Modelling the short-term interest rate with stochastic differential equation in continuous time: linear and nonlinear models

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Abstract

Recently, financial engineering has brought a significant number of interest rate derivative products. Amongst the variables used in pricing these derivative products is the short-term interest rate. This research article examines various short-term interest rate models in continuous time in order to determine which model best fits the South African short-term interest rates. Both the linear and nonlinear short-term interest rate models were estimated. The methodology adopted in estimating the models was parametric approach using Quasi Maximum Likelihood Estimation (QMLE). The findings indicate that nonlinear models seem to fit the South African short-term interest rate data better than the linear models

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1. INTRODUCTION

The bond market has been experiencing a significant progress in recent years. This market has started to even overtake the stock market, which used to be the main market for raising funds. This was observed by the immense increase in the trading volume of fixed income securities and derivatives, Fan (2005). The market has also begun to play a more prominent role in the South African market. Aling and Hassan (2012) concurred and argued that the South African bond market has become one of the largest amongst the emerging markets, and it has become the world's sixth most liquid turnover market. Svoboda (2002) further adds that growth in this market has brought with it an ever-increasing volume and range of interest rate dependent derivative products known as interest rate derivatives.

Amongst the variables used in pricing the derivative products is the short-term interest rate. Short term interest rate is complex to model as it comes with different properties. One of the properties being that it follows stochastic process, which present random variable that changes overtime. Such processes are then modelled in continuous time, which explains why most of the short-term interest rates are set in continuous time framework.

An amount of work on modelling the short-term interest rate has been performed with the intention of understanding its stochastic behaviour. More work was also done in determining the model that can capture particular features of observed interest rate movements using different datasets. This is because short-term interest rate serves as a more fundamental instrument in many financial applications. For instance, Chan, Karolyi, Longstaff and Sanders (1992), Longstaff and Schwartz (1992) compared the performance of eight parametric short-term interest rate models using US Treasury Bill to determine how they capture the stochastic behaviour of the short-term interest rate. Likewise, Sanford and Martin (2006) used Australian data to compare alternative single-factor models that can fit the Australian data.

Niizeki (1998) further utilised Japan and United Kingdom (UK) to fit various models, and found that Constant Elasticity of Volatility (CEV) model explains the UK short-term interest rate better, while Vasicek (1977) model was found to be better in explaining the Japanese short-term interest rate. Sun (2003) compared single-factor interest rate models in five countries (US, UK, Canada, Germany and Japan) and found different results across countries. In addition to that, Gray and Treepongkaruna (2006) made a comparison in eleven countries (US, UK, Japan, France, Germany, Italy, Switzerland, Australia, Hong

Kong, Singapore and Thailand) and came with the same conclusion as Sun (2003) that different markets require different models.

It is increasingly clear that a majority of the above-mentioned countries are developed countries. To the researcher's knowledge, South Africa is also one of the countries where limited research has been conducted on understanding the dynamics of short-term interest rates. Owing to the fact that more studies on modelling the short-term interest rates have focused on a few lead countries, the interest rate characteristics of those countries are well known. For example, it is well known that US interest rate datasets exhibit a mean reversion and non-constant volatility. However, it is difficult to confidently state the characteristics for most developing countries due to limited studies conducted. It thus remains essential for developing countries to start understanding the dynamics followed by the short-term interest rate of their countries so that their statistical features can be known. More precisely, the importance for each country to conduct such a study comes from the fact that no country can rely on the model that fits other countries, as the dynamics and context of countries differ.

The rest of the paper will be structured as follows: Section 2 will be devoted to reviewing various literatures that have been conducted with almost exclusive focus on linear models. Section 3 will introduce the data and methodology of the study. Section 4 will report on the results. Finally, Chapter 5 will summarise the key findings.

2. THEORETICAL BACKGROUND

This section describes the theory behind the short-term interest rate modelling, starting with the expression of the continuous time models. Continuous-time models are presented in the form of stochastic differential equation (SDE), where SDE is a mathematical equation used to model the stochastic process in continuous time.

$$dX_t = \mu(X_t, \theta)dt + \sigma(X_t, \theta)dW_t, \qquad (1)$$

Equation (1) consists of two components; the first being the drift (conditional mean) and the second being the diffusion (conditional variance) function. Applying the SDE to the short-term interest rate requires one to simply put the specifications on the drift and diffusion. The drift is typically specified as linear, nonlinear, or constant, while the specification for the diffusion is either constant or heteroskedastic. Various models are uncovered through applying these different specifications, which is what differentiate these models. A number of single-factor models are illustrated in table 1, with more description found in Annexure 1.

Table 1: Linear and nonlinear short-term interest rate theoretical models

Models	Models Specifications
Merton (1973)	$dr_t = (\alpha_0)dt + \beta_2 dW_t$
CEV[1] (1975)	$dr_t = (\alpha_1 r_t)dt + \beta_2 r_t^{\beta_3} dW_t$
Vasicek (1977)	$dr_t = (\alpha_0 + \alpha_1 r_t)dt + \beta_2 dW_t$
Dothan (1978)	$dr_t = \beta_2 r_t dW_t$
B & S[2] (1980)	$dr_t = (\alpha_0 + \alpha_1 r_t)dt + \beta_2 r_t dW_t$
GBM[3] (1983)	$dr_t = (\alpha_1)dt + \beta_2 r_t dW_t$
CIR[4] (1985)	$dr_t = (\alpha_0 + \alpha_1 r_t)dt + \beta_2 r_t^{1/2} dW_t$
CKLS[5] (1992)	$dr_t = (\alpha_0 + \alpha_1 r_t)dt + \beta_2 r_t^{\beta_3} dW_t$
AS[6] (1996)	$dr_t = \left(\alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \frac{\alpha_3}{r_t}\right)dt + \beta_0 + \beta_1 r_t + \beta_2 r_t^{\beta_3} dW_t$
CHLS[7] (1997)	$dr_t = \left(\alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \frac{\alpha_3}{r_t}\right)dt + \beta_2 r_t^{\beta_3} dW_t$
AG[8] (1999)	$dr_t = (\alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2)dt + \beta_2 r_t^{1/2} dW_t$

Short-term interest rate models consist of a number of parameters, and each parameter has an intuitive meaning. These parameters are explained by using CKLS as a special case model since it has been used widely in the literature.

[1]Constant Elasticity of Variance

[2]Brennan and Schwartz

[3] Geometric Brownian Motion

[4]Cox-Ingersoll-Ross

[5]Chan, Karolyi, Longstaff and Sanders

[6]Ait-Sahalia

[7]Conley, Hansen, Luttmer and Scheinkman

[8]Ahn and Gao

$$dr_t = (\alpha_0 + \alpha_1 r_t)dt + \beta_2 r_t^{\beta_3} dW_t \quad (2)$$

CKLS model, as presented in equation (2), is made up of the drift and diffusion.

Drift parameters are given as follows:

α_0= drift

$-\alpha_1$ = mean reversion (speed of adjustment)

${}^{\alpha_0}/_{\alpha_1}$ = long-run mean of the short-term interest

Mean reverting means that the process tends to revert back to its constant long-run mean. More specifically, ${}^{\alpha_0}/_{\alpha_1}$implies that, when α_1 has larger values, the response of the short-term rate to any deviation from the long run will be quick relative to when the value is small (Koedijk et al., 1997).

Diffusion parameters are given as follows:

β_2 = volatility of the short-term rate

β_3 = level effect of the short-term rate

The level effect of the short-term rate allows volatility to depend on the level of interest rate. In instances where $\beta_3 > 1$, the short rate becomes highly sensitive to the level of interest rate and often leads to a non-stationary process. When the level effect is zero, it makes the variance component to be constant.

Finally, the stochastic process is modelled using dW_t, which is a Wiener process used to model random movements in financial engineering, where $dW_t = \varepsilon(t)dt$ and $\varepsilon(t)$ is the white noise (generalised stochastic process).

Annexure 2 and 3 illustrate the restrictions imposed on various models. A clear depiction made from Annexure 3 and 4 is that the most suitable general linear model is the CKLS, as it nests all the linear models. The AS model, on the other hand, is a general model for both the linear and nonlinear models, as it nests all the linear and nonlinear models. AS is the unrestricted model for all models, while CKLS being an unrestricted model for linear models.

2.1 EMPIRICAL LITERATURE ON THE SHORT-TERM INTEREST RATE

The work of Merton (1973) and Black and Scholes (1973) laid the foundation for the theory of pricing derivatives securities using continuous time models. Later on developments of short-term interest rate models which were also set in continuous time increased. These models were developed with the aim of obtaining better results that can explain the behaviour of interest rates. Hong, Li and Zhao (2004) categorised these models into single-factor models, where the level of interest rate is the only factor allowed to affect the short-term rate. GARCH models, which model persistence volatility clustering in interest rates. Markov Regime-Switching models, which capture the time-varying behaviour of interest rates such as business cycle and changes in monetary policy. Finally, the Jump-diffusion models, which caters for economic shocks, government interventions and news announcements. The GARCH models, Markov Regime-Switching models and the Jump-diffusion models are extended from the single-factor models.

Single-factor models were the first arbitrage free factor models to be used in the history of short-term interest rate models. Jiang (1998) favoured the single-factor models for the reasons that these models offer a stable and consistent model with a parsimonious structure for the fundamental behaviour of interest rates and term structure, they are easy to implement from a computational point of view and also that they provide sanity checks on complex models. Critics came from Hong et al. (2004) who argued that the single-factor models are unable to capture the rich behaviour of interest rate volatility. In addition, Jones (2003) affirms that single-factor models are unsatisfactory in their description of short-rate dynamics, and their implications for other security prices are severe. In addition to the single-factor model, other models suggested such GARCH models, Markov Regime-Switching models Jump models, and non-parametric regression (Muteba Mwamba, 2011) had some advantages and also received critics.

The extended models (GARCH, Markov Regime-Switching and Jump) often containing more complex data-generating processes and are complicated to model as compared to single-factor models. The complexity arising from over-parameterising as they contain more parameters than simple models. Hong et al. (2004) stressed that an extensive search for more complicated models that are over-parameterised could lead to excessive in-sample data snooping, and the resulting model might not work well in an out-sample forecast. The question of which models to choose, amongst others, poses a serious concern. Models can be selected, but what is vital is to know which model is more appropriate than the other in various cases. Chapman and Pearson (2000) also mentioned that determining the appropriateness of the models also comes down in the estimation results of the drift and diffusion functions.

Estimations of the drift and diffusion parameters are the most critical steps in any SDE modelling. The most common assumption on the drift is the linear mean reversion property. Hull (2009) elaborated that, it is expected for the interest rate to experience a mean reversion, as when interest rates goes up or down, this rate will always go back to its long-run mean reversion through the Central Bank's intervention.

However, the mean-reversion characteristic is hardly observed throughout the entire distribution. Stanton (1997) and Jiang (1998) evidenced that the short-rate exhibits very little mean reversion or behave like a random walk below the 14% level but have an extreme mean reversion beyond that. Moreover, Conley, Hansen, Luttmer and Scheinkman (1997) found a nonlinear drift, where the drift function was non-zero only for rates below 3% or above 11%. Ait-Sahalia (1996) also found a nonlinear drift since the interest rate behaves like a random walk over the entire historical range, and then reverts towards the middle of this range only when the rates become exceptionally low or exceptionally high.

Jones (2003) was amongst those who favoured the nonlinearity as he believed nonlinearity to be an indispensable and most relevant feature for many economic issues. Furthermore, he argued that, nonlinearity can offer a potential improvement in fixed income pricing, as it has the potential to explain a number of the outstanding puzzles about the term structure. Gray and Treepongkaruna (2006) also concurred with Jones' (2003) view by stressing that models with nonlinearity in both drift and diffusion are needed to fully capture the important features of the behaviour of a short-term rate.

In South Africa, studies conducted thus far include Aling and Hassan (2012), who compared selected single-factor linear drift models to determine which of these models fit the South African interest rate data. In addition, Svoboda (2002) investigated various interest rate models and their calibration in the South African market, with special focus on the development of interest rate models. It should be noted that these few studies conducted in South Africa focused only on comparing the single-factor short-term interest rate linear models and none on nonlinear models. Nonlinearity is one of the fundamental issues that came out often in the literature and majority of the studies mostly assume linear specifications and left out nonlinearity. This paper then aims to extend part of the work that has been conducted, more in particular by fitting the nonlinear models which are yet to be widely explored in developing countries. Thus, it will provide the first comprehensive empirical analysis in this research area in South Africa.

This study will thus answer the following questions:

- Whether linear or nonlinear short-term interest rate models fit the South African interest rate data.
- Which short-term interest rates models performs better between the linear and nonlinear models?
- What are the key drift and diffusion features that capture the South African interest rate data?

3. DATA AND METHODOLOGY

3.1 Data Analysis

The study is conducted using three types of South African interest rate time series, namely three months Treasury Bill (TBR3), Repurchase rate (REPO), and the Johannesburg Interbank Agreed Rate (JIBAR). These are the commonly used interest rates in South Africa, with the TBR3 being used as a proxy for the short-term risk-free rate in South Africa. All the data series were sourced from I-Net Bridge[9]. The frequency used in all interest rates was weekly, with the sample period covering from the third week of March 1998 to the second week of April 2013.

The time series trends are also plotted to visualise how interest rates evolve with time.

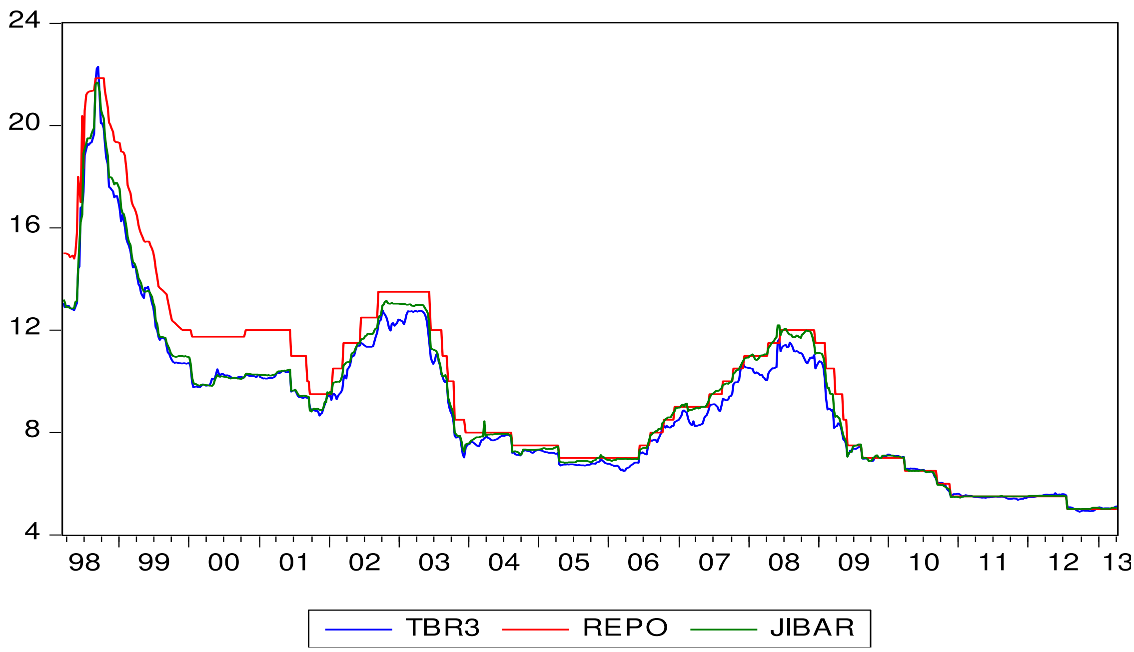

Figure 1: Short-term Interest Rates Evolution

[9] I-Net Bridge is a South African Financial Service company, with the core business of providing economic data, financial market data, and corporate market intelligence in South Africa, (en.wikipedia.org)

Note: The time series evolution covers three interest rates. For all the interest rates, the data starts from March 1998 to April 2013.

As illustrated in Figure 1, interest rates moved together in the same direction even though there was a slight timing difference in their movements. These three interest rates series reached their respective historical high levels during mid-1998, mid-2001 and mid-2008. The trend in mid-1998 and mid-2001 corresponds to the rand crises that took place in South Africa. Since it was the same crisis that occurs in different periods, they were thereafter named the first and second episode of the rand crises.

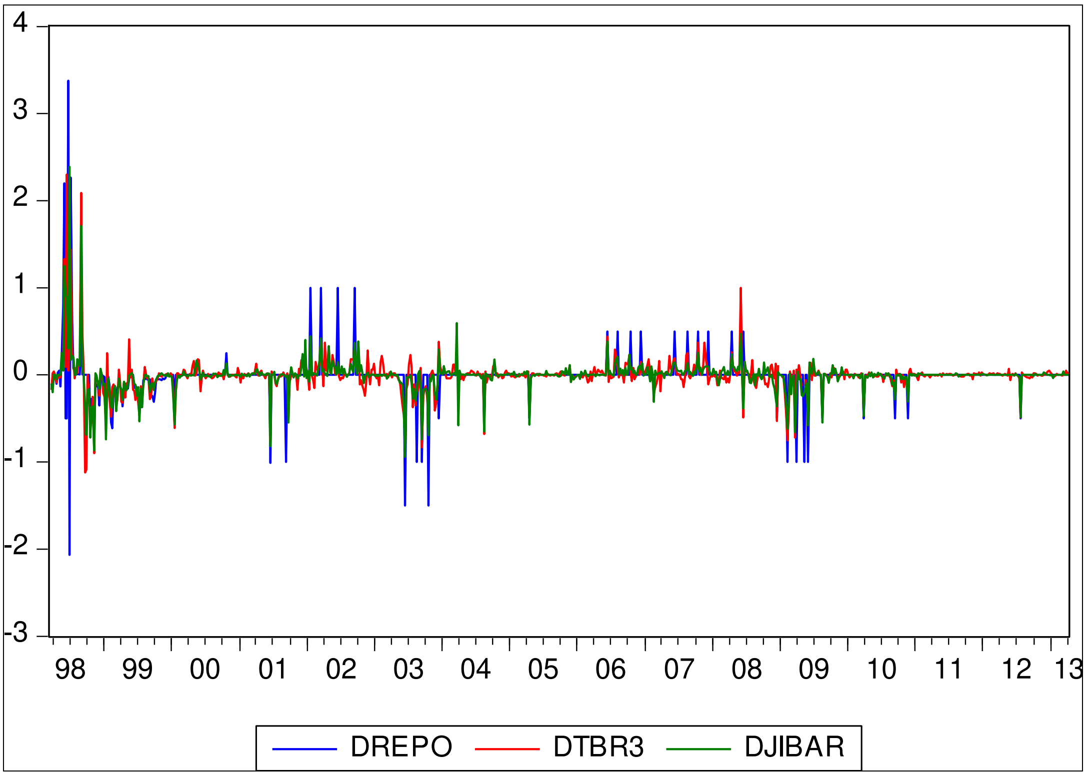

Figure 2: Differenced Short-term Interest Rates Evolution

Note: The data of the original time series was transformed to assess the change from period to period. Transforming entails taking the difference of current and previous period data.

Figure 2 represents the short-term interest rates after transforming the data using the first difference. As compared to Figure 1, the persistence of autocorrelation disappears after taking the first difference. High interest rates are suddenly followed by low interest rates in all cases. From a monetary policy perspective, it is known that when interest rates reach their historical heights, there is less demand. To further stimulate the economy, central banks intervene by taking dramatic measures of controlling inflation. The bank does so by indirectly reducing the interest rates – the opposite also applies. Such a concept explains why short-term interest rates often fall after reaching their highest levels.

The unit root and autocorrelation tests were run on the data using Augmented Dickey-Fuller (ADF) and Phillips and Perron (PP) tests, (Annexure 4). The ADF and PP test shows that the null hypopaper of unit root was rejected which implied that the interest rates are stationary. After taking the first difference, all the rates which were non-stationary became stationary. Meanwhile, Autocorrelations were assessed using the Autocorrelation function (ACF). The p-values from the Autocorrelation were all less than 0.05, which means that the null hypopaper of stationary was rejected, and concluded that the data was non-stationary.

3.2 Descriptive Statistics

A descriptive statistics for the three months Treasury Bill (TBR3), Repo rate (REPO) and Johannesburg Interbank Agreed Rate (JIBAR) together with their graphs in levels and differences is outlined in (Annexure 5). All the interest rates demonstrate positive skewness, which confirmed that the interest rates were not normal. Moreover, the p-values of the Jarque-Bera were also less than 0.05, which confirmed that the null hypopaper of normality distribution should be rejected. The non-normality distributions were also supported by the higher statistic moments such as positive skewness and leptokurtic behaviour.

3.3 Methodology

In this study, eight short-term interest rate linear models together with three nonlinear models are assessed (table 1). Selected short-term interest rate linear models estimated were Merton (1973), Vasicek (1977), CIR (1985) and CKLS, (1992), GBM (1983), B & S (1980), CEV (1975) and Dothan (1978). In addition to these linear models, the well-known nonlinear models estimated were AS (1996), CHLS (1997), and AG (1999).

Interest rate models have traditionally been expressed and modelled in continuous time, as in table 1. However, most financial data are available in discrete time. Since it is impossible to model continuous time equations practically, as a first step, the continuous time models were discretised using Euler-Maruyama method to approximate the continuous time models. Discretisation is the processes used to convert a continuous time equation into a form that can be used to obtain numerical solutions. The discretised models are represented in table 2.

Table 2: Discretised short-term interest rate model equations

Models	Discretised Equations
Merton (1973)	$r_{t+1} - r_t = (\alpha_0)\Delta t + \beta_{2(t+1)} r_t \sqrt{\Delta t}\varepsilon_{t+1}$
CEV (1977)	$r_{t+1} - r_t = (\alpha_1 r_t)\Delta t + \beta_{2(t+1)} r_t^{\beta_3} \sqrt{\Delta t}\varepsilon_{t+1}$
Vasicek (1977)	$r_{t+1} - r_t = (\alpha_0 + \alpha_1 r_t)\Delta t + \beta_{2(t+1)} \sqrt{\Delta t}\varepsilon_{t+1}$
Dothan (1978)	$r_{t+1} - r_t = \beta_{2(t+1)} r_t \sqrt{\Delta t}\varepsilon_{t+1}$
GBM	$r_{t+1} - r_t = (\alpha_1 r_t)\Delta t + \beta_{2(t+1)} r_t \sqrt{\Delta t}\varepsilon_{t+1}$
B & S (1980)	$r_{t+1} - r_t = (\alpha_0 + \alpha_1 r_t)\Delta t + \beta_{2(t+1)} r_t \sqrt{\Delta t}\varepsilon_{t+1}$
CIR (1985)	$r_{t+1} - r_t = (\alpha_0 + \alpha_1 r_t)\Delta t + \beta_{2(t+1)} r_t^{\frac{1}{2}} \sqrt{\Delta t}\varepsilon_{t+1}$
CKLS (1992)	$r_{t+1} - r_t = (\alpha_0 + \alpha_1 r_t)\Delta t + \beta_{2(t+1)} r_t^{\beta_3} \sqrt{\Delta t}\varepsilon_{t+1}$
AS (1996)	$r_{t+1} - r_t = \left(\alpha_0 + \alpha_1 r_t + \alpha_0 r_t^2 + \frac{\alpha_3}{r_t}\right)\Delta t$ $+ \beta_0 + \beta_1 r_t + \beta_{2(t+1)} r_t^{\beta_3} \sqrt{\Delta t}\varepsilon_{t+1}$
CHLS (1997)	$r_{t+1} - r_t = \left(\alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2 + \frac{\alpha_3}{r_t}\right)\Delta t + \beta_{2(t+1)} r_t^{\beta_3} \sqrt{\Delta t}\varepsilon_{t+1}$
AG (1999)	$r_{t+1} - r_t = (\alpha_0 + \alpha_1 r_t + \alpha_2 r_t^2)\Delta t + \beta_{2(t+1)} r_t^{1/2} \sqrt{\Delta t}\varepsilon_{t+1}$

Note: The continuous time short-term interest rate equations were discretised so that numerical solutions of the parameters can be obtained. ε_t is the error term and assumed to be IID~N(0,1), while Δt is the time between each interval. The approximation will be more accurate if Δt is small.

Once the continuous time equations were discretised, the Quasi Maximum Likelihood Estimation (QMLE) technique was employed to obtain the parameters which were estimated using R[10] programme. Unlike the maximum likelihood, which should strictly be based on the

[10] R programme is a language and environment for statistical computing and graphics. It provides a wide variety of statistical (linear and nonlinear modelling, classical statistical tests, time-series analysis, classification, clustering) and graphical techniques, and is highly extensible (http://www.r-project.org)

correct distribution, this technique allows a departure from the true distribution. The method itself entails finding the most likely value for the parameter based on the dataset available.

Other tests conducted were the Likelihood Ratio Test (LRT) and Akaike Information Criterion (AIC) and Schwartz Bayesian Information Criterion (SBIC). LRT was used to test the parameter restrictions reported. This test is a convenient way of checking whether certain parameter restrictions are supported by the data through comparing the restricted and unrestricted models. LRT was conducted separately on the linear models, and thereafter on linear models combined with nonlinear models in order to understand their statistical significance and the effect of adding more parameters on the models. LRT makes use of the estimated maximum log-likelihood values from the models, as illustrated in equation 3.1. For each test, the log-likelihood values for the unrestricted and restricted models were used. The LRT equation is defined as follows:

$$LRT = -2\,log\left(\frac{L_R}{L_u}\right) = 2[log(L_u) - log(L_R)] \sim \chi^2_m \qquad (3)$$

where

m is the number of restrictions imposed

L_R is the log-likelihood for restricted model

L_u is the log-likelihood model for unrestricted model

The hypopaper test was checking whether restrictions imposed were valid. The null hypopaper being that restrictions are valid, while the alternative being that restrictions are not valid and are statistically significantly different from the imposed restriction. The decision of whether to reject or not to reject the null hypopaper was based on the chi-square values and their critical values.

Meanwhile, two of the information criteria used were AIC and SBIC. These methods also rely on the estimated log likelihood and the number of parameters. It is known that a model with more parameters is more likely to fit the in-sample data better than the restricted model; these methods perform the same task of penalising models with more parameters. Unlike the AIC, the SBIC imposes a larger penalty on additional parameters than AIC. SBIC was also used as an additional criterion to overcome such problems as the models used in this paper, which have different numbers of parameters.

$$AIC = 2k - 2log(L) \qquad (4)$$

$$SBIC = klog(n) - 2logL \qquad (5)$$

where

n is the number of observations
k is the number of free parameters

Models with the smallest AIC and SBIC are preferred. With AIC, the lowest model to be selected implies that it is closer to the true estimates. Meanwhile, SBIC, which is a Bayesian measure, implies that the lowest model to be chosen is more likely to be true.

4. RESULTS

The results cover parameter estimations for three interest rates for various models, the likelihood ratio test, and the AIC and BIC results.

Table 3: TBR3 Parameter estimates for the short-term interest rate models

	α_0	α_1	α_2	α_3	β_0	β_1	β_2	β_3	Log-likelihood
Merto	-0.01	-	-	-	-	-	0.21	-	-779.24
n	(-1.37)	-	-	-	-	-	(39.4)***	-	
CEV	-	-0.001	-	-	-	-	-0.01	1.500	-838.78
	-	(-1.42)	-	-	-	-	(-30.)***	(97.98)***	
Vasic	0.01	-0.002	-	-	-	-	0.24	-	-225.25
	(0.37)	(-0.8)	-	-	-	-	(31.6)***	-	
CIR	0.006	-0.002	-	-	-	-	-0.07	-	-523.61
	(0.28)	(-0.69)	-	-	-	-	(-30.)***	-	
Doth	-	-	-	-	-	-	-0.02	-	-776.44
	-	-	-	-	-	-	(-47)***	-	
GBM	-	-0.001	-	-	-	-	-0.02	-	-737.92
	-	(-1.41)	-	-	-	-	(-35)***	-	
B & S	0.004	-0.001	-	-	-	-	-0.02	-	-721.29
	(0.22)	(-0.61)	-	-	-	-	(32.9)***	-	
CKLS	0.005	-0.002	-	-	-	-	0.01	1.39	-802.92
	(0.26)	(-0.63)	-	-	-	-	(47.1)***	(68.5)***	
CHLS	-0.00	-0.00	-0.00	-0.00	-	-	0.018	1.81	1223.48
	(0.00)	(-0.00)	(-0.01)	(-0.00)	-	-	(0.00)	(25.30)**	

								*	
A & G	-0.000	-0.000	-0.00	-	-	-	-0.034	-	1239.06
	(-0.00)	(-0.01)	(-0.02)	-	-	-	(0.00)	-	
AS	-0.000	-0.000	-0.00	-0.001	-0.64	0.18	-2.73	-5.00	1234.57
	(-0.00)	(-0.00)	(-0.01)	(-0.00)	(0.00)	(0.0)	(0.00)	(-345)***	

Table 4: REPO Parameter estimates for the short-term interest rate models

	α_0	α_1	α_2	α_3	β_0	β_1	β_2	β_3	Log likelihood
Merton	-0.013	-	-	-	-	-	0.258	-	-424.66
	(-1.4)	-	-	-	-	-	(39.)***	-	
CEV	-	-0.001	-	-	-	-	0.0081	1.459	-507.64
	-	(-1.5)	-	-	-	-	(73.)***	(88)***	
Vasice	0.013	-0.003	-	-	-	-	0.288	-	-115.55
k	(0.45)	(-1.0)	-	-	-	-	(33)***	-	
CIR	0.008	-0.002	-	-	-	-	0.0801	-	-180.57
	(0.315)	(-0.8)	-	-	-	-	(32.)***	-	
Dothan	-	-	-	-	-	-	-0.020	-	-422.01
	-	-	-	-	-	-	(-46)***	-	
GBM	-	-0.001	-	-	-	-	-0.023	-	-399.11
	-	(-1.45)	-	-	-	-	(-36.)***	-	
B & S	0.005	-0.001	-	-	-	-	-0.024	-	-389.24
	(0.239)	(-0.7)	-	-	-	-	(34.)***	-	
CKLS	0.004	-0.002	-	-	-	-	0.009	1.426	-487.45
	(0.202)	(-0.63)	-	-	-	-	(49.)***	(77)***	
CHLS	-0.000	-0.000	-0.000	-0.000	-	-	0.017	1.826	1424.81
	(-0.00)	(-0.00)	(-0.01)	(-0.00)	-	-	(0.000)	(29)***	
A & G	-0.003	-0.000	-0.000	-	-	-	-0.035	-	1444.83
	(-0.01)	(-0.00)	(-0.03)	-	-	-	(0.000)	-	
AS	-0.000	-0.001	-0.000	-0.000	-4.609	0.330	6.337	-0.407	1424.41
	(-0.00)	(-0.00)	(-0.00)	(-0.00)	(-0.00)	(-0.00)	0.000	-830***	

Table 5: JIBAR Parameter estimates for the short-term interest rate models

	α_0	α_1	α_2	α_3	β_0	β_1	β_2	β_3	Log-likelihood
Merton	-0.010	-	-	-	-	-	0.189	-	-864.31
	(-1.54)	-	-	-	-	-	(39.4)***	-	
CEV	-	-0.001	-	-	-	-	0.009	1.326	-864.99
	-	(-1.6)	-	-	-	-	(40.7)***	(64)***	
Vasice	0.004	-0.002	-	-	-	-	0.222	-	-357.80
	(0.169)	(-0.63)	-	-	-	-	(29.9)***	-	
CIR	0.002	-0.001	-	-	-	-	-0.065	-	-624.89
	(0.110)	(-0.58)	-	-	-	-	(-29)***	-	
Doth	-	-	-	-	-	-	-0.015	-	-860.22
	-	-	-	-	-	-	(-50)***	-	
GBM	-	-0.001	-	-	-	-	-0.019	-	-810.55
	-	(-1.57)	-	-	-	-	(-34.)***	-	
B & S	0.002	-0.001	-	-	-	-	-0.020	-	-790.57
	(0.099)	(-0.56)	-	-	-	-	(-31.)***	-	
CKLS	0.002	-0.001	-	-	-	-	0.011	1.291	-837.73
	(0.134)	(0.58)	-	-	-	-	(27.9)***	(51.2)***	
CHLS	0.009	-0.003	0.000	0.006	-	-	0.017	1.814	1267.58
	(0.004)	(0.01)	(0.58)	(-0.01)	-	-	(0.000)	(23.6)***	
A & G	-0.000	-0.001	-	-	-	-	-0.034	-	1296.17
	(0.002)	(-0.01	0.000	-	-	-	(0.00)	-	
			(0.01)						
AS	0.007	-0.003		0.005	-	0.31	6.322	-0.326	1277.01
	(0.00)	(-0.01)	0.000	(0.00)	5.000	8	(0.000)	(-99)***	
			(0.01)		(0.00)	(0.0)			

Note: Tables 3 to 5 report the parameter estimation of single-factor models, which includes the linear and nonlinear models. The estimated parameters represent the parameters of the drift and diffusion. These parameters were estimated using the Quasi Maximum Likelihood Estimation method. Numbers in parentheses are t-statistics. (***) represent significance level at 1%.

Tables 3 to 5 report parameter estimates from the discretised short-term interest rate models. The main elements to capture from these results are the mean reversion, the volatility performance and the impact of level effect on volatility. The estimates of mean reversion across the linear and nonlinear models represented by α_0, α_1 and α_2 are all found to be statistically insignificantly different from zero. As expected, the α_1 parameters in all models have negative values. Sun (2003) mentioned that the negative values ensure that the parameter is consistent with the interpretation that it represents the mean-reverting coefficient. Even though the sign is consistent with the theory, these coefficients still remain insignificant. This was also the case for nonlinear models, where their individual drift parameters all came out to be insignificant. These findings are similar to that of Chan et al. (1992), who concluded that there was a weak evidence of mean reversion in all the short-term interest rate models, implying that the drift component might not be as relevant as expected.

The diffusion (β_2) parameter results, on the other hand, are found to be highly significant, with their t-statistics ranging from 30 to 90 for all the linear models across different interest rates. With nonlinear models, different results are observed. Their diffusion parameters are all highly insignificant and even zero in some cases. These results might be attributed to over-parameterisation which has been shown to affect the significance of the estimates. Thus adding more parameter on the diffusion has lessened the significance of parameters. When reviewing models separately in order to determine where the diffusion parameters are more significant, tables 4.1, 4.2 and 4.3 reveal that models such as CKLS and CEV, which allow volatility to be a function of the level effect, have the highest volatilities.

In addition, the level effect (β_3), which measures the sensitivity of interest rate volatility with respect to the interest rate, was also intensely analysed. The level effect is represented in two situations. First, there are models that restrict the level effect to a particular value (CIR, Dothan and GBM). Secondly, there are models in which the level effect is estimated directly from the data (CKLS, CEV, AS and CHLS). The analysis of the level effect is mainly to check the dependence of volatility on the level effect.

An obvious observation across these models is that models tend to improve in the presence of level effect, regardless of whether the level effect is restricted by some values or it is estimated within the model. The CEV, CKLS, AS and CHLS are models that required the level effect parameter to be estimated by the data. Interestingly, they all reported values greater than one, with nonlinear models even higher than two. These values are higher as compared to the restricted level effect values on other models. Often, larger values of level effect imply non-stationary volatility. Comparison of the overall level effect models points that those with level effect less than one tend to be highly significant as compared to those with level effect of less and equals to one. Even when level effect is restricted to be less and equals to one, like in other models, the performance of these models are better than when the level effect is zero. This suggests that level effect is essential in modelling the interest rate dynamics in South Africa. Another interesting observation across the three interest rates is that REPO tends to differ significantly to the TBR3 and JIBAR. Values of REPO, in all cases, were found to differ highly with those of TBR3 and JIBAR.

In terms of the maximum log-likelihood values, as models becomes more complex, that is, moving from linear to nonlinear models, their log-likelihood values improve. Nonlinear models seem to have larger log-likelihood values as compared to the linear models. According to Das (2002), larger and positive values of log-likelihood are due to variance of conditional changes in interest rate which is of order Δt, and less than 1. These higher values suggest that nonlinear models provide better fit than linear models.

4.2 Likelihood Ratio Test

This test was conducted to test whether restrictions imposed by various models were statistically significantly different from their assumed parameter restrictions. The test makes use of the log-likelihood values as reported in tables 3 to 5. In conducting this test, the LRT for only the linear models was firstly considered in isolation in order to determine the validity of their parameter restriction with the CKLS model. Secondly, LRT was conducted across the linear and nonlinear models as linear models were found to be special cases of the nonlinear models.

Table 6: Likelihood Ratio Test for linear short-term interest rate models

	TBR3		REPO		JIBAR			
Model	LRT	P-value	LRT	P-value	LRT	P-value	d.o.f	Crit-value
Merton	47.37***	0.00045	125.58***	0.00006	53.15***	0.00035	2	5.99
CEV	71.72**	0.00019	40.37**	0.00061	54.52***	0.00034	1	3.84
Vasicek	1155.34***	0.00000	1206.00***	0.00000	959.86***	0.00000	1	3.84
CIR	558.61***	0.00000	613.76***	0.00000	425.76***	0.00001	1	3.84
Dothan	52.95***	0.00036	130.88***	0.00006	44.97***	0.00049	3	7.81
GBM	130.00***	0.00006	176.68***	0.00003	54.32***	0.00034	2	5.99

B & S	163.25***	0.00004	196.42***	0.00003	94.32***	0.00011	1	3.84

Note: LRT was calculated using the following equation: $LRT = -2\log\left(\frac{L_R}{L_u}\right) = 2[\log(L_u) - \log(L_R)] \sim \chi^2_m$. These results are based on restriction table 3.2 in Chapter 3. (*),(**) and (***) represent significance level at 10%, 5% and 1% respectively.

Table 6 reports the critical values of the chi-squared at different significance levels. These values are way below the calculated chi-squared values. Based on the decision rule, a large value of the chi-squared value indicates that the alternative hypopaper should be favoured over the null hypopaper. In this case, the null hypotheses are rejected at 1% significance level since the computed chi-squared are greater than their corresponding critical values. The rejection of the null hypopaper implies that the restrictions for all models are statistically significantly different from zero. This implies that the restrictions imposed by the restricted model were not valid; thus, the test favours the CKLS unrestricted modelling of other linear models, i.e. with the joint test, there is still no evidence of linear mean reversion in Vasicek, B & S and CIR models.

Table 7: Likelihood Ratio Test for linear and nonlinear short-term interest rate models

	TBR3		REPO		JIBAR			
	LRT	P-value	LRT	P-value	LRT	P-value	d.o.f	Crit-values
Merton	4027.7***	0.0000	3698.2***	0.0000	4282.62***	0.0000	6	12.6
CEV	4146.7***	0.0000	3864***	0.0000	4283.9***	0.0000	5	11.1
Vasicek	2919.7***	0.0000	2618***	0.0000	3269.6***	0.0000	5	11.1
CIR	3516***	0.0000	3210***	0.0000	3803.72***	0.0000	5	11.1
Dothan	4022.1***	0.0000	3692.0***	0.0000	4274.4***	0.0000	7	14.1
GBM	3945.0***	0.0000	3647.1***	0.0000	4175.2***	0.0000	6	12.6
B & S	3911.8***	0.0000	3627.3***	0.0000	4135.2***	0.0000	5	11.1
CKLS	4075.0***	0.0000	3823.7***	0.0000	4229.5***	0.0000	4	9.5

CHLS	22.2***	0.0000	0.79	0.4564	18.8***	0.0000	2	5.9
A & G	8.9***	0.0001	40.8***	0.0000	38.3***	0.0001	4	9.5

Note: LRT was calculated using the following equation: $LRT = -2\log\left(\frac{L_R}{L_u}\right) = 2[\log(L_u) - \log(L_R)] \sim \chi^2_m$. These results are based on restriction table 3.3 in Chapter 3. (*),(**) and (***) represent significance level at 10%, 5% and 1% respectively. CHLS and A & G are nonlinear models.

The results in table 7 have been estimated using the same approach as in table 6, but adding the nonlinear models. When adding the nonlinear models, the unrestricted model becomes the AS model, while the rest of the models are restricted models. The significant difference between the linear and nonlinear models is identified in table 8. This is evident from large LRT values on linear models and small LRT values on nonlinear models. Similar to table 6, the null hypotheses for all the models were rejected at 1% level with the exception of CHLS model. CHLS model, which capture the nonlinear mean reversion (α_2 and α_3) indicates that, the nonlinear mean reversion, are both individually and jointly statistically insignificant for the REPO. However, there is evidence of nonlinear mean reversion in other nonlinear models, and all the linear models are rejected. When focusing on volatility, LRT also reveals that volatility parameters jointly are also statistically different from zero. Overall, the rejection of the null hypopaper implies that the restrictions are not valid, and therefore nonlinear models perform better than linear models.

Table 8: AIC and SBIC for short-term interest rate models

		AIC			SBIC (n=787)		
	Parameters	TBR	REPO	JIBAR	TBR	REPO	JIBAR
Merton	2	1562.48	853.33	1732.61	1564.27	855.12	1734.40
CEV	3	1683.56	1021.27	1735.99	1686.25	1023.96	1738.67
Vasicek	3	456.51	237.11	721.61	459.19	-222.41	724.29
CIR	3	1053.23	367.14	1255.71	1055.92	369.83	1258.39
Dothan	1	1554.89	846.02	1722.43	1555.78	846.92	1723.33
GBM	2	1479.84	802.22	1625.15	1481.63	804.01	1626.94

B&S	3	1448.59	784.49	1587.14	1451.28	787.17	1589.83
CKLS	4	1613.84	982.90	1683.47	1617.43	986.49	1687.05
CHLS	6	-2434.95	-2837.62	-2523.17	-2429.58	-2832.25	-2517.79
A & G	7	-2464.12	-2875.66	-2578.34	-2457.85	-2869.39	-2572.07
AS	8	-2453.17	-2453.17	-2538.01	-2446.00	-2825.66	-2530.84

Note: AIC and SBIC were calculated using the following equations: $AIC = 2k - 2log(L)$, and $SBIC = klog(n) - 2logL$, where n is the number of observations, k, is the number of free parameters and logl is log-likelihood. CHLS, A & G and AS represent nonlinear models.

Table 8 reports the values of AIC and SBIC. Models with the smallest AIC and SBIC are considered to be the best fitting models according to these criteria. When analysing linear models individually, it came out that Vasicek was the best performing model in all the interest rates as it had the lowest AIC and SBIC. Meanwhile, the worst performing model was CEV as it has the highest values in both the AIC and SBIC. However, when incorporating the nonlinear models, they all reported the lowest AIC and SBIC as compared to the linear models, with A & G leading them all. Even though SBIC tends to put more penalties on over-parameterised models than AIC, the choices of the models were consistent. Nonetheless, when ranking the models, nonlinear models came on top of the list, suggesting that South African data is explained better by the nonlinear models.

5. CONCLUSION

The aim of this study was to determine the best model that can fit the South African short-term interest rates. This is of crucial importance as the short-term interest rate is the main input in pricing a number of derivatives. Results of the parameters showed that the diffusion component was more important than the drift component in modelling South African data. Furthermore, models which assumed volatility to be a function of the level of interest rate were found to perform better than models which assumed constant volatility. In addition, models with level effect values of greater than one were better than those that restrict the level effect to be less than one. That being the case, the level effect was also considered to be the key feature that should not be left out when modelling South African interest rate data. The overall comparison of the linear and nonlinear models revealed that the nonlinear models seem to explain the stochastic process of the South African interest rate data better than the linear models. Therefore, it will be more appropriate to use nonlinear models when modelling the short-term interest rate model in South Africa.

The present study relies on a set of assumptions such as normality and constant volatility which are not always realistic. The study was limited to these assumptions so that the basic single-factor models can be understood before considering extended models. Volatility and level effect came out to be important features in modelling the stochastic short-term interest rate data. It was also observed that estimated level effect becomes so high that it might lead to stochastic volatility. For that reason, future studies should consider modelling stochastic volatility. The data analysis also showed that the data had leptokurtic behaviour. This kind of behaviour is often modelled using Jump models. On that account, Jump models should be used to capture these stylised facts. Other features of the financial variables such as regimes switching could have been modelled as South African is mainly affected by structural changes and announcements.

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Appendix 1:

Models Description

Model	Specification $\mu(r)$	Specification $\sigma(r)$	Restrictions	Advantages	Limitations
Merton (1973)	α_0	β_2	$\alpha_1 = \beta_3 = 0$		1. Constant drift and diffusion parameters. 2. The model allows negative interest rates.
Cox and Ross (1975)	$\alpha_1 r_t$	$\beta_2 r_t^{\beta_3}$	$\alpha_0 = 0$	Does not place parameter restrictions on the level of interest rate sensitivity	
Vasicek (1977)	$\alpha_0 + \alpha_1 r_t$	β_2	$\beta_3 = 0$	Have mean-reverting characteristics	1. Constant diffusion parameter. 2. The model allows the interest rate to be negative.
Dothan (1978)	0	$\beta_2 r$	$\alpha_0 \alpha_1 = 0$, $\beta_3 = 1$	Interest rate can never be negative	1. The model is driftless. 2. The model is inadequate to represent the long-term

					behaviour of interest rate.
B & S[11] (1980)	$\alpha_0 + \alpha_1 r_t$	$\beta_2 r$	$\beta_3 = 1$	Have mean-reverting characteristics	The distribution of r(t) is unknown
CIR[12] (1985)	$\alpha_0 + \alpha_1 r_t$	$\beta_2 r_t^{1/2}$	$\beta_3 = 1/2$	1. Have a mean reversion, and volatility is heteroske-dastic. 2. Does not allow negative interest rates.	Restrict the level effect to 1/2
CKLS[13] (1992)	$\alpha_0 + \alpha_1 r_t$	$\beta_2 r_t^y$	0	Have mean reversion, and volatility is heteroske-dastic.	

[11]B & S = Brennan & Schwartz
[12]CIR = Cox-Ingersoll-Ross
[13]CKLS = Chan, Karolyi, Longstaff and Sanders

Appendix 2:

Parameter restrictions imposed by short-term interest rate models on CKLS model

	α_0	α_1	β_2	β_3	Parameter Restrictions
Merton (1973)	-	0	-	0	2
CEV (1975)	0	-	-	-	1
Vasicek (1977)	-	-	-	0	1
Dothan (1978)	0	0	-	1	3
GBM (1983)	0	-	-	1	2
B & S (1980)	-	-	-	1	1
CIR (1985)	-	-	-	1/2	4
CKLS (1992)	-	-	-	-	0

Note: Linear single-factor models of the short-term interest rate nested in CKLS model $dr_t = (\alpha_0 + \alpha_1 r_t)dt + \beta_2 r_t^{\beta_3} dW_t$. CKLS is the unrestricted model, and the remaining models are restricted models.

Appendix 3:

Parameter restrictions imposed by short-term interest rate models on AS model

	α_0	α_1	α_2	α_3	β_0	β_1	β_2	β_3	Parameter Restrictions
Merton (1973)	-	0	0	0	0	0	-	0	6
CEV (1975)	0	-	0	0	0	0	-	-	5
Vasicek (1977)	-	-	0	0	0	0	-	0	5
Dothan (1978)	0	0	0	0	0	0	-	1	6
GBM (1983)	0	-	0	0	0	0	-	1	5
B & S (1980)	-	-	0	0	0	0	-	1	4
CIR (1985)	-	-	0	0	0	0	-	½	4
CKLS (1992)	-	-	0	0	0	0	-	-	4
AS (1996)	-	-	-	-	-	-	-	-	0
CHLS (1997)	-	-	-	-	0	0	-	-	2
AG (1999)	-	-	-	0	0	0	-	½	3

Note: Single-factor models of the short-term interest rate nested in AS model $dr_t = \left(\alpha_0 + \alpha_1 r_t + \alpha_3 r_t^2 + \frac{\alpha_3}{r_t}\right)dt + \beta_0 + \beta_1 r_t + \beta_2 r_t^{\beta_3} dW_t$. AS is the unrestricted model; it is for this reason that there are no parameter restrictions on AS items. The rest of the models act as restricted models; it is for this reason that they contain zeros in their line items.

Appendix 4:

ADF and PP unit root tests

Series	Model	Augmented Dickey Fuller		Phillips and Perron	Conclusion
		τ_τ,τ_μ,τ	Φ_3,Φ_1	τ_τ,τ_μ,τ	
TBR3	τ_τ	-4.3162***	12.7188***	-2.5310	Non-stationary
	τ_μ	-1.2363	37.6954	-1.6760	
	τ	-1.3319	-	-1.2700	
DTBR3	τ_τ	-20.6772***	213.7747***	-23.1747***	Stationary
	τ_μ	-20.6905***	428.0955***	-23.1847***	
	τ	-20.6677***	-	-23.1979***	
REPO	τ_τ	-3.4984**	13.3094***	-2.2894	Non-stationary
	τ_μ	-1.0566		-1.4455	
	τ	-1.6702*	-	-1.4226	
DREPO	τ_τ	-33.1521***	549.5320***	-34.6937***	Stationary
	τ_μ	-33.1721***	1100.429***	-34.7079***	
	τ	-5.9358***	-	-34.7604***	
JIBAR	τ_τ	-3.3370*	18.9915***	-2.4797	Non-stationary
	τ_μ	-2.2556	20.1571**	-1.6507	
	τ	-1.2838	-	-1.2543	
DJIBAR	τ_τ	-5.8269***	54.3491***	-24.1999***	Stationary
	τ_μ	-5.8314***	60.4665***	-24.2077***	
	τ	-5.8046***	-	-24.2269***	

Notes: τ_τ represents the trend plus intercept, τ_μ the intercept, τ the constant, Φ_3 represent F-statistics for trend and intercept, Φ_1 the F-statistics for the intercept. (***) unit means that unit root is rejected at 1% level of significance, (**) unit means that unit root is rejected at 5% level of significance, (*) unit means that unit root is rejected at 10% level of significance.

Appendix 5:

Summary of the interest rate data

	TBR3	REPO	JIBAR	DTBR3	DREPO	DJIBAR
Mean	9.034719	9.767275	9.236857	-0.009757	-0.011509	-0.009836
Median	8.555000	9.000000	8.929500	0.000000	0.000000	0.000000
Maximum	22.30000	21.85500	21.68000	2.300000	3.377000	2.390000
Minimum	4.900000	5.000000	4.999000	-1.120000	-2.066000	-0.942000
Std. Dev.	3.130351	3.716423	3.226320	0.206264	0.255819	0.188113
Skewness	1.255846	1.013036	1.131232	3.062179	3.318743	3.224334
Kurtosis	5.215961	4.053737	4.721835	44.98157	67.01236	52.03505
Jarque-Bera	365.5549	169.9328	263.3861	58648.71	134948.4	79699.52
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Sum	7065.150	7638.009	7223.222	-7.630000	-9.000000	-7.692000
Sum Sq. Dev.	7653.096	10787.02	8129.537	33.22765	51.11141	27.63673
Observations	782	782	782	781	781	781

Note: Appendix 4 reports the summary statistics of three datasets, namely, TBR3, REPO and JIBAR at levels together with their differences. The frequency used for these rates was weekly, starting from 15 March 1998 to 07 April 2013. The four statistical moments are presented as mean, standard deviation, skewness and kurtosis. Skewness is the measure of symmetry; kurtosis is a measure of peakness, and Jarque-Bera is the statistical measure of normality.