Extreme conditional value at risk: a coherent scenario for risk management

Muteba Mwamba, John and Mhlanga, Isaah

University of Johannesburg, University of Johannesburg

13 August 2013

EXTREME CONDITIONAL VALUE AT RISK: A COHERENT SCENARIO FOR RISK MANAGEMENT

Muteba Mwamba, John[1]

and

Mhlanga, Isaah[2]

Department of Economics and Econometrics

Office: D-ring 218

Tel: +27115594371

Fax: +27115593039

Email: johnmu@uj.ac.za

University of Johannesburg

Auckland Park Campus

South Africa

[1] Corresponding author, Associate Professor in the department of economics, University of Johannesburg
[2] Masters' student in financial economics, department of economics, University of Johannesburg

ABSTRACT

This paper empirically compares the static unconditional Value-at-Risk (VaR) and conditional Value-at-Risk (CVaR) estimates based on two extreme value theory (EVT) distributions: the generalized extreme value distribution (GEV) and the generalized Pareto distribution (GPD); and two other traditional methodologies: the historical simulation and the variance covariance method as a benchmark models. Using daily equity and exchange rate data from the United States, Japan, Europe, Brazil, Hong-Kong and South Africa covering the pre-crisis period (2004 to 2006), the crisis period (2007 to 2008) and the recovery period (2009 to 2011), we consider both the downside and upside risk to evaluate extreme losses for both long and short positions held by investors. The paper has several findings. Firstly, we find that the conditional GEV model outperforms all the other models at all the quantiles; however it overestimates risk especially the upside risk. Secondly, the conditional GPD does not perform significantly different from the unconditional historical simulation. Thirdly, as expected of models that ignore the fact that returns are fat tailed by assuming normally distributed returns, the unconditional variance-covariance model underestimates risk in both directions and at all quantiles. Fourthly, risk levels were highest during the crisis period, and decreased significantly in the recovery period however to levels still above the pre-crisis period. Lastly, regarding risk levels in advanced economies compared to emerging economies, a reverse of the pre- crisis period scenario occurred since the onset of the financial crisis, advanced economies are now riskier than emerging economies.

JEL Classification: G01, G15, G32

Keywords: Risk management; value-at-risk; conditional value-at-risk, extreme value theory; generalized extreme value distribution; generalized Pareto distribution, historical simulation; variance-covariance; fat-tails

1. INTRODUCTION

This paper compares the static unconditional VaR with conditional VaR estimates based on the historical simulation (HS), variance covariance (VC) and two EVT distributions, namely, the GEV and GPD during the pre-crisis period, the crisis period and the recovery period. We consider both the downside and upside risk for investors with long and short positions respectively in stocks and currencies. This paper is interesting in that it employs EVT distributions (GPD and GEV) and traditional risk models to estimate simultaneously upside and downside risk measures and assess their implication on the global economy; to our best of knowledge this is the first time such empirical analysis is carried out.

Traditional market risk models include the variance covariance (VC) and historical simulation (HS) models. The VC method assumes that asset returns are normally distributed, however empirical evidence against normality assumption has been largely provided in the literature by Geary (1947), Mandelbrot (1963), Duffie and Pan (1997), McNeil (1997), Da Silva and Mendes (2003), Muteba Mwamba (2011); Worthington and Higgs (2009), and Sheikh and Qiao (2010). Although the HS is more flexible than the VC method in that it does not make any assumption regarding the distribution of returns; its major drawback is the assumption that the past is the best predictor of the future. Bekiros and Georgoutsos (2005) argue that extrapolating the past into the future is not the correct method given that in finance, new instruments that bring new risks are created continuously. This method can also come short especially when there are correlations breakdown in assets (Sheikh and Qiao 2010).

To account for fat-tail characteristics in asset returns Longin (1997a, b), McNeil (1998), McNeil and Frey (2000) and more recently Muteba Mwamba (2012) propose and promote the use of EVT distribution in risk management. They argue that VaR estimates based on EVT distributions are more reliable, and that at higher quantiles, EVT based models produce VaR estimates that cover entirely financial losses observed during extreme market conditions.

Most of the above mentioned studies used only one of the two EVT distributions (i.e. GPD or GEV) to model extreme losses during extreme market volatility. In particular, the GPD is preferred owing to its efficient use of limited data. However, in today's high frequency trading environment, this need not be the case as millions of trading can take place in just a tenth of a second. Therefore, these two EVT distributions can be used together to complement each other if the block size (for the GEV distribution with block of maxima method) and the threshold value (for the GPD distribution with the peak over threshold method) are selected in such a way to give approximately the same sample size of extreme returns.

This paper uses both EVT distributions in order to compute conditional and unconditional VaR estimates for upside and downside risks. The data set used in this paper comprises of daily closing prices of the following equity indices spanning from January 2004 to October 2011; NASDAQ, S&P 500, CAC 40, FTSE 100, NIKKEI 225, HANG SENG, BOVESPA, JSE ALSI, JSE Top 40, and the South African rand per United States dollar exchange rate. This data set is representative of global financial markets in that it considers both emerging and advanced economies from the United States, Europe, Asia, South America and Africa.

We simultaneously fit these return series to two different EVT distributions: GEV which models the maximums of blocks of returns, and GPD which models the returns that are above a certain threshold. For the GEV, each return series is divided into equal non-overlapping and independent blocks of 10 days. We are of the view that a ten day block is quite reasonable in today's high frequency trading environment where millions of trading can be executed in just a tenth of a second. In constructing the series of block maxima, the highest loss (downside risk) or gain (upside risk) is taken from each block. Depending on the number of losses that occur in a particular sub sample (pre-crisis, crisis or recovery) period, each series of block maxima consists of at least 30 extreme losses or gains. We thereafter fit the GEV to these series of block maxima using maximum likelihood method to obtain the shape, scale and location parameters. For the peak over threshold (POT) method, we define all observed returns above the 95th quantile as extreme returns and fit them to the GPD distribution to obtain the scale and shape parameter using the maximum likelihood methods.

Table 5 and Table 6 in appendix 2 report the maximum likelihood estimators corresponding to GPD and GEV distribution respectively. The larger the shape parameter the larger the losses we would expect to incur. Looking at the shape parameters in appendix 2 Table 6 and Table 5 that represent the fat-tailedness of the data from both the GEV and GPD respectively reveals some characteristics of returns that we would expect during the three different sub sample (pre-crisis, crisis or recovery) periods. In general, the shape parameter for the crisis period is larger, followed by that of the recovery period, and then by that of the pre-crisis period. From this observation, we would expect greater extreme losses during the crisis period, followed by the recovery period, and less extreme losses in the pre-crisis period. More so, the GEV has somewhat greater shape parameters than the GPD. This would imply greater risk estimates from the GEV than from the GPD.

Our empirical results show that conditional VaR estimates based on GEV and GPD are higher than unconditional VaR based on the same EVT distributions. These results are consistent with the findings by other researchers (Artzner et al. 1999; Acerb and Tasche 2002) who showed that the unconditional VaR is not coherent risk measure since it is not sub-additive. Sub-additivity means that VaR of a portfolio of different assets must be less than the sum of VaR of individual assets. These results highlight also the importance of conditional VaR as a coherent measure for risk mitigation and diversified portfolio construction. Furthermore, we find a significant decrease of downside risk during the recovery period, albeit to levels that are still higher than the pre-crisis period. Since the onset of the financial crisis, that advanced economies have become riskier than emerging markets.The rest of the paper proceeds as follows. Section two present the methodology of EVT as well as the GEV and the GPD. Section three presents the data and empirical results for different quantiles. Lastly, section four concludes the paper.

2. METHODOLOGY: EXTREME VALUE THEORY

Extreme value theory provides a convenient way to model the tails of distributions. Since it concentrates on the tails of distributions, it has been adopted to model asset returns in time of extreme market activity (see Embrechts et al., 1997; McNeil and Frey, 2000; Muteba Mwamba, 2012; and Danielsson and de Vries, 2000).

Gilli and Kellezi (2003), and Bensalah (2000) points out two related methods of modelling extreme losses. The first method describes the extreme losses through a limit distribution known as the generalized extreme value distribution (GEV), which is a family of asymptotic distributions that describe normalized maxima or minima. The second method provides asymptotic distribution that describes the limit distribution of scaled excesses over high thresholds, and is known as the generalized Pareto distribution (GPD). The two limit distributions results into two approaches of EVT-based modeling: the block of maxima method and the peaks over threshold method respectively[3].

Asymptotic Model Formulation

[3] For an in-depth discussion of these methods, the reader can see Shanbhang and Rao (2003), eds, Handbook of Statistics, Vol.21.

Let us consider independent and identically distributed (i.i.d) random returns $X_{1,}X_2,...,$ with common distribution function F. Let $M_n = \max(X_1,...,X_n)$ be the maximum of the first n random returns. Also, let us suppose $w(F) = \sup\{x : F(x) < 1\}$ is the upper end of F. For $n > 2$, the corresponding results for the minima can be obtained from the following identity

$$\min(X_1, X_2,..., X_n) = -\max(-X_1, -X_2,..., -X_n) \tag{1}$$

M_n almost surely converges to $w(F)$ whether it is finite or infinite since,

$$\begin{aligned}
&\Pr(M_n \le x) = \Pr(X_1 \le x, X_2 \le x,..., X_n \le x) \\
&= \Pr(X_1 \le x)\text{x...x}\Pr(X_n \le x) \\
&= F^n(x)
\end{aligned} \tag{2}$$

$\forall x \in \mathbb{R}, n \in N$, and as $n \to \infty$

Normally, we would estimate F and substitute it into equation (2). The assumption here is that F is known, however it is often unknown. Small errors in estimating F will result is larger errors in F^n thus the result in (2) is not very useful. We want a method to estimate F^n directly. To do that, we observe the behavior of F^n as $n \to \infty$. For any $x < x_+$, with x_+ being the smallest value of x such that $F(x) = 1$, $F^n(x) \to 0$ as $n \to \infty$. The implication is that the distribution of M_n degenerates to a point mass on x_+. A common method used to avoid this is to linearly renormalize M_n (Embrechts et al. 1997; Shanbhang and Rao 2003). The limit theory finds norming constants $\{a_n > 0\}$ and $\{b_n\}$ such that

$$M_n^* = \frac{M_n - b_n}{a_n} \quad , \tag{3}$$

a_n and b_n are a series of constants chosen to stabilize the location and scale parameters of the distribution of M_n^* as n increases.

Extremal Theorems

If there exists a series of constants $\{a_n > 0\}$ and b_n such that

$$\Pr(M_n^* \le x) = \Pr(\frac{M_n - b_n}{a_n} \le x) \to G(x) \text{, as } n \to \infty, \tag{4}$$

G is a non-degenerate distribution function, and belongs to one of the following families of distributions (Fisher and Tippett 1928, De Haan 1970, De Haan 1976, Weissman 1978, Embrechts et al. 1997)

$$G(x) = \exp\{-\exp[-(\frac{x-b}{a})]\}, \qquad -\infty < x < \infty \tag{5}$$

$$G(x) = \begin{cases} 0, & x \le b \\ \exp\{-(\frac{x-b}{a})^{-\alpha}\}, & x > b \end{cases} \tag{6}$$

$$G(x) = \begin{cases} \exp\{-[-(\frac{x-b}{a})^{\alpha}] & x < b \\ 1, & x \ge b \end{cases} \tag{7}$$

Any extreme value distribution can be classified as one of the three types above. Type 5, 6 and 7 are known as the Gumbel, Frechet and Weibull family of distribution. They are the standard extreme value distribution and the corresponding random variables are called standard extreme random variables. For alternative characterization of the three distributions, see Nagaraja (1988), and Khan and Beg (1987).

2.1 The Generalized Extreme Value Distribution

The three distribution functions given in equation 5, 6 and 7 above can be combined into one three-parameter distribution called the generalized extreme value distribution (GEV) given by,

$$G(\mu,\sigma,\xi) = \exp\{-(1+\xi\tfrac{x-\mu}{\sigma})^{-\frac{1}{\xi}}\}, \tag{8}$$

Provided that $1+\xi\frac{x-\mu}{\sigma} > 0, \mu \in \mathbb{R}, \sigma > 0 \,\&\, \xi \in \mathbb{R}.$ $G(\mu,\sigma,\xi)$ is a one parameter representation of the three standard extreme value distributions.

In equation (8) above, μ,σ and ξ represent the location parameter, the scale parameter, and the tail-shape parameter respectively. The tail parameter indicates how 'fat' the distribution tail is. The large the tail parameter, the 'fatter' the distribution tail is. $G(1,1,\alpha^{-1}), \alpha > 0$ corresponds to the Frechet, and distribution $G(-1,-1,-\alpha^{-1}), \alpha > 0$ corresponds to the Weibull distribution. The case where $\xi = 0$ reduces to the Gumbel distribution.

To obtain the estimates of (μ,σ,ξ), we first fit the sample of maximum losses to a GEV, after which we use maximum likelihood estimation method (MLE) to estimate the parameters. A logarithmic likelihood function of the following form is obtained;

$$l((\mu,\sigma,\xi);X) = -n\log(\sigma) - (1+\frac{1}{\xi})\sum_{i=1}^{n}\log[1+\xi(\frac{x_i-\mu}{\sigma})] - \sum_{i=1}^{n}[1+\xi(\frac{x_i-\mu}{\sigma})]^{-1/\xi}, \tag{9}$$

provided that $1+\xi(\frac{x_i-\mu}{\sigma}) > 0,$ for $i = 1,...,n.$

The case for $\xi = 0$ gives rise to the following log-likelihood

$$l((\mu,\sigma,);X) = -n\log(\sigma) - \sum_{i=1}^{n} (\frac{x_i - \mu}{\sigma}) - \sum_{i=1}^{n} \exp\{-(\frac{x_i - \mu}{\sigma})\tag{10}$$

To obtain the estimates of (μ,σ,ξ), we maximize equation (9) and (10) with respect to the entire GEV.

2.2 The Generalized Pareto Distribution (Peak-Over Threshold Method)

One approach in EVT-based modeling is implemented by estimating the conditional distribution of losses given that the losses exceed a high threshold. This approach is sometimes referred to as peak-over threshold (POT), and has the advantage of using more data than the GEV depending on the threshold chosen. It uses more data because it does not only consider only the maximum losses in each block but all the losses that exceed a threshold that is considered very high enough to satisfy some technical condition.

Let $X_1, X_2,...$ be a sequence of i.i.d random returns with common marginal distribution F . We define extreme variables to be those that exceed some threshold τ that is regarded very high. Let

$$M_n = \max\{X_1,..., X_n\} \tag{11}$$

M_n is the series of maximum of the first n observations. We denote an arbitrary term in the sequence X_i by X and suppose that F satisfies equation (4), so that for large enough n ,

$$\Pr\{M_n \leq z\} \approx G(z) \tag{12}$$

And $G(z) = \exp\{-[1+\xi(\frac{z-\mu}{\sigma})]^{-\frac{1}{\xi}}$ for some μ , $\sigma > 0$ and ξ . For a large τ , the distribution of $(X-\tau) \mid X > \tau$ is approximated by

$$H(y) = 1-(1+\tfrac{\xi y}{\hat{\sigma}})^{-\frac{1}{\xi}} \tag{13}$$

defined on $\{y : y > 0\}$, $\{y : 1+\frac{\xi y}{\hat{\sigma}} > 0\}$ and $\hat{\sigma} = \sigma + \xi(\tau - \mu)$.

The family of distributions defined by equation (13) is known as the generalized Pareto distribution (GPD).

Threshold selection

Selecting the threshold τ is a very important step in applying the GPD. It is analogous to selecting the block size in the GEV. Selecting too high a threshold leads to too few extreme observations leading to biased parameters, while selecting very low threshold results in more extreme observations leading to high variance. The selection of the threshold therefore has to balance between these two.

For the purpose of our study, a very simplistic and practical threshold selection method is adopted. We consider all the observations that are above the 95th percentile of ordered data as extreme observations.[4]

Parameter estimation

The parameters of the GPD are estimated by maximum likelihood method (MLE). We suppose that the values $y_1,...,y_n$ are the n excess of a threshold τ. For $\xi \neq 0$, the log-likelihood of equation (13) is given by

$$l(\sigma,\xi) = -n\log(\sigma) - (1+\tfrac{1}{\xi})\sum_{i=1}^{n}\log(1+\tfrac{\xi y_i}{\sigma}) \quad (14)$$

provided that $1+\frac{\xi y_i}{\sigma} > 0$ for $i = 1,...,n$; or else $l(\sigma,\xi) = -\infty$

for $\xi = 0$, the log-likelihood of equation (13) is given by

$$l(\sigma) = -n\log(\sigma) - \sigma^{-1}\sum_{i=1}^{n} y_i \quad (15)$$

2.3 Extreme Value Theory Risk Measures

The EVaR defined as the maximum likelihood alpha quantile estimator of $G_{(\xi,\mu,\sigma)}(x)$, which is by definition given by

$$G^{-1}_{(\xi,\mu,\sigma)}(\alpha) = \inf\left\{x \in R, G_{(\xi,\mu,\sigma)}(x) \geq \alpha\right\}, 0 < \alpha < 1 \quad (16)$$

The quantity $x(\alpha) = G^{-1}(\alpha)$ is the a^{th} quantile of equation 8 and 13 for the GEV and GPD respectively, and is the alpha percent EVaR, which is given by,

$$EVaR_{(\alpha,GEV)}(x) = x(\alpha) = \hat{\mu} + \frac{\hat{\sigma}_t}{\hat{\xi}}\left\{((-\ln(\alpha)^{-\xi}) - 1\right\} \quad (17)$$

$$EVaR_{(\alpha,GPD)}(x) = x(\alpha) = \hat{\mu} + \frac{\hat{\sigma}_t}{\hat{\xi}}[(\tfrac{n}{N\mu}(1-p)^{-\xi} - 1] \quad (18)$$

After obtaining the EVaR, the extreme CVaR is calculated using the following equation.

$$ECVaR_{\alpha}(x) = EVaR_{\alpha}(x) + E[x \mid x \geq EVaR_{\alpha}(x)] \quad (19)$$

3. DATA AND EMPIRICAL RESULTS

3.1 Data Description

---

[4] The reader is however referred to Rebib and Rubin (2007) and Bensalah (2000), and references therein for other techniques of selecting the threshold in this methodology.

The data set utilized in this study consist of nine indexes: NASDAQ, S&P 500 , CAC 40, FTSE 100, NIKKEI 225, HANG SENG , BOVESPA, JSE ALSI , JSE Top 40, and the rand per US$ exchange rate. The data was divided into three sub-samples, the pre-crisis period spanning from Jan 2004 to December 2006, the crisis period covering January 2007 to December 2008 and the recovery period covering January 2009 to October 2011.

Once the parameters have been estimated, we used equation 17 and 18 to compute the EVaR for the GEV and the GPD respectively, and we use equation 19 to compute the ECVaR for both distributions.

3.2 Empirical results

The VaR estimates are interpreted as follows. Take for example, a value of 3.00% representing a downside unconditional VaR (at 99.9% confidence level) of NASDAQ during the crisis period based on the GEV. This would mean that we would have expected the losses for an investor holding a long position in the NASDAQ index to exceed 3.00% of value of NASDAQ in 0.01 percent of the time over the pre-crisis period. Parallel interpretations for the upside VaR hold.

We compare the risks of the assets against each other at different confidence levels, and the risks estimated by the different models against each other and evaluate the adequacy of the models in a form of a static in-sample backtest. More so, we discriminate the risk of advanced economies from that of emerging economies between the sample periods.

Table 1A, 1B and 1C (appendix 1) presents downside risk measures while table 2A, 2B and 2C presents upside risk measures estimated from the four models considered for the three sub-samples considered. In general, emerging market indices have higher unconditional VaR while advanced economies indices have lower unconditional VaR during the pre-crisis period. This is in line with financial market sentiments which viewed emerging markets as risky; however advanced economies became more risky than emerging markets since the financial crisis period. Also, important to note with regard to advanced economies, the Nikkei 225 VaR appear to be reasonably comparable to emerging market economies, as such, it is riskier than other advanced economies. Among advanced economies indices and in most cases, the S&P 500 index has the lowest VaR based on all methodologies and at all quantiles. If we consider the emerging market indices, the rand/US$ has the lowest risk based on all methodologies while the BOVESPA has higher risk.

A comparison of the different methodologies against each other reveals that the variance covariance method underestimates risk. This is not surprising since it is based on the assumption of the returns being normally distributed, of which empirical evidence presented in this paper and elsewhere in the literature strongly reject. The unconditional VaR based on the HS method performs equal with the unconditional VaR based on the GPD, except at 99.9% confidence level where the HS accurately estimates possible losses. At 99.9% confidence level, the HS unconditional VaR performs adequately and equally with the GPD CVaR. Both the unconditional and conditional GEV VaR are adequate, however the unconditional VaR tends to underestimate risk at 95.0% and 99.0%.

To backtest our results, we compare the estimated VaR's with the highest loss that was recorded over the sample period. A look at the pre-crisis risk estimates for all indices reveals that the variance covariance method underestimates risk, while the GEV overestimate risk especially in the upside direction.. The historical simulation, the GPD-based VaR and CVaR only adequately estimate risk at high quantiles of 99% and 99.9%, however at 95% confidence level they underestimate risk.

In practice, the 95% confidence level is normally used. Both the traditional methodologies variance covariance and the HS underestimate risk at this confidence level. The main result to take from this analysis is that the GEV-based CVaR adequately estimate risk for the purpose of setting aside capital to be drawn when extreme losses occur. The disadvantage is that this method would lead to more capital being tied up in unproductive reserves for a rainy day, however, as experience has shown during the financial crisis, such days are sure to come therefore it is worth having enough capital in place to safeguard the survival of institutions in such times.

4. CONCLUSION AND DIRECTIONS FOR FURTHER RESEARCH

This paper assessed the estimates of VAR based on the HS and two EVT-based methodologies, the GPD and the GEV. The variance covariance model was used as a benchmark. The findings of the paper indicate that the conditional VaR based on the GEV is the best model especially at high quantile, however it overestimates risk especially at very high quantiles and in the upside direction. More so, the results also indicate that EVT based VaR is better at estimating the risk of fat tailed financial returns than the traditional variance covariance that underestimates risk due to assuming normality.

The risk management community can accurately estimate risk by using conditional VaR under EVT especially when estimating VaR at high quantiles. This will better prepared them to absorb extreme losses when they occur, and thereby protecting investor capital and the financial system as a whole.

Advances in statistical methodologies for risk estimation enable better characterisation of dependencies among assets. Copula theory is one such methodology that takes into account nonlinearities among asset returns. This is the gap that future research can be directed.

REFERENCES

Acerbi, T. and Tasche, D. (2002). On the coherence of Expected Shortfall. [Available on line at: http://www-m4.ma.tum.de/pers/tasche/shortfall.pdf . Accessed May 2011.

Alexander, C. (2001). Market Models: A guide to Financial Data Analysis. Chichester; New York: Wiley.

Artzner, P., Delbaen, F., Eber, J-M., And Heath , D. (1999) Thinking Coherently. *Mathematical Finance*, 9(3): 203–228.

Bekiros, S.D. and Georgoutsos, D.A. (2005). Estimation of Value-at-Risk by Extreme Value Theory and Conventional Methods: A Comparative Evaluation of Their Predictive Performance. *Journal of International Financial Markets, Institutions and Money, 15: 209-228.*

Bensalah, Y. (2000). Steps in Applying Extreme Value Theory to Finance: A Review. Available on line at: http://test.prismsonline.com/fr/res/wp/2000/wp00-20.pdf

Da Silva, L.C.A., and de Melo Medez, B.V. (2003). Value-at-Risk and Extreme Returns in Asian Markets. *International Journal of Business*, 8 (1): 17-40.

Danielsson, J. and de Vries, C. (2000). Value-at-Risk and Extreme Returns, *Annales d'Economie et deb Statistique*, 60: 239-270.

De Haan, L. (1970). On Regular Variation and Its Application to the Weak Convergence of Sample Extremes*. Mathematical Centre Tract*, 32. Mathematisch Centmm, Amsterdam

De Haan, L.: 1976, Sample extremes: An Elementary Introduction. *Statistica Neerlandica,* 30 (4): 161-172.

Duffie, D., and J. Pan, J. (1997). An overview of value at risk. *Journal of Derivatives*, 4(3): 7-49.

Embrechts, P. (2000). Potential and Limitations as an Integrated Risk Management Tool. *Derivatives Use, Trading & Regulation* 6: 449-456

Embrechts, P. Klüppelberg, C. and Mikosch, T. (1997). Modelling Extremal Events for Insurance and Finance Springer-Verlag, London.

Fama, E. (1965). The behaviour of stock market prices. *Journal of Business*, 38(1): 34-105.

Fama, E.F. (1965). Random Walks in Stock Market Prices. *Financial Analysts Journal* 21 (5): 55–59.

Fisher, R. A. and Tippett, L.H.C. (1928). Limiting forms of the frequency distribution of the largest or smallest member of a sample. *Proc. Cambridge Philos. Soc.* 24: 180-190.

Geary, R.C.: 1947, Testing for Normality, *Biometrika,* 34 (3/4): 209-242.

Gilli, G., and Kellezi, E. (2003). An Application of Extreme Value Theory for Measuring Risk, Department of Econometrics, University of Geneva, Switzerland. Available from: http://www.gloriamundi.org/picsresources/mgek.pdf Accessed March 2011.

Jorion, P. (2001). Value at Risk, The New Benchmark for Managing Financial Risk, 2nd Edition, McGraw-Hill, United States.

J.P. Morgan and Reuters. (1996). RiskMetrics Technical document. Available at http://pascal.iseg.utl.pt/~aafonso/eif/rm/TD4ePt_4.pdf

Kimball, R.C.. (2000). Failures in Risk Management. *New England Economic Review*. Available on line at: http://www.bos.frb.org/economic//neer/neer2000/neer100a.pdf.

Khan, A. H. and Beg, M.I. (1987). Characterization of The Weibull Distribution By Conditional Variance. *Snaky A* 4: 268-271.

Longin, F. (1997a). Beyond the Va, Discussion Paper 97-011, Ceressec.

Longin, F. (1997b). From Value-at-Risk to Stress testing, the Extreme value approach, Discussion Paper 97-004, Ceressec.

Mandelbrot, B. (1963). The Variation of Certain Speculative Prices. *Journal of Business*, 36: 394-419.

Manganelli, S. and Engle, R.F. (2001). Value at Risk models in Finance. *European Working Paper Series, Working Paper Number 75.*

McNeil, A.J. (1997). Estimating the Tails of Loss Severity Distributions Using Extreme Value Theory. ASTIN Bull., 27 (1): 1117-1137.

McNeil, A.J. (1998). Calculating Quantile Risk Measures for Financial Return Series using Extreme Value Theory. Department of Mathematics, ETH E-Collection, Swiss Federal technical University, Zurich.

McNeil, A.J and Frey, R. (2000). Estimation of Tail-Related Risk Measures for Heteroscedastic Financial Time Series: An Extreme Value Approach, *Journal of Empirical Finance*, 7(3/4): 271-300.

Muteba Mwamba J. (2011). Predictability of stock price behaviour in South Africa: a non-parametric approach, *African Finance Journal*, Vol 13, part 1, 2011

Muteba Mwamba, J (2012). Implementing a robust risk model for South African equity markets: a peak-over-threshold approach*. South African journal of economics*, Vol. 80: 4, December 2012

Nagaraja, H. N. (1988) Some characterizations of continuous distributions based on regressions of adjacent order statistics and record values. *Sankhy A* 50: 70-73.

Rebib, R,. and Rubin, M. (2007). Maximum Likelihood Estimation of Parameters for an Extreme Value Data. Available online at: http://n.ethz.ch/student/rebibr/projects/Statistics-MaxLikelihood.pdf.

Rootzen, H., and Kluppelberg, C. (1999). A Single Number cant Hedge against Economic Catastrophes. *Ambio* 28 (6): 550-555. Royal Swedish academy of Sciences.

Shanbhag, D.N., and Rao, C.R. (2003). Extreme Value Theory, Models and Simulation. *Handbook of Statistics,* 21:607-691.

Sheikh, A.Z. and Qiao, H. (2010). Non-Normality of Market Returns: A Framework for Asset allocation Decision-Making. *The Journal of Alternative Investments,* Winter, 12 (3): 8-35.

Weissman, I. (1978). Estimation of parameters and large quantiles based on the k largest observations. *J. Amer. Statist. Assoc.* 73: 812-815.

Worthington, A.C. and Higgs. H. (2006). Efficiency in the Australian Stock Market, 1875-2006: A Note on Extreme Long-Run Random Walk Behaviour. *Faculty of Commerce - Papers* (2006). Available online at: http://works.bepress.com/acworthington/15

APPENDICES

Table 1A. Downside Risk: Pre-crisis Period

	CL	NASDAQ	S&P 500	CAC 40	FTSE 100	NIKKEI 225	HANG SENG	BOVESPA	JSE All Share	JSE Top 40	R/US
VC	95.0	0.22	0.16	0.33	0.26	0.42	0.38	0.49	0.46	0.47	0.2
	99.0	0.61	0.43	0.73	0.58	0.91	0.80	1.20	0.97	1.02	0.6
	99.9	1.04	0.75	1.18	0.94	1.45	1.27	1.99	1.54	1.64	1.1
HS	95.0	1.87	1.35	1.84	1.38	2.13	1.95	3.29	1.94	2.13	1.9
	99.0	2.32	1.68	2.76	2.32	3.21	2.79	4.49	3.86	4.18	2.6
	99.9	2.54	1.84	3.27	2.99	4.80	3.54	6.06	6.41	6.84	3.9
GPD0	95.0	1.85	1.36	1.85	1.39	2.14	1.52	3.26	1.96	2.16	1.9
	99.0	2.36	1.69	2.78	2.28	3.27	2.76	4.61	3.98	4.23	2.7
	99.9	2.52	1.83	3.23	2.93	4.85	3.52	5.98	6.05	6.44	3.7
GPD1	95.0	2.17	1.56	2.42	1.92	2.84	2.45	4.08	3.18	3.42	2.4
	99.0	2.45	1.76	3.01	2.58	3.95	3.11	5.22	4.91	5.22	3.2
	99.9	2.54	1.84	3.29	3.07	5.52	3.73	6.39	6.68	7.13	4.0
GEV0	95.0	2.28	1.61	2.40	1.98	3.09	2.63	4.53	2.97	3.22	2.7
	99.0	2.66	1.94	3.21	2.93	4.39	3.47	6.07	4.47	4.76	3.4
	99.9	3.00	2.27	4.34	4.66	6.44	4.62	8.26	7.29	7.57	4.4
GEV1	95.0	4.72	3.33	5.17	4.40	7.06	5.66	9.92	6.85	7.35	6.3
	99.0	4.75	1.94	6.48	5.92	9.19	7.01	6.07	10.89	11.60	7.4
	99.9	4.80	2.27	4.34	4.66	6.44	4.62	8.26	7.29	7.57	4.4
ML		2.54	1.84	3.27	2.99	4.80	3.54	6.06	6.42	6.84	3.9

Estimated risk is greater than or equal to the maximum loss or gain (adequate)
Estimated risk is more than double the maximum loss or gain (overestimated)
0= unconditional VaR, 1 = conditional VaR, ML= Maximum Loss, CL= Confidence Level

Table 1B. Downside Risk: Crisis Period

	CL	NASDAQ	S&P 500	CAC 40	FTSE 100	NIKKEI 225	HANG SENG	BOVESPA	JSE All Share	JSE Top 40	R/US$
VC	95.0	1.04	1.24	1.05	1.00	1.37	1.19	1.21	0.72	0.77	0.45
	99.0	2.06	2.30	2.04	1.92	2.54	2.40	2.47	1.59	1.70	0.93
	99.9	3.20	3.50	3.14	2.95	3.85	3.76	3.89	2.56	2.75	1.46
HS	95.0	4.58	4.67	4.25	3.93	5.34	5.09	5.47	3.83	4.11	2.04
	99.0	7.94	8.48	6.88	6.88	9.64	8.15	9.22	5.64	6.00	3.55
	99.9	8.98	8.87	9.00	8.82	11.44	12.14	11.28	7.54	7.99	5.20
GPD0	95.0	4.54	1.36	4.32	4.03	5.32	5.04	5.49	3.81	4.14	1.99
	99.0	7.37	1.69	7.24	6.72	8.50	7.79	9.15	6.11	6.56	3.62
	99.9	9.40	1.83	8.94	8.98	12.76	14.78	11.23	7.84	8.27	5.14
GPD1	95.0	6.24	1.56	6.08	5.65	7.29	6.93	7.70	5.19	5.60	2.97
	99.0	8.32	1.76	8.06	7.76	10.36	10.80	10.16	6.92	7.37	4.31
	99.9	9.82	1.84	9.22	9.54	14.47	20.61	11.55	8.21	8.62	5.56
GEV0	95.0	6.19	6.52	5.75	5.48	6.61	7.18	7.83	5.34	5.77	2.90
	99.0	10.47	11.80	8.88	7.74	10.49	11.53	11.21	7.28	7.84	4.29
	99.9	20.45	25.28	14.68	11.01	18.14	20.76	16.45	9.97	10.70	6.88
GEV1	95.0	14.34	14.69	12.95	12.83	15.26	15.79	17.42	11.87	12.46	6.72
	99.0	10.47	11.80	17.88	16.10	21.93	23.67	11.21	14.78	15.79	9.49
	99.9	10.41	9.75	10.44	8.93	11.73	9.20	10.35	6.88	7.54	5.55
ML		8.98	8.87	9.00	8.82	11.44	12.14	11.28	7.54	7.99	5.20

Estimated risk is greater than or equal to the maximum loss or gain (adequate)
Estimated risk is more than double the maximum loss or gain (overestimated)
0= unconditional VaR, 1 = conditional VaR, ML= Maximum Loss, CL= Confidence Level

	CL	NASDAQ	S&P 500	CAC 40	FTSE 100	NIKKEI 225	HANG SENG	BOVESPA	JSE All Share	JSE Top 40	R/US$
		Table 1C. Downside Risk: Recovery Period									
VC	95.0	0.66	0.71	0.63	0.54	0.64	0.57	0.66	0.33	0.35	0.20
	99.0	1.41	1.44	1.39	1.16	1.37	1.28	1.42	0.87	0.94	0.55
	99.9	2.26	2.26	2.24	1.86	2.18	2.07	2.27	1.48	1.60	0.95
HS	95.0	3.32	3.12	3.66	2.84	2.82	3.16	3.29	2.71	2.95	1.75
	99.0	4.65	4.79	4.89	4.59	4.51	4.71	5.07	3.37	3.60	2.30
	99.9	6.99	6.91	5.56	5.32	10.36	5.51	8.07	3.78	3.95	3.97
GPD0	95.0	3.33	3.10	4.32	2.80	2.83	3.10	3.32	2.69	2.93	1.75
	99.0	4.72	4.84	7.24	4.25	4.77	4.63	5.07	3.51	3.71	2.44
	99.9	6.93	6.70	8.94	5.42	9.43	5.42	7.94	3.75	3.93	3.58
GPD1	95.0	4.82	4.16	6.08	3.68	4.14	4.02	4.42	3.19	3.41	2.19
	99.0	5.67	5.67	8.06	4.79	6.77	5.02	6.30	3.64	3.83	2.93
	99.9	8.01	7.28	9.22	5.68	13.08	5.54	9.38	3.77	3.94	4.14
GEV0	95.0	4.58	4.55	4.99	4.01	4.60	4.32	4.83	3.35	3.64	2.46
	99.0	6.28	6.54	7.26	5.79	6.45	5.47	6.84	4.19	4.53	3.36
	99.9	8.68	9.70	11.15	8.76	9.34	6.87	10.15	5.14	5.53	4.73
GEV1	95.0	10.41	9.75	10.44	8.92	9.34	9.20	10.35	6.88	7.54	5.55
	99.0	13.27	13.45	7.26	5.79	16.81	10.98	14.91	4.53	4.19	7.33
	99.9	8.68	9.70	11.15	8.76	11.73	6.87	10.15	5.14	5.53	4.73
ML		6.99	6.91	5.56	5.32	10.36	5.51	8.07	3.78	3.95	3.97

Estimated risk is greater than or equal to the maximum loss or gain (adequate)
Estimated risk is more than double the maximum loss or gain (overestimated)
0= unconditional VaR, 1 = conditional VaR, ML= Maximum Loss, CL= Confidence Level

	CL	NASDAQ	S&P 500	CAC 40	FTSE 100	NIKKEI 225	HANG SENG	BOVESPA	JSE All Share	JSE Top 40	R/US$
		Table 2A. Upside Risk: Pre-crisis Period									
VC	95.0	0.25	0.19	0.17	0.16	0.29	0.27	0.42	0.38	0.43	0.51
	99.0	0.64	0.47	0.50	0.43	0.75	0.64	1.09	0.85	0.95	1.08
	99.9	1.07	0.78	0.87	0.73	1.27	1.06	1.84	1.38	1.53	1.72
HS	95.0	1.84	1.31	1.57	1.22	2.10	1.68	3.04	1.96	2.17	2.49
	99.0	2.51	1.71	2.28	1.99	2.93	2.45	4.71	3.57	4.01	4.06
	99.9	3.05	2.12	2.51	2.65	3.51	3.55	5.60	5.19	5.73	4.49
GPD0	95.0	1.85	1.32	1.57	1.23	2.10	1.66	3.06	1.97	2.16	2.49
	99.0	2.45	1.78	2.28	1.90	2.85	2.45	4.57	3.36	3.73	4.19
	99.9	3.14	2.13	2.48	2.59	3.56	3.35	5.54	5.59	6.05	4.47
GPD1	95.0	2.21	1.60	2.01	1.63	2.55	2.14	3.97	2.84	3.14	3.57
	99.0	2.75	1.94	2.39	2.21	3.17	2.85	5.03	4.32	4.74	4.35
	99.9	3.38	2.20	2.50	2.80	3.76	3.66	5.71	6.69	7.08	4.48
GEV0	95.0	2.47	1.75	2.13	1.76	2.81	2.44	4.44	3.12	3.46	3.63
	99.0	2.92	2.07	2.55	2.25	3.34	3.08	5.65	4.64	5.15	4.92
	99.9	3.36	2.38	2.98	2.87	3.88	3.88	7.20	7.46	8.27	6.76
GEV1	95.0	5.32	3.75	4.48	3.87	5.98	5.26	9.32	7.10	7.87	7.70
	99.0	5.92	4.17	2.55	4.90	6.77	6.63	5.65	9.61	10.88	4.92
	99.9	3.36	2.38	2.98	2.87	3.89	3.89	7.20	7.46	8.27	6.76
MG		3.05	2.12	2.51	2.65	3.51	3.55	5.60	5.19	5.73	4.49

Estimated risk is greater than or equal to the maximum loss or gain (adequate)
Estimated risk is more than double the maximum loss or gain (overestimated)
0= unconditional VaR, 1 = conditional VaR, MG= Maximum Gain, CL= Confidence Level

	Table 2B. Upside Risk: Crisis Period										
	CL	NASDAQ	S&P 500	CAC 40	FTSE 100	NIKKEI 225	HANG SENG	BOVESPA	JSE All Share	JSE Top 40	R/US$
VC	95.0	1.19	1.27	1.23	1.02	1.03	1.62	1.53	0.76	0.85	0.94
	99.0	2.16	2.24	2.22	1.92	1.98	2.95	2.83	1.57	1.74	1.74
	99.9	3.25	3.32	3.33	2.92	3.04	4.45	4.29	2.47	2.73	2.64
HS	95.0	3.93	3.86	3.24	3.22	3.93	4.16	5.02	3.74	3.92	2.96
	99.0	6.28	6.56	9.13	7.84	7.52	11.15	9.64	5.86	6.56	5.40
	99.9	11.62	11.15	10.73	9.47	9.96	15.01	14.91	7.08	7.96	11.70
GPD0	95.0	3.89	3.85	3.25	3.22	3.98	4.33	5.07	3.73	4.00	2.96
	99.0	7.08	6.57	8.87	7.49	6.90	10.87	9.98	5.98	6.62	5.34
	99.9	12.22	16.20	10.59	9.47	10.48	15.08	16.19	7.01	7.88	14.28
GPD1	95.0	5.90	5.95	6.71	5.81	5.77	8.27	8.09	5.09	5.59	4.85
	99.0	9.30	10.91	9.79	8.49	8.47	12.88	12.71	6.49	7.25	9.43
	99.9	14.78	28.46	10.72	9.73	11.79	15.84	18.54	7.14	8.04	26.63
GEV0	95.0	7.24	6.88	7.35	7.45	6.02	9.46	8.89	6.31	7.02	4.62
	99.0	17.02	16.02	18.55	20.54	10.54	23.68	17.72	13.81	15.76	7.38
	99.9	20.20	51.87	69.74	87.96	21.84	88.53	46.09	41.28	48.89	12.85
GEV1	95.0	17.84	17.77	16.80	15.92	13.63	21.31	20.75	13.39	14.98	12.22
	99.0	17.02	16.02	18.55	20.54	10.54	23.68	17.72	13.81	15.76	19.08
	99.9	57.15	51.87	69.74	87.96	21.84	88.53	46.09	41.28	48.89	12.85
MG		11.62	11.15	10.73	9.47	9.96	15.01	14.91	7.08	7.96	11.70

Estimated risk is greater than or equal to the maximum loss or gain (adequate)
Estimated risk is more than double the maximum loss or gain (overestimated)
0= unconditional VaR, 1 = conditional VaR, MG= Maximum Gain, CL= Confidence Level

	Table 2C. Upside Risk: Recovery period										
	CL	NASDAQ	S&P 500	CAC 40	FTSE 100	NIKKEI 225	HANG_SENG	BOVESPA	JSE All Share	JSE Top 40	R/US$
VC	95.0	0.67	0.65	0.62	0.45	0.47	0.57	0.65	0.46	0.52	0.34
	99.0	1.36	1.29	1.34	1.01	1.09	1.29	1.38	1.02	1.14	0.81
	99.9	2.14	2.02	2.14	1.65	1.79	2.10	2.19	1.65	1.84	1.34
HS	95.0	3.03	2.89	3.09	2.58	2.79	3.29	3.00	2.54	2.89	2.09
	99.0	4.79	4.45	5.10	4.13	4.56	4.81	5.34	3.42	3.78	3.01
	99.9	6.96	6.94	9.44	5.02	5.94	7.45	6.02	5.73	6.39	5.31
GPD0	95.0	3.05	2.89	3.11	2.59	2.81	3.30	5.07	2.54	2.90	2.10
	99.0	4.86	4.48	4.61	4.06	4.07	4.76	9.98	3.71	4.03	3.01
	99.9	7.08	7.06	10.95	4.96	7.55	7.23	16.19	5.95	6.93	5.67
GPD1	95.0	4.16	3.89	4.38	3.48	3.70	4.22	8.09	3.29	3.68	2.75
	99.0	5.84	5.59	7.60	4.49	5.58	5.82	12.71	4.67	5.28	4.17
	99.9	7.90	8.35	21.38	5.11	10.78	8.53	18.54	7.30	9.40	8.26
GEV0	95.0	4.74	4.61	4.73	3.74	4.11	4.60	4.83	3.66	4.07	3.03
	99.0	7.26	7.40	7.32	5.28	5.87	6.46	7.24	5.34	5.91	4.22
	99.9	12.17	13.32	12.82	7.85	9.01	9.50	11.80	8.36	9.20	6.06
GEV1	95.0	10.68	10.60	11.23	8.08	9.14	10.37	10.32	8.72	9.67	7.12
	99.0	7.26	7.40	16.76	5.28	11.81	13.91	7.24	11.07	12.30	9.53
	99.9	12.17	13.32	12.82	7.85	9.01	9.50	11.80	8.36	9.20	6.06
MG		6.96	6.94	9.44	5.02	5.94	7.45	6.02	5.73	6.39	5.31

Estimated risk is greater than or equal to the maximum loss or gain (adequate)
Estimated risk is more than double the maximum loss or gain (overestimated)
0= unconditional VaR, 1 = conditional VaR, MG= Maximum Gain, CL= Confidence Level

Appendix 2 Descriptive Statistics and Estimated Parameters

	NASDAQ	S&P 500	CAC 40	FTSE 100	Nikkei 225	Hang Seng	BOVESPA	JSE All Share	JSE Top 40	R/US$
	Table 3: Downside Descriptive Statistics									
	Pre-crisis Period									
Mean	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
Median	-0.01	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.01	-0.01
Maximum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Minimum	-0.03	-0.02	-0.03	-0.03	-0.05	-0.04	-0.06	-0.06	-0.07	-0.04
Std. Dev.	0.01	0.00	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01
Skewness	-0.96	-0.93	-1.64	-1.81	-1.87	-1.63	-1.39	-2.76	-2.66	-1.48
Kurtosis	3.32	3.29	6.16	7.42	7.86	5.98	5.23	15.37	14.64	5.99
Jarque-Bera	58.51	51.18	297.93	467.32	581.58	296.84	196.36	2676.53	2394.43	312.49
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Observations	370.00	349.00	345.00	344.00	371.00	365.00	371.00	350.00	351.00	424.00
	Crisis Period									
Mean	-0.01	-0.01	-0.01	-0.01	-0.01	-0.02	-0.02	-0.01	-0.01	-0.01
Median	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
Maximum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Minimum	-0.09	-0.09	-0.09	-0.09	-0.11	-0.12	-0.11	-0.08	-0.08	-0.05
Std. Dev.	0.01	0.02	0.01	0.01	0.02	0.02	0.02	0.01	0.01	0.01
Skewness	-2.20	-2.40	-2.26	-2.41	-2.75	-1.93	-1.93	-1.79	-1.74	-2.52
Kurtosis	9.30	9.97	9.25	10.50	12.55	8.56	7.84	7.35	7.04	12.26
Jarque-Bera	617.57	739.28	654.45	870.37	1310.17	489.70	398.15	337.18	301.24	1357.62
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Observations	251.00	248.00	264.00	263.00	259.00	257.00	249.00	255.00	254.00	293.00
	Recovery period									
Mean	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
Median	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
Maximum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Minimum	-0.07	-0.07	-0.06	-0.05	-0.10	-0.06	-0.08	-0.04	-0.04	-0.04
Std. Dev.	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Skewness	-1.59	-1.75	-1.54	-1.70	-2.91	-1.37	-1.98	-1.18	-1.14	-1.77
Kurtosis	6.11	6.86	5.34	6.56	20.41	4.72	8.80	3.99	3.83	8.29
Jarque-Bera	257.86	362.60	213.88	340.10	4830.02	156.56	712.94	93.63	84.39	698.07
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Observations	314.00	320.00	344.00	337.00	344.00	360.00	347.00	346.00	344.00	414.00

	NASDAQ	S&P 500	CAC 40	FTSE 100	Nikkei 225	Hang Seng	BOVESPA	JSE All Share	JSE Top 40	R/US
	Table 4: Upside Descriptive Statistics									
	Pre-crisis Period									
Mean	0.01	0.00	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01
Median	0.01	0.00	0.01	0.00	0.01	0.00	0.01	0.01	0.01	0.01
Maximum	0.03	0.02	0.03	0.03	0.04	0.04	0.06	0.05	0.06	0.04
Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Std. Dev.	0.01	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01
Skewness	1.32	1.31	1.22	1.62	1.10	1.42	1.36	2.36	2.34	1.85
Kurtosis	4.73	4.61	4.63	7.08	3.88	5.73	5.35	11.75	11.54	6.88
Jarque-Bera	170.90	170.80	156.96	495.25	95.83	269.55	220.53	1778.66	1701.89	428.6
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Observations	412.00	433.00	437.00	438.00	411.00	417.00	411.00	432.00	431.00	358.0
	Crisis Period									
Mean	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.01
Median	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Maximum	0.12	0.11	0.11	0.09	0.10	0.15	0.15	0.07	0.08	0.12
Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Std. Dev.	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.01
Skewness	3.28	3.54	3.91	3.36	2.62	3.44	3.39	2.20	2.22	4.26
Kurtosis	18.58	20.46	22.15	17.94	12.44	19.00	18.83	8.59	8.79	32.98
Jarque-Bera	3239.70	4067.31	4615.97	2909.83	1282.66	3362.56	3384.74	565.97	597.10	06.08
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Observations	272.00	275.00	259.00	260.00	264.00	266.00	274.00	268.00	269.00	230.0
	Recovery period									
Mean	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Median	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Maximum	0.07	0.07	0.09	0.05	0.06	0.07	0.06	0.06	0.06	0.05
Minimum	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Std. Dev.	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Skewness	2.21	2.33	2.41	1.83	1.68	1.70	1.86	1.85	1.89	1.94
Kurtosis	9.78	10.80	14.42	7.28	7.18	7.54	7.39	8.19	8.43	9.79
Jarque-Bera	1125.89	1395.43	2446.28	513.67	457.79	490.69	522.59	643.20	696.42	796.7
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Observations	412.00	406.00	382.00	389.00	382.00	366.00	379.00	380.00	382.00	312.0

Table 5: Generalized Pareto Distribution: Maximum Likelihood Estimated Parameters

			Downside			Upside	
Asset	Parameter/1	Pre-Crisis	Crisis	Post Crisis	Pre-Crisis	Crisis	Post Crisis
	Shape	-0.797 (0.040)	-0.359 (0.548)	0.054 (0.283)	-0.109 (0.276)	0.061 (0.312)	0.081 (0.257)
NASDAQ	Location	0.007	0.014	0.012	0.007	0.012	0.01
	Scale	0.005 (0.000)	0.023 (0.014)	0.008 (0.003)	0.004 (0.001)	0.019 (0.008)	0.012 (0.004)
	Shape	0.650 (0.057)	-0.065 (0.244)	-0.150 (0.244)	0.335 (0.109)	0.451 (0.502)	0.065 (0.262)
S&P 500	Location	0.005	0.013	0.011	0.005	0.011	0.009
	Scale	0.003 (0.000)	0.003 (0.000)	0.012 (0.004)	0.004 (0.000)	0.011 (0.006)	0.009 (0.003)
	Shape	-0.578 (0.068)	0.470 (0.327)	-0.47 (0.327)	-0.875 (0.021)	-0.829 (0.044)	0.539 (0.413)
CAC 40	Location	0.006	0.013	0.012	0.006	0.012	0.011
	Scale	0.009 (0.000)	0.027 (0.010)	0.027 (0.010)	0.008 (0.000)	0.064 (0.000)	0.006 (0.002)
	Shape	-0.344 (0.105)	-0.274 (0.338)	-0.303 (0.3826)	-0.168 (0.269)	-0.597 (0.821)	-0.449 (0.089)
FTSE 100	Location	0.005	0.012	0.01	0.005	0.011	0.009
	Scale	0.007 (0.000)	0.021 (0.009)	0.011 (0.005)	0.005 (0.001)	0.041 (0.035)	0.013 (0.000)
	Shape	-0.008 (0.289)	0.034 (0.589)	0.262 (0.275)	0.207 (0.338)	0.079 (0.386)	0.331 (0.439)
NIKKEI 225	Location	0.008	0.015	0.011	0.008	0.013	0.01
	Scale	0.007 (0.002)	0.020 (0.013)	0.010 (0.003)	0.005 (0.002)	0.019 (0.009)	0.006 (0.003)
	Shape	-0.217 (0.286)	0.288 (0.611)	0.535 (0.070)	-0.119 (0.217)	-0.418 (0.354)	0.086 (0.268)
HANG SENG	Location	0.006	0.017	0.011	0.006	0.016	0.012
	Scale	0.006 (0.002)	0.013 (0.009)	0.014 (0.000)	0.005 (0.001)	0.057 (0.024)	0.008 (0.003)
	Shape	0.174 (0.253)	-0.486 (0.342)	0.069 (0.301)	-0.420 (0.231)	-0.064 (0.381)	-0.064 (0.381)
BOVESPA	Location	0.012	0.019	0.012	0.012	0.016	0.011
	Scale	0.010 (0.003)	0.033 (0.014)	0.010 (0.004)	0.013 (0.003)	0.032 (0.015)	0.032 (0.015)
	Shape	-0.169 (0.209)	0.337 (0.410)	-0.850 (0.022)	0.060 (0.322)	-0.600 (0.073)	0.149 (0.290)
JSE All Share	Location	0.008	0.014	0.01	0.008	0.012	0.009
	Scale	0.014 (0.004)	0.019 (0.009)	0.010 (0.000)	0.008 (0.003)	0.022 (0.000)	0.006 (0.002)
	Shape	-0.153 (0.207)	0.368 (0.425)	-0.891 (0.021)	0.014 (0.315)	0.580 (0.126)	0.295 (0.313)

JSE Top 40	Location	0.009	0.015	0.011	0.008	0.013	0.01
	Scale	0.015 (0.005)	0.020 (0.010)	0.010 (0.020)	0.010 (0.004)	0.026 (0.003)	0.005 (0.002)
	Shape	-0.146 (0.199)	-0.219 (0.302)	0.066 (0.208)	0.118 (0.000)	0.480 (0.500)	0.352 (0.360)
R/US$	Location	0.007	0.007	0.007	0.009	0.01	0.008
	Scale	0.006 (0.002)	0.012 (0.005)	0.004 (0.001)	0.024 (0.000)	0.010 (0.005)	0.004 (0.002)
1. Standard errors in brackets							

Table 6: Generalized Extreme Value Distribution: Maximum Likelihood Estimated Parameters

			Downside			Upside	
Asset	Parameter/1	Pre-Crisis	Crisis	Post Crisis	Pre-Crisis	Crisis	Post Crisis
	Shape	-0.190 (0.065)	0.525 (0.169)	0.161 (0.091)	-0.228 (0.113)	0.251 (0.115)	-0.001 (0.104)
NASDAQ	Location	0.012 (0.001)	0.015 (0.001)	0.016 (0.001)	0.011 (0.001)	0.017 (0.002)	0.015 (0.001)
	Scale	0.006 (0.000)	0.008 (0.001)	0.008 (0.001)	0.006 (0.000)	0.010 (0.001)	0.011 (0.001)
	Shape	-0.187 (0.060)	0.504 (0.162)	0.203 (0.105)	-0.161 (0.081)	0.295 (0.137)	0.057 (0.113)
S&P 500	Location	0.009 (0.000)	0.013 (0.001)	0.014 (0.001)	0.008 (0.000)	0.015 (0.002)	0.014 (0.001)
	Scale	0.004 (0.000)	0.008 (0.001)	0.008 (0.001)	0.004 (0.000)	0.011 (0.001)	0.010 (0.001)
	Shape	-0.151 (0.072)	0.577 (0.161)	0.203 (0.097)	-0.011 (0.074)	0.136 (0.112)	0.096 (0.129)
CAC 40	Location	0.011 (0.001)	0.015 (0.001)	0.017 (0.001)	0.009 (0.001)	0.016 (0.002)	0.017 (0.001)
	Scale	0.005 (0.000)	0.007 (0.001)	0.007 (0.001)	0.005 (0.000)	0.011 (0.001)	0.010 (0.001)
	Shape	-0.047 (0.054)	0.634 (0.203)	0.085 (0.092)	0.127 (0.089)	0.013 (0.083)	0.082 (0.102)
FTSE 100	Location	0.008 (0.000)	0.013 (0.001)	0.014 (0.001)	0.007 (0.000)	0.015 (0.002)	0.013 (0.001)
	Scale	0.004 (0.000)	0.007 (0.001)	0.007 (0.001)	0.004 (0.000)	0.013 (0.001)	0.008 (0.001)
	Shape	-0.174 (0.067)	0.284 (0.134)	0.119 (0.087)	0.056 (0.094)	0.167 (0.094)	0.048 (0.065)
NIKKEI 225	Location	0.014 (0.001)	0.016 (0.001)	0.017 (0.001)	0.010 (0.001)	0.018 (0.002)	0.016 (0.001)
	Scale	0.006 (0.000)	0.009 (0.001)	0.007 (0.001)	0.006 (0.001)	0.013 (0.001)	0.010 (0.001)
	Shape	-0.058 (0.070)	0.576 (0.191)	0.072 (0.078)	-0.022 (0.088)	0.204 (0.120)	-0.080 (0.087)
HANG SENG	Location	0.011 (0.001)	0.020 (0.002)	0.017 (0.001)	0.010 (0.001)	0.022 (0.002)	0.018 (0.001)
	Scale	0.005 (0.000)	0.010 (0.001)	0.009 (0.001)	0.006 (0.000)	0.012 (0.001)	0.010 (0.001)
	Shape	-0.048 (0.084)	0.4074(0.130)	0.147 (0.092)	0.004 (0.095)	0.047 (0.089)	0.078 (0.087)
BOVESPA	Location	0.020 (0.001)	0.023 (0.002)	0.017 (0.001)	0.018 (0.001)	0.023 (0.003)	0.018 (0.001)
	Scale	0.009 (0.001)	0.011 (0.001)	0.008 (0.001)	0.009 (0.001)	0.017 (0.002)	0.009 (0.001)
	Shape	0.138 (0.074)	0.470 (0.162)	0.122 (0.091)	0.144 (0.072)	-0.009 (0.096)	-0.114 (0.118)
JSE All Share	Location	0.011 (0.001)	0.014 (0.001)	0.013 (0.001)	0.010 (0.001)	0.018 (0.002)	0.014 (0.001)
	Scale	0.006 (0.000)	0.008 (0.001)	0.007 (0.001)	0.005 (0.000)	0.012 (0.001)	0.008 (0.001)
	Shape	0.137 (0.075)	0.488 (0.166)	0.116 (0.090)	0.128 (0.070)	-0.013 (0.098)	-0.117 (0.124)
JSE Top 40	Location	0.012 (0.001)	0.015 (0.001)	0.015 (0.001)	0.011 (0.001)	0.019 (0.002)	0.015 (0.001)
	Scale	0.006 (0.000)	0.008 (0.001)	0.007 (0.001)	0.006 (0.000)	0.013 (0.001)	0.009 (0.001)
	Shape	0.005 (0.084)	0.170 (0.078)	0.045 (0.081)	-0.065 (0.071)	0.136 (0.094)	0.040 (0.076)
R/US$	Location	0.013 (0.001)	0.012 (0.001)	0.011 (0.001)	0.012 (0.001)	0.010 (0.001)	0.010 (0.001)
	Scale	0.008 (0.001)	0.009 (0.001)	0.006 (0.001)	0.006 (0.000)	0.005 (0.000)	0.005 (0.000)
	1. Standard errors in brackets						