parents skilled whereas others had unskilled parents. So, levels of bequests received by today's skilled (resp. unskilled) workers vary according to their parental income status. Consequently, within skilled (resp. unskilled) human capital, wealth gap rises depending on whether the parent

was educated or not. This leads to *an increase in the life time utility gap within both skilled and unskilled human capital due to parental income differences in the diversified regime*. There exists *a constant level of life time utility gap within skilled and unskilled human capital due to parental income differences in the imitation-only and innovation-only regimes* (since there exists a constant between group wage inequality).

8. In the diversified regime, relative skilled composition of human capital rises as an economy moves toward the world technology frontier, which implies that individuals with relatively less cognitive ability now become skilled. As a result of this, wage inequality within skilled workers rises due to difference in cognitive ability. This in turn implies that *life time utility gap within skilled workers increases due to difference in cognitive ability irrespective of whether the parent was skilled or unskilled in the imitation-innovation regime*. There exists *a constant level of life time utility gap within skilled group due to difference in the cognitive ability in the imitation-only and innovation-only regimes* (since there exists constant proportion of skilled and unskilled human capital in the specialized regimes).

The paper is organized as follows. In Section 2, the basic structure of the model is discussed. Section 3 contains the key analytical results for a decentralized market economy. The labor market clearing condition and the equilibrium growth rate of the economy and wage paths of both skilled and unskilled workers for three different regimes are derived. Followed by this the balanced growth path for the economy in the decentralized equilibrium is characterized. The intergenerational upward and downward mobilities and the life time utility gap due to parental income differences as well as due to difference in cognitive ability have also been studied. Additionally, some comparative static analysis has been done with respect to the cost of education. Section 4 concludes this work. Moreover, direction future research is provided. In what follows immediately, is the structure of the economy.

2 The Economic Environment

To begin with the production structure of the economy, which resembles the Aghion and Howitt [1992] creative destruction model with quality ladders has been elaborated. Next, the focus is placed on the structure of the dynamics of technology improvement of different economies depending on their distance from the world technology frontier. This structure is in line with Vandenbussche et al. [2006], where the technology enhancement depends on both the imitation and the innovation activities. Followed by this, the consumption side of the economy has been demonstrated. Individuals care for their children and the capital market is imperfect, as in Maoz and Moav [1999]. First, the discussion starts with the production side.

2.1 Production

There are finite numbers of small open economies. Similar to Aghion and Howitt [1992], in each economy there is an entrepreneur, who is engaged in the production of a final output in a perfectly competitive market. There are continuum of mass one of intermediate input producers, who produce the monopoly output, and invest their monopoly profit in the R & D activity. In each time period t, with certainty one intermediate producer invents the highest available technology/ blueprint for each of the intermediate goods and after that he/ she produces the good with this technology/ blueprint. In the next period, he/ she leaves the market and a new intermediate input producer arrives. There exists free entry and exit in the R & D sector. R & D sector is perfectly competitive. Price of final good is normalized to one. All modeling is done using discrete time interval.

The final output is produced competitively, by using land and continuum of mass one of intermediate inputs. Cobb-Douglas production function of the following form is considered:

$$Y_{t+1} = l_{t+1}^{1-\alpha} \int_0^1 A_{i,t+1}^{1-\alpha} x_{i,t+1}^{\alpha} di, \quad 0 < \alpha < 1,$$

where i denotes the i^{th} intermediate sector, Y_{t+1} is the final output in period $t+1$, l_{t+1} is the total amount of land, $A_{i,t+1}$ is the technology level in sector i in period $t+1$ and $x_{i,t+1}$ is the amount of intermediate input used in sector i in period $t+1$. For simplicity, the total supply of land is normalized to one.

Final good sector produces under perfect competition. Therefore, the price of each of the intermediate inputs i is equal to its marginal product, that is,

$$p_{\mathrm{i,t+1}} = \frac{\partial Y_{\mathrm{t+1}}}{\partial x_{\mathrm{i,t+1}}} = \alpha A_{\mathrm{i,t+1}}^{1-\alpha} x_{\mathrm{i,t+1}}^{\alpha-1},$$

where $p_{i,t+1}$ denotes price of the intermediate input in sector i in period $t+1$. Each intermediate input producer chooses output by maximizing the present discounted value of future profits. Since each intermediate input producer works for one time period only the maximization exercise is equivalent to maximize the profit period by period. Monopolist chooses $x_{i,t+1}$ by solving

$$\max_{x_{\mathrm{i,t+1}}} (p_{\mathrm{i,t+1}} x_{\mathrm{i,t+1}} - x_{\mathrm{i,t+1}}).$$

Accordingly, the monopolist produces the following amount of the intermediate good in sector i in period $t+1$:

$$x_{\mathrm{i,t+1}} = \alpha^{\frac{2}{1-\alpha}} A_{\mathrm{i,t+1}}.$$

The profit of the intermediate input producer is:

$$\pi_{\mathrm{i,t+1}} = (p_{\mathrm{i,t+1}} - 1) x_{\mathrm{i,t+1}} = \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}} A_{\mathrm{i,t+1}} = \delta A_{\mathrm{i,t+1}}, \tag{1}$$

where $\delta = \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}}$.

Note that both the equilibrium level of production and the profit of the intermediate input producer in sector i in period $t+1$ are linearly dependent on the local/ national technology level in sector i in that period. Both the technology adjusted intermediate inputs and the profit are same for all the sectors in every period.

2.2 Dynamics of Productivity

Technological progress depends not only on the innovation upon local/ national technology level but also on the imitation of technology from the world technology frontier. This is similar to Benhabib and Spiegel [1994] and Acemoglu et al. [2006]. However, in both these papers, technology improvement depends on the total stock of human capital and not on its composition. This implies that whether skilled or unskilled human capital are engaged in imitation or innovation activities, does not have any impact on technology enhancement. This is a rather restrictive assumption. So, by improving upon this, a specification as in Vandenbussche et al. [2006] and Aghion et al. [2009] is considered. Imitation and innovation activities require both skilled and unskilled human capital, but with differing intensity of use for each type of activity. It is assumed that innovation is relatively skilled human capital intensive. In a CRS frame work, it implies that imitation is unskilled human capital intensive. This entails that a technologically backward (resp. advanced) economy specializes in imitation (resp. innovation) activity. The intermediate economies perform both the activities.[5]

2.2.1 Imitation-Only Regime

The technology improvement specification of an economy which is in the imitation-only regime is:

$$A_{i,t+1} = A_{i,t} + \lambda \widetilde{U}^{\sigma}_{\mathrm{i,t+1}} \widetilde{S}^{1-\sigma}_{\mathrm{i,t+1}} \frac{1}{\overline{A}_{t+1}} (\overline{A}_t - A_t), \quad \lambda > 0, \tag{2}$$

where, $\widetilde{U}_{\mathrm{i,t+1}}$ and $\widetilde{S}_{\mathrm{i,t+1}}$ respectively measure the level of unskilled and skilled human capital in the imitation-only regime, σ is the elasticity of unskilled human capital in the imitation activity and λ captures the efficiency of the overall technology improvement. A_{t} measures the aggregate technology level of the concerned economy in period t, where, $A_{\mathrm{t}} = \int_0^1 \mathrm{A_{it}} di$. $\overline{A}_{\mathrm{t}}$ measures the aggregate technology level of the world leader, such that, $\overline{A}_{\mathrm{t}} = \int_0^1 \overline{A}_{\mathrm{it}} di$. $(\overline{A}_{\mathrm{t}} - A_{\mathrm{t}})$ captures the scope of imitation, that is, the gap of the concerned economy's technology level from the world leader. Along with the advantage of backwardness, there also exists a disadvantage of backwardness, as mentioned by Gerschenkron et al. [1962] and Howitt [2000]. This is captured by the scope of imitation being divided by its targeted world technology level, that is, $\overline{A}_{t+1}$. More advanced the world leader, more difficult it is to imitate for a backward economy.

[5] **Lemma 3** in Subsection 3.4 in page 30 shows that this is true in equilibrium.

2.2.2 Imitation-Innovation Regime

The technology improvement pattern for an economy which is in the imitation-innovation regime is postulated as:

$$A_{\mathrm{i,t+1}} = A_{\mathrm{i,t}} + \lambda \left[u^{\sigma}_{\mathrm{mi,t+1}} s^{1-\sigma}_{\mathrm{mi,t+1}} \frac{1}{\overline{A}_{\mathrm{t+1}}} (\overline{A}_{\mathrm{t}} - A_{\mathrm{t}}) + \gamma \; u^{\phi}_{\mathrm{ni,t+1}} s^{1-\phi}_{\mathrm{ni,t+1}} A_{\mathrm{t}} \right], \quad \gamma > 0, \tag{3}$$

where, $u_{\mathrm{mi,t+1}}$ and $s_{\mathrm{mi,t+1}}$ respectively denote the amount of unskilled and skilled human capital engaged in the imitation activity in the diversified regime, $u_{\mathrm{ni,t+1}}$ and $s_{\mathrm{ni,t+1}}$ respectively measure the amount of unskilled and skilled human capital employed in the innovation activity in the diversified regime, ϕ is the elasticity of unskilled human capital in the innovation activity and γ measures the relative efficiency of innovation as compared to imitation. This implies that an economy can improve its technology level through two channels: imitation captured by $(\overline{A}_{\mathrm{t}} - A_{\mathrm{t}})$ and innovation reflected in the level of A_{t}.

2.2.3 Innovation-Only Regime

In the innovation-only regime, an economy is so advanced that technology enhancement depends on innovation only – the efficiency by which skilled and unskilled human capital innovate determines the next period technology level.[6] The technology evolution process for this specialized advanced economy is characterized by:

$$A_{\mathrm{i,t+1}} = A_{\mathrm{i,t}} + \lambda\gamma \widehat{U}^{\phi}_{\mathrm{i,t+1}} \widehat{S}^{1-\phi}_{\mathrm{i,t+1}} A_{\mathrm{t}}, \tag{4}$$

where, $\widehat{U}_{\mathrm{i,t+1}}$ and $\widehat{S}_{\mathrm{i,t+1}}$ respectively measure the level of unskilled and skilled human capital in the innovation-only regime.

To satisfy the basic assumption that innovation is relatively skilled human capital intensive than imitation, the following specific assumption is made:

A1. *The elasticity of skilled human capital is higher in the innovation activity than in the imitation activity, that is, $\sigma > \phi$. In the same vein, under the imitation-only*

[6] Here efficiency or productivity of skilled and unskilled human capital are measured in terms of the elasticity of skilled and unskilled human capital in the imitation and innovation activities.

regime, imitation is unskilled human capital intensive, implying that $\sigma > \frac{1}{2}$ *and in the innovation-only regime, innovation is skilled human capital intensive, such that* $\phi < \frac{1}{2}$.[7]

A2. *World technology frontier is growing at a constant exogenous rate* $\bar{g}$.

2.3 Consumption Side

An individual lives for two time periods in an overlapping generations model. He/ she has a log-linear utility function. Utility depends on individual's consumption in both the periods and the level of bequest that he/ she leaves for his/ her child. In the first period of the life, an individual takes a decision on whether to opt for education or not. In the second period, depending on the education decision taken, he/ she works as skilled/ unskilled worker. Like Maoz and Moav [1999], complete absence of capital market is assumed, so that individuals cannot borrow or lend. In other words, income and expenditure in any two periods are independent. An individual spends the bequest received on the first period consumption and education (if opted for it) and allocates the second period income on own consumption and leaves a bequest for his/ her child. Individuals vary in their cognitive ability, captured by parameter θ, which is uniformly distributed over the interval $[0, 1]$. The cost of education is negatively related to the individual's cognitive ability and positively with the wage rate of unskilled worker. This is considered as the opportunity cost of an individual to become skilled, that is,

$$E(\theta, A_{t-1}) = \frac{H w_{\mathrm{ut}}}{\theta}, \tag{5}$$

where $E(\theta, A_{t-1})$ captures the cost of education of an individual with θ cognitive ability and H is any positive constant and w_{u_t} is the wage rate of unskilled workers in period t. Both skilled and unskilled workers maximize their lifetime utility subject to the budget constraint. Each individual maximizes the following lifetime utility function:

$$\mathbb{W}_k = \; c_{\mathrm{k,t,t}} \sqrt{c_{\mathrm{k,t,t+1}} \; x_{\mathrm{k,t,t+1}}}, \tag{6}$$

[7] In the diversified regime this work does not require any assumption on the absolute intensity of skilled or unskilled human capital in the imitation or innovation activities. Hence, these parametric restrictions pertain only to the specialized economies.

where $k = s, u$. s and u respectively denote skilled and unskilled workers. $\mathbb{W}_k$ measures the lifetime utility of the k^{th} individual, $c_{\text{k,t,t}}$ is the consumption level of the k^{th} individual in period t who is born in period t, $c_{\text{k,t,t+1}}$ is the consumption level of k^{th} individual in period $(t+1)$ who is born in period t, and $x_{\text{k,t,t+1}}$ is the level of bequest that k^{th} individual who is born in period t leaves for his/ her child in period $(t+1)$. The budget constraint of skilled worker who is born in period t is given as:

$$c_{\text{s,t,t}} + \frac{Hw_{\text{ut}}}{\theta} = x_{\text{t,t}};$$

$$c_{\text{s,t,t+1}} + x_{\text{s,t,t+1}} = w_{\text{s,t+1}},$$

where $w_{s,t+1}$ measures the wage rate of skilled worker at period $(t+1)$, $x_{t,t}$ is the level of bequest that an individual received from his/ her parent. It depends on whether his/ her parent was skilled or unskilled worker. The budget constraint of unskilled worker who is born in period t is

$$c_{\text{u,t,t}} = x_{\text{t,t}};$$

$$c_{\text{u,t,t+1}} + x_{\text{u,t,t+1}} = w_{\text{u,t+1}}.$$

Perfectly competitive labor market is assumed. Individuals have perfect foresight. Total population is normalized to one. There is no population growth. Each parent has one child. At the end of the t^{th} generation, a new $(t+1)^{th}$ generation appears.

The interaction of production and consumption activities determine the equilibrium composition of human capital. This, in turn, determines the allocation of skilled and unskilled human capital between the imitation and innovation activities that ascertains the overall technology improvement. Consequently, that determines the growth path, convergence condition, wage, inequality and intergenerational mobilities paths of the economy as time progresses.

3 Analytical Results

Key analytical findings of this research have been derived in this section. First, labor supply curve is obtained. Next, the focus of the analysis is on the imitation-only regime followed by that on the innovation-only regime and finally on the imitation-innovation regime. Under each case, the equilibrium composition of human capital, growth curve and the wage paths of both the factors have been characterized. Furthermore, the long run steady state condition has been examined. In addition, the relation between intergenerational mobilities (upward and downward mobilities) and the wage inequality of an economy depending on its distance to frontier have been illustrated. Finally, comparative dynamics are worked out to understand the importance of the composition of human capital at different stages of development.

3.1 Labor Supply

A detailed analysis has been made to determine the labor supply curve of the economy. As already mentioned, income and consumption in two periods are not interrelated (since credit market does not exist.). Thus, the second period utility function subject to the second period budget constraint has been maximized. A log-linear utility function ensures that an individual spends his/ her income equally on second period consumption and bequest, that is, $c_{\mathrm{k,t,t+1}} = x_{\mathrm{k,t,t+1}} = \frac{w_{\mathrm{k,t+1}}}{2}$. An Individual opts for education if his/ her lifetime income as skilled worker is greater than unskilled worker, specifically,

$$\mathbb{W}_s \geq \mathbb{W}_u$$

$$\Rightarrow \quad \left[x_{\mathrm{t,t}} - \frac{Hw_{\mathrm{ut}}}{\theta}\right]\frac{w_{\mathrm{s,t+1}}}{2} \geq x_{\mathrm{t,t}}\frac{w_{\mathrm{u,t+1}}}{2} \qquad \text{[Using eq.(6)]}$$

$$\Rightarrow \quad \theta_{\mathrm{t+1}} \geq \frac{Hw_{\mathrm{ut}}}{x_{\mathrm{t,t}}\left[1 - \frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}}\right]}. \tag{7}$$

An individual avails of the education option if his/ her cognitive ability is higher than a threshold level as mentioned in eq. (7). As expected, this depends on the future wage gap between skilled and unskilled human capital (incentive effect), level of bequest that an individual received from his/ her parent (wealth effect) and also on the cost of education (opportunity cost). If an individual's parent was skilled, he/she receives a higher bequest (that is, $\frac{w_{st}}{2}$) than an individual whose parent was unskilled (that is, $\frac{w_{ut}}{2}$). Therefore, the cutoff level of cognitive ability above which an individual goes for education depends on whether his/ her parent was educated or not.

$$\theta^{\mathrm{u}}_{\mathrm{t+1}} = \frac{2H}{1 - \frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}}} \quad \text{and} \quad \theta^{\mathrm{s}}_{\mathrm{t+1}} = \frac{2H\,\frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}}{1 - \frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}}}, \tag{8}$$

where $\theta^{u}_{\mathrm{t+1}}$ and $\theta^{s}_{\mathrm{t+1}}$ respectively measure the cut off cognitive ability above which an individual goes for education if his/ her parent was unskilled and skilled. Note that $\theta^{\mathrm{s}}_{\mathrm{t+1}} < \theta^{\mathrm{u}}_{\mathrm{t+1}}$. It implies that child of an educated parent has higher opportunity of acquiring education than child of an uneducated parent. So, education decision is not only correlated with the cognitive ability of an individual but is also related to the parental education decision and income. This finding is in line with Maoz and Moav [1999]. Therefore, the proportion of unskilled (resp. skilled) human capital in period $(t+1)$ is a weighted average of proportion of uneducated (resp. educated) individuals in period $(t+1)$ having educated parent in period t and the proportion of having uneducated parent in period t.[8] The proportion of unskilled and skilled human capital in period $(t+1)$ are respectively:

$$U_{\mathrm{t+1}} = \theta^{\mathrm{u}}_{\mathrm{t+1}} U_{\mathrm{t}} + \theta^{\mathrm{s}}_{\mathrm{t+1}} S_{\mathrm{t}} = \frac{2H\,\left[U_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}} S_{\mathrm{t}}\right]}{1 - \frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}}};$$

$$S_{\mathrm{t+1}} = 1 - U_{\mathrm{t+1}}. \tag{9}$$

The proportion of unskilled (resp. skilled) human capital in period $(t+1)$ depends on the composition of human capital in period t and also on the expected future wage gap of skilled and unskilled human capital. So, there is a trade off between *history vs. expectation*, as mentioned in Krugman [1991]. Historically if high wage inequality prevails in the economy (that is, wealth effect is significant) that leads to a high proportion of unskilled human capital. Similarly, if previous period's relative composition of skilled human capital is low that also implies a high proportion of unskilled human capital in the next period. Both of these two factors capture the history effect. However, if expected wage rate of skilled is higher than unskilled (incentive effect) that leads to a lower proportion of unskilled human capital. Thus, it is important to understand the dominating effect of the history vs. expectation.

[8] Educated individuals constitute the skilled set. So, this part of research use these two terminologies as synonyms.

3.2 Imitation-Only Regime

The features of the economies which are in the imitation-only regime have been characterized in this subsection. First, the demand for skilled and unskilled workers are derived. Next, the equilibrium proportion of skilled and unskilled workers are ascertained. Finally, the growth path of an economy and the wage paths of skilled and unskilled workers are determined for the imitation-only regime.

3.2.1 Demand for Skilled and Unskilled Human Capital

The demand curve of skilled and unskilled human capital in the imitation-only regime can be derived as follows. From eqs. (1) and (2), the profit maximizing exercise of the intermediate input producer is:

$$\max_{\widetilde{U}_{i,t+1},\widetilde{S}_{i,t+1},} \delta\ A_{\mathrm{it}} + \lambda\delta \widetilde{U}^{\sigma}_{\mathrm{i,t+1}} \widetilde{S}^{1-\sigma}_{\mathrm{i,t+1}} \frac{1}{\overline{A}_{\mathrm{t+1}}} (\overline{A}_{\mathrm{t}} - A_{\mathrm{t}}) - \left[w_{\mathrm{u,t+1}} \widetilde{U}_{\mathrm{i,t+1}} + w_{\mathrm{s,t+1}} \widetilde{S}_{\mathrm{i,t+1}} \right]. \tag{10}$$

From eq. (10), the first order conditions of the maximization exercise of R & D activity have been derived in the imitation-only regime

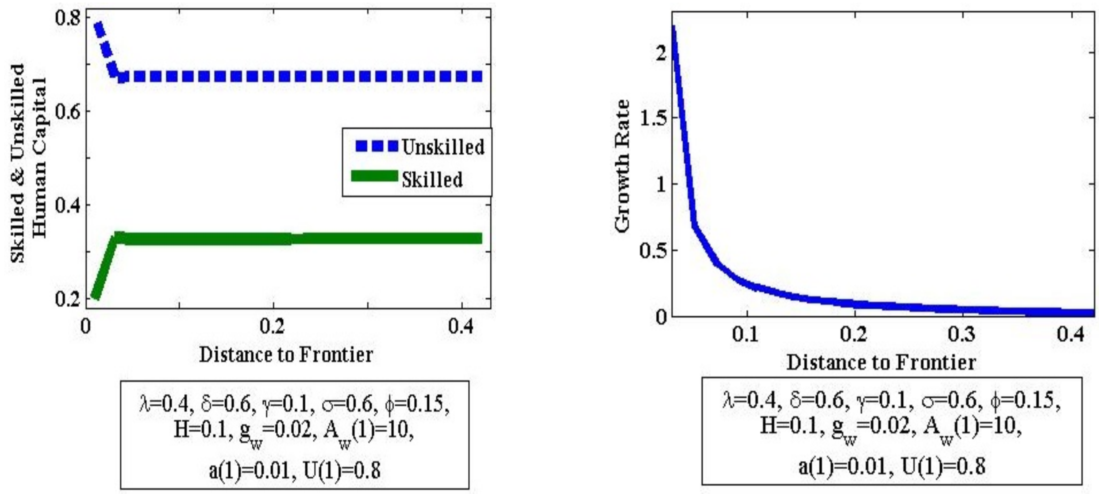
$$\frac{\partial \mathbb{L}^{\mathrm{M}}_{\mathrm{1,t+1}}}{\partial \widetilde{U}_{\mathrm{i,t+1}}} = \lambda\delta_1 \sigma \widetilde{U}^{\sigma-1}_{\mathrm{i,t+1}} \widetilde{S}^{1-\sigma}_{\mathrm{i,t+1}} \frac{1}{(1+\bar{g})} (1 - a_{\mathrm{t}}) - w_{\mathrm{u,t+1}} = 0;$$
$$\frac{\partial \mathbb{L}^{\mathrm{M}}_{\mathrm{1,t+1}}}{\partial \widetilde{S}_{\mathrm{i,t+1}}} = \lambda\delta_1 (1-\sigma) \widetilde{U}^{\sigma}_{\mathrm{i,t+1}} \widetilde{S}^{-\sigma}_{\mathrm{i,t+1}} \frac{1}{(1+\bar{g})} (1 - a_{\mathrm{t}}) - w_{\mathrm{s,t+1}} = 0, \tag{11}$$

where $a_{\mathrm{t}} = \frac{A_{\mathrm{t}}}{\overline{A}_{\mathrm{t}}}$ measures the inverse distance of an economy from the world technology frontier. In this research it is described as *distance to frontier.*

From the above first order conditions in a cluster of eqs. represented as (11), the relative demand curve for skilled and unskilled human capital in the imitation-only regime can be expressed as:

$$\frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}} = \frac{\sigma}{(1-\sigma)} \frac{\widetilde{S}_{\mathrm{i,t+1}}}{\widetilde{U}_{\mathrm{i,t+1}}}. \tag{12}$$

Eq. (12) says that the equilibrium relative wage rate of skilled worker decreases as the relative demand for skilled human capital rises. So, the relative demand curve is negatively sloped in the relative wages.



(a) Skilled and Unskilled Human Capital

(b) Growth Rate

Figure 1: Imitation-Only Regime – Composition of Skilled and Unskilled Human Capital and Growth Rate

3.2.2 Equilibrium

Next, equilibrium proportion of skilled and unskilled human capital in the imitation-only regime has been analyzed. A perfectly competitive labor market ensures that at a competitive wage rate, labor demand equates labor supply. From eqs. (9) and (12), the proportion of unskilled human capital in the imitation-only regime is:

$$\widetilde{U}_{\mathrm{t+1}} = \sigma + 2H\ (1-\sigma)\left[\widetilde{U}_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}\widetilde{S}_{\mathrm{t}}\right]. \tag{13}$$

Eq. (13), yields the proportion of skilled human capital as:

$$\widetilde{S}_{\mathrm{t+1}} = 1 - \widetilde{U}_{\mathrm{t+1}} = (1-\sigma)\left[1 - 2H\ \left(\widetilde{U}_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}\widetilde{S}_{\mathrm{t}}\right)\right]. \tag{14}$$

Eq. (13), exhibits that there exists a positive proportion of unskilled human capital in the imitation-only regime. To ensure the essentiality of skilled input, the following condition is required:

$$\widetilde{S}_{\mathrm{t+1}} > 0 \quad \Rightarrow \quad H < \frac{1}{2\left[\widetilde{U}_{\mathrm{t}} + \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}\widetilde{S}_{\mathrm{t}}\right]}. \tag{15}$$

Condition in eq. (15) is not binded.[9] Now, the question is how equilibrium proportion of skilled or unskilled human capital changes as an economy bridges the gap from the frontier. It depends on the last period's wage inequality and the earlier period's proportion of skilled and unskilled human capital. Last period's equilibrium values depend on the last to last period and the process continues. Thus, the labor market equilibrium condition of the initial period determines today's outcome. These are history dependent in the imitation-only regime. Therefore, simulation technique has been used. The arbitrary parameter values are [λ, δ, γ, σ, ϕ, H, $\bar{g}$, $\overline{A}_1$, U(1), a(1)] = [0.4, 0.6, 0.1, 0.6, 0.15, 0.1, 0.02, 10, 0.8, 0.01]. Specific arbitrary parameter values to represent the initial condition for the imitation-only regime are [U(1), a(1)] = [0.8, 0.01]. These arbitrary parameter values satisfied the regularity condition mentioned in **Lemma 3** in Subsection 3.4 in page 30. These specific parameter values imply that the economy is sufficiently backward and it has a relatively high composition of unskilled human capital.[10] Given that the initial parameter values are arbitrarily given, there exists an initial change in the equilibrium proportion of unskilled human capital. Given diminishing return of the imitation activity (since scope of imitation falls) as an economy progresses, marginal productivity of both skilled and unskilled human capital fall. However, that negative effect is similar to both the factors. Thus, *there exists a constant composition of human capital* (as shown in Fig. 1a in page 18). By **A1**, imitation is unskilled human capital intensive. This implies that *equilibrium level of unskilled human capital is higher than skilled human capital in the imitation-only regime* (as illustrated in Fig. 1a in page 18).

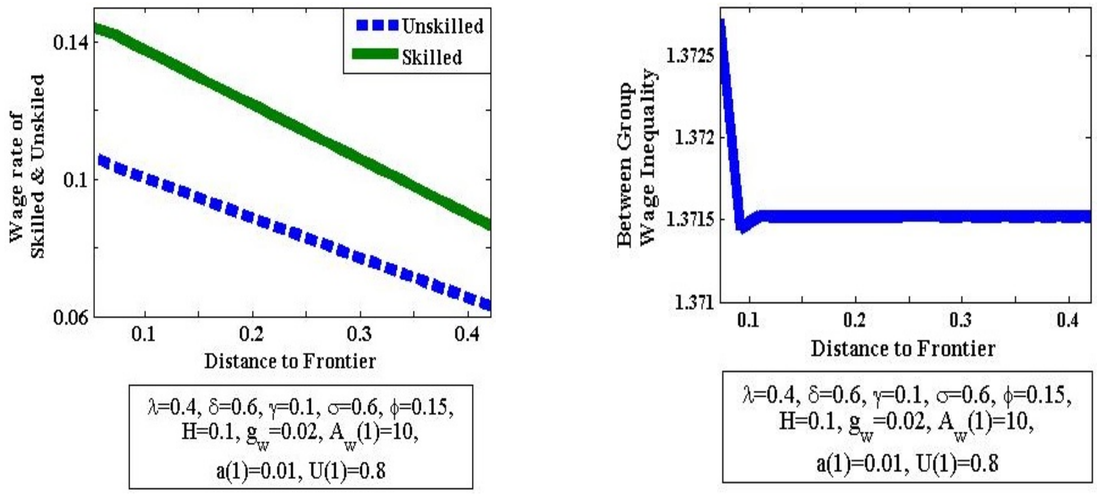
3.2.3 Growth Rate

The growth rate of an economy in the imitation-only regime has been characterized. From eq. (2), one gets,

$$\widetilde{g}_{t+1} = \int_0^1 \frac{A_{i,t+1} - A_{i,t}}{A_{it}} di = \frac{\lambda}{(1+\bar{g})\overline{A}_t} \widetilde{U}^{\sigma}_{t+1} \widetilde{S}^{(1-\sigma)}_{t+1} \frac{(1-a_t)}{a_t},$$

[9]One can show the nonbindedness of this condition by using **Lemma 3** in Subsection 3.4 in page 30.

[10]**Lemma 3** in Subsection 3.4 in page 30 postulates these characteristics for the imitation-only regime.



(a) Wage Paths

(b) Between Group Wage Inequality

Figure 2: Imitation-Only Regime – Wage and Inequality Paths of Skilled-Unskilled Human Capital

where, $\widetilde{g}_{t+1}$ measures the growth rate of an economy in period $t+1$. Growth rate of an economy depends on the composition of skilled and unskilled human capital and its distance to frontier. This implies that growth path is history dependent (since composition of human capital is history dependent). Therefore, analytical solution is hard to get. So, numerical simulation is again used. As the relative gap of an economy from the world technology frontier decreases, the scope of imitation decreases. Consequently, increment of technology is lower. That leads to a *lower growth rate as an economy progresses in the specialized backward regime* (as shown in Fig. 1b in page 18).

3.2.4 Wage Rate

The wage paths of skilled and unskilled workers in the imitation-only regime have been illustrated. Furthermore, the focus is on the analysis of the relative wage gap between skilled and unskilled workers. From a cluster of eqs. named as (11) and eq. (13), wage rate of the different composition of workers are also history dependent as is similar to the equilibrium proportion of skilled and unskilled human capital. Therefore, analytical solutions are not feasible, and hence simulation technique has been used. As an economy progresses, the scope of imitation falls, consequently marginal productivity of both skilled and unskilled workers fall and so do the *wage paths of both the factors fall* (also depicted in Fig. 2a in page 20). However, that diminishing effect is same to both the factors. Consequently, there exists a *constant level of wage inequality between skilled and unskilled workers in the imitation-only regime* (Fig. 2b in page 20 also supports that).

3.3 Innovation-Only Regime

Now, the focus of this study is on the innovation-only regime. The following subsection derives the equilibrium proportion of skilled and unskilled human capital. Consequently, these determine the growth rate of an economy. Furthermore, the wage paths of an economy in the innovation-only regime are also derived.

3.3.1 Demand for Skilled and Unskilled Human Capital

First, the demand curve for skilled and unskilled workers have been examined. From eqs. (1) and (4), maximization exercise of the R & D activity of the intermediate input producers in the innovation-only regime is:

$$\max_{\widehat{U}_{i,t+1},\widehat{S}_{i,t+1},} \delta A_{\mathrm{i,t}} + \lambda\gamma\delta_1 \widehat{U}^{\phi}_{\mathrm{i,t+1}} \widehat{S}^{1-\phi}_{\mathrm{i,t+1}} A_{\mathrm{t}} - \left[w_{\mathrm{u,t+1}} \widehat{U}_{\mathrm{i,t+1}} + w_{\mathrm{s,t+1}} \widehat{S}_{\mathrm{i,t+1}} \right].$$

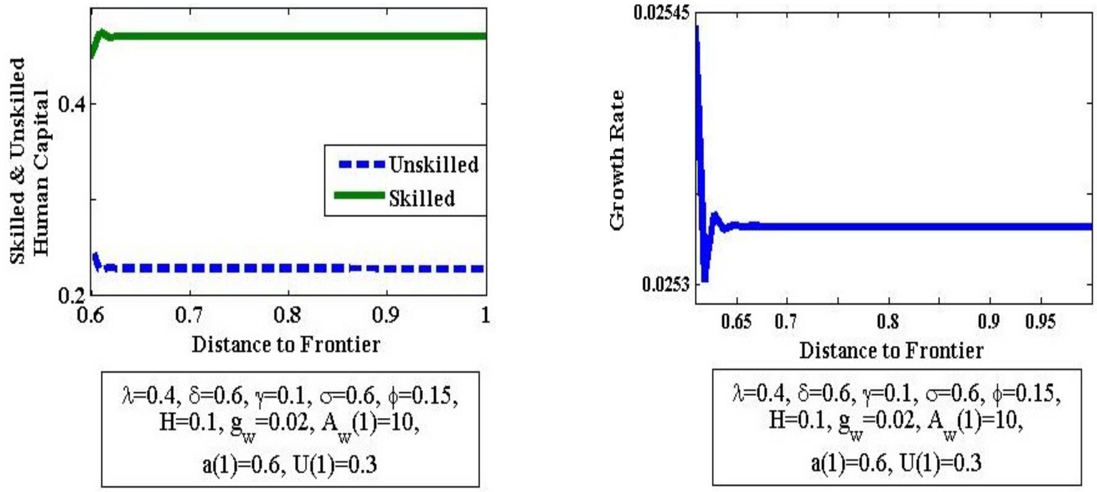
The first-order conditions associated with this maximization exercise are:

$$\begin{aligned} \frac{\partial \mathbb{L}^{\mathrm{N}}_{1,\mathrm{t+1}}}{\partial \widehat{U}_{\mathrm{i,t+1}}} &= \lambda\delta\gamma\phi \widehat{U}^{\phi-1}_{\mathrm{i,t+1}} \widehat{S}^{1-\phi}_{\mathrm{i,t+1}} A_{\mathrm{t}} - w_{\mathrm{u,t+1}} = 0; \\ \frac{\partial \mathbb{L}^{\mathrm{N}}_{1,\mathrm{t+1}}}{\partial \widehat{S}_{\mathrm{i,t+1}}} &= \lambda\gamma\delta(1-\phi) \widehat{U}^{\sigma}_{\mathrm{i,t+1}} \widehat{S}^{-\phi}_{\mathrm{i,t+1}} A_{\mathrm{t}} - w_{\mathrm{s,t+1}} = 0. \end{aligned} \tag{16}$$

From the cluster of eqs. named as (16), the relative demand for skilled-unskilled human capital in the innovation-only regime is

$$\frac{w_{\mathrm{u,t+1}}}{w_{\mathrm{s,t+1}}} = \frac{\phi}{(1-\phi)} \frac{\widehat{S}_{\mathrm{i,t+1}}}{\widehat{U}_{\mathrm{i,t+1}}}. \tag{17}$$

A negatively sloped demand curve exists. As the relative wage rate of skilled worker rises the relative demand for skilled human capital falls.



(a) Skilled and Unskilled Human Capital

(b) Growth Rate

Figure 3: Innovation-Only Regime – Composition of Skilled and Unskilled Human Capital and Growth Rate

3.3.2 Equilibrium

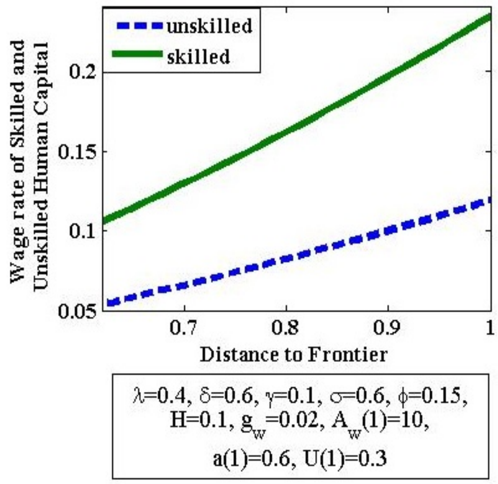
By equating the demand and supply curve, the equilibrium proportion of skilled and unskilled human capital in the innovation-only regime have been derived from eqs. (9) and (17):

$$\widehat{U}_{t+1} = \phi + 2H\ (1-\phi)\left[\widehat{U}_t + \frac{x_{u,t}}{x_{s,t}}\widehat{S}_t\right];$$
$$\widehat{S}_{t+1} = (1-\phi)\left[1 - 2H\ \left(\widehat{U}_t + \frac{x_{u,t}}{x_{s,t}}\widehat{S}_t\right)\right]. \tag{18}$$

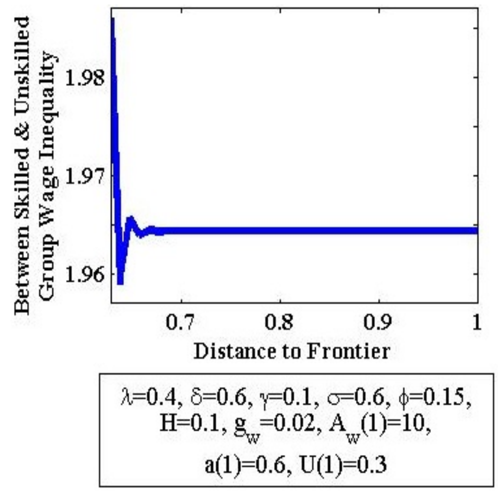
Equilibrium proportion of unskilled human capital is always positive. The regularity condition for the positive stock of skilled human capital is the following:

$$\text{Now,}\quad \widehat{S}_{t+1} > 0 \quad \Rightarrow \quad H < \frac{1}{2\left[\widehat{U}_t + \frac{x_{u,t}}{x_{s,t}}\widehat{S}_t\right]}. \tag{19}$$

Similar to the imitation-only regime, here also the stock of skilled and unskilled human capital in period $(t+1)$ are history dependent. Thus, behaviour of the composition of human capital have been characterized by applying simulation. The specific parametric values for the innovation-only regime are $[U(1),\ a(1)] = [0.3,\ 0.6]$. These capture the



(a) Wage Paths



(b) Between Group Wage Inequality

Figure 4: Innovation-Only Regime – Wage and Inequality Paths of Skilled and Unskilled Human Capital

feature of an economy which is in the advanced specialized regime. It implies that economy is sufficiently advanced and it has a high proportion of skilled to unskilled ratio. It also satisfies all the regularity conditions imposed by the model as illustrated in eq. (19) and also in **Lemma 3** in Subsection 3.4 in page 30. By **A1**, innovation is skilled intensive. Therefore, *the proportion of skilled human capital is higher than unskilled human capital in the innovation-only regime* (as clear from Fig. 3a in page 22). Moreover, as an economy progresses, marginal productivity of both skilled and unskilled human capital rise. However, that increment has similar effect to both the factors. Therefore, *there exists constant proportion of skilled and unskilled human capital.*

3.3.3 Growth Rate

From eq. (4), the growth rate of an economy in the innovation-only regime (denoted as $\widehat{g}_{t+1}$) is $\widehat{g}_{t+1} = \lambda\gamma\widehat{U}^{\phi}_{t+1}\widehat{S}^{(1-\phi)}_{t+1}$. The growth rate depends on the composition of human capital, such that, this too turns out to be history dependent. Given that there exists a fixed proportion of skilled and unskilled human capital, *there exists a constant level of growth rate in the innovation-only regime* (as shown in Fig. 3b in page 22).

3.3.4 Wage Rate

Next, wage paths of skilled and unskilled workers are discussed. From the cluster of eqs. named as (16) imply that as an economy progresses, the technology level rises, which entails an increment in efficiency of the innovation activity. This raises the marginal productivity of both skilled and unskilled workers and consequently, increase the wage rate of both types of workers. This is also depicted in Fig. 4a in page 23. However, these increment have similar effect to both the factors. This implies that after the initial adjustment (due to choice of initial values) there exists a constant level of between group wage inequality in the innovation-only regime. Numerical simulation also exhibits similar result in Fig. 4b in page 23.

3.4 Imitation-Innovation Regime

This subsection, first, derives the demand curve for both skilled and unskilled human capital in the imitation and in the innovation activities. Subsequently, the equilibrium allocation of both types of human capital are ascertained. Finally, the growth rate and wage paths of skilled and unskilled workers for an economy which is in the diversified regime have been characterized.

3.4.1 Demand for Skilled and Unskilled Human Capital

From eqs. (1) and (3), maximization exercise of the R & D producer in the diversified regime is

$$\max_{u_{\mathrm{mi,t+1}},u_{\mathrm{ni,t+1}},s_{\mathrm{mi,t+1}},s_{\mathrm{ni,t+1}}} \lambda\delta\left[u^{\sigma}_{\mathrm{mi,t+1}}s^{1-\sigma}_{\mathrm{mi,t+1}}\frac{1}{\overline{A}_{\mathrm{t+1}}}(\overline{A}_{\mathrm{t}}-A_{\mathrm{t}})+\gamma u^{\phi}_{\mathrm{ni,t+1}}s^{1-\phi}_{\mathrm{ni,t+1}}A_{\mathrm{t}}\right] - \left[w_{\mathrm{u,t+1}}(u_{\mathrm{mi,t+1}}+u_{\mathrm{ni,t+1}})+w_{\mathrm{s,t+1}}(s_{\mathrm{mi,t+1}}+s_{\mathrm{ni,t+1}})\right]. \qquad (20)$$

First-order maximizing conditions in the R & D sector have been derived for the imitation-innovation regime. From eq. (20), one gets that:

$$\frac{\partial\mathbb{L}_{1,\mathrm{t+1}}}{\partial u_{\mathrm{mi,t+1}}}=\lambda\delta\sigma u^{\sigma-1}_{\mathrm{mi,t+1}}s^{1-\sigma}_{\mathrm{mi,t+1}}\frac{1}{\overline{A}_{\mathrm{t+1}}}(\overline{A}_{\mathrm{t}}-A_{\mathrm{t}})-w_{\mathrm{u,t+1}}=0;$$

$$\frac{\partial \mathbb{L}_{1,t+1}}{\partial u_{ni,t+1}} = \lambda\delta\gamma\phi u_{ni,t+1}^{\phi-1} s_{ni,t+1}^{1-\phi} A_t - w_{u,t+1} = 0;$$
$$\frac{\partial \mathbb{L}_{1,t+1}}{\partial s_{mi,t+1}} = \lambda\delta(1-\sigma) u_{mi,t+1}^{\sigma} s_{mi,t+1}^{-\sigma} \frac{1}{\overline{A}_{t+1}}(\overline{A}_t - A_t) - w_{s,t+1} = 0;$$
$$\frac{\partial \mathbb{L}_{1,t+1}}{\partial s_{ni,t+1}} = \lambda\delta\gamma(1-\phi) u_{ni,t+1}^{\phi} s_{ni,t+1}^{-\phi} A_t - w_{s,t+1} = 0. \tag{21}$$

Given that all intermediate good producers are ex-ante identical, they face the same maximization problem. Thus, in equilibrium:

$$u_{mi,t+1} = u_{m,t+1}, \qquad u_{ni,t+1} = u_{n,t+1}, \qquad s_{mi,t+1} = s_{m,t+1} \qquad \text{and} \qquad s_{ni,t+1} = s_{n,t+1}. \tag{22}$$

There is mass 1 of intermediate firms, so that labor market equilibrium condition is

$$S_{t+1} = s_{m,t+1} + s_{n,t+1}, \qquad \text{and} \qquad U_{t+1} = u_{m,t+1} + u_{n,t+1}. \tag{23}$$

From the first-order conditions in the cluster of eqs. named as (21) and by using eq. (22), one gets the relative demand curves for skilled and unskilled human capital in the imitation and innovation activities respectively as:

$$\frac{w_{s,t+1}}{w_{u,t+1}} = \frac{(1-\sigma)}{\sigma}\frac{u_{m,t+1}}{s_{m,t+1}}; \qquad \text{and} \qquad \frac{w_{s,t+1}}{w_{u,t+1}} = \frac{(1-\phi)}{\phi}\frac{u_{n,t+1}}{s_{n,t+1}}. \tag{24}$$

Equalization of the relative wage rate in eq. (24) implies:

$$\psi \frac{s_{m,t+1}}{u_{m,t+1}} = \frac{s_{n,t+1}}{u_{n,t+1}}, \tag{25}$$

where, $\psi = \frac{\sigma(1-\phi)}{\phi(1-\sigma)} > 1$, by **A1**.

Accordingly, the demand for skilled and unskilled human capital in the imitation and innovation activities are worked out to be:

$$s_{n,t+1} = \frac{\psi\, S_{t+1} - h(a_t)\, U_{t+1}}{\psi - 1}; \qquad s_{m,t+1} = \frac{h(a_t)\, U_{t+1} - S_{t+1}}{\psi - 1};$$
$$u_{n,t+1} = \frac{\psi S_{t+1} - h(a_t)\, U_{t+1}}{(\psi-1)\, h(a_t)}; \qquad u_{m,t+1} = \frac{\psi[h(a_t)\, U_{t+1} - S_{t+1}]}{(\psi-1)\, h(a_t)}. \tag{26}$$

where $h(a_t) = \left[\frac{(1-\sigma)\psi^{\sigma}(1-a_t)}{\gamma(1-\phi)(1+\bar{g})\overline{A}_t a_t}\right]^{\frac{1}{(\sigma-\phi)}}$, which is a decreasing function of the distance to frontier. That is, $h'(a_t) = -\frac{h(a_t)}{(\sigma-\phi)a_t(1-a_t)} < 0$.

From eq. (26), the relative demand for skilled and unskilled human capital in the imitation and in the innovation activities are estimated to be:

$$\frac{s_{\mathrm{m,t+1}}}{u_{\mathrm{m,t+1}}} = \frac{h(a_{\mathrm{t}})}{\psi}; \qquad \frac{s_{\mathrm{n,t+1}}}{u_{\mathrm{n,t+1}}} = h(a_{\mathrm{t}}). \tag{27}$$

3.4.2 Equilibrium

By equating the demand and supply curves of skilled and unskilled human capital, the equilibrium level of both types can be ascertained in the diversified regime. Furthermore, the equilibrium allocation of skilled and unskilled human capital in the imitation and innovation activities can also be derived.

First the cutoff level of cognitive ability above which an individual goes for education given that his/ her parent was educated or not is determined. Substituting eqs. (24) and (27) in eq. (8), one gets,

$$\theta^{\mathrm{u}}_{\mathrm{t+1}} = \frac{2H}{1 - \frac{\phi}{(1-\phi)} h(a_{\mathrm{t}})}; \qquad \theta^{\mathrm{s}}_{\mathrm{t+1}} = \frac{2H \, \frac{x_{\mathrm{u,t}}}{x_{\mathrm{s,t}}}}{1 - \frac{\phi}{(1-\phi)} h(a_{\mathrm{t}})}. \tag{28}$$

From eq. (28), the proportion of unskilled human capital in the imitation-innovation regime is derived to be:

$$U_{\mathrm{t+1}} = \theta^{\mathrm{s}}_{\mathrm{t+1}} S_{\mathrm{t}} + \theta^{\mathrm{u}}_{\mathrm{t+1}} U_{\mathrm{t}} = \frac{2H(1-\phi)\left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} S_{\mathrm{t}}\right]}{[(1-\phi) - \phi h(a_{\mathrm{t}})]}. \tag{29}$$

Next, the equilibrium proportion of skilled human capital in the imitation-innovation regime can be derived from eq. (29) as:

$$S_{\mathrm{t+1}} = 1 - U_{\mathrm{t+1}} = 1 - \frac{2H(1-\phi)\left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} S_{\mathrm{t}}\right]}{[(1-\phi) - \phi h(a_{\mathrm{t}})]}. \tag{30}$$

Given the essentiality of both the inputs the following conditions are needed as well:

$$U_{t+1} > 0 \quad \Rightarrow \quad [(1-\phi) - \phi h(a_{\mathrm{t}})] > 0;$$

$$S_{\mathrm{t+1}} > 0 \quad \Rightarrow \quad 1 - \frac{2H(1-\phi)\left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} S_{\mathrm{t}}\right]}{[(1-\phi) - \phi h(a_{\mathrm{t}})]} > 0 \quad \Rightarrow \quad H < \frac{[(1-\phi) - \phi h(a_{\mathrm{t}})]}{2(1-\phi)\left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} S_{\mathrm{t}}\right]}. \tag{31}$$

Regularity condition in eq. (31) is not binded.[11] Some comparative dynamics analyses have been attempted. The change in the total stock of skilled and unskilled human capital as an economy progresses has been examined. Since, the equilibrium proportion of skilled and unskilled human capital are history dependent, analytical solutions are not feasible. Therefore, numerical simulation has been done. The following specific parameter values assumed in the diversified regime are [a(1), U(1)] = [0.5, 0.65]. It implies that economy is neither sufficiently backward nor sufficiently advanced. It has intermediate values in terms of the distance to frontier. Moreover, it neither has relatively high nor relatively low composition of skilled human capital. Arbitrary parameter values also satisfy the regularity condition mentioned in **Lemma 3** in Subsection 3.4 in page 30.

The catch-up component is high for a technologically backward economy. As an economy progresses, the relative gap of that economy from the world technology frontier decreases. Consequently, the relative importance of imitation activity decreases and that of innovation activity rises. From **A1**, in equilibrium the proportion of unskilled human capital falls and skilled human capital rises as an economy progresses, that is, $\frac{\partial\ U_{t+1}}{\partial\ a_t} < 0$ and $\frac{\partial\ S_{t+1}}{\partial\ a_t} > 0$, as shown in Fig. 5a in page 28.

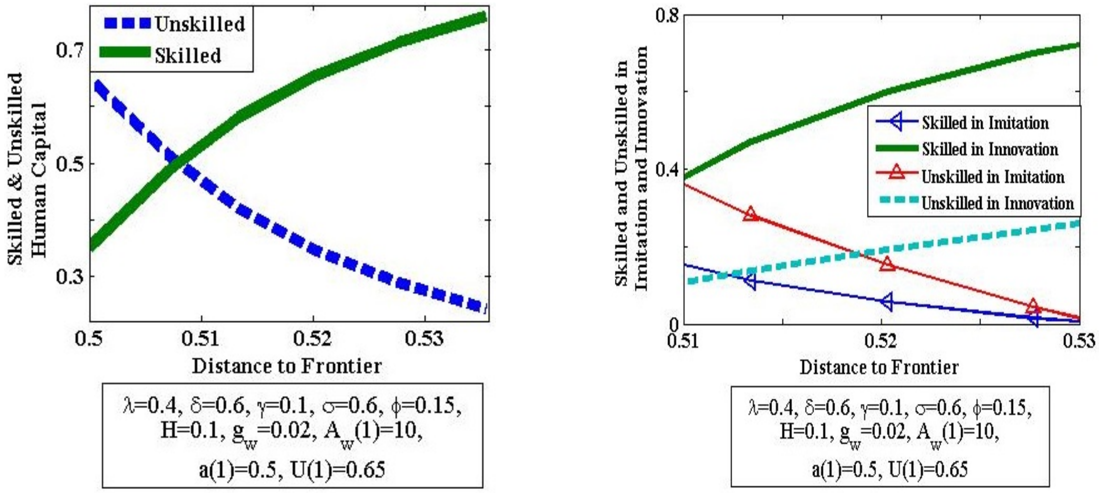
Lemma 1 *Under* **A1**,

- *In the imitation-only regime, there exists a fixed composition of skilled and unskilled human capital. Moreover, equilibrium proportion of skilled human capital is lower than unskilled human capital.*

- *In the innovation-only regime also there exists a constant composition of skilled and unskilled human capital. Additionally, equilibrium proportion of skilled human capital is higher than unskilled human capital in the innovation-only regime.*

- *For a country which is in the imitation-innovation regime, proportion of skilled human capital increases and unskilled human capital decreases as a country moves to the world technology frontier.*

[11] One can show the nonbindedness of this condition by using **Lemma 3** in page 30.



(a) (b)

Figure 5: Diversified Regime – Skilled-Unskilled Human Capital and Allocation of it in Imitation and Innovation Activities

From the cluster of eqs. named as (26) and eqs. (29) and (30), the equilibrium allocation of skilled and unskilled human capital can be ascertained in the imitation and in the innovation activities.

$$
\begin{aligned}
s_{\mathrm{m,t+1}} &= \frac{2\ H\ (1-\phi)[1+h(a_{\mathrm{t}})]\ \left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} S_{\mathrm{t}}\right]}{(\psi-1)\left[(1-\phi)-\phi h(a_{\mathrm{t}})\right]} - \frac{1}{(\psi-1)};\\
u_{\mathrm{m,t+1}} &= \frac{2\ H\ \psi\ (1-\phi)[1+h(a_{\mathrm{t}})]\ \left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} U_{\mathrm{t}}\right]}{(\psi-1)h(a_{\mathrm{t}})\left[(1-\phi)-\phi h(a_{\mathrm{t}})\right]} - \frac{\psi}{(\psi-1)\ h(a_{\mathrm{t}})};\\
s_{\mathrm{n,t+1}} &= \frac{\psi}{(\psi-1)} - \frac{2\ H\ (1-\phi)\left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} S_{\mathrm{t}}\right]\left[\psi + h(a_{\mathrm{t}})\right]}{(\psi-1)\ \left[(1-\phi)-\phi h(a_{\mathrm{t}})\right]};\\
u_{\mathrm{n,t+1}} &= \frac{\psi}{(\psi-1)h(a_{\mathrm{t}})} - \frac{2\ H\ (1-\phi)\left[U_{\mathrm{t}} + \frac{x_{\mathrm{ut}}}{x_{\mathrm{st}}} S_{\mathrm{t}}\right]\left[\psi + h(a_{\mathrm{t}})\right]}{(\psi-1)\ h(a_{\mathrm{t}})\ \left[(1-\phi)-\phi h(a_{\mathrm{t}})\right]}.
\end{aligned}
\tag{32}
$$

Comparative dynamics have been carried out to capture change in allocation of skilled and unskilled human capital in both the imitation and the innovation activities as an economy progresses. By **Lemma 1** and **A1**, as the proportion of skilled (resp. unskilled) human capital increases (resp. decreases) innovation attracts more skilled human capital than imitation as the gap from the world technology frontier falls. Due to complementarity, unskilled human capital also shifts from imitation to innovation. This attracts even more skilled human capital into innovation activity and so on. Therefore, in equilibrium, both skilled and unskilled human capital increase in the innovation activity and decrease in the imitation activity, as is shown in Fig. 5b in page 28. That is, $\frac{\mathrm{d}\ s_{\mathrm{m,t+1}}}{\mathrm{d}\ a_{\mathrm{t}}} < 0$, $\frac{\mathrm{d}\ u_{\mathrm{m,t+1}}}{\mathrm{d}\ a_{\mathrm{t}}} < 0$, $\frac{\mathrm{d}\ s_{\mathrm{n,t+1}}}{\mathrm{d}\ a_{\mathrm{t}}} > 0$ and $\frac{\mathrm{d}\ u_{\mathrm{n,t+1}}}{\mathrm{d}\ a_{\mathrm{t}}} > 0$.[12]

Lemma 2 *Under* **A1***,*

In the imitation-innovation regime, the proportion of both skilled and unskilled human capital shift away from the imitation activity to the innovation activity as an economy bridges its gap from the world technology frontier.

Next, the regularity conditions for the existence of positive amounts of both skilled and unskilled human capital in the imitation and innovation activities have been derived. That is, it is required that $s_{\mathrm{m,t+1}} > 0$, $s_{\mathrm{m,t+1}} < S_{\mathrm{t+1}}$, $s_{\mathrm{n,t+1}} > 0$, $s_{\mathrm{n,t+1}} < S_{\mathrm{t+1}}$, $u_{\mathrm{m,t+1}} > 0$, $u_{\mathrm{m,t+1}} < U_{\mathrm{t+1}}$, $u_{\mathrm{n,t+1}} > 0$ and $u_{\mathrm{n,t+1}} < U_{\mathrm{t+1}}$. From eq. (32), this entails the following regularity condition:

$$\frac{h(a_{\mathrm{t}})}{\psi} < \frac{S_{\mathrm{t+1}}}{U_{\mathrm{t+1}}} < h(a_{\mathrm{t}}) \qquad \text{[From eqs.(26)]}$$

$$\Rightarrow \quad \frac{[(1-\phi) - \phi h(a_{\mathrm{t}})]}{2\ (1-\phi)\ [1+h(a_{\mathrm{t}})]\ \left[S_{\mathrm{t}} + \frac{x_{\mathrm{st}}}{x_{\mathrm{ut}}}\ U_{\mathrm{t}}\right]} < H < \frac{\psi\ [(1-\phi) - \phi\ h(a_{\mathrm{t}})]}{2\ (1-\phi)[\psi + h(a_{\mathrm{t}})]\ \left[S_{\mathrm{t}} + \frac{x_{\mathrm{st}}}{x_{\mathrm{ut}}}\ U_{\mathrm{t}}\right]}.$$

It is known that $h(a_{\mathrm{t}})$ is a decreasing function of a_{t}. In one hand, given any fixed composition of skilled-unskilled human capital (that is, with a fixed value of $\frac{S_{\mathrm{t+1}}}{U_{\mathrm{t+1}}}$), a significantly technologically backward economies, (that is, economies with enough low a_{t}), specialize in the imitation activity, sufficiently technologically advanced economies (that is, economies with enough high a_{t}) specialize in the innovation activity and the intermediate economies perform both the activities. On the other hand, by **A1**, given any fixed distance to frontier, an economy with significantly high (resp. low) composition of skilled to unskilled human capital ratio specializes in the innovation (resp. imitation) activity and the intermediate economies perform both the activities. In

[12] Allocation of skilled and unskilled human capital in the imitation and innovation activities are also history dependent. Thus, for dynamics analysis, one needs to take the help of numerical simulation.

this analysis, parametric value H represents the cost of education. A higher (resp. lower) H implies higher (resp. lower) cost of education and lower (resp. higher) equilibrium proportion of skilled human capital in the economy, entailing that the economy depends more on the imitation (resp. innovation) activity.

Lemma 3 *Under **A1**,*

- *For technology improvement an economy performs both imitation and innovation activities if and only if*

$$\frac{[(1-\phi)-\phi h(a_t)]}{2\;(1-\phi)\;[1+h(a_t)]\;\left[S_t+\frac{x_{st}}{x_{ut}}\;U_t\right]} < H < \frac{\psi\;[(1-\phi)-\phi\;h(a_t)]}{2\;(1-\phi)[\psi+h(a_t)]\;\left[S_t+\frac{x_{st}}{x_{ut}}\;U_t\right]};$$

- *an economy specializes in imitation-only regime if and only if*

$$\frac{[(1-\phi)-\phi h(a_t)]}{2\;(1-\phi)\;[1+h(a_t)]\;\left[S_t+\frac{x_{st}}{x_{ut}}\;U_t\right]} > H;$$

- *and an economy specializes in innovation-only regime if*

$$\frac{\psi\;[(1-\phi)-\phi\;h(a_t)]}{2\;(1-\phi)[\psi+h(a_t)]\;\left[S_t+\frac{x_{st}}{x_{ut}}\;U_t\right]} < H.$$

3.4.3 Growth Rate

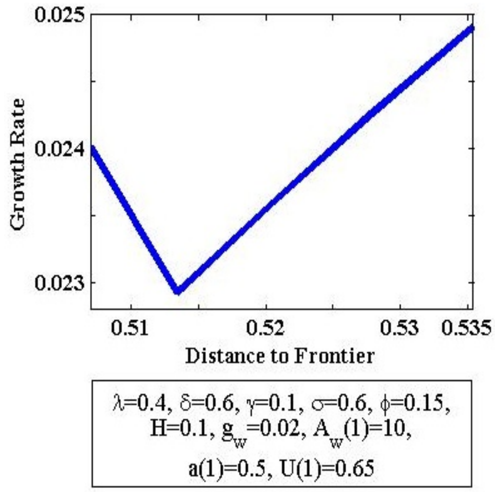
Now, the growth rate has been derived for an economy which is in the diversified regime. From eq. (3), (26), (27) and (29), one gets,

$$\begin{aligned}
g_{\mathrm{t+1}} &= \lambda\left[u^{\sigma}_{\mathrm{m,t+1}}\;s^{1-\sigma}_{\mathrm{m,t+1}}\frac{1}{\overline{A}_{\mathrm{t+1}}}\left(\frac{1-a_{\mathrm{t}}}{a_{\mathrm{t}}}\right)+\gamma\;u^{\phi}_{\mathrm{n,t+1}}s^{1-\phi}_{\mathrm{n,t+1}}\right]\\
&= \lambda\left[\underbrace{\left(\frac{u_{\mathrm{m,t+1}}}{s_{\mathrm{m,t+1}}}\right)^{\sigma}}_{+ve}\;\underbrace{s_{\mathrm{m,t+1}}}_{-ve}\;\frac{1}{\overline{A}_{\mathrm{t+1}}}\;\underbrace{\frac{(1-a_{\mathrm{t}})}{a_{\mathrm{t}}}}_{-ve}+\gamma\;\underbrace{\left(\frac{u_{\mathrm{n,t+1}}}{s_{\mathrm{n,t+1}}}\right)^{\phi}\quad s_{\mathrm{n,t+1}}}_{+ve}\right]\\
&= \lambda\;\gamma(1-\phi)\;h^{-\phi}(a_{\mathrm{t}})\left[1-2H\left(S_{\mathrm{t}}+\frac{x_{\mathrm{st}}}{x_{\mathrm{ut}}}U_{\mathrm{t}}\right)\right]. \qquad (33)
\end{aligned}$$

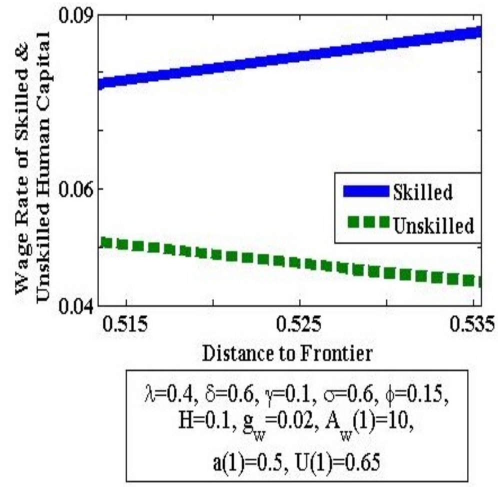
Positive growth rate entails that $H < \frac{1}{2\;\left(S_{\mathrm{t}}+\frac{x_{\mathrm{st}}}{x_{\mathrm{ut}}}U_{\mathrm{t}}\right)}$. However, this condition is not binded. From eq. (33), it is easy to see that the growth rate of an economy in the diversified regime depends on the relative intensity of unskilled and skilled human capital in the imitation and innovation activities, the allocation of skilled human capital in these two activities and the distance of an economy from the world technology frontier. From eq. (27), it is found that the relative intensity of unskilled-skilled human capital in these two activities depend positively on the distance to frontier. The allocation of skilled human capital in the imitation (resp. innovation) activity depends negatively (resp. positively) on the distance to frontier. Obviously, the relative gap from the frontier declines as an economy progresses. Interaction of all of these factors determine the growth rate of an economy which is in the diversified regime. Eq. (33) reveals that growth rate is history dependent. Therefore, numerical simulation is required to understand the growth path. From Fig. 6a in page 32, it is clear that the growth rate initially falls and thereafter rises as an economy progresses. It is a U-shaped growth curve. As an economy shifts from the imitation-only regime to the diversified regime, the scope for both imitation and innovation are low. In one hand, scope of imitation is low since the advantage of backwardness is falling as an economy progresses. On the other hand, scope of innovation is also low at the initial stages of development of the diversified regime, since technology level is not sufficiently high. Further, from **Lemma 1**, the relative proportion of skilled human capital is also short in supply. This causes the falling part of the growth path in the diversified regime. As the time progresses, the opportunity for innovation rises and again from **Lemma 1**, the proportion of skilled human capital rises that leads to an incrementally higher rate of growth.

Proposition 1 *Under* **A1**,

- *In the imitation-only regime, growth rate falls as an economy progresses.*
- *In the diversified regime, growth rate initially falls and thereafter rises as an economy progresses, that is, there exists a U-shaped growth path.*
- *In the innovation-only regime, there exists a constant growth rate.*



(a) Growth Rate



(b) Wage Paths

Figure 6: Diversified Regime – Growth Rate and Wage Paths of Skilled and Unskilled Human Capital

3.4.4 Wage Rate

The discussion now shifts to the dynamics paths of the wage rate of skilled and unskilled workers as an economy bridges its gap from the world frontier. Substituting eq. (27) in the cluster of eqs. named as (21), one gets,

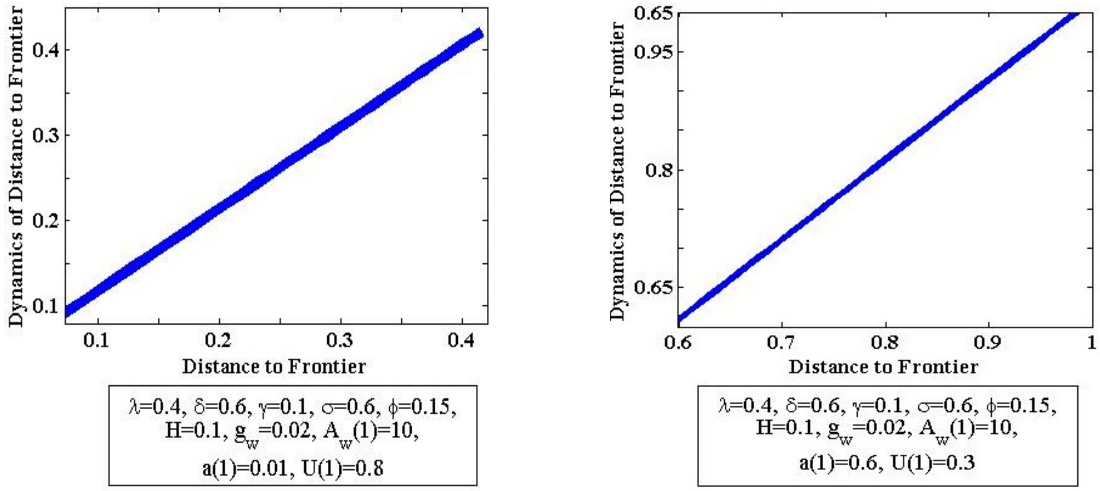
$$w_{\mathrm{u},\ t+1} = \lambda\delta\gamma\phi h^{1-\phi}(a_{\mathrm{t}})a_{\mathrm{t}}\overline{A}_{\mathrm{t}};$$

$$w_{\mathrm{s},\ t+1} = \lambda\delta\gamma(1-\phi)h^{-\phi}(a_t)a_{\mathrm{t}}\overline{A}_{\mathrm{t}}.$$

In the diversified regime, as an economy progresses, the relative importance of innovation increases and imitation decreases. From **A1**, marginal productivity of skilled human capital increases and unskilled human capital decreases and so the wage rate of skilled workers rises and unskilled workers falls. Consequently, the wage gap between skilled and unskilled workers grows, as demonstrated in Fig. 6b in page 32.

Proposition 2 *Under **A1**,*

- *In the imitation-only regime, wage rate of skilled and unskilled workers fall as an economy progresses. There exists a constant level of wage inequality between skilled and unskilled groups.*



(a) Imitation-Only Regime

(b) Innovation-Only Regime

Figure 7: Steady State – Specialized Regimes

- *In the Diversified regime, wage rate of skilled workers rises and unskilled workers falls as an economy steps forward to the world frontier. Wage inequality between skilled and unskilled workers rises as an economy bridges the gap from the world technology frontier.*
- *In the innovation-only regime, wage rate of skilled and unskilled workers rise as an economy progresses. There exists a constant level of wage inequality between skilled and unskilled workers.*

3.5 Steady State

Long run equilibrium condition of an economy has been illustrated in this subsection. It attempts to find an answer to: as the time progresses does an economy converge its gap from the world technology frontier, depending on its distance to the frontier. The definition of growth rate can be specified as:

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t} = \frac{A_{t+1}}{\overline{A}_{t+1}} \frac{\overline{A}_t}{A_t}(1+\bar{g}) - 1$$

$$\Rightarrow \quad a_{t+1} = \frac{(1+g_{t+1})}{(1+\bar{g})} a_t \tag{34}$$

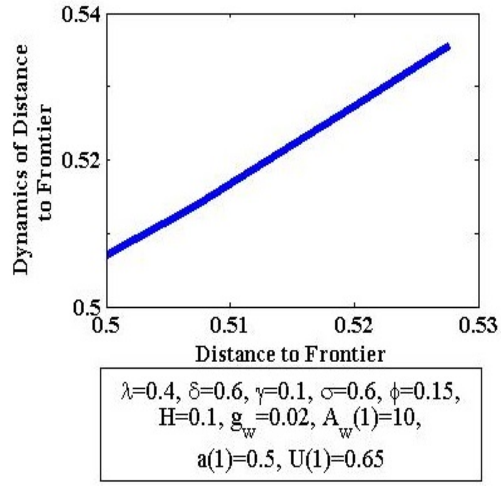


Figure 8: Steady State – Diversified Regime

If the growth rate of an economy is higher than the growth rate of the world leader, then the economy will be able to converge to the frontier, and in the long run, it will catch up with the frontier technology level.[13] Numerical simulation also corroborates this. From Fig. 7a in page 33, Fig. 7b in page 33 and Fig. 8 in page 34, it is clear that as an economy progresses it closes its distance from the world technology frontier. In the long run all the economies will converge to the world technology level. Steady state implies that a_t will converge to a^*, that is, $a_t \to a^*$ and the growth rate of the economy will converge to g^*, that is, $g_t \to g^*$. Therefore, eq. (34) implies that either $g^* = \bar{g}$ or $a^* = 0$. In the long run all the economies will grow at the same rate. That is, there exists **absolute convergence** of the economies in the long run.

Proposition 3 *In the long-run all the economies will converge to the world technology frontier irrespective of its distance to frontier. Moreover, in the steady state, all the economies will grow at the same rate.*

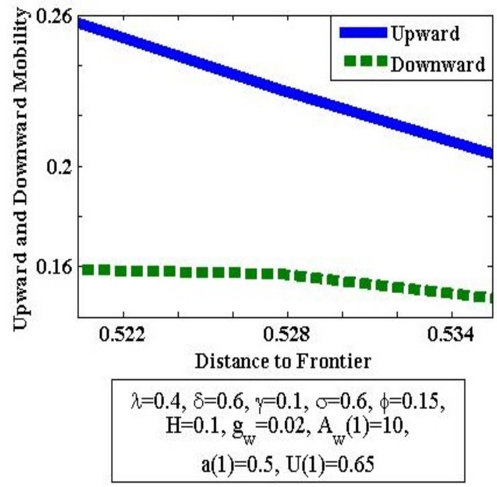


Figure 9: Upward and Downward Mobility – Diversified Regime

3.6 Intergenerational Mobility

This subsection involves an analysis on upward and downward mobilities of individuals. Upward mobility is implied by the phenomena that an individual works as a skilled worker given that his/ her parent was unskilled, that is,

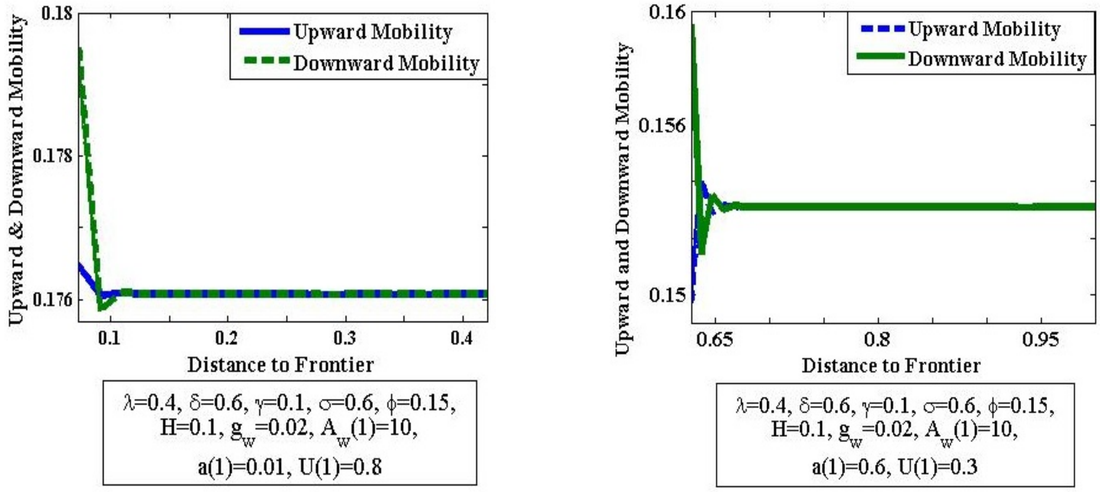
$$\mathrm{UM}_{t+1} = U_t(1 - \theta^{u}_{t+1}),$$

where, UM_{t+1} measures upward mobility in period $(t+1)$. It captures the probability of moving from low equilibrium to high equilibrium. Proportion of skilled human capital (that is, $(1-\theta^{u}_{t+1})$) whose parents were unskilled (that is, U_t). It is a cross product of today's skilled people with earlier period's unskilled individuals. Next, downward mobility implies that parent was skilled but child is working as an unskilled worker, that is,

$$\mathrm{DM}_{t+1} = S_t\theta^{s}_{t+1},$$

where DM_{t+1} measures downward mobility in period $(t+1)$. It is the opposite of the upward mobility. It captures the probability of moving from high to low equilibrium. It is a cross product of the proportion of today's unskilled human capital (that is, θ^{s}_{t+1}) whose parent was skilled (that is, S_t). Intergenerational mobilities assist to analyze the

[13] The economy will catch up if next period's distance to frontier (that is, a_{t+1}) is higher than the earlier period's distance from the world technology frontier (that is, a_t).

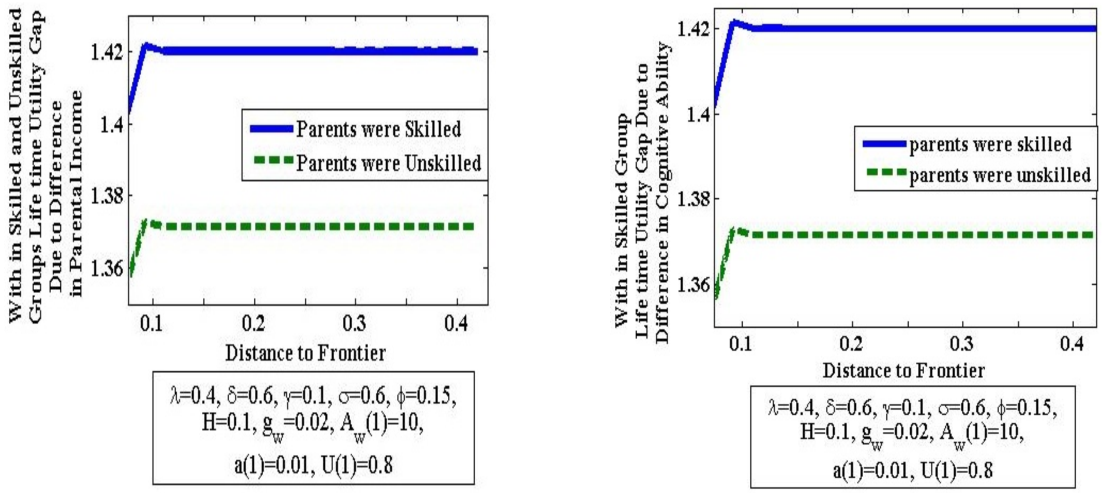


(a) Imitation-Only Regime

(b) Innovation-Only Regime

Figure 10: Upward and Downward Mobility – Specialized Regimes

correlation between cognitive ability and income of an individual. Low mobility implies that individuals whose parents have high (resp. low) income have high chance of working as skilled (resp. unskilled). Now, the focus of the study is to identify the dynamics paths of upward and downward mobilities in the diversified regime. From **Proposition 2**, in the diversified regime, the wage gap between skilled and unskilled human capital rise as an economy progresses technologically. Individuals whose parents were unskilled and left a lesser amount of bequest has lower probability of becoming educated than the individuals's whose parents were skilled. Subsequently, upward and downward mobilities fall as an economy progresses (as shown in Fig. 9 in page 35). As gap from the world frontier falls chances of shifting from low equilibrium to high equilibrium as well as from high equilibrium to low equilibrium fall in the diversified regime. In the specialized regimes, from **Lemma 1**, there exists a fixed proportion of the composition of human capital and from **Proposition 2**, there exists constant wage inequality. This implies a constant level of upward and downward mobilities in the specialized regimes, as is also shown in Fig. 10a in page 36 and Fig. 10b in page 36. To conclude, if parents were educated (skilled) the probability of the children being educated rises and if parents were unskilled then the opportunity for the children becoming educated falls. That is, education becomes



(a) Due to Parental Income Difference

(b) Due to Cognitive Ability Difference

Figure 11: Imitation-Only Regime – Life time Utility Gaps within Skilled and Unskilled Human Capital

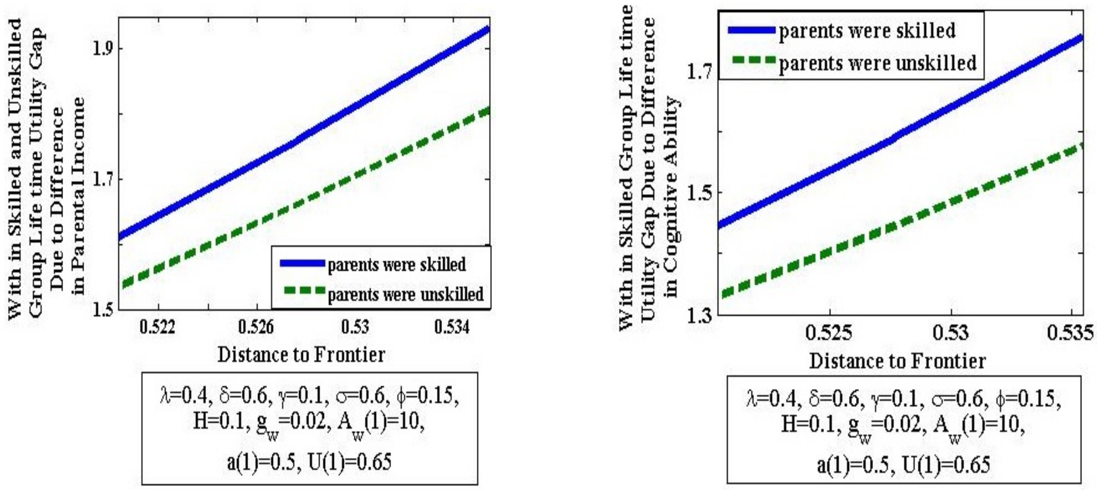
more correlated with parental income and less correlated with child's cognitive ability. It implies that as wage inequality between groups rises mobility falls. There exists a negative relation between intergenerational mobilities and wage inequality.

Proposition 4 *Under* **A1**,

- *In the imitation-only and innovation-only regimes there exists a constant level of upward and downward mobilities as an economy bridges the gap from the world technology frontier.*
- *In the Diversified regime, both upward and downward mobilities fall as an economy progresses.*

3.7 Life Time Utility

First, average life time utility of skilled and unskilled human capital given their parental income have been determined in this subsection. Second, within skilled and unskilled groups life time utility gap due to parental income differences have been worked out.



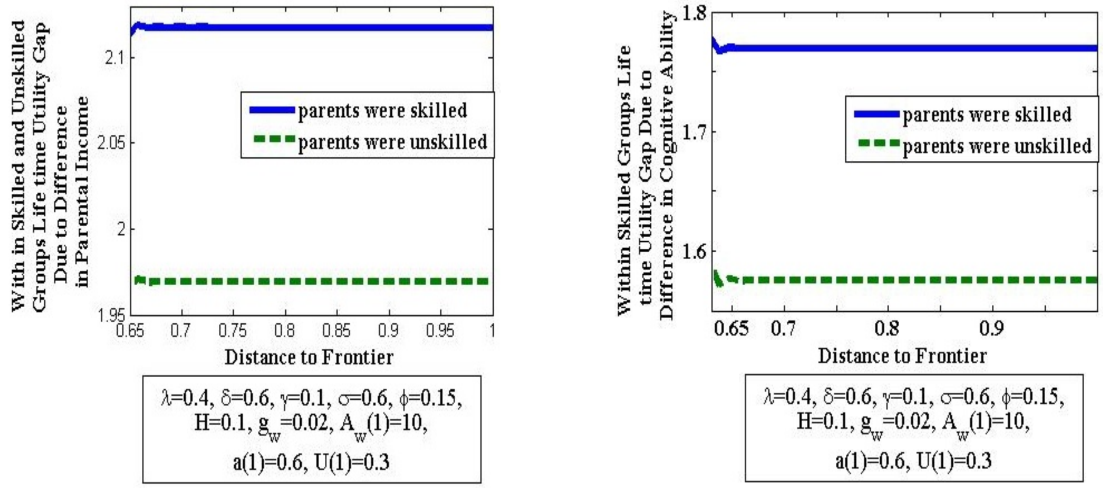
(a) Due to Parental Income Difference

(b) Due to Cognitive Ability Difference

Figure 12: Diversified Regime – Life time Utility Gaps within Skilled and Unskilled Human Capital

Finally, within skilled group life time utility gap due to differences in the cognitive ability among individuals has been ascertained. From eq. (6), life time utility of skilled and unskilled human capital are defined as a function of consumption levels in both the periods of life and the level of bequest that they leave for their children. This in turn depends on the wage income of that individual and also on the level of bequest that he/ she receives from parent and also on the cost of education (if he/ she is skilled). Level of bequest received and cost of education respectively vary among individuals depending on the parental income and cognitive ability. Average cost of education of individuals whose parents were skilled and unskilled are respectively the weighted average of the cost of education of an individual with highest and lowest cognitive ability who go for education depending on parents were skilled and unskilled. Average life time utility of a skilled individual whose parents were skilled and unskilled respectively denoted by W^{ss}_{t+1} and W^{su}_{t+1}, specifically, from eq. (6), are given by:

$$W^{ss}_{t+1} = \left[\frac{ws_t}{2} - \frac{\frac{H\ wu_t}{\theta}\ |_{\theta=1} + \frac{H\ wu_t}{\theta}\ |_{\theta=\ \theta^s_{t+1}}}{2} \right] \frac{ws_{t+1}}{2};$$



(a) Due to Parental Income Difference (b) Due to Cognitive Ability Difference

Figure 13: Innovation-Only Regime – Life time Utility Gap within Skilled and Unskilled Human Capital

$$W^{su}_{t+1} = \left[\frac{wu_t}{2} - \frac{\frac{H\ wu_t}{\theta}\ |_{\theta=1} + \frac{H\ wu_t}{\theta}\ |_{\theta=\ \theta^u_{t+1}}}{2}\right]\frac{ws_{t+1}}{2}.$$

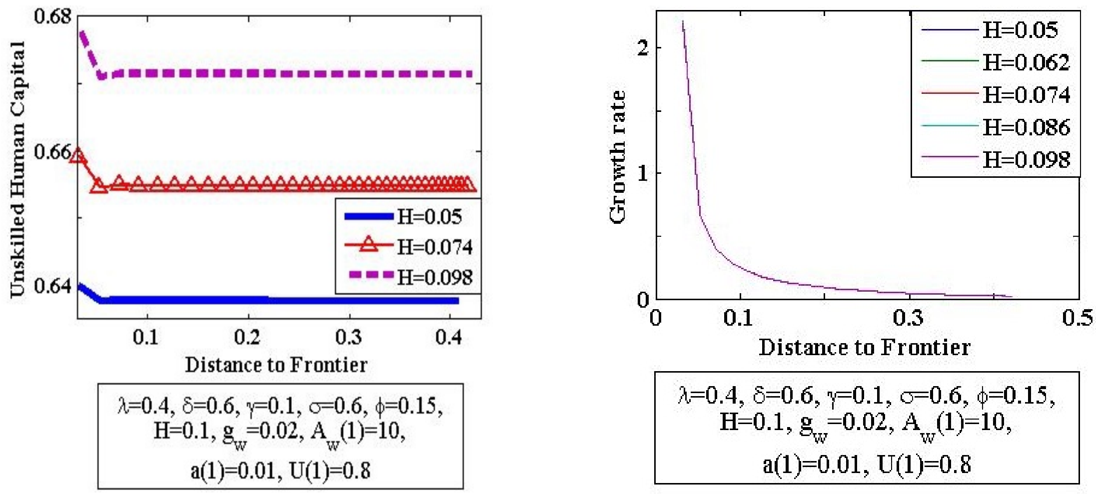
Given the same level of bequest, consumption of k^{th} individual in first period of life is positively related with the cognitive ability of an individual (since first period consumption is the gap between the level of bequest received from parents and the cost of education.) Average life time utility of an unskilled individual whose parent was skilled and unskilled respectively denoted by W^{us}_{t+1} and W^{uu}_{t+1}, specifically, from eq. (6), are given by:

$$W^{us}_{t+1} = \frac{ws_t}{2}\frac{wu_{t+1}}{2} \quad \text{and} \quad W^{uu}_{t+1} = \frac{wu_t}{2}\frac{wu_{t+1}}{2}.$$

Now, life time utility gap within skilled and unskilled human capital have been defined. First, life time utility gap due to parental income differences have been analyzed.

$$\text{Win}^s_{t+1} = \frac{W^{ss}_{t+1}}{W^{su}_{t+1}}; \qquad \text{Win}^u_{t+1} = \frac{W^{us}_{t+1}}{W^{uu}_{t+1}},$$

where Win^s_{t+1} and Win^u_{t+1} respectively measure life time utility gap within skilled and unskilled human capital due to differences in the parental education level. From **Proposition 2**, as an economy moves toward the world technology frontier, wage gap between skilled and unskilled workers rise in the diversified regime. Therefore, gap



(a) Unskilled Human Capital

(b) Growth Rate

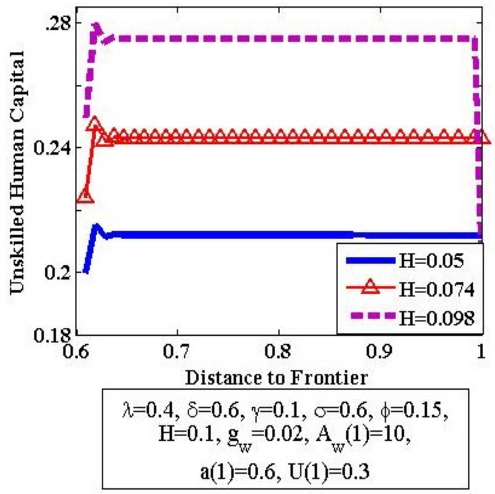
Figure 14: Imitation-Only Regime – Comparative Static wrt Cost of Education

between the level of bequest that a skilled as well as unskilled individuals get from parents due to difference in the parental income rise. This leads to a higher life time utility gap within skilled as well as unskilled human capital due to differences in the parental income as is shown in Fig. 12a in page 38. From **Proposition 2**, in the specialized regimes, there exists a constant wage inequality between skilled and unskilled workers. This implies that there exists a constant gap between the level of bequest due to difference in parental income. This leads to a constant life time utility gap within skilled as well as unskilled human capital due to difference in parental income, as are visible from Fig. 11a in page 37 and Fig. 13a in page 39.

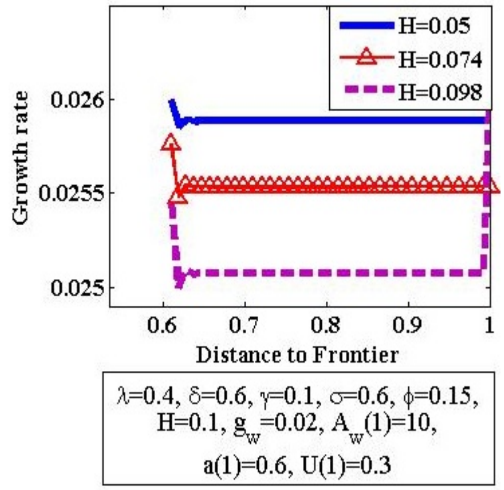
Next, life time utility gap among skilled human capital due to differences in the cognitive ability has been illustrated.

$$\mathrm{Win}^{\theta s}_{\mathrm{t}+1} = \frac{W^{ss}_{\mathrm{t}+1}|_{\theta=1}}{W^{ss}_{\mathrm{t}+1}|_{\theta=\theta^s}}; \qquad \mathrm{Win}^{\theta u}_{\mathrm{t}+1} = \frac{W^{su}_{\mathrm{t}+1}|_{\theta=1}}{W^{su}_{\mathrm{t}+1}|_{\theta=\theta^u}},$$

where $\mathrm{Win}^{\theta s}_{\mathrm{t}+1}$ and $\mathrm{Win}^{\theta u}_{\mathrm{t}+1}$ respectively measure life time utility gap due to cognitive ability differences among skilled human capital even if all of their parents were skilled and unskilled. These capture the life time utility gap due to difference in cognitive ability through the lifetime utility gap of skilled human capital with highest and lowest cognitive ability. By **Lemma 1**, as an economy progresses skilled human rises in the



(a) Unskilled Human Capital



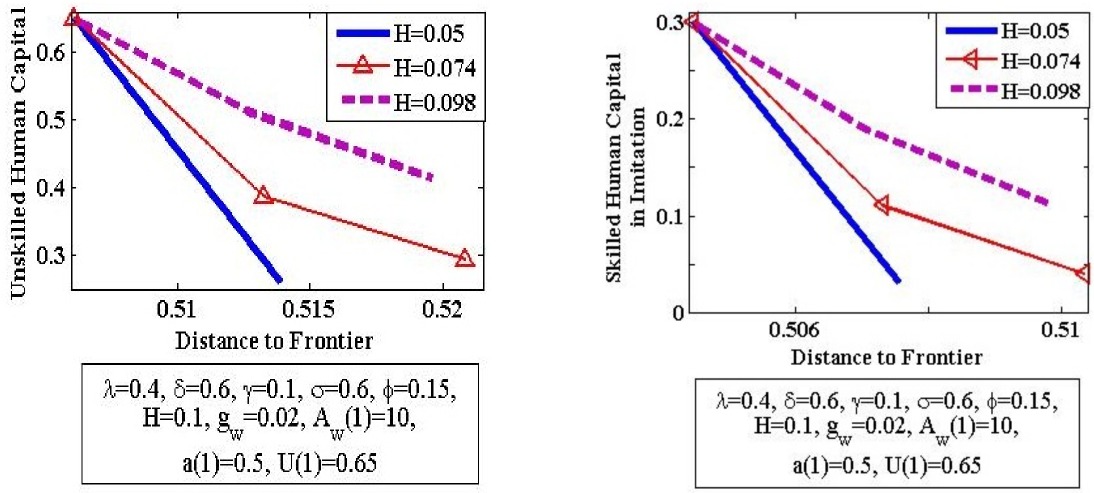
(b) Growth Rate

Figure 15: Innovation-Only Regime – Comparative Static wrt Cost of Education

diversified regime. This in turn implies that individuals with relatively low cognitive ability now become educated. As a result of which cost gap among skilled human capital rises irrespective of their parental income levels. This leads to a higher wealth inequality within skilled human capital due to difference in cognitive ability. To conclude, in the diversified regime, due to difference in cognitive ability, lifetime utility gap within skilled human capital rises irrespective of their parental education status as an economy progresses, as is depicted in Fig. 12b in page 38. Moreover, from eq. (8), it is known that $\theta^{s}_{t+1} < \theta^{u}_{t+1}$. This implies that children of skilled parents work as skilled even with relatively low cognitive ability than individuals with unskilled parents. Therefore, among skilled human capital life time utility gap is high due to difference in cognitive ability for the individuals whose parents were skilled than whose parents were unskilled. Given that there exists a fixed composition of human capital in the specialized regimes, there also exists constant level of life time utility gap within skilled human capital due to difference in cognitive ability. This is true for both the cases – where parents were either skilled or unskilled, as are shown in Fig. 11b in page 37 and Fig. 13b in page 39.

Proposition 5 *Under* **A1**,

- *In the imitation-only and in the innovation-only regimes, there exists constant level*



(a) Unskilled Human Capital (b) Skilled in Imitation

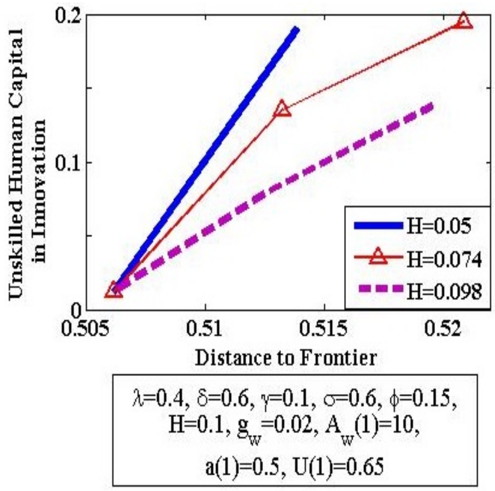
Figure 16: Diversified Regime – Comparative Static wrt Cost of Education

of life time utility gap within skilled as well as unskilled human capital due to parental income differences. Whereas there exists constant level of life time utility gap within skilled human capital due to cognitive ability differences, in the specialized regimes.

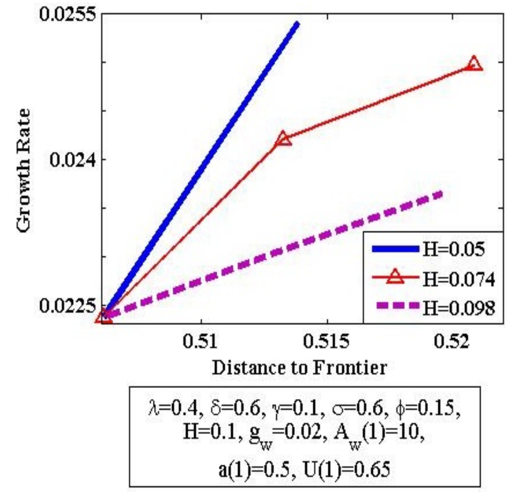
- *In the imitation-innovation regime, life time utility gap within skilled and unskilled human capital rise due to parental income differences. Moreover, life time utility gap within skilled human capital rises due to cognitive ability differences. However, lifetime utility gap within skilled human capital due to cognitive ability differences is higher if parents were skilled than unskilled workers.*

3.8 Comparative Static Analysis w.r.t Cost of Education

The main focus is to look at the impact of an increment in the cost of education (in terms of a change in the parametric value H) on the composition of human capital and also on the growth rate of an economy. Due to a parametric positive shift in cost of education, income of an individual who works as a skilled worker decreases whereas income of that individual by working as an unskilled worker remains unchanged. Therefore, increment in cost of education reduces proportion of skilled human capital and increases unskilled



(a) Unskilled in Innovation



(b) Growth Rate

Figure 17: Diversified Regime – Comparative Static wrt Cost of Education

human capital, irrespective of its distance to frontier, as are shown in Fig. 14a in page 40, in Fig. 15a in page 41 and in Fig. 16a in page 42. By **A1**, imitation depends more on unskilled human capital. Therefore, a hike in cost of education raises the proportion of unskilled human capital and consequently the growth rate of an economy which is in the imitation-only regime. This implies that unskilled human capital is growth enhancing in the imitation-only regime. Due to reduction in the proportion of skilled human capital by **A1**, growth rate falls in the innovation-only regime, as is illustrated in Fig. 15b in page 41. This in turn implies that skilled human capital is growth enhancing in the innovation-only regime.

In the diversified regime by **A1** and **Lemma 2**, both skilled and unskilled human capital shift from innovation to imitation activities and consequently growth rate falls, as are shown in Fig. 16b in page 42, in Fig. 17a in page 43 and in Fig. 17b in page 43. Skilled human capital is growth enhancing in the imitation-innovation regime.

Proposition 6 *Under* **A1**,

- *In the imitation-only regime unskilled human capital is growth enhancing.*
- *In the imitation-innovation and innovation-only regimes, skilled human capital is growth enhancing.*

.

4 Conclusion

Technological progress is a dual phenomenon. A country can improve its technology level by imitating from the world technology frontier or by innovating new knowledge. An economy which is lagging far behind the frontier can improve its technology level by allocating its labor force mainly into imitation. Similarly, an advanced economy can progress technologically by innovating. Under the assumption that different types of human capital are efficient in different activities, Vandenbussche et al. [2006], Aghion et al. [2009] and Basu and Mehra [2014] show that unskilled human capital is the main source of growth for the technologically backward economy and skilled human capital is the main source of growth for the technologically advanced economy. Now, by utilizing an endogenous growth model, with complete absence of credit market, it is shown that growth maximizing level of skilled and unskilled human capital is different for economies depending on its distance from the world technology frontier. Moreover, in the diversified regime probability of becoming rich (resp. poor) given that parents were rich (resp. poor) rise as an economy progresses. That is, as an economy progresses, correlation between income and cognitive ability falls. Moreover, between skilled and unskilled group wage inequality rises in the diversified regime as an economy bridges its gap from the frontier. There exists a positive correlation between equality and intergenerational upward and downward mobilities. Along with that, there exists a U-shaped growth path depending on its distance to frontier. Additionally, by applying growth enhancing education policy, all the economies converge to the world frontier. Without any distortions, like migration and outsourcing, there exists absolute convergence of all the economies irrespective of its distance to frontier.

Bibliography

D. Acemoglu, P. Aghion, and F. Zilibotti. Distance to frontier, selection, and economic growth. *Journal of the European Economic association*, 4(1):37–74, 2006. 10

P. Aghion and P. Howitt. A model of growth through creative destruction. *Econometrica*, 60(2):323–351, 1992. 9

P. Aghion and P. Howitt. Joseph schumpeter lecture appropriate growth policy: A unifying framework. *Journal of the European Economic Association*, 4(2-3):269–314, 2006. 5

P. Aghion, L. Boustan, C. Hoxby, and J. Vandenbussche. Exploiting states mistakes to identify the causal impact of higher education on growth. *NBER Working Paper. Forthcoming*, 2009. 1, 4, 11, 44

D. Andrews and A. Leigh. More inequality, less social mobility. *Applied Economics Letters*, 16(15):1489–1492, 2009. 2

R. J. Barro, X. Sala-i Martin, O. J. Blanchard, and R. E. Hall. Convergence across states and regions. *Brookings papers on economic activity*, pages 107–182, 1991. 4

R. J. Barro, X. Sala-i Martin, O. J. Blanchard, and R. E. Hall. Convergence. *Journal of Political Economy*, 100(2):223–251, 1992. 4

S. Basu and M. K. Mehra. Endogenous human capital formation, distance to frontier and growth. *Research in Economics*, 68(2):117–132, 2014. 1, 4, 5, 44

G. S. Becker and N. Tomes. An equilibrium theory of the distribution of income and intergenerational mobility. *The Journal of Political Economy*, pages 1153–1189, 1979. 2

G. S. Becker and N. Tomes. Human capital and the rise and fall of families. *Journal of labor economics*, pages S1–S39, 1986. 2

J. Benhabib and M. M. Spiegel. The role of human capital in economic development evidence from aggregate cross-country data. *Journal of Monetary economics*, 34(2): 143–173, 1994. 10

A. B. Bernard and S. N. Durlauf. Convergence in international output. *Journal of applied econometrics*, 10(2):97–108, 1995. 4

G. J. Borjas. Ethnic capital and intergenerational mobility. *The Quarterly Journal of Economics*, 1(1):123–150, 1992. 3

M. Das. Persistent inequality: An explanation based on limited parental altruism. *Journal of Development Economics*, 84(1):251–270, 2007. 3

C. Di Maria and P. Stryszowski. Migration, human capital accumulation and economic development. *Journal of Development Economics*, 90(2):306–313, 2009. 5

S. N. Durlauf and P. A. Johnson. Multiple regimes and cross-country growth behaviour. *Journal of Applied Econometrics*, 10(4):365–384, 1995. 3

O. Galor and D. Tsiddon. Technological progress, mobility, and economic growth. *The American Economic Review*, pages 363–382, 1997. 2

A. Gerschenkron et al. Economic backwardness in historical perspective. *Economic backwardness in historical perspective.*, 1962. 11

G. Grossman and E. Helpman. Innovation and growth in the world economy, 1991. 3

J. Hassler, J. V. R. Mora, and J. Zeira. Inequality and mobility. *Journal of Economic Growth*, 12(3):235–259, 2007. 2

P. Howitt. Endogenous growth and cross-country income differences. *American Economic Review*, pages 829–846, 2000. 5, 11

P. Howitt and D. Mayer-Foulkes. R&d, implementation, and stagnation: A schumpeterian theory of convergence clubs. *Journal of Money, Credit and Banking*, 37(1):147–177, 2005. 4, 5

P. Kalaitzidakis, T. P. Mamuneas, A. Savvides, and T. Stengos. Measures of human capital and nonlinearities in economic growth. *Journal of Economic Growth*, 6(3): 229–254, 2001. 3

A. B. Krueger and M. Lindahl. Education for growth: Why and for whom? *Journal of Economic Literature*, 39(4):1101–1136, 2001. 3

P. Krugman. History versus expectations. *The Quarterly Journal of Economics*, 106(2): 651–667, 1991. 16

C.-I. Lee and G. Solon. Trends in intergenerational income mobility. *The Review of Economics and Statistics*, 91(4):766–772, 2009. 3

G. C. Loury. Intergenerational transfers and the distribution of earnings. *Econometrica: Journal of the Econometric Society*, pages 843–867, 1981. 2

E. Maasoumi, J. Racine, and T. Stengos. Growth and convergence: A profile of distribution dynamics and mobility. *Journal of Econometrics*, 136(2):483–508, 2007. 4

Y. D. Maoz and O. Moav. Intergenerational mobility and the process of development. *The Economic Journal*, 109(458):677–697, 1999. 2, 9, 13, 16

D. Mayer-Foulkes. Global divergence. *Available at SSRN 335140*, 2002. 5

R. R. Nelson and E. S. Phelps. Investment in humans, technological diffusion, and economic growth. *The American Economic Review*, pages 69–75, 1966. 3

A. L. Owen and D. N. Weil. Intergenerational earnings mobility, inequality and growth. *Journal of Monetary Economics*, 41(1):71–104, 1998. 2

D. T. Quah. Convergence empirics across economies with (some) capital mobility. *Journal of Economic Growth*, 1(1):95–124, 1996a. 4

D. T. Quah. Empirics for economic growth and convergence. *European economic review*, 40(6):1353–1375, 1996b. 4

D. T. Quah. Twin peaks: growth and convergence in models of distribution dynamics. *The Economic Journal*, pages 1045–1055, 1996c. 4

D. T. Quah. Empirics for growth and distribution: stratification, polarization, and convergence clubs. *Journal of economic growth*, 2(1):27–59, 1997. 4

D. T. Quah. *Ideas determining convergence clubs*. Citeseer, 1999. 4

P. M. Romer. Endogenous technological change. *The Journal of Political Economy*, 98 (5):S71–S102, 1990. 3

X. Sala-i Martin. Cross-sectional regressions and the empirics of economic growth. *European Economic Review*, 38(3):739–747, 1994. 4

X. X. Sala-i Martin. Regional cohesion: evidence and theories of regional growth and convergence. *European Economic Review*, 40(6):1325–1352, 1996. 4

G. Solon. Intergenerational income mobility in the united states. *The American Economic Review*, pages 393–408, 1992. 3

G. Solon. Cross-country differences in intergenerational earnings mobility. *The Journal of Economic Perspectives*, 16(3):59–66, 2002. 3

J. Vandenbussche, P. Aghion, and C. Meghir. Growth, distance to frontier and composition of human capital. *Journal of economic growth*, 11(2):97–127, 2006. 1, 3, 4, 9, 11, 44