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Abstract: In this paper, we study the convergence of 40 main bilateral real exchange rates (RERs) constructed by actual price levels to be consistent with absolute purchasing power parity (PPP), rather than by price indexes as used in popular studies. The time series ADF and KPSS unit root tests reveal that 39 RERs are stationary in their periods of about 60 years or less. The half-lives span from 1 to 40 years and are mostly outside of the consensus range of 3–5 years. Some cases suggest that the half-life may be not an appropriate method for measuring the convergence of absolute PPP. In contrast, some statistic indexes (e.g., root mean squared error) can be applied for this use and may have more power than the half-life.

Keywords: absolute purchasing power parity; real exchange rate; convergence; half-life

JEL Classification: F30; F31

1. Introduction

As an important aspect of understanding the theory of purchasing power parity (PPP), the parity reversion of PPP has been much studied (e.g., Frankel and Rose, 1996; Lothian and Taylor, 1996; Taylor et al., 2001; Koedijk et al., 2004; Bergin et al., 2014; Huang and Yang, 2015). Rogoff (1996) and Macdonald (2007, Chapter 2, Purchasing power parity and the PPP puzzle) conduct a survey. In these popular studies, the real exchange rate (RER) is constructed by the consumer, wholesale, and producer price indexes rather than actual price levels. Such a constructed RER is used in studying relative PPP rather than absolute PPP (Cheung et al., 2005, p. 1153). That is, though the parity reversion of relative PPP is well studied, that of absolute PPP has some blanks.

Thus, we construct the RER by actual price levels and study the convergence of absolute PPP. First, we wonder what the convergence of absolute PPP is (and how the convergence of absolute PPP is different from that of relative PPP). To examine the half-life, different econometric dimensions (time series, panel, whole period, and sub-period) are used. Second, after comparing different half-lives, we wonder whether the half-life is an appropriate method in measuring the validity of absolute PPP. At last, a new method that may be more powerful than the half-life is proposed.

The rest of the paper proceeds as below. Section 2 presents the definition and data. Section 3 presents the main econometric result of the half-life. Section 4 analyzes how the logarithmic form and the lagged difference terms influence the calculated result of the half-life. Section 5 analyzes whether the half-life is an appropriate method. Section 6 gives the application of the new proposed method. Finally, Section 7 concludes the paper.

2. Definition and data

In this paper, RER is defined by Eq. (1), where P_i is the domestic (general) price level of country i, P^* is the price level of the specific foreign country (in this paper, the United States), PPP_i rate is

P_i divided by P^*, and NER_i is expressed as the domestic currency units per fixed foreign currency unit (the domestic currency units per US dollar). In this definition, a greater value of *RER* represents the local currency's appreciation (against the US dollar), and the value of *RER* will be equal to 1 if absolute PPP strictly holds.

$$RER_i = \frac{PPP_i}{NER_i} = \frac{P_i/_{P^*}}{NER_i} = \frac{P_i}{NER_i \times P^*} \quad (1)$$

We follow Zhang and Zou (2014) in collecting the data. That is, all data are from University of Pennsylvania's Penn World Table (PWT) 7.1 online database and the World Bank's World Development Indicators (WDI) online database. We first sequence all the global countries by their GDPs (in constant 2005 US dollars) in 2012 from the WDI database, and then choose the largest 41 among them; the GDP of each country represents greater than 0.3% of the world GDP. These bilateral RERs are of the 40 largest countries (listed in Table 1) against the United States. The PWT supplies the RERs (the variable "p" in the database divided by 100) for the period 1950–2010. The WDI supplies the RERs (the variable "PPP conversion factor to official exchange rate ratio" in the database) for 1980–2012. But the concrete values of RERs in the two databases are not completely the same. Thus, we combine a RER's value in 2010 in the PWT and its growth ratio in 2011–2012 in the WDI to obtain the consistent values in 2011–2012. Such obtained values of RERs in 2011–2012 and those in 1950–2010 in the PWT constitute the total values in the whole period 1950–2012. Though the longest sample is 1950–2012, for concrete countries, the available samples are shorter because of some blank data in some years; see Table 1.

Table 1. Countries and their sample periods.

1950–2012	Other periods
Australia, Austria, Belgium, Canada, Brazil, Colombia, Denmark, Finland, France, India, Ireland, Italy, Japan, Mexico, Netherlands, Nigeria, Norway, Portugal, South Africa, Spain, Sweden, Switzerland, Thailand, Turkey, UK, Venezuela	Chile and Greece: 1951–2012, China: 1952–2012, Germany: 1970–2012, Hong Kong, Indonesia and Singapore: 1960–2012, Israel: 1950–2011, Korea: 1953–2012, Malaysia: 1955–2012, Poland: 1970–2012, Russia: 1990–2012, Saudi Arabia: 1986–2012, United Arab Emirates: 1986–2011

Notes: For China (mainland), version 1 in the PWT is used. Hong Kong refers to Hong Kong SAR.

3. Half-life in absolute PPP

In this section, we will investigate (1) the half-life in each RER's whole period in Section 3.1, (2) the half-life in each RER's sub-period in Section 3.2, and (3) the half-life in the panel data in Section 3.3.

3.1. Half-life in the whole period

As the ADF test tends to accept the null hypothesis that a variable has a unit root (Kwiatkowski, Phillips, Schmidt, and Shin, briefly KPSS, 1992, p. 160), we also use the KPSS test that has a contrary null hypothesis and can overcome such fault of the ADF test in some degree. Concretely, we first use the ADF test. If an RER is confirmed as I(0) by one model specification (with constant, or with constant and trend, or without constant) at one usual level (10%, or 5%, or 1% level), we deem it is I(0). If an RER is not confirmed by the ADF test but by one specification of the KPSS test (with constant or with constant and trend) at one usual level, we also deem it is I(0). If an RER is not confirmed as I(0) by both the ADF and KPSS tests, we deem it is not I(0). The detailed unit root test results are listed in Appendix Table 1, whose conclusions are listed in Table 2.

After an RER is confirmed as I(0), we use Eq. (2) to estimate the coefficient β, where the Newey-West heteroskedasticity and autocorrelation consistent standard error is used. The half-life is calculated by using the most basic form (half-life = log(0.5) / log(β)).[1] The results are displayed in Table 2. Each β is significant at the 1% level, which is consistent with that in Lothian and Taylor (1996, p. 501) who study relative PPP. In the United Arab Emirates, though the KPSS test indicates the RER is I(0), β is greater than 1 and the RER does not converge; this may be caused by the low power of the unit root test. Except in the United Arab Emirates, all the other 39 RERs converge, with the half-lives spanning from 1 to 40 years. Concretely, 3 half-lives are shorter than 3 years, 10 half-lives are within the range of 3–5 years, and 26 half-lives are greater than 5 years.

$$RER_t = \alpha + \beta RER_{t-1} + u_t \quad (2)$$

Table 2. Stationarity and the half-life for each country.

Country	RER	β	half-life	Country	RER	β	half-life
Australia	$I(0)_{KPSS}$	0.950	13.5	Korea	$I(0)_{ADF}$	0.842	4.0
Austria	$I(0)_{ADF}$	0.946	12.5	Malaysia	$I(0)_{KPSS}$	0.933	10.0
Belgium	$I(0)_{ADF}$	0.913	7.6	Mexico	$I(0)_{KPSS}$	0.851	4.3
Brazil	$I(0)_{KPSS}$	0.923	8.7	Netherlands	$I(0)_{KPSS}$	0.948	13.0
Canada	$I(0)_{KPSS}$	0.897	6.4	Nigeria	$I(0)_{KPSS}$	0.858	4.5
Chile	$I(0)_{ADF}$	0.825	3.6	Norway	$I(0)_{KPSS}$	0.970	22.8
China	$I(0)_{KPSS}$	0.983	40.4	Poland	$I(0)_{KPSS}$	0.885	5.7
Colombia	$I(0)_{ADF}$	0.869	4.9	Portugal	$I(0)_{KPSS}$	0.963	18.4
Denmark	$I(0)_{KPSS}$	0.944	12.0	Russia	$I(0)_{ADF}$	0.718	2.1
Finland	$I(0)_{ADF}$	0.905	6.9	Saudi Arabia	$I(0)_{KPSS}$	0.863	4.7
France	$I(0)_{ADF}$	0.870	5.0	Singapore	$I(0)_{KPSS}$	0.945	12.3
Germany	$I(0)_{ADF}$	0.760	2.5	South Africa	$I(0)_{ADF}$	0.767	2.6
Greece	$I(0)_{ADF}$	0.942	11.6	Spain	$I(0)_{ADF}$	0.957	15.8
Hong Kong	$I(0)_{KPSS}$	0.953	14.4	Sweden	$I(0)_{KPSS}$	0.899	6.5
India	$I(0)_{KPSS}$	0.952	14.1	Switzerland	$I(0)_{KPSS}$	0.968	21.3
Indonesia	$I(0)_{KPSS}$	0.893	6.1	Thailand	$I(0)_{ADF}$	0.868	4.9
Ireland	$I(0)_{ADF}$	0.961	17.4	Turkey	$I(0)_{KPSS}$	0.819	3.5
Israel	$I(0)_{ADF}$	0.617	1.4	U. A. Emirates	$I(0)_{KPSS}$	1.140	none
Italy	$I(0)_{ADF}$	0.943	11.8	UK	$I(0)_{ADF}$	0.898	6.4
Japan	$I(0)_{KPSS}$	0.956	15.4	Venezuela	$I(0)_{KPSS}$	0.849	4.2

Notes: $I(0)_{ADF}$ and $I(0)_{KPSS}$ indicate that the I(0) is obtained from the ADF and KPSS tests respectively.

Thus, two conclusions are obtained.

(1) In relative PPP studies (where the RER is constructed by price indexes), a short-run (e.g., several decades) RER is often not stationary when the time series ADF test is used; to have an RER stationary, one often needs long-run (e.g., more than 100 years) data, or otherwise, panel

[1] Chortareas and Kapetanios (2013) discuss different measures in calculating the half-life, but they (p. 438) acknowledge that in the majority of papers dealing with half-lives the measure is inextricably linked to the AR(1) model of Eq. (2).

data or another special unit root test if the data is not long enough. But in absolute PPP of this survey (where the RER is constructed by actual prices) where each RER is about 60 years or less, all the RERs are stationary by the ADF or KPSS test. One eminent example is Russia where the period is 1990 through 2012, but the ADF test with constant and trend confirms that its RER is I(0) at the 1% level.

(2) In relative PPP studies, the "consensus view" of half-life is 3 to 5 years (Rogoff, 1996). In absolute PPP of this survey, however, only 10 (out of 39) RERs have a half-life within 3–5 years; the other 29 (out of 39) RERs have a half-life smaller or greater than 3–5 years. This means that the consensus view of the half-life does not hold in this survey and the convergence of absolute PPP may be very different from that of relative PPP.

3.2. Half-life in the sub-period

For some countries whose whole period is 1950–2012, the exchange rate system changed in the period. The most important change is the move from fixed to flexible exchange rate in major industrial countries in 1973. This change also influences the developing countries to some extent. Thus we divide each period that begins from the 1950s or 1960s into two periods, before and after 1973, for 35 countries. The periods in Germany, Poland, Saudi Arabia, Russia, and the United Araba Emirates are not divided because their periods begin from the 1970s or even later years. The calculated half-lives for the 35 countries, together with the unit root test conclusions for these countries, are listed in Table 3. The detailed unit root test results are listed in Appendix Table 2.

Table 3. Stationarity and the half-life for 35 countries in their sub-periods.

Country	1950–1972			1973–2012		
	RER	β	half-life	RER	β	half-life
Australia	$I(0)_{KPSS}$	0.905^{***}	6.9	$I(0)_{KPSS}$	0.869^{***}	4.9
Austria	$I(0)_{KPSS}$	0.706^{**}	2.0	$I(0)_{ADF}$	0.813^{***}	3.3
Belgium	$I(0)_{KPSS}$	1.138^{***}	none	$I(0)_{ADF}$	0.794^{***}	3.0
Brazil	$I(0)_{ADF}$	0.929^{***}	9.4	$I(0)_{KPSS}$	0.952^{***}	14.1
Canada	$I(0)_{KPSS}$	0.651^{***}	1.6	$I(0)_{KPSS}$	0.924^{***}	8.8
Chile	$I(0)_{KPSS}$	0.796^{***}	3.0	$I(0)_{ADF}$	0.748^{***}	2.4
China	$I(0)_{KPSS}$	0.846^{***}	4.1	$I(0)_{ADF}$	0.924^{***}	8.8
Colombia	$I(0)_{ADF}$	0.860^{***}	4.6	$I(0)_{KPSS}$	0.950^{***}	13.5
Denmark	$I(0)_{KPSS}$	1.084^{***}	none	$I(0)_{ADF}$	0.791^{***}	3.0
Finland	$I(0)_{ADF}$	0.624^{***}	1.5	$I(0)_{ADF}$	0.752^{***}	2.4
France	$I(0)_{ADF}$	0.514^{***}	1.0	$I(0)_{ADF}$	0.747^{***}	2.4
Greece	$I(0)_{ADF}$	0.492^{*}	1.0	$I(0)_{KPSS}$	0.902^{***}	6.7
Hong Kong	$I(0)_{KPSS}$	0.722^{***}	2.1	$I(0)_{KPSS}$	0.917^{***}	8.0
India	$I(0)_{KPSS}$	0.808^{***}	3.3	$I(0)_{KPSS}$	0.945^{***}	12.3
Indonesia	$I(0)_{ADF}$	0.08	none	$I(0)_{KPSS}$	0.915^{***}	7.8
Ireland	$I(0)_{ADF}$	0.855^{***}	4.4	$I(0)_{ADF}$	0.910^{***}	7.3
Israel	$I(0)_{KPSS}$	0.609^{**}	1.4	$I(0)_{KPSS}$	0.764^{***}	2.6
Italy	$I(0)_{ADF}$	0.998^{***}	346.2	$I(0)_{KPSS}$	0.853^{***}	4.4
Japan	$I(0)_{KPSS}$	0.999^{***}	692.8	$I(0)_{KPSS}$	0.838^{***}	3.9
Korea	$I(0)_{KPSS}$	0.583^{***}	1.3	$I(0)_{ADF}$	0.777^{***}	2.7
Malaysia	$I(0)_{ADF}$	0.869^{***}	4.9	$I(0)_{KPSS}$	0.943^{***}	11.8

Country	1950–1972			1973–2012		
	RER	β	half-life	RER	β	half-life
Mexico	$I(0)_{KPSS}$	0.843^{***}	4.1	$I(0)_{KPSS}$	0.799^{***}	3.1
Netherlands	$I(0)_{KPSS}$	1.118^{***}	none	$I(0)_{ADF}$	0.777^{***}	2.7
Nigeria	$I(0)_{KPSS}$	1.091^{***}	none	$I(0)_{KPSS}$	0.779^{***}	2.8
Norway	$I(0)_{KPSS}$	1.029^{***}	none	$I(0)_{KPSS}$	0.875^{***}	5.2
Portugal	$I(0)_{ADF}$	0.580^{***}	1.3	$I(0)_{KPSS}$	0.912^{***}	7.5
Singapore	$I(0)_{KPSS}$	0.450^{**}	0.9	$I(0)_{ADF}$	0.898^{***}	6.4
South Africa	$I(0)_{ADF}$	0.730^{***}	2.2	$I(0)_{ADF}$	0.716^{***}	2.1
Spain	$I(0)_{ADF}$	0.704^{***}	2.0	$I(0)_{ADF}$	0.866^{***}	4.8
Sweden	$I(0)_{KPSS}$	1.027^{***}	none	$I(0)_{ADF}$	0.729^{***}	2.2
Switzerland	$I(0)_{KPSS}$	1.215^{***}	none	$I(0)_{ADF}$	0.805^{***}	3.2
Thailand	$I(0)_{ADF}$	0.680^{***}	1.8	$I(0)_{KPSS}$	0.897^{***}	6.4
Turkey	$I(0)_{KPSS}$	0.785^{***}	2.9	$I(0)_{KPSS}$	0.789^{***}	2.9
UK	$I(0)_{KPSS}$	0.812^{***}	3.3	$I(0)_{ADF}$	0.764^{***}	2.6
Venezuela	$I(0)_{ADF}$	0.666^{***}	1.7	$I(0)_{KPSS}$	0.878^{***}	5.3

Notes: *, **, and *** indicate that β is significant at the 10%, 5%, and 1% level, respectively. For some countries, the first year in the period 1950–1972 is not 1950, and/or the last year in the period 1973–2012 is not 2012; see Table 1 for details.

The econometric results in the sub-periods confirm the conclusions that we just obtained from the whole periods in Section 3.1.

(1) As in the whole period, in each sub-period, the RER is stationary by the ADF or KPSS test. This indicates that the stationarity of a RER of absolute PPP may be easily obtained no matter whether it is in major industrial countries' fixed exchange rate period (1950–1972, less than 25 years) or in major industrial countries' flexible exchange rate period (1973–2012).

(2) As in the whole period, in the sub-period, only a few (no more than one third) half-lives are within 3–5 years, and most half-lives are out of the range of 3–5 years. We first examine the period 1950–1972. For Indonesia, β is not significant even at the 10% level, and we think that the half-life does not exist. In addition, β is greater than 1 for 7 RERs and the half-life does not exist for these RERs either. Thus, the half-life exists in 27 (out of 35) RERs. Among the 27 RERs, 8 half-lives are within 3–5 years and 19 half-lives are smaller or greater than 3–5 years. Then we examine the period 1973–2012. In this period, the half-life exists in all the 35 RERs. Among the 35 RERs, 9 half-lives are within 3–5 years and 26 half-lives are out of the range of 3–5 years.

It should be noted that in the sub-period 1950–1972 there are 7 special RERs (the RERs for Belgium, Denmark, the Netherlands, Nigeria, Norway, Sweden, and Switzerland). For these RERs, though the ADF or KPSS test confirms that they are stationary, the βs are greater than 1 and the half-lives do not exist. In the whole period, we also find one such RER that is stationary but does not converge (the RER for the United Araba Emirates). This phenomenon may be caused by the different model specifications in the unit root test and in Eq. (2) .

3.3. Half-life in the panel data

After knowing the half-life in each country in the time series dimension, we turn to the panel data dimension, which can give a general or averaged measure of the 40 RERs.

In the whole period 1950–2012, a summary of panel unit root tests (LLC, IPS, ADF-Fisher and

PP-Fisher) reveals that the RER is I(0). Then we estimate Eq. (2) (its panel data form) by the two-way fixed effects method which is justified by the redundant fixed effects tests. β = 0.904 with the *p*-value = 0, and the half-life = 6.9.

In the sub-period 1950–1972, the PP-Fisher test reveals that the RER is I(0). Then we estimate Eq. (2) by the two-way fixed effects method which is justified by the redundant fixed effects tests. β = 0.762 with the *p*-value = 0, and the half-life = 2.6. In the sub-period 1973–2012, the IPS and PP-Fisher tests reveal that the RER is I(0). Then we estimate Eq. (2) by the two-way fixed effects method which is justified by the redundant fixed effects tests. β = 0.840 with the *p*-value = 0, and the half-life = 4.0.

Thus, two conclusions can be obtained. (1) As in the time series cases, the RER can still be confirmed as being stationary (by at least one test) whether in the whole period or in the sub-period. (2) Though the half-life is within 3–5 years in one case of the sub-period 1973–2012, the half-life is out of the range of 3–5 years in two cases of the whole period and the sub-period 1950–1972, thus the half-life in the panel data still tends to be out of the range of 3–5 years.

4. Does the logarithmic form or lagged difference term matter?

In popular studies (e.g., Lothian and Taylor (1996) and Koedijk et al. (2004)) the variable is used in its logarithmic form; but we follow Zhang and Zou (2014, p. 828) (who argue that using the original form is more direct than using the logarithmic form in testing the validity of absolute PPP) and use the original form in Section 3. In addition, in calculating the coefficient of β in Eqs. (4)–(5), Lothian and Taylor (1996, p. 501) use Eq. (4) where lagged difference terms are not included, while Koedijk et al. (2004, p. 1091, Table 2) use Eq. (5) where lagged difference terms are included. Thus, in this section, we analyze how the half-lives differ across the different model specifications, Eqs. (2)–(5). As in Section 3.1, we also use the whole period for each RER.

First, we examine the time series case. In Table 4, each logarithmic RER in Eqs. (4) and (5) is also confirmed as I(0) by the ADF or KPSS test. In Eqs. (3) and (5), the number of lagged difference terms is set to be one, which is mostly supported by the Schwarz information criterion. The Newey-West heteroskedasticity and autocorrelation consistent standard error is used in estimating each equation. Except in Eq. (5) for Russia where β is not significant even at the 10% level, each β is significant at the 1% or 5% level. For the United Arab Emirates, though the KPSS test indicates that both RER and log(RER) are I(0), β in each equation is greater than 1, thus the half-life does not exist. For all the other countries, β in each equation is smaller than 1 and the half-life exists.

$$RER_t = \alpha + \beta RER_{t-1} + \sum_{k=1}^{n} \gamma_k \Delta RER_{t-k} + u_t \quad (3)$$

$$log(RER)_t = \alpha + \beta log(RER)_{t-1} + u_t \quad (4)$$

$$log(RER)_t = \alpha + \beta log(RER)_{t-1} + \sum_{k=1}^{n} \gamma_k \Delta log(RER)_{t-k} + u_t \quad (5)$$

Table 4. Half-lives derived from different equations.

Country	half-life				Country	half-life			
	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)		Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)
Australia	13.5	8.0	10.8	8.4	Korea	4.0	4.3	4.1	4.3
Austria	12.5	10.5	13.5	12.7	Malaysia	10.0	9.6	11.0	10.2
Belgium	7.6	5.2	8.4	5.8	Mexico	4.3	3.7	4.1	3.6

Country	half-life				Country	half-life			
	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)		Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)
Brazil	8.7	5.1	8.2	5.0	Netherlands	13.0	10.7	15.1	14.1
Canada	6.4	4.0	6.2	4.1	Nigeria	4.5	3.6	6.2	5.1
Chile	3.6	2.5	4.3	3.1	Norway	22.8	17.0	18.4	17.9
China	40.4	27.4	36.1	28.5	Poland	5.7	5.6	5.8	5.6
Colombia	4.9	6.3	6.1	6.6	Portugal	18.4	10.7	17.9	9.7
Denmark	12.0	9.0	13.5	11.0	Russia	2.1	1.4	1	none
Finland	6.9	5.5	7.3	6.3	Saudi Arabia	4.7	4.3	4.6	4.4
France	5.0	4.2	4.9	4.3	Singapore	12.3	7.3	10.7	7.6
Germany	2.5	1.8	2.5	1.8	South Africa	2.6	1.7	2.4	1.6
Greece	11.6	8.4	10.7	7.5	Spain	15.8	9.8	16.2	10.2
Hong Kong	14.4	9.4	13.5	8.9	Sweden	6.5	5.4	7.1	6.6
India	14.1	15.1	21.3	22.0	Switzerland	21.3	15.8	26.3	20.0
Indonesia	6.1	5.1	4.2	4.3	Thailand	4.9	3.7	5.6	4.1
Ireland	17.4	13.0	16.6	13.2	Turkey	3.5	2.9	4.2	3.5
Israel	1.4	1.2	2.1	1.5	U. A. Emirates	none	none	none	none
Italy	11.8	9.6	12.7	10.7	UK	6.4	5.2	7.0	5.9
Japan	15.4	13.8	15.1	18.9	Venezuela	4.2	4.5	4.7	5.2

Notes: For each country, the whole period is used.

We can see that in most countries the half-lives differ slightly in different model specifications. For example, in Korea, the half-lives are all about 4 years in Eqs. (2)–(5); in Malaysia, the half-lives are all about 10 years in Eqs. (2)–(5). Only in a few countries, the half-lives from different equations differ obviously, for example in Portugal.

Then in the panel case, the *log(RER)* is also I(0). Each equation is estimated by the two-way fixed effects method which is justified by the redundant fixed effects tests. Each β is significant at the 1% level. The half-lives for Eq. (2), Eq. (3), Eq. (4), and Eq. (5) are 6.9, 6.2, 6.7, and 6.3, respectively. The half-lives from different model specifications are also very near.

Thus, considering both the time series and panel dimensions, the half-life is generally robust whether or not we use the logarithmic form and lagged difference term. In retrospect, when using Eq. (2) in Section 3.1 we find that 10 out of the total 39 RERs have a half-life within 3–5 years. Correspondingly, the number of RERs that have a half-life within 3–5 years is 8 if we use Eq. (3), 8 if we use Eq. (4), and 9 if we use Eq. (5) out of the total 39 or 38 RERs, respectively. That is, if we use other model specifications, the consensus view of the half-life (3–5 years) does not hold either.

5. Half-life may be not an appropriate method

In relative PPP studies, one thinks that the smaller the half-life is, the faster the RER converges or the more valid the PPP theory is. In this section, we analyze whether this view is true in absolute PPP studies.[2]

[2] To be clear, what we discuss is whether the use of the half-life in *absolute* PPP studies is appropriate; whether the use of the half-life in *relative* PPP studies is appropriate is beyond our discussion.

5.1. Some cases illustrated

Table 2 shows that the half-life of the RER of France is 5 and that of South Africa is 2.6. This gives us an inference that absolute PPP should be more valid in South Africa than in France. Likewise, the half-life of the RER of Australia is 13.5 and that of Russia is 2.1, which means that absolute PPP should be more valid in Russia than in Australia. However, such inferences are against the visual examination and formal econometric test.

Fig. 1 gives the RERs of these two pairs of countries (France and South Africa on the left, Australia and Russia on the right) in their whole periods. Seen from the left figure, the RER of France fluctuates around the horizontal line of 1 (especially after 1973), but the RER of South Africa fluctuates around the horizontal line of 0.7 and is invariably smaller than 1. Given that the equilibrium value of absolute PPP is 1, the figure shows that absolute PPP should be more valid in France than in South Africa. Likewise, in the right figure, the RER of Australia fluctuates around the horizontal line of 1, but the RER of Russia fluctuates around the horizontal line of 0.5 and is invariably smaller than 1, which means that absolute PPP should be more valid in Australia than in Russia. As France and Australia both have a level of GDP per capita (GDPP) close to the US, and South Africa and Russia both have a very low GDPP level compared with the US, the visual conclusion (that absolute PPP should be more valid in France and Australia than in South Africa and Russia) is also consistent with the well-known regularity, absolute PPP's system deviation——the closer a country's GDPP is to the US's GDPP, the more valid absolute PPP is between that country and the US.

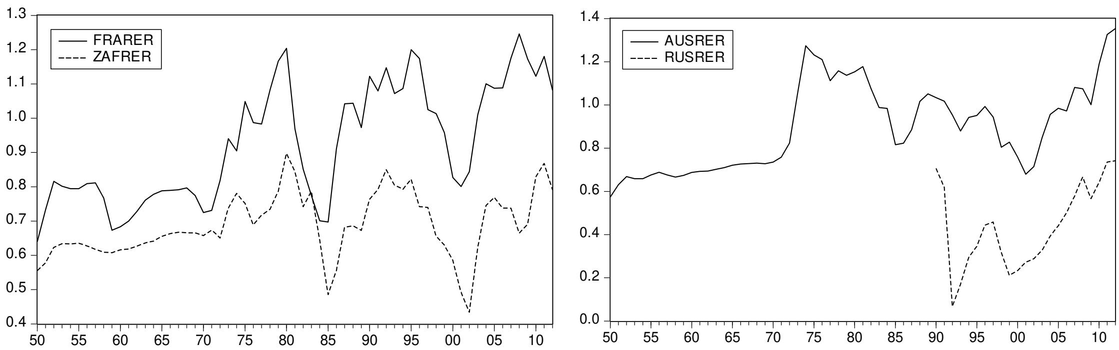

Fig. 1. RERs of the four countries.

Notes: FRARER, ZAFRER, AUSRER, and RUSRER refer to the RER of France, South Africa, Australia, and Russia, respectively.

Sources: World Development Indicators, Penn World Table 7.1, and the authors' calculation.

Zhang and Zou (2014) point out that the unit root and cointegration tests have low power in assessing the validity of absolute PPP and they propose two formal tests, the coefficient restriction and RER misalignment distribution tests. In the coefficient restriction test, we test whether $b = 1$ in Eq. (6), where *NER* is the nominal exchange rate and *PPP* is the PPP rate defined in Eq. (1).[3] According to this model specification, the closer b is to 1, the more valid absolute PPP is. In the RER misalignment distribution test (RER misalignment = $RER - 1$, *RER* is defined in Eq. (1)), the closer the RER misalignment is to a normal distribution with zero mean, the more valid absolute

[3] An alternative way to test $b = 1$ in Eq. (6) is to test $(a, b) = (0, 1)$ in the equation $NER_t = a + b \cdot PPP_t + u_t$, which is used by Zhang and Zou (2014).

PPP is. Table 5 shows that, for Australia or France, the value of b (1.08 or 1.02) is close to 1, the null hypothesis b = 1 is accepted by the Wald coefficient restriction test at the 0.05 level, the RER misalignment is a normal distribution at the 0.05 level, and the RER misalignment mean (-0.10 or -0.07) is close to zero. For Russia or South Africa, the value of b (1.85 or 1.42) is far away from 1, the null hypothesis b = 1 is rejected by the Wald coefficient restriction test, the RER misalignment is a normal distribution, but the RER misalignment mean (-0.56 or -0.31) is far from zero. Thus, the tests show that absolute PPP is obviously more valid in Australia and France than in Russia and South Africa, which confirms the intuitive conclusion from Fig. 1.

$$NER_t = b \cdot PPP_t + u_t \tag{6}$$

Table 5. Coefficient restriction and RER misalignment distribution tests for the four countries.

Country	Period	b (*p*-value)	Test for b = 1: χ^2 statistic (*p*-value)	RER misalignment distribution test	
				Mean	JB statistic (*p*-value)
Australia	1950–2012	1.08	3.41	-0.10	4.01
		(0.00)	(0.06)		(0.13)
France	1950–2012	1.02	0.21	-0.07	5.01
		(0.00)	(0.64)		(0.08)
Russia	1990–2012	1.85	12.23	-0.56	1.06
		(0.00)	(0.00)		(0.59)
South Africa	1950–2012	1.42	64.68	-0.31	0.07
		(0.00)	(0.00)		(0.96)

Notes: The cointegration relation between *NER* and *PPP* in each country has been confirmed after the stationarity of them is tested. OLS or FMOLS is used to estimate Eq. (6).

Given the contradictive conclusion from the half-life and from the visual examination and formal test, we think that the conclusion from the half-life is less credible. In other words, we think that the half-life is not a proper method in assessing the validity of absolute PPP.

5.2. Account for the reason

Why is the half-life not a proper method in assessing the validity of absolute PPP?

The half-life is obtained from β by using Eq. (2). The smaller β is, the shorter the half-life is, and the faster the RER converges according to the common view. Under the convergence implied by the half-life, if a RER converges, it will fluctuate around a horizontal line of a certain value. But this concept has a fault in measuring the validity of absolute PPP.

Concretely, the four RERs in Fig. 1 all converge, but their convergences are different. For South Africa, its RER fluctuates around the horizontal line of 0.7 and it converges to 0.7. For Russia, its RER fluctuates around the horizontal line of 0.5 and it converges to 0.5. But 0.7 and 0.5 are not the equilibrium value of absolute PPP. Only for Australia and France and Australia, their RERs fluctuate around the horizontal line of 1 and they converge to 1, the equilibrium value of absolute PPP. In other words, if a half-life exists, the RER must converge, but the value that the RER converges to is different for different RERs and it may not be 1 for all RERs. Thus, what the half-life measures is how a RER behaves (how a RER converges to its particular value) but not how absolute PPP is valid for this RER.

Zhang and Zou (2014) point out that absolute PPP may not hold for a RER even if it is stationary. The unit root test has a fault in testing absolute PPP. The half-life is again based on the unit root test because it is obtained from β by using Eq. (2) which is used in the unit root test. That is, the fault of the half-life in measuring the validity of absolute PPP is related to the fault of the unit root test.

6. Some statistic indexes can be applied

In contrast with the fault of the half-life in measuring the validity of absolute PPP, we think that some statistic indexes can be applied in this use and may have more power than the half-life. Such statistic indexes are root mean squared error (RMSE), mean absolute error (MAE), among others. Here we use RMSE to illustrate. In this paper RMSE is defined in Eq. (6), where $\widehat{RER}_t$ denotes the RER's equilibrium value in period t. As the equilibrium value of the RER is 1, it is replaced by the value 1; and RMSE is constructed by the RER and 1 in the equation. The criterion in RMSE is that the smaller RMSE is, the more valid absolute PPP is.

$$RMSE = \sqrt{\sum_{t=1}^{n}\left(RER_t - \widehat{RER}_t\right)^2/n} = \sqrt{\sum_{t=1}^{n}(RER_t - 1)^2/n} \quad (6)$$

Then we sequence all the countries according to their RMSE values. The results are listed in Table 6. As expected by the systemic deviation of absolute PPP, most developed countries have a smaller RMSE value but most developing countries have a larger RMSE value. Concretely, the RMSE values for France, Australia, South Africa, and Russia are 0.1848, 0.2251, 0.3262, and 0.5956, respectively. That is, the validity of absolute PPP decreases from France to Australia, to South Africa, and to Russia. Absolute PPP is more valid in France and Australia than in South Africa and Russia. This is consistent with the intuitive examination in Fig. 1 and with the formal econometric tests in Table 5.

Table 6. RMSE value for each country.

Country	RMSE (No.)	Country	RMSE (No.)	Country	RMSE (No.)
Canada	0.0908 (1)	Malaysia	0.3013 (15)	Portugal	0.3854 (29)
Germany	0.1514 (2)	Denmark	0.3147 (16)	Spain	0.4139 (30)
France	0.1848 (3)	Ireland	0.3149 (17)	Korea	0.4329 (31)
Finland	0.2185 (4)	Italy	0.3158 (18)	Brazil	0.4420 (32)
Belgium	0.2221 (5)	Singapore	0.3163 (19)	Indonesia	0.4769 (33)
U. K.	0.2241 (6)	Austria	0.3186 (20)	Mexico	0.4790 (34)
Australia	0.2251 (7)	South Africa	0.3262 (21)	China	0.4961 (35)
Sweden	0.2475 (8)	Netherlands	0.3384 (22)	Poland	0.5078 (36)
Saudi Arabia	0.2531 (9)	Turkey	0.3479 (23)	Thailand	0.5186 (37)
Venezuela	0.2594 (10)	U. A. Emirates	0.3545 (24)	Nigeria	0.5229 (38)
Hong Kong	0.2635 (11)	Greece	0.3632 (25)	India	0.5282 (39)
Chile	0.2677 (12)	Switzerland	0.3682 (26)	Russia	0.5956 (40)
Norway	0.2921 (13)	Colombia	0.3739 (27)		
Israel	0.2990 (14)	Japan	0.3782 (28)		

Notes: For each country, the whole period is used.

Though in terms of the four countries (Australia, France, Russia, and South Africa) RMSE has more power than the half-life, the power of RMSE (e.g., whether RMSE has more or less power than the coefficient restriction and RER misalignment distribution tests) needs further studies.

7. Conclusion

In this paper, we construct the RER using actual prices to be consistent with absolute PPP and study the convergence of absolute PPP. The conclusions obtained are as below.

(1) In relative PPP studies where the RER is constructed by price indexes, the conclusion that the RER is stationary is often obtained only by using a long-run time series data, panel data, or other special econometric methods. However, the stationarity of the RER in this paper can be easily obtained by a traditional time series ADF or KPSS unit root test in a comparatively short-run period (about 60 years or less).

(2) Different from the consensus view about the half-life of 3–5 years in relative PPP studies, the half-life of the RER in this paper spans from 1 to 40 years, with most half-lives outside of the range of 3–5 years. Different logarithmic forms and lagged difference terms in the equation specification do not affect the value of the half-life in an obvious way.

(3) Some concrete cases illustrate that the half-life is not an appropriate method for measuring the validity of absolute PPP, because the intuitive examination and formal econometric test show the absolute PPP may be more valid for a RER with a greater half-life than for another RER with a smaller half-life.

(4) In contrast with the fault of the half-life in measuring the validity of absolute PPP, some statistic indexes (e.g., RMSE and MAE) can be applied in this use. In terms of the four countries (Australia, France, Russia, and South Africa), RMSE has more power than the half-life.

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Appendix Tables

The detailed results of the unit root tests in the whole period for each of the 40 RERs in Section 3.1 are listed in the Appendix Table 1, which shows that all of the variables are I(0) at one usual level (the 1%, 5%, or 10% level). The unit root test results for the sub-periods for each of the 35 RERs in Section 3.2 are listed in the Appendix Table 2, which also shows that all of the variables are I(0) at one usual level. The unit root test is implemented by using the software EViews 7.

Appendix Table1 1. Unit root test for each country's RER in its whole period.

Country	Type	Statistic	CV	Country	Type	Statistic	CV
Australia	$KPSS_{ct}$	0.13	$0.15_{0.05}$	Korea	ADF_{ct}	-3.93	$-3.49_{0.05}$
Austria	ADF_{ct}	-3.51	$-3.49_{0.05}$	Malaysia	$KPSS_{ct}$	0.08	$0.15_{0.05}$
Belgium	ADF_{ct}	-3.60	$-3.49_{0.05}$	Mexico	$KPSS_{ct}$	0.10	$0.15_{0.05}$
Brazil	$KPSS_{c}$	0.22	$0.46_{0.05}$	Netherlands	$KPSS_{ct}$	0.12	$0.15_{0.05}$
Canada	$KPSS_{ct}$	0.07	$0.15_{0.05}$	Nigeria	$KPSS_{c}$	0.35	$0.46_{0.05}$
Chile	ADF_{ct}	-5.67	$-3.49_{0.05}$	Norway	$KPSS_{ct}$	0.07	$0.15_{0.05}$
China	$KPSS_{ct}$	0.15	$0.22_{0.01}$	Poland	$KPSS_{c}$	0.23	$0.46_{0.05}$
Colombia	ADF_{c}	-5.90	$-2.91_{0.05}$	Portugal	$KPSS_{ct}$	0.14	$0.15_{0.05}$
Denmark	$KPSS_{ct}$	0.09	$0.15_{0.05}$	Russia	ADF_{ct}	-4.89	$-3.64_{0.05}$
Finland	ADF_{ct}	-3.80	$-3.49_{0.05}$	Saudi Arabia	$KPSS_{ct}$	0.10	$0.15_{0.05}$
France	ADF_{ct}	-3.81	$-3.49_{0.05}$	Singapore	$KPSS_{ct}$	0.08	$0.15_{0.05}$
Germany	ADF_{c}	-3.41	$-2.94_{0.05}$	South Africa	ADF_{ct}	-4.13	$-3.49_{0.05}$
Greece	ADF_{ct}	-4.52	$-3.49_{0.05}$	Spain	ADF_{ct}	-3.80	$-3.49_{0.05}$
Hong Kong	$KPSS_{c}$	0.42	$0.46_{0.05}$	Sweden	$KPSS_{ct}$	0.15	$0.22_{0.01}$
India	$KPSS_{ct}$	0.13	$0.15_{0.05}$	Switzerland	$KPSS_{ct}$	0.09	$0.15_{0.05}$
Indonesia	$KPSS_{ct}$	0.11	$0.15_{0.05}$	Thailand	ADF_{ct}	-3.73	$-3.49_{0.05}$
Ireland	ADF_{ct}	-3.75	$-3.49_{0.05}$	Turkey	$KPSS_{ct}$	0.12	$0.15_{0.05}$
Israel	ADF_{ct}	-3.87	$-3.49_{0.05}$	U. A. Emirates	$KPSS_{ct}$	0.14	$0.15_{0.05}$
Italy	ADF_{ct}	-3.77	$-3.49_{0.05}$	UK	ADF_{ct}	-4.79	$-3.49_{0.05}$
Japan	$KPSS_{ct}$	0.15	$0.22_{0.01}$	Venezuela	$KPSS_{c}$	0.42	$0.46_{0.05}$

Notes: $KPSS_c$ (ADF_c) and $KPSS_{ct}$ (ADF_{ct}) refer to the KPSS (ADF) unit root test whose model specification has a constant, and has a constant and linear trend, respectively. CV is the abbreviation for critical value. The subscript 0.01, 0.05, or 0.10 of the value on the CV column refers to the critical value at the 0.01, 0.05, or 0.10 level, respectively.

Appendix Table 2. Unit root test for 35 countries' RERs in their sub-periods.

Country	1950–1972			1973–2012		
	Type	Statistic	CV	Type	Statistic	CV
Australia	$KPSS_{ct}$	0.09	$0.15_{0.05}$	$KPSS_{c}$	0.23	$0.46_{0.05}$
Austria	$KPSS_{ct}$	0.17	$0.22_{0.01}$	ADF_{c}	-2.71	$-2.61_{0.10}$
Belgium	$KPSS_{ct}$	0.17	$0.22_{0.05}$	ADF_{c}	-3.23	$-2.94_{0.05}$

Country	1950–1972			1973–2012		
	Type	Statistic	CV	Type	Statistic	CV
Brazil	DF	-3.19	-1.96	$KPSS_{ct}$	0.09	$0.15_{0.05}$
Canada	$KPSS_{ct}$	0.11	$0.15_{0.05}$	$KPSS_{c}$	0.15	$0.46_{0.05}$
Chile	$KPSS_{ct}$	0.10	$0.15_{0.05}$	ADF_{ct}	-3.47	$-3.19_{0.10}$
China	$KPSS_{c}$	0.36	$0.46_{0.05}$	DF	-2.13	$-1.95_{0.05}$
Colombia	DF	-2.04	$-1.96_{0.05}$	$KPSS_{c}$	0.14	$0.46_{0.05}$
Denmark	$KPSS_{ct}$	0.13	$0.15_{0.05}$	ADF_{c}	-3.08	$-2.94_{0.05}$
Finland	ADF_{c}	-2.86	$-2.65_{0.10}$	ADF_{c}	-3.54	$-2.94_{0.05}$
France	ADF_{c}	-3.29	$-3.01_{0.05}$	ADF_{c}	-3.10	$-2.94_{0.05}$
Greece	ADF_{c}	-3.04	$-2.67_{0.10}$	$KPSS_{ct}$	0.09	$0.15_{0.05}$
Hong Kong	$KPSS_{c}$	0.39	$0.46_{0.05}$	$KPSS_{c}$	0.16	$0.46_{0.05}$
India	$KPSS_{ct}$	0.08	$0.15_{0.05}$	$KPSS_{ct}$	0.19	$0.22_{0.01}$
Indonesia	ADF_{c}	-2.97	$-2.71_{0.10}$	$KPSS_{c}$	0.37	$0.46_{0.05}$
Ireland	ADF_{ct}	-3.84	$-3.67_{0.05}$	ADF_{ct}	-3.20	$-3.19_{0.01}$
Israel	$KPSS_{ct}$	0.13	$0.15_{0.05}$	$KPSS_{c}$	0.43	$0.46_{0.05}$
Italy	ADF_{ct}	-3.46	$-3.26_{0.10}$	$KPSS_{ct}$	0.06	$0.15_{0.05}$
Japan	$KPSS_{ct}$	0.09	$0.15_{0.05}$	$KPSS_{c}$	0.41	$0.46_{0.05}$
Korea	$KPSS_{ct}$	0.14	$0.15_{0.05}$	ADF_{c}	-3.09	$-2.94_{0.05}$
Malaysia	DF	-2.38	$-1.96_{0.05}$	$KPSS_{ct}$	0.15	$0.15_{0.05}$
Mexico	$KPSS_{ct}$	0.08	$0.15_{0.05}$	$KPSS_{c}$	0.41	$0.46_{0.05}$
Netherlands	$KPSS_{ct}$	0.19	$0.22_{0.01}$	ADF_{c}	-3.09	$-2.94_{0.05}$
Nigeria	$KPSS_{ct}$	0.17	$0.22_{0.01}$	$KPSS_{c}$	0.29	$0.46_{0.05}$
Norway	$KPSS_{ct}$	0.10	$0.15_{0.05}$	$KPSS_{ct}$	0.10	$0.15_{0.05}$
Portugal	ADF_{c}	-2.77	$-2.64_{0.10}$	$KPSS_{ct}$	0.09	$0.15_{0.05}$
Singapore	$KPSS_{ct}$	0.12	$0.15_{0.05}$	ADF_{c}	-2.66	$-2.61_{0.10}$
South Africa	ADF_{ct}	-4.21	$-3.69_{0.05}$	ADF_{c}	-3.60	$-2.94_{0.05}$
Spain	ADF_{c}	-2.99	$-2.65_{0.10}$	ADF_{ct}	-3.24	$-3.19_{0.10}$
Sweden	$KPSS_{ct}$	0.13	$0.15_{0.05}$	ADF_{c}	-4.59	$-2.94_{0.05}$
Switzerland	$KPSS_{ct}$	0.17	$0.22_{0.01}$	ADF_{ct}	-3.42	$-3.19_{0.10}$
Thailand	ADF_{ct}	-5.46	$-3.64_{0.05}$	$KPSS_{c}$	0.23	$0.46_{0.05}$
Turkey	$KPSS_{c}$	0.36	$0.46_{0.05}$	$KPSS_{c}$	0.18	$0.46_{0.05}$
UK	$KPSS_{ct}$	0.14	$0.15_{0.05}$	ADF_{c}	-3.28	$-2.94_{0.05}$
Venezuela	ADF_{ct}	-4.78	$-3.63_{0.05}$	$KPSS_{c}$	0.22	$0.46_{0.05}$

Notes: $KPSS_c$ (ADF_c) and $KPSS_{ct}$ (ADF_{ct}) refer to the KPSS (ADF) unit root test whose model specification has a constant, and has a constant and linear trend, respectively. DF refers to the ADF test without constant and trend. CV is the abbreviation for critical value. The subscript 0.01, 0.05, or 0.10 of the value on the CV column refers to the critical value at the 0.01, 0.05, or 0.10 level, respectively. For some countries, the first year in the period 1950–1972 is not 1950, and/or the last year in the period 1973–2012 is not 2012; see Table 1 for details.