A Rational Economic Model of Paygo Tax Rates

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2014
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Abstract

We argue that a rational-economic model of how societies choose their paygo tax rate can explain the cross section variance of these rates in large, developed OECD economies. Using a two-period OLG framework, we suggest that paygo tax rates are determined by a representative agent and a benevolent government jointly maximizing the expected life-time utility of the representative agent. In order to calculate these expected utilities, we construct probability distributions of life-time labor and capital income by simulating annual models of real wages and the return to capital estimated from data on real GDP and the real return to capital from the end of World War II to 2002. The joint distribution of the error terms is bootstrapped from the estimated errors of the annual equations. Expectations are taken over these distributions. The model predicts that each country chooses the paygo tax rate which maximizes the expected life-time utility of its representative agent. Risk aversion, described by a CRRA utility function, is assumed uniform across countries, such that the variance of the predicted rates is due exclusively to cross-country differences in the objective characteristics of the dynamics of wages and the return to capital in each country. These predicted rates are shown to explain 85% of the variance of observed effective-paygo rates. The calculations show that it is cross-country differences in the level and variability of the return to capital which are the most important source of this variance. We use the model to simulate a hypothetical world in which all countries share a unique, global capital market, and show that this scenario leads to a radical convergence of paygo rates. In a further exercise, we add an estimate of the probability of global crises like that of 2008 to the national distributions computed from post-War data, and examine the potential effect on paygo rates of these previously neglected, low probability events.

JEL Codes: C15, D81, D91, H55;
Keywords: Pay-as-you-go, Savings, Risk Aversion, OLG, National Capital Markets

Word count: 7 500 words.

*We are grateful for the suggestions of Gabrielle Demange, Peter Diamond, Fritz Kubler, participants in the Netspar 10th International Workshop of Pension, Insurance and Saving and the members of research seminars at the Paris School of Economics and the Stern School, New York University. The paper has benefited from the excellent research assistance of Tite Yokossi. Any errors are our own. The opinions expressed are those of the authors and do not necessarily reflect the official views of the OECD.
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1 Introduction

One of the puzzles of macroeconomic policy analysis is the extent to which the importance of pay-as-you-go retirement systems varies across countries in the developed world. Table 1, which presents effective tax rates for these systems in the largest countries of the OECD, shows that, in 2002, they varied from 5% in Australia to 33% in Italy.

Analysts have sought to explain this diversity in different ways. Some have argued that it reflects historical differences, such as differences in the degree of social conflict and the power of unions. Others have suggested that it is the mark of cultural factors, some societies being more risk averse than others, and thus more intent on ensuring the stability of retirement incomes.

We seek to demonstrate that differences in this key social parameter—the importance of pay-as-you-go (henceforth paygo) effective contribution rates—are the consequence of rational, welfare maximizing decisions by individuals and societies in countries whose underlying economic characteristics are themselves different. Our hypothesis builds on the Aaron (1966) proposition that a society’s choice between paygo and funded saving should depend on whether the natural rate of growth is greater or less than the rate of return on capital—paygo being optimal in the first case and funded saving in the second. Aaron’s criterion was a knife-edge criterion. We suggest that, in the richer world of stochastic dynamics, it is optimal for societies to choose to rely on both forms of provision (paygo and saving), and that it is the balance of the two that is affected by economic conditions. Simply put, in countries in which labor income is expected to grow slowly and to be subject to recurrent shocks, individuals and society will, ceteris paribus, emphasize personal saving for the provision of retirement income. In countries in which the real return to capital is expected to be low and volatile, they

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1In seeking exogenous explanations for different attitudes toward social policy in the United States and Europe, Alessia and Glaeser (2004) point, among other factors, to the presence of racial tensions in the United States. In their earlier discussion of the divergent evolution of social security provisions in different countries, Bruno and Sachs (1985) emphasized the nature of union pressures.
will, on the contrary, put more weight on paygo transfers.

We use a standard hypothesis testing approach to examine the validity of this rational-economic hypothesis. We first construct a model of how societies and individuals determine their effective paygo tax rate and saving rate. This is a simple, two-period overlapping generations model in which a benevolent public authority and a representative individual jointly choose that tax rate and saving rate which maximize the representative individual’s expected life-time utility. The next step is to construct empirical estimates of the dynamic distributions of labor and capital income in each country. Armed with these distributions, we use the model to generate a cross section of predicted values of tax and saving rates for each country in a reference year, which we take to be 2002. The test of the validity of the model is how well it explains actual, effective paygo tax rates in that year.\(^2\) Though the model describes the dynamic behavior of individuals and societies over time, what it explains is differences in paygo tax rates across countries at a given point in time.

The biggest challenge in this exercise is data construction. In order to analyze the forward implications of saving and paygo provision we must first estimate the distribution of the representative individual’s labor and capital income over a horizon of sixty or more years – active life plus retirement. Moreover, the variability and higher moments of this distribution are decisively important for the computation of expected utility. In order to estimate these, we would ideally like to have historical data on as many life-cycles as possible. The available macroeconomic data, which only stretch back one hundred years at best, do not provide direct evidence of more than two or three lifetimes.

To overcome this obstacle, we resort to a Monte-Carlo approach. Using data since the end of World War II, we estimate simple models of the annual dynamics of labor

\(^2\)We chose 2002 as reference year because it is the first year when comparative and comprehensive data on pension systems in the OECD became available (OECD (2003)). We would like also to test the ability of the model to predict saving rates, but comprehensive, internationally comparable panel data on cohort lifetime savings are not yet available.
and capital income. We then use the parameters of these models, and bootstrapped estimates of the distributions of their error terms, to simulate as many life-time histories as we need. With these, we compute the expected life-time utility of the representative individual corresponding to any pair of paygo tax and saving rates. Our model says that the rates that maximize this expected utility are the rates that society will choose.

We test the model on a subset of the countries of the OECD, focussing on high-income countries which have developed financial markets, and which, though open to trade, are not so small that trade dwarfs domestic production. Specifically, we chose from the 23 developed countries in the OECD in 2003, the eight for which the average value of the ratio of exports plus imports to GDP during the decade of the 1990s was less than 55 percent.3

This paper builds on a long literature about the implications of risk and uncertainty for the efficiency of paygo schemes. Merton (1983), Gordon and Varian (1988), Gale (1990), Demange and Laroque (1999,2000), Demange (2002) and others have shown that a paygo system can enhance welfare in dynamically efficient economies because of the unique opportunity it provides for workers to spread the risk of life-time earnings over different generations. Gottardi and Kubler (2011) extend these analyses and note that the result depends on the degree to which the paygo system reduces capital accumulation. In de Ménil, Murtin and Sheshinski (2005), we derived analytical results relating steady state optimal saving and paygo tax rates (when they exist and are unique) to underlying fundamentals. The present paper uses the same model to obtain empirical estimates of a cross section of economically rational tax and saving rates.

More recently, several papers have used models similar to ours to analyse the properties of public policies designed to provide intergenerational risk sharing. Bohn (2009) 

posits a comprehensive, intergenerational welfare maximization framework, and concludes that social security policies that shift risk from the working years to the the

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3 The countries we selected are Australia, France, Germany, Italy, Japan, Spain, the United Kingdom and the United States. The average ratio of exports plus imports to GDP during the 1990s in these eight countries ranges from 18% in Japan to 53% in the United Kingdom.
years of retirement are preferable to policies that shift risk in the opposite direction. Thogersen (1998) reaches a similar conclusion in an analysis which focuses on the mean and variance of life-time income in both periods combined. Using a model also similar to the one used here, Wagener (2003) argues that this conclusion follows from the use of an ex-ante utility criterion. He shows that the conclusion is reversed if the income of the representative individual in her working years is taken as given, and utility is maximized over expectations of retirement income. Beetsma and Bovenberg (2009) extend this conclusion to economies in which there coexist both paygo and mandatory funded pension systems. They argue that a fixed replacement rule in the funded pillar provides risk sharing benefits which a defined contribution rule can not. Van Hemmert (2005) analyses state-contingent pension transfers. Gollier (2007) abstracts from the uncertainty of labor income in order to focus on the volatility of the return to saving, and shows that pooled pension funds can of themselves provide intergenerational insurance which the kind of individual saving behavior modeled here can not.

Simulation methods have been increasingly used to estimate the welfare effects of either increasing or down-sizing a paygo system such as that of the United States. Krueger and Kubler (2005) posit an economy populated with representative agents living through a life-cycle of nine stages, who face macroeconomic shocks to labor and capital income. They use Monte-Carlo techniques to compute the expected lifetime utility of each cohort of agents. Their focus is on the Pareto optimality of a single experiment – the introduction of Social Security with a 2% tax rate in the United States in 1935. In their most general cases, they conclude that the experiment was not Pareto improving.

Nikishama and Smetters (2007) emphasize individual diversity (neglected in this study and many of those cited). In their model of the U.S. economy, individuals have different skill levels, and are subject to idiosyncratic shocks. Their model does not allow for macroeconomic shocks to the wage rate and the return to capital. They find
that progressively reducing the U.S. paygo system to half the size it began with in 2003 would diminish welfare in the sense of Hicks, unless the down-sizing were accompanied with labor tax reforms that compensated for the insurance against idiosyncratic risk provided by the original paygo system.

Other authors - Dutta, Kappur and Orszag (2000) and Matsen and Thogersen (2004) have used historically observed growth rates and rates of investment return to calculate optimal paygo tax rates and saving rates for different developed countries. However, our Monte-Carlo approach to estimating the relevant moments of these variables is different from theirs. Dutta, Kappur and Orszag (DKO) and Matsen and Thogersen (MT) take decades as representative of life histories.

DKO and MT’s “portfolio approach” further reduces the design of a social security system to a decision regarding the optimal weights to be given, in a fixed portfolio of social security assets, to investment in capital and investment in an asset whose rate of return is equal to the growth of the wage bill. They deal only in rates of growth and thus implicitly assume that the real wage bill follows a random walk with a drift. They do not allow for the possibility that real wages may tend to regress to their long-term trend. As MT themselves point out, in such a world, there is no way to insure against the risk of a bad draw in the lottery for life-time labor income. Their approach thus assumes away one of the principal early arguments in favor of paygo retirement schemes. Our approach is to model the real wage bill as a process of stochastic deviations around a deterministic growth path, and test for the stationarity of that process. When nonstationarity is rejected, as it is in our estimates, paygo can provide intergenerational insurance against the variability of real wage income. We calculate the amount of such insurance which would be optimal for each country.

This paper is structured as follows: Section 2 discusses the empirical evidence of diversity in effective paygo tax rates in the OECD. Section 3 presents a model of how societies and individuals determine paygo tax and saving rates. Section 4 estimates the
annual dynamics of average earnings and the average return to saving from macroeco-
nomic data from 1950 through 2002. This section includes a test of the stationarity of
the wage process. Section 5 aggregates these annual income components into summary
measures of the labor and capital income of the representative agent in the active and
retirement years. In section 6, we derive the tax and saving rates which maximize the
expected utility of the representative individual in each country. Section 7 tests the abil-
ity of these predicted rates to explain the cross-section variation of the effective paygo
tax rates of the OECD countries in our sample. Section 8 addresses two hypothetical
questions: What would the effect on paygo tax rates be of a global convergence of
national capital markets? How would paygo tax rates have differed if individuals had
anticipated the possibility (but not predicted the occurrence) of a crisis like that of 2008?
Section 9 concludes.

2 The Diversity of Effective Paygo Rates

Table 1 presents in country alphabetical order the effective paygo tax rates which
we seek to explain. A comment is in order about the nature of this variable and the way
it is measured. It is well known that social security budgets are often out of balance,
that paygo pensions are frequently partially financed out of general revenue, and that,
as a consequence wage tax rates are a poor measure of the importance of a paygo
system. The answer is to construct alternative, effective paygo tax rates. A country’s
effective rate measures what the wage tax would have to be for the revenue it generates
to finance the paygo disbursements to which that country is committed. 4 Let \( \frac{b}{w} \), be the
average pension an employee is entitled to at retirement – relative to the then average
economy wide wage, and \( \frac{p}{c} \) be the ratio of the number of pensioners to the number of
active employees at that time, then the effective tax rate required to balance the system
is

\[ 4 \text{See Disney (2004).} \]
Appendix A discusses the estimates that we use of \( \frac{b}{w} \), the "relative pension level," and \( \rho \), the dependency rate, in order to construct effective paygo tax rates.

The salient characteristic of the distribution presented in Table 1 is its diversity. The observations form three distinct clusters. Three countries – Australia, the United Kingdom and the United States – have low effective tax rates. At the other end of the spectrum are Italy and Spain – countries whose effective rates are more than double those of the first group. And, in between, is the third cluster – Germany, Japan and France – with effective rates roughly half way between the top group and the bottom group. What explains this pattern? Cultural and historical differences? Or a shared pattern of common response to differences in underlying economic conditions?

3 The Model

The central issue individuals and societies face in constructing a pension system is how to allocate income between the active years and the retirement years. The standard, two-period, overlapping generations model describes the basic relationships. We abstract from individual heterogeneity and assume that life-time saving is determined by a representative individual\(^5\). Where we depart from the standard model, is that we treat the paygo tax rate as an endogenous, collective decision. We posit the existence of a benevolent authority which sets the tax rate. The representative individual and the tax authority are assumed both to know the distributions of capital and labor income.

\[ \theta^e = \frac{b}{w} \frac{p}{\rho} \]  

\(^5\)We implicitly assume that all individuals are identical, and thus neglect one of the important rationales for social security – its potential effect on intragenerational redistribution. Disney and Whitehouse (1993) examine the effect of "opting out" in the U.K. on intragenerational distribution. Pestieau (1999) examines the influence of different paygo formulae on distribution within each cohort. Our representative agent assumption facilitates the analysis of cross-country heterogeneity.
Together, acting behind a "veil of ignorance," before lifetime income is known, they jointly choose the tax and saving rates that maximise expected life-time utility. They assume that the economy is on the steady state path towards which it is tending when their decision is made.

Let \( \omega_A \) be the labor income of the representative individual during her active years, and \( \omega_B \) her pro-rata share of the labor income of the individuals who are working during her retirement years. Each individual provides for her retirement by contributing a portion \( \theta \) of her labor income during her active years to a balanced paygo system, and by saving and investing an amount \( S \) of that income. The budget constraint relating consumption during the active and retirement years, \( c_A \) and \( c_B \), to her labor income, her pension, and her saving is:

\[
    c_A = (1 - \theta)\omega_A - S \quad (2)
\]
\[
    c_B = \theta\omega_B + RS \quad (3)
\]

\( R \) is the gross return on the saving of the working years. The model assumes that the individual plans to consume all her saving during retirement, and does not plan to leave a bequest.

We emphasize the fact that labor income (\( \omega_A \) and \( \omega_B \)) and \( R \) are random variables. The choice of any \( S \) and \( \theta \) by the individual and society therefore entails evaluating

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6We model a decision making process of a consensual kind, in which societies and individuals base their choices on the expected utility of a representative individual over her entire life-cycle. Galassos and Profetta (2002, 2004) and others focus instead on conflicts between categories of the population, notably between the young and the old. The dynamics in their models depend critically on the demographic structure of the population.

7We assume that \( 0 < \theta < 1 \), that liquidity and institutional constraints impose \( S > 0 \), and that \( \theta \) and \( S \) remain constant throughout the individual’s life. As van Hemert (2005) points out, state contingent decisions would improve welfare. Though some governments have enacted state-contingent paygo mechanisms, these are rare.

8The two period structure of the model abstracts from much of the complexity of an optimal consumption path. We assume that all saving is used exclusively for retirement consumption.
the risks associated with that choice. Let

\[ V = E[u(c_A)] + \beta E[u(c_B)] \]  

be the ex-ante expected utility of the representative individual, where \( u(\cdot) \) is the separate utility function applicable to income in each phase of life and \( \beta \) is a discount factor reflecting personal time preference\(^9\). The tax authority and the individual jointly choose \( \theta \) and \( S \) by maximizing \( V \) in (3) subject to (1) and (2). We refer to their choices as \( \theta^* \) and \( S^* \).

We leave the possible feedback from \( S \) to \( R \) and \( \omega \) (through a closed economy aggregate production function, for instance) out of the analysis. One way to interpret this simplification is to argue that free trade determines factor prices up to structural and institutional country-specific effects\(^{10}\).

4 Annual Income dynamics

4.1 Wages and the return to capital

The challenge in using the simple structure of the preceding section is to construct reasonable empirical representations of the life time volatility of labor and capital earnings in these countries. We do so by estimating simple annual models of these income generating processes, and using them to simulate large numbers of hypothetical life histories for each country.

In each country, we use data on annual real GDP from OECD (2005) as a proxy for the real wage bill, \( y_t \)^{11}, and measure the average wage rate as real wages per employee

\(^9\)In empirical work below, we shall assume that \( u(\cdot) \) is a constant relative risk aversion function (CRRA).

\(^{10}\)We will represent these effects with random variables whose distributions vary from country to country. The important simplifying assumption is that these country-specific effects are not influenced by domestic capital accumulation.

\(^{11}\)As do many of the authors we have cited, we use real GDP as a proxy for real total compensation, because it is difficult to obtain long, comparable historical series on total compensation for many countries.
\[ w_t = y_t / e_t. \]  

We identify the annual return to saving in each country with the total real return on stocks, as measured by the broadest stock index incorporating dividends available in each country, which we obtain from Global Financial Data. Monthly stock indices are averaged to obtain an annual index. Real total annual returns, \( r_t \), are calculated by using the national CPI from IMF (2004) to deflate nominal returns\(^{13}\).

We model average annual earnings per employee \( w_t \) as varying around a growth path which converges to a country-specific, long-run trend. Differences in these trend growth rates of labor income\(^{14}\) will be one of the differences conditioning the relative desirability of paygo and funded saving across countries. In order to estimate these trends, and the variability of earnings around them, without bias and efficiently, we must take account of a possible phenomenon of convergence. We allow for this possibility by including slow-down effects represented by the function \( f(t) \) in our equation for the real wage:

\[
\ln w_t = \alpha + gt + f(t) + x_t \tag{5}
\]

\[
\lim_{t \to \infty} f(t) = 0
\]

where \( g \) is the rate of growth on the long-term exponential growth path toward which \( w_t \) converges, \( w_1 = 1 \), and \( x_t \) is the innovation on annual wages.\(^{15}\) Different specifications of the function \( f \) were tested, and two functional forms have been applied.\(^{16}\)

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\(^{12}\)With the help of Mitchell (2003) we are able to extend OECD data back to 1950 for all countries except Australia and Spain, for which we were obliged to begin in 1964 and 1970, respectively.

\(^{13}\)We have experimented with using a measure of the return to saving which incorporates returns to bonds as well as to equities. However, the only bond return data we were able to find on a comparable basis for our eight countries was data on the return to government bonds. These are an inadequate measure of the return to corporate bonds. We do not obtain satisfactory results with mixed portfolios of stocks and government bonds.

\(^{14}\)For these country to country differences in long run wage trends to be consistent with our open economy assumption, it must be the case that countries differ in product (and therefore technology) mix or in the skill composition of the labor force.

\(^{15}\)Setting \( w_1 = 1 \) at the beginning of the active life of each cohort ensures that every cohort in every country, no matter when it is born, views and analyzes its life-time prospects in the same way as every other.

\(^{16}\)For the US, the UK and Australia we used a simple exponential form \( \mu e^{- \rho t} \) and for France, Japan, Spain and Germany, which displayed a larger slowdown in growth from 1950 to 1980, more degrees of freedom were needed. As a consequence, we used the functional form \( \mu \log \left[ 1 - \nu e^{-\rho t} \right] \), which is the solution of non-linearized convergence equations.
We represent the log-returns to equity simply as the sum of a constant mean return \( \bar{r} \) and an innovation \( \epsilon_t \).

\[
\ln(1 + r) = \bar{r} + \epsilon_t
\]

Though we treat the long-term trend of wages and the long-term return on investment as independent of one another, we allow for cyclical interaction between the deviations from long-term values of both factor incomes. We model this interaction with an estimated vector auto-correlation of errors from each income generating process. The wage income process and the variability of investment returns in “normal” times are estimated with data from 1950 through 2002 (we discuss the implications of low probability catastrophes in Appendix B).

Table 2 and the first four columns of Table 3 describe the results we obtain when we estimate (4) and (5). Table 2 shows that, with the non-linear trend specification just mentioned, residuals \( x_t \) pass stationarity tests for all countries at a 5% confidence level and for most countries at a 1% confidence level. This implies, as we emphasized in the introduction, that a paygo system is capable of providing intergenerational insurance against the variability of real wages. The motivation for our model is to explain how different countries choose different tax and saving rates for this purpose.

Table 3 shows that country to country differences in the mean and variability of real investment returns dwarf differences in the growth and variability of wages. The contrast between Spain – where the real return to investment averages 3.4% and has a coefficient of variation of 7.2 – and the United States – where it averages 7.5% and has a coefficient of variation of 1.8 – is striking. Italy resembles Spain. Australia and the United Kingdom resembles the United States. The other countries lie in between.

Our discussion so far has not yet taken account of serial and cross correlations in
the dynamics of wage and investment returns. We estimate a first-order VAR process to model the dynamic interaction between \( x_t \) and \( \epsilon_t \). Specifically, we suppose that

\[
\begin{pmatrix}
    x_t \\
    \epsilon_t
\end{pmatrix} = \begin{bmatrix}
    \rho_{x_t,x_{t-1}} & \rho_{x_t,\epsilon_{t-1}} \\
    \rho_{\epsilon_t,x_{t-1}} & \rho_{\epsilon_t,\epsilon_{t-1}}
\end{bmatrix}\begin{pmatrix}
    x_{t-1} \\
    \epsilon_{t-1}
\end{pmatrix} + \begin{pmatrix}
    \eta_t \\
    \nu_t
\end{pmatrix}
\]

(7)

where \( \eta_t \) and \( \nu_t \) are two white noises. Results are presented in columns 5 - 8 of Table 3.

We find a significant degree of serial correlation of wage innovations - which is consistent with the existence of a business cycle - in all countries (between 0.39 in Australia and 0.85 in the United States). We also find moderate but significant serial correlation of investment returns in four countries with volatile stock markets (Japan, Italy, Spain and Germany). The other four countries all exhibit a significant negative correlation between wage innovations in one year and real stock market returns in the following year. We interpret this lagged, negative feedback as reflecting the dynamics of profit margins over the business cycle.\(^{17}\) In these countries (the U.S., the U.K., France and Australia), the cross correlation transmits the serial correlation observed in wages to investment returns. This turns out to have a significant effect on the stochastic characteristics of the life-time return to investment. The latter is, by its nature, an average product of cumulative annual returns. If the annual returns are serially uncorrelated, the variability of their sum benefits from the Law of Large numbers.\(^{18}\) Positive serial correlation will, on the other hand, amplify the variability of the sum.

As discussed in the next section, our assessment of the variability of labor income in different countries also takes account of differences in the probability of being unemployed. We therefore complete our annual model with the simplest possible repres-

\(^{17}\)Investigating this dynamic further would take us far afield from the focus of our study. Suffice it to say that other scholars analysing comparable data bases have found a similar negative, delayed cross correlation. See Bottazi et al. (1996) in their macroeconomic study of international portfolio decisions. The large absolute size of the estimated cross correlation – when it is significant – reflects the fact that wage deviations are much smaller than investment return deviations.

\(^{18}\)Though aggregate life-time returns are cumulative products of annual growth factors, the logarithm of the former is the sum of the logarithms of the latter.
The presentation of the unemployment rate, $u_t$, which we model as the sum of a constant mean $\pi$ and an innovation $\zeta_t$.

$$ u_t = \pi + \zeta_t $$ (8)

We restrict estimation of equation (8) to the last twenty years of our sample of “normal” years (1982-2002) in order to avoid difficulties associated with the secular increase in unemployment rates in Europe after the first oil shock.

### 4.2 Unemployment

If the risk of unemployment is borne by the affected individual, it can add significantly to the volatility of her wage earnings. A simple example illustrates the point.

Assume that $w$, the annual wage earned by the representative individual when working, is constant. If this individual was never unemployed, her labor income would be constant. Assume, however, that the individual faces a probability $u$ of being unemployed, and receiving no pay. Her expected annual income is, therefore, $(1-u)w$. The variance of annual income is now $w^2 u (1-u)$. This volatility, which is notable during the individual’s active years, does not affect her paygo retirement income. When she is retired and receiving a pension, she is not affected by the specific identity of the unemployed persons in the successor cohort. She simply receives her fraction of the pool of wages being paid at that time. Thus a paygo system transfers income from a period of individual volatility to a period when, because it is pooled, it is more stable. This feature tends to increase its attractiveness.\(^{20}\)

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19We abstract from the heterogeneity which is an important feature of unemployment, and assume that all individuals face the same probability of being unemployed.

20In principle, unemployment insurance could eliminate this individual specific variability of earnings. If individuals who worked paid a tax of $uw$, whose proceeds were transferred to the unemployed, then all individuals would receive $(1-u)w$ in after-tax income, regardless of their employment status. However, the moral hazard created by such a system would be substantial. In principle also, a paygo system which linked pension benefits strictly to life-time earnings would eliminate some of the asymmetry described in the text. In a point system like the German one, or a notional account system such as the one operating in Italy, pension benefits are directly affected by the employment history of the retiree in her active years. In practice, different countries compensate income lost to unemployment to different, partial degrees. For simplicity, we ignore these refinements in our calculations.
The fact that the risk of unemployment is spread over many active years attenuates its effect on the variability of total life-time earnings. But it remains significant. If $T$ is the number of years of active life, the expected life-time income of the individual in the previous example is $T(1 - u)w$. The variance of her life-time income is $Tw^2u(1 - u)$. Its coefficient of variation is $\frac{1}{\sqrt{T}}\sqrt{\frac{u}{1-u}}$, which is 0.05 if $T$ is 40 and $u$ 10%. This individual unemployment volatility is as large if not larger than the volatility which comes from the cyclical movements of average real wages.

5 Life-time income dynamics

The purpose of this section is to show how the annual income histories which we simulate below can be condensed into summary measures of income in the active and retirement years. We proceed by deriving expressions for the average income of the representative individual during her active years and her retirement years.

Redefine $\omega_A$ as her average wage income in a specific life history or trajectory. In a similar manner, redefine $\omega_B$ as her share of the wages on the same trajectory of the individuals who will be working when she is retired. Though $\omega_A$ and $\omega_B$ are given for any specified trajectory, ex-ante, before a specific trajectory has been drawn, they are random variables.

For the moment, assume that the maximum length of a working period is $T$ years and the maximum length of retirement is also $T$ (we will introduce survival probabilities later). Then $^{21}$

$$\omega_A = \sum_{t=1}^{T} \frac{\phi_t w_t}{T} \tag{9}$$

The parameter $\phi_t$ equals 1 if the individual is employed, 0 if she is not, and has the

$^{21}$In the following expression, $w_t$ stands for the real wage of the representative individual in the $t$th year of her active life. We assume (for lack of better information) that this wage does not depend on seniority, and, therefore, that the representative individual earns the economy-wide average wage rate every year.
binomial distribution

\[ P(\phi_t = 0) = u_t \]
\[ P(\phi_t = 1) = (1 - u_t) \]  

(10)

Let \( p_t \) be the number of retired persons, and recall that \( y_t \) is the wage bill. Then:

\[ \omega_B = \sum_{t=T+1}^{2T} \frac{y_t/p_t}{T} \]

(11)

Noting that one can decompose

\[ \frac{y_t}{p_t} = \frac{y_t e_t l_t}{e_t l_t p_t} \]

(12)

where \( e_t \) is total employment, \( l_t \) the labor force, \( w_t = y_t/e_t \) is average annual earnings per employee, \( e_t/l_t = 1 - u_t \), and \( l_t/p_t = 1/d_t \), with \( d_t \) the dependancy ratio (or ratio of retirees to the active labor force), one can rewrite (10) as

\[ \omega_B = \sum_{t=T+1}^{2T} \frac{(1 - u_t)w_t/d_t}{T} \]

(13)

The dependency ratio, \( d_t \), differs from country to country because of differences in the growth rate of the population and differences (due to custom and mortality) in the ratio of retirement years to active years. On any given historical trajectory, \( d_t \) varies as demographic patterns change. But, on a steady state path, \( d_t \) is necessarily constant.22

Consider now the computation of the average, life-time return to saving. We assume, for simplicity, that the purpose of saving is exclusively to provide income during retirement, and we focus on the rate at which annual saving can be transformed into a retirement annuity. If the representative individual saves \( S \) every year of her active life, she will have accumulated

\[ 22 \text{The dependency rates used in our simulations are computed from OECD demographic and labor force participation data for the periods 2042-2062. The projected averages for this period are: Australia – 0.24; France – 0.35; Germany – 0.45; Italy – 0.47; Japan – 0.39; Spain – 0.44; U.K. – 0.29; U.S.A. – 0.24. See Appendix A.} \]
The document states that by the time she retires, the individual will purchase an annuity $A$, which satisfies the following "fair value" constraint:

$$F = A \sum_{t=T+1}^{2T} \frac{\sigma_t}{\prod_{r=t+1}^{T}(1 + r_r)}$$

where $\sigma_t$ is the representative individual’s probability of survival to age $t$. We can write the resulting annuity as

$$A = \left[ \frac{\sum_{t=1}^{T} \prod_{r=t+1}^{T}(1 + r_r)}{\sum_{t=T+1}^{2T} \frac{\sigma_t}{\prod_{r=t+1}^{T}(1 + r_r)}} \right] S$$

or

$$A = RS$$

with $R$ equal to the expression in brackets in (15). Note that $R$ measures both accumulation and annuitization. It is, as before, a random variable.

With these empirical definitions of $\omega_A$, $\omega_B$ and $R$, the budget constraints (1) and (2) can be interpreted as depicting the relationship between average consumption during the representative individual’s active years and average consumption during her retirement years. Our central hypothesis is that, in each country, the representative individual and the tax authority jointly choose the paygo tax rate and the saving rate, $\theta^*$ and $S^*$, which maximize the representative individual’s ex-ante, expected life-time utility.

In calculating this expected utility, the individual and the tax authority take account of the fact that the individual will enjoy it conditional on her continuing to be alive. The survival rates are a function of spending on health, which in turn depends on pension provisions.
and that a positive rate of time preference, $\delta$, leads her to discount more distant instantaneous utilities more heavily. The probability of survival, $\sigma_t$, and the rate of time preference will affect the weight attributed to more distant utilities in the same manner. The planner’s maximand can therefore be written:

$$V = E \left[ \sum_{t=1}^{T} \frac{\sigma_t}{(1 + \delta)^t} u(c_A) + \sum_{t=T+1}^{2T} \frac{\sigma_t}{(1 + \delta)^t} u(c_B) \right]$$

(18)

We assume that $u(c)$ is characterized by constant relative risk aversion, and refer to that parameter as $RRA$. $\theta^*$ and $S^*$ are the values which maximize $V$ in (17).

6 Predicting Tax and Saving Rates

We are now in a position to use (4), (5), (6) and (7) to simulate many life histories of $w_t$, $r_t$ and $u_t$ (we chose to generate 1000). For each history, we draw values at random from empirical distributions of $\eta_t$, $v_t$, and $\zeta_t$ constructed by boot-strapping our estimates of these error terms.

Our objective is to construct values of $\theta^*$ and $S^*$. We assume that risk preferences are the same across countries, and argue that it is country differences in the distributions of labor and capital income which determine these rational paygo and saving rates. The question nonetheless remains: at what value should we fix the common relative rate of risk aversion, $RRA$?

The empirical litterature provides a wide range of estimates. Some are inferred

25In order to remain within the framework of a two-period model, we have to make the simplifying assumption that income and consumption in the active years and income and consumption in the retirement years are equal every year to their respective average values.

26We also assume that $T = 40$, and that $\delta = .02$. We have tested that results are not sensitive to these assumptions.
from survey data on the economic decisions of households. Others are based on the observed responses of individuals to hypothetical or real lotteries. A third and very different body of estimates is derived from macroeconomic time series analyses of the rate of risk aversion implied by observed capital asset pricing.

We choose to set $RRA = 12$, half way between the values suggested by quiz show evidence and the value used by Kruger and Kubler (2005) in their analysis of the U.S. social security system. We then proceed to compute $\theta^*$ and $S^*$, using the algorithm described in Appendix B.

Table 4 presents the values of $\theta^*$ which we obtain, and compares them with the values of effective tax rates, $\theta^e$, reported in Table 1. Appendix B reports alternative values of $\theta^*$ based on a range of different assumptions regarding $RRA$.

### 7 Testing the Model with Historical Data

The test of our rational, economic model is the degree to which $\theta^*$ predicts actual values of $\theta^e$ in a cross-section regression,

$$\theta^e_k = \theta^* + \varepsilon_k,$$

where $k$ is a country index. In Figure 1, we plot $\theta^*$ on the horizontal axis and $\theta^e$ on the vertical axis. There is clearly a strong relationship between the variables. $R^2$ of the

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27 In an early and frequently cited study based on portfolio allocation evidence, Friend and Blume (1975) infer values of $RRA$ between 2 and 3.


29 Beetsma and Schotman (2001) study responses to a popular Dutch television program in which participants are offered a “double or nothing” choice on cumulated winnings often exceeding $5000. Their simplest and most basic, utility maximization model leads to an estimate of $RRA = 7$.

30 Campbell (2003), who uses national financial data similar to ours to estimate consumption-based asset pricing models, finds implied values of $RRA$ between 50 and 600 for several of the countries in our sample.
linear regression drawn in the Figure is .845. This result confirms our basic hypothesis that objective, economic differences explain the diverse pattern of effective paygo tax rates in our sample.

However, the regression implies that actual rates are too sensitive to predicted rates. In order to fit the highest actual rates (Italy and Spain) the slope of the relationship has to be excessively high. We conjecture that the underlying relationship is non-linear, because weak capital market conditions and low saving rates are correlated with weak regulatory infrastructure – poor policing of insider trading, inadequate protection of minority stockholder rights, etc. These institutional effects further discourage reliance on capital markets, and promote paygo retirement provision. Because they are not accounted for in our model, but are correlated with its explanatory variable, these factors cause paygo provisions at high levels of $\theta^*$ to be higher than the the linear model predicts.

On the basis of this argument we estimate the following Poisson relationship:

$$\theta_k^* = \exp(a + b\theta_k^*) + \varepsilon_k$$  \hspace{1cm} (20)

Predicted and actual values are plotted in Figure 2. In the Poisson model, the estimated slope is 1.0 – as posited in equation (19) – when $\theta_k^*$ is low, and increases as it rises.\textsuperscript{31} The Poisson model also appears to fit the observations better than the linear version.\textsuperscript{32} For both reasons, we prefer the Poisson model.

Appendix C discusses the implications of the Great Recession of 2008 for our rational economic model and its predictions.

\textsuperscript{31}In the Poisson model, the slope is 1.0 at $\theta_k^* = 6.6$, 1.7 at $\theta_k^* = 10$, and higher for higher values. In the linear model, the slope is everywhere 3.7

\textsuperscript{32}Although the F test of the linear model and the Chi2 test of the Poisson model are not strictly comparable, the fact a RMSE corrected for degrees of freedom, computed for the Poisson case, is smaller than the RMSE of the linear estimate suggests that overall goodness of fit is better in the Poisson case.
8 Further Results

8.0.1 The Integration of Financial Markets

If the capital markets of the large OECD countries we consider were to become fully integrated, what would the consequences be for national patterns of optimal paygo tax and saving rates? The fact that capital returns have been the most important source of observed international differences in income dynamics in the post-War period makes this a particularly relevant question.

We address the issue by performing the following thought experiment. Suppose that our eight countries continue to have distinct and separate labor markets, but that they share a common, pooled capital market. Assume further, for simplicity, that the stochastic characteristics of the return to investment in that pooled capital market are the same as those we have observed for the United States. It is straightforward to compute the predicted, optimal, steady state, paygo tax and saving rate for each country in such a world simply by replacing its own distribution of capital returns by that of the United States.\(^{33}\) The result is summarized in Table 6. Not surprisingly, the pattern of retirement provision does converge in the eight countries. Access to the relative stability of a global capital market would bring about declines in paygo rates which vary inversely with the previous volatility of national capital markets.

8.0.2 Global Crises

We assume that in 2002 individuals and the pension authority did not allow for the possibility of crises like that which unfolded in 2008. If they had, would they have made the pension commitments implicit in the predictions of section 6? Clearly, the

\(^{33}\)The radical nature of this hypothetical exercise should be emphasized. It abstracts from all of the institutional differences that characterized OECD capital markets in the second half of the twentieth Century. It also assumes away national heterogeneity of capital. For the real rate of return to be identical across countries, they would all have to share a common capital market. In addition, those which had common currencies or were linked by fixed exchange rates would have to share a common inflation rate, and exchange rate depreciation would have to exactly compensate for inflation differences between countries with floating rates (these inflation assumptions are necessary to ensure that real as well as nominal returns are equated.)
realization of such a possibility would have reduced expectations of life-time utility. But, even though faced with a more somber future, each society would still have had to choose the best possible balance between paygo and saving. Would awareness of the possibility of a crisis have affected those calculations? Calculations in Appendix C suggest that paygo rates would not have been fundamentally altered.

9 Conclusion

The central result of this paper is that a rational economic model of how societies set paygo tax rates replicates the diverse pattern of effective paygo rates in large OECD countries at the beginning of the 21st Century. The model is a simple, two-period OLG model, in which a representative individual and a benevolent tax authority jointly choose the tax rate and the saving rate which maximizes the expected life-time utility of the representative individual. We assume that the individual and the tax authority both have complete knowledge of the distribution of labor and capital income over the individual’s life time. We construct this distribution by estimating annual equations for the wage rate and the return to capital, and using them to simulate large numbers of life histories. Taking expectations over these life histories, we compute the expected life-time utility of the representative individual as a function of the paygo tax and saving rates. The model predicts that society will choose the tax and saving rate which maximize the individual’s expected life-time utility.

We find that societies in which capital markets are relatively stable, and offer rates of return well in excess of the rate of growth of labor income, tend to have high saving rates and moderate paygo tax rates. By contrast, societies in which capital markets are relatively volatile, and the growth of labor income is high and stable, choose high

34Data on life-time saving rates, which are necessarily estimated by cohort rather than by calendar year, exist in only a few countries. On the other hand, effective paygo tax rates can be calculated for every country in our sample. Therefore the only comparison we are able to make between computed optimal rates and actual rates refers to paygo taxes.
paygo tax rates and low saving rates. These considerations alone – the most important of which are differences in the mean and variability of the return to capital – explain 85% of the cross section variance of observed effective paygo tax rates in 2002. If one considers that the dependent variable is constructed from detailed analyses of legislative provisions and demographic trends, and the independent variable is inferred from statistics on wage and capital return data going back 50 years, the probability that this relationship reflects a chance correlation is low.
A Constructing Estimates of Effective Paygo Tax Rates in 2002

Following Disney (2004), we note that the condition for budgetary balance of each national system implies that the effective tax rate for that country is

\[ \theta^e = \frac{b}{w} \frac{p}{e}. \]  

(A-1)

Of the two parameters on the right hand side, the more difficult to estimate is \( \frac{b}{w} \), "the relative pension level", because it depends on numerous, detailed, country-specific regulations. We use estimates of this parameter first published in 2003 in a new OECD publication, Pensions at a Glance. Noting that laws in effect at a given moment generally entail future commitments to increases or reductions of benefits spread out over many years, the OECD computes, on the basis of legislation in effect in 2002, the average pension which an employee who entered the labor force that year, and subsequently fulfilled all relevant working requirements, would be entitled to at the statutory retirement age. To the extent that a social security system is contributory, different individuals (all of whom have met all the work requirements) will receive different pensions, because of differences in their life-time earnings. The OECD therefore takes the average of individual pension entitlements over the range which it projects for life-time earnings. It presents this average pension as a percentage of its projection of the economy wide average wage during the years when the individual in question will be in retirement.

The forward looking nature of the OECD measure of \( \frac{b}{w} \) requires for consistency that our measure of \( \frac{L}{e} \) also be forward looking. It should reflect the balance between retirees and workers for the duration of the retirement of the cohort whose entitlements we are estimating. We base our estimate of \( \frac{L}{e} \) in each country on OECD projections of the number of retirees and the number of active employees in each country between 2042 and 2062.

We adjust for the statutory nature of the OECD’s measure of relative pension levels by recognizing that many individuals who are working at the time they retire will nonetheless not receive a full, potential pension, because they have incurred spells of maternity leave, unemployment, or other interruptions from active work. We estimate that the average, normal pension is 20 % lower than the potential level calculated by the OECD. As in Disney (2004), we further argue that persons who were not employed

\[ 35 \text{OECD (2003) presents projected pension levels for men and not for women. Though, in all the countries} \]
during the last decade of their working lives receive half of a normal pension, this
additional discount corresponding to average widow and survivor provisions.

The effective tax rates thus obtained are not current tax rates. They reflect the
future commitments that legislation currently in effect implies for the cohort entering
employment, and measure the burden of those commitments on those who will be
employed during this cohort’s retirement. Their forward looking nature corresponds
to the forward looking nature of the collective decision process hypothesized in our
model.

B Simulation Algorithm

We construct measures of the rational, economic, effective paygo tax rates which our
model predicts by using the annual equations for labor and capital income estimated in
section 4 to simulate numerous life cycle histories. The algorithm has seven steps:

1. At each step \( j \), we draw a full history, \( 0 < t < 2T \), from the empirical distributions
   of \( \hat{\eta}_t, \hat{\nu}_t \) and \( \hat{\zeta}_t \). We use the first two to construct a sample of innovations
   \((x_t^{(j)}, t_t^{(j)})_{2T \geq t \geq 0}\) using the vector auto-regression (6).

2. Capital returns are simply deduced from (5), whereas annual wages in the steady-state regime are given by

\[
\ln w_t^{(j)} = \hat{a} + \hat{g}t + x_t^{(j)}
\]  

(21)

The convergence of earnings is assumed completed.

3. Using (8) and (12), we compute \( \omega_A^{(j)}, \omega_B^{(j)} \), allowing the unemployment rate to
   have the differential effects described in these equations. We use (15) to compute
   \( R(j) \).

4. We select values of the saving and tax rates \((\theta, S)\), and compute \( c_A^{(j)}, c_B^{(j)} \) and the
   lifetime utility associated with that history and those values of \((\theta, S)\),

\[
\sum_{t=T+1}^{2T} \frac{\sigma^j}{(1+r)^t} u(c_A^j) + \sum_{t=T+1}^{2T} \frac{\sigma^j}{(1+r)^{t+1}} u(c_B^j).
\]

In our sample, statutory pension provisions are the same for women as for men, the actual pensions women
receive are lower than those received by men, because their annual earnings are lower. Much of our discount
reflects the lower average career earnings of women.
5. We go to step 1 and loop 1000 times.

6. We compute expected utility $V$ for this pair $(\theta, S)$, using (17).

7. We scan over the space $1 > \theta, S > 0$, allowing each variable to increase in steps of 0.01. The values $\theta^*$ and $S^*$ which maximize $V$ are the values which the model predicts that society and the individual will adopt.36

The values of $S^*$ and $\theta^*$ depend on $RRA$. Table B1 displays the values obtained when $RRA$ varies between 5 and 20. The last row in the Table, under the predicted rates, gives the cross-section correlation of these predicted rates with the actual, effective rates in the first column. The correlation is high for a wide range of values of relative risk aversion. This suggests that the goodness of fit of our rational-economic model is not very sensitive to $RRA$.

**C The Implications of the Great Recession of 2008**

This appendix examines the possible effects on paygo tax and saving rates of incorporating the previously ignored probability of a crisis. Using Barro and Ursua (2008) and Barro (2009), we infer a binomial variable with a known probability of realization which simultaneously generates two outcomes - a decline of GDP (our proxy for wage income) and a drop in the return to capital.37 In steps 1 and 2 of the algorithm in Appendix B, we add draws from this binomial distribution to the values of $\tilde{x}_t$ and $\tilde{\varepsilon}_t$ derived from the population of shocks previously estimated from data from 1950 to 2002. Each of the eight countries is characterized by its own population of "normal" shocks $\tilde{x}_t$ and $\tilde{\varepsilon}_t$. The binomial distribution describing crises, which is added to those "normal" shocks, is the same across countries (we only have one estimate), but we continue to simulate independent life histories for each country.

36We also use this simulation algorithm to calculate quasi confidence intervals, conditional on the choice of $RHA$, for the values of $\theta^*$ and $S^*$ obtained. We take the maximum value of the estimated life-time utility of the representative agent in each country, and trace pairs of paygo tax and saving rates that cause expected utility to be 99% of that maximum value. The exercise shows that the estimated utility function is relatively flat, and that variations of +/- 10% of $\theta^*$ are within this 99% confidence interval.

37Specifically, we refer to the data in Barro and Ursúa (2008) on macroeconomic crises since 1870. According to Table 8 p. 279, 70 crises have taken place in 17 OECD countries in the 136 years of history examined, implying a probability of crisis equal to 3.0%. The corresponding average decline in GDP per capita was 17.4% (Table 9 p. 283). We estimate the shock to the stock price to be 22.9%, which is the average decline among OECD countries during GDP crises as calculated from Table C2 p. 323.
Comparison of Table C1 with Table 4 in the text suggests that incorporating the expectation of low probability periodic crises does not substantially change the calculus of rational individuals and authorities. The new predicted paygo tax rates are similar to the old, though they are everywhere slightly higher. Though expected utility declines, the trade-off between paygo and saving remains substantially the same.

References


[38] OECD, unpublished projections of laborforce and participation rates by country.


