Smets-Wouters '03 model revisited - an implementation in gEcon

Klima, Grzegorz and Podemski, Karol and Retkiewicz-Wijtiwiak, Kaja and Sowińska, Anna E.

Department for Strategic Analyses, Chancellery of the Prime Minister of the Republic of Poland

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Smets–Wouters '03 model revisited — an implementation in `gEcon`*†

Grzegorz Klima‡
Department for Strategic Analyses,
Chancellery of the Prime Minister
of the Republic of Poland

Karol Podemski
Department for Strategic Analyses,
Chancellery of the Prime Minister
of the Republic of Poland

Kaja Retkiewicz-Wijtiwiak
Department for Strategic Analyses,
Chancellery of the Prime Minister
of the Republic of Poland

Anna E. Sowińska
Department for Strategic Analyses,
Chancellery of the Prime Minister
of the Republic of Poland

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Abstract

This paper presents an implementation of the well-known [Smets & Wouters 2003] model for Euro Area using the `gEcon` package — what we call the "third generation" DSGE modelling toolbox. Our exercise serves three goals. First, we show how `gEcon` can be used to implement an important — from both applications and historical perspective — model. Second, through rigorous exposition enforced by the `gEcon`'s block-agent paradigm we analyse all the Smets-Wouters model's building blocks. Last, but not least, the implementation presented here serves as a natural starting point for important from applications point of view extensions, like opening the economy, introducing non-lump-sum taxes, or adding sectors to the model economy. Full model implementation is attached.

Keywords: DSGE, monetary policy, staggered prices, staggered wages.

JEL classification: C63, C88, E3, E4, E5.

1 Introduction

The idea that the economy is a complex system propagating exogenous random disturbances dates back to the works of [Frish 1933] and [Slutsky 1937] in the '30s of the past century. It re-emerged after the Keynesian Revolution in the '70s with the stochastic growth model of [Brock & Mirman 1972] and the works

*The views expressed herein are solely of the authors and do not necessarily reflect those of the Chancellery of the Prime Minister of the Republic of Poland.

†`gEcon` is an open-source package and can be downloaded from http://gecon.r-forge.r-project.org/.

‡E-mail: gklima@users.sourceforge.net.

of Lucas [Lucas Jr. 1972], [Lucas Jr. 1975]. These papers were theoretical in nature. It was the seminal work of [Kydland & Prescott 1982] that tried to quantify the implications of dynamic general equilibrium model with stochastic shocks and gave birth to the real business cycle (RBC) literature. However, the assumptions and properties of RBC models like perfect competition and money neutrality were questioned from the very beginning. As a result, in the '90s RBC models were modified to incorporate monopolistic competition and price rigidities [Rotemberg & Woodford 1997] leading to the so-called New Keynesian / dynamic stochastic general equilibrium (DSGE) modelling school. The model of [Christiano *et al.* 2005] is widely recognised as the canonical DSGE model, which accounts for money non-neutrality and prolonged but eventually decaying ("hump-shaped") response to monetary policy shocks. Following this work and using Bayesian techniques [Smets & Wouters 2003] estimated the first DSGE model to be used in policy making (the model was used by the European Central Bank).

As RBC/DSGE models are complex and non-linear, they have to be solved using numerical methods. In the '80s each researcher had to implement the entire model from scratch, most often in FORTRAN.[1] The numerical approach usually involved the linear-quadratic (LQ) approximation, which reduced the problem to solving the algebraic Riccati equation, familiar from control theory. The technological revolution of the '80s and '90s — growing popularity of personal computers, exponentially increasing computational power, rapid growth of the Internet, and emergence of software packages like MATLAB — was bound to reduce the barriers to entry to dynamic general equilibrium modelling. In the '90s Harald Uhlig's tool-kit [Uhlig *et al.* 1995] was released, followed by the solver by Christopher Sims [Sims 2002]. It was no longer necessary to implement one's own dynamic linear rational expectations solver (or LQ optimal control problem solver) and the cost of model implementation was dramatically reduced.[2] Still, researchers had to derive model equations and perturbation matrices, as the numerical approach was 1st order perturbation around the deterministic steady state. From today's perspective these packages can be viewed as the "first generation" of DSGE modelling tools — although they significantly reduced the cost of applying numerical methods, they still required a lot of pen & paper derivations. Deriving the 1st order perturbation matrices (not to mention higher order approximation) is a tedious task even for a model with only 10 variables. Automation of this process was a natural next step in the evolution of DSGE modelling software. This step was taken with the Dynare project [Adjemian *et al.* 2013]. This "second generation" toolbox internally handles the creation of perturbation matrices (symbolic differentiation) and then passes them on to the embedded solvers. The (almost) only input required is the system of non-linear equations describing the behaviour of the model economy. It should come as no surprise that over time Dynare has become the *de facto* standard in DSGE modelling.

Deriving first order conditions for agents' optimisation problems in standard economic models is not a difficult task to a trained economist, yet it becomes tedious and error-prone when the size of the model increases or many amendments are made in the model construction process. Changes in preferences, technology, or introduction of taxes require part of the model equations to be derived again. In addition, the costs of model debugging increase at least quadratically with the model size. These considerations have led us to developing `gEcon` [Klima *et al.* 2015] — the "third generation" DSGE modelling package. In `gEcon`, models are written in the form of decision problems of the economic agents (consumers, producers, etc.) and market clearing conditions — all equilibrium equations are derived automatically. The algorithm employed is described in detail in [Klima & Retkiewicz-Wijtiwiak 2014]. There are many advantages of writing models in such a way. First of all, the organisation of the input model file is clear and logical. Writing optimisation problems explicitly makes it easier to grasp the structure of the model. Each agent is described in a separate block, which simplifies the process of modifying and extending the model. Since the implementation of the model is independent of the chosen form of the first-order approximation (linear or log-linear), changing the approach does not involve any additional effort, as opposed to e.g. Dynare.

This paper serves three goals. First, we show how `gEcon` can be used to implement an important —

[1] Thus fulfilling Robert Lucas' call for economists "to write a FORTRAN program that will accept specific economic policy rules as 'inputs' and will generate as 'output' statistics describing the operating characteristics of times series we care about, which are predicted to result from these policies". [Lucas Jr. 1980]

[2] Nowadays, solving small scale RBC/DSGE model using these toolboxes is a standard homework assignment in graduate schools.

from both applications and historical perspective — model. Second, through rigorous exposition enforced by the gEcon's block-agent paradigm we analyse all the [Smets & Wouters 2003] model's building blocks. Last, but not least, the implementation presented here serves as a natural starting point for important from applications point of view extensions, like opening the economy, introducing non-lump-sum taxes or adding sectors to the model economy.

The paper is organised as follows. Section 2 contains the description of the consumers' problem. It involves the choice between labour and consumption, the determination of optimal amount of investments and the utilisation rate of capital, and the determination of optimal wage in Calvo wage setting mechanism. Labour is heterogeneous and bundled by a representative firm. Only part of households can request new level of wages in each period. In section 3 firms are introduced to the model and the Calvo price setting mechanism is described. Two types of firms are present in the model economy — intermediate and final, with only part of intermediate firms being able to fully adjust prices. The behaviour of the government and of the monetary authority is described in section 4. Market clearing and aggregation conditions complement the model description (section 5). Each section presents code listings with the gEcon implementation of the relevant optimisation problems. Since the output gap in the Taylor rule is calculated relative to the economy without frictions or mark-up shocks, along with each component of the model with rigidities its flexible counterpart is described, as well as its gEcon implementation. It has to be stressed that the model presented here is not a one-to-one replica of the original [Smets & Wouters 2003] model. The differences are summarised in section 6. Some of them are minor modifications that have no effect on the remaining solution or dynamics of the model. However, a few alterations result from the different approach to solving the problem or act as corrections. Section 7 briefly describes the solution procedure and obtained results. Section 8 concludes. The appendices contain the full list of variables in the model (Appendix A), the full gEcon implementation of the model with accompanying R code (Appendix B),[3] steady-state values and standard deviations of variables (Appendix C), basic model statistics (Appendix D), and comparison between responses to the shocks in economies with and without nominal rigidities (Appendix E).

Signature [SW'03] will be understood as a reference to [Smets & Wouters 2003] hereafter. The equations presented in [SW'03] will be referred to as (X–SW'03), X being the ordinal number of the equation in [SW'03].

2 Households

General consumer problem

In the model economy there is a continuum of identical infinitely lived households (indexed by $i \in [0, 1]$). Households maximise their expected lifetime utility $U_t(i)$ given by $\mathrm{E}_t[\sum_{\tau=0}^{\infty} \beta^{\tau} u_{t+\tau}(i)]$, where β is the discount factor and u_t is the instantaneous utility. The objective can be rewritten in a recursive manner as:

$$U_t(i) = u_t(i) + \beta \mathrm{E}_t[U_{t+1}(i)]\,. \tag{1}$$

The instantaneous utility function u is separable in consumption (C) and labour (L), and it assumes the following form[4]:

$$u_t(i) = \varepsilon_t^b \left(\frac{1}{1-\sigma_c} \left(C_t(i) - H_t\right)^{1-\sigma_c} - \frac{\omega \varepsilon_t^L}{1+\sigma_l} \left(L_t(i)\right)^{1+\sigma_l} \right), \tag{2}$$

with consumption habit given by $H_t = hC_{t-1}$, where h is the intensity of habit formation. The utility function is increasing in the difference between the current household's consumption and the average consumption in the economy from the previous period.[5] σ_c represents the coefficient of relative risk aversion, while σ_l is the reciprocal of the elasticity of labour with respect to the wage. The instantaneous utility

[3] The code can also be downloaded from the gEcon website: http://gecon.r-forge.r-project.org/.

[4] The parameter of labour disutility ω is set to 1 in [SW'03].

[5] This means that the so-called external habit formation is introduced. A common alternative is the internal habit formation, in which agents recognize their own influence on the formation of the habit.

of the household may be affected by any of the two preference shocks: ε_t^b is a shock to the discount rate influencing the intertemporal elasticity of substitution, while ε_t^L represents a shock to the labour supply. Both shocks follow the first order autoregressive processes (with i.i.d. innovations):[6]

$$\log \varepsilon_t^b = \rho_b \log \varepsilon_{t-1}^b + \eta_t^b,$$
$$\log \varepsilon_t^L = \rho_L \log \varepsilon_{t-1}^L - \eta_t^L.$$

The problem is solved subject to the budget constraint and the law of motion for capital.

The household's total income comes from the following sources:

- Labour income modified by the net cash-flow from the common insurance. Real wage $W_t(i)$ is earned for each unit of labour $L_t(i)$ supplied. Payments $A_t(i)$ ensure that the wage bill is equally distributed among all households and $\int A_t(i)di = 0$ for each period.
- Return on capital reduced by the cost associated with the level of capacity utilisation. The amount of capital offered by the household is determined by the capital stock $K_{t-1}(i)$ accumulated in the previous period, and the level of capacity utilisation $z_t(i)$. The cost of capacity utilisation is given by the function Ψ.
- Dividends $Div_t(i)$ paid out by firms.
- Proceeds from bonds purchased in the previous period.

The disposable income is reduced by the lump-sum tax T_t levied by the government.

The cost function Ψ associated with the level of capacity utilisation is strictly increasing ($\Psi' > 0$), convex ($\Psi'' > 0$), and equals 0 if the argument equals 1. The form adopted here (after [Adjemian *et al.* 2007]) is:

$$\Psi(z_t) = \frac{\bar{r}^k}{\psi}(e^{\psi(z_t-1)} - 1), \tag{3}$$

where $\bar{r}^k$ is the steady-state level of return on capital and ψ is the scale parameter.

Households rent capital to producers on a perfectly competitive market at rate r_t^k. In order to increase their income from renting capital, individual households can either intensify the use of the available capital stock or invest. Both operations are costly.

Households divide their income and financial wealth between consumption, investment $I_t(i)$ and the purchase of one-period bonds. Let $B_t^N(i)$ denote the (nominal) face value of bonds purchased at time t. The cost (in nominal terms) of purchasing bonds is equal to $B_t^N(i)/R_t$, where R_t is the (gross) nominal interest rate. The cost of purchasing $B_t^N(i)$ bonds in real terms is equal to $B_t^N(i)/(R_t P_t)$, where P_t is the price level. At time t bonds bought in the previous period bring $B_{t-1}^N(i)$ nominally and $B_{t-1}^N(i)/P_t$ in real terms.

The household's budget constraint stated in real terms is given by:

$$C_t(i) + I_t(i) + \frac{B_t^N(i)}{R_t P_t} =$$
$$= (W_t(i)L_t(i) + A_t(i)) + (r_t^k z_t(i)K_{t-1}(i) - \Psi(z_t(i))K_{t-1}(i)) + Div_t(i) + \frac{B_{t-1}^N(i)}{P_t} - T_t.$$

In general, the price level P_t is non-stationary. Using $B_t(i) = B_t^N(i)/P_t$ to denote the value of bonds in real terms and $\pi_t = P_t/P_{t-1}$ to denote inflation, one can rewrite the household's budget constraint as follows:

$$C_t(i) + I_t(i) + \frac{B_t(i)}{R_t} =$$
$$= (W_t(i)L_t(i) + A_t(i)) + (r_t^k z_t(i)K_{t-1}(i) - \Psi(z_t(i))K_{t-1}(i)) + Div_t(i) + \frac{B_{t-1}^N(i)}{P_{t-1}}\frac{P_{t-1}}{P_t} - T_t$$

[6] A minor change was made relative to [SW'03] in order to achieve the desired interpretation of η_t^L as a positive labour supply shock.

and finally:

$$
\begin{aligned}
&C_t(i) + I_t(i) + \frac{B_t(i)}{R_t} = \\
&= (W_t(i)L_t(i) + A_t(i)) + (r_t^k z_t(i) K_{t-1}(i) - \Psi(z_t(i))K_{t-1}(i)) + Div_t(i) + \frac{B_{t-1}(i)}{\pi_t} - T_t.
\end{aligned}
\tag{4}
$$

The law of motion for capital is given by:

$$
K_t(i) = (1-\tau)K_{t-1}(i) + \left[1 - S\left(\varepsilon_t^I \frac{I_t(i)}{I_{t-1}(i)}\right)\right] I_t(i), \tag{5}
$$

where τ is the capital depreciation rate and I_t denotes gross investment. Changes in investment are associated with additional costs. The adjustment cost function S($\cdot$) for investment is positive and its first derivative equals zero in the steady state (i.e. $S(1) = 0$, $\forall_{x \neq 1} S(x) > 0$ and $S'(1) = 0$). ε_t^I follows a first order autoregressive process: $\log \varepsilon_t^I = \rho_I \log \varepsilon_{t-1}^I + \eta_t^I$.[7]

The following cost function is adopted:

$$
S(x) = \frac{\varphi}{2}(x-1)^2, \tag{6}
$$

where φ is the scale parameter.

Since the problem described above is faced by a representative household, the net cash inflow resulting from common insurance equals zero. `gEcon` implementation of the optimisation problem of a representative household is presented in Listing 1.

```
block CONSUMER
{
    definitions
    {
        u[] = epsilon_b[] * ((C[] - H[]) ^ (1 - sigma_c) / (1 - sigma_c) -
              omega * epsilon_L[] * (L_s[]) ^ (1 + sigma_l) / (1 + sigma_l));
    };
    controls
    {
        C[], K[], I[], B[], z[];
    };
    objective
    {
        U[] = u[] + beta * E[][U[1]];
    };
    constraints
    {
       C[] + I[] + B[] / R[] =
          W[] * L[] +
           r_k[] * z[] * K[-1] - r_k[ss]  / psi * (exp(psi * (z[] - 1)) - 1)  * K[-1] +
           Div[] + B[-1] / pi[] - T[] : lambda[];
       K[] =  (1 - tau) * K[-1]  +
              (1 - varphi / 2 * (epsilon_I[] * I[] / I[-1] - 1) ^ 2) * I[] : q[];
    };
    identities
    {
        H[] = h * C[-1];
        Q[] = q[] / lambda[];
    };
    calibration
    {
        beta = 0.99;              # Discount factor
```

[7]One has to be cautious about the interpretation of the ε_t^I shock. As ε_t^I is a shock to the investment cost function, its positive values could be thought of as a negative shock to investment.

```
        tau = 0.025;              # Capital depreciation rate
        varphi = 6.771;           # Parameter of investment adjustment cost function
        psi = 0.169;              # Capacity utilisation cost parameter
        sigma_c = 1.353;          # Coefficient of relative risk aversion
        h = 0.573;                # Habit formation intensity
        sigma_l = 2.4;            # Reciprocal of labour elasticity w.r.t. wage
        omega = 1;                # Labour disutility parameter
    };
};

block PREFERENCE_SHOCKS
{
    identities
    {
        log(epsilon_b[]) = rho_b * log(epsilon_b[-1]) + eta_b[];
        log(epsilon_L[]) = rho_L * log(epsilon_L[-1]) - eta_L[];
    };
    shocks
    {
        eta_b[],     # Preference shock
        eta_L[];     # Labour supply shock
    };
    calibration
    {
        rho_b = 0.855;
        rho_L = 0.889;
    };
};

block INVESTMENT_COST_SHOCKS
{
    identities
    {
        log(epsilon_I[]) = rho_I * log(epsilon_I[-1]) + eta_I[];
    };
    shocks
    {
        eta_I[];     # Investment shock
    };
    calibration
    {
        rho_I = 0.927;
    };
};
```

Listing 1: Consumer problem

In the listing above Tobin's q (the value of installed capital in terms of its replacement cost, denoted by `Q[]`) has been introduced as a ratio between the two Lagrange multipliers.

The fact that the labour supplied does not appear in the problem formulation (and is not listed as a control variable) deserves explanation. In [SW'03], households supply differentiated labour, which gives them monopolistic power and makes them, to some extent, the wage setters. This could be reflected by making the wage one of the decision variables. In an economy without nominal rigidities, wages can be freely set. However, in a Calvo wage-setting scheme, a household cannot reset its wage in every period, and the time between any two consecutive re-optimisations is random. In the next two subsections the monopolistically competitive labour is introduced and the optimal wage setting rule for the Calvo scheme is derived explicitly.

Labour market structure

It is assumed that households are hired by a representative competitive firm (e.g. labour agency) that bundles differentiated labour using the following technology[8]:

$$L_t = \left[\int_0^1 L_t(i)^{\frac{1}{1+\lambda_w}} di \right]^{1+\lambda_w}. \tag{7}$$

The labour bundler's maximisation problem is as follows:

$$\max_{L_t(i)} W_t L_t - \int_0^1 W_t(i) L_t(i) di.$$

The first order condition with respect to the labour supplied by each household i yields:

$$L_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\frac{1+\lambda_w}{\lambda_w}} L_t. \tag{8}$$

Equation (8) determines the amount of labour of household i that the labour agency demands in response to the wage set by the household. Substituting the obtained demand function into the zero-profit condition allows to determine the aggregate wage:

$$W_t = \left[\int_0^1 W_t(i)^{-\frac{1}{\lambda_w}} di \right]^{-\lambda_w}. \tag{9}$$

Wage setting in Calvo scheme

The Calvo scheme implies that only a fraction of households receive a random wage-change signal that is necessary to re-optimise the wage. The probability that a specific household will receive such signal is constant and equals $1 - \xi_w$. The remaining households, which did not receive the wage-change signal, are allowed to partially index their wages to past inflation. The indexed wages are adjusted according to:

$$W_t(i) = \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} W_{t-1}(i), \tag{10}$$

where γ_w is the degree of wage indexation, i.e. for $\gamma_w = 1$ perfect indexation takes place, while if $\gamma_w = 0$ wages remain constant until the signal is received.

This implies that if a household cannot change its wage for τ periods, its normalized (real) wage after τ periods is equal to:

$$W_{t+\tau}(i) = \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} \right) W_t(i), \tag{11}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$. The expression on the right hand side of equation (11) is equivalent to $\frac{(P_{t-1+\tau}/P_{t-1})^{\gamma_w}}{P_{t+\tau}/P_t} W_t(i)$, cf. (12-SW'03).

Substituting (11) in (8), we arrive at:

$$L_{t+\tau}(i) = \left(\left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} \right) \frac{W_t(i)}{W_{t+\tau}} \right)^{-\frac{1+\lambda_w}{\lambda_w}} L_{t+\tau}. \tag{12}$$

[8] The parameter $1 + \lambda_w$ determines the wage mark-up over the labour disutility.

The maximisation problem of the household that received a signal to re-optimise its wage is:

$$\max_{W_t(i)} \mathrm{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\xi_w)^\tau \left[\lambda_{t+\tau}(i) \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} \right) W_t(i) L_{t+\tau}(i) - \varepsilon_{t+\tau}^b \varepsilon_{t+\tau}^L \omega \frac{(L_{t+\tau}(i))^{1+\sigma_l}}{1+\sigma_l} \right] \right], \tag{13}$$

where $\lambda_t(i)$ is the Lagrange multiplier from the general consumer problem representing the marginal utility of consumption. Each component of the sum is the difference between the utility of consumption gained from selling labour and the utility loss related to giving up leisure. The net utilities in future periods are discounted with β^τ and weighted by the cumulative probabilities of not receiving the wage-change signal ξ_w^τ. Substituting for $L_{t+\tau}(i)$ from equation (12), one obtains:

$$\begin{aligned} \max_{W_t(i)} \mathrm{E}_t \Bigg[\sum_{\tau=0}^{\infty} (\beta\xi_w)^\tau \Bigg[& \lambda_{t+\tau}(i) \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} \right) W_t(i) \left(\left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} \right) \frac{W_t(i)}{W_{t+\tau}} \right)^{-\frac{1+\lambda_w}{\lambda_w}} L_{t+\tau} \\ & -\varepsilon_{t+\tau}^b \frac{\varepsilon_{t+\tau}^L \omega}{1+\sigma_l} \left(\left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} \right) \frac{W_t(i)}{W_{t+\tau}} \right)^{-\frac{(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} {L_{t+\tau}}^{1+\sigma_l} \Bigg] \Bigg]. \end{aligned}$$

All households set the same wage, so the indices i can be dropped. Since the markets are assumed to be complete (implying perfect risk sharing), the marginal utility of consumption is also the same for any household. The first order condition with respect to $W_t(i)$ is:

$$\begin{aligned} \mathrm{E}_t \Bigg[\sum_{\tau=0}^{\infty} (\beta\xi_w)^\tau \Bigg[& -\frac{\lambda_{t+\tau}}{\lambda_w} \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} \right)^{-\frac{1}{\lambda_w}} \left(\frac{W_t^\star}{W_{t+\tau}} \right)^{-\frac{1+\lambda_w}{\lambda_w}} L_{t+\tau} \\ & + \frac{(1+\lambda_w)}{\lambda_w W_t^\star} \varepsilon_{t+\tau}^b \varepsilon_{t+\tau}^L \omega \left(\left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} \right) \frac{W_t^\star}{W_{t+\tau}} \right)^{-\frac{(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} {L_{t+\tau}}^{1+\sigma_l} \Bigg] \Bigg] = 0, \end{aligned} \tag{14}$$

where $W_t^\star$ denotes the optimal wage set at time t.

The first order condition involves infinite sums. In order to solve the wage-setting problem one might proceed as in [SW'03], where the equation is log-linearised. In this way one can obtain the real wage equation (33–SW'03). Another possibility is to transform (14) into a recursive form, following for example [Schmitt-Grohe & Uribe 2004]. This method does not involve any approximations and allows to derive the equations determining the steady state explicitly. The latter approach is adopted in this paper.

Rewriting (14) as two separate sums and multiplying both sides by $\frac{\lambda_w W_t^\star}{1+\lambda_w}$ yields:

$$\begin{aligned} & \frac{1}{1+\lambda_w} W_t^\star \mathrm{E}_t \left[\sum_{\tau=0}^{\infty} \left[(\beta\xi_w)^\tau \lambda_{t+\tau} \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} \right)^{-\frac{1}{\lambda_w}} \left(\frac{W_t^\star}{W_{t+\tau}} \right)^{-\frac{1+\lambda_w}{\lambda_w}} L_{t+\tau} \right] \right] = \\ & = \mathrm{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\xi_w)^\tau \left[\varepsilon_{t+\tau}^b \varepsilon_{t+\tau}^L \omega \left(\left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}} \right) \frac{W_t^\star}{W_{t+\tau}} \right)^{-\frac{(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} {L_{t+\tau}}^{1+\sigma_l} \right] \right]. \end{aligned} \tag{15}$$

Let us denote the left-hand side as f_t^1 and the expectation on the right-hand side as f_t^2. We have:

$$f_t^1 = f_t^2. \tag{16}$$

The left-hand side of (15) can be transformed into a recursive form in the following manner:

$$
\begin{aligned}
f_t^1 =& \frac{1}{1+\lambda_w} W_t^\star \lambda_t \left(\frac{W_t^\star}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} L_t + \\
& \frac{1}{1+\lambda_w} W_t^\star \mathrm{E}_t \left[\sum_{\tau=1}^{\infty} (\beta\xi_w)^\tau \lambda_{t+\tau} \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}}\right)^{-\frac{1}{\lambda_w}} \left(\frac{W_t^\star}{W_{t+\tau}}\right)^{-\frac{1+\lambda_w}{\lambda_w}} L_{t+\tau}\right] \\
=& \frac{1}{1+\lambda_w} (W_t^\star)^{-\frac{1}{\lambda_w}} \lambda_t W_t^{\frac{1+\lambda_w}{\lambda_w}} L_t + \beta\xi_w W_t^\star \mathrm{E}_t \left[\frac{W_{t+1}^\star}{W_{t+1}^\star} \left(\frac{\pi_t^{\gamma_w}}{\pi_{t+1}}\right)^{-\frac{1}{\lambda_w}} \times \right. \\
& \left. \times \frac{1}{1+\lambda_w} \sum_{\tau=0}^{\infty} \left[(\beta\xi_w)^\tau \lambda_{t+\tau+1} \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s}^{\gamma_w}}{\pi_{t+s+1}}\right)^{-\frac{1}{\lambda_w}} \left(\frac{W_t^\star}{W_{t+\tau+1}}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \left(\frac{W_{t+1}^\star}{W_{t+1}^\star}\right)^{-\frac{1+\lambda_w}{\lambda_w}} L_{t+\tau+1}\right]\right] \\
=& \frac{\lambda_t}{1+\lambda_w} (W_t^\star)^{-\frac{1}{\lambda_w}} W_t^{\frac{1+\lambda_w}{\lambda_w}} L_t + \beta\xi_w \mathrm{E}_t \left[\left(\frac{\pi_t^{\gamma_w}}{\pi_{t+1}}\right)^{-\frac{1}{\lambda_w}} \left(\frac{W_{t+1}^\star}{W_t^\star}\right)^{\frac{1}{\lambda_w}} f_{t+1}^1\right] \\
=& \frac{\lambda_t}{1+\lambda_w} L_t^{\star d} W_t^\star + \beta\xi_w \mathrm{E}_t \left[\left(\frac{\pi_t^{\gamma_w}}{\pi_{t+1}}\right)^{-\frac{1}{\lambda_w}} \left(\frac{W_{t+1}^\star}{W_t^\star}\right)^{\frac{1}{\lambda_w}} f_{t+1}^1\right],
\end{aligned} \tag{17}
$$

where

$$
L_t^{\star d} = \left(\frac{W_t^\star}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} L_t. \tag{18}
$$

Analogously, for the right-hand side we obtain:

$$
\begin{aligned}
f_t^2 =& \varepsilon_t^b \varepsilon_t^L \omega \left(\frac{W_t}{W_t^\star}\right)^{\frac{(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} L_t^{1+\sigma_l} + \mathrm{E}_t \left[\sum_{\tau=1}^{\infty} (\beta\xi_w)^\tau \left[\varepsilon_{t+\tau}^b \varepsilon_{t+\tau}^L \omega \left(\left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}}\right) \frac{W_t^\star}{W_{t+\tau}}\right)^{-\frac{(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} {L_{t+\tau}}^{1+\sigma_l}\right]\right] \\
=& \varepsilon_t^b \varepsilon_t^L \omega \left(\frac{W_t}{W_t^\star}\right)^{\frac{(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} L_t^{1+\sigma_l} + \beta\xi_w \left(\frac{\pi_t^{\gamma_w}}{\pi_{t+1}}\right)^{-\frac{1}{\lambda_w}} \times \\
& \times \mathrm{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\xi_w)^\tau \left[\varepsilon_{t+\tau+1}^b \varepsilon_{t+\tau+1}^L \omega \left(\left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_w}}{\pi_{t+s}}\right) \frac{W_t^\star}{W_{t+\tau+1}}\right)^{-\frac{(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} \left(\frac{W_{t+1}^\star}{W_{t+1}^\star}\right)^{-\frac{(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} {L_{t+\tau+1}}^{1+\sigma_l}\right]\right] \\
=& \varepsilon_t^b \varepsilon_t^L \omega \left(\frac{W_t}{W_t^\star}\right)^{\frac{(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} L_t^{1+\sigma_l} + \beta\xi_w \mathrm{E}_t \left[\left(\frac{\pi_t^{\gamma_w}}{\pi_{t+1}}\right)^{-\frac{(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} \left(\frac{W_{t+1}^\star}{W_t^\star}\right)^{\frac{(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} f_{t+1}^2\right] \\
=& \varepsilon_t^b \varepsilon_t^L \omega (L_t^{\star d})^{1+\sigma_l} + \beta\xi_w \mathrm{E}_t \left[\left(\frac{\pi_t^{\gamma_w}}{\pi_{t+1}}\right)^{-\frac{(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} \left(\frac{W_{t+1}^\star}{W_t^\star}\right)^{\frac{(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} f_{t+1}^2\right].
\end{aligned} \tag{19}
$$

In the actual model implementation we will introduce a random (i.i.d Gaussian) disturbance η_t^w to equation (16):

$$
f_t^1 = f_t^2 + \eta_t^w. \tag{20}
$$

η_t^w can be interpreted as a labour market inefficiency shock[9] — it serves as a substitute for a monopolistic power (mark-up) shock, i.e. a shock to λ_w (for details see section 6).

The recursive equations (17) and (19) can be expressed in `gEcon` as in Listing 2 presented below.

[9] This is similar to the price mark-up shock introduced on page 19.

```
block WAGE_SETTING_PROBLEM
{
    definitions
    {
        L_star[] = (pi_star_w[]) ^ (-((1 + lambda_w) / lambda_w)) * L[];
    }
    identities
    {
        f_1[] = 1 / (1 + lambda_w) * w_star[] * lambda[] * L_star[] +
              beta * xi_w * E[][((pi[] ^ gamma_w) / pi[1])^(-1 / lambda_w) *
              (w_star[1] / w_star[])^(1 / lambda_w) * f_1[1]];
        f_2[] = epsilon_L[] * omega * epsilon_b[] * (L_star[]) ^ (1 + sigma_l) +
              beta * xi_w * E[][((pi[] ^ gamma_w) / pi[1])^
              (-((1 + lambda_w) / lambda_w)*(1 + sigma_l)) *
              (w_star[1] / w_star[])^(((1 + lambda_w) / lambda_w) * (1 + sigma_l)) * f_2[1
                  ]];
        f_1[] = f_2[] + eta_w[];
        pi_star_w[] = w_star[] / W[];
    };
    shocks
    {
        eta_w[];                    # Wage mark-up shock
    };
    calibration
    {
        gamma_w = 0.763;            # Indexation parameter for non-optimising workers
        lambda_w = 0.5;             # Wage mark-up
        xi_w = 0.737;               # Probability of not receiving ``wage-change signal''
    };
};
```

Listing 2: Wage setting problem

The optimal wage $W_t^\star$ determined by (17) and (19) is set by all households that have received the wage-change signal. For the rest of the households, wages are determined by the previous-period wages and inflation as in (11). Since the aggregate wage W_t is given by the Dixit-Stiglitz-type function (9), the equation describing the aggregate wage evolution assumes the following form:

$$W_t^{-\frac{1}{\lambda_w}} = \xi_w \left(\frac{\pi_{t-1}^{\gamma_w}}{\pi_t} \right)^{-\frac{1}{\lambda_w}} W_{t-1}^{-\frac{1}{\lambda_w}} + (1 - \xi_w) W_t^{\star -\frac{1}{\lambda_w}},$$

which can be transformed to:

$$1 = \xi_w \left(\frac{\pi_{t-1}^{\gamma_w}}{\pi_t} \right)^{-\frac{1}{\lambda_w}} \left(\frac{W_{t-1}}{W_t} \right)^{-\frac{1}{\lambda_w}} + (1 - \xi_w) \left(\pi_t^{\star w} \right)^{-\frac{1}{\lambda_w}}, \tag{21}$$

with $\pi_t^{\star w} = \frac{W_t^\star}{W_t}$.

Equation (21) can be written in `gEcon` as in Listing 3.

```
block WAGE_EVOLUTION
{
    identities
    {
        1 = xi_w * ((pi[-1] ^ gamma_w) / pi[]) ^ (-1 / lambda_w) *
            (W[-1] / W[])^(-1 / lambda_w) + (1 - xi_w) * (pi_star_w[]) ^ (-1 / lambda_w);
    };
};
```

Listing 3: Evolution of wages

In order to formulate the labour market clearing conditions, it is helpful to introduce the index of wage dispersion that links the demand for the aggregate labour with the total supply of all types of differentiated labour. Integrating both sides of equation (8) gives the aggregate condition:

$$\int_0^1 L_t(i)di = \left(\int_0^1 \left(\frac{W_t(i)}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} di\right) L_t.$$

Let us introduce the wage dispersion index $\nu_t^w = \int_0^1 \left(\frac{W_t(i)}{W_t}\right)^{-\frac{(1+\lambda_w)}{\lambda_w}} di$ and denote total labour services provided by all households as $L_t^s = \int_0^1 L_t(i)di$. We have:

$$L_t^s = \nu_t^w L_t. \tag{22}$$

Recall that in each period $1-\xi_w$ fraction of households can re-optimise their wages and ξ_w households only index them to past inflation. This leads to the recursive formula for the evolution of the wage dispersion ν_t^w:

$$\begin{aligned}
\nu_t^w = &\int_0^1 \left(\frac{W_t(i)}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} di = \\
=&(1-\xi_w)\left(\frac{W_t^\star}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} + (1-\xi_w)\xi_w\left(\frac{W_{t-1}^\star}{W_t}\frac{\pi_{t-1}^{\gamma_w}}{\pi_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} + (1-\xi_w){\xi_w}^2\left(\frac{W_{t-2}^\star}{W_t}\frac{\pi_{t-1}^{\gamma_w}}{\pi_t}\frac{\pi_{t-2}^{\gamma_w}}{\pi_{t-1}}\right)^{-\frac{1+\lambda_w}{\lambda_w}} + \ldots \\
=&(1-\xi_w)\left(\frac{W_t^\star}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} + \xi_w\left(\frac{W_{t-1}}{W_t}\frac{\pi_{t-1}^{\gamma_w}}{\pi_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \times \\
&\times\left((1-\xi_w)\left(\frac{W_{t-1}^\star}{W_{t-1}}\right)^{-\frac{1+\lambda_w}{\lambda_w}} + (1-\xi_w)\xi_w\left(\frac{W_{t-2}^\star}{W_{t-1}}\frac{\pi_{t-2}^{\gamma_w}}{\pi_{t-1}}\right)^{-\frac{1+\lambda_w}{\lambda_w}} + \ldots\right) \\
=&(1-\xi_w)\left(\pi_t^{\star w}\right)^{-\frac{1+\lambda_w}{\lambda_w}} + \xi_w\left(\frac{W_{t-1}}{W_t}\frac{\pi_{t-1}^{\gamma_w}}{\pi_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \nu_{t-1}^w.
\end{aligned} \tag{23}$$

```
block LABOUR_AGGREGATION
{
    identities
    {
        nu_w[] = (1 - xi_w) * pi_star_w[] ^(-((1 + lambda_w) / lambda_w)) +
                 xi_w * ((W[-1]/W[]) * ((pi[-1]^gamma_w) / pi[])) ^
                   (-((1 + lambda_w) / lambda_w)) * nu_w[-1];
        L[] = L_s[] / nu_w[];
    };
};
```

Listing 4: Labour aggregation

Consumer problem without wage rigidities

The flexible wages economy can be regarded as the specific case of the Calvo scheme with the probability of receiving the wage-change signal equal to 1 in each period. Equations describing the economy without rigidities could be derived by setting $\xi_w = 0$. However, the fact that all consumers may set their wage in each period makes it possible to describe the behaviour of agents as a solution to an explicitly stated optimisation problem. The latter approach is adopted in this subsection.

Variables describing the behaviour of this economy can be distinguished from their counterparts in the economy with rigidities by the superscript f.

The consumer problem is given by:

$$\max_{C_t^f(i), K_t^f(i), I_t^f(i), B_t^f(i), z_t^f(i), L_t^f(i)} U_t^f(i) = \varepsilon_t^b \left(\frac{1}{1-\sigma_c} \left(C_t^f(i) - H_t^f \right)^{1-\sigma_c} - \frac{\omega \varepsilon_t^L}{1+\sigma_l} (L_t^f(i))^{1+\sigma_l} \right) + \beta \mathrm{E}_t \left[U_{t+1}^f(i) \right] \tag{24}$$

s.t.:

$$C_t^f(i) + I_t^f(i) + \frac{B_t^f(i)}{R_t^f} = Inc_t^f(i) + Div_t^f(i) + B_{t-1}^f(i) - T_t^f, \tag{25}$$

$$K_t^f(i) = (1-\tau) K_{t-1}^f(i) + \left[1 - S\left(\varepsilon_t^I \frac{I_t^f(i)}{I_{t-1}^f(i)} \right) \right] I_t^f(i), \tag{26}$$

where $Inc_t^f(i) = (W_t^{f,disutil} L_t^f(i) + \Pi_t^{f,ws}(i)) + (r_t^{k,f} z_t^f(i) K_{t-1}^f(i) - \Psi(z_t^f(i)) K_{t-1}^f(i))$ and $\Pi_t^{f,ws}$ is the monopolistic profit of workers. Contrary to the model with rigidities, the wage $W_t^{f,disutil}$ is understood here as the marginal cost of labour in terms of consumption and not the actual wage that the individuals receive. Note that here the labour is among the control variables.

Since it is assumed that all households are symmetric their indices can be dropped. The problem is then expressed in `gEcon` in the following way:

```
block CONSUMER_FLEXIBLE
{
    definitions
    {
        u_f[] = epsilon_b[] * ((C_f[] - H_f[]) ^ (1 - sigma_c) / (1 - sigma_c) -
                omega * epsilon_L[] * L_s_f[] ^ (1 + sigma_l) / (1 + sigma_l));
        Inc_i_f[] =  W_disutil_f[] * L_s_f[]  + Pi_ws_f[] +
                     r_k_f[] * z_f[] * K_f[-1] -
                     r_k_f[ss]  / psi * (exp(psi * (z_f[] - 1)) - 1)  * K_f[-1];
    };
    controls
    {
        C_f[], K_f[], I_f[], B_f[], z_f[], L_s_f[];
    };
    objective
    {
        U_f[] = u_f[] + beta * E[][U_f[1]];
    };
    constraints
    {
        C_f[] + I_f[] + B_f[] / R_f[]  =
            Inc_i_f[] + Div_f[] + B_f[-1] - T_f[]: lambda_f[];
        K_f[] = (1 - tau) * K_f[-1]  +
                (1 - varphi / 2 * (epsilon_I[] * I_f[] / I_f[-1] - 1)^2) * I_f[]  : q_f[];
    };
    identities
    {
        H_f[] = h * C_f[-1];
        Q_f[] = q_f[] / lambda_f[];
    };
};
```

Listing 5: Consumer problem, flexible wages economy

The problem of a labour bundler is similar to the one in the economy with wage rigidities. Specifically, the labour agency aggregates the labour provided by households using the following aggregation technology:

$$L_t^f = \left[\int_0^1 L_t^f(i)^{\frac{1}{1+\lambda_w}} di \right]^{1+\lambda_w}. \tag{27}$$

This results in similar equations for the aggregate wage and the demand for individual labour. In the flexible wages economy, all workers demand the same wage and therefore, they face the same demand for their services. The aggregate labour demand and supply will then be equal to the individual household's choice. Since households provide the same amount of labour, equation (27) can be simplified as in the following listing.

```
block LABOUR_AGGREGATION_FLEXIBLE
{
    identities
    {
        L_s_f[] = L_i_f[];
        L_f[] = L_s_f[];
    };
};
```

Listing 6: Labour aggregation, flexible wages economy

The main difference from the nominal rigidities economy is in the optimal wage setting problem. The monopolistic profit of workers, introduced in (25), is given by:

$$\Pi_t^{f,ws}(i) = \left(W_t^f(i) - W_t^{f,disutil}\right) L_t^f(i), \tag{28}$$

where $W_t^f(i)$ is the actual wage charged by workers. The monopolistic profit is the surplus of the wage over individuals' (marginal) leisure cost multiplied by the hours worked ($L_t^f(i)$). The workers' aim is to maximise their monopolistic profit, while the labour agency's demand for their kind of labour is given by the flexible wages counterpart of equation (8).

```
block FLEXIBLE_MONOPOLISTIC_WORKER
{
    controls
    {
        W_i_f[], L_i_star_f[];
    };
    objective
    {
        Pi_ws_f[] = (W_i_f[] - W_disutil_f[]) * L_i_star_f[];
    };
    constraints
    {
        L_i_star_f[] = L_f[] * (W_i_f[] / W_f[])^(-(1 + lambda_w) / lambda_w);
    };
    identities
    {
        L_i_star_f[] = L_i_f[];
    };
};
```

Listing 7: Worker's problem, flexible wages economy

3 Firms

Households own all firms in the economy. The principal-agent problem is assumed away, so firms maximise the present value of the profits (Π_t) adjusted by the discount factor and marginal utilities of households: $\mathrm{E}_t\left[\sum_{\tau=0}^{\infty} \beta^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \Pi_{t+\tau}\right]$. Firms attach greater importance to the profits in the states in which households' consumption is low.

Final good sector

The final good sector is perfectly competitive. It combines a continuum of differentiated intermediate goods $Y_t(j)$ (indexed by $j \in [0,1]$) into the final good denoted as Y_t and sells it to households and the governement. Households can use the final good for both consumption and investment. The final good production technology is given by:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{1}{1+\lambda_p}} dj \right]^{1+\lambda_p}. \tag{29}$$

Because there are no intertemporal effects, the final good producer's problem is actually static. It maximises profit[10] given by:

$$\Pi_t = P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj,$$

where $P_t(j)$ is the price of the jth intermediate good and P_t is the aggregate good price.

The first order condition with respect to $Y_t(j)$ implies the following demand function for the jth intermediate good:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\frac{1+\lambda_p}{\lambda_p}} Y_t. \tag{30}$$

The parameter λ_p determines the steady-state mark-up of firms producing intermediate goods and can be interpreted as a measure of their market power. The equation for aggregate price level P_t can be derived by using the zero-profit condition for perfectly competitive firms:

$$P_t = \left[\int_0^1 P_t(j)^{-\frac{1}{\lambda_p}} dj \right]^{-\lambda_p}. \tag{31}$$

Intermediate goods producers

Each intermediate good $Y_t(j)$ is produced by a monopolistically competitive firm j facing the Cobb–Douglas production function with fixed costs Φ[11]:

$$Y_t(j) = \varepsilon_t^a K_t^d(j)^\alpha L_t^d(j)^{1-\alpha} - \Phi. \tag{32}$$

$K_t^d(j)$ and $L_t^d(j)$ denote the demand of the jth firm for capital and labour respectively, α stands for the capital share in the product, and ε_t^a is a stochastic productivity level. The productivity level is common for all firms and follows the autoregressive process:

$$\log(\varepsilon_t^a) = \rho_a \log(\varepsilon_{t-1}^a) + \eta_t^a, \tag{33}$$

where η_t^a is an i.i.d. Gaussian shock. The firms employ bundled labour by paying the wage W_t for each unit and rent capital from households at the rental rate r_t^k.

The intermediate good producer's optimisation problem may be divided into two parts:

- the problem of determining the demand for labour (L_t^d) and capital (K_t^d) so as to minimise the cost of producing $Y_t(j)$ amount of good,
- the problem of setting the optimal price (and indirectly production level), maximising the expected profit.

[10] In equilibrium the final good producer will earn zero profits.

[11] The fixed costs are frequently introduced in order to ensure zero profits of monopolistically competitive firms.

The first problem can be formulated as:

$$\min_{L_t^d(j), K_t^d(j)} W_t L_t^d(j) + r_t^k K_t^d(j) \quad \text{s.t.} \quad Y_t(j) = \varepsilon_t^a K_t^d(j)^\alpha L_t^d(j)^{1-\alpha} - \Phi \tag{34}$$

or equivalently as:

$$\max_{L_t^d(j), K_t^d(j)} -W_t L_t^d(j) - r_t^k K_t^d(j) \quad \text{s.t.} \quad Y_t(j) = \varepsilon_t^a K_t^d(j)^\alpha L_t^d(j)^{1-\alpha} - \Phi. \tag{35}$$

Thanks to the constant returns to scale, the marginal cost is independent of the produced amount of good and equal among all firms.[12] This observation significantly simplifies the solution of the optimal pricing problem by allowing to treat marginal cost as given to the firm when it decides on price.

The optimisation problem may be stated in `gEcon` language as in the following code listing. In [SW'03] the fixed cost parameter has not been given explicitly. Instead, it has been assumed that the steady-state ratio of final output increased by the fixed cost to the final output is equal to 1.408. This condition is imposed in the calibration section.

[12] This is a standard result. Consider the problem:

$$\min_{K,L} rK + wL \;\; \text{s.t.} \;\; F(K,L) = \bar{y} \; (= y + \Phi),$$

where F satisfies $F(tK, tL) = tF(K,L)$ (is homogeneous of order 1). Lagrange function is given by:

$$\mathcal{L} = rK + wL + \lambda(\bar{y} - F(K,L))$$

and the first order conditions are:

$$r = \lambda F_1(K,L), \quad w = \lambda F_2(K,L)$$

implying:

$$\frac{r}{w} = \frac{F_1(K,L)}{F_2(K,L)}.$$

Let K', L' denote the solution when $\bar{y} = 1$. Consider $(K'', L'') = \bar{y}(K', L')$. We claim that (K'', L'') is the solution to the general problem. (K'', L'') satisfies the constraint:

$$F(K'', L'') = F(\bar{y}K', \bar{y}L') = \bar{y}F(K', L') = \bar{y}$$

and the first order condition (we use the fact that first derivatives are homogeneous of order 0):

$$\frac{F_1(K'', L'')}{F_2(K'', L'')} = \frac{F_1(\bar{y}K', \bar{y}L')}{F_2(\bar{y}K', \bar{y}L')} = \frac{F_1(K', L')}{F_2(K', L')} = \frac{r}{w}.$$

Now we have:

$$rK'' + wL'' = r\bar{y}K' + w\bar{y}L' = (rK' + wL')\bar{y}.$$

This implies that the cost is proportional to $\bar{y}$.

Note that the value of the Lagrange multiplier does not depend on $\bar{y}$ (by homogeneity of order 0 of the first derivatives). Moreover, we have (using the first order conditions and Euler's homogeneous function theorem):

$$rK' + wL' = \lambda F_1(K', L')K' + \lambda F_2(K', L')L' = \lambda(F_1(K', L')K' + F_2(K', L')L') = \lambda F(K', L') = \lambda.$$

The Lagrange multiplier is therefore equal to the marginal cost.

For the specific case of Cobb–Douglas technology $F(K,L) = AK^\alpha L^{1-\alpha}$, the marginal cost is actually given by:

$$\frac{1}{A}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{r}{\alpha}\right)^\alpha.$$

```
block FIRM
{
    controls
    {
        K_j_d[], L_j_d[];
    };
    objective
    {
        tc_j[] = - L_j_d[] * W[] - K_j_d[] * r_k[];
    };
    constraints
    {
        Y_j[] = epsilon_a[] * K_j_d[] ^ alpha * L_j_d[]^(1 - alpha) - Phi :        mc[];
    };
    calibration
    {
        alpha = 0.3;                                   # Capital share in output
        (Y_j[ss] + Phi) / Y_j[ss] = 1.408 -> Phi;    # Calibration of fixed costs
    };
};

block TECHNOLOGY
{
    identities
    {
        log(epsilon_a[]) = rho_a * log(epsilon_a[-1]) + eta_a[];
    };
    shocks
    {
        eta_a[];     # Productivity shock
    };
    calibration
    {
        rho_a = 0.823;
    };
};
```

Listing 8: Intermediate good firm problem

Price setting in Calvo scheme

Price setting is the second part of the firms' problem. Our presentation of the price setting Calvo scheme will closely mimic that of the wage setting problem.

The firms producing intermediate goods set prices for their products to maximise profit. However, they require a price-change signal in order to be able to reset their prices. The probability of receiving such signal $(1 - \xi_p)$ is equal for each firm and constant in each period. In particular, it does not depend on the time elapsed from the last reoptimisation. The firms that cannot reset their prices in a given period apply (partial) price indexation based on inflation from the previous period ($\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$). The actual rate of indexation is equal to $\pi_{t-1}^{\gamma_p}$, where γ_p is the indexation parameter. Consequently, if the firm cannot change its price set in period t for τ periods, then the price for its products in period $t + \tau$ is equal to $\prod_{s=1}^{\tau} \pi_{t+s-1}^{\gamma_p} P_t(j)$.

The optimisation problem of firms which are allowed to reset their prices can be stated using the difference between the revenues from sales and costs:

$$\max_{P_t(j)} \mathrm{E}_t \left[\sum_{\tau=0}^{\infty} (\beta \xi_p)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\frac{\prod_{s=1}^{\tau} \pi_{t+s-1}^{\gamma_p}}{P_{t+\tau}} P_t(j) - mc_{t+\tau} \right) Y_{t+\tau}(j) \right] \tag{36}$$

given the following demand function:

$$Y_{t+\tau}(j) = \left(\frac{\left(\prod_{s=1}^{\tau} \pi_{t+s-1}^{\gamma_p}\right) P_t(j)}{P_{t+\tau}} \right)^{-\frac{(1+\lambda_p)}{\lambda_p}} Y_t. \tag{37}$$

The term ξ_p^τ in the discount factor is used to weigh payoffs using cumulative probability of not receiving the price-change signal. By substituting the expression for the demand (37) into the maximisation problem (36), one can rewrite the latter as:

$$\max_{P_t(j)} \mathrm{E}_t \left[\sum_{\tau=0}^{\infty} (\beta \xi_p)^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \left[\left(\left(\left(\prod_{s=1}^{\tau} \pi_{t+s-1}^{\gamma_p} \right) \frac{P_t(j)}{P_{t+\tau}} \right)^{-\frac{1}{\lambda_p}} - mc_{t+\tau} \left(\left(\prod_{s=1}^{\tau} \pi_{t+s-1}^{\gamma_p} \right) \frac{P_t(j)}{P_{t+\tau}} \right)^{-\frac{(1+\lambda_p)}{\lambda_p}} \right) Y_{t+\tau} \right] \right]. \tag{38}$$

Differentiating the expression (38) with respect to the price and equating the derivative to zero yields:

$$\begin{aligned} \mathrm{E}_t \Bigg[\sum_{\tau=0}^{\infty} (\beta \xi_p)^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \Bigg[\Bigg(& -\frac{1}{\lambda_p} \left(\left(\prod_{s=1}^{\tau} \pi_{t+s-1}^{\gamma_p} \right) \frac{P_t^\star}{P_{t+\tau}} \right)^{-\frac{1}{\lambda_p}} \frac{1}{P_t^\star} \\ + & \frac{(1+\lambda_p)}{\lambda_p} \frac{mc_{t+\tau}}{P_t^\star} \left(\left(\prod_{s=1}^{\tau} \pi_{t+s-1}^{\gamma_p} \right) \frac{P_t^\star}{P_{t+\tau}} \right)^{-\frac{(1+\lambda_p)}{\lambda_p}} \Bigg) Y_{t+\tau} \Bigg] \Bigg] = 0, \end{aligned} \tag{39}$$

where $P_t^\star$ denotes the optimal price set at time t. As firms are identical, they set the same prices and their indices can be dropped.

Similarly to the wage setting problem, the first order condition is expressed as an infinite sum. In [SW'03], the log-linearisation of (39) leads to what is commonly referred to as the "new-Keynesian Phillips curve" (32–SW'03). However, our approach will follow [Schmitt-Grohe & Uribe 2004], i.e. we shall rewrite the first order condition in a recursive way.

Let us rewrite (39) as:

$$\begin{aligned} \mathrm{E}_t \Bigg[\sum_{\tau=0}^{\infty} (\beta \xi_p)^\tau \lambda_{t+\tau} \Bigg[\Bigg(& -\frac{1}{\lambda_p} \left(\left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_p}}{\pi_{t+s}} \right) \frac{P_t^\star}{P_t} \right)^{-\frac{1}{\lambda_p}} P_t^{\star -1} \\ + & \frac{(1+\lambda_p)}{\lambda_p} mc_{t+\tau} \left(\left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_p}}{\pi_{t+s}} \right) \frac{P_t^\star}{P_t} \right)^{-\frac{(1+\lambda_p)}{\lambda_p}} P_t^{\star -1} \Bigg) Y_{t+\tau} \Bigg] \Bigg] = 0. \end{aligned} \tag{40}$$

Multiplying both sides by $P_t^\star \left(\frac{P_t^\star}{P_t}\right)^{\frac{(1+\lambda_p)}{\lambda_p}}$ gives:

$$\begin{aligned} \mathrm{E}_t \Bigg[\sum_{\tau=0}^{\infty} (\beta \xi_p)^\tau \lambda_{t+\tau} \Bigg[\Bigg(& -\frac{1}{\lambda_p} \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_p}}{\pi_{t+s}} \right)^{-\frac{1}{\lambda_p}} \frac{P_t^\star}{P_t} \\ + & \frac{(1+\lambda_p)}{\lambda_p} mc_{t+\tau} \left(\prod_{s=1}^{\tau} \frac{\pi_{t+s-1}^{\gamma_p}}{\pi_{t+s}} \right)^{-\frac{(1+\lambda_p)}{\lambda_p}} \Bigg) Y_{t+\tau} \Bigg] \Bigg] = 0. \end{aligned} \tag{41}$$

Rearranging we arrive at the condition:

$$\begin{aligned}
&\mathrm{E}_t\left[\sum_{\tau=0}^{\infty}(\beta\xi_p)^{\tau}\,\lambda_{t+\tau}\left(\prod_{s=1}^{\tau}\frac{\pi_{t+s-1}^{\gamma_p}}{\pi_{t+s}}\right)^{-\frac{1}{\lambda_p}}\frac{P_t^{\star}}{P_t}Y_{t+\tau}\right]\\
&=(1+\lambda_p)\mathrm{E}_t\left[\sum_{\tau=0}^{\infty}(\beta\xi_p)^{\tau}\,\lambda_{t+\tau}mc_{t+\tau}\left(\prod_{s=1}^{\tau}\frac{\pi_{t+s-1}^{\gamma_p}}{\pi_{t+s}}\right)^{-\frac{(1+\lambda_p)}{\lambda_p}}Y_{t+\tau}\right],
\end{aligned} \tag{42}$$

which can be expressed in a recursive fashion. Let us denote the left-hand side as g_t^1 and the expectation on the right-hand side as g_t^2. We have:

$$g_t^1=(1+\lambda_p)g_t^2. \tag{43}$$

The left-hand side can be written as:

$$\begin{aligned}
g_t^1&=\mathrm{E}_t\left[\sum_{\tau=0}^{\infty}(\beta\xi_p)^{\tau}\,\lambda_{t+\tau}\left(\prod_{s=1}^{\tau}\frac{\pi_{t+s-1}^{\gamma_p}}{\pi_{t+s}}\right)^{-\frac{1}{\lambda_p}}\frac{P_t^{\star}}{P_t}Y_{t+\tau}\right]\\
&=\lambda_t\frac{P_t^{\star}}{P_t}Y_t+\mathrm{E}_t\left[\sum_{\tau=1}^{\infty}(\beta\xi_p)^{\tau}\,\lambda_{t+\tau}\left(\prod_{s=1}^{\tau}\frac{\pi_{t+s-1}^{\gamma_p}}{\pi_{t+s}}\right)^{-\frac{1}{\lambda_p}}\frac{P_t^{\star}}{P_t}Y_{t+\tau}\right]\\
&=\lambda_t\frac{P_t^{\star}}{P_t}Y_t+(\beta\xi_p)\,\frac{P_t^{\star}}{P_t}\mathrm{E}_t\left[\frac{P_{t+1}}{P_{t+1}^{\star}}\left(\frac{\pi_t^{\gamma_p}}{\pi_{t+1}}\right)^{-\frac{1}{\lambda_p}}\sum_{\tau=0}^{\infty}(\beta\xi_p)^{\tau}\,\lambda_{t+\tau+1}\left(\prod_{s=1}^{\tau}\frac{\pi_{t+s}^{\gamma_p}}{\pi_{t+s+1}}\right)^{-\frac{1}{\lambda_p}}\frac{P_{t+1}^{\star}}{P_{t+1}}Y_{t+\tau+1}\right]\\
&=\lambda_t\frac{P_t^{\star}}{P_t}Y_t+(\beta\xi_p)\,\frac{P_t^{\star}}{P_t}\mathrm{E}_t\left[\frac{P_{t+1}}{P_{t+1}^{\star}}\left(\frac{\pi_t^{\gamma_p}}{\pi_{t+1}}\right)^{-\frac{1}{\lambda_p}}g_{t+1}^1\right]\\
&=\lambda_t\pi_t^{\star}Y_t+(\beta\xi_p)\,\pi_t^{\star}\mathrm{E}_t\left[\frac{1}{\pi_{t+1}^{\star}}\left(\frac{\pi_t^{\gamma_p}}{\pi_{t+1}}\right)^{-\frac{1}{\lambda_p}}g_{t+1}^1\right].
\end{aligned} \tag{44}$$

$\pi_t^{\star}=\frac{P_t^{\star}}{P_t}$ has been introduced in order to eliminate the possibly nonstationary variable P_t from the model. In the log-linear approximation, it can be interpreted as the percentage deviation of optimal price from the average price in period t.

The expectation on the right-hand side of equation (42) can be transformed into:

$$\begin{aligned}
g_t^2&=\mathrm{E}_t\left[\sum_{\tau=0}^{\infty}(\beta\xi_p)^{\tau}\,\lambda_{t+\tau}mc_{t+\tau}\left(\prod_{s=1}^{\tau}\frac{\pi_{t+s-1}^{\gamma_p}}{\pi_{t+s}}\right)^{-\frac{(1+\lambda_p)}{\lambda_p}}Y_{t+\tau}\right]\\
&=\lambda_t mc_t y_t+\mathrm{E}_t\left[\sum_{\tau=1}^{\infty}(\beta\xi_p)^{\tau}\,\lambda_{t+\tau}mc_{t+\tau}\left(\prod_{s=1}^{\tau}\frac{\pi_{t+s-1}^{\gamma_p}}{\pi_{t+s}}\right)^{-\frac{(1+\lambda_p)}{\lambda_p}}Y_{t+\tau}\right]\\
&=\lambda_t mc_t y_t+(\beta\xi_p)\,\mathrm{E}_t\left[\left(\frac{\pi_t^{\gamma_p}}{\pi_{t+1}}\right)^{-\frac{(1+\lambda_p)}{\lambda_p}}\sum_{\tau=0}^{\infty}(\beta\xi_p)^{\tau}\,\lambda_{t+\tau+1}mc_{t+\tau+1}\left(\prod_{s=1}^{\tau}\frac{\pi_{t+s}^{\gamma_p}}{\pi_{t+s+1}}\right)^{-\frac{(1+\lambda_p)}{\lambda_p}}Y_{t+\tau+1}\right]\\
&=\lambda_t mc_t y_t+(\beta\xi_p)\,\mathrm{E}_t\left[\left(\frac{\pi_t^{\gamma_p}}{\pi_{t+1}}\right)^{-\frac{(1+\lambda_p)}{\lambda_p}}g_{t+1}^2\right].
\end{aligned} \tag{45}$$

The optimal price determined by (42) can be used to simplify the equation (31) that determines the price index. Rewriting the equation as follows:

$$P_t^{-\frac{1}{\lambda_p}} = \int_0^1 P_t(j)^{-\frac{1}{\lambda_p}} dj \tag{46}$$

and then dividing firms into the optimising and non-optimising sets, we obtain:

$$P_t^{-\frac{1}{\lambda_p}} = \xi_p \left(\pi_{t-1}^{\gamma_p}\right)^{-\frac{1}{\lambda_p}} P_{t-1}^{-\frac{1}{\lambda_p}} + (1 - \xi_p) \left(P_t^\star\right)^{-\frac{1}{\lambda_p}}. \tag{47}$$

The average price is influenced by the indexed average price from the previous period, optimal price in the current period, and the probability of receiving the price-change signal. In general, the price level can be nonstationary and therefore, the expression has to be transformed by dividing both sides by $P_t^{-\frac{1}{\lambda_p}}$ and introducing $\pi_t^\star$:

$$1 = \xi_p \left(\frac{\pi_{t-1}^{\gamma_p}}{\pi_t}\right)^{-\frac{1}{\lambda_p}} + (1 - \xi_p) \left(\pi_t^\star\right)^{-\frac{1}{\lambda_p}}. \tag{48}$$

`gEcon` code for the Calvo pricing mechanism described above (equations (43), (44), (45), and (48)) is presented in Listing 9. Note that the so-called price mark-up shock (η_t^p) has been introduced to equation (43) as an i.i.d Gaussian disturbance:

$$g_t^1 = (1 + \lambda_p) g_t^2 + \eta_t^p. \tag{49}$$

This shock can be interpreted as a deviation of intermediate goods firms' margins from their steady-state level (see [Gali 2009] for more detailed explanation).

```
block PRICE_SETTING_PROBLEM
{
    identities
    {
        g_1[] = (1 + lambda_p) * g_2[] + eta_p[];
        g_1[] = lambda[] * pi_star[] * Y[] + beta * xi_p *
                E[][(pi[] ^ gamma_p / pi[1]) ^ (-1 / lambda_p) *
                   (pi_star[] / pi_star[1]) * g_1[1]];
        g_2[] = lambda[] * mc[] * Y[] + beta * xi_p *
                E[][(pi[] ^ gamma_p / pi[1]) ^ (-((1 + lambda_p) / lambda_p)) * g_2[1]];
    };
    shocks
    {
        eta_p[];                    # Price mark-up shock
    };
    calibration
    {
        xi_p = 0.908;               # Probability of not receiving the ``price-change signal''
        gamma_p = 0.469;            # Indexation parameter for non-optimising firms
    };
};

block PRICE_EVOLUTION
{
    identities
    {
        1 = xi_p * (pi[-1] ^ gamma_p / pi[]) ^ (-1 / lambda_p) +
            (1 - xi_p) * pi_star[] ^ (-1 / lambda_p);
    };
};
```

Listing 9: Price setting problem

Similarly to the wage setting Calvo scheme, in order to state the market clearing condition for intermediate products we shall introduce an additional variable — the price dispersion index.

The demand for the jth firm's product in terms of aggregate product can be written as:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} Y_t. \tag{50}$$

Integrating equation (50) over all firms $j \in [0,1]$ and using Y_t^s to denote the total amount of intermediate goods $\int_0^1 Y_t(j)dj$ we arrive at:

$$Y_t^s = \left(\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} dj\right) Y_t. \tag{51}$$

Let us set:

$$\nu_t^p = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} dj.$$

ν_t^p is a measure of price dispersion (for its interpretation and implications see [Schmitt-Grohe & Uribe 2004]). Equation (51) simplifies to:

$$Y_t^s = \nu_t^p Y_t. \tag{52}$$

The law of motion for ν_t^p can be derived as follows:

$$\begin{aligned}
\nu_t^p =& \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} dj \\
=&(1-\xi_p)\left(\frac{P_t^\star}{P_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} + (1-\xi_p)\xi_p\left(\frac{P_{t-1}^\star \pi_{t-1}^{\gamma_p}}{P_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} + (1-\xi_p)\xi_p^2\left(\frac{P_{t-2}^\star \pi_{t-1}^{\gamma_p}\pi_{t-2}^{\gamma_p}}{P_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} + \ldots \\
=&(1-\xi_p)\left(\frac{P_t^\star}{P_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} + \xi_p\left(\frac{P_{t-1}\pi_{t-1}^{\gamma_p}}{P_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} \times \\
&\times\left((1-\xi_p)\left(\frac{P_{t-1}^\star}{P_{t-1}}\right)^{-\frac{1+\lambda_p}{\lambda_p}} + (1-\xi_p)\xi_p\left(\frac{P_{t-2}^\star \pi_{t-2}^{\gamma_p}}{P_{t-1}}\right)^{-\frac{1+\lambda_p}{\lambda_p}} + \ldots\right) \\
=&(1-\xi_p)\left(\frac{P_t^\star}{P_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} + \xi_p\left(\frac{\pi_{t-1}^{\gamma_p}}{\pi_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} \nu_{t-1}^p \\
=&(1-\xi_p)\pi_t^{\star -\frac{1+\lambda_p}{\lambda_p}} + \xi_p\left(\frac{\pi_{t-1}^{\gamma_p}}{\pi_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} \nu_{t-1}^p.
\end{aligned} \tag{53}$$

As all intermediate firms face the same prices of production factors and employ identical constant-returns-to-scale technology, the aggregate demand for production factors may be regarded as the demand for the production factors of a representative firm. In other words, even though firms produce different amounts of intermediate goods, the same relations between $Y_t(j)$, $K_t^d(j)$, and $L_t^d(j)$ hold for all j and for aggregates $\int_0^1 Y_t(j)dj$, $\int_0^1 K_t^d(j)dj$, and $\int_0^1 L_t^d(j)dj$. With a slight abuse of notation, one can state market clearing conditions as:

$$Y_t^s = Y_t(j), \tag{54}$$

$$K_t^d = K_t^d(j), \tag{55}$$

$$L_t^d = L_t^d(j). \tag{56}$$

The aggregation conditions discussed above are presented in Listing 10.

```
block FACTOR_DEMAND_AGGREGATION
{
    identities
    {
        K_d[] = K_j_d[];
        L_d[] = L_j_d[];
    };
};

block PRODUCT_AGGREGATION
{
    identities
    {
        Y_s[] = Y_j[];
        nu_p[] = (1 - xi_p) * pi_star[] ^ (-((1 + lambda_p) / lambda_p))
                  + xi_p * (pi[-1] ^ gamma_p / pi[]) ^
                  (-((1 + lambda_p) / lambda_p)) * nu_p[-1];
        Y[] * nu_p[] = Y_s[];
    };
};
```

Listing 10: Product and factor demand aggregation

Monopolistic competition in the flexible prices economy

Firms in the final good sector use the same aggregation technology as in (29), which results in identical equations for aggregate price level and for the demand for the jth intermediate good. In the flexible prices economy, all intermediate good producers operate under the same conditions and set the same price, so the demand for their output is uniform. The amount of intermediate good produced by a representative firm equals the amount of the final good. Model is closed by setting the aggregate price level to 1.

```
block FACTOR_DEMAND_AGGREGATION_FLEXIBLE
{
    identities
    {
        K_d_f[] = K_j_d_f[];
        L_d_f[] = L_j_d_f[];
    };
};

block PRODUCT_AGGREGATION_FLEXIBLE
{
    identities
    {
        Y_s_f[] = Y_j_f[];
        Y_f[] = Y_s_f[];
    };
};

block PRICE_EVOLUTION_FLEXIBLE
{
    identities
    {
        P_f[] = 1;
    };
};
```

Listing 11: Product and production factor demand aggregation, flexible prices economy

The cost minimisation problem of firms producing intermediate goods is the same as in the sticky price economy. The relevant gEcon code can be found in Appendix B. The optimal price setting problem is given by a version of equations (36) and (37) with $\xi_p = 0$, i.e.:

$$\max_{P_t^f(j)} \mathrm{E}_t \left[\left(P_t^f(j) - mc_t^f \right) Y_t^f(j) \right]$$

s.t.:

$$Y_t^f(j) = \left(\frac{P_t^f(j)}{P_t^f} \right)^{-\frac{(1+\lambda_p)}{\lambda_p}} Y_t^f.$$

gEcon implementation of the price setting problem in the flexible prices economy is given below.

```
block PRICE_SETTING_PROBLEM_FLEXIBLE
{
    controls
    {
        Y_j_f[], P_j_f[];
    };
    objective
    {
        Pi_ps_f[] = (P_j_f[] - mc_f[]) * Y_j_f[];
    };
    constraints
    {
        Y_j_f[] = (P_j_f[] / P_f[])^(-((1 + lambda_p) / lambda_p)) * Y_f[];
    };
    calibration
    {
        C_f[ss] / Y_f[ss] = 0.6 -> lambda_p;      # Calibration of the price mark-up
    };
};

block FACTOR_DEMAND_AGGREGATION_FLEXIBLE
{
    identities
    {
        K_d_f[] = K_j_d_f[];
        L_d_f[] = L_j_d_f[];
    };
};
```

Listing 12: Price setting problem, flexible prices economy

The calibration section above deserves a brief comment. Following [SW'03] the value of λ_p is calibrated to impose steady-state consumption share in the output (0.6).

4 Fiscal and monetary authorities

Government

The government collects lump sum taxes (T_t) from households to finance its expenditure (G_t). Additionally, it can raise money by issuing bonds (B_t^N). Its behaviour is taken as exogenous by households and firms. The government spending is subject to the stochastic disturbance (ε_t^G):

$$G_t = \bar{G}\varepsilon_t^G, \tag{57}$$

where ε_t^G follows the first-order autoregression process:

$$\log \varepsilon_t^G = \rho_G \log \varepsilon_{t-1}^G + \eta_t^G \tag{58}$$

with i.i.d. Gaussian error η_t^G. $\bar{G}$ denotes the steady-state government expenditures, so the value of disturbance may be interpreted as the percentage deviation from the steady state.

The budget constraint of the government (in real terms) is given by:

$$G_t + \frac{B_{t-1}^N}{P_t} = T_t + \frac{B_t^N}{P_t R_t},$$

where R_t is the gross nominal rate of return. Using $B_t = \frac{B_t^N}{P_t}$ and $\pi_t = \frac{P_t}{P_{t-1}}$, the government budget constraint can be restated as follows:

$$G_t + \frac{B_{t-1}}{\pi_t} = T_t + \frac{B_t}{R_t}. \tag{59}$$

In equilibrium, bonds are in zero net supply and the value of collected taxes is equal to the expenditures.

`gEcon` implementation of equations (57), (58), and (59) is presented in Listing 13. The parameter $\bar{G}$ is calibrated so that the total government spending in the steady state takes 18% of the economy output (as in [SW'03]).

```
block GOVERNMENT
{
    identities
    {
        G[] = G_bar * epsilon_G[];
        G[] + B[-1] / pi[] = T[] + B[] / R[];
    };
    calibration
    {
        G[ss] / Y[ss] = 0.18 -> G_bar; # Calibration of the steady state government
            expenditures
    };
};

block GOVERNMENT_SPENDING_SHOCK
{
    identities
    {
        log(epsilon_G[]) = rho_G * log(epsilon_G[-1]) + eta_G[];
    };
    shocks
    {
        eta_G[];    # Government spending shock
    };
    calibration
    {
        rho_G = 0.949;
    };
};
```

Listing 13: Government behaviour

The government behaviour in the economy without rigidities is identical. The relevant `gEcon` code can be found in Appendix B.

The monetary authority

When setting the nominal (gross) interest rate R_t, the monetary authority takes the following factors into account[13]:

- the deviation of the interest rate in the previous period ($\hat{R}_{t-1}$) from its steady-state value (interest rate smoothing),
- the deviation of lagged inflation from the inflation objective ($\hat{\pi}_{t-1} - \mathring{\pi}_t$)[14],
- the output gap, defined as the percentage difference between the actual level of output and potential level of output, i.e. in the absence of rigidities and mark-up shocks ($\hat{Y}_t - \hat{Y}_t^f$),
- the dynamics of the output gap $\left(\left(\hat{Y}_t - \hat{Y}_t^f\right) - \left(\hat{Y}_{t-1} - \hat{Y}_{t-1}^f\right)\right)$,
- inflation dynamics ($\hat{\pi}_t - \hat{\pi}_{t-1}$).

The monetary policy function (commonly referred to as the generalized Taylor rule) is given by:

$$\begin{aligned}\hat{R}_t =& \rho \hat{R}_{t-1} + (1-\rho)\{\mathring{\pi}_t + r_\pi(\hat{\pi}_{t-1} - \mathring{\pi}) + r_Y(\hat{Y}_t - \hat{Y}_t^f)\} \\ &+ r_{\Delta_\pi}(\hat{\pi}_t - \hat{\pi}_{t-1}) + r_{\Delta_y}(\hat{Y}_t - \hat{Y}_t^f - (\hat{Y}_{t-1} - \hat{Y}_{t-1}^f)) + \eta_t^R.\end{aligned} \tag{60}$$

Elasticities r_π, r_Y, r_{Δ_π}, r_{Δ_y} are determined by the relative weights attached by the monetary authority to the respective deviations. The parameter ρ measures the degree of the interest rate smoothing.

The inflation objective evolves according to:

$$\mathring{\pi}_t = (1 - \rho_{\bar{\pi}})\mathring{\pi} + \rho_{\bar{\pi}}\mathring{\pi}_{t-1} + \eta_\pi, \tag{61}$$

where η_π is a white noise, and $\mathring{\pi}$ a long-run inflation objective.

In the listing below, the log-deviations have been written explicitly. The inflation objective has been calibrated to 1. The calibrated parameters `calibr_pi` and `calibr_pi_obj` are used to impose the steady-state relationships on the model.

```
block MONETARY_POLICY_AUTHORITY
{
    identities
    {
        log(R[] / R[ss]) + calibr_pi = rho * log(R[-1] / R[ss]) +
                                       (1 - rho) * (log(pi_obj[]) +
                                       r_pi * (log(pi[-1] / pi[ss]) - log(pi_obj[])) +
                                       r_Y * (log(Y[]/ Y[ss]) - log(Y_f[] / Y_f[ss]))) +
                                       r_Delta_pi * (log(pi[]/pi[ss]) - log(pi[-1]/pi[ss])) +
                                       r_Delta_y * (log(Y[] / Y[ss]) -
                                                         log(Y_f[] / Y_f[ss]) -
                                                   (log(Y[-1] / Y[ss]) -
                                                         log(Y_f[-1] / Y_f[ss]))) +
                                       eta_R[];
        log(pi_obj[]) = (1 - rho_pi_bar) * log(calibr_pi_obj) +
                        rho_pi_bar * log(pi_obj[-1]) + eta_pi[];
    };
    shocks
    {
        eta_R[],     # Interest rate shock
        eta_pi[];    # Inflation objective shock
    };
```

[13] $\hat{x}_t$ for any variable x_t denotes the percentage deviation from the steady-state value (approximated by $\log x_t - \log \bar{x}$).
[14] Inflation objective $\mathring{\pi}_t$ is expressed as the logarithm of the actual gross inflation target.

```
    calibration
    {
        r_Delta_pi = 0.14;                          # Weight on the dynamics of inflation
        r_Delta_y = 0.159;                          # Weight on the dynamics of output gap
        rho = 0.961;                                # Interest rate smoothing parameter
        r_Y = 0.099;                                # Weight on the output gap
        r_pi = 1.684;                               # Weight on the inflation gap
        pi_obj[ss] = 1      -> calibr_pi_obj;       # Calibration of the inflation objective
        pi[ss] = pi_obj[ss] -> calibr_pi;           # Calibration of the inflation
        rho_pi_bar = 0.924;                         # AR parameter for the inflation objective
    };
};
```

Listing 14: The monetary policy rule

5 Market clearing conditions

The following relation holds between the demand and supply of capital (due to variable capital utilisation):

$$K_t^d = z_t K_{t-1}.$$

Labour market clears:

$$L_t^d = L_t.$$

Bond market clearing condition is:

$$B_t = 0.$$

Firms pay out their profits as dividends (Div_t). The total dividend is therefore equal to the value of the total output less compensation of labour and capital:

$$Div_t = Y_t - L_t^d W_t - K_t^d r_t^k. \tag{62}$$

The final good market clears by Walras' law.

Equilibrium conditions described in this section are collected in Listing 15.

```
block EQUILIBRIUM
{
    identities
    {
        K_d[] = z[] * K[-1];
        L[] = L_d[];
        B[] = 0;
        Div[] = Y[] - L_d[] * W[] - K_d[] * r_k[];
    };
};
```

Listing 15: Equilibrium conditions

Similar conditions hold for the flexible prices economy, cf. `gEcon` model implementation in Appendix B.

6 Differences relative to [SW'03]

Intentional changes

One of the ways to introduce market inefficiency shocks is to make the mark-up parameter a stochastic variable, e.g. by setting $\lambda_{w,t} = \lambda_w + \eta^w_t$, where η^w_t is a white noise process. However, such formulation would make it impossible to write the first order conditions and the laws of motion for the price and wage dispersion in a recursive form — one would not be able to factor out expressions involving $\lambda_{w,t}$ or $\lambda_{p,t}$ in relevant equations. It has been decided that the aforesaid mark-up shocks will be introduced indirectly, in an equivalent way (up to the first order approximation), by adding shocks to the price and wage equations. Shocks to the labour and goods market inefficiencies are introduced in the form of stochastic disturbances η^w_t and η^p_t to the first order conditions for the optimal wage (20) and price (49) in Calvo setting. The variances of both shocks have been rescaled so that the impulse response functions are identical to those in [SW'03].

In this implementation of the [SW'03] model, the shock to the rate of return on equity investment has been omitted. This shock is introduced in a non-structural way, it does not appear in any optimization problem.

Corrections

For the purpose of verifying the correctness of the model described in this paper and its `gEcon` implementation, a benchmark code consisting of the log-linearised equations quoted from [SW'03] and their counterparts in the flexible prices economy has been written. This exercise allowed us to find two mistakes in the original paper.

The log-linearised law of motion for capital (5) should be written as:

$$\hat{K}_t = (1-\tau)\hat{K}_{t-1} + \tau \hat{I}_t, \tag{63}$$

where the variables with hats over them denote the percentage deviations from the steady state. In the original paper investment ($\hat{I}$) enters equation (31–SW'03) in lag.

Additionally, the log-linearised equilibrium condition should take the costs of variable capacity utilisation into account. This can be noticed by combining budget constraints for all agents and market clearing conditions. The log-linearised goods market equilibrium condition should therefore take the following form (the missing term is underlined)[15]:

$$\hat{Y}_t = \left(1 - \tau\frac{K_{ss}}{Y_{ss}} - \frac{\bar{G}}{Y_{ss}}\right)\hat{C}_t + \tau\frac{K_{ss}}{Y_{ss}}\hat{I}_t + \frac{\bar{G}}{Y_{ss}}\hat{\varepsilon}^G_t + \underline{\frac{K_{ss}}{Y_{ss}}\bar{r}^k\hat{r}^k_t\psi}. \tag{64}$$

There were no differences in the remaining equations.[16]

The first-order approximation of the [SW'03] model with equations (31–SW'03) and (35–SW'03) modified as in (63) and (64) gave identical results as the model described in this paper.

7 Solution and simulation

Dynamic stochastic models are solved in `gEcon` in four steps. First, model equations, steady-state relationships, and first order perturbation matrices are derived based on the model formulation. Then the steady state is determined numerically. In the third step, first order approximation equations are solved. Finally, based on the results of the third step and shock distribution parameters, model statistics are computed.

[15]Subscript ss denotes the steady-state values of variables.

[16]We have actually also changed the sign in front of the η^L_t shock for consistency between shock sign and its interpretation, see footnote on page 4.

gEcon automatically generates model equations, creating system of first order conditions, constraints, and market clearing conditions. The model presented in Appendix B produces a system of 78 equations with 78 variables.[17] The symbolic reduction algorithm implemented in gEcon allows to eliminate redundant variables from the model, cf. the tryreduce section in the model code. As a result, the number of variables and equations was reduced to 54.[18]

Eliminating some of the model variables and equations reduces the computational complexity of solving for the steady state. Still, finding the solution using the nleqslv package [Hasselman 2013] called by gEcon requires providing initial guesses for some of the variables. Note that the deterministic steady-state values of capacity utilisation (z_t, z_t^f), Tobin's q (Q_t, Q_t^f), inflation and its objective (π_t, $\mathring{\pi}_t$), as well as disturbances $\bar{\varepsilon}_t{}^b$, $\bar{\varepsilon}_t{}^L$, $\bar{\varepsilon}_t{}^a$, $\bar{\varepsilon}_t{}^I$, $\bar{\varepsilon}_t{}^G$ will all be equal to 1. In addition, as rates of return do not usually exceed few percentage points, we have set initial values of r_t^k and $r_t^{k,f}$ to 0.01. All these settings can be found in the R code in Listing 17 in Appendix B. The obtained steady-state values of model variables are presented in Table 3 in Appendix C.

gEcon uses Christopher Sims' solver (gensys, see [Sims 2002]) to find the first order approximation around the steady state. The model variables were automatically log-linearised in gEcon by appropriate transformation of perturbation matrices.[19] Perturbation solution indicates that the model has 19 state variables: $\bar{\varepsilon}^b$, $\bar{\varepsilon}^L$, $\bar{\varepsilon}^a$, $\bar{\varepsilon}^I$, $\bar{\varepsilon}^G$, ν_t^p, ν_t^w, π_t, $\mathring{\pi}_t$, C_t, C_t^f, I_t, I_t^f, K_t, K_t^f, Y_t, Y_t^f, W_t, and R_t.

The computation of model statistics requires providing the variance-covariance matrix of shocks. Following [SW'03] it has been assumed that shocks are uncorrelated — variance-covariance matrix is diagonal with the entries given in Table 1.

η^b	η^L	η^I	η^a	η^w	η^p	η^G	η^R	η^π
$(0.3360)^2$	$(3.52)^2$	$(0.085)^2$	$(0.598)^2$	$(0.685)^2$	$(0.790)^2$	$(0.325)^2$	$(0.081)^2$	$(0.017)^2$

Table 1: Variances of model shocks

Second moments of variables were computed by gEcon using spectral methods. Standard deviations of (log-linearised) model variables are presented in Table 3 in Appendix C. Correlations between variables, autocorrelations and the decomposition of the variance for model variables can be found in Tables 4-7 in Appendix D.

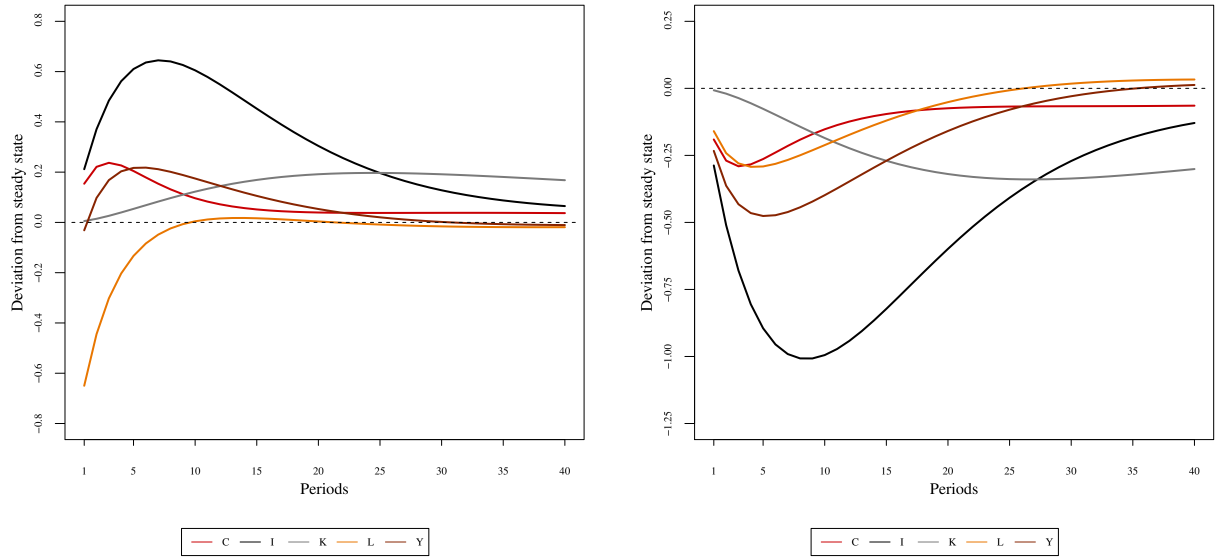
Figure 1 shows the impulse response functions to a positive productivity shock and to a positive interest rate shock. Impulse responses were computed under the assumption that the magnitudes of shocks are equal to their standard deviations.

Appendix E contains the comparison of impulse response functions to all shocks in economies with and without nominal rigidities.

[17] The initial model size is actually larger as gEcon automatically reduces Lagrange multipliers that were not explicitly named by the user.

[18] Manual derivations, like those performed in [SW'03], allow to further reduce the number of variables, yet they are associated with additional effort and involve risk of making a mistake as demonstrated in the case of equation (64).

[19] For detailed description of the method used for log-linearisation, see gEcon Users' Guide [Klima *et al.* 2015], section 8.1.



a) **b)**

Figure 1: (a) Response to a productivity shock
(b) Response to an interest rate shock

8 Summary

In this paper we have shown how the benchmark DSGE model can be written and solved using the `gEcon` package. We hope that our exercise highlights the advantages of using `gEcon` in DSGE modelling: model implementation reflects the economic structure of the model, final set of equations describing model dynamics is derived automatically, and any extensions or amendments to the model are easy to implement. We also hope that our exposition and model implementation are clear and comprehensive enough to serve both educational purposes and as a point of departure for building more complex models.

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Appendix A. List of variables in the model

Variables appearing in the flexible prices and wages economy are marked with a dagger (†).

`.gcn` file	Paper	Short description
–	B_t^N	Nominal bond holding
–	$B_t^{f,N}$	Nominal bond holding†
–	P_t	Aggregate price level
`B`	$B_t = B_t^N / P_t$	Real bond holding
`B_f`	$B_t^f = B_t^{f,N} / P_t^f$	Real bond holding†
`C`	C_t	Real consumption
`C_f`	C_t^f	Real consumption†
`Div`	Div_t	Real dividend
`Div_f`	Div_t^f	Real dividend†
`epsilon_a`	ε_t^a	Technology level
`epsilon_b`	ε_t^b	Level of shock to the discount rate
`epsilon_G`	ε_t^G	Level of shock to the government spending
`epsilon_I`	ε_t^I	Level of shock to the adjustment cost function
`epsilon_L`	ε_t^L	Level of shock to the labour supply
`f_1`	$f_t^1 = \frac{\lambda_t}{1+\lambda_w} \tilde{L}_t^d \tilde{W}_t + \beta \xi_w \mathrm{E}_t \left[\left(\frac{\pi_t^{\gamma_w}}{\pi_{t+1}} \right)^{-\frac{1}{\lambda_w}} \left(\frac{\tilde{W}_{t+1}}{\tilde{W}_t} \right)^{\frac{1}{\lambda_w}} f_{t+1}^1 \right]$	Recursive formulation of (17)
`f_2`	$f_t^2 = \varepsilon_t^b \varepsilon_t^L \omega (\tilde{L}_t^d)^{1+\sigma_l} + \beta \xi_w \mathrm{E}_t \left[\left(\frac{\pi_t^{\gamma_w}}{\pi_{t+1}} \right)^{\frac{-(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} \left(\frac{\tilde{W}_{t+1}}{\tilde{W}_t} \right)^{\frac{(1+\lambda_w)(1+\sigma_l)}{\lambda_w}} f_{t+1}^2 \right]$	Recursive formulation of (19)
`G`	G_t	Real government expenditures
`g_1`	$g_t^1 = \lambda_t \pi_t^\star Y_t + (\beta \xi_p) \pi_t^\star \mathrm{E}_t \left[\frac{1}{\pi_{t+1}^\star} \left(\frac{\pi_t^{\gamma_p}}{\pi_{t+1}} \right)^{-\frac{1}{\lambda_p}} g_{t+1}^1 \right]$	Recursive formulation of (44)
`g_2`	$g_t^2 = \lambda_t mc_t y_t + (\beta \xi_p) \mathrm{E}_t \left[\left(\frac{\pi_t^{\gamma_p}}{\pi_{t+1}} \right)^{-\frac{(1+\lambda_p)}{\lambda_p}} g_{t+1}^2 \right]$	Recursive formulation of (45)
`G_f`	G_t^f	Real government expenditures†
`H`	H_t	Level of habit
`H_f`	H_t^f	Level of habit†
`I`	I_t	Real investments
`I_f`	I_t^f	Real investments†
`Inc_f`	$Inc_t^f(i)$	Incomes of household from renting production factors†
`K`	K_t	Real capital stock
`K_d`	K_t^d	Aggregate demand for capital
`K_d_f`	$K_t^{f,d}$	Aggregate demand for capital†
`K_f`	K_t^f	Real capital stock
`K_j_d`	$K_t^d(j)$	Demand for capital of jth firm
`K_j_d_f`	$K_t^{f,d}(j)$	Demand for capital of jth firm†
`L`	L_t	Aggregate labour supply
`L_d`	L_t^d	Aggregate labour demand
`L_d_f`	$L_t^{f,d}$	Aggregate labour demand†

`L_f`	L_t^f	Aggregate labour supply†
`L_i_f, L_i_star_f`	$L_t^f(i)$	Labour supplied by ith household†
`L_i_tilde`	$\tilde{L}_t(i)$	Demand for labour of households reoptimising at t period
`L_j_d`	$L_t^d(j)$	Demand of the jth firm for labour
`L_j_d_f`	$L_t^{f,d}(j)$	Demand of the jth firm for labour†
`L_s`	L_t^s	Total supply of all types of differentiated labour
`L_s_f`	$L_t^{f,s}$	Total supply of all types of differentiated labour†
`lambda`	λ_t	Marginal utility of consumption
`lambda_f`	λ_t^f	Marginal utility of consumption†
`mc`	mc_t	Intermediate firm's marginal costs
`mc_f`	mc_t^f	Intermediate firm's marginal costs†
`nu_p`	ν_t^p	Price dispersion
`nu_w`	ν_t^w	Wage dispersion
`P_f`	P_t^f	Aggregate price level†
`P_j_f`	$P_t^f(j)$	Price of the jth good†
`pi`	$\pi_t = \frac{P_t}{P_{t-1}}$	Inflation rate
`pi_obj`	$\mathring{\pi}_t$	Inflation rate objective
`Pi_ps_f`	$\Pi_t^{f,ps}$	Monopolistic price setter profit†
`pi_star`	$\pi_t^\star = \frac{\tilde{P}_t}{P_t}$	Optimal-to-aggregate price ratio
`pi_star_w`	$\pi_t^{\star w} = \frac{\tilde{W}_t}{W_t}$	Optimal-to-aggregate wage ratio
`Pi_ws_f`	$\Pi_t^{f,ws}$	Monopolistic wage setter profit†
`q`	q_t	Lagrange multiplier on the law of motion for the capital
`Q`	Q_t	Tobin's q
`q_f`	q_t^f	Lagrange multiplier on the law of motion for the capital†
`Q_f`	Q_t^f	Tobin's $q^\dagger$
`R`	R_t	Nominal (gross) interest rate
`R_f`	R_t^f	Nominal (gross) interest rate†
`r_k`	r_t^k	Rate of return from capital
`r_k_f`	$r_t^{f,k}$	Rate of return from capital†
`T`	T_t	Real value of taxes
`T_f`	T_t^f	Real value of taxes†
`tc_j`	$tc_t(j)$	Total cost of the jth firm
`tc_j_f`	$tc_t^f(j)$	Total cost of the jth firm†
`U`	U_t	Expected utility of households
`u`	U_t	Instantaneous utility
`u_f`	u_t^f	Instantaneous utility†
`U_f`	U_t^f	Expected utility of households†
`W`	W_t	Aggregate real wage
`W_disutil_f`	$W_t^{f,disutil}$	Marginal disutility from labour†
`W_f`	W_t^f	Aggregate real wage†
`W_i_f`	$W^f(i)$	Wage for the labour supplied by ith household†

`w_star`	$w_t^{\star}$	Optimal price set in period t
`Y`	Y_t	Aggregate product
`Y_f`	Y_t^f	Aggregate product†
`Y_j`	$Y_t(j)$	The product of the jth firm
`Y_j_f`	$Y_t^f(j)$	The product of the jth firm†
`Y_s`	Y_t^s	Total amount of intermediate goods produced
`Y_s_f`	$Y_t^{f,s}$	Total amount of intermediate goods produced†
`z`	z_t	Capacity utilization
`z_f`	z_t^f	Capacity utilization†

Table 2: Model variables

Appendix B. The full gEcon implementation of the model with accompanying R code

```
# ############################################################################
# (c) Chancellery of the Prime Minister 2012-2015                           #
# Licence terms can be found in the file:                                   #
# http://gecon.r-forge.r-project.org/files/gEcon_licence.txt                #
#                                                                           #
# Authors: Grzegorz Klima, Karol Podemski,                                  #
#          Kaja Retkiewicz-Wijtiwiak, Anna Sowińska                         #
# ############################################################################
# DSGE model based on Smets Wouters (2003), with corrected equations:
# - (31) The correct loglinearised law of motion for the capital
#        should be written as: K[] = (1 - tau) * K[-1] + tau * I[].
# - (35) The goods market equilibrium condition should be written as:
#        Y_P[] = (1 - tau * k_Y - g_Y) * C[] + tau * k_Y * I[] +
#                g_Y * epsilon_G[] + k_Y * r_k_bar * r_k[] * psi
#        In the [SW'03] the last term accounting for
#        the cost of capacity utilisation was missing.
# The shock \eta^Q_t from equation (30) has not been introduced.
# ############################################################################

options
{
    output LaTeX = TRUE; output LaTeX landscape = TRUE;
    output logfile = TRUE;
    verbose = TRUE;
};

tryreduce
{
    H[], B[], H_f[], B_f[], K_j_d[], K_j_d_f[], K_d[], K_d_f[], P_f[], pi_star_w[], tc_j[],
        tc_j_f[], Y_j[],
    Y_j_f[], Div[], Div_f[], L_j_d_f[], L_j_d[], L_d[], L_d_f[], lambda[], lambda_f[], L_i_f
        [], L_i_star_f[];
};

block CONSUMER
{
    definitions
    {
        u[] = epsilon_b[] * ((C[] - H[]) ^ (1 - sigma_c) / (1 - sigma_c) -
              omega * epsilon_L[] * (L_s[]) ^ (1 + sigma_l) / (1 + sigma_l));
    };
    controls
    {
        C[], K[], I[], B[], z[];
    };
    objective
    {
        U[] = u[] + beta * E[][U[1]];
    };
    constraints
    {
       C[] + I[] + B[] / R[] =
          W[] * L[] +
          r_k[] * z[] * K[-1] - r_k[ss]  / psi * (exp(psi * (z[] - 1)) - 1)  * K[-1] +
          Div[] + B[-1] / pi[] - T[] : lambda[];
       K[] =  (1 - tau) * K[-1]  +
             (1 - varphi / 2 * (epsilon_I[] * I[] / I[-1] - 1) ^ 2) * I[] : q[];
    };
    identities
    {
        H[] = h * C[-1];
```

```
        Q[] = q[] / lambda[];
    };
    calibration
    {
        beta = 0.99;            # Discount factor
        tau = 0.025;            # Capital depreciation rate
        varphi = 6.771;         # Parameter of investment adjustment cost function
        psi = 0.169;            # Capacity utilisation cost parameter
        sigma_c = 1.353;        # Coefficient of relative risk aversion
        h = 0.573;              # Habit formation intensity
        sigma_l = 2.4;          # Reciprocal of labour elasticity w.r.t. wage
        omega = 1;              # Labour disutility parameter
    };
};

block PREFERENCE_SHOCKS
{
    identities
    {
        log(epsilon_b[]) = rho_b * log(epsilon_b[-1]) + eta_b[];
        log(epsilon_L[]) = rho_L * log(epsilon_L[-1]) - eta_L[];
    };
    shocks
    {
        eta_b[],    # Preference shock
        eta_L[];    # Labour supply shock
    };
    calibration
    {
        rho_b = 0.855;
        rho_L = 0.889;
    };
};

block INVESTMENT_COST_SHOCKS
{
    identities
    {
        log(epsilon_I[]) = rho_I * log(epsilon_I[-1]) + eta_I[];
    };
    shocks
    {
        eta_I[];    # Investment shock
    };
    calibration
    {
        rho_I = 0.927;
    };
};

block WAGE_SETTING_PROBLEM
{
    definitions
    {
        L_star[] = (pi_star_w[]) ^ (-((1 + lambda_w) / lambda_w)) * L[];
    }
    identities
    {
        f_1[] = 1 / (1 + lambda_w) * w_star[] * lambda[] * L_star[] +
              beta * xi_w * E[][((pi[] ^ gamma_w) / pi[1])^(-1 / lambda_w) *
              (w_star[1] / w_star[])^(1 / lambda_w) * f_1[1]];
        f_2[] = epsilon_L[] * omega * epsilon_b[] * (L_star[]) ^ (1 + sigma_l) +
              beta * xi_w * E[][((pi[] ^ gamma_w) / pi[1])^
              (-((1 + lambda_w) / lambda_w)*(1 + sigma_l)) *
```

```
              (w_star[1] / w_star[])^(((1 + lambda_w) / lambda_w) * (1 + sigma_l)) * f_2[1
                  ]];
        f_1[] = f_2[] + eta_w[];
        pi_star_w[] = w_star[] / W[];
    };
    shocks
    {
        eta_w[];                # Wage mark-up shock
    };
    calibration
    {
        gamma_w = 0.763;        # Indexation parameter for non-optimising workers
        lambda_w = 0.5;         # Wage mark-up
        xi_w = 0.737;           # Probability of not receiving ``wage-change signal''
    };
};

block WAGE_EVOLUTION
{
    identities
    {
        1 = xi_w * ((pi[-1] ^ gamma_w) / pi[]) ^ (-1 / lambda_w) *
           (W[-1] / W[])^(-1 / lambda_w) + (1 - xi_w) * (pi_star_w[]) ^ (-1 / lambda_w);
    };
};

block LABOUR_AGGREGATION
{
    identities
    {
        nu_w[] = (1 - xi_w) * pi_star_w[] ^(-((1 + lambda_w) / lambda_w)) +
                 xi_w * ((W[-1]/W[]) * ((pi[-1]^gamma_w) / pi[])) ^
                   (-((1 + lambda_w) / lambda_w)) * nu_w[-1];
        L[] = L_s[] / nu_w[];
    };
};

block CONSUMER_FLEXIBLE
{
    definitions
    {
        u_f[] = epsilon_b[] * ((C_f[] - H_f[]) ^ (1 - sigma_c) / (1 - sigma_c) -
                omega * epsilon_L[] * L_s_f[] ^ (1 + sigma_l) / (1 + sigma_l));
        Inc_i_f[] =  W_disutil_f[] * L_s_f[]  + Pi_ws_f[] +
                 r_k_f[] * z_f[] * K_f[-1] -
                 r_k_f[ss]  / psi * (exp(psi * (z_f[] - 1)) - 1)  * K_f[-1];
    };
    controls
    {
        C_f[], K_f[], I_f[], B_f[], z_f[], L_s_f[];
    };
    objective
    {
        U_f[] = u_f[] + beta * E[][U_f[1]];
    };
    constraints
    {
        C_f[] + I_f[] + B_f[] / R_f[]  =
            Inc_i_f[] + Div_f[] + B_f[-1] - T_f[]: lambda_f[];
        K_f[] = (1 - tau) * K_f[-1]  +
                (1 - varphi / 2 * (epsilon_I[] * I_f[] / I_f[-1] - 1)^2) * I_f[]  : q_f[];
    };
    identities
    {
        H_f[] = h * C_f[-1];
```

```
        Q_f[] = q_f[] / lambda_f[];
    };
};

block FLEXIBLE_MONOPOLISTIC_WORKER
{
    controls
    {
        W_i_f[], L_i_star_f[];
    };
    objective
    {
        Pi_ws_f[] = (W_i_f[] - W_disutil_f[]) * L_i_star_f[];
    };
    constraints
    {
        L_i_star_f[] = L_f[] * (W_i_f[] / W_f[])^(-(1 + lambda_w) / lambda_w);
    };
    identities
    {
        L_i_star_f[] = L_i_f[];
    };
};

block LABOUR_AGGREGATION_FLEXIBLE
{
    identities
    {
        L_s_f[] = L_i_f[];
        L_f[] = L_s_f[];
    };
};

block FIRM
{
    controls
    {
        K_j_d[], L_j_d[];
    };
    objective
    {
        tc_j[] = - L_j_d[] * W[] - K_j_d[] * r_k[];
    };
    constraints
    {
        Y_j[] = epsilon_a[] * K_j_d[] ^ alpha * L_j_d[]^(1 - alpha) - Phi :        mc[];
    };
    calibration
    {
        alpha = 0.3;                                   # Capital share in output
        (Y_j[ss] + Phi) / Y_j[ss] = 1.408 -> Phi;    # Calibration of fixed costs
    };
};

block TECHNOLOGY
{
    identities
    {
        log(epsilon_a[]) = rho_a * log(epsilon_a[-1]) + eta_a[];
    };
    shocks
    {
        eta_a[];     # Productivity shock
    };
    calibration
```

```
    {
        rho_a = 0.823;
    };
};

block PRICE_SETTING_PROBLEM
{
    identities
    {
        g_1[] = (1 + lambda_p) * g_2[] + eta_p[];
        g_1[] = lambda[] * pi_star[] * Y[] + beta * xi_p *
                E[][(pi[] ^ gamma_p / pi[1]) ^ (-1 / lambda_p) *
                    (pi_star[] / pi_star[1]) * g_1[1]];
        g_2[] = lambda[] * mc[] * Y[] + beta * xi_p *
                E[][(pi[] ^ gamma_p / pi[1]) ^ (-((1 + lambda_p) / lambda_p)) * g_2[1]];
    };
    shocks
    {
        eta_p[];                # Price mark-up shock
    };
    calibration
    {
        xi_p = 0.908;           # Probability of not receiving the ``price-change signal''
        gamma_p = 0.469;        # Indexation parameter for non-optimising firms
    };
};

block PRICE_EVOLUTION
{
    identities
    {
        1 = xi_p * (pi[-1] ^ gamma_p / pi[]) ^ (-1 / lambda_p) +
           (1 - xi_p) * pi_star[] ^ (-1 / lambda_p);
    };
};

block FACTOR_DEMAND_AGGREGATION
{
    identities
    {
        K_d[] = K_j_d[];
        L_d[] = L_j_d[];
    };
};

block PRODUCT_AGGREGATION
{
    identities
    {
        Y_s[] = Y_j[];
        nu_p[] = (1 - xi_p) * pi_star[] ^ (-((1 + lambda_p) / lambda_p))
                 + xi_p * (pi[-1] ^ gamma_p / pi[]) ^
                 (-((1 + lambda_p) / lambda_p)) * nu_p[-1];
        Y[] * nu_p[] = Y_s[];
    };
};

block FIRM_FLEXIBLE
{
    controls
    {
        K_j_d_f[], L_j_d_f[];
    };
    objective
    {
```

```
        tc_j_f[] = - L_j_d_f[] * W_f[] - K_j_d_f[] * r_k_f[];
    };
    constraints
    {
        Y_j_f[] = epsilon_a[] * K_j_d_f[]^alpha * L_j_d_f[]^(1 - alpha) - Phi     : mc_f[];
    };
};

block PRICE_SETTING_PROBLEM_FLEXIBLE
{
    controls
    {
        Y_j_f[], P_j_f[];
    };
    objective
    {
        Pi_ps_f[] = (P_j_f[] - mc_f[]) * Y_j_f[];
    };
    constraints
    {
        Y_j_f[] = (P_j_f[] / P_f[])^ (-((1 + lambda_p) / lambda_p)) * Y_f[];
    };
    calibration
    {
        C_f[ss] / Y_f[ss]  = 0.6  -> lambda_p;      # Calibration of the price mark-up
    };
};

block FACTOR_DEMAND_AGGREGATION_FLEXIBLE
{
    identities
    {
        K_d_f[] = K_j_d_f[];
        L_d_f[] = L_j_d_f[];
    };
};

block PRODUCT_AGGREGATION_FLEXIBLE
{
    identities
    {
        Y_s_f[] = Y_j_f[];
        Y_f[] = Y_s_f[];
    };
};

block PRICE_EVOLUTION_FLEXIBLE
{
    identities
    {
        P_f[] = 1;
    };
};

block GOVERNMENT
{
    identities
    {
       G[] = G_bar * epsilon_G[];
       G[] + B[-1] / pi[] = T[] + B[] / R[];
    };
    calibration
    {
       G[ss] / Y[ss] = 0.18  -> G_bar; # Calibration of the steady state government
           expenditures
```

```
    };
};

block GOVERNMENT_SPENDING_SHOCK
{
    identities
    {
        log(epsilon_G[]) = rho_G * log(epsilon_G[-1]) + eta_G[];
    };
    shocks
    {
        eta_G[];     # Government spending shock
    };
    calibration
    {
        rho_G = 0.949;
    };
};

block GOVERNMENT_FLEXIBLE
{
    identities
    {
        G_f[] = G_bar * epsilon_G[];
        G_f[] + B_f[-1] = T_f[] + B_f[] / R_f[];
    };
};

block MONETARY_POLICY_AUTHORITY
{
    identities
    {
        log(R[] / R[ss]) + calibr_pi = rho * log(R[-1] / R[ss]) +
                                      (1 - rho) * (log(pi_obj[]) +
                                      r_pi * (log(pi[-1] / pi[ss]) - log(pi_obj[])) +
                                      r_Y * (log(Y[]/ Y[ss]) - log(Y_f[] / Y_f[ss]))) +
                                      r_Delta_pi * (log(pi[]/pi[ss]) - log(pi[-1]/pi[ss])) +
                                      r_Delta_y * (log(Y[] / Y[ss]) -
                                                        log(Y_f[] / Y_f[ss]) -
                                                  (log(Y[-1] / Y[ss]) -
                                                        log(Y_f[-1] / Y_f[ss]))) +
                                      eta_R[];
        log(pi_obj[]) = (1 - rho_pi_bar) * log(calibr_pi_obj) +
                        rho_pi_bar * log(pi_obj[-1]) + eta_pi[];
    };
    shocks
    {
        eta_R[],     # Interest rate shock
        eta_pi[];    # Inflation objective shock
    };
    calibration
    {
        r_Delta_pi = 0.14;                          # Weight on the dynamics of inflation
        r_Delta_y = 0.159;                          # Weight on the dynamics of output gap
        rho = 0.961;                                # Interest rate smoothing parameter
        r_Y = 0.099;                                # Weight on the output gap
        r_pi = 1.684;                               # Weight on the inflation gap
        pi_obj[ss] = 1      -> calibr_pi_obj;       # Calibration of the inflation objective
        pi[ss] = pi_obj[ss] -> calibr_pi;           # Calibration of the inflation
        rho_pi_bar = 0.924;                         # AR parameter for the inflation objective
    };
};

block EQUILIBRIUM
{
```

```
    identities
    {
        K_d[] = z[] * K[-1];
        L[] = L_d[];
        B[] = 0;
        Div[] = Y[] - L_d[] * W[] - K_d[] * r_k[];
    };
};

block EQUILIBRIUM_FLEXIBLE
{
    identities
    {
        K_d_f[] = z_f[] * K_f[-1];
        L_f[] = L_d_f[];
        B_f[] = 0;
        Div_f[] = Y_f[] - L_d_f[] * W_f[] - K_d_f[] * r_k_f[];
    };
};
```

Listing 16: The full `gEcon` implementation of the model

```
# ###################################################################
# (c) Chancellery of the Prime Minister 2012-2015                  #
# Licence terms can be found in the file:                          #
# http://gecon.r-forge.r-project.org/files/gEcon_licence.txt        #
#                                                                   #
# Authors: Grzegorz Klima, Karol Podemski,                         #
#          Kaja Retkiewicz-Wijtiwiak, Anna Sowińska                #
# ###################################################################

library(gEcon)

sw_gecon <- make_model('SW_03.gcn')

initv <- list(z = 1, z_f = 1, Q = 1, Q_f = 1, pi = 1, pi_obj = 1,
              epsilon_b = 1, epsilon_L = 1, epsilon_I = 1, epsilon_a = 1, epsilon_G = 1,
              r_k = 0.01, r_k_f = 0.01)
sw_gecon <- initval_var(sw_gecon, init_var = initv)

sw_gecon <- steady_state(sw_gecon)

sw_gecon <- solve_pert(sw_gecon, loglin = TRUE)

summary(sw_gecon)

a <- c(eta_b = 0.336 ^ 2, eta_L = 3.52 ^ 2, eta_I = 0.085 ^ 2, eta_a = 0.598 ^ 2,
       eta_w = 0.6853261 ^ 2, eta_p = 0.7896512 ^ 2,
       eta_G = 0.325 ^ 2, eta_R = 0.081 ^ 2, eta_pi = 0.017 ^ 2)
sw_gecon  <- set_shock_cov_mat(sw_gecon, shock_matrix = diag(a), shock_order = names(a))

shock_info(sw_gecon, all_shocks = TRUE)

sw_gecon <- compute_moments(sw_gecon)

get_moments(sw_gecon)

sw_gecon_irf <- compute_irf(sw_gecon, var_list = c('C', 'Y', 'K', 'I', 'L'), chol = T,
                            shock_list = list('eta_a', 'eta_R'), path_length = 40)
plot_simulation(sw_gecon_irf)
```

Listing 17: The accompanying R code

Appendix C. Steady-state values of model variables and standard deviations of the log-linearised variables

Variable	Steady-state value	Standard deviation
epsilon_L	1	4.4929
epsilon_b	1	0.4207
epsilon_I	1	0.1103
epsilon_a	1	0.7338
epsilon_G	1	0.4236
f_1	8.7708	0.8265
f_2	8.7708	0.8314
g_1	48.8253	1.8874
g_2	35.7045	1.8874
mc	0.7313	0.8206
mc_f	0.7313	0
nu_w	1	0
nu_p	1	0
pi	1	0.1145
pi_obj	1	0.0220
pi_star	1	0.7093
q	2.4577	0.3662
q_f	2.4577	0.4179
r_k	0.0351	0.1520
r_k_f	0.0351	0.1934
w_star	1.1227	0.6559
z	1	0.8992
z_f	1	1.1442
C	1.2049	0.7154
C_f	1.2049	1.4816
G	0.3615	0.4236
G_f	0.3615	0.4236
I	0.4418	1.8279
I_f	0.4418	2.6127
K	17.6712	0.2275
K_f	17.6712	0.2981
L	1.2891	0.9894
L_s	1.2891	0.9894
L_s_f	1.2891	1.1104
L_f	1.2891	1.1104
P_j_f	1	0
Pi_ws_f	0.4824	1.1952
Pi_ps_f	0.5396	1.6828
Q	1	0.9374
Q_f	1	2.7021
R	1.0101	0.2153
R_f	1.0101	1.1384
T	0.3615	0.4236
T_f	0.3615	0.4236
U	-427.937	0.0694
U_f	-427.937	0.0685
W	1.1227	0.3847
W_disutil_f	0.7485	1.0047
W_i_f	1.1227	1.0047

W_f	1.1227	1.0047
Y	2.0081	0.9158
Y_s	2.0081	0.9158
Y_f	2.0081	1.6828
Y_s_f	2.0081	1.6828

Table 3: The steady-state values of model variables and standard deviations of the log-linearised variables

Appendix D. Correlations and the decomposition of variance

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
π	0.8807	0.6745	0.4521	0.2453	0.0675
r^{k}	0.7406	0.5165	0.3259	0.1668	0.0369
z	0.7406	0.5165	0.3259	0.1668	0.0369
C	0.8710	0.6373	0.3843	0.1546	-0.0336
I	0.9450	0.8182	0.6499	0.4634	0.2760
K	0.9796	0.9208	0.8287	0.7102	0.5730
L	0.7052	0.4567	0.2534	0.0919	-0.0325
Q	0.6716	0.3950	0.1791	0.0185	-0.0958
R	0.7709	0.4956	0.2532	0.0622	-0.0789
T	0.7130	0.4706	0.2705	0.1093	-0.0169
W	0.9502	0.8308	0.6685	0.4852	0.2981
Y	0.9049	0.7264	0.5191	0.3142	0.1286

Table 4: Autocorrelations of selected variables

	Y_{t-5}	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Y_{t+5}
π	-0.0302	0.0149	0.0695	0.1342	0.2089	0.2918	0.3054	0.2829	0.2421	0.1928	0.1412
r^{k}	0.0634	0.1539	0.2602	0.3818	0.5177	0.6641	0.6386	0.5393	0.4065	0.2649	0.1293
z	0.0634	0.1539	0.2602	0.3818	0.5177	0.6641	0.6386	0.5393	0.4065	0.2649	0.1293
C	0.2858	0.4541	0.6276	0.7837	0.8829	0.8571	0.6559	0.4004	0.1541	-0.0538	-0.2132
I	-0.0533	0.1257	0.3315	0.5496	0.7521	0.8889	0.9085	0.8464	0.7317	0.5871	0.4300
K	-0.6653	-0.6234	-0.5412	-0.4173	-0.2558	-0.0709	0.1134	0.2806	0.4205	0.5279	0.6011
L	-0.0246	0.0658	0.1773	0.3103	0.4633	0.6313	0.6174	0.5280	0.4056	0.2752	0.1513
Q	0.4149	0.5390	0.6490	0.7187	0.7038	0.5322	0.1992	-0.0471	-0.2176	-0.3261	-0.3858
R	-0.3352	-0.4182	-0.4867	-0.5194	-0.4803	-0.3114	-0.0556	0.1372	0.2635	0.3345	0.3632
T	-0.0142	-0.0061	0.0057	0.0224	0.0456	0.0778	0.0584	0.0416	0.0274	0.0156	0.0061
W	-0.1606	-0.1421	-0.1138	-0.0747	-0.0236	0.0398	0.1153	0.1829	0.2337	0.2647	0.2764
Y	0.1286	0.3142	0.5191	0.7264	0.9049	1	0.9049	0.7264	0.5191	0.3142	0.1286

Table 5: Correlations of selected variables with product

	π	r^{k}	z	C	I	K	L	Q	R	T	W	Y
π	1	0.6760	0.6760	0.0210	0.1725	0.0987	0.4656	-0.0467	0.5942	0.0147	0.6883	0.2918
r^{k}	0.6760	1	1	0.3287	0.4613	-0.0583	0.9301	0.0562	0.3296	0.0504	0.2489	0.6641
z	0.6760	1	1	0.3287	0.4613	-0.0583	0.9301	0.0562	0.3296	0.0504	0.2489	0.6641
z^{f}	-0.5270	-0.0588	-0.0588	0.7621	0.4294	-0.4265	-0.0147	0.7299	-0.906	0.0204	-0.4339	0.5296
C	0.0210	0.3287	0.3287	1	0.6635	-0.3251	0.3000	0.7968	-0.6658	-0.0318	-0.1514	0.8571
I	0.1725	0.4613	0.4613	0.6635	1	0.2233	0.4653	0.3260	-0.2300	-0.0133	0.0775	0.8889
K	0.0987	-0.0583	-0.0583	-0.3251	0.2233	1	0.0477	-0.4934	0.4258	0.0138	0.2971	-0.0709
L	0.4656	0.9301	0.9301	0.3000	0.4653	0.0477	1	0.0024	0.2632	0.0518	-0.058	0.6313
Q	-0.0467	0.0562	0.0562	0.7968	0.3260	-0.4934	0.0024	1	-0.8223	-0.0178	-0.1917	0.5322
R	0.5942	0.3296	0.3296	-0.6658	-0.2300	0.4258	0.2632	-0.8223	1	0.0206	0.51	-0.3114
T	0.0147	0.0504	0.0504	-0.0318	-0.0133	0.0138	0.0518	-0.0178	0.0206	1	0.0145	0.0778
W	0.6883	0.2489	0.2489	-0.1514	0.0775	0.2971	-0.058	-0.1917	0.5100	0.0145	1	0.0398
Y	0.2918	0.6641	0.6641	0.8571	0.8889	-0.0709	0.6313	0.5322	-0.3114	0.0778	0.0398	1

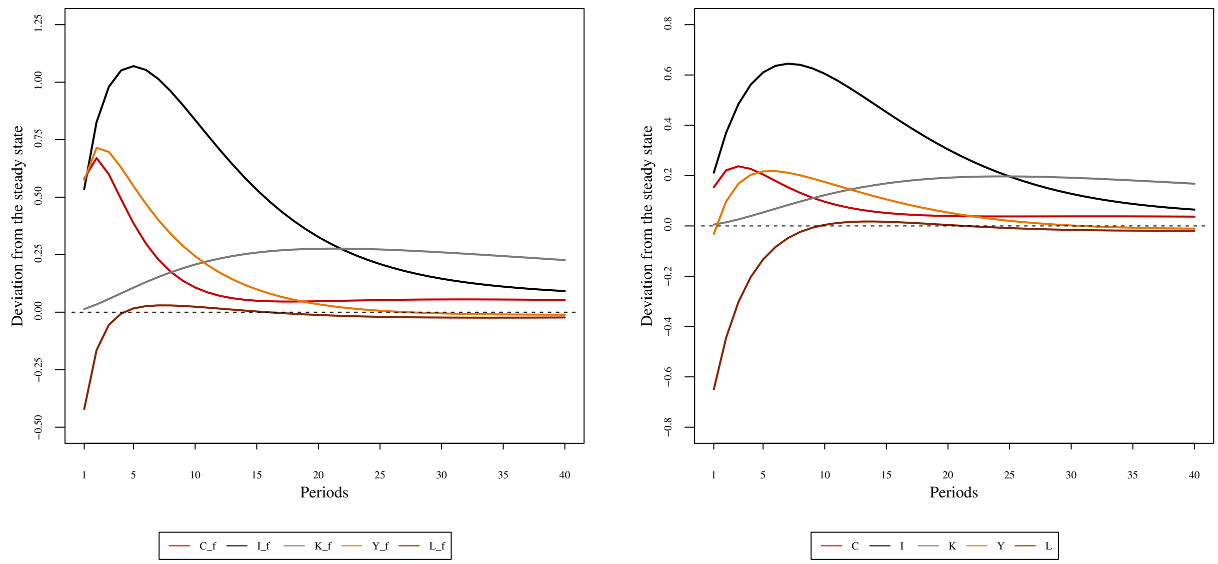
Table 6: Correlations between selected variables

	η^{π}	η^{b}	η^{L}	η^{I}	η^{w}	η^{a}	η^{p}	η^{G}	η^{R}
π	0.0036	0.0034	0.1088	0.0002	0	0.2145	0.0002	0.0003	0.6691
r^{k}	0.0011	0.0050	0.1087	0.0004	0	0.4632	0	0.0026	0.4189
z	0.0011	0.0050	0.1087	0.0004	0	0.4632	0	0.0026	0.4189
C	0.0007	0.0462	0.4269	0.0003	0	0.2144	0	0.0013	0.3102
I	0.0010	0.0116	0.4314	0.0032	0	0.1782	0	0.0008	0.3737
K	0.0011	0.0113	0.4397	0.0034	0	0.1643	0	0.0010	0.3793
L	0.0004	0.0034	0.2564	0.0002	0	0.5695	0	0.0029	0.1672
Q	0.0006	0.0104	0.3712	0.0005	0	0.2537	0	0.0003	0.3633
R	0.0005	0.0108	0.3976	0.0001	0	0.4912	0	0.0004	0.0994
T	0	0	0	0	0	0	0	1	0
W	0.0018	0.0113	0.4010	0.0002	0.0001	0.0120	0	0.0004	0.5731
Y	0.0011	0.0090	0.4081	0.0005	0	0.1125	0	0.0065	0.4624

Table 7: The decomposition of variance for selected variables

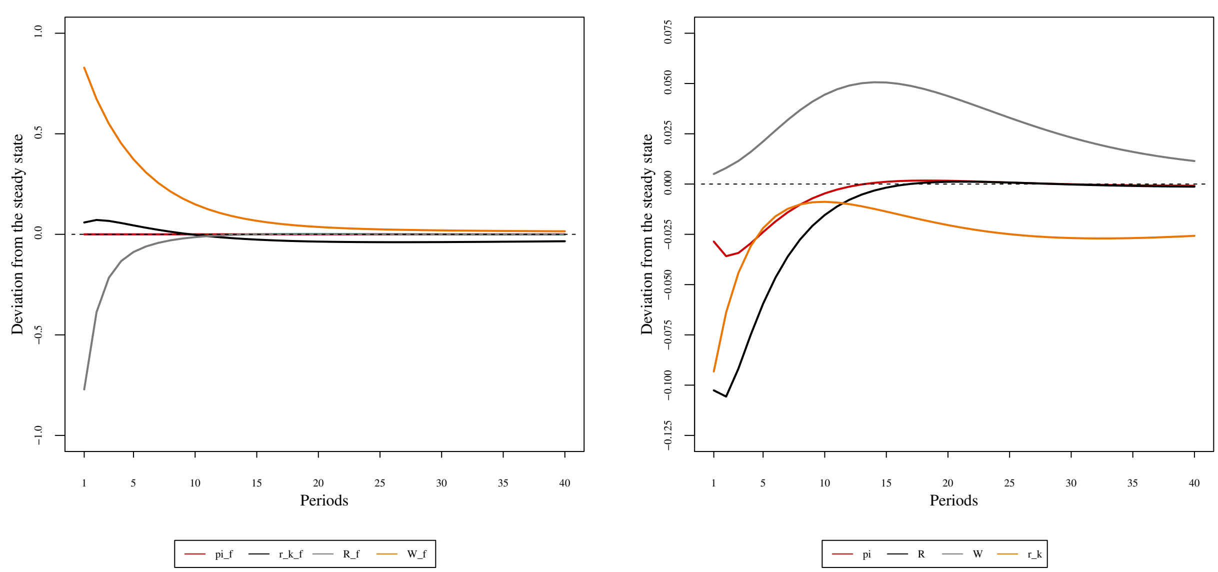
Appendix E. Impulse response functions in economies with and without nominal rigidities

Impulse responses to a positive productivity shock



a) **b)**

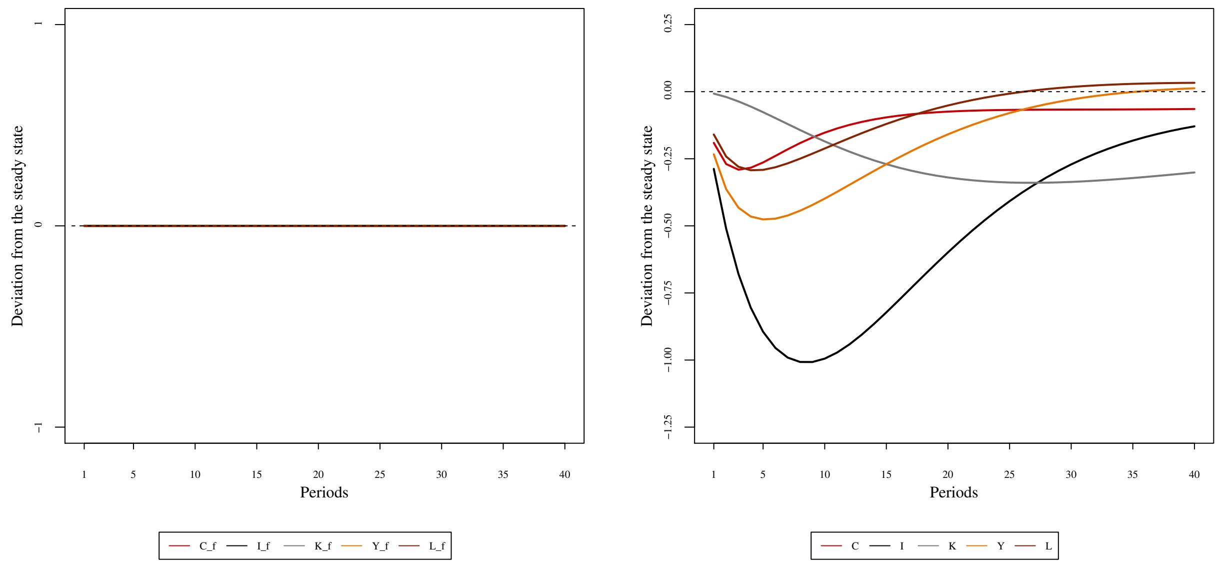
Figure 2: (a) Response to a productivity shock in the economy without rigidities
(b) Response to a productivity shock in the economy with rigidities



a) **b)**

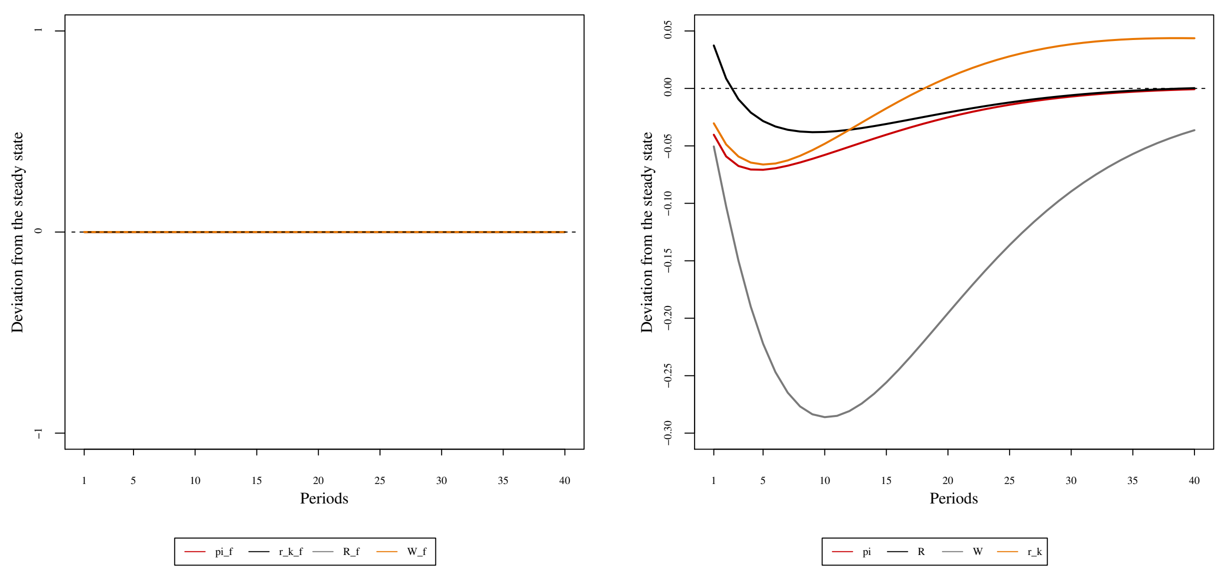
Figure 3: (a) Response to a productivity shock in the economy without rigidities
(b) Response to a productivity shock in the economy with rigidities

Impulse responses to a positive interest rate shock



a) **b)**

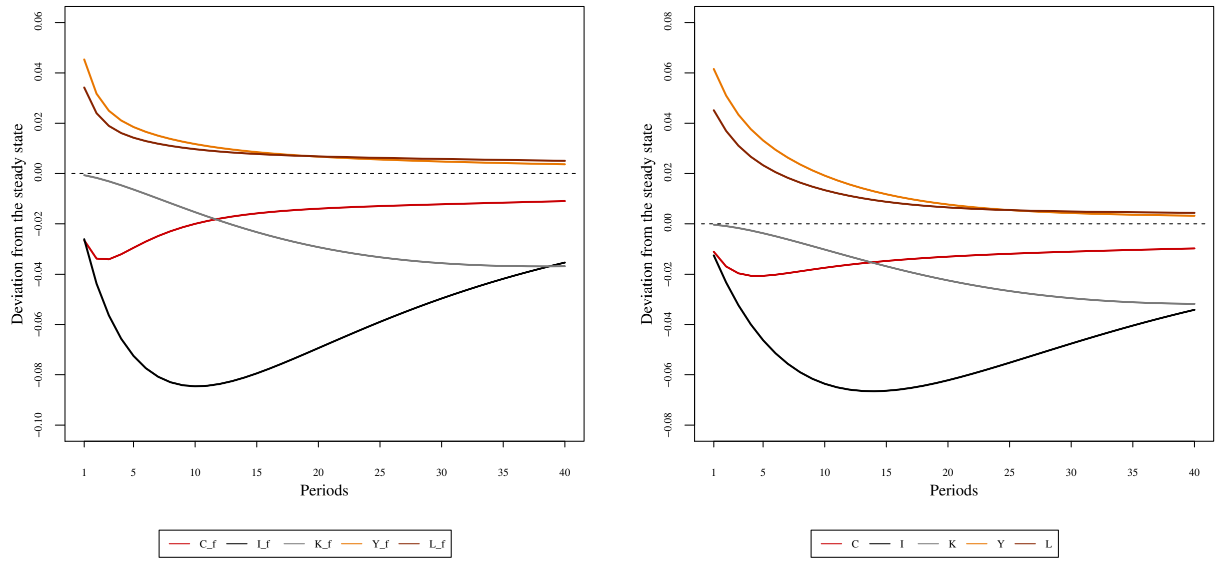
Figure 4: (a) Response to an interest rate shock in the economy without rigidities
(b) Response to an interest rate shock in the economy with rigidities



a) **b)**

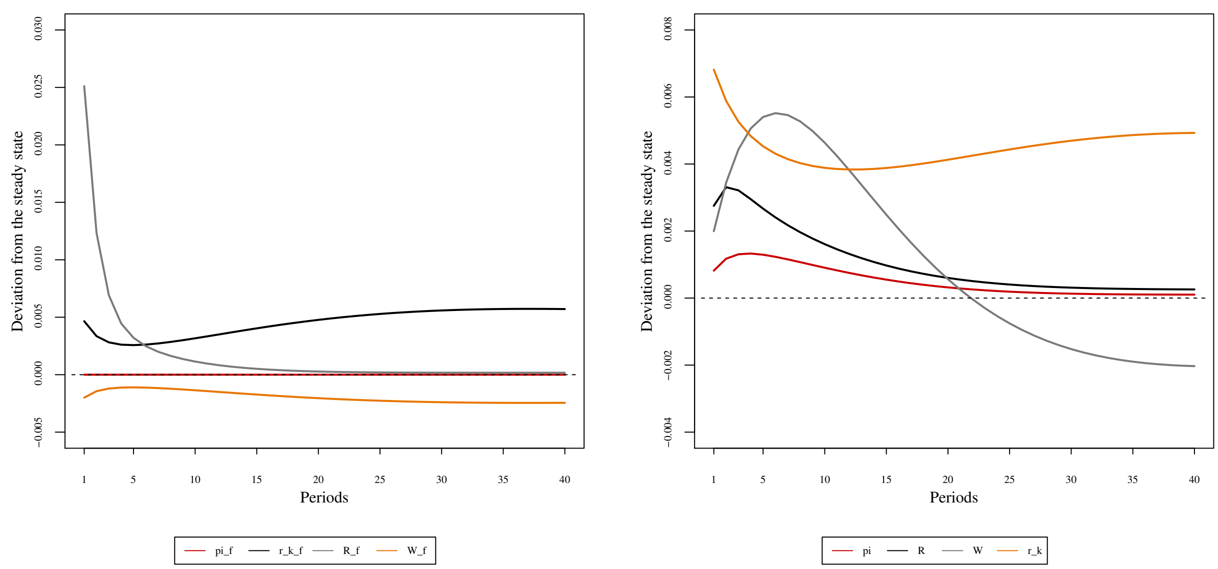
Figure 5: (a) Response to an interest rate shock in the economy without rigidities
(b) Response to an interest rate shock in the economy with rigidities

Impulse responses to a positive government spending shock



a) b)

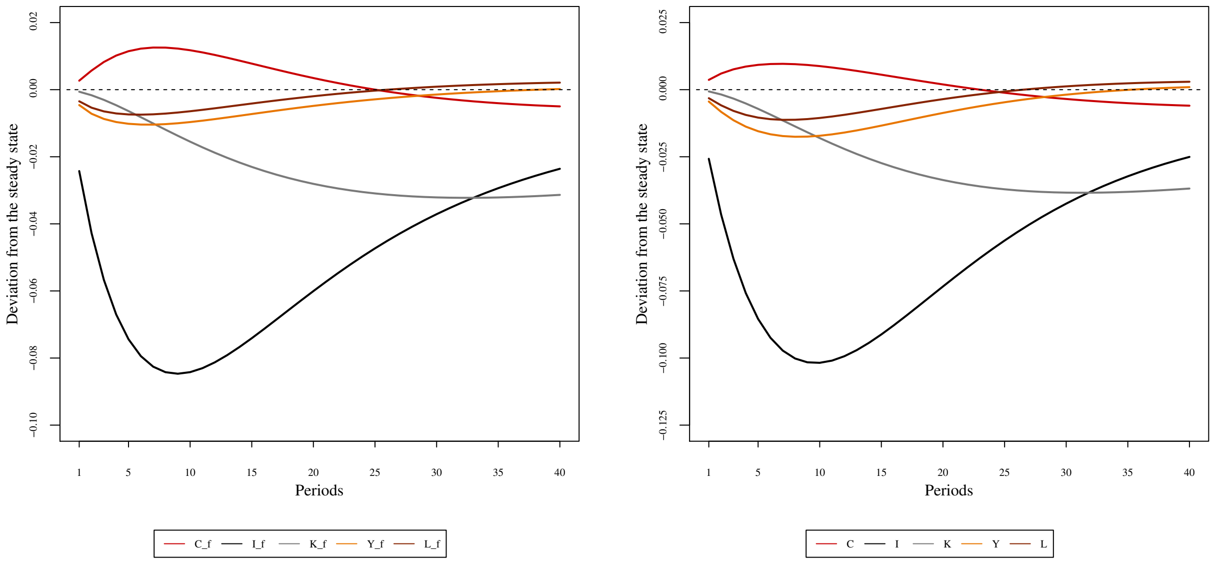
Figure 6: (a) Response to a government spending shock in the economy without rigidities
(b) Response to a government spending shock in the economy with rigidities



a) b)

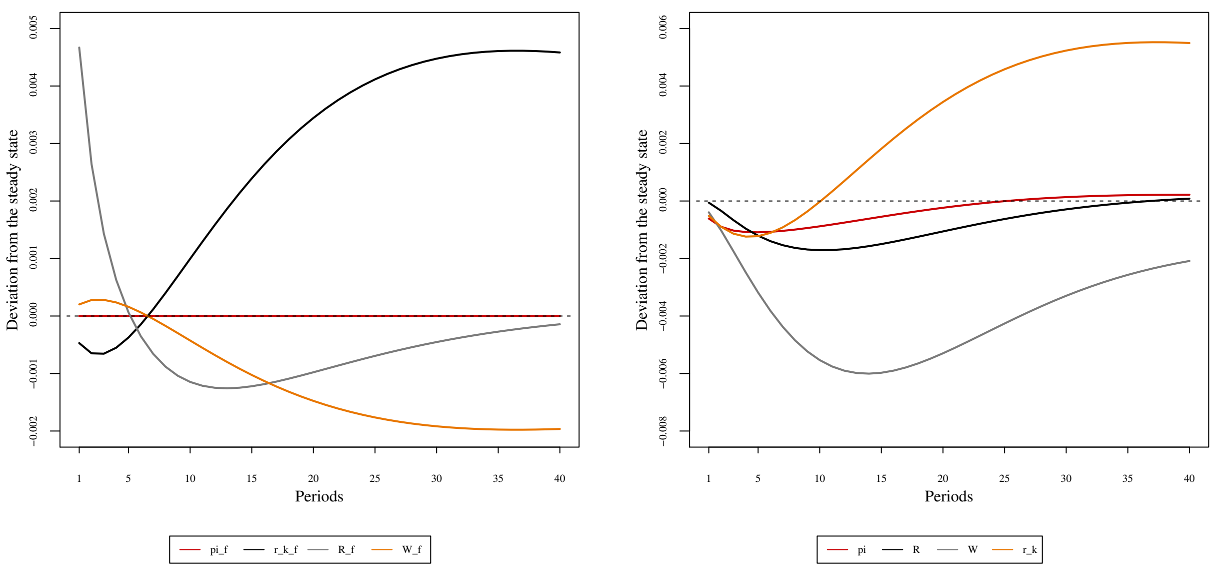
Figure 7: (a) Response to a government spending shock in the economy without rigidities
(b) Response to a government spending shock in the economy with rigidities

Impulse responses to a negative investment shock



a) **b)**

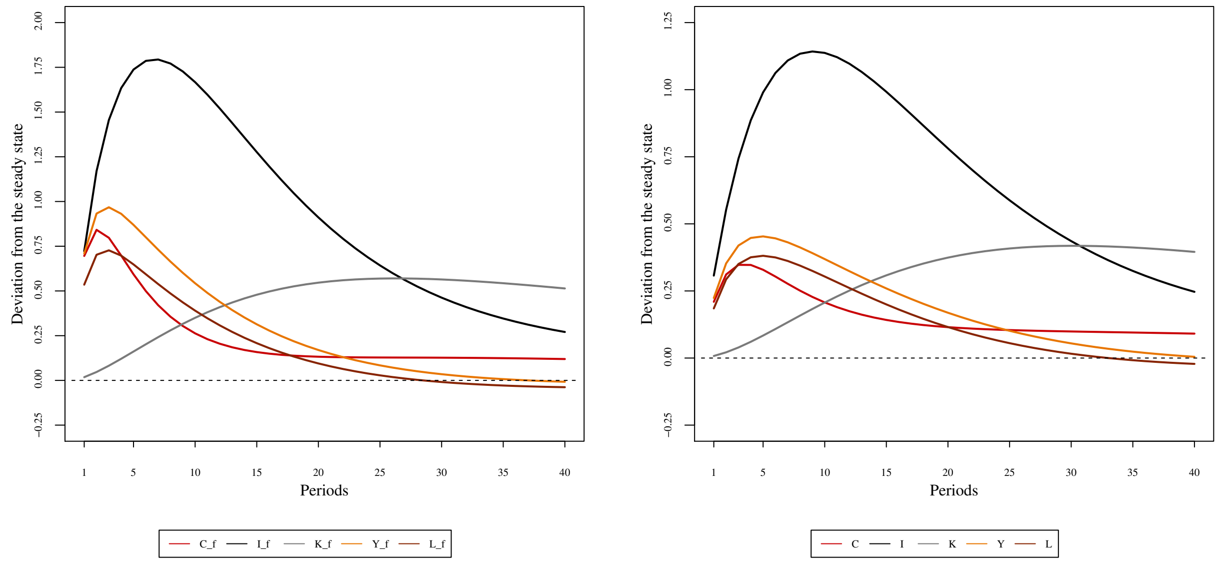
Figure 8: (a) Response to a negative investment shock in the economy without rigidities
(b) Response to a negative investment shock in the economy with rigidities



a) **b)**

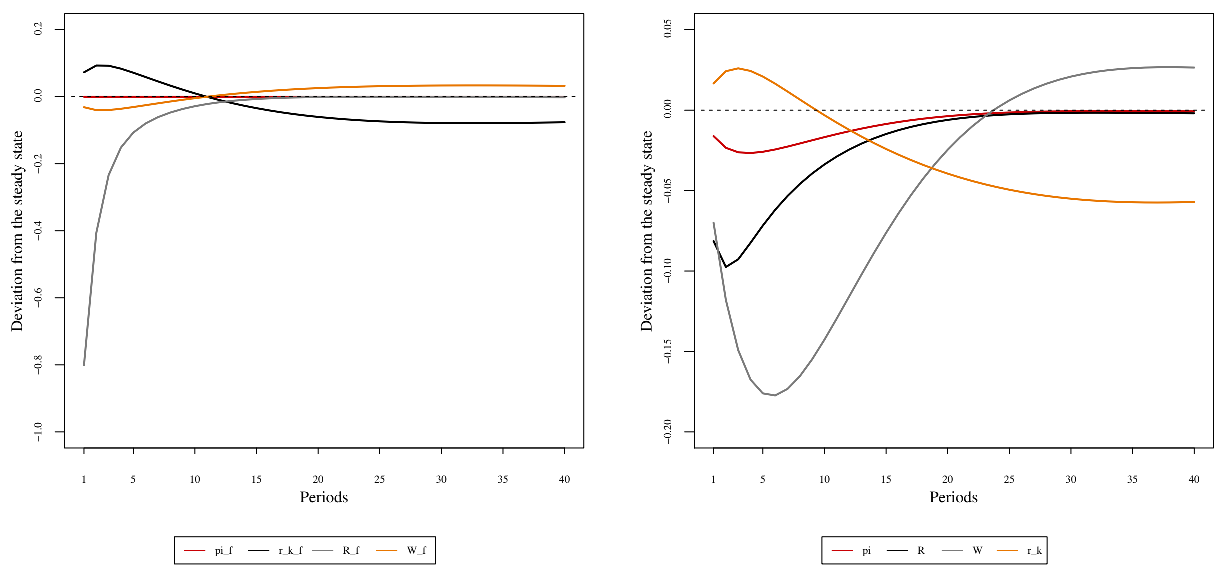
Figure 9: (a) Response to a negative investment shock in the economy without rigidities
(b) Response to a negative investment shock in the economy with rigidities

Impulse responses to a positive labour shock



a) **b)**

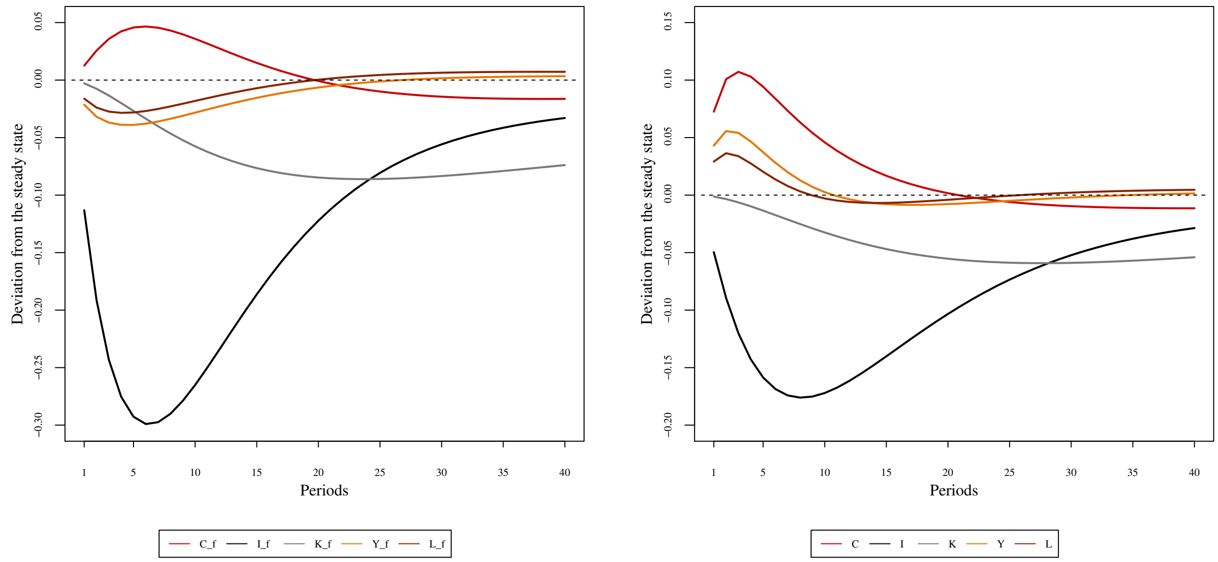
Figure 10: (a) Response to a labour shock in the economy without rigidities
(b) Response to a labour shock in the economy with rigidities



a) **b)**

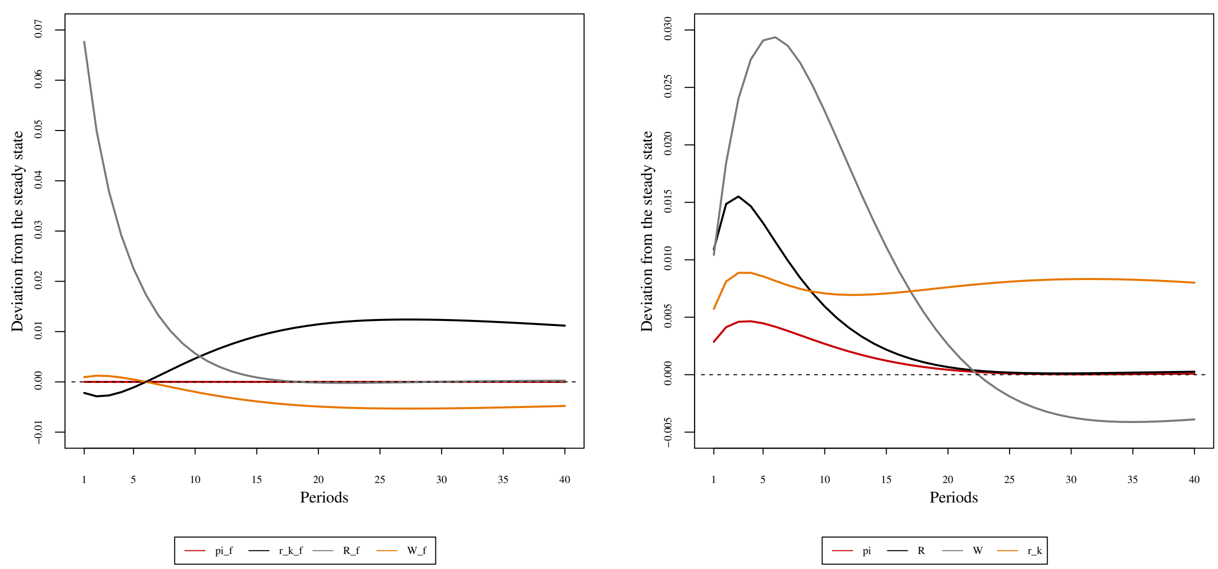
Figure 11: (a) Response to a labour shock in the economy without rigidities
(b) Response to a labour shock in the economy with rigidities

Impulse responses to a positive time preference shock



a) **b)**

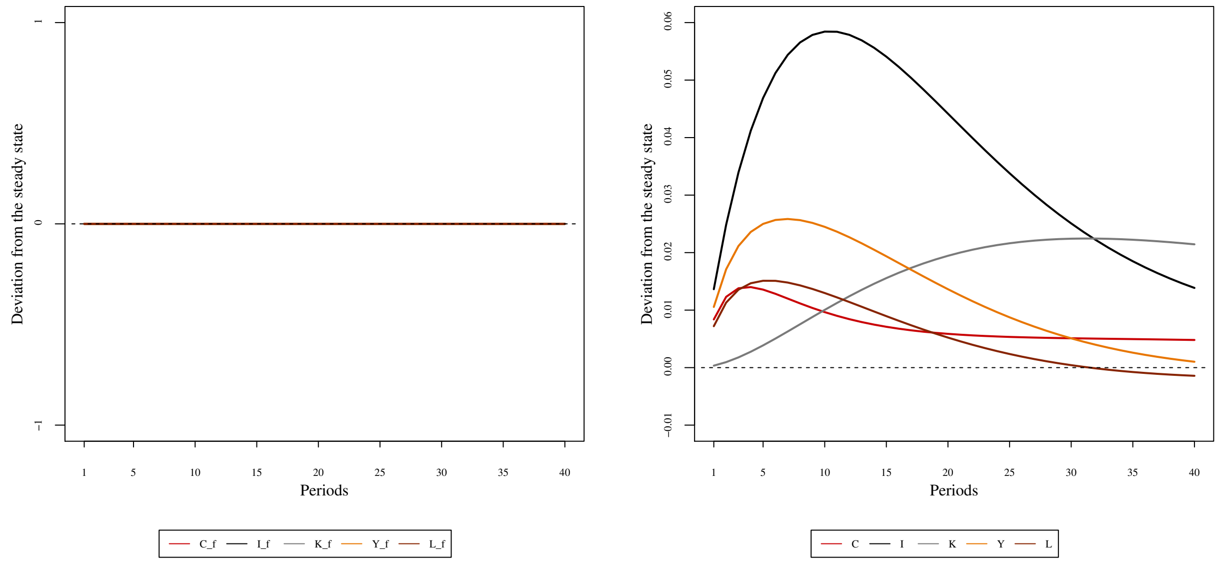
Figure 12: (a) Response to a time preference shock in the economy without rigidities
(b) Response to a time preference shock in the economy with rigidities



a) **b)**

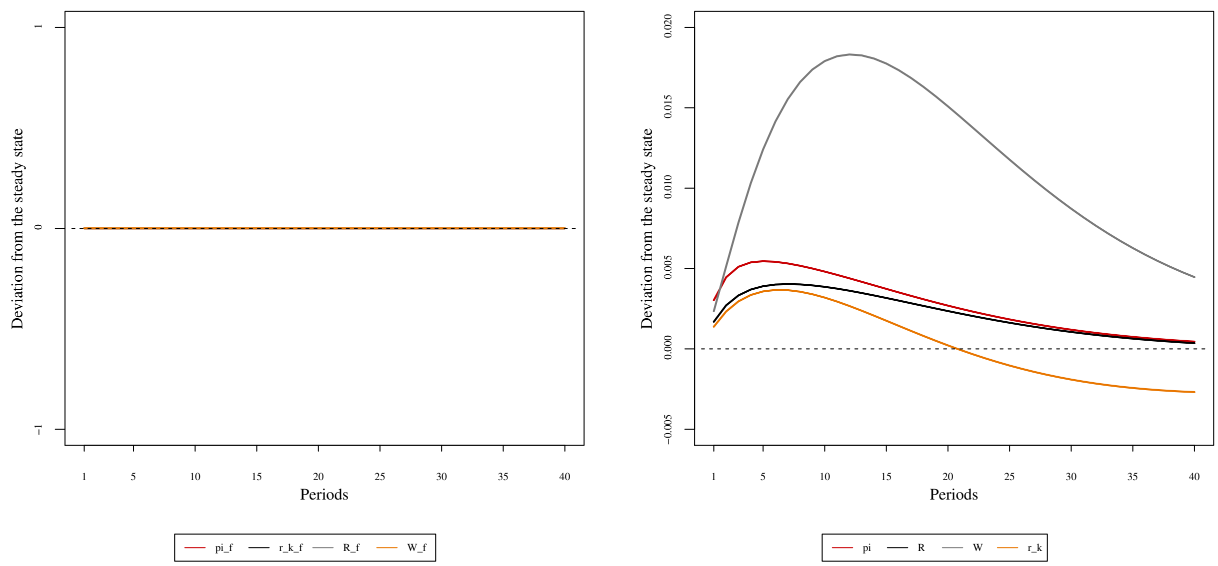
Figure 13: (a) Response to a time preference shock in the economy without rigidities
(b) Response to a time preference shock in the economy with rigidities

Impulse responses to a positive inflation objective shock



a) **b)**

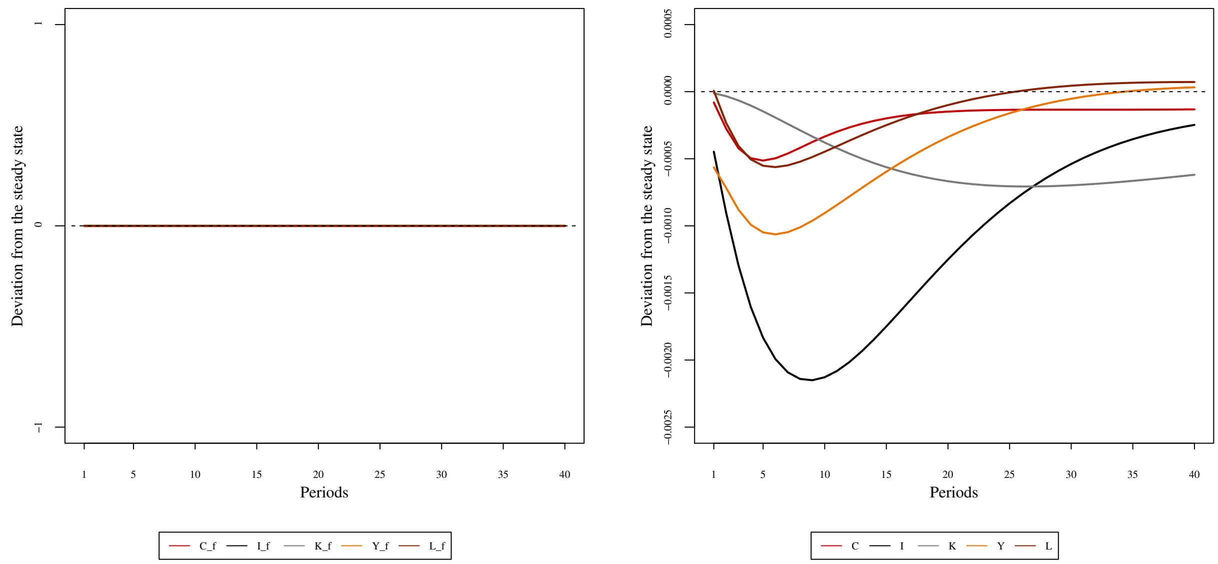
Figure 14: (a) Response to an inflation objective shock in the economy without rigidities
(b) Response to an inflation objective shock in the economy with rigidities



a) **b)**

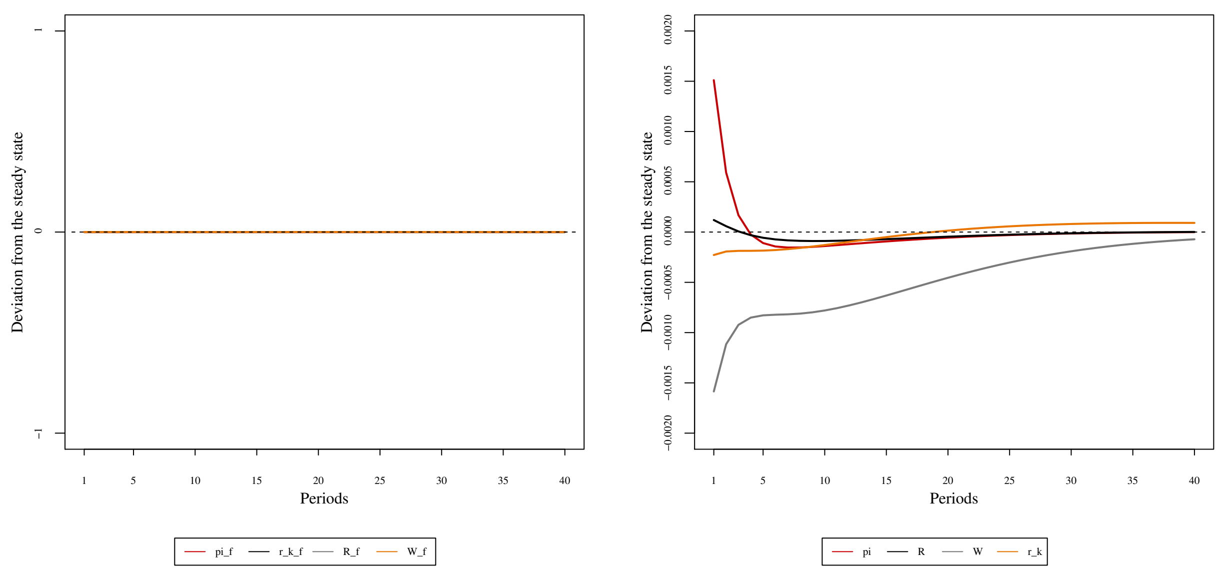
Figure 15: (a) Response to an inflation objective shock in the economy without rigidities
(b) Response to an inflation objective shock in the economy with rigidities

Impulse responses to a positive price mark-up shock



a) **b)**

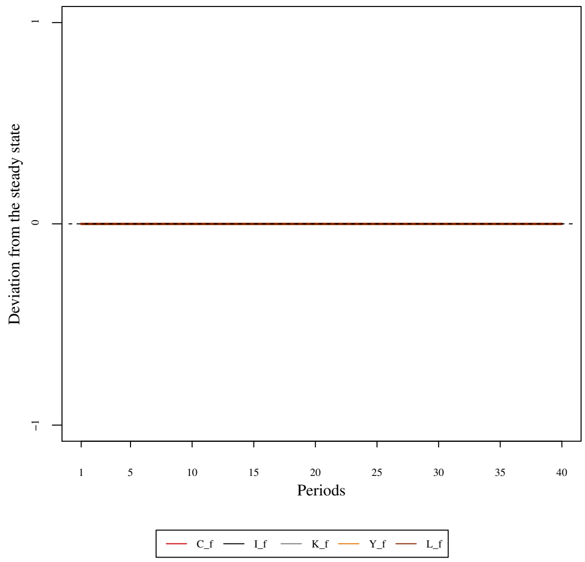
Figure 16: (a) Response to a price mark-up shock in the economy without rigidities
(b) Response to a price mark-up shock in the economy with rigidities



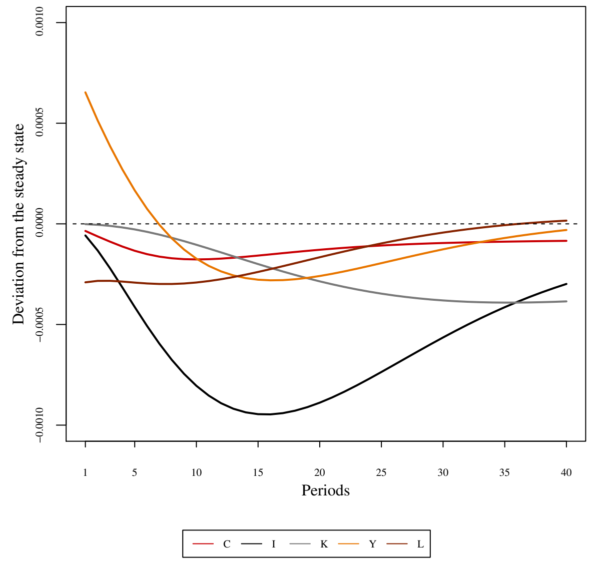
a) **b)**

Figure 17: (a) Response to a price mark-up shock in the economy without rigidities
(b) Response to a price mark-up shock in the economy with rigidities

Impulse responses to a positive wage mark-up shock

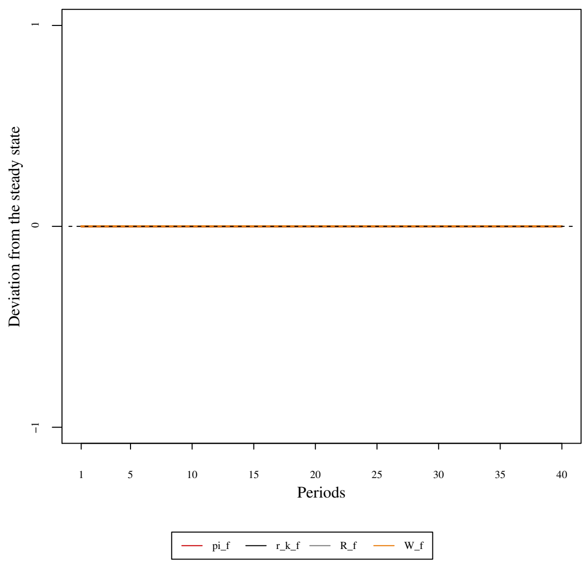


a)

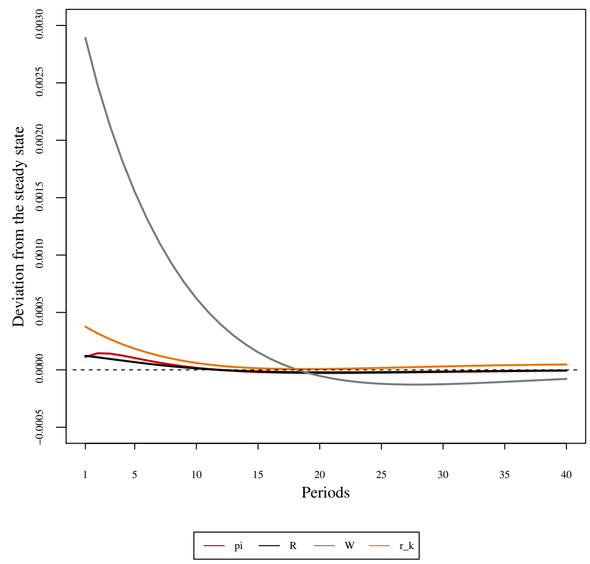


b)

Figure 18: (a) Response to a wage mark-up shock in the economy without rigidities
(b) Response to a wage mark-up shock in the economy with rigidities



a)



b)

Figure 19: (a) Response to a wage mark-up shock in the economy without rigidities
(b) Response to a wage mark-up shock in the economy with rigidities