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20 February 2015

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Abstract

This paper theoretically investigates how trade affects skill intensity at firm level. In order to analyze this, we develop a model in which firms engages in the division of labor within firms by in putting two types of labor. Unskilled labor is inputted into the production line of the production division and skilled labor is inputted into the production division to conduct the production line. Firms can reduce marginal cost by promoting the division of labor in the production division. Both types of labor are also inputted into head office for domestic market and for export market. These head offices are different in skill intensity. Though all firms are ex-ante identical, the division of labor of exporters is stronger than that of non-exporters on the unique equilibrium. That fixed labor input of headquarter division for export market is more skill intensive than that for domestic market is equivalent to the fact that total labor input of exporters is more skilled intensive than that of non-exporters. Furthermore, all firms reduce the type of labor inputted intensively into head quarter division for the export market while raising the type of labor inputted less intensively into that division.

Keywords: export decision; division of labor within firms; skill intensity

JEL classification numbers : F12

1 Introduction

Traditional studies of international trade have investigated the following questions. Which industries are relatively skill intensive? How does trade liberalization enhance certain

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industries' skill intensity? By contrast, some recent studies emphasize the changes in skill intensity at firm level. For example, Bustos (2011) indicates that the changes in relative demand for skilled labor cannot be explained by labor reallocation across industries and firms but by skill upgrading within firms. Using microdata from Sweden, Davidson et al. (2013) indicate that exporters have more skill-intensive organizations than nonexporters, and furthermore, multinational enterprises have more skill-intensive organizations than exporters.

Then, how should firms decide to export and reorganize their structure ? We particularly focus on the division of labor in this regard. This paper presents a simple model that theoretically investigates the relationship between export decisions and decisions on the extent of the division of labor within firms. We incorporate two types of labor and two types of fixed costs composed of the two types of labor into the model of Chaney and Ossa (2013). Following Medein (2003), fixed costs for the domestic and export markets differ in skill intensity.

This paper's main results are as follows. To guarantee a unique equilibrium in which exporters and nonexporters coexist, skill intensity of the two types of fixed costs need to be different. The division of labor of exporters is stronger than that of nonexporters. That the fixed labor input of head offices for the export market is more skill-intensive than that for the domestic market is equivalent to the total labor input of exporters being more skill-intensive than that of nonexporters. Regardless of decreases in variable and fixed trade costs, or the structures of head offices, the number of exporters increases while the numbers of nonexporters and all firms decrease. Furthermore, all firms reduce the type of labor inputted intensively into head quarter division for the export market while raising the type of labor inputted less intensively into that division. These results are robust in this model. Whether the inequalities in labor input, output, and skill intensity between exporters and nonexporters expand depends critically on whether trade liberalization affects a firm's marginal revenue directly.

In this model, if skill intensity of the two types of fixed costs is the same, there is little possibility of an equilibrium in which exporters and nonexporters coexist. This property is shared with Medin (2003) and Yeaple (2005). In those models, there are only ex-ante identical firms. We show that such models need the two types of fixed costs in skill intensity to guarantee equilibrium in which exporters and nonexporters coexist. Yeaple (2005) shows exporters adopt more high technology based on more skilled-intensive firm structure than nonexporters. This result is consistent with this paper's model to a certain degree although this paper's model depends on differences in skill intensities of the two types of fixed costs.

This paper's research is related to trade and firm structure, in particular, skill intensity. Zadeh (2013) focuses on the extent of the division of labor of each type of labor independently and shows a relationship between relative specialization and trade. Holmes and Mitchell (2008) consider the relationship between trade and mechanization. Davidson et al. (2013) present a model in which firms input only skilled workers into head offices for the export market. Yeaple (2005) considers the relationship between trade and skill formation. Ekholm and Midelfart (2005), Bustos (2011), and Harrigan and Reshef (2011) consider trade-induced skill-biased technological change. All these studies show that trade raises the skill intensity of exporters.

The rest of this paper is organized as follows. Section 2 analyzes aurtarkic equilibrium. Section 3 analyzes how opening up to trade promotes the division of labor and increases welfare. Section 4 analyzes how trade liberalization promotes the division of labor. Finally, we present the Conclusion and Appendix.

2 Autarkic economy

We introduce the division of labor into the trade model of monopolistic competition with fixed export costs. The setup of the model is based on Chaney and Ossa (2013). In this section, we develop a model of an autarkic economy.

2.1 Representative household

There are L units of unskilled workers and K units of skilled workers. They supply one unit of labor inelastically at wage rates w and v, respectively. The preference of the representative households is given by a constant elasticity of substitution utility function over a continuum of goods indexed by θ, as follows $U = \left[\int_{\theta\in\Theta} c(\theta)^{\rho} d\theta\right]^{1/\rho}$, $0 < \rho < 1$, where the measure of the set Θ represents the mass of available differentiated goods, and $c(\theta)$ represents the consumption of variety θ. Since all workers belong to the representative household, its budget constraint is given by $\int_{\theta\in\Theta} p(\theta)c(\theta)d\theta \leq wL + vK + M\pi$, where π is firm profit and M is the number of firms. From standard utility maximization, the price index can be obtained as follows $P = \left[\int_{\theta\in\Theta} (p(\theta))^{1-\sigma} d\theta\right]^{1/(1-\sigma)}$, where $\sigma = 1/(1-\rho) > 1$ is the elasticity of substitution between any two varieties and also represents the price elasticity of demand for each variety.

2.2 Firm structure

Firms input labor into a production division and head office. Firms input l_p^u units and l_d^u of unskilled labor into the production division and head office, respectively. Similarly, firms input l_p^s units and l_d^s of skilled labor into the production division and head office, respectively. The total labor input of unskilled and skilled labor is given by $l_t^u \equiv l_p^u + l_d^u$ and $l_t^s \equiv l_p^s + l_d^s$, respectively.

Firms can operate by inputting a combination of unskilled and skilled labor into the head office, regardless of output. We let f_d be the combination and f_d is defined as $f_d = (l_d^s/\alpha)^\alpha \left[l_d^u/(1-\alpha)\right]^{1-\alpha}$, where $\alpha \in (0,1)$. Firms minimize the cost of head offices, FC_d, defined as $FC_d = vl_d^s + wl_d^u$. Hence, firms face the following minimization problem under given (w, v)

$$\min_{l_d^s, l_d^u} FC_d = vl_d^s + wl_d^u, \quad s.t.\ f_d = (l_d^s/\alpha)^\alpha \left[l_d^u/(1-\alpha)\right]^{1-\alpha}.$$

The solution to this problem is given as follows

$$l_d^s(w,v) = \frac{\alpha f_d}{(w/v)^{1-\alpha}},$$

and

$$l_d^u(w,v) = \frac{(1-\alpha) f_d}{(v/w)^{\alpha}}.$$

These equations give

$$FC_d(w,v) = w^{1-\alpha} v^\alpha f_d.$$

That is, firms must pay $FC_d(w,v)$ as fixed costs.

The structure of firms' production divisions is similar to that of Shintaku (2015). Unskilled labor is inputted into production lines. Skilled labor coordinates teams in production divisions.

From the result of Shintaku (2015), unskilled labor, which is inputted into a product line for y units of a final good, is given by $l_p^u(t,y) = (\gamma y)/(2t)$. Skilled labor, inputted as coordinators, is given by $l_p^s(t,y) = tf$.

Firms select the number of teams t such that variable cost $VC(t,y,w,v)$ is minimized for given (y,w,v), where $VC(t,y,w,v) = vl_p^s(t,y) + wl_p^u(t,y)$. From $\partial VC(y,w,v)/\partial t = vf - w\gamma y/(2t^2)$, we can obtain the optimal number of teams, as follows $t(y,w,v) = \left[(w/v)(\gamma y)/(2f)\right]^{1/2}$.

By substituting $t(y,w,v)$ for $l_p^s(t,y)$ and $l_p^u(t,y)$, we can obtain the following equations

$$l_p^s(y,w,v) = \left(\frac{w}{v}\frac{\gamma f y}{2}\right)^{1/2},$$

and

$$l_p^u(y,w,v) = \left(\frac{v}{w}\frac{\gamma f y}{2}\right)^{1/2},$$

respectively. $l_p^s(y,w,v)$, $l_p^u(y,w,v)$, and $VC(t,y,w,v)$ give $VC(y,w,v) = (2wv\gamma fy)^{1/2}$. $VC(y,w,v)$ and $FC_d(w,v)$ give the following total cost function

$$TC(y,w,v) \equiv VC(y,w,v) + FC_d(w,v) = (2wv\gamma fy)^{1/2} + w^{1-\alpha}v^{\alpha}f_d.$$

From $TC(y,w,v)$, the marginal cost function, $MC(y,w,v)$, is given by $MC(y,w,v) = (1/2)\,(2vw\gamma f/y)^{1/2}$. From $TC(y,w,v)$, the average cost function, $AC(y,w,v)$, is given by $AC(y,w,v) = (2wv\gamma f/y)^{1/2} + w^{1-\alpha}v^{\alpha}f_d/y$. Note that $\partial MC/\partial y < 0$ and $\partial AC/\partial y < 0$ hold.

From $VC(t,y,w,v)$, we can obtain the following proposition.

Proposition 1. *As the division of labor is promoted (t increases), the real marginal cost measured by unskilled labor ($MC(y,w,v)/w$) decreases, given wages of unskilled and skilled labor.*

Proof: See Appendix A.

Proposition 1 indicates that we can use the number of teams to represent firm productivity.

2.3 Autarkic equilibrium

Autarkic equilibrium is characterized by the optimal pricing rule, $PP: p = \mu MC(y,w,v)$, where $\mu \equiv \epsilon/(\epsilon-1)$ and the free-entry and free-exit condition, $FE: p = AC(y,w,v)$, under given (w,v) are as follows

$$PP_{A|v/w}: \frac{p}{w} = \frac{\mu}{2}\left(\frac{v}{w}\frac{2\gamma f}{y}\right)^{1/2}, \tag{1}$$

$$FE_{A|v/w}: \frac{p}{w} = \left(\frac{v}{w}\frac{2\gamma f}{y}\right)^{1/2} + \left(\frac{v}{w}\right)^{\alpha}\frac{f_d}{y}. \tag{2}$$

Subscript $A|v/w$ represents the conditions at autarkic equilibrium under given (w,v).

For our later analysis, we introduce notations B and G_d.

Definition 1. *We define B and G as follows*

$$B \equiv \frac{\mu}{2} - 1,$$

$$G_d \equiv \frac{v}{w}^{\alpha} f_d.$$

$PP_{A|v/w}$ of (1) and $FE_{A|v/w}$ of (2) give $y_{A|v/w} = (w/v)G_d^2/(2\gamma f B^2)$ and $(p/w)_{A|v/w} = (v/w)[B(B+1)\gamma f]/G_d$, respectively. By substituting $y_{A|v/w}$ for $t(y,w,v)$, $l^s(y,w,v)$, and $l^u(y,w,v)$, we can obtain $t_{A|v/w} = (w/v)[G_d/(2fB)]$ and the following equations

$$l^s_{t,A|v/w} = \frac{w}{v} G_d \left(\alpha + \frac{1}{2B} \right),$$

$$l^u_{t,A|v/w} = G_d \left(1 - \alpha + \frac{1}{2B} \right).$$

The market-clearing conditions of unskilled and skilled labor are given by $L = Ml^u_{t,A|v/w}$ and $K = Ml^s_{t,A|v/w}$, respectively. These equations formulate simultaneous equations, in which M and (v/w) are unknown variables. We can solve these equations by using the abovementioned $l^s_{t,A|v/w}$ and $l^u_{t,A|v/w}$ as follows

$$\left(\frac{v}{w}\right)_A = \frac{L}{K} \frac{\left(\alpha + \frac{1}{2B}\right)}{\left(1 - \alpha + \frac{1}{2B}\right)},$$

$$M_A = \frac{1}{f_d} \left(\frac{K}{\alpha + 1/(2B)} \right)^{\alpha} \left(\frac{L}{1 - \alpha + 1/(2B)} \right)^{1-\alpha}.$$

Then, we have characterized all endogenous variables.[1]

We impose the following assumption to obtain the internal solution.

Assumption 1. *$B > 0$ holds. That is, $2 < \mu$ and $1 < \epsilon < 2$ hold.*

Assumption 1 is a necessary and sufficient condition for the existence of the internal solution.

Proposition 2. *If and only if Assumption 1 holds, endogenous variables M_A, $(v/w)_A$, y_A, $(p/w)_A$, t_A, $l^s_{t,A}$, and $l^u_{t,A}$ are positive.*

Proof. To guarantee that $(v/w)_A$ and M_A are positive, we do not always need $B > 0$. However, if y_A, $(p/w)_A$, t_A, $l^s_{t,A}$, and $l^u_{t,A}$ are positive, we need $B > 0$. Hence, if all endogenous variables are positive, we need $B > 0$. Conversely, if $B > 0$, all endogenous variables are positive. Q.E.D.

3 Opening up to trade

We extend the model reported in the previous section to the case of trade between two identical countries with fixed export costs. Without the loss of generality, we focus on the

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[1] From Walras' law, we have not analyzed an income–expenditure clearing condition.

home country's allocation.

3.1 Firms' decisions

Firms choose among the following options: to exit or enter the domestic market (nonexporters), or to exit or enter both the domestic and foreign markets (exporters). Hereafter, we use superscript "ne", which is the nonexporter variable, and "e", which is the exporter variable.

Exporters experience two types of trade costs. First, they must export $\tau \in [1, \infty)$ units of product to send one unit to a foreign market (iceberg trade cost). Second, to enter export markets, exporters must input a combination of unskilled labor (l_x^u) and skilled labor (l_x^s) into their head offices, regardless of output. We let f_x be the combination and f_x is defined as $f_x = (l_x^s/\phi)^\phi \left[l_x^u/(1-\phi)\right]^{1-\phi}$, where $\phi \in (0,1)$. Firms minimize the cost of their head offices, FC_x, where FC_x is defined as $FC_x = vl_x^s + wl_x^u$.

In a similar manner to derivation of $l_d^s(w,v)$ and $l_d^u(w,v)$, we can obtain $l_x^s(w,v)$ and $l_x^u(w,v)$ as follows

$$l_x^s(w,v) = \frac{\phi f_x}{(w/v)^{(1-\phi)}},$$

$$l_x^u(w,v) = \frac{(1-\phi) f_x}{(v/w)^{\phi}},$$

where $\phi \in (0,1)$. Hence, exporters must pay the following fixed costs in addition to FC_d

$$FC_x(w,v) = w^{1-\phi} v^{\phi} f_x.$$

Then, we can obtain the total cost function of exporters as follows

$$TC^e(y_t^e, w, v) = (2wv\gamma f y_t)^{1/2} + w^{1-\alpha} v^{\alpha} f_d + w^{1-\phi} v^{\phi} f_x,$$

where y_t^e represents the total output of exporters. Exporters sell to consumers in domestic and foreign countries by using y_t^e units of output. To produce y_t^e units of output, exporters input $l_t^{e,u}$ units of unskilled labor and $l_t^{e,s}$ units of skilled labor. $l_t^{e,u}$ and $l_t^{e,s}$ are defined as $l_t^{e,u} = l_p^{e,u} + l_d^{e,u} + l_x^{e,u}$ and $l_t^{e,s} = l_p^{e,s} + l_d^{e,s} + l_x^{e,s}$, respectively

The final good market-clearing conditions for nonexporters and exporters of the home country are given by $c_{ne} = y^{ne}$ and $y_t^e = y_d^e + y_x^e = c_e + \tau c'^*_e$, respectively, where c'^*_e represents the consumption by foreign consumers of imported brands from the home country. The superscript * represents the economic entities of the foreign country and the superscript $'$ represents imported brands.

Price index P_T (dual to the aggregator C), which representative households face, is given by $P_T = \left[\int_{\theta\in\Theta} (p_d(\theta))^{1-\sigma}\, d\theta + \int_{\theta^*\in\Theta^*} [\tau p_d(\theta^*)]^{1-\sigma}\, d\theta^*\right]^{1/(1-\sigma)}$, where nonexporters and exporters of the home country set a price for consumers in the home country of p_d^{ne} and p_d^e, respectively. Exporters in the home country set a mill price of τp_d^e for consumers in the foreign country. Hence, the optimal pricing rules of nonexporters and exporters are given by $p_d^{ne} = \mu MC(y^{ne}, w, v)$ and $p_d^e = \mu MC(y^e, w, v)$, respectively. The free-entry and free-exit conditions of nonexporters and exporters are given by $p_d^{ne} = AC(y^{ne}, w, v)$ and $p_d^e = AC(y^e, w, v)$, respectively.

3.2 Trading equilibrium

Nonexporters face the same type of optimal pricing rule and free-entry and free-exit conditions as $PP_{A|v/w}$ of (1) and $FE_{A|v/w}$ of (2). Hence, $y^{ne}_{T|v/w}$, $(p_d/w)^{ne}_{T|v/w}$, $t^{ne}_{T|v/w}$, $l^{ne,s}_{t,T|v/w}$, and $l^{ne,u}_{t,T|v/w}$ are the same values as those of $y_{A|v/w}$, $(p/w)_{A|v/w}$, $t_{A|v/w}$, $l^s_{t,A|v/w}$, and $l^u_{t,A|v/w}$, respectively.

On the other hand, exporters face a different type of free-entry and free-exit condition from $FE_{A|v/w}$ of (2) while they face the same type of optimal pricing rule as $PP_{A|v/w}$ of (1). We can obtain the free-entry and free-exit condition that exporters face by replacing $(v/w)^\alpha(f_d/y)$ with $(v/w)^\alpha(f_d/y) + (v/w)^\phi(f_x/y)$ in $FE_{A|v/w}$ of (2). Hence, we can obtain $y^e_{T|v/w}$, $(p_d/w)^e_{T|v/w}$, $t^e_{T|v/w}$, $l^{e,s}_{t,T|v/w}$, and $l^{e,u}_{t,T|v/w}$ by replacing $(v/w)^\alpha(f_d/y)$ with $(v/w)^\alpha(f_d/y) + (v/w)^\phi(f_x/y)$ in $y_{A|v/w}$, $(p/w)_{A|v/w}$, $t_{A|v/w}$, $l^s_{t,A|v/w}$, and $l^u_{t,A|v/w}$, respectively.

For our later analysis, we introduces notation G_x.

Definition 2.

$$G_x \equiv \left(\frac{v}{w}\right)^\phi f_x$$

By using G_x, the free-entry and free-exit condition that exporters face can be rewritten as follows

$$FE^e_T : \left(\frac{p_d}{w}\right)^e_{T|v/w} = \left(\frac{v}{w}\frac{2\gamma f}{y^e_t}\right)^{1/2} + \frac{G_d + G_x}{y^e_t}. \tag{3}$$

Hence, we can obtain $y^e_{T|v/w}$, $(p_d/w)^e_{T|v/w}$, $t^e_{T|v/w}$, $l^{e,s}_{t,T|v/w}$, and $l^{e,u}_{t,T|v/w}$ by replacing G_d with $G_d + G_x$ in $y_{A|v/w}$, $(p/w)_{A|v/w}$, $t_{A|v/w}$, $l^s_{t,A|v/w}$, and $l^u_{t,A|v/w}$, respectively, as follows: $y^e_{t,T|v/w} = (w/v)(G_d + G_x)^2/(2\gamma f B^2)$, $\left(\frac{p^e_d}{w}\right)_{T|v/w} = (v/w)[B(B+1)\gamma f]/(G_d+G_x)$, $t^e_{T|v/w} = (w/v)(G_d + G_x)/(2fB)$, $l^s_{t,T|v/w} = (w/v)(G_d + G_x)[\alpha + 1/(2B)]$, and $l^u_{t,T|v/w} = (w/v)(G_d + G_x)[1 - \alpha + 1/(2B)]$.

For all $(v/w) > 0$, $y^e_{t,T} > y^{ne}_T$, $(p^e_d/w)_T < (p^{ne}_d/w)_T$, $t^e_T > t^{ne}_T$, $l^{e,s}_{t,T} > l^{ne,s}_{t,T}$, and $l^{e,u}_{t,T} > l^{ne,u}_{t,T}$ hold.

$y^e_{t,T} > y^{ne}_T$ and $(p^e_d/w)_T < (p^{ne}_d/w)_T$ are explained in Figure 1.

$y^e_{t,T} > y^{ne}_T$ implies $t^e_T > t^{ne}_T$. Hence, we can obtain the following proposition.

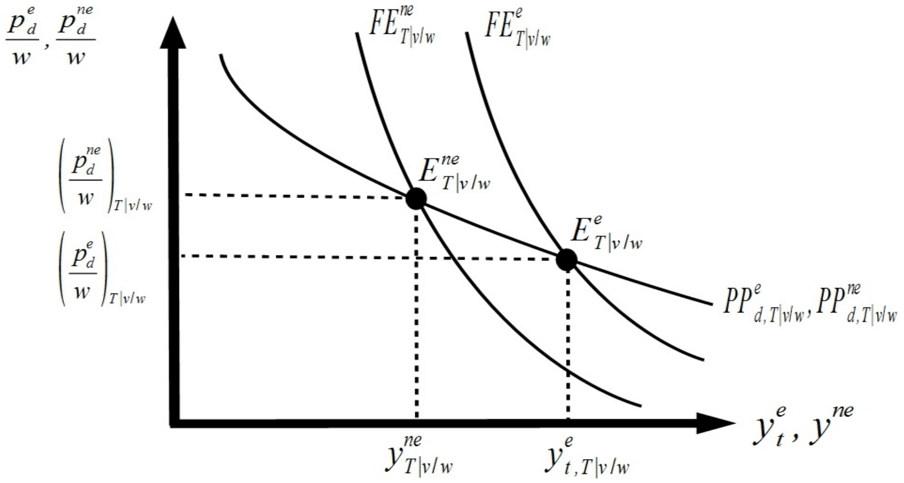

Figure 1: Tarding equilibrium in $(y, p/w)$ space.

Proposition 3. *For any* $(v/w) > 0$*, the division of labor of exporters is stronger than that of nonexporters.*

From Proposition 1, $t_T^e > t_T^{ne}$ implies $MC_T^e < MC_T^{ne}$. Hence, productivity of exporters is higher than that of nonexporters. This result is similar to Yeaple (2005) and Bustos (2011) in the sense that exporters have more skill-intensive structures and adopt higher technology than nonexporters. This is because exporters must obtain more revenue than nonexporters in order to pay fixed export costs, FC_x, in addition to FC_d. This implies that the average productivity of this industry at trading equilibrium is higher than that at autarkic equilibrium.

In order to make $l^u_{t,T|v/w}$ and $l^s_{t,T|v/w}$ simple, we introduce the following notations.

Definition 3.

$$R_1 \equiv \alpha + \frac{1}{2B}, \;\; R_2 = 1 - \alpha + \frac{1}{2B}$$

$$Q_1 \equiv \phi + \frac{1}{2B}, \;\; Q_2 = 1 - \phi + \frac{1}{2B}$$

Using R_1, R_2, Q_1, and Q_2, we can rewrite $l^u_{t,T|v/w}$ and $l^s_{t,T|v/w}$ as follows

$$l^{e,s}_{t,T|v/w} = \frac{w}{v}\left(G_d R_1 + G_x Q_1\right),$$

$$l^{e,u}_{t,T|v/w} = G_d R_2 + G_x Q_2.$$

For the relationship among R_1, R_2, Q_1, and Q_2, we can obtain the following properties.

Lemma 1. *Under Assumption 1, the following conditions hold.*

1. $Q_1/Q_2 \geq R_1/R_2$ *is equivalent to* $\phi \geq \alpha$.

2. $Q_1/Q_2 < R_1/R_2$ is equivalent to $\phi < \alpha$.

Proof

$Q_1/Q_2 \geq R_1/R_2$ is equivalent to $(\phi - \alpha)(1 + 1/B) \geq 0$. $(\phi - \alpha)(1 + 1/B) \geq 0$ is equivalent to $\phi \geq \alpha$ under $B > 0$. $Q_1/Q_2 < R_1/R_2$ is equivalent to $(\phi - \alpha)(1 + 1/B) < 0$. $(\phi - \alpha)(1 + 1/B) < 0$ is equivalent to $\phi < \alpha$ under $B > 0$. Q.E.D.

By using Lemma 1, we can obtain the following properties for the relationship between skill intensity of headquarter division for the domestic market, α, and that for the export market, ϕ.

Proposition 4. *For any $v/w > 0$, the following conditions hold under Assumption 1.*

1. That the labor input of f_x is more skilled labor-intensive than that of f_d is equivalent to the total labor input of exporters being more skilled labor-intensive than that of nonexporters. That is, $\phi > \alpha$ is equivalent to $(l^{e,s}_{t,T|v/w} > l^{e,u}_{t,T|v/w}) > (l^{ne,s}_{t,T|v/w} > l^{ne,u}_{t,T|v/w})$.

2. That the labor input of f_x is more unskilled labor-intensive than that of f_d is equivalent to the total labor input of exporters being more unskilled labor-intensive than that of nonexporters. That is, $\phi < \alpha$ is equivalent to $(l^{e,s}_{t,T|v/w} > l^{e,u}_{t,T|v/w}) < (l^{ne,s}_{t,T|v/w} > l^{ne,u}_{t,T|v/w})$.

Proof. *$(l^{e,s}_{t,T|v/w} > l^{e,u}_{t,T|v/w}) > (l^{ne,s}_{t,T|v/w} > l^{ne,u}_{t,T|v/w})$ is equivalent to $(R_1 + Q_1)/(R_2 + Q_2) > R_1/R_2$. $(R_1 + Q_1)/(R_2 + Q_2) > R_1/R_2$ is equivalent to $Q_1/Q_2 > R_1/R_2$. $Q_1/Q_2 > R_1/R_2$ is equivalent to $\phi > \alpha$ under $B > 0$ from property 1 of Lemma 1. Similarly, $(l^{e,s}_{t,T|v/w} > l^{e,u}_{t,T|v/w}) < (l^{ne,s}_{t,T|v/w} > l^{ne,u}_{t,T|v/w})$ is equivalent to $Q_1/Q_2 < R_1/R_2$. $Q_1/Q_2 < R_1/R_2$ is equivalent to $\phi < \alpha$ under $B > 0$ from property 2 of Lemma 1. Q.E.D.*

The results of the case $\phi > \alpha$ are consistent with that of Davidson et al. (2013). In the model of Davidson et al. (2013), in order to enter the export market, exporters input the only skilled labor into the headquarter division for the export market. Then, exporters is more skilled intensive than non-exporters.

These results can be explained as follows. From $l^u_p(y, w, v) = [(v/w)(\gamma f y)/2]^{1/2}$ and $l^s_p(y, w, v) = [(w/v)(\gamma f y)/2]^{1/2}$, we can obtain $l^s_p(y, w, v)/l^u_p(y, w, v) = w/v$. This indicates that for all firms, the skill intensities of the labor input into the production division do not depend on firm size measured by output. That is, $l^{e,s}_p(y, w, v)/l^{e,u}_p(y, w, v) = l^{ne,s}_p(y, w, v)/l^{ne,u}_p(y, w, v)$ holds. For all firms, the skill intensities of f_d are the same. Therefore, differences in the skill intensity of total labor input between exporters and nonexporters arises from that of f_x.

Now, we characterize $(v/w)_T$. The relative number of exporters to nonexporters can be obtained from the final good market-clearing conditions of the exporters' good, $y^e_{T|v/w} = c_{e|v/w} + \tau c'^{*}_{e|v/w}$, and those of the nonexporters' good, $y^{ne}_{T|v/w} = c_{ne|v/w}$. These conditions and the optimal pricing conditions give the condition $RGMC$ (See Appendix B for the derivation), as follows

$$RGMC : \frac{y^e_{t,T|v/w}}{y^{ne}_{T|v/w}} = (1 + \tau^{1-\sigma})^{\frac{2}{2-\sigma}}. \tag{4}$$

In addition, optimal pricing conditions, free-entry and free-exit conditions of exporters, $FE^e_{T|v/w}$, and free-entry and free-exit conditions of nonexporters, $FE^{ne}_{T|v/w}$, give the relative number of exporters to nonexporters, RFE, as follows

$$RFE : \frac{y^e_{t,T|v/w}}{y^{ne}_{T|v/w}} = \left(1 + \frac{G_x}{G_d}\right)^2. \tag{5}$$

RGMC and RFE gives the following equation:

$$1 + \tau^{1-\sigma} = \left(1 + \frac{G_x}{G_d}\right)^{2-\sigma}. \tag{6}$$

(6) characterizes G_x/G_d. From the definition of G_x/G_d, when G_x/G_d is determined, $(v/w)_T$ is determined simultaneously.

We define H as $H \equiv (1 + \tau^{1-\sigma})^{1/(2-\sigma)}$, where $H > 1$ holds from $\tau > 1$ and $1 < \sigma < 2$ of Assumption 1. By using H, we can rewrite (6) as follows:

$$\left(\frac{v}{w}\right)_T = \left[(H - 1)\frac{f_d}{f_x}\right]^{1/(\phi-\alpha)}. \tag{7}$$

(7) characterizes $(v/w)_T$. When $(v/w)_T$ is determined, $y^e_{t,T}$, y^{ne}_T, t^e_T, t^{ne}_T, $l^{e,s}_{t,T}$, $l^{ne,s}_{t,T}$, $l^{e,u}_{t,T}$, $l^{ne,u}_{t,T}$ $(p^e_d/w)_T$, and $(p^{ne}_d/w)_T$ can be characterized.

To focus on G_x/G_d clarifies the comparison of the behavior of exporters and nonexporters. Figure 2 describes how G_x/G_d is determined and shows the relationship between the RGMC of (4) and the RFE of (5).

$RGMC$ describes the vertical line in the following result. $y^e_{t,T|v/w}/y^{ne}_{T|v/w}$ depends on $(1 + \tau^{1-\sigma})$ and $p^e_{d,T|v/w}/p^{ne}_{d,T|v/w}$ because $y^e_{t,T|v/w}/y^{ne}_{T|v/w}$ is equal to $c_{e|v/w} + \tau c'^*_{e|v/w}/c_{ne|v/w}$. $p^e_{d,T|v/w}/p^{ne}_{d,T|v/w}$ does not depend on v/w. Therefore, $y^e_{t,T|v/w}/y^{ne}_{T|v/w}$ sticks to $(1 + \tau^{1-\sigma})$.

RFE describes the upward right curve in the following result. As G_x/G_d increase, so does FC_x relative to FC_d. Then, exporters must expand firm size and reduce their average costs to survive. Therefore, as G_x/G_d increases, so does $y^e_{t,T|v/w}/y^{ne}_{T|v/w}$.

We let z represent the proportion of exporters to all firms. We focus an an internal solution and let z belong to a set of $(0, 1)$. The labor market-clearing conditions of unskilled

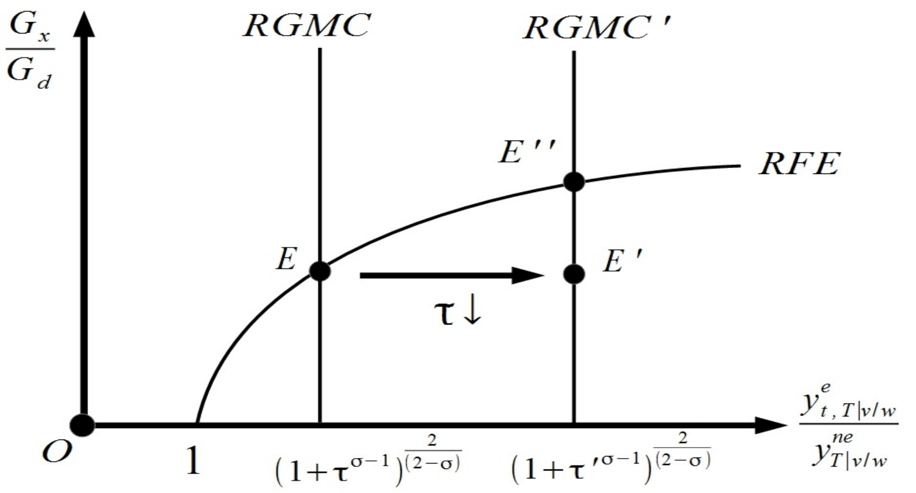

Figure 2: Relative final good market-clearing and free-entry and free-exit conditions.

and skilled labor are given by

$$L = (1-z)Ml^{ne,u}_{t,T} + zMl^{e,u}_{t,T},$$

and

$$K = (1-z)Ml^{ne,s}_{t,T} + zMl^{e,s}_{t,T},$$

respectively. These equations formulate simultaneous equations, in which z and M are unknown variables. We can solve these equations as follows

$$z_T = \frac{G_d}{G_x}\frac{R_1 - R_2\frac{vK}{wL}}{Q_2\frac{vK}{wL} - Q_1}, \tag{8}$$

$$M_T = \frac{1}{G_d}\frac{KQ_2\frac{v}{w} - LQ_1}{Q_2R_1 - Q_1R_2}. \tag{9}$$

z_T of (8) or M_T of (9) may be negative. Furthermore, z_T of (8) may be more than 1. Then, in order to guarantee the existence of the internal solution, we impose the following assumption. .

Assumption 2. *We assume $\phi \neq \alpha$ and the following conditions hold for v/w characterized by (7).*

1. *When $\phi > \alpha$ holds, $(R_1/R_2) < (vK/wL) < (Q_1+R_1)/(Q_2+R_2)$ holds.*
2. *When $\phi < \alpha$ holds, $(R_1/R_2) > (vK/wL) > (Q_1+R_1)/(Q_2+R_2)$ holds.*

Then, we can obtain the internal solution as follows.

Proposition 5. *If and only if Assumption 1 and 2 hold, an equilibrium that certifies an internal point is determined uniquely.*

Proof: See Appendix C.

These results can be explained as follows. Proposition 5 requires $(vK/wL) < (Q_1 + R_1)/(Q_2 + R_2)$ under $\phi > \alpha$. This requires that the factor rewards, vK, are sufficiently small relative to the factor rewards, wL. That is, it requires that K is sufficiently small relative to L, and that τ and f_x are sufficiently large relative to f_d. Otherwise, all firms export.

On the other hand, Proposition 5requires that $(vK/wL) > (R_1/R_2)$ holds for (vK/wL). That is, this requires that the factor rewards, vK, are sufficiently large relative to the factor rewards, wL. Otherwise, all firms enter only the domestic market.

That is, Proposition 5 indicates that under $\phi > \alpha$, if (vK/wL) is not too large and c is also sufficiently small, exporters and nonexporters coexist. We can consider the case of $\phi < \alpha$ in a similar manner.

What about the case of $\phi = \alpha$? There is little possibility that the internal solution exists in which exporters and nonexporters coexist; this is because such a solution exists only when $1+\tau^{1-\sigma} = (1+f_x/f_d)^{2-\sigma}$ holds. Furthermore, even if only $1+\tau^{1-\sigma} = (1+f_x/f_d)^{2-\sigma}$ holds, the internal solution is not determined uniquely. In this sense, the assumption of $\phi \neq \alpha$ is critical. That is, the difference in skill intensities between exporters and nonexporters is necessary to guarantee the unique internal solution.

4 Trade Liberalization

We define trade liberalization as a decrease in variable trade cost, τ, or fixed trade cost, f_x. We consider the effects of these changes on the division of labor, relative firm size, skill intensity, relative skill intensity, number of firms, share of the number of exporters, and so on.

Before such an analysis, we consider the impacts of a decrease in τ and f_x on $(G_x/G_d)_T$ and $(v/w)_T$.

Lemma 2. *Under Assumption 1, the following properties hold.*

1. A decrease in τ raises $(G_x/G_d)_T$ in both case of $\phi > \alpha$ and $\phi < \alpha$.

2. When $\phi > \alpha$ holds, a decrease in τ raises $(v/w)_T$, while when $\phi < \alpha$ holds, a decrease in τ reduces $(v/w)_T$.

3. A decrease in f_x does not change $(G_x/G_d)_T$.

4. When $\phi > \alpha$ holds, a decrease in f_x raises $(v/w)_T$, while when $\phi < \alpha$ holds, a decrease in f_x reduces $(v/w)_T$

Proof. See Appendix D.

These properties can be explained as follows.

We consider property 1. This represents an increase in the marginal revenue of exporters relative to that of nonexporters. Hence, a decrease in τ shifts the $RGMC$ line to the right. If we fix $(G_x/G_d)_T$, we can obtain a new point E'. At this point, exporters have profits while nonexporters have losses. Hence, some nonexporters enter export markets while some nonexporters exit these markets. In order to stop this entry and exit, $(G_x/G_d)_T$ needs to rise.

We consider property 2. Although a decrease in τ has the same impact on $(G_x/G_d)_T$, regardless of $\phi > \alpha$ and $\phi < \alpha$, the change in $(G_x/G_d)_T$ has different impacts on $(v/w)_T$, depending on the relationship between $\phi > \alpha$ and $\phi < \alpha$. Hence, a decrease in τ raises $(G_x/G_d)_T$ in both case of $\phi > \alpha$ and $\phi < \alpha$. This means that a decrease in τ raises the relative factor reward of the type of labor inputted intensively in head offices for export markets. Note that trade liberalization brings skill premiums under $\phi > \alpha$ while it removes skill premiums under $\phi < \alpha$.

We consider properties 3 and 4. A decrease in f_x does not change directly the marginal revenue of exporters relative to that of nonexporters. Then, changes in f_x do not affect $(G_x/G_d)_T$. Since eq (6) keeps $(G_x/G_d)_T$ constant, changes in f_x affect $(v/w)_T$, depending on the relationship between $\phi > \alpha$ and $\phi < \alpha$, so that a decrease in f_x raises the relative factor reward of the type of labor inputted intensively in the head offices for export markets.

We now consider the effect of a decrease in τ on allocations.

Proposition 6. *Under Assumption 1 and 2, we can obtain the following properties.*

1. In the case of $\phi > \alpha$, a decrease in τ raises the labor input of unskilled workers and reduces the number of teams, as well as the labor input of skilled workers, for both exporters and nonexporters. In the case of $\phi < \alpha$, a decrease in τ reduces the labor input of unskilled workers and raises the number of teams, as well as the labor input of skilled workers, for both exporters and nonexporters. In the case of $\alpha > 1/2$, a decrease in τ raises the output of nonexporters. In the case of $(2\alpha - 1)G_d + (2\phi - 1)G_x > 0$, a decrease in τ raises the output of exporters.

2. In both cases of $\phi > \alpha$ and $\phi < \alpha$, a decrease in τ raises the ratio of exporters to nonexporters in terms of output, number of teams, and labor input for both skilled workers and unskilled workers.

3. In the case of $\phi > \alpha$, a decrease in τ raises the ratio of exporters to nonexporters in terms of skill intensity, $(l^{e,s}_{t,T|v,w}/l^{e,u}_{t,T|v,w})/(l^{ne,s}_{t,T|v,w}/l^{ne,u}_{t,T|v,w})$. In the case of $\phi < \alpha$, a decrease in τ reduces that ratio.

4. In both cases of $\phi > \alpha$ and $\phi < \alpha$, a decrease in τ raises the number of exporters *and reduces the number of nonexporters and all firms.*

Proof. See Appendix E.

These properties can be explained as follows.

We consider property 1. Under $\phi > \alpha$, a decrease in τ raises $(v/w)_T$, and hence, raises the labor input of unskilled workers and reduces the number of teams and the labor input for skilled workers for both exporters and nonexporters. Under $\phi < \alpha$, a similar mechanism works.

We consider property 2. A decrease in τ raises G_x/G_d from Lemma 2. This expands the firm size of exporters relative to that of nonexporters, and hence, raises both l^s and l^u relative to nonexporters.

We consider property 3. Under $\phi > \alpha$, a decrease in τ raises $(v/w)_T$. This raise the ratio of exporters to nonexporters in skill intensity. Under $\phi > \alpha$, we can undertake a similar analysis.

We consider property 4 by focusing on labor reallocation. In the case of $\phi > \alpha$, the skilled labor input of all firms decreases while the unskilled labor input increases. That is, new exporters that have skill-intensive head offices for the export market absorb skilled labor from all other firms, while all the surviving firms absorb unskilled labor from exiting nonexporters that have unskilled labor-intensive head offices for the domestic market. In the case of $\phi < \alpha$, the skilled labor input of all firms increases while the unskilled labor input increases. That is, new exporters that have unskilled labor-intensive head offices for the export market absorb unskilled labor from all other firms, while all surviving firms absorb skilled labor from exiting nonexporters that have skilled labor-intensive head offices for the domestic market.

We now consider the effect of a decrease in f_x on allocations.

Proposition 7. *Under Assumption 1 and 2, we obtain the following properties.*

1. *The same properties as 1 and 4 of Proposition 6 hold.*
2. *A decrease in f_x/f_d does not change relative skill intensity.*

Proof

Property 1: In the case of $\phi > \alpha$, a decrease in f_x/f_d raises $(v/w)_T$ while in the case of $\phi < \alpha$, a decrease in f_x/f_d reduces $(v/w)_T$. These changes are the same as the results of Property 1 of Lemma 2.

Property 2: A decrease in f_x/f_d does not change $(G_x/G_d)_T$, as shown in Property 3 of Lemma 2, and hence, does not change relative skill intensity.

Q.E.D.

These results indicate that a decrease in f_x/f_d affects each type of firm's allocation but does not affect the ratio of both types of labor inputs and skill intensities. This is because a decrease in f_x/f_d does not change the marginal revenue of exporters relative to nonexporters and hence, also does not change $(G_x/G_d)_T$.

We summarize the abovementioned analysis. Propositions 6 and 7 indicate the following properties. Regardless of a decrease in variable and fixed trade cots, or the structure of head offices, the number of exporters increases while the numbers of nonexporters and all firms decrease. Furthermore, all firms reduce the type of labor inputted intensively into head quarter division for the export market while raising the type of labor inputted less intensively into that division. These results are robust in this model. Whether the inequalities in labor input, output, and skill intensity between exporters and nonexporters expand depends critically on whether trade liberalization affects firms' marginal revenue directly.

5 Conclusion

This paper theoretically investigates the relationship between export decisions and decisions about the extent of the division of labor within firms. We incorporate two types of labor and two types of fixed costs composed of the two types of labor into the model of Chaney and Ossa (2013).

All firms are ex-ante identical. To guarantee a unique equilibrium in which exporters and nonexporters coexist, skill intensity of the two types of fixed costs needs to be different. The division of labor of exporters is stronger than that of nonexporters. That the fixed labor input of head offices for the export market is more skill intensive than that for the domestic market is equivalent to the total labor input of exporters being more skill intensive than that of nonexporters. The number of exporters increases while the numbers of nonexporters and all firms decrease. Furthermore, all firms reduce the type of labor inputted intensively into head quarter division for the export market while raising the type of labor inputted less intensively into that division. These results are robust in this model. Whether the inequalities in labor input, output, and skill intensity between exporters and nonexporters expand depends critically on whether trade liberalization affects firms' marginal revenue directly.

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Appendix

Appendix A: Proof of Proposition 1

$VC(t,y)/w$ can be rearranged as follows:

$$\frac{VC(t,y)}{w} = l_p^u(t(y),y) + \frac{v}{w}ft(y) = \frac{\gamma y}{2t(y)} + \frac{v}{w}ft(y).$$

By using this equation, we can obtain the following equations:

$$\begin{aligned} MC(t,y)/w &= \frac{d[VC(t,y)/w]}{dy} \\ &= \frac{\partial[VC(t,y)/w]}{\partial y} + \underbrace{\frac{\partial[VC(t,y)/w]}{\partial t}}_{0}\frac{dt(y)}{dy} \\ &= \frac{\partial[VC(t,y)/w]}{\partial y} \\ &= \frac{\gamma}{2t}. \end{aligned}$$

Q.E.D.

Appendix B: Derivation of RGMC (4)

We derives optimal consumptions to derive eq (4). We define income of representative households in home country as I, where $I = wL + vK$. Then, optimal consumptions of representative households in home country, $c_{ne|v/w}$ and $c_{e|v/w}$, are given as follows:

$$c_{ne|v/w} = (p_d^{ne})^{-\sigma}(P_T)^{\sigma-1}I, \tag{B.1}$$

$$c_{e|v/w} = (p_d^{e})^{-\sigma}(P_T)^{\sigma-1}I. \tag{B.2}$$

We can obtain optimal consumption of representative households in foreign country for the imported brands, ${c'}^*_{e|v/w}$, as follows:

$$\begin{aligned} {c'}^*_{e|v/w} &= {c'}_{e|v/w} && \text{by symmetry of countries} \\ &= (\tau p_d^{e*})^{-\sigma}(P_T)^{\sigma-1}I \\ &= (\tau p_d^{e})^{-\sigma}(P_T)^{\sigma-1}I. && \text{by symmetry of countries} \end{aligned} \tag{B.3}$$

Equation (4) can be obtained from theses optimal consumptions, $c_{e|v/w}$, $\tau {c'}^*_{e|v/w}$, $c_{ne|v/w}$, final good market clearing conditions of exporter's good, $y^e_{T|v/w} = c_{e|v/w} + \tau {c'}^*_{e|v/w}$, those of non-exporter's good, $y^{ne}_{T|v/w} = c_{ne|v/w}$, and optimal pricing, $(p^e_d/w)_{T|v/w}$, $(p^{ne}_d/w)_{T|v/w}$ as follows.

$$\begin{aligned}
\frac{y^e_{T|v/w}}{y^{ne}_{T|v/w}} &= \frac{c_{e|v/w} + \tau c'^{*}_{e|v/w}}{c_{ne|v/w}} \quad \text{by final market conditions} \\
&= (1+\tau^{1-\sigma}) \left(\frac{p^e_{d,T|v/w}}{p^{ne}_{d,T|v/w}} \right)^{-\sigma} \quad \text{by optimal consumptions} \\
&= (1+\tau^{1-\sigma}) \left(\frac{(\mu/2)[(v/w)(2\gamma f)/y^e_{T|v/w}]^{1/2}}{(\mu/2)[(v/w)(2\gamma f)/y^{ne}_{T|v/w}]^{1/2}} \right)^{-\sigma} \quad \text{by optimal pricing rules} \\
&= (1+\tau^{1-\sigma}) \left(\frac{y^e_{T|v/w}}{y^{ne}_{T|v/w}} \right)^{\sigma/2}.
\end{aligned}$$

This gives eq (4).

Appendix C: Proof of Proposition 5

Assumption 1 implies $H > 0$ and $H > 0$ implies $(v/w)_T > 0$. $(v/w)_T > 0$ implies that variables of $y^e_{t,T}$, y^{ne}_T, t^e_T, t^{ne}_T, $l^{e,s}_{t,T}$, $l^{ne,s}_{t,T}$, $l^{e,u}_{t,T}$, $l^{ne,u}_{t,T}$, $(p^e_d/w)_T$, and $(p^{ne}_d/w)_T$ are positive.

We indicates the following conditions under 1 and 2.

- ***Property 1.*** *$z_T > 0$ implies $M_T > 0$*

- ***Property 2.*** *Under $\phi < \alpha$,*

$$0 < z_T < 1 \;\; \leftrightarrow \;\; \frac{Q_1 + R_1}{Q_2 + R_2} < \frac{vK}{wL} < \frac{R_1}{R_2}.$$

- ***Property 3.*** *Under $\phi > \alpha$,*

$$0 < z_T < 1 \;\; \leftrightarrow \;\; \frac{R_1}{R_2} < \frac{vK}{wL} < \frac{Q_1 + R_1}{Q_2 + R_2}.$$

- ***Property 4.*** *Under $\phi = \alpha$, an equilibrium which grantees the internal solution exists only when the following condition holds*

$$1 + \tau^{1-\sigma} = \left(1 + \frac{f_x}{f_d}\right)^{2-\sigma}.$$

Even if the following condition just holds, the internal solution is not uniquely determined.

Proof of Property 1.

$M_T > 0$ holds when the numerator and the denominator of M_T in (9) are positive, or negative together. Hence, $M_T > 0$ is equivalent to the following condition.

$$\left[\frac{R_1}{R_2} > \frac{Q_1}{Q_2} \ \wedge \ \frac{vK}{wL} > \frac{Q_1}{Q_2}\right] \ \vee \ \left[\frac{R_1}{R_2} < \frac{Q_1}{Q_2} \ \wedge \ \frac{vK}{wL} < \frac{Q_1}{Q_2}\right]. \tag{C.1}$$

$z_T > 0$ holds when the numerator and the denominator of z_T in (8) are positive, or negative together. Hence, $z_T > 0$ is equivalent to the following condition.

$$\left[\frac{vK}{wL} < \frac{R_1}{R_2} \ \wedge \ \frac{vK}{wL} > \frac{Q_1}{Q_2}\right] \ \vee \ \left[\frac{vK}{wL} > \frac{R_1}{R_2} \ \wedge \ \frac{vK}{wL} < \frac{Q_1}{Q_2}\right].$$

This condition can be rewritten as follows:

$$\left[\frac{Q_1}{Q_2} < \frac{vK}{wL} < \frac{R_1}{R_2}\right] \ \vee \ \left[\frac{R_1}{R_2} < \frac{vK}{wL} < \frac{Q_1}{Q_2}.\right] \tag{C.2}$$

(C.2) implies (C.1) and hence, $z_T > 0$ implies $M > 0$. Q.E.D.

Proof of Property 2.

When the numerator and the denominator of z_T in (8) are positive together, $z_T > 0$ is equivalent to $(Q_1/Q_2) < (vK)/(wL) < (R_1/R_2)$ under $\phi < \alpha$ from property 2 of Lemma 1. Then, $z < 1$ is equivalent to the following condition:

$$\frac{vK}{wL} > \frac{Q_1 + R_1}{Q_2 + R_2}. \tag{C.3}$$

We should note that the following condition holds:

$$\frac{Q_1 + R_1}{Q_2 + R_2} - \frac{Q_1}{Q_2} = \frac{Q_2 R_1 - Q_1 R_2}{Q_2(Q_2 + R_2)} > 0.$$

This indicates that $z_T < 1$ implies $z_T > 0$. Hence, Property 2 holds. Q.E.D.

Proof of Property 3.

When the numerator and the denominator of z_T in (8) are negative together, $z_T > 0$ is equivalent to $(Q_1/Q_2) > (vK)/(wL) > (R_1/R_2)$ under $\phi > \alpha$ from property 1 of Lemma

1. Then, $z < 1$ is equivalent to the following condition:

$$z < 1 \ \leftrightarrow \ \frac{vK}{wL} < \frac{Q_1 + R_1}{Q_2 + R_2}.$$

We should note that the following condition holds:

$$\frac{Q_1 + R_1}{Q_2 + R_2} - \frac{Q_1}{Q_2} = \frac{Q_2 R_1 - Q_1 R_2}{Q_2(Q_2 + R_2)} < 0.$$

This indicates that $z_T < 1$ implies $z_T > 0$. Hence, Property 3 holds. Q.E.D.

Proof of Property 4.

Under $\phi = \rho$, $R_1 = R_2$ and $Q_1 = Q_2$ hold from results of Lemma 1. Then, we may drop subscripts of R and Q. We can obtain $G_x/G_d = f_x/f_d$ from the definition of G_d and G_x. Then, (6) can be rewritten as follows:

$$1 + \tau^{1-\sigma} = \left(1 + \frac{f_x}{f_d}\right)^{2-\sigma}. \tag{C.4}$$

This indicates that if (C.4) does not just hold, there is not the internal solution.

Furthermore, (C.4) does not determine $(v/w)_T$ uniquely. This result is different form (6). Therefore, in this case, the equilibrium are determined uniquely. Q.E.D.

Appendix D: Proof of Lemma 2

Properties 1 and 2

From $1 < \sigma < 2$ of Assumption 1 , we can get the following condition:

$$\frac{dH}{d\tau} = \frac{1}{2-\sigma}(1 + \tau^{1-\sigma})^{(\sigma-1)/(2-\sigma)} \underbrace{(1-\sigma)}_{-} \tau^{-\sigma} < 0.$$

First, we consider a case of $\phi > \alpha$. Then, we can get the following conditions:

$$\frac{d(v/w)}{dH} = \frac{1}{\underbrace{\phi - \alpha}_{+}} \left[(H-1)\frac{f_d}{f_x}\right]^{1/(\phi-\alpha)} \frac{f_d}{f_x} > 0,$$

$$\frac{d(G_x/G_d)}{d(v/w)} = \underbrace{(\phi - \alpha)}_{+} \left(\frac{v}{w}\right)^{(\phi-\alpha)-1} \frac{f_x}{f_d} > 0.$$

From these conditions, we can obtain

$$\frac{d(G_x/G_d)}{d\tau} = \underbrace{\frac{d(G_x/G_d)}{d(v/w)}}_{+} \underbrace{\frac{d(v/w)}{dH}}_{+} \underbrace{\frac{dH}{d\tau}}_{-} < 0.$$

Next, we consider a case of $\phi < \alpha$. From the above relations, we can immediately obtain $d(v/w)/dH < 0$ and $d(G_x/G_d)/d(v/w) < 0$. Hence, we can get the following condition:

$$\frac{d(G_x/G_d)}{d\tau} = \underbrace{\frac{d(G_x/G_d)}{d(v/w)}}_{-} \underbrace{\frac{d(v/w)}{dH}}_{-} \underbrace{\frac{dH}{d\tau}}_{-} < 0.$$

Q.E.D.

Properties 3 and 4

Eq (6) indicates that $(G_x/G_d)_T$ does not depend on f_x. Hence, properties 3 is proved.

By differentiating eq (7) for f_x, we can obtain

$$\frac{d\left(\frac{v}{w}\right)_T}{df_x} = \frac{1}{\phi - \alpha}\left[(H-1)\frac{f_d}{f_x}\right]^{1/(\phi-\alpha)-1}(-1)\frac{(H-1)f_d}{f_x^2}.$$

Hence, under $\phi > \alpha$, $d[(v/w)_T]/df_x < 0$ while under $\phi < \alpha$, $d[(v/w)_T]/df_x > 0$. Q.E.D.

Appendix E: Proof of Proposition 6

Proof of Property 1.

From $y^{ne}_{T|v/w}$, we can obtain the following equation:

$$\frac{dy^{ne}_{T|v/w}}{d(v/w)} = (2\alpha - 1)\left(\frac{v}{w}\right)^{2(\alpha-1)}\frac{f_d}{2\gamma f B^2}.$$

This indicates that under $\alpha > 1/2$, $dy^{ne}_{t,T|v/w}/d(v/w) > 0$ while under$\alpha \leq 1/2$, $dy^{ne}_{t,T|v/w}/d(v/w) \leq 0$.

From $y^{e}_{T|v/w}$, we can obtain the following equation:

$$\frac{dy^{e}_{t,T|v/w}}{d(v/w)} = \frac{(v/w)^2(G_d + G_x)}{2\gamma f B^2}[(2\alpha - 1)G_d + (2\phi - 1)G_x].$$

This indicates that under $(2\alpha - 1)G_d + (2\phi - 1)G_x > 0$, $dy^{e}_{t,T|v/w}/d(v/w) > 0$ while under $(2\alpha - 1)G_d + (2\phi - 1)G_x \leq 0$, $dy^{e}_{t,T|v/w}/d(v/w) \leq 0$.

By differentiating $t^{ne}_{T|v/w}$, $t^{e}_{T|v/w}$, $l^{ne,s}_{T|v/w}$, $l^{e,s}_{T|v/w}$, $l^{ne,u}_{T|v/w}$, and $l^{e,u}_{T|v/w}$ for (v/w), we can obtain the following relations:

$$\frac{dt^{ne}_{T|v/w}}{d(v/w)} = (\alpha-1)\left(\frac{v}{w}\right)^{\alpha-2}\frac{f_d}{2fB} < 0,$$

$$\frac{dt^{e}_{T|v/w}}{d(v/w)} = \frac{(\alpha-1)\left(\frac{v}{w}\right)^{\alpha-2}f_d + (\phi-1)\left(\frac{v}{w}\right)^{\phi-2}f_x}{2fB} < 0,$$

$$\frac{dl^{ne,s}_{T|v/w}}{d(v/w)} = (\alpha-1)\left(\frac{v}{w}\right)^{\alpha-2}f_d R_1 < 0,$$

$$\frac{dl^{e,s}_{t,T|v/w}}{d(v/w)} = (\alpha-1)\left(\frac{v}{w}\right)^{\alpha-2}f_d R_1 + (\phi-1)\left(\frac{v}{w}\right)^{\phi-2}f_x Q_1 < 0,$$

$$\frac{dl^{ne,u}_{T|v/w}}{d(v/w)} = \alpha\left(\frac{v}{w}\right)^{\alpha-2}f_d R_2 > 0,$$

$$\frac{dl^{e,u}_{t,T|v/w}}{d(v/w)} = \alpha\left(\frac{v}{w}\right)^{\alpha-2}f_d R_2 + \phi\left(\frac{v}{w}\right)^{\phi-2}f_x Q_2 > 0.$$

Q.E.D.

Proof of Property 2.

In Lemma (2), we analyzed $d(G_x/G_d)/d\tau$. In the following analysis, we can make analysis easier and clearer by using $d(G_x/G_d)/d\tau$.

$y^{e}_{t,T|v/w}$ can be rewritten as follows:

$$y^{e}_{t,T|v/w} = \frac{w}{v}\frac{(G_d+G_x)^2}{2\gamma f B^2} = \left(1+\frac{G_x}{G_d}\right)y^{ne}_{t,T}.$$

Hence, we can obtain

$$\frac{d(y^{e}_{t,T|v/w}/y^{ne}_{t,T|v/w})}{d\tau} = \underbrace{\frac{d(y^{e}_{t,T|v/w}/y^{ne}_{t,T|v/w})}{d(G_x/G_d)}}_{+}\underbrace{\frac{d(G_x/G_d)}{d\tau}}_{-} < 0.$$

$\left(\frac{p}{w}\right)^{e}_{d,T|v/w}$ can be rewritten as follows:

$$\left(\frac{p}{w}\right)^{e}_{d,T|v/w} = \frac{v}{w}\frac{B(B+1)\gamma f}{G_d+G_x} = \left(\frac{p}{w}\right)^{ne}_{T|v/w}\frac{1}{1+G_x/G_d}.$$

Hence, we can obtain

$$\frac{d\left[\left(\frac{p}{w}\right)^{e}_{d,T|v/w} / \left(\frac{p}{w}\right)^{ne}_{T|v/w}\right]}{d\tau} = \underbrace{\frac{d\left[\left(\frac{p}{w}\right)^{e}_{d,T|v/w} / \left(\frac{p}{w}\right)^{ne}_{T|v/w}\right]}{d(G_x/G_d)}}_{-} \underbrace{\frac{d(G_x/G_d)}{d\tau}}_{-} > 0.$$

$t^{e}_{T|v/w}$ can be rewritten as follows:

$$t^{e}_{T|v/w} = \frac{w}{v}\frac{G_d + G_x}{2fB} = \left(1 + \frac{G_x}{G_d}\right) t^{ne}_{T|v/w}.$$

Hence, we can obtain

$$\frac{d(t^{e}_{T|v/w}/t^{ne}_{T|v/w})}{d\tau} = \underbrace{\frac{d(t^{e}_{T|v/w}/t^{ne}_{T|v/w})}{d(G_x/G_d)}}_{+} \underbrace{\frac{d(G_x/G_d)}{d\tau}}_{-} < 0.$$

$\frac{l^{e,s}_{t,A|v/w}}{l^{ne,s}_{t,A|v/w}}$ can be rewritten as follows:

$$\frac{l^{e,s}_{t,A|v/w}}{l^{ne,s}_{t,A|v/w}} = 1 + \frac{G_x}{G_d}\frac{Q_1}{R_1}.$$

Hence, we can obtain

$$\frac{d(l^{e,s}_{t,A|v/w}/l^{ne,s}_{t,A|v/w})}{d\tau} = \underbrace{\frac{d(l^{e,s}_{t,A|v/w}/l^{ne,s}_{t,A|v/w})}{d(G_x/G_d}}_{+} \underbrace{\frac{d(G_x/G_d)}{d\tau}}_{-} < 0.$$

$\frac{l^{e,u}_{t,A|v/w}}{l^{ne,u}_{t,A|v/w}}$ can be rewritten as follows:

$$\frac{l^{e,u}_{t,A|v/w}}{l^{ne,u}_{t,A|v/w}} = 1 + \frac{G_x}{G_d}\frac{Q_2}{R_2}.$$

Hence, we can obtain

$$\frac{d(l^{e,u}_{t,A|v/w}/l^{ne,u}_{t,A|v/w})}{d\tau} = \underbrace{\frac{d(l^{e,u}_{t,A|v/w}/l^{ne,u}_{t,A|v/w})}{d(G_x/G_d}}_{+} \underbrace{\frac{d(G_x/G_d)}{d\tau}}_{-} < 0.$$

Proof of Property 3.

In the following analysis, we can make analysis easier and clearer by using $d(G_x/G_d)/d\tau$ as proof of property 2.

$\frac{l^{e,s}_{t,T|v,w}/l^{e,u}_{t,T|v,w}}{l^{ne,s}_{t,T|v,w}/l^{ne,u}_{t,T|v,w}}$ can be rewritten as follows:

$$\frac{l^{e,s}_{t,T|v,w}/l^{e,u}_{t,T|v,w}}{l^{ne,s}_{t,T|v,w}/l^{ne,u}_{t,T|v,w}} = \frac{1+\frac{G_x}{G_d}\frac{Q_1}{R_1}}{1+\frac{G_x}{G_d}\frac{Q_2}{R_2}}.$$

Hence, we can obtain

$$d\left(\frac{l^{e,s}_{t,T|v,w}/l^{e,u}_{t,T|v,w}}{l^{ne,s}_{t,T|v,w}/l^{ne,u}_{t,T|v,w}}\right)/d(G_x/G_d) = \frac{\frac{Q_1}{R_1}-\frac{Q_2}{R_2}}{\left(1+\frac{G_x}{G_d}\frac{Q_2}{R_2}\right)^2}.$$

We should note

$$\frac{Q_1}{R_1}-\frac{Q_2}{R_2}>0 \ \leftrightarrow \ \frac{Q_1}{Q_2}>\frac{R_1}{R_2}.$$

Hence, under a case of $\phi > \alpha$, we can obtain

$$d\left(\frac{l^{e,s}_{t,T|v,w}/l^{e,u}_{t,T|v,w}}{l^{ne,s}_{t,T|v,w}/l^{ne,u}_{t,T|v,w}}\right)/d\tau = \underbrace{d\left(\frac{l^{e,s}_{t,T|v,w}/l^{e,u}_{t,T|v,w}}{l^{ne,s}_{t,T|v,w}/l^{ne,u}_{t,T|v,w}}\right)/d\left(\frac{G_x}{G_d}\right)}_{+} \times \underbrace{\frac{d(G_x/G_d)}{d\tau}}_{-} < 0,$$

and under a case of $\phi < \alpha$, we can obtain

$$d\left(\frac{l^{e,s}_{t,T|v,w}/l^{e,u}_{t,T|v,w}}{l^{ne,s}_{t,T|v,w}/l^{ne,u}_{t,T|v,w}}\right)/d\tau = \underbrace{d\left(\frac{l^{e,s}_{t,T|v,w}/l^{e,u}_{t,T|v,w}}{l^{ne,s}_{t,T|v,w}/l^{ne,u}_{t,T|v,w}}\right)/d\left(\frac{G_x}{G_d}\right)}_{-} \times \underbrace{\frac{d(G_x/G_d)}{d\tau}}_{-} > 0.$$

Proof of Property 4.

$z_T M_T$ can be rewritten as follows:

$$z_T M_T = \frac{R_1 L - K R_2 \frac{v}{w}}{G_x(Q_2 R_1 - Q_1 R_2)}.$$

We consider a case of $\alpha > \phi$.

$$\frac{d(z_T M_T)}{d(v/w)} = \underbrace{\frac{1}{Q_2 R_1 - Q_1 R_2}}_{+} \underbrace{\frac{-K R_2 G_x - \overbrace{(R_1 L - K R_2 \frac{v}{w})}^{+} f_x \phi (\frac{v}{w})^{\phi-1}}{G_x^2}}_{-} < 0.$$

Then, we can obtain

$$\frac{d(z_T M_T)}{d\tau} = \underbrace{\frac{d(z_T M_T)}{d(v/w)}}_{-} \underbrace{\frac{d(v/w)}{d\tau}}_{+} < 0.$$

We consider a case of $\phi > \alpha$.

$$\frac{d(z_T M_T)}{d(v/w)} = \underbrace{\frac{1}{(Q_2 R_1 - Q_1 R_2)}}_{-} \underbrace{\frac{-KR_2 G_x(1-\phi) - R_1 L f_x \phi(\frac{v}{w})^{\phi-1}}{G_x^2}}_{-} > 0.$$

Then, we can obtain

$$\frac{d(z_T M_T)}{d\tau} = \underbrace{\frac{d(z_T M_T)}{d(v/w)}}_{+} \underbrace{\frac{d(v/w)}{d\tau}}_{-} < 0.$$

We consider a case of $\phi > \alpha$. From

$$\frac{dM_T}{d(v/w)} = \underbrace{\frac{1}{Q_2 R_1 - Q_1 R_2}}_{-} \underbrace{\frac{KQ_2 G_d - \overbrace{(KQ_2 \frac{v}{w} - LQ_1)}^{-} \alpha f_d \left(\frac{v}{w}\right)^{\alpha-1}}{G_d^2}}_{+} < 0,$$

we can obtain

$$\frac{dM_T}{d\tau} = \underbrace{\frac{dM_T}{d(v/w)}}_{-} \underbrace{\frac{d(v/w)}{d\tau}}_{-} > 0$$

We consider a case of $\alpha > \phi$. From

$$\frac{dM_T}{d(v/w)} = \underbrace{\frac{1}{Q_2 R_1 - Q_1 R_2}}_{+} \underbrace{\frac{KQ_2 G_d(1-\alpha) + LQ_1 \alpha f_d \left(\frac{v}{w}\right)^{\alpha-1}}{G_d^2}}_{+} > 0,$$

we can obtain

$$\frac{dM_T}{d\tau} = \underbrace{\frac{dM_T}{d(v/w)}}_{+} \underbrace{\frac{d(v/w)}{d\tau}}_{+} > 0.$$