Animal spirits, investment and unemployment: An old Keynesian view of the Great Recession

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Animal Spirits, Investment and Unemployment: 
An Old Keynesian View of the Great Recession* 

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Abstract

This paper develops a DSGE model with investment and capital accumulation build along demand-driven explanations of the Great Recession. Specifically, following Farmer (2013), I set forth a search framework in which households decide about consumption while firms decide about recruiting effort as well as investment. This setting closed with market clearing in good and asset markets has one less equation than unknowns. Therefore, in order to solve such an indeterminacy, I assume that investment is driven by self-fulfilling expectations about the adjustment cost of capital. Consistently with the view of business cycles pushed by stock price fluctuations, this model has the potential to provide a more comprehensive rationale of the consumption-investment patterns observed during the years of the crisis.

JEL Classification: E24, E32, E52, J64.

Keywords: Investment; Capital accumulation, Finance-induced recession; Search, DSGE Models.

1 Introduction

According to a widespread view, the Great Recession of 2007-2008 can be thought as the upshot of a dramatic loss of confidence triggered by the burst of a financial bubble that abruptly reduced house and stock prices (cf. Hurd and Rohwedder, 2010; Bell and Blanchflower, 2011; Christelis

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et al., 2011). A prominent backer of this view is Farmer (2012a-b, 2013, 2015), who depicts the finance-induced recession as a self-fulfilling reduction of households’ wealth value that led to a sudden consumption contraction that, in turn, drove GDP (unemployment) downwards (upwards).

Farmer’s (2012a-b, 2013) theoretical framework reformulates into a Walrasian setting two important ideas from Keynes’s (1936) General Theory. The first is that the economy can be consistent with a continuum of steady-state unemployment equilibria, while the second is that beliefs of asset market participants might have an independent influence on the economic activity by selecting a perfect-foresight equilibrium in which private consumption, according to its dominant weight in GDP quotas, is assumed to be the crucial component of aggregate demand.

This theoretical proposal, sometimes referred as new ‘Farmerian’ economics, provides new interesting insights on business cycles fluctuations and gives the chance to dig out into the Keynesian view according to which market confidence is essential in determining realized macroeconomic outcomes.\(^1\) However, it is well known that in the General Theory the component of private expenditure mainly driven by market psychology instead of economy’s fundamentals is not consumption but corporate investment; indeed, Keynes (1936) coined the term ‘animal spirits’ just to describe the non-fundamental based behaviour of entrepreneurs regarding investment spending. Moreover, according to Keynes (1936), private investment - via the multiplier effect - was one of the main drivers of business cycles (cf. Smith and Zoega, 2009).

<table>
<thead>
<tr>
<th></th>
<th>$\Delta \ln (Y)$</th>
<th>$\Delta \ln (C)$</th>
<th>$\Delta \ln (I)$</th>
<th>$\Delta \ln (U)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td>0.945</td>
<td>0.842</td>
<td>4.450</td>
<td>6.827</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.388</td>
<td>0.088</td>
<td>0.199</td>
<td>0.616</td>
</tr>
<tr>
<td>Correlation matrix</td>
<td>$\Delta \ln (Y)$</td>
<td>1</td>
<td>0.617</td>
<td>0.782</td>
</tr>
<tr>
<td>$\Delta \ln (C)$</td>
<td>$-1$</td>
<td>$0.257$</td>
<td>$-0.473$</td>
<td></td>
</tr>
<tr>
<td>$\Delta \ln (I)$</td>
<td>$-1$</td>
<td>$1$</td>
<td>$-0.558$</td>
<td></td>
</tr>
<tr>
<td>$\Delta \ln (U)$</td>
<td>$-1$</td>
<td>$-1$</td>
<td>$1$</td>
<td></td>
</tr>
</tbody>
</table>

**Table 1:** US data (1950-2012), quantity indexes

As far as US data are concerned, the importance of investment in explaining macroeconomic fluctuations is still hard to neglect. For instance, table 1 collects the volatility, the persistence and the correlation matrix of GDP ($Y$), consumption ($C$), private investment ($I$) and unemployment ($U$) over the last sixty years on a quarterly basis.\(^2\) The figures show that the correlation of investment both to GDP and unemployment - in absolute value - is slightly

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\(^1\)An extensive review of the new Farmerian approach is given by Guerrazzi (2012).

\(^2\)Data on GDP, consumption and investments are retrieved from the seasonally adjusted quantity indexes provided by the Bureau of Economic Analysis (Index Numbers, 2009=100). See www.bea.gov. Moreover, data on unemployment are retrieved from the Bureau of Labor Statistics. See www.bls.gov.
higher than the one of consumption. Moreover, among the components of private aggregate demand, investment appears as the more volatile variable so, at least in principle, the more prone to mirror sudden switches in market confidence.

**Figure 1:** US consumption and investment (2000-2012)

Additional intriguing elements about investment behaviour can also be derived from the inspection of recent data. Specifically, the two panels of figure 1 draws the paths of the real values of consumption (left scale) and investment (right scale) both in levels and as percentage of GDP starting from the beginning of the century. The left-hand side diagram shows that the wave of pessimism triggered by the finance-induced recession of 2008-2009 had a strong impact on the two components of private aggregate spending. However, while consumption already recovered its pre-crisis level at the end of 2010, investment, as pointed out by Lavander and Parent (2012-2013), is still below its 2007 magnitude. Furthermore, the right-hand side diagram shows that - in relative terms - the wealth effect on consumption triggered by the Great Recession is almost negligible while the depressing effect on investment is more serious and quite persistent.

In this paper, taking into account the macroeconomic patterns sketched above, I introduce productive investment and capital accumulation in the one-sector framework developed by Farmer (2013). To the best of my knowledge, this is the first new Farmerian contribution in which both consumption and investment are addressed within an inter-temporal optimization framework. Specifically, I build a demand-driven search DSGE model in which households put forward an optimal trajectory for consumption while, at the same time, consumer-owned firms decide about optimal recruiting effort as well as an optimal trajectory for investment along the

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3 Along these lines, Zoega (2010) points out the simultaneous deficiency of employment and investment that characterized the latest financial crisis.


Given the presence of search frictions, the model economy closed with market clearing in asset and good markets is characterized by one more unknown than equations so that both its dynamics and its stationary solution remain indeterminate. In my own proposal, such an indeterminacy is solved by assuming that entrepreneurs form self-fulfilling expectations about the adjustment cost of capital. This variable is assumed to convey the Keynesian state of long-term expectations that selects equilibrium unemployment period by period by delivering market allocations that are not necessary efficient outcomes.

From a theoretical point of view, my model implies that whenever entrepreneurs perceive capital adjustments as more (less) expensive, the model economy experiences a sudden decrease (increase) of investment flows. Thereafter, lower (higher) investment depresses (boosts) capital accumulation by reducing (increasing) the wealth of households. This in turn triggers a negative (positive) wealth effect that leads to a decrease (increase) in private consumption. On the whole, lower (higher) investment and lower (higher) consumption push unemployment upwards (downwards). As a consequence, this setting seems to have the potential to provide a more comprehensive rationale of the consumption-investment patterns observed during the Great Recession.

In addition, from a quantitative perspective, I show that the long-run behaviour of the model economy mirrors the observed co-movements of GDP, consumption and investment. Moreover, I give robust evidence that the transmission mechanism of confidence shocks implied by this theoretical framework appears quite consistent with business cycles driven by self-fulfilling asset price fluctuations.

The paper is arranged as follows. Section 2 develops the social planner problem. Section 3 offers a decentralized version. Section 4 analyses some quantitative implications of the model. Finally, section 5 concludes by giving some policy implications.

2 Social planner problem

Following Farmer (2013), I begin by introducing the problem of a benevolent social planner whose goal is the maximization of the individual welfare of a representative household endowed with certain preferences. Such a social planner is constrained by two distinct technologies: the former describes how labour and capital combine themselves in order to produce output, the latter conveys the way in which unemployed workers can be recruited in the productive side of the economy.

In what follows, I provide a description of household preferences and binding technologies. Moreover, I solve the social planner problem and I give its stationary solution.
2.1 Household preferences and labour market participation

I will assume that the model economy is populated by a continuum of identical households endowed with logarithmic preferences that do not yield utility (disutility) from leisure (work).\(^5\) As a consequence, the present value of households discounted utility can be written as

\[
E_0 \left[ \sum_{t=0}^{+\infty} \beta^t \log(C_t) \right] \quad 0 < \beta < 1
\]

where \(E[\cdot]\) is the expectation operator, \(\beta\) is the discount factor and \(C_t\) is current real consumption.

The dimension of the representative household is normalized to one. Moreover, in each period, its members can be alternatively employed or unemployed. Therefore, denoting employed household members by \(L_t\), it follows that the unemployment rate can be conveyed as

\[
U_t = 1 - L_t
\]

2.2 Production technology and capital accumulation

Output in this model economy is produced by means of a Cobb-Douglas technology by combining capital and labour in a stochastic manner. As a consequence,

\[
Y_t = S_t K_t^\alpha X_t^{1-\alpha} \quad 0 < \alpha < 1
\]

where \(Y_t\) is the level of production, \(S_t\) is a supply shock, \(K_t\) is the stock of capital, \(\alpha (1 - \alpha)\) is the elasticity of output with respect to capital (labour) and \(X_t\) is the amount of labour used in production.

Consistently with Farmer (2013), I assume that employed workers can be alternatively allocated in recruiting or production activities. Therefore,\(^4\)

\[
L_t = X_t + V_t
\]

where \(V_t\) is the share of employed workers allocated to recruiting.

Moreover, in contrast to Farmer (2013), the amount of output which is not consumed is assumed to boost capital accumulation. Therefore, the stock of capital evolves according to the usual dynamic law. Hence,

\[
K_{t+1} = Y_t - C_t + (1 - \delta)K_t \quad 0 < \delta < 1
\]

where \(\delta\) is the capital depreciation rate.

\(^5\)In an unpublished appendix, Farmer shows that controlling for labour supply does not significantly alter the results achieved in this simplest context. See www.rogerfarmer.com.
2.3 Search technology and employment dynamics

Symmetrically with production, the technology that moves unemployed workers from home to work is a stochastic Cobb-Douglas combination between recruiters and jobless workers. This assumption leads to the following employment evolution law:

\[ L_{t+1} = B_t V_t^\theta (1 - L_t)^{1-\theta} + (1 - \sigma) L_t \quad 0 < \theta < 1, 0 < \sigma < 1 \] (6)

where \( B_t \) is a matching shock, \( \theta (1 - \theta) \) is the elasticity of matching with respect to recruiters (unemployment) and \( \sigma \) is the exogenous job destruction rate.\(^6\)

2.4 Solution of the social planner problem

Taking into account the building blocks described above, the social planner problem can be written as

\[
\max_{\{C_t, V_t, K_{t+1}, L_{t+1}\}} \mathbb{E}_0 \left[ \sum_{t=0}^{+\infty} \beta^t \log (C_t) \right]
\]

s.t.

\[
K_{t+1} = S_t K_t^\alpha (L_t - V_t)^{1-\alpha} - C_t + (1 - \delta) K_t
\]

\[
L_{t+1} = B_t V_t^\theta (1 - L_t)^{1-\theta} + (1 - \sigma) L_t
\]

\[
K_0 = \bar{K}, \quad L_0 = \bar{L}
\]

where \( \bar{K} \) and \( \bar{L} \) are, respectively, the initial conditions for capital and employment.

The first-order conditions (FOCs) for the problem in (7) are the following:

\[
\frac{1}{C_t} = \beta E_t \left[ \frac{\alpha S_{t+1} \Phi_t^{\alpha-1} + 1 - \delta}{C_{t+1}} \right]
\] (9)

\[
\frac{S_t \Phi_t^\alpha \Psi_t^{1-\theta}}{\theta B_t C_t} = \beta E_t \left[ \frac{S_{t+1} \Phi_t^{\alpha} \Psi_{t+1}^{1-\theta} + (1 - \sigma) \Psi_{t+1}^{1-\theta} B_{t+1} \Psi_{t+1}}{\theta B_{t+1}} \right]
\] (10)

\[
K_{t+1} = S_t \Phi_t^\alpha (L_t - V_t) - C_t + (1 - \delta) K_t
\] (11)

\[
L_{t+1} = B_t \Psi_t^\theta (1 - L_t) + (1 - \sigma) L_t
\] (12)

\[
\lim_{t \to +\infty} \beta^t \lambda_t K_t = 0
\] (13)

\[
\lim_{t \to +\infty} \beta^t \mu_t L_t = 0
\] (14)

\(^6\)In the context of the standard search and matching model à la Pissarides (2000), an equivalent stochastic dynamics for (un)employment is set forth by Andolfatto (1996).
where $\Phi_t \equiv K_t (L_t - V_t)^{-1}$, $\Psi_t \equiv V_t (1 - L_t)^{-1}$ and $\{\lambda_t\}_{t=0}^{+\infty}$ ($\{\mu_t\}_{t=0}^{+\infty}$) is the sequence of Lagrange multipliers on the capital accumulation constraint (employment evolution law).\footnote{It is worth noting that $\Phi_t$ and $\Psi_t$ convey, respectively, a measure of the capital-labour ratio and a measure of labour market tightness.}

The interpretation of the FOCs of the social planner problem is straightforward. Eq.s (9) and (10) are the Euler equations for the two control variables, namely, consumption and recruiters. Moreover, eq.s (11) and (12) reproduce the dynamics of the two state variables. Furthermore, (13) and (14) are the required transversality conditions.

The solution of the social planner’s problem is quite relevant; indeed, the implied trajectories for $Y_t$ and $U_t$ define, respectively, the new Farmerian counterparts of the potential output and the natural rate of unemployment retrieved from new Keynesian DSGE models (e.g. Gali, 2008).

### 2.5 Steady-state of the social planner problem

The social planner problem is a concave maximization problem constrained by two convex technology constraints. As a consequence, (7) has a unique meaningful saddle-path stationary solution towards which all the endogenous variables asymptotically have to converge in order to verify the transversality conditions in eq.s (13) and (14) (cf. Cass, 1966).

Adopting the notational convention such that variables without time indexes denote steady-state values, the stationary solution of the social planner problem is defined by the following proposition:

**Proposition 1** The employment steady-state solution of the social planner problem is given by

$$L = \frac{B \hat{\Psi}^\theta}{\sigma + B \hat{\Psi}^\theta}$$

where $\hat{\Psi}$ is defined by the positive root of the following non-linear hyperbolic expression:

$$\Psi B (1 - \theta) \beta + \Psi^{1-\theta} (1 - \beta (1 - \sigma)) - B \theta \beta = 0$$

Thereafter, the steady-state levels of the other endogenous variables can be retrieved from

$$V = \left( \frac{\sigma L}{B (1 - L)^{1-\sigma}} \right)^{\frac{1}{\beta}}$$

$$K = \left( \frac{\alpha \beta S}{1 - \beta (1 - \delta)} \right)^{\frac{1}{1-\alpha}} (L - V)$$

$$C = S \left( \frac{\alpha \beta S}{1 - \beta (1 - \delta)} \right)^{\frac{1}{1-\alpha}} (L - V) - \delta K$$

The proof is given in Appendix.

Proposition 1 has two important implications. First, equilibrium (un)employment is not affected by technology shocks. Consequently, the steady-state value of the wandering natural
rate of unemployment implied by the solution of the social planner problem is driven by matching shocks only and it is not affected by technology trends (cf. Layard et al., 1991). Moreover, equilibrium (un)employment spills over into the other endogenous variables but not the other way round; indeed, equilibrium (un)employment is completely determined by the discount rate and the parameters underlying employment dynamics. In section 4, this result will be quite useful for calibrating the market version of the model.

3 A decentralized version

In this section, drawing on the theoretical works on investment by Jorgenson (1963), Abel and Blanchard (1983) and Chirinko (1993), I extend the decentralized version of the framework developed by Farmer (2013) by taking into account that productive firms have to decide about the optimal amount of recruiters as well as the optimal trajectory of investment. As I will show below, this setting closed with market-clearing in asset and good markets displays steady-state and dynamic indeterminacy because it has one less equation than unknowns.

3.1 Households

In the decentralized economy households maximize their discounted flow of utility under a wealth-accumulation path. Moreover, consistently with the matching mechanism described in the previous section, their choices are constrained by the fact that, in each period, a market-determined share of their unemployed members will find a job while a fixed share of their employed members will lose its position. Therefore, the representative household is assumed to solve the following problem:

$$\max_{\{C_t, A_{t+1}\}_{t=0}^{\infty}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t \log (C_t) \right]$$

s.t.

$$A_{t+1} = (1 + r_t) A_t + w_t L_t - C_t$$
$$L_{t+1} = \tilde{q}_t (1 - L_t) + (1 - \sigma)L_t$$
$$A_0 = \overline{A}, \quad L_0 = \overline{L}$$

where $C_t$ is the real value of consumption expenditure, $A_t$ is the current value of household’s wealth, $r_t$ is the real interest rate, $w_t$ is the real wage, $\tilde{q}_t$ is the endogenous probability to find a job and $\overline{A}$ is the initial level of wealth.

8A by-product of this feature is that whenever $\theta = 0.5$, equilibrium (un)employment collapses to the value derived by Farmer (2013).

9Such an indeterminacy does not arise in the general equilibrium model by Abel and Blanchard (1983) because they implicitly assume that the labour market always clear. By contrast, in the present framework, search frictions usually prevent this to happen.
The FOCs for the household problem can be written as

\[
\frac{1}{C_t} = \beta E_t \left[ \frac{1 + r_{t+1}}{C_{t+1}} \right]
\]

(21)

\[A_{t+1} = (1 + r_t) A_t + w_t L_t - C_t \]

(22)

\[
\lim_{t \to \infty} \beta^t \varphi_t A_t = 0
\]

(23)

where \(\{\varphi_t\}_{t=0}^{+\infty}\) is the sequence of Lagrange multipliers on the wealth accumulation constraint .

Eq. (21) is the Euler equation for consumption. Moreover, eq.s (22) reproduces the dynamics of state variables. Furthermore, (23) is the required transversality condition.

Since employment dynamics enters the problem of the household as an exogenous shock and production technology is stochastic, I need to assume that there exists a complete set of Arrow securities indexed for each possible realization of the states of the world. Under those circumstances, the Euler equation in (21) implies that payments streams will be discounted period by period with the following price kernel:

\[
Q_t = \beta \left( \frac{C_t}{C_{t+1}} \right)
\]

(24)

Taking into account that households are assumed to be the owners of firms, the expression in eq. (24) will be implemented below to evaluate the present value of expected cash-flows generated by the production activity.

### 3.2 Firms

Productive firms are assumed to set recruiters and investment by maximizing their discounted cash-flows under the capital accumulation constraint. Moreover, symmetrically with households, they will take into account that in each period recruiters can hire a market-determined share of workers while a fixed share of employees quits for exogenous redundancy. Therefore, the problem of the representative firm can be written as

\[
\max_{\{V_t, I_t, K_{t+1}, L_{t+1}\}_{t=0}^{+\infty}} E_0 \left[ \sum_{t=0}^{+\infty} Q_t \left( S_t K_t^\alpha (L_t - V_t)^{1-\alpha} - (1 + p_{I,t}) I_t - w_t L_t \right) \right]
\]

s.to

\[
\begin{align*}
K_{t+1} &= I_t + (1 - \delta) K_t \\
L_{t+1} &= q_t V_t + (1 - \sigma) L_t \\
K_0 &= K, \ L_0 = L
\end{align*}
\]

(25)

where \((1 + p_{I,t})\) is the gross cost of investment, \(I_t\) is real investment and \(q_t\) is the endogenous hiring effectiveness of each corporate recruiter.
Depending on the degree of aggregation, the factor that gives a value to investment expenditure can be interpreted in different ways. On the one hand, according to the seminal work by Jorgenson (1963) on investment at the plant level, \( 1 + p_{I,t} \) can be thought as an exogenously given price of capital goods divided by the GDP deflator. However, in a frictionless one-good economy, \( p_{I,t} \) should be always equal to 0 (cf. Wang and Wen, 2012). On the other hands, in the spirit of the macroeconomic contribution by Abel and Blanchard (1983), \( p_{I,t} \) can be interpreted as a time-dependent adjustment cost conveyed in real terms that the firm has to pay in order to modify the level of its capital stock. For instance, when a firm decide to invest a given fraction of its production output it may have to bear a certain amount of installation and testing costs that must be added to the planned investment expenditure. Nevertheless, those installation and testing costs do not augment the productive capital stock of the firm.

In this paper, I plainly adhere to the macroeconomic perspective. However, I do not follow the literature on the installation costs of new capital equipment which usually assumes that \( p_{I,t} \) is determined by fundamentals such as the flow of investment, the stock of capital and technology shocks (e.g. Chirinkio, 1993). On the contrary, I take the adjustment cost of capital as the outcome of self-fulfilling beliefs. This modelling strategy is motivated by the fact that, everything else being equal, the higher (lower) the expected values of \( p_{I,t} \), the lower (higher) the expected cash-flows of the firm. Consequently, sudden changes in the expectations about \( p_{I,t} \) have the potential to convey sharp shifts in corporate expected yield prospects by influencing the willingness to invest of productive firms. Moreover, the introduction of an adjustment cost of capital unrelated to the fundamentals of the economy allows the decentralized version of the model to behave differently from the centralized version so that market allocations do not necessarily coincide with efficient outcomes.

The FOCs of the firm problem are the following:

\[
p_{I,t} = E_t \left[ Q_t \left( \alpha_S t_{t+1} \Phi_{t+1}^{\alpha-1} + (1 + p_{I,t+1}) (1 - \delta) \right) \right] - 1
\]

\[
\frac{(1 - \alpha) S_t \Phi_{t+1}^{\alpha}}{q_t} = E_t \left[ Q_t \left( (1 - \alpha) S_{t+1} \Phi_{t+1}^{\alpha} \left( 1 + \frac{1 - \sigma}{q_{t+1}} \right) - w_{t+1} \right) \right]
\]

\[
K_{t+1} = I_t + (1 - \delta) K_t
\]

\[
L_{t+1} = q_t V_t + (1 - \sigma) L_t
\]

\[
\lim_{t \to +\infty} Q_t \omega_1 K_t = 0
\]

\[
\lim_{t \to +\infty} Q_t \xi_1 L_t = 0
\]
where $\{\omega_t\}_{t=0}^{+\infty}$ ($\{\xi_t\}_{t=0}^{+\infty}$) is the sequence of Lagrange multipliers on the capital accumulation constraint (employment evolution law).

Eqs. (26) and (27) are Euler equations, respectively, for investment and recruiters. Moreover, eqs (28) and (29) reproduces the dynamics of state variables. Furthermore, (30) and (31) are the transversality conditions.

### 3.3 Search probabilities

The probability to find a job as well as the recruiting effectiveness of corporate recruiters are both determined by assuming that in a symmetric equilibrium the employment evolution laws that affect the problems of households and firms describe the same employment path tracked by the employment dynamics that bind the social planner problem. Therefore, in each period, the probability to find a job is given by

$$\tilde{q}_t = B_t \Psi_t^\theta$$  \hspace{1cm} (32)

Moreover, in a similar manner, the recruiting effectiveness of corporate recruiters can be conveyed as

$$q_t = B_t \Psi_t^{\theta-1}$$  \hspace{1cm} (33)

The expressions in eqs (32) and (33) mirror the traditional trading externalities that characterize a textbook search and matching economy; indeed, $\tilde{q}_t$ ($q_t$) is an increasing (decreasing) function of the labour market tightness indicator (cf. Pissarides, 2000).

### 3.4 Characterizing equilibria

Leaving out supply and matching shocks that, by definition, are exogenous factors, the decentralized model is called in to determine period by period the following set of twelve endogenous variables:

$$\{C_t, A_t, L_t, V_t, I_t, K_t, Q_t, \tilde{q}_t, q_t, r_t, w_t, p_{I,t}\}$$  \hspace{1cm} (36)

Straightforward algebra suggests that determinacy of the model requires the same number of equations. First, two of them immediately derive from the definitions of search probabilities, i.e., eqs (32) and (33). Moreover, the FOCs of households and firms problems provide additional seven forward- and backward-looking inter-temporal relationships. In details, the Euler equation for consumption, i.e., eq. (21), the price kernel, i.e., eq. (22), the wealth accumulation path, i.e., eq. (23), the Euler equation for investment, i.e., eq. (28), the Euler equation for recruiters, i.e., eq. (29), the capital evolution law, i.e., eq. (30), and an employment dynamic pattern consistent with the already mentioned search probability, i.e., eq. (12).
To close the model three more equations are called in. On an intra-temporal basis, two important relationships come from the market-clearing conditions on asset and good markets, respectively,

\[ A_t = K_t \]  \hspace{1cm} (37)

and

\[ C_t + (1 + p_{I,t}) I_t = S_t \Phi_t^\alpha (L_t - V_t) \]  \hspace{1cm} (38)

Eq. (37) states that value of households’ wealth has to be equal to the value of firms’ capital. Moreover, according to the national account identity, eq. (38) conveys that the sum of consumption and investment expenditures must be equal to produced output.

Finally, similarly to Farmer (2013), the balance between the number of equations and the number of unknowns is reached by assuming that entrepreneurs form self-fulfilling expectations about the adjustment cost of capital. As a consequence,

\[ E_t [p_{I,t+1}] = x_t \]  \hspace{1cm} (39)

where \( x_t \) is a belief-function which is assumed to map observations of current and past investment costs to expectations about future costs.

It is worth noting that, in each period of time - given the expectation of \( p_I \) - all the other endogenous variable are determined in order to make such an expectation self-validating. In other words, eq. (39) resolves the indeterminacy of the model economy by selecting a perfect-foresight path in which expectations about the adjustment cost of capital are self-fulfilling.

In practice, there is a variety of ways \( x_t \) could be specified. For instance, Farmer (2012b) models beliefs by resorting to a martingale. Moreover, Farmer (2013) assumes that \( x_t \) takes the form of a conventional adaptive expectation equation. Since in the next section I will focus only on steady-state equilibria, I will not provide any specific functional form for \( x_t \). Taking into account that the way in which agents form expectation may change over time, I leave the evolution of beliefs as well the the short dynamics of the model economy to further developments.\(^{10}\)

### 3.5 Steady-state of the decentralized model

In steady-state, households’ Euler equation for consumption implies that the equilibrium real interest rate is given by

\[ r = \frac{1 - \beta}{\beta} \]  \hspace{1cm} (41)

\(^{10}\)A general theory of expectations is beyond the scope of the present contribution.
As far as the result in eq. (41) is concerned, the stationary solution of the other endogenous variables can be retrieved from the following proposition:

**Proposition 2** Define the constants $\Omega_0$, $\Omega_1$, and $\Omega_2$ as follows

\[
\begin{align*}
\Omega_0 &\equiv 1 - \beta (1 - \delta (1 + \alpha)) \frac{\alpha}{\alpha (1 - \beta)} \\
\Omega_1 &\equiv (1 - \alpha) (1 - \beta (1 - \delta)) \frac{\alpha}{\alpha (1 - \beta)} \\
\Omega_2 &\equiv \frac{\sigma B}{B \beta} (1 - \beta (1 - \sigma))
\end{align*}
\]  

(42)

For each value of $p_I \in (-1, (\Omega_0 - \Omega_1)^{-1} - 1)$, the (positive) employment steady-state solution of the decentralized version of the model is given by the root of the following hyperbolic equation:

\[
\frac{1}{1 + p_I} - \Omega_0 + \Omega_1 \left( \frac{L}{1 - L} \right) \frac{1 - \sigma}{\sigma} = 0
\]  

(45)

Thereafter, the steady-state levels of the other endogenous variables can be obtained from the following equations:

\[
\begin{align*}
V &= \left( \frac{\sigma L}{B (1 - L)^{1 - \sigma}} \right)^{\frac{1}{\sigma}} \\
K &= A = \left( \frac{\alpha^\beta S}{(1 + p_I) (1 - \beta (1 - \delta))} \right)^{\frac{1}{\alpha}} (L - V) \\
I &= \delta K \\
w &= (1 - \alpha) S \left( \frac{K}{L - L} \right)^{\alpha} \left( 1 - \left( \frac{L}{1 - L} \right) \frac{1 - \sigma}{\sigma} \Omega_2 \right) \\
C &= K r + w L \\
Q &= \beta
\end{align*}
\]

(46)

In addition, $\Phi$, $\Psi$, $\tilde{q}$ and $q$ can be derived from their respective definitions.

The proof is given in Appendix.

Proposition 2 suggests three interesting conclusions. First, in the decentralized model the equilibrium values of the belief function and the matching shock univocally select equilibrium (un)employment by solving the indeterminacy mentioned above. As a consequence, symmetrically with the employment steady-state solution of the social planner problem, supply shocks do not affect the equilibrium unemployment rate of the decentralized economy. Second, there exists an upper bound for the eligible equilibrium value of the cost of investment that pushes equilibrium employment towards zero. Furthermore, whenever the equilibrium adjustment cost of capital becomes a subsidy that exactly counterbalances its replacement cost, so that $(1 + p_I)^{-1}$ tends to infinity, the employment steady-state solution of the decentralized model tends to the full employment allocation; indeed, the hyperbolic expression on RHS of (45) tends to infinity if and only if $L$ approaches one.\(^{11}\) The determination of equilibrium employment is illustrated in figure 2.

\(^{11}\)In this case households would be allowed to consume all the produced output.
In this theoretical framework, whenever entrepreneurs perceive capital adjustments as more (less) expensive, there are two subsequent effects on the private components of aggregate demand. First, there is a sudden decrease (increase) of the investment flow. Consequently, this setting provides a straightforward formalization for how the credit crunch, i.e., the dramatic worsening of firm access to bank credit experienced over the financial crisis, translated into a fall in firms’ spending on additional physical capital (cf. Haltenhof et al., 2014). Moreover, lower (higher) investment depresses (boosts) capital accumulation by reducing (rising) the wealth of households. Similarly to Farmer (2013), this in turn triggers a negative (positive) wealth effect that leads to a decrease (increase) in private consumption. On the whole, as shown in figure 2, lower (higher) investment and lower (higher) consumption push unemployment upwards (downwards).

All in all, the arguments developed above reveal that this model seems to have the potential to provide a more comprehensive rationale of the consumption-investment patterns observed during the Great Recession without neglecting capital accumulation.

4 Quantitative implications of the model

In this section I explore some quantitative implications of the theoretical framework developed in sections 2 and 3. First, I provide a suitable model calibration. Moreover, I analyse the long-run behaviour of the model economy by deriving the properties of steady-state equilibria. In addition, I discuss the reliance of stock market prices and the relative price of investment as business cycle drivers.
4.1 Calibration

The model is calibrated in order to be consistent with US quarterly figures. Specifically, the capital share, the discount factor and the depreciation rate of capital are set at the same values chosen by Kydland and Prescott (1982) in their real business cycle contribution. Moreover, the parameters of the employment evolution law are fixed according to the JOLT-based estimations retrieved by Shimer (2005). In addition, the equilibrium value of productivity shocks is normalized to one while the corresponding figure for matching shocks is set in order to convey a social optimal unemployment rate equal to the historical unemployment rate implied by the data reported in table 1, i.e., a point value of 5.84%. The whole set of parameter values is collected in table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share</td>
<td>$\alpha$</td>
<td>0.360</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.999</td>
</tr>
<tr>
<td>Capital depreciation</td>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Matching elasticity</td>
<td>$\theta$</td>
<td>0.280</td>
</tr>
<tr>
<td>Job destruction rate</td>
<td>$\sigma$</td>
<td>0.100</td>
</tr>
<tr>
<td>Productivity shock</td>
<td>$S$</td>
<td>1.000</td>
</tr>
<tr>
<td>Matching shock</td>
<td>$B$</td>
<td>2.155</td>
</tr>
</tbody>
</table>

Table 2: Calibration

4.2 Properties of steady-state equilibria

The results in proposition 2 recall that for each eligible value of $p_I$ there exists a unique meaningful steady-state level of (un)employment. Thereafter, given the solution for $L$, the steady-state values of all the other endogenous variables can be easily derived. Taking into account the parametrization in table 2, figure 3 tracks the steady-state relationships of GDP, consumption, investment and its cost over the range of employment rates observed over the last sixty years (dotted lines represent planning optimum). Those relationships can be taken as the theoretical cointegrating relationships existing among the different variables.

All over the past sixty years, the US unemployment rate ranged from a minimum value of 2.57%, reached in the second quarter of 1953, to a maximum value of 10.66% achieved in the forth quarter of 1982. The diagrams in figure 3 reveal that along the range of observed unemployment the cyclical co-movements of the theoretical co-movement of GDP, consumption

---

12The implicit hypothesis for this numerical choice is that all over the concerned period, on average, actual unemployment fluctuated around the value that would have been chosen by the social planner whose behaviour is described in section 2.

13The MATLAB code to derive the panels of figure 3 is available from the author.
and investment is fairly consistent with the figures of the correlation matrix in table 1; indeed, counter-cyclical patterns appear only when unemployment falls below 3%, a figure lower than the planning optimum that is not so recurrent in actual data.

![Figure 3: Steady-state relationships](image)

4.3 What is the driving force of business cycles?

Farmer (2012a-b, 2013, 2015) and the other backers of the finance-induced recession mentioned in the introduction convincingly argue that the stock market crash of 2008 triggered the subsequent macroeconomic downturn. On a closer inspection, the transmission mechanism of beliefs shocks implied by the model outlined in section 3 can easily support this view; indeed, circumstantial evidence analyzed, inter alia, by Fama (1981) and Barro (1990) shows that there is a quite strong positive relation between stock market prices and corporate investment. Obviously, this relation suggests that increases (decreases) in asset market values may lead entrepreneurs to perceive capital adjustment as less (more) costly in a self-fulfilling manner. This, on turn, will increase (decrease) their willingness to hire.\(^{14}\)

On an empirical perspective, this conjecture is corroborated by the negative relation observed between the relative price of investment - that I take as a proxy for the gross cost of investment - and the deflated S&P500 index over the last sixty years depicted in figure 4.\(^{15}\)

\(^{14}\)A similar relation among asset prices, investment and employment have been found by Zoega (2009) in many OECD countries.

\(^{15}\)On the one hand, the relative price of investment is build by dividing the price index of gross private domestic investment by the GDP deflator such as provided by the Bureau of Economic Analysis. On the other hand, the S&P500 index is retrieved by removing seasonal patterns and deflating the figures provided by the Federal Reserve Bank of Saint Louis. See www.research.stlouisfed.org.
Figure 4: Asset prices and the price of investment (1957-2012); first differences of logs

The diagram in figure 4 shows that asset prices and the relative price of investment are linked by a clear-cut negative relation all over the period under examination; indeed, the linear regression line has a slope of $-3.56$ with a standard error of 1.11. Therefore, given the strength of such a relation, the model developed in section 3 appears consistent with business cycles driven by self-fulfilling asset price movements.\footnote{Taking the same variables in levels, such a negative relation is even stronger (slope: $-32.90$; standard error: 0.90). Details are available from the author.} In this light, $p_I$ can be considered as the signal of what Keynes (1936) called the inducement to invest, i.e., the incentive to invest on new projects triggered when companies’ shares are quoted very high.

5 Concluding remarks

In this paper I introduce investment and capital accumulation in the theoretical setting developed by Farmer (2013). Specifically, I build a demand-driven search economy in which households decide their optimal trajectory for consumption while, at the same time, consumer-owned firms decide about optimal recruiting effort as well as the optimal trajectory for productive investment (cf. Jorgenson, 1963; Abel and Blanchard, 1983; Chirinko, 1993).

Given the presence of search frictions, closing the model with market clearing in the assets and goods markets leads to a non-linear system in which there is one more unknown than equations. In the present proposal, such an indeterminacy is solved by assuming that entrepreneurs form self-fulfilling expectations about the adjustment cost of capital.

In this setting, I show that whenever entrepreneurs perceive capital adjustments as more (less) expensive, the model economy experiences a sudden decrease (increase) of the investment flow. Thereafter, lower (higher) investment depresses (boosts) capital accumulation by reducing
(rising) the wealth of households. This on turn triggers a negative (positive) wealth effect that leads to a decrease (increase) in private consumption. On the whole, lower (higher) investment and lower (higher) consumption push employment downwards (upwards). As a consequence, this framework seems to have the potential to provide a more comprehensive explanation of the consumption-investment patterns observed during the Great Recession.

From a quantitative point of view, I show that long run behaviour of the model economy is consistent with the observed co-movements of GDP, consumption and investment. Moreover, I provide evidence that the transmission mechanism of belief shocks implied by the present theoretical framework can mirror business cycles driven by self-fulfilling asset price fluctuations.

What are the suggested economic policies or the regulatory framework that according to the present analysis would be more effective in remedying and preventing crises? Obviously, a fiscal stimulus that sustains aggregate demand would counterbalance the employment effects triggered by a financial turmoil. However, the theoretical model and the empirical evidence presented in this paper suggest that an intervention on the asset market aimed at preventing sudden crashes may be able to produce the same employment effects as well as a stabilization of investment flows. For instance, when there is a loss of confidence that drives asset prices downwards, the central bank could buy shares on the asset market by selling back public securities. Since the return on public securities is guaranteed by the government through the tax system, this intervention should stop the deflation in the asset market by preventing the fall of investment. In the long run, such an intervention of qualitative easing may be more effective than fiscal expansionary policies in boosting economic growth.

Appendix

In what follows, I provide the formal proofs for propositions 1 and 2.

Proof of proposition 1

In steady-state, the Euler equation for recruiters and employment dynamics holding in social planner’s problem imply that

\[
\frac{S\Phi^\alpha\Psi^{1-\theta}}{\theta BC} = \frac{\beta S\Phi^\alpha}{C} \left( 1 + \frac{(1 - \sigma) \Psi^{1-\theta} - (1 - \theta) B\Psi}{\theta B} \right) \tag{A1}
\]

\[
V = \left( \frac{\sigma L}{B(1 - L)^{1-\theta}} \right)^{\frac{1}{\beta}} \tag{A2}
\]

Straightforward algebra reveals that (A1) is equivalent to eq. (16). Consequently, recalling that \( \Psi \equiv V (1 - L)^{-1} \) and denoting by \( \hat{\Psi} \) the positive solution of (A1), the equilibrium level of employment is obtained by combining \( \hat{\Psi} \) with (A2) as conveyed by eq. (15). Thereafter, the
equilibrium levels of capital and consumption can be derived, respectively, from the steady-state versions of eq.s (9) and (11).

**Proof of proposition 2**

On the one hand, in steady-state, the Euler equation for corporate investment and the capital accumulation path are given by

\[ p_I = \beta \left( \alpha S \Phi^\alpha - 1 + (1 + p_I) (1 - \delta) \right) - 1 \]  
\[ K = I + (1 - \delta)K \]

Since \( \Phi \equiv K (L - V)^{-1} \), eq.s (B1) and (B2) imply that

\[ I = \delta \left( \frac{\alpha \beta S}{(1 + p_I) (1 - \beta (1 - \delta))} \right)^{\frac{1}{1 - \alpha}} (L - V) \]  

On the other hand, the equilibrium Euler equation for recruiters and the wealth accumulation path can be written as

\[ \frac{(1 - \alpha) S \Phi^\alpha}{q} = \beta \left( (1 - \alpha) S \Phi^\alpha \left( 1 + \frac{1 - \sigma}{q} \right) - w \right) \]

\[ A = (1 + r) A + wL - C \]

Considering the results in eq.s (41) and (B2) as well as the market-clearing condition for assets in eq. (37), eq. (B5) implies that

\[ C = \frac{I (1 - \beta)}{\beta \delta} + wL \]

Moreover, taking into account the definitions \( \Phi \) and \( q \) and the result in eq. (B2), eq. (B4) leads to

\[ w = (1 - \alpha) S \left( \frac{I}{\delta (L - V)} \right)^{\alpha} \left( 1 - \frac{1 - \beta (1 - \sigma)}{B \left( \frac{V}{1-L} \right)^{\theta} \beta} \right) \]

Plugging the results in eq.s (B3), (B6) and (B7) into the market-clearing condition for the goods market in eq. (38) taking into account the result in eq. (A2) allows to derive the hyperbolic expression in eq. (45). Furthermore, the steady-state value of the shadow value of employment in eq. (51) follows immediately from the respective equilibrium Euler equation.
References


