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# Regulating a Manager-Controlled Monopoly with Unknown Costs

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## Abstract

We study the regulation of a manager-controlled monopoly with unknown costs, borrowing from the earlier work of Baron and Myerson (BM) (1982), where the monopoly is controlled by the owner. Our regulatory environment involves the case where the regulator can tax the owner as well as the case where she cannot. We show that the optimal price schedule in our model generally lies below the one in the BM model. In addition, if the compensation parameter is sufficiently small, the optimal price can be as low as the marginal cost, provided that the regulator cannot tax the owner of the monopoly. We also examine how the size of the managerial compensation affects the welfare of the owner of the monopoly as well as the social welfare. Moreover, we show that in settings where the owner of a manager-controlled monopoly cannot be taxed, the owner prefers to separate management from ownership, provided that the marginal cost of production is sufficiently large. However, the owner always prefers to manage the monopoly herself when the marginal cost of production is sufficiently small.

**Keywords:** Monopoly; Regulation; Firm Ownership; Firm Control

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# 1 Introduction

A great body of the economics of regulation has dealt with models involving monopolistic or oligopolistic firms that are managed by their owners. In many of these models where the regulated firm is assumed to possess private information about cost, demand or effort, regulatory solutions have been proposed as a direct revelation mechanism (see, for example, Baron and Myerson (1982), Baron and Besanko (1984), Laffont and Tirole (1986), Prusa (1990), and Strausz (2011)). A direct revelation mechanism requires the regulated firm to report its private information and it ensures truthful revelation if the regulated firm is provided with sufficiently high incentives. As the owner of a regulated firm maximizes her profits, the incentives offered by a typical regulatory mechanism in the existing literature accordingly takes into account the effect of the owner's potential lies on her profits. However in real life, the ownership and management are separated for many firms, and the objective function of such firms are either revenues, market shares or a fraction of the profits rather than the whole profits.<sup>1</sup> Thus, the incentives of a manager-controlled firm to misrepresent its private information under a direct revelation mechanism may differ from those of an owner-controlled firm, calling for the need of extending the existing theories on monopoly regulation. To this end, our paper examines the implication of separating management from ownership in an incentive model of monopoly regulation. The model we construct borrows from the seminal work of Baron and Myerson (BM) (1982), who characterized an optimal direct-revelation mechanism regulating an owner-managed monopoly with unknown marginal costs.

The model of BM is the first in the whole economics of regulation to define a general social welfare function, defined as the sum of consumers' welfare and a fraction of the monopolist's welfare.<sup>2</sup> The regulatory mechanism proposed by BM

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<sup>1</sup>See Williamson (1963) and Sklivas (1987) for a discussion on the validity of profit maximization hypothesis.

<sup>2</sup>For a particular definition of social welfare function that treats the welfares of consumers and the monopoly equally, a solution to the problem of regulating a monopoly under asymmetric information was earlier provided by Loeb and Magat (1979). Their solution delegates the output decision to the monopoly, which is also offered the right to the whole social surplus. Thus, the monopoly chooses to produce at the socially efficient level. While this delegatory incentive scheme is not a direct-revelation mechanism, its outcome is essentially the same as the one obtained in the

involves four policy functions: a price function and a quantity function which are consistent with the market demand curve, a probability function that specifies at each cost level whether the monopoly will be permitted to operate, and a subsidy function that specifies a money transfer from consumers to the monopoly at each cost level. The construction of the mechanism ensures that the monopoly truthfully reveals the private cost parameter. In fact, this is made possible by guaranteeing to the monopoly an information rent which is maximized only under truthful revelation. Formally, this incentive-compatible rent has to be equal to the area under a (downward-sloping) adjusted demand curve over the range of possible marginal costs not lower than the actual marginal cost of the monopoly. Because this rent is generated through a subsidy from consumers to the monopoly, the consumers and the whole society can be expected to become better off when it is lower, provided that the welfare of the monopoly has a lower weight than the welfare of consumers in the definition of social welfare. Absolutely, to reduce the information rent one needs to reduce the marginal information rent, equalling the quantity of output, at each possible cost level. However, the problem of limiting the output schedule is not trivial, since a contraction of this schedule would suppress not only the information rent but also the gross social surplus net of the total cost of production. Consequently, any change in the output schedule would always create a tradeoff since the (expected) social welfare under regulation turns to be (the expected value of) the weighted difference between ‘the gross social surplus net of the total cost of production’ and ‘the information rent’ received by the monopoly. Indeed, this tradeoff is right balanced by the optimal output policy proposed by BM.

We extend the described regulation model of BM by separating ownership from management; i.e. in our model the monopoly is controlled by a separate manager. As we will assume that the marginal cost information is privately known to the manager, the regulator will elicit the private cost information from the manager instead of the owner of the monopoly. Clearly, the regulator can be expected to use the cost information, once he has extracted it from the manager, to also regulate the earnings of the owner through an optimal tax. In reality, however, this may not

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BM model under the described egalitarian form of social welfare function. However, this scheme is obviously not efficient under other forms of social welfare function.

always be true. The reason is that many firms are widely held corporations, where the owner may involve hundreds or thousands of shareholders, and because of some political concerns, the regulator may be unwilling to directly or indirectly tax a huge number of shareholders. To account for this situation, we will allow in our model the case where the regulator cannot tax the owner of the manager-controlled monopoly, as well as the case where she can.

We will also assume that the manager receives a positive fraction of the profits of the monopoly as a salary while the rest of the profits accrues to the owner. Obviously, in the absence of any regulation the separation of ownership from management would be inconsequential for the market outcome since our managerial objective function is perfectly aligned with the welfare of the owner. However, under regulation the separation will matter despite the assumed form of managerial objectives. The reason is that the incentive to misrepresent the marginal cost will be lower for the manager in our model (where his unregulated salary is a fraction of the profits) than the owner in the BM model (where the whole of the profits constitutes her unregulated gain). Accordingly, the information rent the manager should obtain to truthfully reveal his private cost information can be predicted to be lower than the information rent the owner of the monopoly would receive in the BM model. So, the necessity of suppressing the output schedule to reduce the welfare loss caused by the rent of the private information seems to become less stringent in the case of a manager-controlled monopoly than in the case of an owner-controlled monopoly. Indeed, because of the presence of managerial compensations, the optimal price (output) schedule in our model will always lie below (above) the one in the BM model. Moreover, if the compensation parameter is sufficiently small, the optimal price in our model can be as low as the marginal cost, provided that the regulator cannot tax the owner of the monopoly.

We will also show that irrespective of whether the owner of the monopoly knows the marginal cost of production, it can never be optimal for the owner to choose the size of the managerial compensation (as a fraction of the profits) arbitrarily small or arbitrarily large. In addition, when the regulator can tax the owner of the monopoly, the expected social welfare becomes decreasing in the size of the managerial compensation. However, when the regulator cannot tax the owner of the

monopoly, the expected social welfare becomes increasing over a range of managerial compensations that are sufficiently small.

A second question we attempt to answer in this paper is whether the owner prefers a manager-controlled monopoly to an owner-controlled monopoly in situations where she knows the marginal cost of production. Our results show that this is indeed the case if the marginal cost is sufficiently large and if the regulator cannot tax the owner of a manager-controlled company. On the other hand, the owner prefers to manage the monopoly herself if the marginal cost of production is sufficiently small.

The rest of the paper is organized as follows: Section 2 presents our model, containing basic structures for a manager-controlled monopoly and the problem faced by the regulator. Section 3 contains results for this model, involving the optimal regulatory policy and some welfare analyses. Section 4 studies whether separating management from ownership can be desirable for the owner of a regulated monopoly. Finally, Section 5 concludes.

## 2 Model

The model builds upon the earlier work of Baron and Myerson (BM) (1982). We consider an environment where the operations of a monopoly with private cost information are regulated. However, unlike in the BM model we assume that the monopoly is a manager-controlled firm. Thus, our model involves three agents: the regulator (she), the owner of the monopoly (she) and the manager (he).

The manager faces a negatively sloped inverse demand function

$$P(q) = a - q, \quad \text{for all } q \geq 0, \tag{1}$$

where  $q$  denotes the quantity of output and  $a$  is a positive real. The manager also faces a cost function

$$C(q, \theta) = \theta q, \quad \text{for all } q \geq 0, \tag{2}$$

where  $\theta$  denotes the marginal cost. We assume that  $\theta \in [\theta_0, \theta_1]$  with  $a > \theta_1 > \theta_0 > 0$ . For simplicity, we set the fixed cost of production to zero. The inverse demand function and the form of the cost function are known to the manager and the owner

of the monopoly as well as to the regulator. On the other hand, the marginal cost parameter  $\theta$  is privately known to the manager. However, the owner of the monopoly and the regulator have a prior belief about  $\theta$ , represented by the density function  $f$ , which is positive and continuous over its support  $[\theta_0, \theta_1]$ . Let  $F$  denote the cumulative distribution function associated with  $f$ . We assume that both the belief  $f$  and its support  $[\theta_0, \theta_1]$  are known to all three agents.

Given the described environment, the total value to consumers of an output of quantity  $q$  is  $V(q) = \int_0^q P(x)dx$ , and the consumer surplus is  $V(q) - P(q)q$ . On the other hand, the (operating) profits of the monopoly are  $P(q)q - C(q, \theta)$ . We assume that the manager of the monopoly is entitled to a fraction of the profits, i.e.,  $k [P(q)q - C(q, \theta)]$ , where  $k \in [0, 1]$  is determined by the owner. Accordingly, the rest of the profits, i.e.,  $(1 - k)[P(q)q - C(q, \theta)]$ , accrues to the owner. For simplicity, we assume that the manager does not earn any fixed salary.

To find the optimal regulatory mechanism, we restrict ourselves by the Revelation Principle [Dasgupta, Hammond and Maskin (1979), Myerson (1979), and Harris and Townsend (1981)] to direct mechanisms that require the manager of the monopoly to report his private cost parameter and that gives him no incentive to lie. Extending the direct mechanism of BM (1982), we consider a mechanism involving the outcome functions  $\langle r, p, q, s_m, s_o \rangle$ . For any cost report  $\hat{\theta}$  of the manager,  $p(\hat{\theta})$  and  $q(\hat{\theta})$  are the regulated price and quantity respectively,  $r(\hat{\theta})$  is the probability that the monopoly will be permitted to operate,  $s_m(\hat{\theta})$  and  $s_o(\hat{\theta})$  are the expected values of the subsidies the manager and the owner will receive conditional on the probability that the monopoly is permitted to operate. Here, we should note that in some manager-controlled monopolies, the owner may consist of many public shareholders, and because of some political concerns the regulatory agency may be unwilling to tax shareholders of the regulated monopoly. To account for such cases, our model will involve a binary valued parameter  $\gamma$ , which will take the value of 1 if the regulator can tax the owner and will take the value of 0 otherwise. The value of  $\gamma$  is exogenously given to each of the three agents in the regulatory environment.

We also assume that both the manager and the owner of the monopoly are risk neutral. If the manager reports the marginal cost parameter as  $\hat{\theta}$  when it is actually

$\theta$ , his compensation (or salary)  $S_m(\hat{\theta}, \theta)$  becomes

$$S_m(\hat{\theta}, \theta) = k \left[ p(\hat{\theta})q(\hat{\theta}) - \theta q(\hat{\theta}) \right] r(\hat{\theta}) + s_m(\hat{\theta}), \quad (3)$$

whereas the net earnings of the owner becomes

$$\pi_o(\hat{\theta}, \theta) = (1 - k) \left[ p(\hat{\theta})q(\hat{\theta}) - \theta q(\hat{\theta}) \right] r(\hat{\theta}) + \gamma s_o(\hat{\theta}). \quad (4)$$

A regulatory policy  $\langle r, p, q, s_m, s_o \rangle$  is called *feasible* if it satisfies the following conditions for all  $\theta \in [\theta_0, \theta_1]$ :

(i)  $r(\theta)$  satisfies

$$0 \leq r(\theta) \leq 1, \quad (5)$$

(ii)  $p(\theta)$  and  $q(\theta)$  are consistent with the inverse demand curve, i.e.,

$$p(\theta) = P(q(\theta)), \quad (6)$$

(iii) the regulatory policy is *individually rational* for both the manager and the owner of the monopoly under the truthful revelation of the manager, i.e.,

$$S_m(\theta) \equiv S_m(\theta, \theta) \geq 0, \quad (7)$$

$$\pi_o(\theta) \equiv \pi_o(\theta, \theta) \geq 0. \quad (8)$$

(iv) the regulatory policy is *incentive-compatible* for the manager, i.e.,

$$S_m(\theta, \theta) \geq S_m(\hat{\theta}, \theta), \quad \text{for all } \hat{\theta} \in [\theta_0, \theta_1]. \quad (9)$$

The aim of the regulator is to find a regulatory policy that satisfies the above feasibility conditions and that is optimal from the viewpoint of the society (or the regulator benevolently acting on behalf of it). Below, we will formally define the regulator's objective. Let  $\theta \in [\theta_0, \theta_1]$  be the private cost information of the manager. Given a feasible regulatory policy  $\langle r, p, q, s_m, s_o \rangle$ , the manager will truthfully report  $\theta$ , and the consumer welfare (consumer surplus net of the subsidies paid to the manager and the owner of the monopoly) will be given by

$$W^C(\theta) = [V(q(\theta)) - p(\theta)q(\theta)] r(\theta) - s_m(\theta) - \gamma s_o(\theta). \quad (10)$$

The social welfare  $W^S(\theta)$  is then defined to be the sum of the consumer welfare  $W^C(\theta)$  plus a fraction ( $\alpha \in [0, 1]$ ) of the sum of the welfares of the manager and the owner of the monopoly,  $\alpha[S_m(\theta) + \pi_o(\theta)]$ . Formally,

$$\begin{aligned} W^S(\theta) &= W^C(\theta) + \alpha [S_m(\theta) + \pi_o(\theta)] \\ &= [V(q(\theta)) - (1 - \alpha)(1 - k)p(\theta)q(\theta) - (\alpha + k(1 - \alpha))\theta q(\theta)]r(\theta) \\ &\quad - (1 - \alpha)S_m(\theta) - (1 - \alpha)\gamma s_o(\theta). \end{aligned} \tag{11}$$

The objective of the regulator is to choose a feasible regulatory policy that maximizes the expected value of  $W^S(\theta)$ , conditional on the prior beliefs  $f(\theta)$ . Formally, the regulator's problem is

$$\max_{r, p, q, s_m, s_o} \int_0^{\theta_1} W^S(\theta) f(\theta) d\theta \quad \text{subject to (5) - (9)}. \tag{12}$$

### 3 Results

Below, we will first characterize the optimal regulatory policy, or the solution to the problem in (12). The following assumptions will be helpful for our results.

**Assumption 1.**  $F(\theta)/f(\theta)$  is nondecreasing in  $\theta \in [\theta_0, \theta_1]$ .

**Assumption 2.**  $k(1 - \alpha)F(\theta_1)/f(\theta_1) < [1 - (1 - \alpha)(1 - k)(1 - \gamma)](a - \theta_1)$ .

The first assumption, which is standard in the mechanism design literature, will ensure that the marginal informational rent function will be nonincreasing at the optimal solution.<sup>3</sup> On the other hand, the second assumption will ensure that the optimal output will be positive at the least efficient production technology. Given Assumption 1, if the inequality in Assumption 2 holds for  $\theta_1$ , it will also be true for all  $\theta \in [\theta_0, \theta_1]$ . So, Assumptions 1 and 2 together will guarantee that the optimal output will always be positive.

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<sup>3</sup>We will use Assumption 1 to simplify the derivation of the optimal regulatory policy. BM (1982) characterizes the optimal policy for the case of  $k = 1$  without appealing to Assumption 1.

**Proposition 1.** *Let Assumptions 1 and 2 hold and let  $(1 - \alpha)(1 - k)(1 - \gamma) < 1/2$ . Then, the solution to the regulator's problem in (12) is given by the optimal policy  $\langle \bar{r}, \bar{p}, \bar{q}, \bar{s}_m, \bar{s}_o \rangle$  satisfying equations (13)-(20) for all  $\theta \in [\theta_0, \theta_1]$ :*

$$\bar{p}(\theta) = \max\{\theta, \bar{z}(\theta)\} \quad (13)$$

$$\bar{z}(\theta) = \frac{(1 - M)\theta + k(1 - \alpha)\frac{F(\theta)}{f(\theta)} - Ma}{1 - 2M} \quad (14)$$

$$M = (1 - \alpha)(1 - k)(1 - \gamma) \quad (15)$$

$$\bar{p}(\theta) = P(\bar{q}(\theta)) \quad (16)$$

$$\bar{r}(\theta) = \begin{cases} 1 & \text{if } \bar{\Gamma}(\theta) \geq 0 \\ 0 & \text{otherwise.} \end{cases} \quad (17)$$

$$\bar{\Gamma}(\theta) = V(\bar{q}(\theta)) - M\bar{p}(\theta)\bar{q}(\theta) - [(1 - 2M)\bar{z}(\theta) + Ma]\bar{q}(\theta) \quad (18)$$

$$\bar{s}_m(\theta) = k[\theta\bar{q}(\theta) - \bar{p}(\theta)\bar{q}(\theta)]\bar{r}(\theta) + k\int_{\theta}^{\theta_1} \bar{r}(x)\bar{q}(x)dx \quad (19)$$

$$\bar{s}_o(\theta) = (1 - k)[\theta\bar{q}(\theta) - \bar{p}(\theta)\bar{q}(\theta)]\bar{r}(\theta) \quad (20)$$

**Proof.** We will mimic the proof of the main theorem in Baron and Myerson (1982). The incentive compatibility condition (9) requires that  $r(\theta)q(\theta)$  is nonincreasing, and also that  $S_m(\theta) \equiv S_m(\theta, \theta) = \max_{\hat{\theta} \in [\theta_0, \theta_1]} S(\hat{\theta}, \theta)$ , implying  $S'_m(\theta) = -kq(\theta)r(\theta)$  by the envelope function theorem. Thus,

$$S_m(\theta) = S_m(\theta_1) + k\int_{\theta}^{\theta_1} r(x)q(x)dx. \quad (21)$$

Using integration by parts, we can then calculate

$$\int_{\theta_0}^{\theta_1} S_m(\theta)f(\theta)d\theta = S_m(\theta_1) + k\int_{\theta_0}^{\theta_1} r(\theta)q(\theta)F(\theta)d\theta. \quad (22)$$

On the other hand, from equation (11) and the constraint (8) it follows that the expected social welfare is maximized only if

$$s_o(\theta) = (1 - k)[\theta q(\theta) - p(\theta)q(\theta)]r(\theta), \quad (23)$$

implying  $\pi_o(\theta) = 0$  if  $\gamma = 1$  and  $\pi_o(\theta) = (1 - k)[p(\theta)q(\theta) - \theta q(\theta)]r(\theta)$  if  $\gamma = 0$ . Inserting (22) and (23) into  $\int_{\theta_0}^{\theta_1} W^S(\theta)f(\theta)d\theta$  yields

$$\begin{aligned} & \int_{\theta_0}^{\theta_1} \left[ \left( V(q(\theta)) - (1 - \alpha)(1 - k)(1 - \gamma)p(\theta)q(\theta) \right. \right. \\ & \left. \left. - \left( \alpha + (1 - \alpha)(k + (1 - k)\gamma) \right) \theta q(\theta) \right) - k(1 - \alpha) \frac{F(\theta)}{f(\theta)} q(\theta) \right] r(\theta) f(\theta) d\theta \\ & - (1 - \alpha) S_m(\theta_1). \end{aligned} \quad (24)$$

Maximizing the expected social welfare in (24) requires maximizing the integrand for all  $\theta$ . The first-order condition for the unconstrained optimum would yield  $\bar{p}(\theta) = \bar{z}(\theta)$ , with  $\bar{z}(\theta)$  satisfying (14), given (15). However, the individual rationality condition (8) also requires  $\bar{p}(\theta) \geq \theta$ . Since  $\bar{z}(\theta) \geq \theta$  may not always hold when  $k < 1$ , we must have  $\bar{p}(\theta) = \max\{\theta, \bar{z}(\theta)\}$ , implying (13). Equation (16) follows from (6). Under the individual rationality condition (7), maximizing the integral (24) also implies that  $S_m(\theta_1) = 0$ . (In fact, for  $\alpha = 1$ , any nonnegative value of  $S_m(\theta_1)$  can be optimal; however, for simplicity we set it to zero.) Using the optimal functions  $\bar{p}(\cdot)$  and  $\bar{q}(\cdot)$  that are given by (13)-(16) and also the optimality condition  $S_m(\theta_1) = 0$ , the expected social welfare in (24) can be rewritten as

$$\int_{\theta_0}^{\theta_1} \left[ V(\bar{q}(\theta)) - M\bar{p}(\theta)\bar{q}(\theta) - [(1 - 2M)\bar{z}(\theta) + Ma]\bar{q}(\theta) \right] r(\theta) f(\theta). \quad (25)$$

The probability  $r(\theta)$  must be equal to one if the expression inside the large brackets in the above equation is nonnegative and zero otherwise, implying  $\bar{r}(\theta)$  as given by (17) and (18). It is easy to check that  $\bar{z}(\theta)$  is increasing,  $\bar{p}(\theta)$  is nondecreasing, and  $\bar{q}(\theta)$  is nonincreasing, in  $\theta$ , thanks to Assumption 1. Note also that if  $\bar{p}(\theta) = \theta$ , then it is easy to check in (18) that

$$\bar{\Gamma}(\theta) = (1 - 2M) \left( \frac{a^2 - \theta^2}{2} - \bar{z}(\theta) \right), \quad (26)$$

implying

$$\bar{\Gamma}'(\theta) = (1 - 2M) [-\theta - \bar{z}'(\theta)] < 0, \quad (27)$$

since  $\bar{z}'(\theta) > 0$ . On the other hand, if  $\bar{p}(\theta) = \bar{z}(\theta)$ , then

$$\bar{\Gamma}(\theta) = \frac{a^2 - \bar{z}^2(\theta)}{2} - [(1 - M)\bar{z}(\theta) + Ma](a - \bar{z}(\theta)), \quad (28)$$

implying

$$\bar{\Gamma}'(\theta) = -(1 - 2M)(a - \bar{z}(\theta))\bar{z}'(\theta) < 0, \quad (29)$$

since  $a - \bar{z}(\theta) > 0$  by Assumptions 1 and 2 and also  $\bar{z}'(\theta) > 0$  by Assumption 1. Therefore,  $\bar{r}(\theta)$  is always nonincreasing. Equation (19) follows from the fact that the manager's regulated salary  $\bar{S}_m(\theta)$  is equal to  $k \int_{\theta}^{\theta_1} \bar{r}(x)\bar{q}(x)dx = k [\bar{p}(\theta)\bar{q}(\theta) - C(\bar{q}(\theta), \theta)]\bar{r}(\theta) + \bar{s}_m(\theta)$ . On the other hand, (20) follows from (23) calculated at the optimal policy functions  $\bar{p}(\theta)$ ,  $\bar{q}(\theta)$ , and  $\bar{r}(\theta)$ . Finally note that, (13)-(20) satisfy the feasibility conditions (5)-(9).  $\square$

**Corollary 1.** *Let Assumptions 1 and 2 hold and  $k = 1$ , i.e., the whole of the profits are received by the manager. Then, the optimal levels of price, output, information rent, and the social welfare implied by (13)-(20) at each cost level become identical to those in the BM model (1982), where the monopoly is managed by its owner.*

When  $\alpha = 1$ , the optimal policy in (13)-(20) would reduce to  $\bar{p}(\theta) = \bar{z}(\theta) = \theta$ ,  $\bar{q}(\theta) = a - \theta$ ,  $\bar{r}(\theta) = 1$ ,  $\bar{s}_m(\theta) = k \int_{\theta}^{\theta_1} \bar{q}(x)dx$ , and  $\bar{s}_o(\theta) = 0$ . As the optimal price and output policies obtained under  $\alpha = 1$  are independent of  $k$ , we will restrict ourselves in the rest of this paper to the more interesting case of  $\alpha < 1$ .

We also observe from (15) that when owner of a manager-controlled monopoly can be taxed by the regulator ( $\gamma = 1$ ), the parameter  $M = (1 - \alpha)(1 - k)(1 - \gamma)$  would become zero, and the assumption  $M < 1/2$  in Proposition 1 would hold for all  $k$  and  $\alpha$ . Hence, we can claim the following.

**Corollary 2.** *Let Assumptions 1 and 2 hold,  $\gamma = 1$ , and  $\alpha \in [0, 1)$ . Then, for all  $\theta \in [\theta_0, \theta_1]$ , we have*

$$\bar{p}(\theta) = \bar{z}(\theta) = \theta + k(1 - \alpha)F(\theta)/f(\theta), \quad (30)$$

implying that  $\bar{p}(\theta)$  is increasing and  $\bar{q}(\theta)$  is decreasing in  $k$  for all  $\theta \in (\theta_0, \theta_1]$ . Moreover,  $\bar{r}(\theta) = 1$  for all  $k \in [0, 1]$  and  $\theta \in [\theta_0, \theta_1]$ .

**Proof.** By the stated assumptions, the optimal policy is given by (13)-(20). Let  $\gamma = 1$ . Then,  $M = 0$  from (15). Pick any  $\theta \in [\theta_0, \theta_1]$ . Inserting  $M = 0$  into (14) yields  $\bar{z}(\theta) = \theta + k(1 - \alpha)F(\theta)/f(\theta)$ . Clearly,  $\bar{z}(\theta) \geq \theta$ , implying  $\bar{p}(\theta) = \bar{z}(\theta)$ . Moreover,  $\bar{p}(\theta)$  is increasing and  $\bar{q}(\theta)$  is decreasing in  $k$  if  $\theta > \theta_0$ . Finally, using  $M = 0$  and  $\bar{p}(\theta) = \bar{z}(\theta)$ , we obtain  $\bar{\Gamma}(\theta) = [a - \bar{z}(\theta)]^2/2 > 0$ . Then, (17) implies  $\bar{r}(\theta) = 1$ .  $\square$

The above result implies that in regulatory environments where legal settings allow the regulator to tax the owner of the monopoly, the optimal price (output) in our model is lower (higher) than the optimal price (output) in the BM model, as long as  $k < 1$ . Below, we examine the effect of  $k$  of the optimal regulatory policy when the owner of the monopoly cannot be taxed.

**Corollary 3.** *Let Assumptions 1 and 2 hold,  $\gamma = 0$ , and  $\alpha \in (1/2, 1)$ . Then,  $\bar{z}(\theta)$  is increasing,  $\bar{p}(\theta)$  is nondecreasing, and  $\bar{q}(\theta)$  is nonincreasing in  $k$  for all  $\theta \in [\theta_0, \theta_1]$ .*

**Proof.** Since  $\gamma = 0$  and  $\alpha > 1/2$ , we have  $M = (1 - \alpha)(1 - k) < 1/2$ . Therefore all of the assumptions of Proposition 1 are satisfied thanks to the stated assumptions of the corollary, implying that the optimal policy is given by (13)-(20). Now, pick any  $\theta \in [\theta_0, \theta_1]$ . Differentiating (14) with respect to  $k$  yields

$$\bar{z}_k(\theta) = \frac{(1 - \alpha) \left( a - \theta + (2\alpha - 1) \frac{F(\theta)}{f(\theta)} \right)}{(1 - 2M)^2}. \quad (31)$$

Clearly,  $\bar{z}_k(\theta) > 0$  since  $a > \theta_1 \geq \theta$  and  $\alpha > 1/2$  by assumption. Then, from (13) it follows that  $\bar{p}(\theta)$  is nondecreasing in  $k$ , and from (1) and (16) it follows that  $\bar{q}(\theta)$  is nonincreasing in  $k$ .  $\square$

Corollary 3 implies that the regulated price (output) of the manager-controlled monopoly is never above (below) that of the owner-controlled monopoly in the BM model, where  $k = 1$ . It is of interest to also see whether the optimal price given

by (13) can always be equal to the marginal cost of production. Calculating the difference

$$\bar{z}(\theta) - \theta = \frac{M(\theta - a) + k(1 - \alpha)\frac{F(\theta)}{f(\theta)}}{1 - 2M}, \quad (32)$$

we observe that the denominator of the ratio on the right hand side is positive when  $\alpha \in (1/2, 1)$ . On the other hand, in the nominator the first term is always nonpositive since  $\theta \leq \theta_1 < a$  by assumption, while the second term is always nonnegative. So, the sign of  $\bar{z}(\theta) - \theta$  is in general ambiguous, depending on the parameters in the above equation. However, given the other parameters, the parameter  $k$  may help us to predict the sign of  $\bar{z}(\theta) - \theta$  as will be shown.

Let us denote by  $k^*(\theta)$  the level of  $k$  that solves  $\bar{z}(\theta) - \theta = 0$ . Equating the right hand side of (32) to zero, we can solve for

$$k^*(\theta) = \begin{cases} \frac{(\alpha - \theta)}{(\alpha - \theta) + \frac{F(\theta)}{f(\theta)}} & \text{if } \gamma = 0, \\ 0 & \text{if } \gamma = 1. \end{cases} \quad (33)$$

This leads us to the following result.

**Corollary 4.** *Let Assumptions 1 and 2 hold, and  $\alpha \in (1/2, 1)$ . Then, for all  $\theta \in [\theta_0, \theta_1]$ , the optimal price satisfies*

$$\bar{p}(\theta) = \begin{cases} \theta & \text{if } k \in [0, k^*(\theta)], \\ \bar{z}(\theta) & \text{if } k \in (k^*(\theta), 1]. \end{cases} \quad (34)$$

**Proof.** By the stated assumptions, the optimal policy is given by (13)-(20). Equation (32) implies that

$$\bar{z}(\theta_0) - \theta_0 = \frac{M(\theta_0 - a)}{1 - 2M} \leq 0. \quad (35)$$

So,  $\bar{p}(\theta_0) = \theta_0$  for all  $k \in [0, 1]$ , implying that  $\bar{p}(\theta_0)$  satisfies (34) for  $k^*(\theta_0) = 1$ . Now, pick any  $\theta \in (\theta_0, \theta_1]$ . Since  $k^*(\theta)$  in (33) solves  $\bar{z}(\theta) = \theta$ , to show that the optimal price at  $\theta$  satisfies (34), it will be sufficient to prove that  $\bar{z}_k(\theta) > 0$ . Note that if  $\gamma = 1$ , then  $M = 0$  and  $\bar{z}(\theta) = \theta + k(1 - \alpha)F(\theta)/f(\theta)$ , implying

$\bar{z}_k(\theta) = (1 - \alpha)F(\theta)/f(\theta) > 0$ . On the other hand, if  $\gamma = 0$ , then  $\bar{z}_k(\theta) > 0$  by (31).  
 $\square$

Equation (33) suggests that when  $\gamma = 0$ , we have  $0 < k^*(\theta) < 1$  for all  $\theta > \theta_0$ . Thus, the optimal regulatory policy, as illustrated in Figure 1, recommends marginal cost pricing if the parameter  $k$  is sufficiently small and recommends a positive markup if the parameter  $k$  is sufficiently large. Moreover, we observe that when  $\gamma = 0$ , the breaking point  $k^*(\theta)$  is decreasing in  $\theta$  due to Assumption 1.

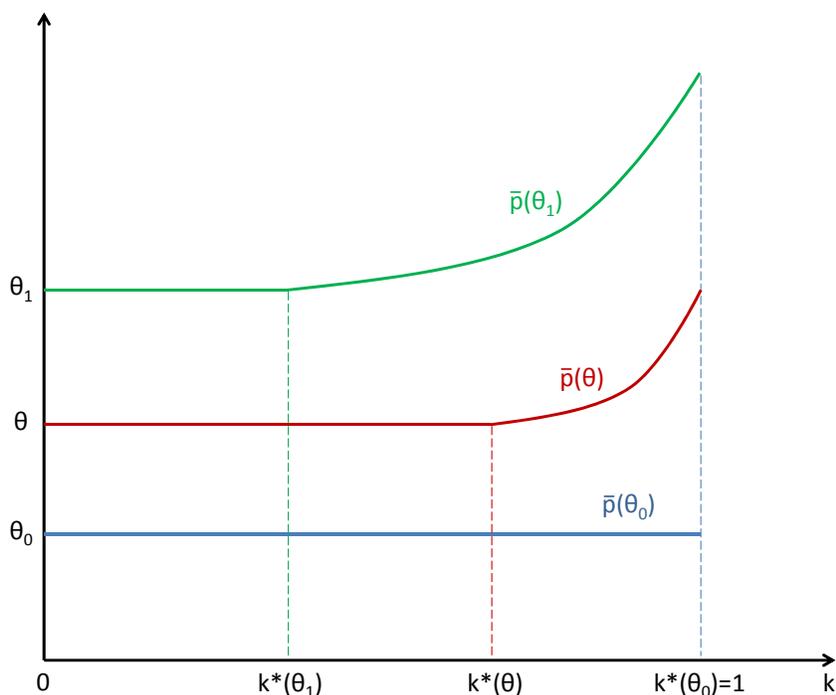


Figure 1. Optimal Regulatory Price as a Function of  $k$

We will now examine how the welfares of different parties in the society change with respect to the parameter  $k$ . Given the optimal regulatory policy (13)-(20), let  $\bar{\pi}_o(\theta)$  and  $\bar{S}_m(\theta)$  denote the regulated net earnings of the owner of the monopoly and the regulated salary of the manager respectively, and let  $\bar{W}^S(\theta)$  denote the regulated

social welfare. When  $\gamma = 1$ , all the earnings of the owner of the monopoly is taxed by the regulator so that  $\bar{\pi}_o(\theta) = 0$  for all  $\theta \in [\theta_0, \theta_1]$ . In that case,  $k$  has no effect on  $\bar{\pi}_o(\cdot)$ . On the other hand, when the regulator is not allowed to tax the owner of the monopoly, i.e.,  $\gamma = 0$ , we have  $\bar{\pi}_o(\theta) = (1 - k)[\bar{p}(\theta)\bar{q}(\theta) - \theta\bar{q}(\theta)]\bar{r}(\theta)$  and differentiating it with respect to  $k$  yields

$$\begin{aligned} \frac{\partial \bar{\pi}_o(\theta)}{\partial k} &= -[\bar{p}(\theta)\bar{q}(\theta) - \theta\bar{q}(\theta)]\bar{r}(\theta) + (1 - k)[\bar{p}(\theta)\bar{q}(\theta) - \theta\bar{q}(\theta)]\bar{r}_k(\theta) \\ &\quad + (1 - k)[\bar{p}_k(\theta)\bar{q}(\theta) + (\bar{p}(\theta) - \theta)\bar{q}_k(\theta)]\bar{r}(\theta). \end{aligned} \quad (36)$$

In the above equation, the first term on the right hand side, the direct effect of  $k$ , is nonpositive, whereas the sign of the second and third terms, constituting indirect effects, are ambiguous. Therefore, the net effect of  $k$  on  $\bar{\pi}_o(\theta)$  is in general indeterminate. However, we are still able to show below that the optimal value of  $k$  for the monopoly owner cannot be zero, i.e., the full ownership of profits can never be optimal for the owner of the regulated monopoly when  $\gamma = 0$ . (Hereafter, for any function  $x(\cdot)$  in our model and for any  $\theta \in [\theta_0, \theta_1]$  and any  $k' \in [0, 1]$ ,  $x(\theta|k = k')$  will denote the value of  $x$  at  $\theta$  when  $k = k'$ .)

**Proposition 2.** *Let Assumptions 1 and 2 hold,  $\gamma = 0$ , and  $\alpha \in (1/2, 1)$ . Pick any  $\theta \in (\theta_0, \theta_1]$ . If  $\hat{k}(\theta) = \operatorname{argmax}_{k \in [0, 1]} \bar{\pi}_o(\theta)$ , then  $\hat{k}(\theta) \in (0, 1)$ ,  $\bar{p}(\theta|k = \hat{k}(\theta)) > \theta$ , and  $\bar{r}(\theta|k = \hat{k}(\theta)) = 1$ .*

**Proof.** By the assumptions stated above, the optimal regulatory policy is given by (13)-(20). Pick any  $\theta \in (\theta_0, \theta_1]$ . Note that  $\bar{\pi}_o(\theta) = (1 - k)[\bar{p}(\theta)\bar{q}(\theta) - \theta\bar{q}(\theta)]\bar{r}(\theta)$ . Equations (14) – (15) imply that  $\bar{z}(\theta|k = 0) - \theta = M(\theta - a)/(1 - 2M) \leq 0$ . So,  $\bar{p}(\theta|k = 0) = \theta$ , implying  $\bar{\pi}_o(\theta|k = 0) = 0$ . It is also clear that  $\bar{\pi}_o(\theta|k = 1) = 0$ . We also have  $\bar{p}(\theta|k = 1) = \theta + (1 - \alpha)F(\theta)/f(\theta) > \theta$ , and  $\bar{r}(\theta|k = 1) = 1$  since  $\bar{\Gamma}(\theta) = (a - \bar{z}(\theta))^2/2 \geq 0$ . Since  $\bar{p}(\theta)$  and  $\bar{r}(\theta)$  are continuous in  $k$ , there exists  $k' \in (0, 1)$  such that  $\bar{p}(\theta|k = k') > \theta$  and  $\bar{r}(\theta|k = k') = 1$ , implying  $\bar{\pi}_o(\theta|k = k') > 0$ . Thus, if  $\hat{k}(\theta) = \operatorname{argmax}_{k \in [0, 1]} \bar{\pi}_o(\theta)$ , then  $\hat{k}(\theta) \in (0, 1)$ ,  $\bar{p}(\theta|k = \hat{k}(\theta)) > \theta$ , and  $\bar{r}(\theta|k = \hat{k}(\theta)) = 1$ .  $\square$

The above proposition will be helpful in Section 4, where the owner of the

monopoly will be assumed to know the marginal cost of production and will choose the compensation parameter that maximizes her own net gains. On the other hand, in this section, the marginal cost information is unknown to the monopoly owner. Therefore, the compensation parameter chosen by the monopoly owner when  $\gamma = 0$  should maximize the *expected* value of her net gains.

**Proposition 3.** *Let Assumptions 1 and 2 hold,  $\gamma = 0$ , and  $\alpha \in (1/2, 1)$ . If  $\hat{k} = \operatorname{argmax}_{k \in [0,1]} \int_{\theta_0}^{\theta_1} \bar{\pi}_o(\theta) f(\theta) d\theta$ , then  $\hat{k} \in (0, 1)$ .*

**Proof.** By the assumptions stated above, the optimal regulatory policy is given by (13)-(20). Note that  $\int_{\theta_0}^{\theta_1} \bar{\pi}_o(\theta) f(\theta) d\theta = (1 - k) \int_{\theta_0}^{\theta_1} [\bar{p}(\theta) \bar{q}(\theta) - \theta \bar{q}(\theta)] \bar{r}(\theta) f(\theta) d\theta$ . Equations (14) – (15) imply that for all  $\theta \in [\theta_0, \theta_1]$  we have  $\bar{z}(\theta|k = 0) - \theta = M(\theta - a)/(1 - 2M) \leq 0$ , implying  $\bar{p}(\theta|k = 0) = \theta$ . Therefore,  $\int_{\theta_0}^{\theta_1} \bar{\pi}_o(\theta|k = 0) f(\theta) d\theta = 0$ . On the other hand,  $\int_{\theta_0}^{\theta_1} \bar{\pi}_o(\theta|k = 1) f(\theta) d\theta = 0$ , trivially. Now pick any  $\theta \in (\theta_0, \theta_1]$ . We have  $M = 0$  when  $k = 1$ , implying  $\bar{p}(\theta|k = 1) = \theta + (1 - \alpha)F(\theta)/f(\theta) > \theta$  by (13) and (14). Moreover, from (18) it follows that

$$\bar{\Gamma}(\theta|k = 1) = \frac{(a - \bar{z}(\theta))^2}{2} \geq 0. \quad (37)$$

So,  $\bar{r}(\theta|k = 1) = 1$  by (17). Since  $\bar{p}(\theta)$  and  $\bar{r}(\theta)$  are continuous in  $k$ , there exists  $k' \in (0, 1)$  such that for all  $\theta \in (\theta_0, \theta_1]$  and for all  $k'' \in (k', 1)$ , we have  $\bar{\pi}_o(\theta|k = k'') > 0$ , implying  $\int_{\theta_0}^{\theta_1} \bar{\pi}_o(\theta|k = k'') f(\theta) d\theta > 0$ . Thus, we have established that if  $\hat{k} = \operatorname{argmax}_{k \in [0,1]} \int_{\theta_0}^{\theta_1} \bar{\pi}_o(\theta) f(\theta) d\theta$ , then  $\hat{k} \in (0, 1)$ .  $\square$

Proposition 3 shows that the optimal managerial compensation chosen by the owner of the monopoly, when she does not know the marginal cost of production, cannot be arbitrarily small or arbitrarily large if  $\gamma = 0$ . Now, in order to find the effect of  $k$  on the manager's regulated salary, we differentiate  $\bar{S}_m(\theta) = k \int_{\theta}^{\theta_1} \bar{q}(x) \bar{r}(x) dx$  with respect to  $k$  and obtain

$$\frac{\partial \bar{S}_m(\theta)}{\partial k} = \int_{\theta}^{\theta_1} \bar{r}(x) \bar{q}(x) dx + k \int_{\theta}^{\theta_1} [\bar{r}_k(x) \bar{q}(x) + \bar{r}(x) \bar{q}_k(x)] dx. \quad (38)$$

The first integral, accounting for the direct effect of  $k$  on  $\bar{S}_m(\theta)$ , is always nonnegative. In fact, for all  $\theta \in [\theta_0, \theta_1]$  we have  $\bar{p}(\theta|k = 0) = \theta$  and it is easy to check that

$\bar{r}(\theta|k = 0) = 1$ , implying  $\int_{\theta}^{\theta_1} \bar{q}(x|k = 0)\bar{r}(x|k = 0)dx > 0$ . Besides, the second integral in (38) vanishes when  $k = 0$ , implying  $\partial\bar{S}_m(\theta)/\partial k|_{k=0} > 0$ . On the other hand, the sign of the second integral in (38), which becomes relevant when  $k > 0$ , is ambiguous. Overall, the net effect of  $k$  on  $\bar{S}_m(\theta)$  is indeterminate when  $k > 0$ . Although it is clear that the manager's regulated salary  $\bar{S}_m$  attains its minimum (the value of zero) when  $k = 0$ , it is not determinate whether or not the manager prefers the full ownership of profits ( $k = 1$ ) to the partial ownership ( $k < 1$ ).

Since the manager's salary  $\bar{S}_m(\theta)$  explicitly enters into the definition of the *actual* social welfare in (11), the effect of  $k$  on the actual social welfare is also indeterminate. However, we below show that the manager's compensation parameter  $k$  has a negative impact on the *expected* social welfare if  $\gamma = 1$ .

**Proposition 4.** *Let Assumptions 1 and 2 hold,  $\gamma = 1$ , and  $\alpha \in (1/2, 1)$ . Then,  $\int_{\theta_0}^{\theta_1} \bar{W}^S(\theta)f(\theta)d\theta$  is decreasing in  $k \in [0, 1]$ .*

**Proof.** By the assumptions stated above, the optimal regulatory policy is given by (13)-(20). Pick any  $\theta \in (\theta_0, \theta_1]$ . Note that with  $\gamma = 1$ , we have  $M = 0$ , and equations (13) and (14) would imply  $\bar{p}(\theta) = \bar{z}(\theta) = \theta + k(1 - \alpha)F(\theta)/f(\theta) > \theta$ . Moreover, from (18) it follows that

$$\bar{\Gamma}(\theta) = \frac{(a - \bar{z}(\theta))^2}{2} \geq 0. \quad (39)$$

So,  $\bar{r}(\theta) = 1$  by (17). Thus, the expected social welfare in (25) reduces to

$$\int_{\theta_0}^{\theta_1} \bar{W}^S(\theta)f(\theta)d\theta = \int_{\theta_0}^{\theta_1} \bar{\Gamma}(\theta)f(\theta)d\theta. \quad (40)$$

Note that

$$\bar{\Gamma}_k(\theta) = -2[a - \bar{z}(\theta)]\bar{z}_k(\theta) < 0 \quad (41)$$

by Assumptions 1 and 2 and Corollary 2. Therefore,  $\int_{\theta_0}^{\theta_1} \bar{W}^S(\theta)f(\theta)d\theta$  is decreasing in  $k \in [0, 1]$ .  $\square$

The above result directly reveals what the socially optimal size of the managerial compensation is when  $\gamma = 1$ .

**Corollary 5.** *Let Assumptions 1 and 2 hold,  $\gamma = 1$ , and  $\alpha \in (1/2, 1)$ . Then, the expected social welfare attains its maximum when all of the profits is received by the owner of the monopoly ( $k = 0$ ).*

Below, we will investigate the effect of  $k$  on the expected social welfare when the regulator cannot tax net earnings of the owner ( $\gamma = 0$ ).

**Assumption 3.**  $F(\theta_1)/f(\theta_1) < (a + \theta_1)(a - \theta_1 - 1)$ .

Note that when  $f$  is the uniform distribution, we have  $F(\theta)/f(\theta) = \theta - \theta_0$  for all  $\theta \in [\theta_0, \theta_1]$ , and Assumption 3 is satisfied if and only if  $(\theta_1 - \theta_0) + \theta_1(\theta_1 + 1) < a(a - 1)$ . In fact, for any belief  $f$  it is always true to say that Assumption 3 would hold when the demand parameter  $a$  is sufficiently high.

**Proposition 5.** *Let Assumptions 1-3 hold,  $\gamma = 0$ , and  $\alpha \in (1/2, 1)$ . Then  $\int_{\theta_0}^{\theta_1} \bar{W}^S(\theta) f(\theta) d\theta$  is increasing in  $k$  over  $[0, k^*(\theta_1))$  where  $k^*(\theta_1) = (a - \theta_1)/[(a - \theta_1) + F(\theta_1)/f(\theta_1)]$ .*

**Proof.** By the assumptions stated above, the optimal regulatory policy is given by (13)-(20). Note also from (25) that the expected social welfare is equal to  $\int_{\theta_0}^{\theta_1} \bar{\Gamma}(\theta) \bar{r}(\theta) f(\theta)$ . Let  $k \in [0, k^*(\theta_1))$  and  $\gamma = 0$ , and pick any  $\theta \in [\theta_0, \theta_1]$ . We must have  $\bar{p}(\theta) = \theta$  since by equation (33) we have  $k^*(\theta) > k^*(\theta_1)$ , thanks to Assumption 1. Therefore, from (18) we have

$$\bar{\Gamma}(\theta) = (1 - 2M) \left( \frac{a^2 - \theta^2}{2} - \bar{z}(\theta) \right). \quad (42)$$

It follows that

$$\bar{\Gamma}_k(\theta) = -(1 - 2M) \bar{z}_k(\theta) + (1 - \alpha) (a^2 - \theta^2 - 2\bar{z}(\theta)). \quad (43)$$

Inserting (31) into the above equation yields

$$\bar{\Gamma}_k(\theta) = (1 - \alpha) \left( a^2 - \theta^2 - a - \theta - \frac{F(\theta)}{f(\theta)} \right). \quad (44)$$

We have  $\bar{\Gamma}_k(\theta_1) > 0$  by Assumption 3. Moreover,  $\bar{\Gamma}_{k\theta}(\theta) < 0$  by Assumption 1. Therefore, it is true that  $\bar{\Gamma}_k(\theta) > 0$ . So, the expected social welfare is increasing in  $k$  over  $[0, k^*(\theta_1))$ .  $\square$

The effect of  $k$  on the expected social welfare is ambiguous over the interval  $[k^*(\theta_1), 1]$ . However, the above result is sufficient to directly imply that when  $\gamma = 0$ , the expected social welfare cannot be maximized by the choice of  $k = 0$ , unlike in the case of  $\gamma = 1$ .

**Corollary 6.** *Let Assumptions 1-3 hold,  $\gamma = 0$ , and  $\alpha \in (1/2, 1)$ . Then, the expected social welfare attains its maximum for some  $k \in [k^*(\theta_1), 1]$ , where  $k^*(\theta_1) = (a - \theta_1) / [(a - \theta_1) + F(\theta_1)/f(\theta_1)] \in (0, 1)$ .*

Up to now, we have seen how the optimal regulatory policy and the induced welfares in the regulatory environment are affected by the value of the managerial compensation parameter  $k$ . Next, we will deal with the question whether the owner of the monopoly would prefer to separate management from ownership when he knows the value of the marginal cost parameter.

## 4 Should the Owner of a Regulated Monopoly Leave the Control to a Manager?

Reconsider the regulatory environment introduced in Section 2 with the following modification. The marginal cost parameter of the monopoly, which remains to be unknown to the regulator, is now known to the owner of the monopoly (as well as to the manager if there is any). However, the regulator's belief as to whether the owner of the monopoly knows the cost information depends on the existence of a manager. If the monopoly has no manager, then (we assume) the regulator will believe that

the marginal cost parameter is privately known to the owner. On the other hand, if the monopoly employs a manager, then the regulator will believe that the cost information is privately held by the manager. Given this modified environment, the owner of the monopoly has to decide whether he should hire, and leave the control to, a manager before the monopoly is optimally regulated.

Clearly, if the monopoly employs no manager, then the regulatory policy will elicit the marginal cost information from the monopoly owner. In this case, the regulated welfare of the monopoly owner with the marginal cost  $\theta$  will be equal to that in the BM case, which corresponds to  $\bar{S}_m(\theta|k = 1)$  in our model. On the other hand, if the monopoly is controlled by a manager, then the regulatory policy will elicit the marginal cost information from the manager. In that case, the owner is expected to choose the managerial compensation parameter as  $\hat{k}(\theta) = \operatorname{argmax}_{k \in [0,1]} \bar{\pi}_0(\theta)$  to maximize her net gain. Consequently, the welfare of the owner will be  $\bar{\pi}_0(\theta|k = \hat{k}(\theta))$ . It will be optimal for the owner of the monopoly to hire a manager (with the compensation parameter  $\hat{k}(\theta)$ ) if and only if  $\bar{\pi}_0^*(\theta|k = k^*(\theta)) \geq \bar{S}_m(\theta|k = 1)$ .

**Proposition 6.** *Let Assumptions 1 and 2 hold and  $\alpha \in (1/2, 1)$ . Then, under the regulatory policy (13)-(20), the monopoly owner (i) strictly prefers to let the control to a manager if  $\gamma = 0$  and the marginal cost of production is sufficiently large, (ii) strictly prefers to control the monopoly herself if the marginal cost of production is sufficiently small.*

**Proof.** By the assumptions stated above, the optimal regulatory policy is given by (13)-(20). First, let  $\gamma = 1$ . Then  $\bar{\pi}_o(\theta) = 0$  for all  $\theta \in [\theta_0, \theta_1]$ .

Now, let  $\gamma = 0$ . If the monopoly is controlled by the owner, then  $k = 1$ , otherwise  $k = \hat{k}(\theta_1) \equiv \operatorname{argmax}_{k \in [0,1]} \bar{\pi}_o(\theta)$ . Below, will calculate the net gain of the monopoly owner at  $k = 1$  and  $k = \hat{k}(\theta)$ .

First consider the case of  $\theta = \theta_1$ . Trivially,  $\bar{S}_m(\theta_1|k = 1) = \int_{\theta_1}^{\theta_1} \bar{r}(x|k = 1)\bar{q}(x|k = 1)dx = 0$ . On the other hand, from Proposition 2,  $\hat{k}(\theta_1) \in (0, 1)$ ,  $\bar{p}(\theta_1|\hat{k}(\theta_1)) > \theta_1$ , and  $\bar{r}(\theta_1|\hat{k}(\theta_1)) = 1$ , implying  $\bar{\pi}_o(\theta_1|k = \hat{k}(\theta_1)) > 0$ .

Now, consider the case of  $\theta = \theta_0$ . Equations (13)-(15) imply that for all  $\theta \in$

$(\theta_0, \theta_1]$ , we have  $\bar{p}(\theta|k=1) = \bar{z}(\theta|k=1) = \theta + (1-\alpha)F(\theta)/f(\theta)$ , implying  $\bar{q}(\theta|k=1) = a - \bar{z}(\theta|k=1) > 0$  by Assumptions 1 and 2. Moreover, for all  $\theta \in (\theta_0, \theta_1]$ , we have  $\bar{r}(\theta|k=1) = 1$  by (17), since  $\bar{\Gamma}(\theta|k=1) = (a - \bar{z}(\theta))^2/2 \geq 0$  by (18). Therefore,  $\bar{S}_m(\theta_0|k=1) = \int_{\theta_0}^{\theta_1} \bar{r}(x|k=1)\bar{q}(x|k=1)dx > 0$ . On the other hand, we have

$$\bar{z}(\theta_0|k=\hat{k}(\theta_0)) - \theta_0 = \frac{(1-\alpha)(1-\hat{k}(\theta_0))(\theta_0-a)}{1-2(1-\alpha)(1-\hat{k}(\theta_0))} < 0, \quad (45)$$

due to the assumptions that  $\alpha \in (1/2, 1)$  and  $\theta_0 < \theta_1 < a$  and the fact that  $\hat{k}(\theta_0) \in (0, 1)$  by Proposition 2. Thus,  $\bar{p}(\theta_0) = \theta_0$ , implying  $\bar{\pi}_o(\theta_0|k=\hat{k}(\theta_0)) = 0$ .

Above, we have established that

$$\bar{\pi}_o(\theta_1|k=\hat{k}(\theta_1)) > 0, \quad \bar{S}_m(\theta_1|k=1) = 0$$

and

$$\bar{\pi}_o(\theta_0|k=\hat{k}(\theta_0)) = 0, \quad \bar{S}_m(\theta_0|k=1) > 0.$$

Because both  $\bar{S}_m(\theta|k=1)$  and  $\bar{\pi}_o(\theta|k=k^*(\theta))$  are continuous in  $\theta$ , we have two conclusions:

*Conclusion 1:* When  $\gamma = 0$ , there exists  $\bar{\theta} \in (\theta_0, \theta_1)$  such that for all  $\theta \in [\bar{\theta}, \theta_1]$  we have  $\bar{\pi}_o(\theta|k=\hat{k}(\theta)) > \bar{S}_m(\theta|k=1)$ .

*Conclusion 2:* Both when  $\gamma = 0$  and  $\gamma = 1$ , there exists  $\tilde{\theta} \in (\theta_0, \theta_1)$  such that for all  $\theta \in [0, \tilde{\theta}]$  we have  $\bar{\pi}_o(\theta|k=\hat{k}(\theta)) < \bar{S}_m(\theta|k=1)$ .

Obviously, parts (i) and (ii) of the proposition follows from Conclusions 1 and 2, respectively.  $\square$

Interestingly, the above proposition shows that when regulated according to an incentive-compatible mechanism, the owner of a technologically efficient monopoly may prefer to possess the control of her company, whereas the owner of a technologically inefficient monopoly may prefer to delegate the control of her company to a manager, provided that the owner is not taxed by the regulator. However, these two forms of control cannot be ranked unambiguously in terms of the expected social welfare.

## 5 Conclusion

In this paper, we have studied the problem of regulating a monopoly controlled by a manager who privately knows the marginal cost of production. The direct revelation mechanism we have constructed elicits the cost information from the manager instead of the owner, unlike in the BM model. As the manager is entitled to a fraction of the profits of the monopoly in the case of no regulation, the amount of rent due to the private cost information is generally found to be smaller for the manager in our model than for the owner in the BM model, alleviating the need for suppressing at each cost level the output, equalling the marginal information rent.

Accordingly, we have found that generally the optimal price is lower and the optimal output is higher in our model than in the BM model. In addition, if the compensation parameter chosen by the owner is sufficiently small, the optimal price recommends marginal cost pricing, provided that the regulator cannot tax the owner of the monopoly. Moreover, regardless whether the owner of the monopoly has complete or incomplete information about the marginal cost parameter, it is never optimal for her to offer an arbitrarily small or large fraction of the profits to the manager. We have also showed that when the marginal cost of production is sufficiently large, the owner of the monopoly chooses to separate management from ownership, provided that she is not taxed by the regulator. In contrast, the owner of the monopoly finds separation undesirable if the marginal cost of production is sufficiently small.

A possible extension to our model may consider the implications of alternative forms of managerial compensations on the optimal regulation. For example, like in Sklivas (1987), the manager may be entitled to a weighted sum of profits and revenues, where the weights to be optimally chosen by the owner. Moreover, extending the monopoly model to a duopoly, one can obtain a strategic game played between the owners of the two firms in the market in choosing optimal managerial compensations for their managers. It may be fruitful to investigate how such a game would affect the regulatory outcome and the social welfare as well as the firms' decisions to separate management from ownership.<sup>4</sup>

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<sup>4</sup>The proposed strategic game was already studied by Sklivas (1987) for a duopolistic market in a non-regulatory environment. They found that the two firms in the market earn higher profits

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