

Quantile forecasts of inflation under model uncertainty

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 ${\rm April}\ 2015$

Online at https://mpra.ub.uni-muenchen.de/64341/ MPRA Paper No. 64341, posted 15 May 2015 04:31 UTC

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April 30, 2015

Abstract

Bayesian model averaging (BMA) methods are regularly used to deal with model uncertainty in regression models. This paper shows how to introduce Bayesian model averaging methods in quantile regressions, and allow for different predictors to affect different quantiles of the dependent variable. I show that quantile regression BMA methods can help reduce uncertainty regarding outcomes of future inflation by providing superior predictive densities compared to mean regression models with and without BMA.

Keywords: Bayesian model averaging; quantile regression; inflation forecasts; fan charts

JEL Classification: C11, C22, C52

1 Introduction

Quantile regression generalizes traditional least squares regression by estimating different values of regression coefficients that allow to make inference on the conditional median and other quantiles of the variable of interest. Least squares regression only produces coefficients that allow to fit the mean of the dependent variable given some explanatory/predictor variables. In that respect, quantile regression is, for obvious reasons, more appropriate for making inferences about predictive distributions and assessing forecast uncertainty. At the same time quantile regression estimates are more robust against outliers in the variable of interest (i.e. the dependent variable). In several fields of statistics, quantile regression is used to discover predictive relationships between the dependent and exogenous variables, when typical regression modelling fails to indicate the existence of predactibility in these exogenous variables; see Koenker (2005).

In this paper I apply model selection methods for regression models to a univariate time-series quantile

regression model for inflation. My ultimate aim is to produce quantile forecasts for inflation using several potential explanatory variables. Bayesian model averaging (BMA) and selection (BMS) methods have been traditionally used to deal with model uncertainty in forecasting regressions. I show that application of BMA in the quantile regression model allows to forecast each quantile of inflation using a different set of predictors. This interesting feature of model selection and averaging in quantile regression means that we can approximate complex forms of the posterior predictive density of inflation, despite the fact that the quantile regression model I specify is inherently linear. Although a large empirical literature using quantile regression exists, applications of (Bayesian) model averaging are scarce. exception is the study of Crespo-Cuaresma, Foster and Stehrer (2011), however, these authors do not rely on Bayesian estimation, rather they approximate Bayesian inference by using Least Squares and the Bayesian Information Criterion (BIC). I provide an efficient Gibbs sampling alogorithm that allows to jointly estimate full posterior distributions of the parameters of the quantile regression model, and at the same time to obtain full posterior distirbutions of the uncertainty of each predictor (in the form of "posterior probabilities of inclusion" of each predictor).

This paper comes to integrate two vastly expanding literatures. On the one hand, there are several studies which develop estimation, inference and forecasting in Bayesian quantile regression models, such as Bernardi, Casarin and Petrella (2014), Gaglianone and Lima (2012), Geraci and Bottai (2007), Gerlach, Chen and Chan (2011), Lancaster and Jun (2010), Meligkotsidou, Vrontos and Vrontos (2009), Schüler (2014), Tsionas (2003) and Yu and Moyeed (2001). On the other hand, there is a vast literature in macroeconomic and financial forecasting that shows the superiority of Bayesian model averaging and selection methods over other alternatives; see Koop and Korobilis (2012) and Wright (2008), among several others.

Empirical evaluation of the quantile regression BMA method is based on forecasting monthly US consumer price index inflation observed for the period 1978m1-

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2013m7, using 32 potential predictors. I show which predictors are relevant for each quantile of inflation at various forecast horizons, and I compare my results with Bayesian model averaging in the mean regression specification. Based on predictive likelihoods (Geweke and Amisano, 2011) the quantile regression BMA provides superior density forecasts compared to regular regression BMA, and naive quantile regression methods without BMA.

In the next Section I present the model and the BMA prior, and in Section 3 I present the empirical results. Section 4 concludes the paper and discusses further extensions.

2 Bayesian quantile regression

Following Yu and Moyeed (2001) the quantile regression model has a convenient mixture representation which, as explained below, is particularly convenient for Bayesian estimation using the Gibbs sampler. In particular, I consider the linear model

$$y_t = x_t' \beta_p + \varepsilon_t, \tag{1}$$

where x_t is a $n \times 1$ vector of explanatory variables, and β_p is a vector of coefficients dependent on the p-th quantile of the random error term ε_t which is defined as the value q_p for which $\Pr(\varepsilon_t < q_p) = p$. In typical specifications of quantile regression (Koenker, 2005) the distribution of ε_t is left unspecified (that is, it is a nonparametric distribution F_p), and estimation of β_p is the solution to the following minimization problem

$$\min_{\beta} \sum_{t=1}^{T} \rho_p(\varepsilon_t), \qquad (2)$$

where the loss function is $\rho_p(u) = u(p - I(u < 0))$ and I(A) is an indicator function which takes value one if event A is true, and zero otherwise.

The major contribution of Yu and Moyeed (2001) was to show that the minimization in equation (2) is equivalent to maximizing a likelihood function under the asymmetric Laplace error distribution; see also Tsionas (2003). Reed and Yu (2011) have recently established, both theoretically and empirically, that the assymetric Laplace likelihood accurately approximates the true quantiles of many distributions having different properties. At the same time, as shown in Kotz et al. (1998), the asymmetric Laplace distribution can admit various mixture representations. In Bayesian analysis a popular representation is that of a scale mixture of normals with scale parameter following the exponential distribution¹.

This mixture formulation allows the likelihood function to be written in *conditionally* normal form, and inference based on conditional posterior distributions is straightforward. Even when the joint posterior distribution of model parameters is of complex form (as it is the case when the likelihood is asymmetric Laplace - no matter what the prior is), one can rely on the Gibbs sampler (Reed and Yu, 2011) in order to sample from these conditional posteriors. When the conditional likelihood admits a normal or a mixture of normals form, these conditional posteriors belong to known distributions and, thus, easy to draw samples from; see the Techincal Appendix for details.

Following Kozumi and Kobayashi (2011) we can represent the error distribution ε_t using the form

$$\varepsilon_t = \theta z_t + \tau \sqrt{z_t} u_t, \tag{3}$$

where $z_t \sim Exponential$ (1), that is, a variate from an exponential distribution with rate parameter one, and u_t is distributed standard normal. In this formulation it holds that $\theta = (1-2p)/p(1-p)$, and $\tau^2 = 2/p(1-p)$, for a given quantile $p \in [0,1]$. Supplanting the formula for ε_t into equation (1) gives the new quantile regression form

$$y_t = x_t' \beta_p + \theta z_t + \tau \sqrt{z_t} u_t, \tag{4}$$

and the conditional density of y_t given the Exponential variates z_t is Normal and is of the form

$$f(\mathbf{y}|\beta(p), \mathbf{z}) \propto \left(\prod_{i=1}^{T} z_{t}^{-\frac{1}{2}}\right) \times \exp\left\{-\frac{1}{2} \sum_{i=1}^{T} \frac{\left(y_{t} - x_{t}'\beta_{p} - \theta z_{t}\right)^{2}}{\left(\tau \sqrt{z_{t}}\right)^{2}}\right\},$$

where $\mathbf{y} = (y_1, ..., y_T)'$ and $\mathbf{z} = (z_1, ..., z_T)'$.

Given this likelihood formulation we can now define prior distributions. Bayes theorem says that the posterior distribution - the penultimate quantity of interest during the estimation part of statistical inference - is simply the product of the (conditionally) Normal likelihood and the prior. In particular, Yu and Moyeed (2001) prove that all the posterior moments of β_p exist when the prior for β_p is Normal. In this paper I consider the conditionally Normal prior

$$\begin{array}{lll} \beta_{i,p} & \sim & N\left(0,\gamma_{i,p}\delta_{i,p}^2\right), \\ \delta_{i,p}^{-2} & \sim & Gamma\left(a,b\right), \\ \gamma_{i,p} & \sim & Bernoulli\left(\pi_0\right), \\ \pi_0 & \sim & Beta\left(c,d\right). \end{array}$$

Looking only at the first line of the above formulas, the prior for each $\beta_{i,p}$, i=1,...,n, looks like a typical Normal prior, however, it is the case that this prior with

¹A typical application of this mixture representation is in the Bayesian lasso prior; see Park and Casella (2008).

the many hierarchies is a mixture of Normals prior. When the indicator variable $\gamma_{i,p}=1$ then $\beta_{i,p}$ has a Normal prior with variance $\delta_{i,p}^2$. When $\gamma_{i,p}=1$ then $\beta_{i,p}$ has a Normal prior with mean zero and variance zero, i.e. a point mass at zero. Such an extremely informative prior means that predictor $x_{i,t}$ is not relevant for the p-th quantile. The indicators $\gamma_{i,p}$ are estimated from the data, thus they have their own Bernoulli prior with probability π_0 . Additionally, in order to avoid subjectively selecting the hyperparameters $\pi_0, \delta_{i,p}^2$, we introduce hyper-prior distributions on them so that they are estimated by evidence in the data.

Posterior computation is relatively simple. We need to sequentially sample from the posteriors of each unkown quantity, namely $(\beta_p, z_t, \delta_p^{-2}, \gamma_p, \pi_0)$ conditional on all the other ones. These conditional posteriors can be sampled using the Gibbs sampler algorithm described in detail in the Technical Appendix.

3 Empirics

In this section I examine which predictors affect inflation quantiles at forecast horizons h=1,3,6,12, and whether QR-BMA can provide superior density forecasts compared to BMA in the regular regression model. For that reason I consider total CPI for the period 1978m1-2013m7 as the dependent variable and two own lags of inflation as well as 32 exogenous variables, as potential predictors. The data and transformations are explained in the data appendix. For the purpose of forecasting, the model in equation (1) is re-written as

$$y_{t+h} = x_t' \beta_p + \varepsilon_{t+h}, \tag{5}$$

for t=1-h,...,T-h so that a "direct" point forecast of the p-th quantile at the end of the sample is of the form $y_{T+h|T}^p = x_T'\beta_p$. Tables 1 and 2 show selected predictors by applying the BMA prior of the previous section to the regression and quantile regression models. Table 1 refers to CPI 1-step ahead and Table 2 refers to CPI 12 months ahead. Only predictors with mean posterior probability of inclusion which is higher than 0.5 are presented in these tables. Such probabilities can be computed by using the posterior of the indicators $\gamma_{i,p}$, which are sequences of zeros and ones so that its posterior mean is the desired probability of inclusion. The results in these two Tables clearly indicate that there is heterogeneity in selecting predictors for each quantile, as well as between the mean regression and the median (p=0.5) regression.

In order to evaluate the forecast performance of each model I consider a recursive procedure, starting with estimating parameters using 40% of the total sample, forecast out-of-sample for each horizon h=1,3,6,12, then add one observation, estimate parameters and forecast again,

and continue like that until the sample is exhausted. This procedure allows me to evaluate forecasts using the final 60% of the sample, that is the period 1992m2-2013m7-h. When computing quantile forecats, I follow Gaglianone et al (2012) and collect the quantities $y_{T+h|T}^p$ for several quantiles and construct the full predictive density using an Epanechnikov kernel. Results from all models are based on 20,000 iterations of the Gibbs sampler after discarting the first 5,000 iterations which are more prone to the effects of the initial conditions of all parameters. These choices are driven by the fact that convergence of the quantile regression model is excellent, however, computation is quite expensive as it requires to update all coefficients for each separate quantile.

Given that the predictive density of the Bayesian quantile regression is of non-standard form, I use Average Predictive Likelihoods (APLs) as the most numerically reliable method for evaluating density forecasts. These are defined as the average over the evaluation period of all predictive densities $f(y_{t+h})$ evaluated at the outof-sample observation y_{t+h}^o ; see Geweke and Amisano (2010). I evaluate forecasts of the Bayesian mean regression as well as the Bayesian quantile regression models with various predictors. The first case considers the simple AR(2) model estimated with noninformative prior ("AR(2)" case). The second case estimates the regression and quantile regression models using two lags plus all exogenous predictors with a noninformative prior ("full" case). Finally, both the mean regression and quantile regression are estimated using the Bayesian model averaging prior on the 32 exogenous predictors, while the two lags of inflation are unrestricted using a noninformative prior ("BMA" case). While in Tables 1 and 2 I showed the results of model selection (i.e. selecting predictors with important probability of inclusion in a forecasting model), in Table 3 I present results from the average model, i.e. a model which allows even less important predictors to enter the final forecasting model (but with a low weight) 3 .

Results are presented in Table 3 for all six different models and all four forecast horizons. The quantile regression with Bayesian model averaging is the clear winner of this comparison. First it is clear that quantile regression models (whether BMA is present or not), perform on average better than traditional regression models. This is because traditional Bayesian regression mod-

²For each draw from the Gibbs sampler I generate forecasts of quantiles $p \in [0.05, 0.06, ..., 0.94, 0.95]$, i.e. I obtain 91 quantiles. I do not consider the 5% probability from each tail of the predictive density for reasons explained in Gaglianone et al. (2012).

³The qualitative results are not affected by this choice. Model averaging in general has lower risk compared to model selection, but which one is better is an empirical issue that is not the purpose of this paper; see the discussion in Koop and Korobilis (2012) for more information.

els produce predictive densities which are conditionally Gaussian, while the Bayesian quantile regression produces predictive densities which are mixtures of Gaussians - thus more flexible and can capture higher kurtosis in inflation during the recent financial crisis. However, it is impressive how allowing for model averaging in each individual quantile of the predictive distribution gives vast increase in average predictive likelihoods, showing the potential benefits of this method.

4 Conclusions

This paper proposes a new empirical procedure for implementing Bayesian model averaging, which allows different predictor variables to affect different quantiles of the dependent variable. The benefits of this flexible approach are evaluated using data for CPI inflation for the US and a relatively large number of predictor variables. Results indicate that the quantile regression BMA approach indeed finds that different predictors are relevant for each quantile of inflation, and that by taking this feature into account predictive distributions are superior. The good empirical performance of the proposed method suggests that there are clear benefits from considering quantile-specific Bayesian inference that could potentially generalize to other settings, e.g. specifying Bayesian shrinkage priors in quantile vector autoregressions, or having different degree of time-variation in the parameters of a time-varying parameter quantile regression. Such extensions are beyond the purpose of this note and are left for future research.

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Table 1: Selected predictors of CPI inflation, h = 1

Predictor	CD1M	FEDFUNDS	OILPRICE	INFEXP
Regression BMA			•	•
Quantile Regression BMA				
p=0.05				•
p=0.25	•	•		•
p=0.5				•
p=0.75	•	•		•
p=0.95	•			•

Note: Predictors with probability of inclusion <0.5 for any of the models, are not included in the table.

Table 2: Selected predictors of CPI inflation, h = 12

	UNRATE	MORTG	MPRIME	CD1M	FEDFUNDS	UEMP15OV	INFEXP
Regression BMA	•	•				•	•
Quantile Regression BMA							
p=0.05	•					•	
p=0.25							•
p=0.5		•		•	•		•
p=0.75			•		•		
p=0.95				•	•		

Note: Predictors with probability of inclusion <0.5 for any of the models, are not included in the table.

Table 3: Average Predictive Likelihoods

	h = 1	h = 3	h = 6	h = 12		
REGRES	SSION					
AR(2)	0.1214	0.1209	0.1183	0.1156		
full	0.1132	0.1087	0.1075	0.1050		
BMA	0.1259	0.1232	0.1190	0.1173		
QUANTILE REGRESSION						
AR(2)	0.1242	0.1199	0.1180	0.1161		
full	0.1126	0.1103	0.1100	0.1063		
BMA	0.1425	0.1384	0.1361	0.1323		

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Technical Appendix

The transformed quantile regression model is given in equation (4) which we rewrite here for convenience

$$y_t = x_t' \beta_p + \theta z_t + \tau \sqrt{z_t} u_t, \tag{A.1}$$

with x_t' being the (fixed) exogenous variables, and $z_t \sim Exponential(1)$ and $u_t \sim N(0,1)$ are new variables introduced when transforming the likelihood (see main text for more details). The prior we use is of the form

$$\beta_{i,p} \sim N\left(0, \gamma_{i,p} \delta_{i,p}^{2}\right),$$

$$\delta_{i,p}^{-2} \sim Gamma\left(\underline{a}, \underline{b}\right),$$

$$\gamma_{i,p} \sim Bernoulli\left(\pi_{0}\right),$$

$$\pi_{0} \sim Beta\left(\underline{c}, \underline{d}\right),$$

where $(\underline{a}, \underline{b}, \underline{c}, \underline{d})$ are prior hyperparameters chosen by the researcher. In order to obtain draws from the posteriors of all the unknown parameters, we sample sequentially 20,000 times from the following conditional distirbutions

1. Sample $\beta(p)$ conditionally on knowing all other parameters (incl z_t) and, of course, the data x_t, y_t , from:

$$\beta_p|\gamma_p,\tau^2,\mathbf{z},\mathbf{x},\mathbf{y}\sim N\left(\overline{\beta},\overline{V}_{\beta}\right),$$
 where $\overline{V}_{\beta}=\left(\sum_{t=1}^T\frac{\widetilde{x}_t'\widetilde{x}_t}{\tau^2z_t}+\Delta_p^{-1}\right)^{-1}$ and $\overline{\beta}=\overline{V}_{\beta}\left[\sum_{t=1}^T\frac{\widetilde{x}_t(y_t-\theta z_t)}{\tau^2z_t}\right]$, and Δ is a diagonal prior

variance matrix with diagonal element $\delta_{i,p}^2$. Note that in the formulas above we need to replace x_t with \tilde{x}_t where $\tilde{x}_{i,t} = x_{i,t}\gamma_{i,p}$, i.e. whenever $\gamma_{i,p} = 0$, \tilde{x}_t has its *i*-th element replaced with zero (for all t = 1, ..., T).

2. Sample $\delta_{i,p}^{-2}$ conditional on other parameters and data from:

$$\delta_{i,p}^{-2}|\boldsymbol{\beta}_{i,p},\mathbf{x},\mathbf{y} \sim Gamma\left(\overline{a},\overline{b}\right),$$

where
$$\overline{a} = \underline{a} + \frac{1}{2}$$
, $\overline{b} = \frac{\left(\beta_{i,p}\right)^2}{2} + \underline{b}$.

3. Sample $\gamma_{i,p}$ conditional on other parameters and data from:

$$\gamma_{i,p}|\gamma_{-/i,p},\beta_{i,p},\mathbf{z},\mathbf{x},\mathbf{y}\sim Bernoulli\left(\overline{\pi}\right),$$

where
$$\overline{\pi} = \frac{\pi_0 f\left(\gamma_{i,p} = 1 | \gamma_{-/i,p}; \mathbf{x}, \widetilde{\mathbf{y}}\right)}{\pi_0 f\left(\gamma_{i,p} = 1 | \gamma_{-/i,p}; \mathbf{x}, \widetilde{\mathbf{y}}\right) + (1 - \pi_0) f\left(\gamma_{i,p} = 0 | \gamma_{-/i,p}; \mathbf{x}, \widetilde{\mathbf{y}}\right)},$$
 $\widetilde{\mathbf{y}} = \mathbf{y} - \theta \mathbf{z}$, and $\gamma_{-/i,p}$ denotes the vector γ_p with its i -th element removed (i.e. condition $\gamma_{i,p}$ on all remaining $n-1$ elements in γ_p). The function

$$f\left(\gamma_{i,p}=1|\gamma_{-/i,p};\mathbf{x},\widetilde{\mathbf{y}}\right)$$
 is the likelihood of the model

$$\widetilde{y}_t = y_t - \theta z_t = x_t' \beta_p + \tau \sqrt{z_t} u_t,$$

evaluated assuming $\gamma_{i,p} = 1$, and similarly for the function $f\left(\gamma_{i,p} = 0 | \gamma_{-/i,p}; \mathbf{x}, \widetilde{\mathbf{y}}\right)$.

4. Sample π_0 conditional on other parameters and data from:

$$\pi_0 | \gamma_p, \beta_p, \mathbf{z}, \mathbf{x}, \mathbf{y} \sim Beta(\overline{c}, \overline{d}),$$

where $\overline{c}=n_{\gamma}+\underline{c}$ and $\overline{d}=n-n_{\gamma}+\underline{d}$, and n_{γ} denotes the number of elements in γ_p which are one, i.e. $n_{\gamma}=\sum_i \gamma_{i,p}=1$.

5. Sample z_t conditional on other parameters and data from:

$$\mathbf{z}|\boldsymbol{\beta}_{p},\boldsymbol{\gamma}_{p},\mathbf{x},\mathbf{y}\sim GIG\left(\frac{1}{2},\overline{\kappa}_{1},\overline{\kappa}_{2}\right),$$

where
$$\overline{\kappa}_1 = \left[\sum_{t=1}^T (y_t - x_t \beta_p) / \tau\right]$$
 and

 $\overline{\kappa}_2 = \sqrt{2 + \theta^2}/\tau$. The p.d.f of the Generalized Inverse Gaussian density is of the form

$$f\left({x|v,a,b} \right) = \frac{{{{\left({b/a} \right)}^v}}}{{2K\left({ab} \right)}}{x^{v - 1}}\exp \left\{ { - \frac{1}{2}\left({{a^2}{x^{ - 1}} + {b^2}x} \right)} \right\},$$

with x > 0, $-\infty < v < \infty$, a, b > 0.

Data Appendix to: "Quantile forecasts of inflation under model uncertainty"

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Data are from FRED (http://research.stlouisfed.org/fred2/) and are shown in the table below. The dependent variable is CPIAUCSL (Consumer Price Index for All Urban Consumers: All Items). All variables are transformed to be approximate stationary. In particular, if $z_{i,t}$ is the original untransformed series, the transformation codes are (column Tcode below): 1 - no transformation (levels), $x_{i,t} = z_{i,t}$; 4 - logarithm, $x_{i,t} = \ln z_{i,t}$; 5 - first difference of logarithm, $x_{i,t} = 1200 \times \ln(z_{i,t}/z_{i,t-1})$.

No	Mnemonic	Description	Tcode
1	INDPRO	Industrial Production Index	5
2	HOUST	Housing Starts: Total: New Privately Owned Units Started	4
3	HSN1F	New One Family Houses Sold: United States	4
4	NAPM	ISM Manufacturing: PMI Composite Index	4
5	TCU	Capacity Utilization: Total Industry	4
6	UNRATE	Civilian Unemployment Rate	1
7	PAYEMS	All Employees: Total nonfarm	5
8	CIVPART	Civilian Labor Force Participation Rate	4
9	AWHI	Index of Aggregate Weekly Hours	5
10	MORTG	30-Year Conventional Mortgage Rate	1
11	MPRIME	Bank Prime Loan Rate	1
12	CD1M	1-Month Certificate of Deposit: Secondary Market Rate	1
13	FEDFUNDS	Effective Federal Funds Rate	1
14	M1SL	M1 Money Stock	5
15	M2SL	M2 Money Stock	5
16	BUSLOANS	Commercial and Industrial Loans, All Commercial Banks	5
17	CONSUMER	Consumer Loans at All Commercial Banks	5
18	REALLN	Real Estate Loans, All Commercial Banks	5
19	EXGEUS	Germany / U.S. Foreign Exchange Rate	5
20	EXJPUS	Japan / U.S. Foreign Exchange Rate	5
21	EXCAUS	Canada / U.S. Foreign Exchange Rate	5
22	EXUSUK	U.S. / U.K. Foreign Exchange Rate	5
23	OILPRICE	Spot Oil Price: West Texas Intermediate	5
24	MVATOTASSS	Motor Vehicle Assemblies: Total motor vehicle assemblies	1
25	UEMP15OV	Number of Civilians Unemployed for 15 Weeks & Over	4
26	UEMPLT5	Number of Civilians Unemployed - Less Than 5 Weeks	4
27	CONSENT	Index of Consumer Sentiment	1
28	INFEXP	Expected Changes in Inflation Rates	1