Another reason why the efficient market hypothesis is fuzzy

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Another Reason why the Efficient Market Hypothesis is Fuzzy

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ABSTRACT:

This paper makes use of the performance evaluation to test the validity of the efficient market hypothesis (EMH) in hedge fund universe. The paper develops a fuzzy set based performance analysis and portfolio optimisation and compares the results with those obtained with the traditional probability methods (frequentist and Bayesian models). We consider a data set of monthly investment strategy indices published by Hedge Fund Research group. The data set spans from January 1995 to June 2012. We divide this sample period into four overlapping sub-sample periods that contain different economic market trends. To investigate the presence of managerial skills among hedge fund managers we first distinguish between outperformance, selectivity and market timing skills. We thereafter employ three different econometric models: frequentist, Bayesian and fuzzy regression, in order to estimate outperformance, selectivity and market timing skills using both linear and quadratic CAPM models. Persistence in performance is carried out in three different fashions: contingence table, chi-square test and cross-sectional auto-regression technique. The findings obtained with probabilistic methods contradict the EMH and suggest that the "market is not always efficient," it is possible to make abnormal rate of returns if one exploits mispricing in the market, and makes use of specific investment strategies. However, the results obtained with the fuzzy set based performance analysis support the appeal of the EMH according to which no economic agent can make risk-adjusted abnormal rate of return. The set of optimal invest strategies under fuzzy set theory results in a well-diversified portfolio of investment with an expected mean return equal to that of the efficient frontier portfolio under the Markowitz' mean-variance.

Keywords: fuzzy set theory, probability, uncertainty, hedge fund, investment strategies

INTRODUCTION

The EHM is a central theme in modem finance (Fama, 1991). It argues that due to information dissemination, there is no arbitrage opportunity. When the informed trader acts in a certain direction, that would push the prices in the same direction and in return, this would eliminate his/her advantage. Analytical models such as rational expectations equilibrium models are founded on this assumption. They are mainly outcome-based models and they do not focus on the decision-making process. Rather they provide a characterization of the end result at the macro level which does not capture the behaviour of individual decision-makers.

The EMH has been criticized recently by behavioural finance researchers and experimental economists (Barberis et al., 2001; Kahnam and Tversky, 1982). They argue that the "irrationality" or "bounded rationality" of the traders would trigger inefficiency in the market. This paper is a nice addition to this literature since it provides an empirical analysis of how individual behaviour (of the hedge fund managers) impact the market structure and performance and whether the success of some fund managers depends on their skill sets, rather than on random luck. The paper is interesting not only because it discusses an important and relevant topic in finance but also in the last decade or so, the hedge fund industry has become a central player in the financial markets (Lo, 2008). There is still a lot of research to be done, which would have significant academic contributions, as well as policy implications. There is a growing concern that the hedge fund industry is not regulated sufficiently. The empirical results obtained in this paper may contribute in this regard. In addition, the robustness of the optimal investment strategies obtained in this study under different methods would be an interesting finding for the hedge fund managers.

This paper provides an empirical investigation of the decision-making of investment strategies employed by hedge fund managers under the EMH assumptions. The study is based on monthly investment returns recorded over a period of 17 years. The key finding of the study is the demonstration of the difference between decision-making under risk (where probabilistic models such as the frequentist and the Bayesian models can be used) and decision-making under uncertainty (where fuzzy credibility theory would be relevant). Probabilistic models are based on the assumption of normality and precise probability distributions, whereas fuzzy models allow for a higher degree of ambiguity. The study shows that, under probabilistic framework, the success of a fund manager depends on her selectivity skill, and market timing skill (during recovery periods). These managerial skills are used to generate abnormal rate of returns for their client by employing an optimal set of investment strategies made of equity hedge (EH), emerging markets (EM), relative values (RV) and funds of weighted currencies (FWC). These results highlight the importance of emerging markets (China India, Brazil, South Africa, etc) has a preferable investment destination by many fund managers from across the globe.

These findings contradict the rational expectations model and the Efficient Market Hypothesis – EMH. Indeed, if markets are efficient, fund managers cannot take advantage of any differences in the securities market expectations regarding returns and risk to generate abnormal excess returns from active trading (see Blake, 1994). We argue that the market is itself made up of some irrational agents (see for example Kahneman and Tversky, 1982) that cause inefficiencies in the markets, and that investors do have heterogeneous expectations regarding securities risk and returns. As a result fund managers frequently adjust their portfolio weights to follow different investment strategies and identify any opportunities to "*beat the market*".

However, under a fuzzy set theory this effect of bounded rationality disappears! The paper finds found no evidence of managerial skills (neither selectivity nor market skill are found). In addition under fuzzy set theory, the paper finds that investment allocations is well diversified across all investment strategies than in a probabilistic framework where optimal investments tend to tilt toward the fund manager's source of mispricings.

The question of whether or not the EMH holds in practice particularly in mutual funds has been largely investigated in recent year; most of studies on this topic have been carried out under probabilistic framework. These include Brown and Goetzman (1995), Carhart (1997), Agarwal and Naik (2000), Kat and Menexe (2003), Malkiel (1995) and De Souza and Gokcan (2004). These studies have all resulted in two types of inconclusive findings. The first type of findings (Brown et al., 1999; Kosowski, Malkiel, 1995; Naik and Teo, 2007) support the EMH by arguing that since the market is informationally efficient, hedge fund managers do not have skills to make abnormal rate of returns. The second type of findings (Agarwal and Naik, 2000; Hwang and Salmon, 2002; Capocci and Hubner, 2004; and Carhart, 1997) argues that due to information dissemination and to limited arbitrage hedge fund managers do have skills to outperform the market.

This paper takes a different approach by developing a performance evaluation framework based on fuzzy set theory in order to test the validity of the EMH in mutual (hedge) fund particularly. To the best of our knowledge, this study is first of its kind to using fuzzy set theory in order to assess the validity of the EMH. The paper develops a fuzzy set based performance analysis (using both linear and quadratic CAPM models), and portfolio selection problems (using both possibility and credibility theories). The paper starts by building a fuzzy set based CAPM model able to deal with the uncertainty and vagueness surrounding hedge fund returns modelling. We follow Tanaka et al. (1982) who translated a probabilistic regression model into a fuzzy regression model. In Tanaka's model the distributional assumption of probabilistic regression model is relaxed and uncertainty about coefficient estimates is represented by a fuzzy relationship. The coefficient estimates of their fuzzy model were assumed to be symmetrical fuzzy numbers of a triangular form. The basic idea of Tanaka's model is often referred to as possibilistic regression based on possibility theory pioneered by Zadeh (1965, 1978). The possibilistic regression methodology aims at minimizing the total spread of the fuzzy coefficients (decision criterion) subject to including all the given data. Further development of possibilistic regression models were presented by Diamond (1988) who used the least square errors as a decision criterion to be minimized.

Although possibility measure has been widely used in fuzzy set modelling, it has however presented some limitations. One great limitation is that possibility measure is not self-dual. Using possibility measure which has no self-duality property, one can find that two fuzzy events with different occurring chances may have the same possibility value. This limitation led recently to a significant increase in the amount of research on fuzzy possibilistic regression modelling. Some of this includes remarkable work by Xizhao and Minghu (1992) who use min-max procedure via possibility distribution to estimate a generalized fuzzy linear regression model where all beta coefficients are considered as fuzzy numbers. Yen, Ghoshray and Roig (1999) extend the results of a fuzzy linear regression model that uses symmetrical triangular coefficient to one with non-symmetrical fuzzy triangular coefficients.

This paper contributes to the ongoing research in fuzzy set theory by presenting a triangular credibilistic rather than possibilistic regression model based on Liu's (2004, 2007) credibility measure theory. The Credibilistic regression model overcomes some of the shortcomings of its counterpart (the possibilistic regression) such as sensitivity to outliers (Peters, 1994) and self-duality property (Liu 2007). In addition, the paper develops an optimisation problem based on fuzzy set theory in order to determine the optimal combination of investment strategies used by skilled managers to outperform the market. We believe that returns in the hedge fund industry are generated by random and uncertain variables as well as non-random variables such as the psychological state of the manager, (bullish, bearish or neutral). A bullish (bearish or neutral) manager will behave in such a way that the profit from her/his position reflects her/his psychological state. These events are not random; hence the use of probability measures in modelling returns in the hedge fund industry is somewhat dubious. Thus, the paper proposes two fuzzy bi-objective models for hedge fund strategies allocation in order to deal not only with uncertain, but also fuzzy events that affect the returns of a manager who employs a certain investment strategy. We derive two utility functions, namely the interval valued and the possibilitic crisp objective functions. These crisp functions are then respectively subjected to four different types of constraint that give managers different horizons to manoeuvre for obtaining their desired level of absolute returns.

Recently fuzzy theory, especially possibility theory, has been applied to portfolio selection in order to extend the mean-variance portfolio selection of Markowitz. A number of fuzzy portfolio selection problems have been proposed by researchers such as Inuiguchi and Ramik (2000) and Tanaka et al. (2000) who applied possibilistic measures to portfolio selection problems. Further studies on portfolio selection problems using fuzzy set theory include among others Tanaka, Guo and Türksen (2000) who propose two kinds of portfolio selection models based on fuzzy probabilities and possibility distributions respectively, rather than conventional probability distributions as in Markowitz's mean-variance model. They argue that since fuzzy probabilities and possibility distributions are obtained depending on possibility grades of security data offered by experts, investment experts' knowledge can be reflected in their portfolio selection model.

The rest of this paper is organized as follows: section two introduces some notions of fuzzy numbers of trapezoidal form, fuzzy set regression, and fuzzy set optimization. Section three discusses briefly two fundamental probabilistic portfolio selection problems considered in this paper as benchmark models, namely Markowitz's mean-variance and the Bayesian portfolio optimisation. Section four present the empirical results and section five concludes the paper.

METHODOLOGY

In a two period framework, we make use of both the linear and quadratic CAPM in order to generate managerial skills.

$$\mathrm{R_{it}} - \mathrm{R_f} = \alpha + \beta_{i1}(\mathrm{R_{mkt}} - \mathrm{R_f}) + \varepsilon_{it} \quad (1)$$

In equation (1), R_{it} represents the rate of returns on main strategy i, $\mathrm{R_{mkt}}$ represents the rate of returns on the market portfolio, R_f is the risk-free rate of returns, and β_{ij} represents sensitivity of expected returns of factor i to market factors.

The intercept term in (1), α, is referred to as *alpha* and measures the skills of the hedge fund manager. This model is based on the assumption that markets are efficient in the famous Fama (1984) efficient market hypothesis context which relies on "*normality of asset return distribution"* and "*absence of transaction costs"*. In this context all market participants have the same beliefs about asset prices, which presumably suggest no mispricing in the market; that is, alpha and beta in (1) are statistically equal to zero and one respectively.

A skilled manager attempts to exploit any mispricing that occurs in the market, thereby generating a certain value of alpha statistically different from zero. Where the value of alpha is positive (negative) it is a signal that the investment strategy whose rate of returns is R_{it} is underpriced (overpriced) and the fund manager would gain from the strategy if s/he takes a long (short) position. A skilled manager will exhibit persistence in outperforming the market in different sample periods.

For a fund manager with market timing skill, the returns on the managed portfolio will not be linearly related to the market return. This arises because the manager will gain more than the market does when the market return is forecast to rise and he will lose less than the market does when the market is forecast to fall. Thus, his portfolio returns will be a concave function of the market returns. Treynor and Mazuy (1966) presented the following quadratic CAPM in order to capture the return of a skilled manager:

$$r_{it} - r_f = \alpha_i + \beta_{1i}(r_{mt} - r_f) + \beta_{2i}(r_{mt} - r_f)^2 + \varepsilon_{it} \quad (2)$$

Treynor and Mazuy (1966) showed how the significance of β_{2i} provides evidence of the over-performance of a portfolio. Admati et al. (1986) suggested that α_i in equation (2) can be interpreted as the selectivity component of performance (i.e. the ability to select outperforming investments) and the $E[\beta_{2i}(r_{mt} - r_f)^2]$ interpreted as the timing component of performance (i.e. the ability to forecast the return on individual assets). The Treynor and Mazuy (1966) performance measure (TM) is therefore:

$$TM = \alpha_i + \beta_{2i}E[(r_{mt} - r_f)^2] \quad (3)$$

The estimation of the managerial skill coefficients (outperformance, selectivity, and market timing) in the two abovementioned CAPM models relies on strong[1] and unrealistic assumption[2]. The aim of this paper is to present a fuzzy set version of the two CAPM models that can produce more robust and reliable managerial skill coefficients. We solely rely on credibility theory developed by Liu (2004, 2007).

Credibility theory

A fuzzy random variable is a function from a measurable space to the set of fuzzy variables. Let Θ be a non-empty set of events, and $P(\Theta) = 2^{\Theta}$ a power set on Θ. Let again A be an event from Θ; credibility measure theory assigns to each event A a number $\mathrm{Cr}\{\mathrm{A}\}$ which indicates the credibility that $A \in 2^{\Theta}$ will occur. Following Liu (2004, 2007) the following axioms apply:

Axiom 1: $Cr\{\Theta\} = 1$

Axiom 2: Cr is increasing, i.e. for all $A \subset B,\ \mathrm{Cr}\{\mathrm{A}\} \leq Cr\{B\}$

Axiom 3: Cr is self-dual, i.e. $\mathrm{Cr}\{\mathrm{A}\} + Cr\{A^C\} = 1, \forall \mathrm{A} \in 2^{\Theta}$

Axiom 4: $\mathrm{Cr}\left\{\bigcup_i A_i\right\} \wedge 0.5 = Sup_i\{Cr\{A_i\}\},$ for any $\{\mathrm{A_i}\}$ that has $\mathrm{Cr}\{\mathrm{A_i}\} \leq 0.5$

Axiom 5: Let Θ_k be a non-empty set of events on which Cr_k satisfy the first four axioms. ($k = 1,2...,n$) and $\Theta = \Theta_1 \times \Theta_2 \times .... \times \Theta_n$, then;

$$Cr\{(\theta_1,....,\theta_n)\} = Cr_1\{\theta_1\} \wedge ..... \wedge Cr_n\{\theta_n\} \tag{4}$$

For each $(\theta_1,....,\theta_n) \in \Theta.$

Definition 1: (Liu 2004, 2007); any set function Cr satisfying the first four axioms is called "credibility measure." Moreover, the credibility measure of empty set is zero, i.e. $Cr\{\Phi\} = 0$

Definition 2: (Liu, 2007)

A fuzzy random variable ξ can be defined in Liu's framework as a mapping from credibility space $(\Theta, P(\Theta), Cr)$ to the space of real numbers i.e. $\xi : (\Theta, P(\Theta), Cr \to \Re)$.

[1] The exact form of the functional relationship between dependent and independent variables is often unknown, hence creating uncertainty and vagueness in the phenomena under investigation.

[2] Recently an amount of research has been carried out to assess the credibility of normal distribution with financial data. Much of this research (Harvey and Newbold, 2003; Peiró, 1999, Gehin, 2006) shows that financial time series, especially hedge fund data, presents excess skewness and kurtosis which often leads to the rejection of normality assumption.

Example 1: Let $\Theta = \{\theta_1, \theta_2\}$ and $\mathrm{Cr}\{\theta_1\} = Cr\{\theta_2\} = 0.5$ then $(\Theta, \mathrm{P}(\Theta), \mathrm{Cr})$ is a credibility space, and the function

$$\xi(\theta) = \begin{cases} 0, \text{ if } \theta = \theta_1 \\ 1, \text{ if } \theta = \theta_2 \end{cases}$$

is a fuzzy variable in the credibility theory framework.

Example 2: Let $\theta = [0,1]$ and $\mathrm{Cr}\{\theta\} = \dfrac{\theta}{2},\ \forall \theta \in \Theta$. Then $(\Theta, P(\Theta), Cr)$ is a credibility space and the function $\xi(\theta) = \theta$ is a fuzzy variable in the sense of credibility theory.

NB: A crisp number C may be regarded as a special fuzzy variable. In fact, it is the constant function $\xi(\theta) = C$ on the credibility space $(\Theta, P(\Theta), Cr)$.

A fuzzy variable ξ is said to be

(i) Non-negative if $Cr\{\xi < 0\} = 0$

(ii) Positive if $Cr\{\xi \le 0\} = 0$

(iii) Continuous if $Cr\{\xi = x\}$ is a continuous function of x;

(iv) Simple if there exists a finite sequence $\{x_1, ..., x_n\}$ such that $Cr\{\xi \ne x_1, ..., \xi \ne x_n\} = 0$

Definition 3: (Liu, 2007)

A n dimensional fuzzy vector $\xi = \{\xi_1, ..., \xi_n\}$ is defined as a function from a credibility space $(\Theta, P(\Theta), Cr)$ to the set of n dimensional real vectors $X = (x_1, ..., x_n)$.

Fuzzy arithmetic is similar to that of real numbers. The sum (product) of two fuzzy numbers ξ_1 and ξ_2 is also a fuzzy number. The product (sum) of a fuzzy number with a scalar number is also a fuzzy number. We refer interested readers to Liu (2004) for proof.

Membership Function

The membership function represents the degree of possibility that the fuzzy variable ξ takes some prescribed value. If a fuzzy variable is defined on the triplet $(\Theta, P(\Theta), Cr)$ then its membership function is derived from the credibility measure by

$$\mu(x) = (2\mathrm{Cr}\{\xi = x\}) \wedge 1;\ \forall x \in \Re \quad (5)$$

Special Membership Functions

Liu (2007) considers the following membership functions as special function for fuzzy variable:

- An equipossible fuzzy variable on $[0,1]$ is a fuzzy variable whose membership function is given by

$$\mu(x)=\begin{cases}1, \text{if } a \le x \le b \\ 0, \text{otherwise}\end{cases} \tag{6}$$

Graphically this membership function has the following shape:

$\mu(x)$

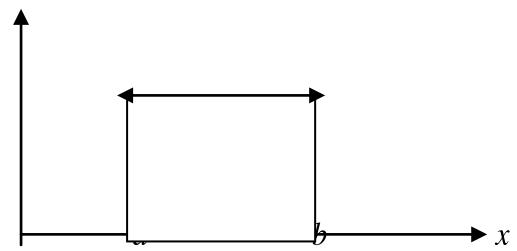

Figure 1: Membership function for an equipossible fuzzy variable

- A Triangular fuzzy variable is defined by the triplet (a,b,c) where crisp values $a<b<c$; has the following membership function;

$$\mu(x)=\begin{cases}\dfrac{x-a}{b-a}, \text{if } a \le x \le b \\ \dfrac{x-c}{b-c}, \text{if } b \le x \le c \\ 1, \text{if } x=b \\ 0, \text{otherwise}\end{cases} \tag{7}$$

Graphically a membership function for a triangular fuzzy variable has the following form:

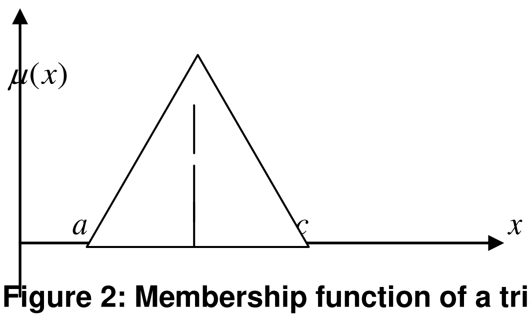

Figure 2: Membership function of a triangular fuzzy variable

- A trapezoidal fuzzy variable defined by the quadruplet (a,b,c,d) where crisp values $a < b < c < d$; has a membership function defined by:

$$\mu(x)=\begin{cases}\dfrac{x-a}{b-a}, \text{if a} \le \text{x} \le \text{b} \\ 1, \text{ if b} \le \text{x} \le \text{c} \\ \dfrac{x-d}{c-d}, \text{if c} \le \text{x} \le \text{d} \\ 0, \text{ otherwise}\end{cases} \tag{8}$$

Graphically the membership function of a trapezoidal has the following form:

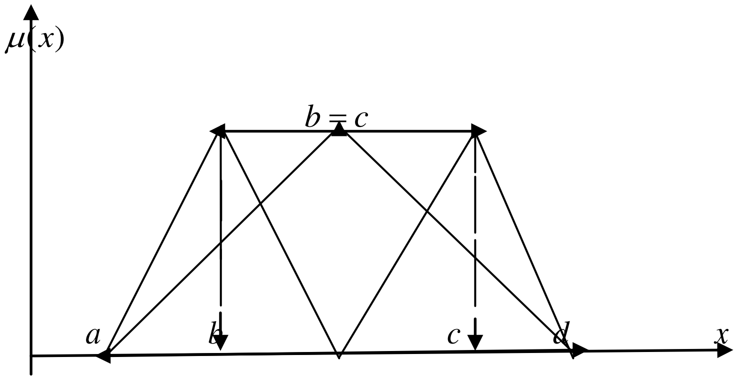

Figure 3: Membership function of a trapezoidal fuzzy variable

Credibility Distribution

Liu (2002) defined the credibility distribution $\Phi : \Re \to [0,1]$ of a fuzzy variable ξ to take a value less or equal to x as

$$\Phi(x) = Cr\{\theta \in \Theta / \xi(\theta) \le x\} \tag{9}$$

Let ξ be a fuzzy variable with membership function μ , then Liu (2007) defines its credibility distribution as

$$\Phi(x) = \frac{1}{2}\left(\underset{Y \le \infty}{Sup}\, \mu(\mathrm{y}) + 1 - \underset{Y > \infty}{Sup}\, \mu(\mathrm{y})\right);\ \forall \mathrm{x} \in \Re \tag{10}$$

Example 4: let a and b be two numbers such that $0 \le \mathrm{a} \le 0.5 \le \mathrm{b} \le 1$. We define a fuzzy variable by the following membership function

$$\mu(x)=\begin{cases}2a, \text{if x} < 0 \\ 1, \text{if x} = 0 \\ 2-2b, \text{if x} > 0\end{cases}$$

Hence from (10) we obtain the credibility distribution:

$$\Phi(x) = \begin{cases} a, \text{if} \quad \mathrm{x} < 0 \\ b, \text{if} \quad \mathrm{x} > 0 \end{cases}$$

Thus the left and right limits are $\lim_{\mathrm{X} \to -\infty} \Phi(\mathrm{x}) = a; \lim_{\mathrm{X} \to +\infty} \Phi(\mathrm{x}) = b$

Example 5: Let ξ be the equipossible fuzzy variable on $\Re$ then its credibility distribution is $\Phi(x) = 0.5$.

Chance Distribution

A random fuzzy variable is a function from the credibility space $(\Theta, P(\Theta), Cr)$ to the set of random variables. It is worth noting that two measures are involved in chance distribution, namely the credibility measure defined in $(\Theta, P(\Theta), Cr)$ and the probability measure defined in $(\Omega, A, \Pr)$ where Ω and A are defined as the fundamental non-empty set events and the event defining the sigma algebra. The combination of credibility and probability measures leads to a hybrid theory referred to (Zhu and Liu, 2004) as the chance measure theory that models both random and fuzzy events simultaneously. The chance of a random fuzzy event $\xi \in \beta$ is a function from $[0,1]$ to $[0,1]$ defined as

$$Ch\{\xi \in \beta\}(\alpha) = \underset{Cr\{A\} \geq \alpha}{Sup} \; Inf \; \Pr\{\xi(\theta) \in \beta\} \tag{11}$$

Zhu and Liu (2004) define the chance distribution as

$$\Phi(x, \alpha) = Ch\{\xi \leq x\}(x) \tag{12}$$

The first and second moments of a triangular fuzzy variable are shown by Liu, (2007) to be given by

The mean $E(\xi) = \dfrac{a + 2b + c}{4}$ (13)

And the variance $Var(\xi) = \dfrac{(c - a)^2}{24}$ (14)

The average chance distribution of a random fuzzy variable is a chance distribution involving credibility and probability measure theories. It is used in this chapter to model both randomness and fuzziness (vagueness) in the hedge fund universe. Liu (2007) defined the average chance distribution of a random fuzzy variable ξ by

$$\Psi(\xi) = Ch(\xi \leq x) = \int_0^1 Cr\{\theta \in \Theta : \Pr\{\xi(\mathrm{x}) \leq x\} \geq \alpha\} dx \tag{15}$$

Credibility Regression Analysis

Credibility regression analysis with average chance distribution describes the relationship of both random uncertainty variables (such as investment strategy returns, asset prices, etc.) and fuzzy variables (such as the fund manager's belief about the general market trends, the degree with which a fund manager is bullish, how higher a fund manager is bearish on a given investment strategy, etc.).

This paper develops a fuzzy set version of the linear CAPM in equation (1) and quadratic CAPM in equation (2) above to assess the persistence in performance analysis of hedge fund managers, and the optimality of investment strategies they use to outperform the market. These two equations can be rewritten as follows:

$$y_i = \alpha + \sum_{k=1}^{n} \beta_k x_{ki} + e_i \qquad (16)$$

where $n = 1$ or 2, $x_1 = (r_{mt} - r_f)$ for n=1 or $x_2 = (r_{mt} - r_f)^2$ for n=2; $\forall i = 1,2,.....,T$

If $n = 1$, then this model nests the linear CAPM model shown in equation (1), if $n = 2$, the model in equation (16) nests a quadratic CAPM shown in equation (2).

In equation (16) $y_i, x_i, \alpha, \beta_i, e_i$ represent the excess return on the main style, excess return on the factor i, the alpha, sensitivity of x_{ki} to changes in y_i and the disturbance term respectively.

To express the uncertainty about alpha, the ambiguity surrounding the coefficients generating process, the difficulties in verifying the validity of assumptions of the underlying data distribution, the inaccuracy and the distortion introduced by linearization, we present a corresponding fuzzy credibility regression model of the form:

$$y_i = \hat{\hat{\alpha}} + \sum_{k=1}^{n} \hat{\hat{\beta}}_k x_{ki} + \tilde{e}_i \qquad (17)$$

where $\hat{\hat{\alpha}}, \hat{\hat{\beta}}_i, \tilde{e}_i$ are triangular fuzzy numbers under credibility measure theory representing the alpha, the sensitivity of x_{ki} to changes in y_i and the disturbance term respectively. In matrix form equation (17) can be written as

$$Y = X\hat{\hat{\beta}} + \tilde{e}\,; \qquad (18)$$

where $\hat{\hat{\beta}} = \left(\hat{\hat{\alpha}},\ \hat{\hat{\beta}}_1\right)$ for a linear CAPM, $\hat{\hat{\beta}} = \left(\hat{\hat{\alpha}},\ \hat{\hat{\beta}}_1,\ \hat{\hat{\beta}}_2\right)$ for a quadratic CAPM i.e. $\hat{\hat{\beta}}$ is a vector of triangular fuzzy coefficients estimated under credibility measures. The triangular fuzzy disturbance term is defined in such a way that it contains both randomness and fuzziness i.e. probability and uncertainty information respectively. Hence it can be written as

$$\tilde{e} = \varepsilon + \xi \qquad (19)$$

where ε is a random disturbance component defined on a probability space $(\Omega, A, \Pr)$ and ξ a fuzzy (uncertainty) disturbance component defined on $(\Theta, P(\Theta), Cr)$.

Following Liu (2007) we define a triangular fuzzy set of event Θ corresponding to the disturbance term

$$\Theta = \left\{ \tilde{e}_i / \tilde{e}_i = y_i - x_i^t \hat{\hat{\beta}};\ i = 1,2,....,T \right\} \tag{20}$$

Liu (2004) shows that the expected mean and variance of such triangular fuzzy disturbance term $\tilde{e}_i$ are given by

$$E(\tilde{e}_i) = \frac{-h + (2 \times 0) + h}{4} = 0 \tag{21}$$

$$\mathrm{Var}(\tilde{e}_1) = \frac{h^2}{6} \tag{22}$$

where $h > 0$ is the spread of a triangular fuzzy number centred at zero. To estimate the parameters $\hat{\hat{\beta}}$ and $\mathrm{Var}(\hat{\hat{\beta}}_i) = \sigma_\xi$ we use the maximum uncertainty principle proposed by Liu and Liu (2003), which states that for any fuzzy event, if there are multiple reasonable values that a chance measure may take, then the value as close to 0.5 as possible is assigned to the event. Under this maximum uncertainty principle, the average chance M-estimation of parameters $\hat{\hat{\beta}}$, a, b, c, and $\mathrm{Var}(\hat{\hat{\beta}}_i) = \sigma_\xi$ consists in minimizing the following objective function:

$$\begin{aligned} &\mathit{Minimize}\ \mathrm{Q}\left(\hat{\hat{\beta}}^t, \mathrm{a, b, c}, \sigma_\xi / y_1, y_2, ..., y_T\right) = \sum_{i=1}^{n} \left\{ \Psi\left(\left(y_i - x_i^t \hat{\hat{\beta}} \right);\ \mathrm{a, b, c}, \sigma_\xi \right) - 0.5 \right\}^2 \\ &\quad \text{Subject to } \mathrm{E}(\tilde{e}) = \frac{a + 2b + c}{4} = 0 \end{aligned} \tag{23}$$

where $\hat{\hat{\beta}}^t = \left(\hat{\hat{\beta}}_0, ..., \hat{\hat{\beta}}_k \right)$; k=1 or 2 ($k = 1$ for linear CAPM or 2 for quadratic CAPM).

Notice that to obtain $\hat{\hat{\beta}}_0$ for example, we need to get the first derivatives of the objective function $Q(.)$ with respect to $\tilde{\beta}_0$ subject to $a + 2b + c = 0$ and equalize it to zero. The same for $\hat{\hat{\beta}}_1$, $\hat{\hat{\beta}}_2$, a, b, c, and σ_ξ. In fact we need to obtain $k+1$ parameter estimates from the fuzzy credibility regression model in equation (17) and four other parameters a, b, c, and σ_ξ related to the triangular mean and standard deviation of the coefficient estimates. Hence we need to solve a system of $(k+5)$ non-linear equations subject to $a + 2b + c = 0$.

The solution to this optimization problem is obtained using the non-linear least square optimization method a numerical optimisation technique proposed by Dennis (1977).

Investment Allocation under Fuzzy Set Theory

To deal with uncertainty and fuzziness in hedge fund investing, we represent each hedge fund strategy returns (r_i) by a (left-right) LR-fuzzy number of a trapezoidal form shown in Figure 4 below:

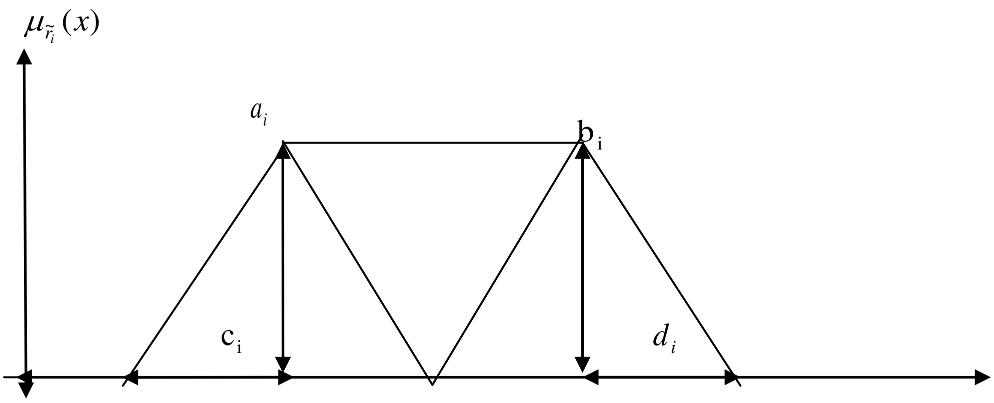

Figure 4: Trapezoidal left-right fuzzy returns

Following Vercher (2007) we denote by $\tilde{r}_i(a_i,b_i,c_i,d_i)_{LR}$ a LR-fuzzy return generated with strategy i; Ψ a set of LR-fuzzy returns and $\lambda \in R$; where a_i, b_i, c_i, d_i are real numbers on the trapeze representing the core spread [$a_i =$P40th , $b_i =$P60th] and the extremes $c_i =$P40th-P5th , and $d_i =$P95th-P60th where Pk^{th} is the k^{th} percentile of historical returns distribution. The following definitions apply.

***Definition** 1*: A fuzzy number $\tilde{r}_i$ is said to be a LR-fuzzy return i.e. $\tilde{r}_i(a_i,b_i,c_i,d_i)_{LR}$ generated with strategy i, if and only if its membership function is defined by:

$$\mu_{\tilde{r}_i}(x) = \begin{cases} L\left(\dfrac{a_i - x}{c_i}\right); \text{ if } \quad a_i - c_i \le x \le a_i \\ 1 \ ; \text{ if } \quad a_i \le x \le b_i \\ R\left(\dfrac{x - b_i}{d_i}\right); \text{ if } \quad b_i \le x \le b_i + d_i \end{cases} \tag{24}$$

where $a_i \le x \le b_i$ is the modal internal of real returns; $L(.)$ and $R(.)$ are reference linear functions strictly decreasing and upper semi-continuous defined from $[0,1] \xrightarrow{\text{to}} [0,+\infty[$ respectively. These functions verify the condition of the trapezoidal symmetry such that $L(x) = L(-x)$; $R(x) = R(-x)$ and $L(0) = R(0) = 1$. According to Lodwich and Bachman (2005) trapezoidal fuzzy numbers are a cut representation of any asymmetrical distribution of different shape forms

Definition 2: Let $\tilde{r}_1$ and $\tilde{r}_2$ two fuzzy returns (generated with two different investment strategies 1 and 2); with membership functions $\mu_{\tilde{r}_1}(x)$ and $\mu_{\tilde{r}_1}(y)$ respectively, where $x, y, \lambda \in R$. Following Vercher (2007) the possibility that the statement "return generated with strategy 2 is higher than that generated with strategy 1," is true is given by:

$$Pos(\tilde{r}_1 < \tilde{r}_2) = Sup\{\min(\mu_{\tilde{r}_1}(x), \mu_{\tilde{r}_2}(y)) / x, y \in R, x \le y\} \tag{25}$$

In the same way the possibility that the statement "return generated with strategy 2 is the same as that generated with strategy 1," is true is given by:

$$Pos(\tilde{r}_1 = \tilde{r}_2) = Sup\{\min(\mu_{\tilde{r}_1}(x), \mu_{\tilde{r}_2}(y)) / x, y \in R\}. \tag{26}$$

The possibility that the statement "return generated with strategy 1 is less than the manager's targeted rate of returns λ," is true is given by:

$$Pos(\tilde{r}_1 < \lambda) = Sup\{\min(\mu_{\tilde{r}_1}(x)) / x \in R, x < \lambda\}. \tag{27}$$

The possibility that the manager has reached his/her targeted rates of return λ with a given strategy is given by:

$$Pos(\tilde{r}_i = \lambda) = \mu_{\tilde{r}_i}(\lambda). \tag{28}$$

Definition 3: Let $\tilde{r}_{1i}(a_{1i}, b_{1i}, c_{1i}, d_{1i})_{LR}$ and $\tilde{r}_{2i}(a_{2i}, b_{2i}, c_{2i}, d_{2i})_{LR}$ two fuzzy rates of return generated with two different investment strategies 1 and 2. The sum (subtraction and product) of two fuzzy rates of return is also a fuzzy number (Liu, 2004). In fact:

$$\tilde{r}_{1i} + \tilde{r}_{2i} = (a_{1i} + a_{2i}, b_{1i} + b_{2i}, c_{1i} + c_{2i}, d_{1i} + d_{2i})_{LR};$$

$$\tilde{r}_{1i} - \tilde{r}_{2i} = (a_{1i} - a_{2i}, b_{1i} - b_{2i}, c_{1i} - c_{2i}, d_{1i} - d_{2i})_{LR}; \tag{29}$$

$$\lambda\tilde{r} = \begin{cases} (\lambda a_i, \lambda b_i, \lambda c_i, \lambda d_i)_{LR} & if \quad \lambda \ge 0 \\ \lambda b_i, \lambda a_i, |\lambda| d_i, |\lambda| c_i)_{LR} & if \quad \lambda < 0 \end{cases}$$

The alpha level set of a fuzzy return $\tilde{r}_i(a_i, b_i, c_i, d_i)_{LR}$ generated with strategy i is a crisp subset of real numbers (R) denoted by:

$$[\tilde{r}_i]^\alpha = \{x / \mu_{\tilde{r}_i}(x) \ge \alpha, x \in R\}. \tag{30}$$

In this study we use two different representations of expected mean of fuzzy returns; the first is interval valued fuzzy expected mean introduced by Dubois and Prade (1987) and the second is the possibilitic expected mean of fuzzy numbers presented by Carlsson and Fuller (2001) which is consistent with the extension principle[3] and is based on the set of alpha cuts. To obtain the fuzzy expected return for a given portfolio of hedge fund strategies we mimic Vercher (2006) by using membership function for each fuzzy return $\tilde{r}_i(a_i, b_i, c_i, d_i)_{LR}$ from the power utility function ($1-|x|^{\alpha}$) and evaluate all the shape parameters by means of ranking procedure (using percentile) of historical strategy returns.

If hedge fund strategy returns $\tilde{r}_i$ ($i = 1,2...n$ with n the number of investment strategies); are LR-fuzzy returns i.e. $\tilde{r}_i(a_i, b_i, c_i, d_i)_{LR}$ with different shape parameters; then the portfolio expected rate of return $\tilde{R}$ is given by:

$$\tilde{R} = \sum_{i=1}^{n} w_i \tilde{r}_i \tag{31}$$

$$\tilde{R} = \left[\sum_{i=1}^{n} a_i w_i, \sum_{i=1}^{n} b_i w_i, \sum_{i=1}^{n} c_i w_i, \sum_{i=1}^{n} d_i w_i\right] = \left[A(w), B(w), C(w), D(w)\right] \tag{32}$$

Equation (32) can be rewritten in matrix form as:

$$\tilde{R} = \left[A(w), B(w), C(w), D(w)\right] \tag{33}$$

where w represent a vector of investment capital allocated to investment strategy i. Following Dubois and Prade (1987), the interval-valued expected fuzzy mean return $\mathrm{E}(\tilde{\mathrm{R}})$ is given by:

$$E(\tilde{R}) = \left[E_*(\tilde{R}), E^*(\tilde{R})\right] \tag{34}$$

[3] The Zadeh (1965) extension principle is a basic concept in the fuzzy set theory that extends crisp domains of mathematical expressions to fuzzy domains. Suppose $\mathrm{f}(.)$ is a function from X to Y and A is a fuzzy set on X defined as:

$$\mathrm{A} = \mathrm{ma}(\mathrm{x}_1)/\mathrm{x}_1 + \mathrm{ma}(\mathrm{x}_2)/\mathrm{x}_2 + ... + \mathrm{ma}(\mathrm{x}_\mathrm{n})/\mathrm{x}_\mathrm{n}$$

where *ma* is the Membership Function of A . the $+$ sign is a fuzzy OR (Max) and the $/$ sign is a notation (indicated the variable x_i in discourse domain X - NOT DIVISION)

Then the Zadeh (1965) extension principle states that the image of fuzzy set A under the mapping $\mathrm{f}(.)$ can be expressed as a fuzzy set B .

$$\mathrm{B} = \mathrm{f}(\mathrm{A}) = \mathrm{ma}(\mathrm{x}_1)/\mathrm{y}_1 + \mathrm{ma}(\mathrm{x}_2)/\mathrm{y}_2 + ... + \mathrm{ma}(\mathrm{x}_\mathrm{n})/\mathrm{y}_\mathrm{n}$$

where $\mathrm{y}_\mathrm{i} = \mathrm{f}(\mathrm{x}_\mathrm{i}), \mathrm{i} = 1, 2, 3, 4, ..., \mathrm{n}$

where $E_*(\tilde{R}) = \int_0^1 (Inf \quad \tilde{R}_\alpha) d\alpha$; and $E^*(\tilde{R}) = \int_0^1 (Sup \quad \tilde{R}_\alpha) d\alpha$; are lower and upper bound of the interval respectively. Following Carlsson and Fuller (2001), the possibilitic expected fuzzy return $\tilde{R}$ is given by:

$$M(\tilde{R}) = \left[M_*(\tilde{R}), M^*(\tilde{R}) \right] \tag{35}$$

where $M_*(\tilde{R}) = 2\int_0^1 \alpha (Inf \quad \tilde{R}_\alpha) d\alpha$; and $M^*(\tilde{R}) = 2\int_0^1 \alpha (Sup \quad \tilde{R}_\alpha) d\alpha$; are lower and upper bounds of the interval respectively. Bermudez et al. (2005) showed that the possibilistic expected mean is a subset of the interval-valued expected mean.

To model the risk associated with hedge fund investment we use a downside risk measure proposed by Leon et al. (2004) rather than the standard deviation. We believe that fund managers are more worried about the downside risk of their investment positions than the upper side. We view the downside risk as the failure of a manager to deliver on his/her promises. By definition the two downside risk measures corresponding respectively to interval-valued and possibilistic fuzzy returns are as follows:

$$\tilde{\mathrm{V}}_1(\tilde{\mathrm{R}}) = \mathrm{E}[\max \{0, \mathrm{E}(\tilde{\mathrm{R}}) - \tilde{\mathrm{R}}\}] \tag{36}$$

$$\mathrm{V}_2(\tilde{\mathrm{R}}) = \mathrm{E}[\max \{0, \mathrm{M}(\tilde{\mathrm{R}}) - \tilde{\mathrm{R}}\}] \tag{37}$$

Corresponding crisp functions of equations 34, 35, 36, and 37 can be obtained easily as in Vercher (2007) for an LR-fuzzy return $\tilde{r}_i(a_i, b_i, c_i, d_i)_{LR}$ as follows:

$$E(\tilde{R}) = \max\left[0, \sum_{i=1}^{n} \left\{ \frac{1}{2}(a_i + b_i) + \frac{1}{4}(d_i - c_i) \right\} \right] = \sum_{i=1}^{n} \left\{ \frac{1}{2}(a_i + b_i) + \frac{1}{4}(d_i - c_i) \right\} \tag{38}$$

$$M(\tilde{R}) = \max\left[0, \sum_{i=1}^{n} \left\{ \frac{1}{2}(a_i + b_i) + \frac{1}{6}(d_i - c_i) \right\} \right] = \sum_{i=1}^{n} \left\{ \frac{1}{2}(a_i + b_i) + \frac{1}{6}(d_i - c_i) \right\} \tag{39}$$

$$V_1(\tilde{R}) = E\left[\max\left\{ 0, \sum_{i=1}^{n} \left\{ (b_i - a_i) + \frac{1}{2}(c_i + d_i) \right\} \right\} \right] = \sum_{i1}^{n} \left[b_i - a_i + \frac{1}{2}(c_i + d_i) \right] \tag{40}$$

$$V_2(\tilde{R}) = E\left[\max\left\{ 0, \sum_{i=1}^{n} \left\{ (b_i - a_i) + \frac{1}{3}(c_i + d_i) \right\} \right\} \right] = \sum_{i1}^{n} \left[b_i - a_i + \frac{1}{3}(c_i + d_i) \right] \tag{41}$$

We defined two performance measures for investment allocation under fuzzy returns, the first is based on interval valued portfolio returns and the second on possibilistic portfolio returns;

$PI = \dfrac{E(\widetilde{R})}{V_1(\widetilde{R})}$; for an interval valued portfolio returns and; (42)

$PP = \dfrac{M(\widetilde{R})}{V_2(\widetilde{R})}$; for a possibilistic portfolio returns. (43)

The portfolio with the highest PI (PP) measure will be the most preferred.

Formulation of Investment Constraints under Fuzzy Set Theory

We present a bi-objective fuzzy set based investment allocation problem that minimises the downside risk while maximising the portfolio rate of return. The investment allocation problem is subjected to four different types of investment constraints, namely restriction on both short selling and leverage i.e. $\sum_{i=1}^{n} w_i = 1$ and $\mathrm{w_i} \geq 0$.

The second type of constraint restricts only short selling while giving the manager an unlimited leverage manoeuvre in order to achieve his/her targeted returns i.e. $\sum_{i=1}^{n} w_i \neq 1$ and $w_i > 0$,or $w_i < 0$.

The third type of constraint restricts only leverage while allowing the manager to use short selling i.e. $\sum_{i=1}^{n} w_i = 1$, and $w_i > 0$ or $w_i < 0$.

The fourth type of constraint gives the manager great margin of manoeuvre to use leverage and short selling in order to achieve his/her targeted rates of return i.e. $\sum_{i=1}^{n} \neq 1$ or $w_i < 0$ or $w_i > 0$.

The two crisp bi-objective strategies allocation problems that we want to optimize are:

- **Problem 1: Interval-valued problem;**

Maximize $E(\widetilde{R}) = \sum_{i=1}^{n} \left[\frac{1}{2}(a_i + b_i) + \frac{1}{4}(d_i - c_i) \right] w_i$

Minimize $V_1(\widetilde{R}) = \sum_{i=1}^{n} \left[(b_i - a_i) + \frac{1}{2}(c_i + d_i) \right] w_i$

Subject to:

1. $\begin{cases} \sum_{i=1}^{n} w_i \neq 1 \\ w_i > 0, \quad w_i < 0 \end{cases}$

2. $\begin{cases} \sum_{i=1}^{n} w_i \neq 1 \\ w_i > 0 \end{cases}$

3. $\begin{cases} \sum_{i=1}^{n} w_i = 1 \\ w_i > 0, \quad w_i < 0 \end{cases}$

4. $\begin{cases} \sum_{i=1}^{n} w_i = 1 \\ w_i > 0 \end{cases}$

- **Problem 2: Possibilistic problem:**

Maximize $M(\tilde{R}) = \sum_{i=1}^{n} \left[\frac{1}{2}(a_i + b_i) + \frac{1}{6}(d_i - c_i) \right] w_i$

Minimize $V_2(\tilde{R}) = \sum_{i=1}^{n} \left[(b_i - a_i) + \frac{1}{3}(c_i + d_i) \right] w_i$

Subject to: 1. $\begin{cases} \sum_{i=1}^{n} w_i \neq 1 \\ w_i > 0, \quad w_i < 0 \end{cases}$

2. $\begin{cases} \sum_{i=1}^{n} w_i \neq 1 \\ w_i > 0 \end{cases}$

3. $\begin{cases} \sum_{i=1}^{n} w_i = 1 \\ w_i > 0, \quad w_i < 0 \end{cases}$

4. $\begin{cases} \sum_{i=1}^{n} w_i = 1 \\ w_i > 0 \end{cases}$

where w_i; represents the investment capital allocated to strategy i. We make use of the genetic algorithm to solve these bi-objective investment allocation problems.

Investment Allocation under Bayesian Settings

In this section we deal with estimation risk for hedge fund investment allocation by making use Bayesian statistics. Under this framework, we take care not only of the estimation risk but also of the asymmetrical behaviour of strategy returns by using the parameters of the posterior distribution instead of those of historical distribution. We therefore assume that the fund manager has an informative and uninformative prior and can update her/his beliefs about the future expected returns distribution as new information comes into the markets. Furthermore, we extend the Bayesian portfolio selection model by presenting a counterpart model known as the Black-Litterman (1992) model.

We mimic Harvey, Liechty et al. (2004) who address both the estimation risk and the inclusion of higher moments in the portfolio selection. They suggest the use of skew normal distribution to capture the asymmetrical behaviour of returns. In this Bayesian framework, objective functions are optimized using predictive returns generated with the Monte Carlo Markov Chain (MCMC) simulations.

Suppose that a fund manager has a holding period of length τ ; the fund manager's objective is to maximize his/her wealth at the end of the investment period $T+\tau$ where T is the sample period. Denote by $\mathrm{Y}_{T+\tau}$ the unobserved next τ periods' expected returns; the predictive returns distribution can be written as (see Harvey, Liechty et al., 2004):

$$p(Y_{T+\tau} / Y_n) \propto \int p(Y_{T+\tau} / \mu, \Sigma, S) p(\mu, \Sigma, S / Y_n) d\mu d\Sigma dS \quad (44)$$

where Y_{n} is a $(T \times N)$ matrix of historical returns of all investment strategies (N strategies) during the past T periods.

$\mathrm{p}(\mu, \Sigma, \mathrm{S} / \mathrm{Y}_{\mathrm{n}})$ is the joint posterior distribution of strategy returns assumed in this paper to be a skewed student's t-distribution with first, second and third moment given by $\mu, \Sigma, \text{and } S$ respectively. This distribution summarizes uncertainty about the future expected returns distribution.

$p(Y_{T+\tau} / \mu, \Sigma, S)$ is a multivariate skewed student's t-distribution for the next τ period future expected returns. And $\propto$: a proportionality sign.

We account for estimation risk by averaging in (44) over the posterior distribution of the parameters $\mu, \Sigma, \text{and } S$. Therefore the distribution of $Y_{T+\tau}$ will not depend on unknown parameters, but only on the past returns series Y_n assumed to be skewed student's t-distribution.

The analytical solution of equation 44 is often difficult to obtain; often numerical methods such as the MCMC simulations (Metropolis-Hasting or the Gibbs sampler algorithm) are used to obtain the predictive distribution. In this paper the Gibbs sampler algorithm will be used for this purpose.

Substituting the predictive returns distribution into the fund manager's objective functions, the following multi objective portfolio optimization problem is presented:

$$\begin{cases} \max\limits_{W} \int \omega' \tilde{\mu}_{T+\tau} p(Y_{T+\tau} / Y_n) dY_{T+\tau} \\ \min\limits_{W} \lambda \int (\omega' \tilde{\Sigma}_{T+\tau} \omega) p(Y_{T+\tau} / Y_n) dY_{T+\tau} \\ \max\limits_{W} \gamma \int (\omega' \tilde{S}_{T+\tau} \omega \otimes \omega) p(Y_{T+\tau} / Y_n) dY_{T+\tau} \\ \quad \text{Subject to} : \omega \mathrm{I} = 1 \end{cases} \tag{45}$$

where $\tilde{\mu}_{T+\tau}, \tilde{\Sigma}_{T+\tau}, \tilde{S}_{T+\tau}, \lambda, \gamma, \text{and} \otimes$ is the predictive mean, predictive covariance matrix, predictive coskewness matrix of future expected returns, aversion to change in risk, aversion to change in skewness, and the kronecker product.

To obtain the predictive moments of future expected returns, we use a skew t distribution derived from the skew elliptical class of distributions presented by Sahu et al. (2003). Its general form is shown to be:

$$f(X / \mu, \Sigma, g^{(P)}) = |\Sigma|^{1/2} g^{(P)} \{(X - \mu)' \Sigma^{-1/2} (X - \mu)\}; \quad X \in \Re^P \tag{46}$$

with $g^{(P)}(u) = \dfrac{\Gamma(p/2) g(u,p)}{\pi^{p/2} \int_0^\infty r^{\frac{P}{2}-1} g(r,p) dr}$; where; $a \geq 0$; $\int_0^\infty r^{\frac{P}{2}-1} g(r,p) dr \neq 0$

Sahu et al. (2003) show that when $g(u,p) = \left(1 + \dfrac{u}{\upsilon}\right)^{-\frac{(\upsilon+P)}{2}}$; (with $\upsilon > 0$) equation (46) becomes a multivariate student's t-distribution under the condition that the vector of random variables X is transformed as follows:

$$X = \mu + DZ + \varepsilon \tag{47}$$

where Z is a vector of unobservable random variables whose distribution is elliptical with mean zero and identity covariance matrix I_p ; $\mu \in \Re^P$ vector of mean; D, is a $p \times p$ matrix of skewness and co skewness:

$$D = \begin{bmatrix} \delta_{11} & \delta_{12} & .... & \delta_{1p} \\ \delta_{21} & \delta_{22} & .... & \delta_{2p} \\ : & : & .... & : \\ \delta_{p1} & \delta_{p2} & .... & \delta_{pp} \end{bmatrix};$$

with δ_{ij} : representing the coskewness of random variable x_i and x_j for all $i \neq j$; and skewness for $i = j$; and ε a vector of error terms defined as $\varepsilon \rightarrow st(0, \Sigma, \upsilon)$ (i.e. skewed t-student random variable). Consequently Sahu et al. (2003) show that the conditional distribution of random variable $Y = (X / Z > 0)$ given $\mu, \Sigma, D, \text{and}\, \upsilon$ has the following multivariate skewed student's t-distribution:

$$p(Y/\mu,\Sigma,D,\upsilon)=2^{\alpha}t_{\alpha},\upsilon(Y/\mu,\Sigma+D^{2}) \tag{48}$$

where V follows student's t-distribution, $t_{\alpha},\upsilon+\alpha$.

It is now possible to implement a Bayesian portfolio selection under the assumption that hedge fund returns have a skewed student's t-distribution. This implementation is done using the MCMC simulations with a Gibbs sampler that requires us to first specify the likelihood function, the priors and posteriors distribution before computing the predictive moments of future expected returns.

Following German and German (1984) the likelihood of the data can be specified as

$$y_i/z_i,\mu,\Sigma,D,w_i \rightarrow \mathrm{N_p}\left(\mu+Dz_i,\frac{\Sigma}{w}\right) \tag{49}$$

where $z_i \rightarrow N_p(0,I_P);\quad \text{and } \mathrm{w_i} \rightarrow \Gamma\left(\frac{\upsilon}{2},\frac{\upsilon}{2}\right)$

For the informative priors scenario we consider the conjugate priors distribution for the unknown parameter μ given $\Sigma,\upsilon,$ and D; and the unknown parameter Σ which has a multivariate inverted Wishart distribution as in Harvey et al. (2004):

$$\begin{aligned}
&\mu \rightarrow N_p(m,\Sigma_\mu)\\
&\Sigma \rightarrow Inv-W_p(C_\Sigma,\Omega_\Sigma)\\
&D \approx \delta \rightarrow N_p(d,\Sigma_\delta)\\
&\upsilon \rightarrow \Gamma(\gamma,\Sigma_\upsilon)
\end{aligned} \tag{50}$$

Notice that δ is a parameter that adjusts the degree of our beliefs about the skewness in the distribution of the data, a prior value of this parameter must be specified in the informative prior settings; it goes the same with the mean vector d which reflects our prior information.

Following Polson and Tew (2000), and Harvey et al. (2004), we then obtain the predictive moments of future expected distribution as

$$\begin{aligned}
&\tilde{\mu}_{T+\tau}=\mu\\
&\tilde{\Sigma}_{T+\tau}=\Sigma+\mathrm{var}(m/Y)\\
&\tilde{S}_{T+\tau}=S+3E(V\otimes m/Y)-3E(V/Y)\otimes\tilde{\mu}_{T+\tau}-E(m-\tilde{\mu}_{T+\tau})\otimes(m-\tilde{\mu}_{T+\tau})/Y)
\end{aligned} \tag{51}$$

where $\tilde{\mu}_{T+\tau},\tilde{\Sigma}_{\mathrm{T}+\tau},\tilde{\mathrm{S}}_{\mathrm{T}+\tau}$ are the predictive (central) moments, and μ,Σ,S are the posterior means of the moments obtained with the Gibbs sampler (see Geman and Geman, 1984).

To implement the Gibbs sampler algorithm we need to be able to sample from $p(\mu, \Sigma, S / Y)$. The algorithm proceeds by drawing iteratively from this distribution starting from an arbitrary set of values $(\mu^{(0)}, \Sigma^{(0)}, S^{(0)})$

$$
\begin{array}{l}
\mu^{(1)} \to p(\mu / \Sigma^{(0)}, S^{(0)}, Y) \\
\Sigma^{(1)} \to p(\Sigma / \mu^{(1)}, S^{(0)}, Y) \\
S^{(1)} \to p(S / \mu^{(1)}, \Sigma^{(1)}, Y) \\
\ldots\ldots\ldots\ldots\ldots\ldots \\
. \\
. \\
\ldots.\mu^{(N)}, \Sigma^{(N)}, S^{(N)}
\end{array}
\qquad (52)
$$

Geman and Geman (1984) showed that for the $(\mu^{(t)}, \Sigma^{(t)}, S^{(t)})$ sample obtained after N iterations we need:

$$(\mu^{(t)}, \Sigma^{(t)}, S^{(t)}) \xrightarrow{\textit{converge} \text{ to}} (\mu, \Sigma, S) \xrightarrow{\textit{In} \text{ Probability to}} p(\mu, \Sigma, S / Y) \text{ as t} \to \infty$$

Once the predictive means are determined, the optimization problem in equation 45 can be solved with different level of risk and skewness aversion (λ and γ) using the genetic algorithm.

DATASET AND EMPIRICAL ANALYSIS

We consider a set of returns on hedge fund indices provided by Hedge Fund Research Inc. (HFRI). The HFRI is the largest data provider on alternative investment industry. The database contains more than 6 500 hedge funds from all over the world. The monthly returns series are HFRI strategy indices representing the equally weighted returns, net of fees, of hedge funds classified in each strategy. The database is updated bi-weekly with new funds information (removed and/or newly included funds). The data set on these strategy indices spans January 1995 to June 2012; to account for survivorship bias we consider only the sample periods of after 1994. Following Capocci and Hubner (2004) hedge fund data starting after 1994 are more reliable and do not contain any survivorship bias. The entire sample period (January 1995 to June 2012) is then subdivided into four sub-sample periods that include different economic market trends such as the 1998 Japanese crisis, the Dotcom bubble, the 2001 South African currency crisis, and the 2008-2009 sub-prime crisis. The subdivision of our entire sample into four sub-sample periods follows Capocci, Corhay and Hubner (2003). sub-sample period1, January 1995 to March 2000, which represents the economic recovery sample. Sub-sample period2, April 2000 to Dec 2002, representing the low economic growth sample. Sub-sample period3, January 2003 to January 2007, representing the strong economic growth sample. Sub-sample period4, February 2007 to June 2012, representing the economic recession sample.

Table 1 summarises the descriptive statistics of the seven main investment strategies, namely ED, EH, EM, FoF, FWC, MCRO and RV:

Table 1: Descriptive statistics overall sample period

	ED	*EH*	*EM*	*FoF*	*FWC*	*MCRO*	*RV*
Mean	0.9130	0.9559	0.8760	0.5297	0.8097	0.8086	0.7274
Median	1.2695	1.1842	1.5058	0.7400	1.0365	0.6855	0.8400
Std Dev	2.0457	2.7780	4.2083	1.8045	2.1363	1.8932	1.2988
Kurt	4.5035	2.0866	4.3954	3.9964	2.7323	0.5664	16.9558
Skew	-1.3856	-0.2272	-1.0355	-0.7581	-0.6969	0.4197	-3.0935
Min	-8.9000	-9.4584	-21.0200	-7.4700	-8.7000	-3.7700	-8.0312
Max	5.1300	10.8800	14.8000	6.8500	7.6500	6.8200	3.9329

Table 1 shows that emerging markets have the largest negative return and the largest standard deviation. Despite being riskier markets, emerging market exhibits the largest maximum return of all available investment strategies. In addition, Equity Hedge investment strategy is the second investment strategy with the highest rate of return.

Performance Evaluation: Evidence Against the EMH

Using equation (16) and (17), we are able to generate different managerial skill coefficients for each investment strategy used by these fund managers. Under a two-period performance evaluation framework, we classify each skill coefficient as winner and/or loser for each investment category. The existence of persistence in performance over a long period will be enough evidence against the EHM. We therefore define a fund manager as a *winner* if the strategy that he uses generates a managerial skill coefficient i.e. Jensen's alpha (or the Treynor and Mazuy coefficients) that is higher than the median of historical returns; and a *loser* a fund manager whose Jensen's alpha (or Treynor and Mazuy coefficient) is lower than the median of historical returns.

The persistence in performance in this context relates to fund managers that are winners in two consecutive periods denoted by WW, or losers in two consecutive periods, denoted LL. Similarly, winners in the first period and losers in the second period are denoted by WL, and LW denoted the reverse. We use both the cross product ratio (CPR) proposed Christensen (1990) and the chi-square test statistics to detect the persistence in performance of fund managers. The CPR is given by:

$$CPR = \frac{(WW * LL)}{(WL * LW)} \tag{53}$$

The CPR captures the ratio of the funds which show persistence in performance to the ones which do not. Under the null hypothesis of no persistence in performance, the CPR is equal to one. This implies that each of the four categories denoted by WW, WL, LW, LL represent 25% of all funds. To make a decision about the rejection of the null hypothesis, we make use of the Z-statistic given by:

$$\mathrm{Z-statistic} = \frac{\mathrm{Ln(CPR)}}{\sigma_{\mathrm{Ln(CPR)}}} \tag{54}$$

where $\sigma_{Ln(CPR)} = \sqrt{\frac{1}{WW} + \frac{1}{WL} + \frac{1}{LW} + \frac{1}{LL}}$ (55)

For example, a Z-statistic greater than 1.96 indicates evidence of the presence of significant persistence in performance at a 5% confidence level. (For more details see Kat and Menexe, 2003 and De Souza and Gokcan, 2004).

We also use the chi-square test statistic to compare the distribution of observed frequencies for the four categories WW, WL, LW, and LL, for each investment strategy with the expected frequency distribution. Studies carried out in persistence performance using chi-square test statistics (Carpenter and Lynch, 1999 and Park and Staum, 1998) reveal that the chi-square test based on the numbers of winners and losers is well specified, powerful and more robust compared to other test methodologies, as it deals carefully with the presence of survivorship bias. The chi-square test statistic (see Agarwal and Naik, 2000) is given by:

$$\chi^2_{Cal} = \frac{(WW - D_1)^2}{D_1} + \frac{(WL - D_2)^2}{D_2} + \frac{(LW - D_3)^2}{D_3} + \frac{(LL - D_4)^2}{D_4} \tag{56}$$

where $\begin{cases} \mathrm{D}_1 = \frac{(\mathrm{WW} + \mathrm{WL}) * (\mathrm{WW} + \mathrm{LW})}{\mathrm{N}} \\ \mathrm{D}_2 = \frac{(\mathrm{WW} + \mathrm{WL}) * (\mathrm{WL} + \mathrm{LL})}{\mathrm{N}} \\ \mathrm{D}_3 = \frac{(\mathrm{LW} + \mathrm{LL}) * (\mathrm{WW} + \mathrm{LW})}{\mathrm{N}} \\ \mathrm{D}_4 = \frac{(\mathrm{LW} + \mathrm{LL}) * (\mathrm{WL} + \mathrm{LL})}{\mathrm{N}} \end{cases}$

We summarize below our results on the test of the EMH using performance analysis in the following tables.

Table 2: Performance persistence[4]

		Frequentist	Bayesian	Fuzzy
	Outperform	P1-P2;P2-P3	P1-P2;P2-P3	None
Contingence	Selectivity	P2-P3	P2-P3	P2-P3
	Timing	P1-P2	P1-P2	P3-P4
	Outperform	P1-P2;P2-P3	P1-P2;P2-P3	None
Chi-square	Selectivity	P2-P3	P2-P3	P2-P3;P3-P4
	Timing	P1-P2	P1-P2	P3-P4
	Outperform	P2-P3	P2-P3	None
Regression	Selectivity	P2-P3	P2-P3	P2-P3
	Timing	None	None	P3-P4

The results obtained with the fuzzy credibility method show that although some fund managers possess selectivity skill (during sub-sample period 2 to sub-sample period 3) and timing skill (during sub-sample period 3 and sub-sample period 4); fund managers do not possess market outperformance skill to generate enough excess returns during the entire sample period. Therefore fuzzy credibility method supports the EMH according to which the market is always efficient and no market participant can make risk-adjusted abnormal rates of return.

However, the results of the first two methods (frequentist and Bayesian) as presented in Table 2 show that hedge fund managers exhibit persistence in overall market outperformance during the period between sub-sample period1 through sub-sample period3. This outperformance is due to market timing skill during sub-sample period 1 and sub-sample period 2; and to selectivity skill during sub-sample period 2 and sub-sample period 3. These results contradict the EMH paradox and show that the "market is not always efficient" and that it is possible to make abnormal rates of return if one has selectivity skills. In other words, the results show that fund managers who possess selectivity skills can outperform (beat) the market at 7.5% or higher significance level if and only if the economic conditions that governed the financial market during the period between sub-sample period2 and sub-sample period3 remain the same i.e. fast domestic growth coupled with low interest rates.

The difference in findings obtained with the probabilistic method (frequentist and Bayesian) and uncertainty method (fuzzy credibility theory) is primarily due to the way uncertainty is

[4] For more detailed results, please contact the authors

modelled in the hedge fund universe in particular and in financial markets in general. Probability differs fundamentally with uncertainty; probability assumes that the total number of states of economy is known, whereas uncertainty assumes that the total number of states of economy is unknown.

The fuzzy credibility method overcomes these assumptions. However, the presence of selectivity and market timing skills found in fuzzy credibility results doesn't guarantee the overall market outperformance because the method takes into account the lack of probabilities of realizations, the opportunity cost due to poor diversification across assets and over time, the management and incentive fees charged by the manager as well as other transaction costs. When one considers all these costs, the risk-adjusted rate of return generated by fund managers in uncertain market environment would be equal to zero as stipulated by the EMH.

To investigate the optimality of investment strategies used by skilled managers to outperform the market, we first fuzzify each investment strategy return by identifying the 5th, 40th, 60th and 95th degree of percentiles that represents the crisp incredients of the fuzzy return $\tilde{r}_i(a_i, \mathrm{b_i}, \mathrm{c_i}, \mathrm{d_i})$. Table 3 and 6.2 exhibit the percentiles of historical return distribution and the fuzzified returns corresponding to each investment strategy respectively.

Table 3: Historical percentiles

	ED	EH	EM	FoF	FWC	MCRO	RV
5th	-2.4726	-3.5201	-5.5225	-2.4567	-2.5737	-2.0715	-0.681
40th	0.7858	0.4750	0.6100	0.3510	0.5329	0.2500	0.6800
60th	1.5479	1.8500	2.1636	0.9900	1.5163	1.1128	1.0800
95th	3.7175	5.2000	6.2875	3.0189	3.7175	4.1469	2.0538

Table 4: Inputs for fuzzified returns

	ED	EH	EM	FoF	FWC	MCRO	RV
a	0.7858	0.4750	0.6100	0.3510	0.5329	0.2500	0.6800
b	1.5479	1.8500	2.1636	0.9900	1.5163	1.1128	1.0800
c	3.2584	3.9951	6.1325	2.8077	3.1066	2.3215	1.3611
d	2.1696	3.3500	4.1239	2.0289	2.2012	3.0341	0.9738

Based on these inputs we construct the membership function of each investment strategy; for example the membership function of Event-Driven strategy is constructed as:

$$
\mu_{\tilde{r}_i}(x) = \begin{cases} L\left(\dfrac{0.7858 - x}{3.2584}\right); \text{ if } -2.4726 \le x \le 0.7858 \\ 1 \; ; \text{ if } 0.7858 \le x \le 1.5479 \\ R\left(\dfrac{x - 1.5479}{2.1696}\right); \text{ if } 1.5479 \le x \le 3.7175 \end{cases}
$$

These membership functions represent the degree to which each fund manager believes that his/her expected portfolio return is going to be.

The portfolio optimization problem 1 i.e. interval-valued and 2 i.e. possibility above become:

- Interval-valued optimization problem

$$
\begin{aligned}
\text{Max} f &= 0.8947W1 + 1.0012W2 + 0.8847W3 + 0.4758W4 + 0.7983W5 + 0.86W6 + 0.7832W7 \\
\text{Min} f &= 3.4761W1 + 5.0476W2 + 6.6818W3 + 3.0573W4 + 3.6373W5 + 3.5406W6 + 1.5675W7
\end{aligned}
$$

Subject to the set of constraints

- Possibility-based optimization problem

$$
\text{Max} f = 0.9854W1 + 1.055W2 + 1.052W3 + 0.5407W4 + 0.8737W5 + 0.8002W6 + 0.8155W7
$$

$$
\text{Min} g = 2.5714W1 + 3.8234W2 + 4.9724W3 + 2.2512W4 + 2.7527W5 + 2.648W6 + 1.1783W7
$$

Subject to a set of financial constraints

We present four types of financial constraints for each optimization problem i.e. interval-valued and possibility problem, as follows:

1. No leverage and no short selling allowed: $\sum_{i=1}^{n} W_i = 1;$ and $W_i > 0 \;\; \forall i \in N$;
2. Only leverage is allowed; $W_i > 0 \; \forall i \in N$;
3. Only short selling is allowed; $\sum_{i=1}^{n} W_i = 1; \quad \forall i \in N$
4. Leverage and short selling allowed i.e. no limit on leverage and short selling.

With these four types of financial constraints, we investigate the portfolio risk-reward trade-off of each fund manager whose aim is to maximize the portfolio expected return and minimize the overall risk simultaneously. We solve a bi-objective portfolio optimization problem using a genetic algorithm method. We therefore combine the two objective functions above into one in such a way that the constant k represents the aversion (risk attitude) toward the expected return and the constant c represents the aversion toward the downside risk:

- Interval-valued utility function

$$\operatorname{Min} z = -k(0.8947W1 + 1.0012W2 + 0.8847W3 + 0.4758W4 + 0.7983W5 + 0.86W6 + 0.7832W7) + c(3.4761W1 + 5.0476W2 + 6.6818W3 + 3.0573W4 + 3.6373W5 + 3.5406W6 + 1.5675W7)$$

- Possibility-based utility function

$$\operatorname{Min} z = -k(0.9854W1 + 1.055W2 + 1.052W3 + 0.5407W4 + 0.8737W5 + 0.8002W6 + 0.8155W7) + c(2.5714W1 + 3.8234W2 + 4.9724W3 + 2.2512W4 + 2.7527W5 + 2.6448W6 + 1.1783W7)$$

Each objective function is subject to each one of the four constraints above, making a total of four optimization problems. For example, under the first constraint of neither leverage nor short selling allowed, we can solve the bi-objective function of interval-valued problem and obtain a set of optimal weights using a numerical optimization method known as the genetic algorithm optimization technique.

The solution to the bi-objective problem corresponding to interval-valued and possibility respectively are exhibited in Table 5. Our choice of risk aversion coefficients k and c follows the study by Waggle et al. (2005), who found that reasonable values of $k\,(c)$ should be in the range of 1 to 10. They classify an aggressive investor as having a risk aversion coefficient between 1 and 2, a moderate investor one of between 2 and 5, and a conservative investor one of between 5 and 10. They classify an investor with a risk aversion coefficient of 3 as an average investor. In this study we assume that fund managers are classified as aggressive (risk taker) investors with a coefficient of risk aversion between [1, 2].

Table 5: Interval-valued optimal weights

k=c=1	ED	EH	EM	FoF	FWC	MCRO	RV
No leverage no short sell	0.3306	0.0246	0.0285	0.278	0.0284	0.0285	0.2803
Leverage only	0.0147	0.0726	0.0083	0.009	0.0266	0.0256	0.022
Short selling only	1.8202	-2.552	-3.448	2.492	-1.503	-2.76	6.9491
Leverage and short sell	-10.49	-14.66	-17.17	-13.7	-6.045	-6.77	-4.886

This table highlights the difference between financial constraints: for an absolute risk taker fund manager; no leverage no short selling, and we leverage only constraint strategies, which produce positive holdings as shown in Table 5 and Table 6, whereas the rest of constraint strategies result in negative holding.

Table 6: Possibility-based optimal weights

k=c=2	ED	EH	EM	FoF	FWC	MCRO	RV
No leverage no short sell	0.3719	0.0528	0.064	0.0601	0.0644	0.072	0.3138
Leverage only	0.0308	0.0039	1.E-04	0.001	0.0133	0.0058	0.0025
Short selling only	1.5527	-4.379	-4.632	1.324	-2.917	1.4376	8.6125
Leverage and short sell	-11.1	-17.52	-13.61	-11.1	-4.64	-11.24	-1.46

The expected portfolio return and downside risk corresponding to each type of financial constraint is shown in Table 7.

Table 7: Portfolio expected return, downside risk and Portfolio Performance Measure

		No leverage no short sell	Leverage only	Short selling only	Leverage and short sell
	Return	*0.7446*	*0.1580*	*-0.9203*	*-60.2176*
Intv-value	Risk	*2.9576*	*0.7230*	*-26.3166*	*-320.5387*
	PI	**0.2518**	**0.2185**	**0.0350**	**0.1879**
	Return	0.8287	0.0498	-0.8099	-59.2812
Possibility	Risk	2.3488	0.1527	-26.8723	-232.3501
	PP	**0.3528**	**0.3261**	**0.0301**	**0.2551**

From Table 7 the possibility portfolio selection exhibits higher expected returns (except for leverage only portfolio) than the interval-valued portfolio. When we consider the downside risk we find the same results i.e. possibility portfolio provides lower downside risk than the interval-valued portfolio (except for the leverage and short sell portfolio). Hence the portfolio selection problem based on possibility theory generates optimal weights that are better than those generated with the interval-valued theory. The no leverage no short selling constraint strategy provides the highest rate of return, while the leverage and short selling constraint strategy provides the lowest downside risk for the possibility distribution model. The portfolio performance is measured by PI (interval valued portfolio performance) and PP (possibility portfolio performance) measures defined in equations 42 and 43. These measures reveal that portfolio selection based on possibility distribution outperforms the interval-valued problem except for the case where the manager is not allowed to use leverage.

In Table 8 we highlight a set of investment strategies found to be optimal in this study.

Table 8: Optimal strategies per optimization techniques

Method	Optimal Strategies	Port. Retrun	Port. Risk	Comb. Weights
Possibility	ED,RV,MCRO,FWC	0.83%	2.35%	82%
Interval	ED, RV, FoF	0.74%	2.96%	89%
Bayesian	EH, EM, RV, FWC	16.79%	2.62%	75.70%
Black-Litterm.	RV, MCRO	0.75%	1.21%	100%
Mean-var.	RV, ED, MCRO	0.80%	1.41%	100%

Different portfolio selection techniques, namely the fuzzy possibility theory, the fuzzy interval-based theory, the Bayesian skew t distribution, the Black-Litterman and the Mean-variance models are used in order to investigate the optimality of hedge fund investment strategies. Table 6.11 shows a set of optimal strategies obtained with a given optimization method and its corresponding portfolio mean and risk. In the last column of Table 6.11 we show the combined weight of these optimal strategies; for example the optimal weight for the Black-Litterman model is RV and MCRO with 0.75% and 1.21% portfolio return and risk respectively. The two investment strategies represent a combined total weight of 100%. Notice that these optimal investment strategies are ranked by weight in descending order i.e. the first optimal strategy has the highest weight followed by the second, etc. In the previous example of Black-Litterman model optimal strategies (RV and MCRO) RV has the highest weight followed by MCRO.

Our results show that the Bayesian skew t distribution portfolio selection model provides a highly diversified portfolio with a rate of return equal to 16.79%. The optimal set of investment strategies is made up of four out of seven investment strategies, namely EH, EM, RV and FWC. The four strategies represent 75.7% of total weight. There are two investment strategies that make a big difference between the solution obtained with the Bayesian skew t distribution method and other optimization methods. The two investment strategies are EH and EM; these two strategies appear only in the set of optimal strategies obtained with Bayesian skew t distribution method. These results show that equity market investments in emerging markets (China India, Brazil, South Africa, etc) though riskier, can provide potentially high growth more than any other investment strategy if one understands better how to deal with risk and uncertainty in these markets.

CONCLUSION

This paper aimed at testing the validity of the EMH by making use of the performance analysis of hedge fund return. The main objective was to determine whether fund managers have genuine skills to outperform the market; and if they do have those skills, what are the main investment strategies they use to outperform the market? To reach this objective, monthly returns on hedge fund indices collected from Hedge Fund Research group were considered for the period between January 1995 and June 2012. With this main objective in mind, we decided to divide our entire sample into four overlapping sub-samples and see whether skilled fund manager would consistently outperform the market in these different sub-sample periods.

We based our inferences on the efficient market hypothesis as a prediction model by assuming that the market is efficient and that fund managers cannot outperform it. we used the CAPM (Sharpe, 1964) and quadratic CAPM (Treynor and Mazuy, 1966) and developed three different econometric models namely the frequentist, the Bayesian, and the fuzzy credibility models in order to estimate the outperformance, the selectivity and the market timing coefficients. The first two models are referred to as probabilistic models while the last one as uncertainty model. Using three different techniques widely applied in hedge fund performance analysis (see for example Naik, 2009, Agarwal, 2000): contingency table, chi-square test, and cross-section regression we find the following results. probabilistic model show that fund managers have skills to outperform the market this market outperformance is due to market timing skill and to selectivity skill. The set of optimal investment strategies they use to outperform the market is made up of equity hedge, emerging markets, relative values, and (funds of weighted) currencies investment.

However, fuzzy set based model shows that although there are few managers with little selectivity skill; fund managers do not have enough skills to outperform the market because of lack of market timing. We investigate the set of optimal investment strategies used under the assumption of the EHM, and found that the weights were spread across all investment strategies resulting in a well-diversified portfolio whose expected return is fairly not above the efficient frontier of the Markowitz mean-variance optimisation.

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APPENDIX

1. **ED:** HFRI Event-Driven (Total) Index
   - HFRI ED: Distressed/Restructuring Index: **ED_RES**
   - HFRI ED: Merger Arbitrage Index: **ED_MA**
   - HFRI ED: Private Issue/Regulations D Index: **ED_PVT**
2. **EH:** HFRI Equity Hedge (Total) Index:
   - HFRI EH: Equity Market Neutral Index: **EH_EMN**
   - HFRI EH: Quantitative Directional: **EH_QUANT**
   - HFRI EH: Sector - Energy/Basic Materials Index: **EH_ENERG**
   - HFRI EH: Sector - Technology/Healthcare Index: **EH_TECH**
   - HFRI EH: Short Bias Index: **EH_SBIAS**
3. **EM:** HFRI Emerging Markets (Total) Index:

- HFRI Emerging Markets: Asia ex-Japan Index: **EM_ASIA-JP**
- HFRI Emerging Markets: Global Index: **EM_GLOBAL**
- HFRI Emerging Markets: Latin America Index: **EM_LAT_AM**
- HFRI Emerging Markets: Russia/Eastern Europe Index: **EM_EAST-EU**

4. **FoF:** HFRI Fund of Funds Composite Index:
   - HFRI FOF: Conservative Index: **FoF_CONSV**
   - HFRI FOF: Diversified Index: **FoF_DIVERS**
   - HFRI FOF: Market Defensive Index: **FoF_MKT-DFENS**
   - HFRI FOF: Strategic Index: **FoF_STRATG**
5. **FWC:** HFRI Fund Weighted Composite Index:
   - HFRI Fund Weighted Composite Index CHF: **FWC_CHF**
   - HFRI Fund Weighted Composite Index EUR: **FWC_EUR**
   - HFRI Fund Weighted Composite Index GBP: **FWC_GBP**
   - HFRI Fund Weighted Composite Index JPY: **FWC_JPY**
6. **MCRO:** HFRI Macro (Total) Index:
   - HFRI Macro: Systematic Diversified Index: **MCRO_SYST-DIV**
7. **RV:** HFRI Relative Value (Total) Index:
   - HFRI RV: Fixed Income-Asset Backed: **RV_FIAB**
   - HFRI RV: Fixed Income-Convertible Arbitrage Index: **RV_FICA**
   - HFRI RV: Fixed Income-Corporate Index: **RV_FICORP**
   - HFRI RV: Multi-Strategy Index: **RV_MSTRAT**
   - HFRI RV: Yield Alternatives Index: **RV_YEILDA**

NB: each of these investment strategies has 186 observations corresponding to 186 months that have been divided into four sub-sample periods.