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The Cyclical Volatility of Equilibrium Unemployment and Vacancies: Evidence from Italy*

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Abstract

In this paper, we explore the fluctuations of unemployment and vacancies in the Italian labour market over the last twenty years. For reasons of data availability on unfilled job openings, this period is split in two parts. The former is covered by a help-wanted time series, while the latter is analyzed relying on a harmonized vacancy rate. In both periods, in line with previous findings on the unemployment volatility puzzle, we find that the tightness indicator is more volatile than productivity. However, the gap between the respective volatilities achieves the order of magnitude observed in other countries only by using the official measure of vacancies. In addition, we show that a model with segmented labour markets and on-the-job search has the potential to provide a rationale for the pattern disclosed by the most recent data.

JEL Classification: E12; E24; J63; J64.

Keywords: Macroeconomic fluctuations; Italian labour market; Shimer puzzle; Market segmentation; On-the-job search.

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1 Introduction

After the publication of two influential articles by Shimer (2004, 2005) - in which he shows that the standard Mortensen-Diamond-Pissarides (MDP) model is unable to replicate observed fluctuations in unemployment and job vacancies in response to productivity shocks of plausible magnitude - the empirical appraisal of the cyclical behaviour of equilibrium unemployment and vacancies has regained a lot of interest in the macroeconomic debate (e.g. Cardullo, 2010; Guerrazzi, 2011).

Retrieving US quarterly data over a fifty-year time horizon, Shimer (2004, 2005) measures, inter-alia, the autocorrelation and the volatility of unemployment, vacancies and labour productivity. One of the most striking finding of his empirical explorations is that the standard deviation of the vacancy-unemployment ratio, i.e., the labour market tightness indicator, is almost 20 times as large as the standard deviation of labour productivity over the period under examination. The so-called 'Shimer puzzle' (or 'unemployment volatility puzzle') comes from the fact that the MDP model predicts that those two variables should have nearly the same volatility.[1]

A number of contributors tried to address this theoretical inconsistency within the US economy. For instance, Shimer (2004) and Hall (2005a,b) rely on real wage stickiness questioning the Nash bargaining hypothesis. Furthermore, Hagedorn and Manovskii (2008) show that calibrating the value of non-market activities close to labour productivity amplifies the volatility of labour market tightness. Moreover, Pissarides (2009) suggests that taking into account that mainly the wages of newly hired workers respond to productivity shocks brings the model close to available evidence. In addition, Silva and Toledo (2009) and Petrosky-Nadeau and Wasmer (2013) argue, respectively, that labour turnover costs and financial frictions may generate the required amplification mechanism.

Spurred on by such a stream of theoretical efforts aimed at reconciling Shimer's (2004, 2005) empirical findings with the well-established theory of equilibrium unemployment, a number of scholars become active in testing the soundness of the unemployment volatility puzzle even outside the US economy. First, Zhang (2008) explores the cyclical behaviour of unemployment and vacancies in Canada. Moreover, Miyamoto (2011) replicates the experiment with Japan data. Furthermore, Gartner et al. (2012) do the same in Germany. While they provides different theoretical rationales for observed paths, all those authors find that in all the countries taken into consideration the volatility of labour market tightness is much higher than the one attached to productivity.[2]

More recently, Justiniano and Michelacci (2011) as well as Amaral and Tasci (2012) test

[1]According to Shimer (2004, 2005), the intuition for those findings is that wages bargained according to the Nash rule should absorb a great deal of productivity shocks. Therefore, vacancies and unemployment should be only partially affected by erratic disturbances affecting the value of output.

[2]Specifically, the reported ratio between those indexes is about 18 for Canada, 13 in Japan and 50 in Germany.

Shimer's (2004, 2005) empirical results over a set of OECD countries in which a number of EU members is included. However, until now, nothing has been said about the cyclical behaviour of unemployment and vacancies in the Italian context. As a consequence, in this paper, we aim at filling this gap by putting under scrutiny the macroeconomic fluctuations of unemployment, vacancies and productivity in the Italian labour market over the last twenty years.

All over this period, the Italian labour force survey together with official national accounts are able to provide homogeneous quarterly figures for both unemployment and labour productivity measured, respectively, as the fraction of the labour force actively searching for work and the added value per worker.[3] Unfortunately, the same does not hold for job vacancies and the corresponding vacancy rate. Therefore, according to data availability on unfilled job openings, our empirical analysis is split in two balanced sub-periods. Namely, the former (1993-2003) is covered by the help-wanted time series (HWTS) provided by the Italian Institute for the Development of Vocational Training (ISFOL), while the latter (2004-2014) is analyzed by means of the harmonized, or official, vacancy rate worked out by the Italian National Institute of Statistics (ISTAT).[4] Retrieving data from both series, we explore the macroeconomic fluctuations of the Italian labour market by keeping up the same computational approach adopted throughout the literature mentioned above.

The main results achieved in this paper are the following. On the empirical ground, confirming previous findings on the unemployment volatility puzzle, we find that, in both periods, despite some relevant heterogeneities in the co-movements of involved series, the labour market tightness indicator is more volatile than labour productivity. However, the gap between the two measures of volatility achieves the order of magnitude observed in other countries only by using the official measure of vacancies. In addition, from a theoretical point of view, we show that a matching framework with segmented labour markets and on-the-job search build along the lines of Krause and Lubik (2006) has the potential to provide an amplification mechanism that outperforms the baseline MDP model. In details, after the development of a two-sector matching model in which firms differ in terms of the cost paid to keep open a vacancy, we show that search effort spent by workers employed in bad jobs to look for better positions can magnify the effects of productivity shocks on labour market tightness by delivering figures of volatilities close to the most recent empirical evidence.

The paper is arranged as follows. Section 2 analyses the patterns of unemployment, vacancies and productivity in the Italian labour market over the last twenty years by exploiting available data. Section 3 develops a matching model with segmented labour markets and on-the-job search by exploring its amplification mechanism. Finally, section 4 concludes.

[3] Throughout the paper, labour productivity is measured as billions of euros of added value per workers.

[4] In the Italian context a measure of labour scarcity has sometimes been used as a proxy of the vacancy rate (e.g. Sestito, 1991; Padoa-Schioppa, 1999; Destefanis and Fonseca, 2007). The cyclical behaviour of this index is explored in Appendix.

2 Italian labour market facts

In this section we provide an empirical analysis of the Italian labour market dynamics over the period 1993-2014. However, as we stated above, this time-horizon is not covered by a unique series for unfilled job openings. To the best of our knowledge, the longest and more reliable Italian vacancy series available all over the period under examination are essentially two, i.e., the ISFOL HWTS that covers about the first decade and the harmonized ISTAT vacancy rate that holds all over the second one.

As a consequence, in what follows, we provide distinct empirical analyses for the two relevant sub-periods.[5] Specifically, for each selected time-span, we take into consideration the level and trend paths of unemployment, vacancies, labour market tightness and productivity. In addition, a special attention is paid to the Beveridge curve, i.e., the well-known relationship between unemployment and vacancies by testing different specifications. Moreover, for each series, we report detailed statistics on volatility, persistence and correlation.

2.1 The period covered by the ISFOL HWTS (1993.q1-2003.q4)

The ISFOL HWTS is grounded on help-wanted job advertisements collected by the Center for Business Statistics (Centro di Statistica Aziendale) headquartered in Florence. Data collection started about thirty years ago. However, essentially for reasons of attrition, homogeneous figures are available only for the years 1993-2003 (cf. Mandrone, 2012). All over this time-horizon, the ISFOL HWTS retrieves information about job advertisements on the main 32 Italian newspapers.

The basic unit of survey of the ISFOL HWTS are job advertisements. However, the information collected in each newspaper is processed in order to derive an estimation for the corresponding number of job vacancies posted by firms on a quarterly basis.[6] Thereafter, following the official definition also implemented by ISTAT, the corresponding vacancy rate is obtained by dividing unfilled positions by the total number of available jobs, i.e., vacancies plus the total number of employed people.[7]

The level and trend paths of unemployment (u), vacancies (v), labour market tightness (v/u) and labour productivity (a) all over the time-span taken into consideration by the ISFOL

[5]The two series for unfilled job openings do not overlap and this does not allow to implement consistent merging procedures. Moreover, as argued by Valletta (2005), the use of an HWTS to retrieve vacancies beyond the 90s will be rather misleading for the growing reliance of alternative sources such as internet posting.

[6]Since the collected help-wanted job advertisements usually target very generic or very specific profiles, the corresponding vacancy index have been often used as an indicator for the most hunted professions (cf. Mandrone, 2012).

[7]The official ISTAT survey takes into account fixed-term and part-time jobs but it does not consider vacancies for managerial positions. Obviously, the stock of employment is corrected in order to be consistent with these restrictions.

HWTS are illustrated in the four panels of figure 1 (all the series are seasonally adjusted; HP trends obtained by setting the smoothing parameter at $1,600$).

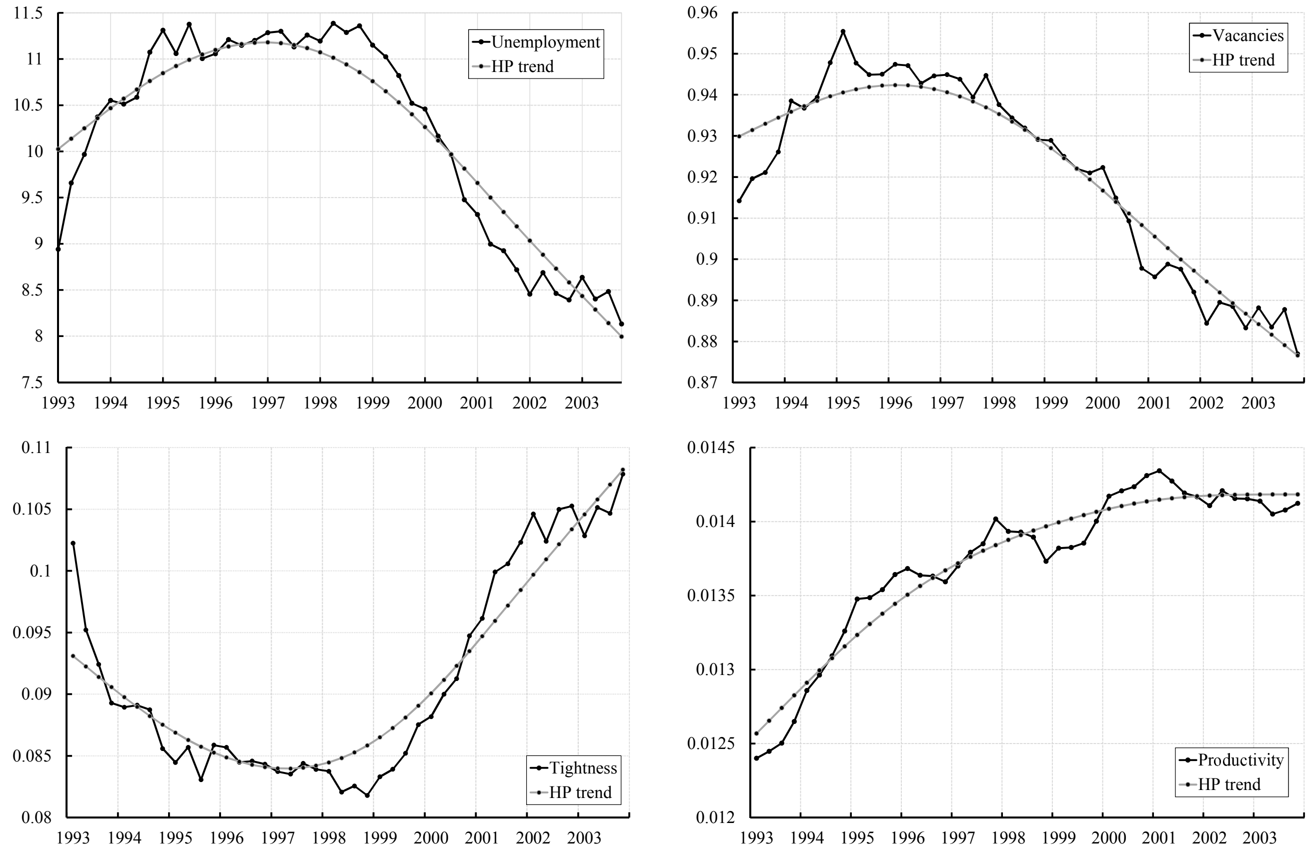

Figure 1: Italian labour market facts (1993.q1-2003.q4)

The four diagrams show that the period covered by the ISFOL HWTS is characterized by a recover of labour market tightness as well as a mildly upward trend in labour productivity. It is quite likely that those encouraging patterns are the joint outcome of labour market reforms aimed at increasing employment flexibility (namely, the Treu Act) and the end of austerity policies carried out after the European monetary crisis burst at the beginning of the 90s.[8]

The level and trend log-deviation relationships between unemployment and vacancies are illustrated in the two panels of figure 2 (both diagrams include the OLS estimation of the slope of the involved relationship together with its standard error).

[8]An empirical assessment of the Treu Act grounded on an efficient matching frontier approach is provided by Destefanis and Fonseca (2007).

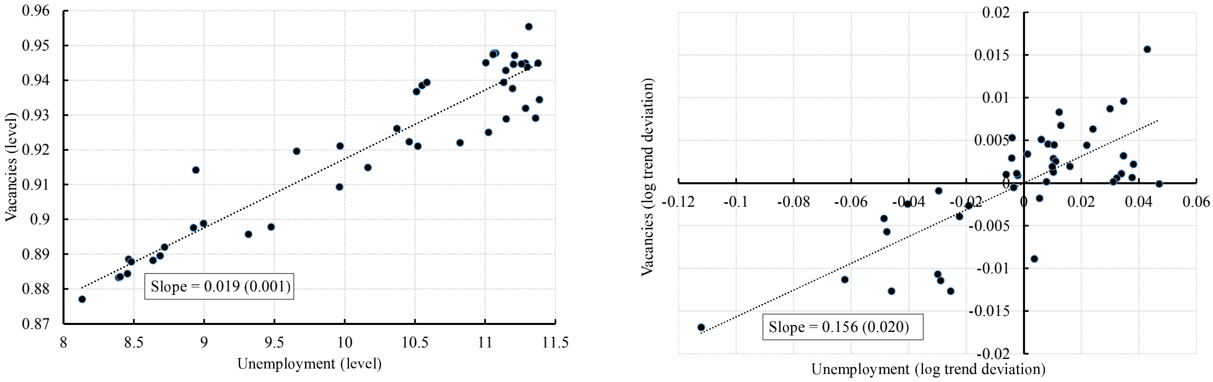

Figure 2: Unemployment versus vacancies (1993.q1-2003.q4)

The two diagrams reveals that the series of vacancies retrieved from the ISFOL HWTS together with official unemployment rates are not consistent with the textbook negative shape of the Beveridge curve; indeed, the relationship between unemployment and vacancies is significantly upward-sloped both in levels and in trend log-deviations. According to the standard DMP model, as far as levels are concerned, such an upward pattern could represent a conventional job-creating condition identified by downward shifts of the Beveridge curve pushed by the already mentioned labour market reforms carried out in the second half of the 90s that probably increased the efficiency of matching. However, the adopted filtering procedure, is unable to detect the negative shape of the Beveridge curve.[9]

A set of summary statistics concerning the trend log-deviation dynamics of the series depicted in figure 1 can be found in table 1.

		$\ln(u)$	$\ln(v)$	$\ln(v/u)$	$\ln(a)$
Standard deviation		0.032	0.006	0.027	0.009
Quarterly autocorrelation		0.642	0.640	0.632	0.816
Correlation matrix	$\ln(u)$	1	0.764	-0.987	0.078
	$\ln(v)$	–	1	-0.655	0.419
	$\ln(v/u)$	–	–	1	0.009
	$\ln(a)$	–	–	–	1

Table 1: Summary statistics, quarterly Italian data (1993.q1-2003.q4)

The figures in table 1 suggest some interesting but rather preliminary conclusions. First, as we already said in commenting figure 2, there is a positive relationship between the trend log-deviations of unemployment and vacancies that is inconsistent with the conventional wisdom

[9] Using the ISFOL HWTS, Sestito (1988) identifies a negative relationship between vacancies and unemployment by assuming that the the latter is driven by a linear trend.

on the shape the Beveridge curve (cf. Shimer 2005). In addition, data reveal an unconventional counter-cyclicality of labour productivity.[10] Second, in contrast to the findings of Justiniano and Michelacci (2011), the implied series of vacancy rates is less volatile than unemployment and productivity. Moreover, the volatility of labour productivity is actually below the volatility of labour tightness; nevertheless, the distance between the two is not that pronounced as we observe in other European countries (cf. Amaral and Tasci, 2012).

Despite the short length of the time-horizon and the dynamic peculiarities of the ISFOL HWTS, the latter result can be taken as a first signal that even in the Italian context a standard MDP model might be not good enough to replicate observed data. Moreover, all the issues related to the derivation of a vacancy index from newspaper advertisements without any reference to shared official criteria are arguments for a further examination.[11]

2.2 The period covered by the ISTAT vacancy rate (2004.q1-2014.q4)

In the third quarter of 2003, ISTAT started a new survey on job vacancies and worked hours identified with the acronym VELA. In compliance with the official EUROSTAT guidelines on job vacancy reporting, this survey aims at measuring vacancies, job flows and worked hours in firms that employ more than 10 employees.[12] As a consequence, this survey provides a harmonized vacancy rate that directly measures the extent of search externalities experienced by Italian firms in their recruiting processes. After the ISFOL HWTS analyzed above, this series is intended to become the longest data source on Italian unfilled job openings.

The paths of unemployment, vacancies, labour market tightness and labour productivity all over the period covered by the ISTAT vacancy rate are illustrated in the four panels of figure 3 (all the series are seasonally adjusted; HP trends obtained by setting the smoothing parameter at $1,600$).

[10] Amaral and Tasci (2012) observe similar patterns in Australia, Poland and Spain.

[11] For instance, Abrahm (1987) argues that occupation composition of employment, equal employment opportunity pressures as well as consolidation in newspaper markets may lead to substantial distortions in the estimation of vacancies from job advertisements. Furthermore, in the Italian labour market additional inconsistencies may derive from the fact that usually recruitment occur through informal links (e.g. Pistaferri, 1999).

[12] The sample size is about $15,000$ firms selected by drawing on the National Statistical Register of Active Firms (ASIA).

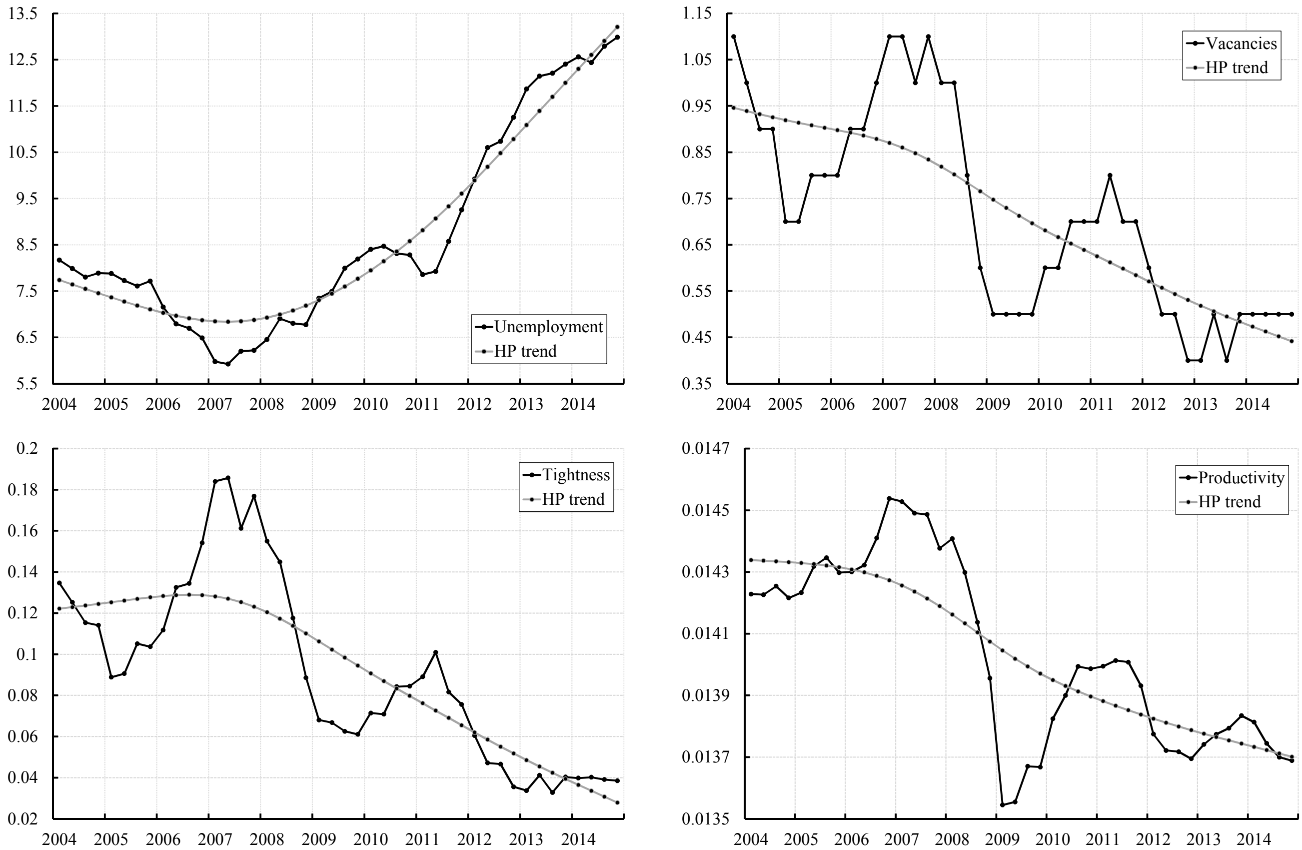

Figure 3: Italian labour market facts (2004.q1-2014.q4)

From a macroeconomic perspective, the time horizon described by the ISTAT vacancy rate is quite different from the one illustrated in figure 1. As shown in the four panels of figure 3, this period is characterized by quite adverse labour market conditions; indeed, after the satisfying performance of the beginning, unemployment begun to grow at increasing rates while labour market tightness displays a quite clean decreasing trend. In addition, the path of labour productivity follows a typical double-deep pattern, indeed, after the dramatic recession of 2008-2009, in 2012 the Italian economy experienced another period of decline after a brief period of very feeble growth.

The level and trend log-deviation relationships between unemployment and vacancies are illustrated in the two panels of figure 4 (both diagrams include the OLS estimation of the slope of the involved relationship together with its standard error).

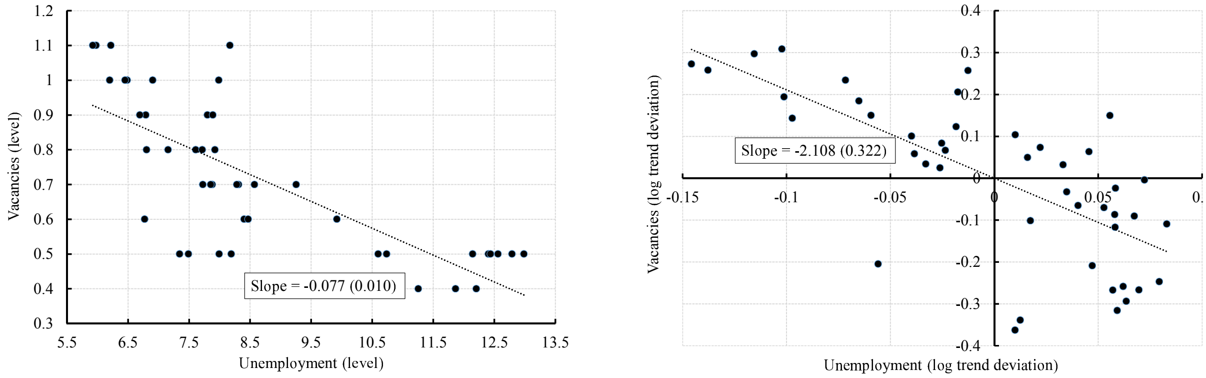

Figure 4: Unemployment versus vacancies (2004.q1-2014.q4)

Even the picture of the relationships between unemployment and vacancies obtained by means of the ISTAT harmonized vacancy rate is completely at odds with the one retrieved in figure 3; indeed, in this case data clearly uncover the textbook negative shape of the Beveridge curve.[13] Those patterns suggest two broad considerations. First, taking the standard DMP model, the level path of the unemployment-vacancy relationship can be interpreted as the upshot of the identification of a conventional Beveridge curve induced by a job creating condition that shifted in clockwise direction in response to the unsatisfactory productivity performance emphasized in the last panel of figure 3. Moreover, on an econometric perspective, the trend deviation paths of unemployment and vacancies reveals that the adopted filtering procedure is actually able to detect the textbook shape of the Beveridge curve.

A set of summary statistics concerning the trend log-deviation dynamics of the series depicted in figure 3 can be found in table 2.

		$\ln(u)$	$\ln(v)$	$\ln(v/u)$	$\ln(a)$
Standard deviation		0.063	0.189	0.238	0.012
Quarterly autocorrelation		0.852	0.782	0.835	0.852
Correlation matrix	$\ln(u)$	1	-0.710	-0.829	-0.615
	$\ln(v)$	–	1	0.982	0.807
	$\ln(v/u)$	–	–	1	0.804
	$\ln(a)$	–	–	–	1

Table 2: Summary statistics, quarterly Italian data (2004.q1-2014.q4)

From the point of view of co-movements, the figures in table 2 are definitely more conventional than the ones in table 1; indeed, all over the concerned period, vacancies and tightness are

[13]This result was already stressed by Baldi and Sorrentino (2010) on a pioneering study on the properties of the harmonized Italian vacancy rate series.

both pro-cyclical while unemployment displays the usual negative correlation with productivity.[14] Moreover, in line with previous findings on the unemployment volatility puzzle reviewed in the introduction, the figures in table 2 convey a clearer picture with respect to results shown in table 1; indeed, the volatility of the official labour market tightness indicator exceeds the one on productivity by an order of magnitude. In other words, the ratio between the latter and the former is around 20, a figure fairly close to the one retrieved by Shimer (2005) and Zhang (2008), respectively, in the US and in Canada. This result is robust to different smoothing procedures.[15]

3 Towards a theoretical explanation of observed volatility

On the whole, data analyzed in section 2 do not provide robust cyclical regularities, especially in terms of co-movements among the involved series; indeed, the only established finding is that even in the Italian context the volatility of the labour market tightness indicator is higher than the volatility of productivity no matter the data source under scrutiny.[16] As a consequence, in order to explain this dynamic pattern, we develop a theoretical framework able to generate a suitable amplification mechanism. Specifically, we present an equilibrium version of the matching framework put forward by Krause and Lubik (2006) and we show that labour market segmentation and the possibility of on-the-job search have the potential to magnify vacancy and unemployment fluctuations in response to productivity shocks.

3.1 Labour market segmentation and on-the-job search

Krause and Lubik (2006) develop a matching model in which consumption goods are produced in two intermediate sectors which are assumed to differ in terms of vacancy posting costs. In details, one of the two intermediate sectors is assumed to be characterized by higher costs of vacancy posting with respect to the other. Therefore, as far as labour market conditions are concerned, the former will be less tight than the latter. Moreover, for a decreasing return argument, the former will be characterized by higher productivity and wages with respect to the

[14] An interesting additional feature conveyed by the figures in table 2 are the large increases observed in the volatilities of unemployment and productivity with respect to their respective values reported in table 1. The seriousness of the shocks that hit the Italian economy during the last decade is probably the responsible for this pattern.

[15] For instance, setting the smoothing parameter of the HP filter to 10^5, i.e., the value adopted by Shimer (2005), the ratio between the volatility of tightness and the corresponding measure for productivity switches to a point value of about 21.

[16] As we show in the Appendix, this stylized fact also appears even when the vacancy rate is proxied by an indicator of labour scarcity.

latter by mirroring the traditional segmentation processes undergoing in actual labour markets usually labeled with the distinction among primary and secondary sectors of the economy.[17] In turn, this segmentation of labour market conditions will lead workers (under)employed in the low-wage, high-tightness sector to make some efforts to find jobs in the high-wage, low-tightness sector in order to upgrade their positions.[18] Calibrating and simulating this model, Krause and Lubik (2006) show that their theoretical framework is able to fairly reproduce the observed path of US vacancy and unemployment fluctuations in response to productivity shocks of plausible magnitude.

In this paper we follow a different approach. Namely, we analytically solve a steady-state version of Krause and Lubik (2006) model and then we assess the elasticities of labour market tightness with respect to productivity under different scenarios. As shown by Shimer (2005) and Mortensen and Nagypal (2007), such elasticities are useful approximations to the volatilities of the corresponding variables in the dynamic stochastic set-up.[19] Furthermore, we formally and numerically show that search effort can effectively increase the response of the labour market tightness indicator to productivity disturbances. To the best of our knowledge, this is the first attempt to evaluate the amplification mechanism implied by the framework of Krause and Lubik (2006) by means of the derivation of the elasticity of the labour market tightness indicator with respect to productivity.

3.2 The model

We assume that the economy is populated by a measure 1 of risk-neutral workers. Time is continuous and the discount rate is denoted by r. In addition, there are two intermediate good sectors in the economy, dubbed as g (good) and b (bad). The two intermediate goods are combined together via a CES aggregator to produce the unique final consumption good. Hence,

$$Y = a \left(Q_g^{\frac{\rho-1}{\rho}} + Q_b^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \qquad \rho > 1 \tag{1}$$

where Y is the quantity of the consumption good, a is a measure of aggregate productivity, Q_i is output in sector i, with $i = \{g, b\}$, and ρ is the elasticity of substitution between the two intermediate goods.

Product markets are perfectly competitive. As a consequence, taking the consumption good as numeraire, cost minimization leads to the following inverse demand functions:

[17] Evidence of labour market segmentation in the Italian context is given by Cipollone (2001) and, more recently, by Battisti (2013).

[18] The determinants of underemployment in Italy and other European countries are discussed by Ruiz-Quintanilla and Claes (1996).

[19] Mortensen and Nagypal (2007) show that the two outcomes coincide in the limit when the arrival rate of shocks is close to zero or changes in productivity are small.

$$p_i(Q_i) \;=\; a\left(\frac{Q_i}{Y}\right)^{-\frac{1}{\rho}} \qquad i = \{g, b\} \tag{2}$$

In each intermediate sector, each firm can hire one worker at most, that in turn produces one unit of the intermediate good. Therefore, if e_i is the level of employment in sector i, then $e_i = Q_i$, with $i = \{g, b\}$.

The only difference between the two sectors is in the flow cost paid to keep a vacancy open, namely, we assume that $c_g > c_b$. As it will be clear later on, the sector with higher vacancy costs will exhibit lower employment and, for decreasing returns in the production function of consumption goods, higher productivity and wages. Therefore, employed workers in sector b exert some effort in searching for a job in sector g.

Defining u as the unemployment rate, the employment flows in the two sectors can be derived in a quite standard manner. On the one hand, the flow of new matches in sector b is denoted by $m(v_b, u) = \mu v_b^{1-\eta} u^{\eta}$, with $\mu > 0$ and $0 < \eta < 1$, where v_b is the vacancy rate in the bad sector. Denoting $\theta_b \equiv v_b/u$, the corresponding job finding rate is given by $f(\theta_b) \equiv m(v_b, u)/u$ and the rate at which vacancies are filled is $q(\theta_b) \equiv m(v_b, u)/v_b$, which is a positive, decreasing and convex function of θ_b.

On the other hand, in sector g, the matching function is given by $m(v_g, u+se_b) = \mu v_g^{1-\eta}(u+se_b)^{\eta}$, where v_g is the the vacancy rate in that sector while s is the amount of search effort spent by the e_b workers employed in sector b to look for a better position. If we define $\theta_g \equiv v_g/(u+se_b)$, then we can again convey the job finding rate and the job filling rate as functions of tightness only, i.e., $f(\theta_g) \equiv m(v_g, u+se_b)/(u+se_b)$ and $q(\theta_g) \equiv m(v_g, u+se_b)/v_g$. Furthermore, in both sectors, at an exogenous rate δ, a firm-worker pair is destroyed in each instant.

Denoting by ϕ the share of employees in sector g, the prices of the intermediate goods can be written as:

$$p_g \;=\; a\left(1+\left(\frac{1-\phi}{\phi}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{1}{\rho-1}} \tag{3}$$

$$p_b \;=\; a\left(1+\left(\frac{\phi}{1-\phi}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{1}{\rho-1}} \tag{4}$$

In steady-state, the flows in and out of employment status must be equal. Therefore,

$$\delta\phi e = \; sf(\theta_g)(1-\phi)e \;+\; f(\theta_g)u_g \tag{5}$$

$$(1-\phi)e\,(\delta +\; sf(\theta_g)) = \; f(\theta_b)u_b \tag{6}$$

where $e \equiv e_b + e_g = 1 - u$ is total employment.

The on-the-job mechanism implies there is a share of workers $sf(\theta_g)(1-\phi)e$ that quit their job in sector b to become employed in sector g. The discounted present value of employment in sector g, i.e., V_{eg}, verifies the following Bellman equation:

$$rV_{eg} \;=\; w_g + \delta\,(V_{ug} - V_{eg}) \tag{7}$$

where w_g is the real wage in intermediate sector g and V_{ug} represents the discounted expected lifetime income of an unemployed worker in the same sector.

Similarly, the discounted present value in employment in sector b, i.e., V_{eb}, is assumed to verify

$$rV_{eb} = \max_{s}\; w_b - \kappa s^{\sigma} + \delta\,(V_{ug} - V_{eb}) + sf(\theta_g)\,(V_{eg} - V_{eb}) \qquad \sigma > 1 \tag{8}$$

Workers employed in sector b may exert some effort in searching for a job in sector g. The disutility of on-the-job search is captured by the convex function κs^{σ}. The first-order condition (FOC) for the problem in (8) is given by

$$\kappa\sigma s^{\sigma-1} = f(\theta_g)\,(V_{eg} - V_{eb}) \tag{9}$$

Denoting the value for leisure by z, the values of searching for a job in the good and in the bad sector, respectively, V_{ug} and V_{ug}, verify the following Bellman equations:

$$rV_{ui} = z + f(\theta_i)\,(V_{ei} - V_{ui}) \qquad i = \{g, b\} \tag{10}$$

Unemployed workers are free to direct their search towards either sector. However, a non-arbitrage condition ensures that the value of searching for a job must be equal across sectors. Hence,

$$rV_{ug} \;=\; rV_{ub} = rV_u \tag{11}$$

On the firms' side, the expected discounted profits of a vacancy takes the following form:

$$rJ_{vi} = -c_i + q(\theta_i)\,(J_{ei} - J_{vi}) \qquad i = \{g, b\} \tag{12}$$

Moreover, the expected utilities of a filled vacancy in the two sectors are equal to

$$J_{eg} = p_g - w_g + \delta\,(J_{vg} - J_{eg}) \tag{13}$$

$$J_{eb} = p_b - w_b + \delta\,(J_{vb} - J_{eb}) + sf(\theta_g)\,(J_{eb} - J_{eb}) \tag{14}$$

A firm-worker pair in sector b breaks down not only at the exogenous destruction rate δ but also whenever the employee finds a job in sector g. In each sector, firms post vacancies as long as the discounted expected profits are non-negative, so that $J_{vi} = 0$, for $i = \{g, b\}$. Using (12), (13), and (14) we derive

$$\frac{p_b - w_b}{r + \delta + sf(\theta_g)} = \frac{c_b}{q(\theta_b)} \tag{15}$$

$$\frac{p_g - w_g}{r + \delta} = \frac{c_g}{q(\theta_g)} \tag{16}$$

The free-entry zero profit condition in sector g, i.e., eq. (16), is qualitatively identical to the one retrieved in the standard MDP model and it equates the expected cost of filling a vacancy (the RHS) with the expected revenues (the LHS). By contrast, in sector b, expected revenues are discounted by a further term, i.e., $sf(\theta_g)$, that takes into account the probability that workers may leave their job in order to switch in the other sector.

When a worker and a firm form a match, the surplus $V_{ei} - V_u + J_{ei}$ with $i = \{g, b\}$, is shared through Nash bargaining. If β denotes the bargaining power of the workers, then the surplus-sharing condition can be written as

$$(1-\beta)\,(V_{ei} - V_u) = \beta J_{ei} \qquad i = \{g, b\}, \quad 0 < \beta < 1 \tag{17}$$

Consistently with Krause and Lubik (2006), we do not consider the option of recall. As a consequence, this means that wages in previous jobs are not part of the outside options of workers.[20]

Using eq.s (10) – (12) and (17) it is possible to derive that

$$\theta_b\, c_b = \theta_g\, c_g \tag{18}$$

Since $c_g > c_b$, then $\theta_g < \theta_b$. Because of the higher vacancy cost, less firms enter sector g leading to lower tightness, lower employment and a higher sector productivity implied by the decreasing marginal returns characterizing the production function of final goods.

Exploiting eq.s (10), (12), and (17), the FOC for the search effort in eq. (9) can be written as

$$s = \left(\frac{c_b\,\theta_b \beta}{\kappa\sigma\,(1-\beta)}\left(1 - \left(\frac{c_b}{c_g}\right)^{1-\eta}\right)\right)^{\frac{1}{\sigma-1}} \tag{19}$$

Furthermore, using the eq.s (7), (8), (10), (13), and (14), the Nash bargaining solution in eq. (17) allows to write the wage equation in both sectors. Specifically,

$$w_g = \beta\,(p_g + c_g\theta_g) + (1-\beta)z \tag{20}$$

[20] Krause and Lubik (2006) extensively discuss the consequences of imposing a recall in the Nash bargaining game.

$$w_b = \beta\left(p_b + (1-s)c_b\theta_b\right) + (1-\beta)\left(z + ks^{\sigma}\right) \tag{21}$$

It is worth noting that eq. (19) implies that $d\,w_b/d\,s = (1-\beta)\kappa\sigma s^{\sigma-1} - \beta c_b\theta_b < 0$. This inequality follows because the stronger the effort spent in searching for a job in sector g, the lower the expected surplus of a match in sector b, as it breaks up more easily. Therefore, this translates into a lower wage w_b.

Definition 1 *A steady-state equilibrium is defined by a vector* $(\theta_i, w_i, p_i(q_i))$*, with* $i = \{g, b\}$*, a value of search effort* s*, a share of employment in the* g *sector* ϕ*, and a value of consumption output* Y *satisfying:*

- *The wage equations* (20) *and* (21)*;*
- *The non-arbitrage condition in eq.* (18)*;*
- *The FOC for the optimal search effort in eq.* (19)*;*
- *The zero-profit conditions in eq.s* (15) *and* (16)*;*
- *The FOCs in the final good sector in eq.* (2) *and eq.* (1) *for* Y*.*

To find the steady-state equilibrium, we insert the expressions for the real wages in eq.s (20) and (21) in the zero-profit conditions conveyed by eq.s (15) and (16) and then we substitute θ_g for θ_b via the non-arbitrage condition in eq. (18). In this way, we derive

$$(1-\beta)(p_b - z) - \beta c_b\theta_b - \frac{c_b(r+\delta)}{q(\theta_b)} - (1-\beta)\kappa s^{\sigma} + c_b\theta_b s\left(\beta - \left(\frac{c_b}{c_g}\right)^{1-\eta}\right) = 0 \tag{22}$$

$$(1-\beta)(p_g - z) - \beta c_b\theta_b - \frac{c_g(r+\delta)}{q(\theta_b)}\left(\frac{c_b}{c_g}\right)^{\eta} = 0 \tag{23}$$

From eq. (19), recall that search effort s is a monotonically increasing function of θ_b. As a consequence, the system in eq.s (22) and (23) is composed by two equations in two unknowns, i.e., θ_b and ϕ.[21] If a (unique) solution of the system exists, then all the other remaining variables of the model can be easily obtained by using the eq.s cited in definition 1. The following proposition summarizes the results:

Proposition 1 *If* $z < 1$*, then there is a unique solution for the system in eq.s* (22) *and* (23)*. Therefore, the steady-state equilibrium of the model exists and it is unique.*

[21] Recall that, according to eq.s (3) and (4), p_g and p_b depend on ϕ only.

The intuition for the statement in proposition 1 can be given as follows. The Cobb-Douglas matching functions as well as the CES aggregator verify the Inada conditions. Thereafter, taking into account that the latter displays decreasing marginal returns, it is easy to show that eq. (22) is an increasing function in (ϕ, θ_b) space that intersects the horizontal axis in a point greater than zero and that goes to $+\infty$ as $\phi = 1$. Moreover, eq. (23) is a decreasing function in (ϕ, θ_b) space that intersects the vertical axis in a point between zero and one and that goes to $+\infty$ as $\phi = 0$. As a consequence, a solution for system in eq.s (22) and (23) exists and it is unique. Technical details can be found in Appendix.

3.3 Amplification of productivity shocks

We now compute the elasticity of tightness with respect to the productivity parameter a. Taking into account the non-arbitrage condition in eq. (18), the elasticity of tightness in sector g has the same magnitude of the one in sector b. In other words,

$$\frac{d\theta_b}{da}\frac{a}{\theta_b} = \frac{d\theta_g}{da}\frac{a}{\theta_g} \equiv \epsilon_a \tag{24}$$

Totally differentiating the system in eq.s (22) and (23) allows to derive the following expression:

$$\epsilon_a = \frac{p_b}{\Phi_0 + \frac{c_b(\eta(r+\delta)+\beta f(\theta_b)+\Phi_1)}{(1-\beta)q(\theta_b)}} \tag{25}$$

where the multipliers Φ_0 and Φ_1 are defined as

$$\Phi_0 \equiv -\frac{\partial p_b}{\partial \phi}\frac{\partial \phi}{\partial \theta_b}\theta_b = \frac{\phi c_b \left(\beta f(\theta_b) + \eta(r+\delta)\left(\frac{c_b}{c_g}\right)^{\eta-1}\right)}{(1-\phi)(1-\beta)q(\theta_b)} \tag{26}$$

$$\Phi_1 \equiv s f(\theta_b)\left(\left(\frac{c_b}{c_g}\right)^{1-\eta}\left(1+\frac{1-\beta}{\sigma-1}\right) - \beta\right) \tag{27}$$

As convincingly argued by Mortensen and Nagypal (2007), the empirical counterpart of eq. (25) is given by the following OLS regression coefficient:

$$\xi \equiv \varrho_{\ln\left(\frac{v}{u}\right),\ln(a)} \frac{\text{standard deviation of } \ln\left(\frac{v}{u}\right)}{\text{standard deviation of } \ln(a)} \tag{28}$$

where $\varrho_{\ln\left(\frac{v}{u}\right),\ln(a)}$ is the observed correlation between the tightness indicator and labour productivity.

According to the figures, respectively, in table 1 and in table 2, ξ takes a value of 0.027 in the period covered by the ISFOL HWTS and a value of 15.946 in the period covered by the ISTAT vacancy rate.

Two terms distinguish ϵ_a from the corresponding expression of the elasticity in a standard MDP model. The first term is the derivative in eq. (26), i.e., the term stemming from the assumption that there is a decreasing demand for the intermediate goods produced by firms. In the textbook MDP model, it is implicitly assumed that firms face an infinitely elastic demand so that output price does not change. In addition, the second term is given by the expression in eq. (27), i.e., the term that conveys the presence of on-the-job search in sector b.

The amplification potential of eq. (25) is conveyed by the following proposition:

Proposition 2 *If* $\left(\frac{c_b}{c_g}\right)^{1-\eta} < \frac{\beta(\sigma-1)}{\sigma-\beta}$, *then* ϵ_a *is increasing in* s.

The proof comes directly from the inspection of eq. (25). If $(c_b/c_g)^{1-\eta} < \beta(\sigma-1)/(\sigma-\beta)$, then $\Phi_1 < 0$ and search effort has a positive impact on ϵ_a. In other words, proposition 2 claims that the introduction of on-the-job search increases the elasticity of tightness in response to an increase in productivity. Under plausible assumptions about the arrival rate of shocks in a and the magnitude of its change, this implies that the introduction of on-the-job search amplifies the volatility of tightness in response to shocks on productivity.

A rationale for this result goes as follows. When a positive productivity shock hits intermediate sectors of the economy, firms post more vacancies both in the high-paid and in the low-paid sector. Higher values of v_g raises search effort s.[22] Thereafter, more job-seekers in sector g imply a lower expected duration of vacancies $q(\theta_g)^{-1}$, so even more vacancies are posted in sector g. Vacancy creation and employees' search effort are strategic complements. In other words, an increase in v_g triggers search effort that, in turn, raises v_g even more. The process ends because of the convexity assumption about the search cost in eq. (8). The final result is a larger amplification in vacancy posting in response to a productivity shock. A complementarity between sectors also arises. If search effort of low-paid employees goes up, then congestion effect in the matching technology will make more difficult for unemployed workers to find a high-paid job. As a consequence, they will direct their search toward the low-paid sector. This in turn will boost vacancy creation in that sector.

3.4 A calibration experiment

The model developed above is quite non-linear. Therefore, in order to provide a quantitative assessment of the theoretical results underlying proposition 2, we rely on a computational experiment. Specifically, we calibrate the model on a quarterly basis and we evaluate how ϵ_a react to search effort spent by workers employed in sector b to look for better positions.[23] Moreover,

[22] Totally differentiating eqs. (18) and (19) leads to $ds/d\theta_g > 0$.

[23] MATLAB$^{\text{TM}}$ codes are available from the authors.

for reasons of comparability with other countries in which there is a clean evidence of the unemployment volatility puzzle, we take as reference period of calibration the last decade only, i.e., the period covered by the ISTAT harmonized vacancy rate by targeting the corresponding value of ξ. This choice is also motivated by the fact that the correlation matrix retrieved in the period covered by the ISFOL HWTS is not consistent co-movements of variables implied by the Krause and Lubik's (2006) framework. Specifically, the model developed above does not admit an upward sloped Beveridge curve neither a counter-cyclical productivity.

The model is calibrated as follows. First, consistently with Shimer (2005), the parameters of the matching function and the job destruction rate, respectively, μ, η and δ, are retrieved by computing job finding and separation rates exploiting OECD data on long-term unemployment.[24] Moreover, in order to consider efficient fluctuations, we set $\beta = \eta$ (cf. Hosios, 1990). The value for productivity (a) is derived by averaging data in the forth panel of figure 3. Similarly, the value of leisure (z) is obtained by averaging labour productivity all over the period and taking into account OECD labour shares and replacement rates (cf. Martin, 1996). The figure of the interest rate is consistent with an annual real interest rate slightly above 4%.

Parameter	**Description**	**Value**
μ	*Scale parameter of the matching function*	1.548
η	*Elasticity of the matching function*	0.519
δ	*Job separation rate*	0.090
β	*Workers' bargaining power*	0.519
a	*Average productivity*	0.014
z	*Value of leisure*	0.005
r	*Interest rate*	0.012
c_b	*Bad job creation cost*	0.200
c_g	*Good job creation cost*	0.800
κ	*Search cost function parameter*	0.040
ρ	*Elasticity of substitution*	2.450

Table 3: Calibration

The remaining model parameters are calibrated following the contribution by Krause and Lubik (2006). Specifically, bad and good job creation costs, namely, c_b and c_g, are set in order to mimic a 1 to 4 ratio while the cost function parameter (κ) is fixed at 4%. Furthermore, consistently with the evidence provided by Falzoni et al. (2007), the value of the elasticity of substitution among the two intermediate goods (ρ) is chosen in order to replicate an equilibrium share of good jobs (ϕ) around 30%. Thereafter, the model is simulated for different values of

[24]Identification issues are addressed by instrumenting with lagged values of the involved variables.

the search elasticity parameter (σ) in order to solicit different search effort levels (s).[25] The whole set of parameter values is collected in table 3 while simulation results are illustrated in figure 5.

The diagram in figure 5 clearly corroborates the theoretical results of proposition 2; indeed, there is a clear positive relation between search effort and the elasticity of the labour market tightness indicator with respect to productivity shocks. Moreover, the simulated values of ϵ_a have the same order of magnitude of the coefficient ξ computed by means of the official data in table 2.

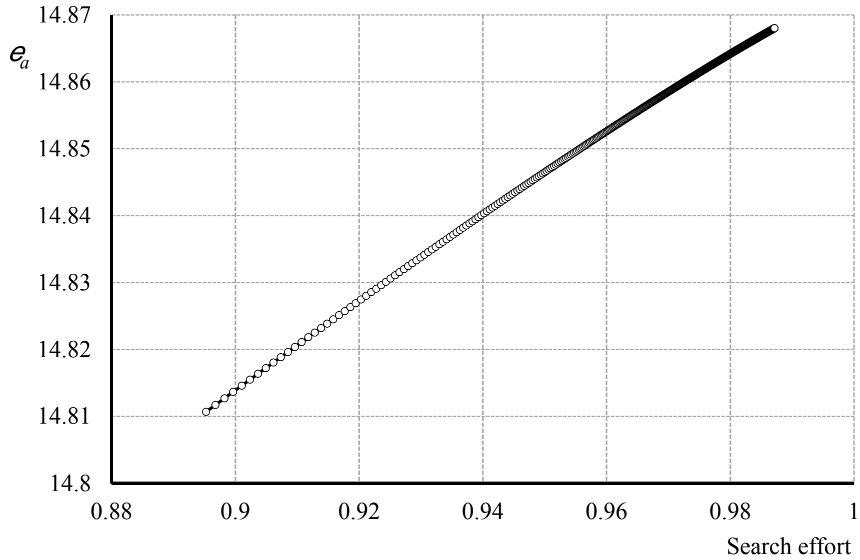

Figure 5: Simulation results

4 Concluding remarks

In this paper, we explore the macroeconomic fluctuations of unemployment, vacancies and productivity in the Italian labour market over the last twenty years. Since the selected time-horizon is not covered by a unique time series for unfilled job openings, our empirical analysis is divided in two parts. The former (1993-2003) is covered by the ISFOL HWTS, while the latter (2004-2014) is analyzed relying on the harmonized ISTAT vacancy rate.

The main results achieved in this paper can be summarized as follows. First, on the empirical ground, we find that in two periods under examination, despite some significant differences in the co-movements of involved series, the volatility of the labour market tightness indicator is higher than the volatility of labour productivity. However, the ratio between the the standard

[25]Given the values of η, β, c_b and c_g, σ is allowed to vary between 55 and $1,000$ in order to meet the requirements of proposition 2.

deviation between the two achieved the order of magnitude observed in other countries only in the second decade when official data on job openings become available. This finding appears quite robust and extends to Italy previous results on the unemployment volatility puzzle derived in other OECD countries (cf. Shimer, 2005; Zhang 2008; Miyamoto, 2012; Gertner et al., 2012). As a consequence, as long as the MDP model is used to design and evaluate labour market policies in the Italian context it is necessary to take into account this stylized feature of the business cycle. For instance, since productivity is less volatile than the tightness indicator and wages are mainly determined by the former, a policy of labour market deregulation is likely to be more effective in reducing unemployment than in affecting existing wage differentials.

Moreover, from a theoretical perspective, we show that a matching model with segmented labour markets and on-the-job search build along the lines of Krause and Lubik (2006) has the potential to provide the required amplification mechanism. Specifically, we show that search effort spent by workers (under)employed in the secondary sector of the economy to look for jobs in the primary sector can exacerbate the impact of productivity disturbances on labour market tightness by delivering results close the official empirical evidence.

References

[1] Abrahm, K.G. (1987), Help-Wanted Advertising, Job Vacancies, and Unemployment, *Brookings Papers on Economic Activity*, Vol. 18, No. 1, pp. 207-248.

[2] Amaral, P.S. and Tasci, M. (2012), The Cyclical Behavior of Equilibrium Unemployment and Vacancies across OECD Countries, *Working Paper of the Federal Reserve Bank of Cleveland*, No. 1236.

[3] Baldi, C. and Sorrentino, M. (2010), I Posti Vacanti in Italia e in Europa. Le Nuove Statistiche Trimestrali Armonizzate: Prime Analisi delle Serie Storiche, *Contributi ISTAT*, No. 2/2010.

[4] Battisti, M. (2013), Reassessing Segmentation in the Labour Market: An Application for Italy 1994-2004, *Bulletin of Economic Research*, Vol. 65, No. s1, pp. s38-s55.

[5] Bragato, S. (1990), La Curva di Beveridge e le Componenti della Disoccupazione in Italia (1980-1988), *Economia & Lavoro*, Vol. 24, No. 4, pp. 111-122.

[6] Cardullo, G. (2010), Matching Model Under Scrutiny: An Appraisal of the Shimer Puzzle, *Journal of Economic Survey*, Vol. 24, No. 9., pp. 622-656.

[7] Cipollone, P. (2001), Is the Italian Labour Market Segmented?, *Temi di Discussione della Banca d'Italia*, No. 400.

[8] Destefanis, S. and Fonseca, R. (2007), Matching Efficiency and Labour Market Reform in Italy: A Macroeconometric Assessment, *Labour: Review of Labour Economics and Industrial Relations*, Vol. 21, No. 1, pp. 57-84.

[9] Falzoni, A.M, Venturini, A. and Villosio, C. (2007), Skilled and Unskilled Wage Dynamics in Italy in the 90s: Changes in the Individual Characteristics, Institutions, Trade and Technology, *Working Paper of the Centre for Employment Studies - Labor*, No. 61.

[10] Gartner, H., Merkl, C. and Rothe, T. (2012), Sclerosis and Large Volatilities: Two Sides of the Same Coin, *Economic Letters*, Vol. 117, No. 1, pp. 106-109.

[11] Guerrazzi, M. (2011), Search and Stochastic Dynamics in the Old Keynesian Economics: A Rationale for the Shimer Puzzle, *Metroeconomica*, Vol. 62, No. 4, pp. 561-586.

[12] Hagedorn, M. and Manovskii, I. (2008), The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited, *American Economic Review*, Vol. 98, No. 4, pp. 1692-1706.

[13] Hall, R.E. (2005a), Employment Fluctuations with Equilibrium Wage Stickiness, *American Economic Review*, Vol. 95, No. 1, pp. 50-65.

[14] Hall, R.E. (2005b), Employment Efficiency and Sticky Wages: Evidence from Flows in the Labor Market, *Review of Economic and Statistics*, Vol. 87, No. 3, pp. 397-407.

[15] Hosios, A.J. (1990), On the Efficiency of Matching and Related Models of Search and Unemployment, *Review of Economic Studies*, Vol. 57, No. 2, pp. 279-298.

[16] Justiniano, A. and Michelacci, C. (2011), The Cyclical Behavior of Equilibrium Unemployment and Vacancies in the US and Europe, *NBER International Seminar on Macroeconomics*, No. 12483.

[17] Krause, M.U. and Lubik, T.A. (2006), The Cyclical Upgrading of Labor and On-the-Job Search, *Labour Economics*, Vol. 13, No. 4, pp. 459-477.

[18] Martin, J.P. (1996), Measure of Replacement Rates for the Purpose of International Comparison: A Note, *OECD Economic Studies*, No. 26.

[19] Miyamoto, H. (2011), Cyclical Behaviour of Unemployment and Job Vacancies in Japan, *Japan and the World Economy*, Vol. 23, No. 3, pp. 214-225.

[20] Mandrone, E. (2012), *Job Vacancies in the Italian Labour Market: The New ISFOL Help Wanted Time Series*, ISFOL Temi & Ricerche, No. 2.

[21] Mortensen, D.T. and Nagypal, E. (2007), More on Unemployment and Vacancy Fluctuations, *Review of Economic Dynamics*, Vol. 10, No. 3, pp. 327-347.

[22] Padoa-Schioppa, F. (1991), A Cross-Country Comparison of Sectoral Mismatch in the 1980s, in Padoa-Schioppa, F. (ed.), *Mismatch and Labour Mobility*, Cambridge University Press, pp. 1-43.

[23] Petrosky-Nadeau, N. and Wasmer, E. (2013), The Cyclical Volatility of Labor Markets under Frictional Financial Markets, *American Economic Journal: Macroeconomics*, Vol. 5, No. 1, pp. 193-221.

[24] Pissarides, C.A. (2000), *Equilibrium Unemployment Theory*, 2nd Edition, MIT Press, Cambridge, Mass.

[25] Pissarides, C.A. (2009), The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?, *Econometrica*, Vol. 77, No. 5, pp. 1339-1369.

[26] Pistaferri, L. (1999), Informal Networks in the Italian Labour Market, *Giornale degli Economisti e Annali di Economia*, Vol. 58, No. 3-4, pp. 355-375.

[27] Ruiz-Quintanilla, S.A. and Claes, R. (1996), Determinants of Underemployment of Young Adults: A Multi-Country Study, *Industrial and Labor Relations Review*, Vol. 49, No. 3, pp. 424-438.

[28] Sestito, P. (1988), Esiste una Curva di Beveridge per l'Italia?, *Temi di Discussione del Servizio Studi della Banca d'Italia*, No. 101.

[29] Sestito, P. (1991), Disoccupazione e Carenze di Personale, *Politica Economica*, Vol. 7, No. 1, pp. 85-103.

[30] Shimer, R. (2004), The Consequences of Rigid Wages in Search Models, *Journal of the European Economic Association*, Vol. 2, No. 2-3, pp. 469-479.

[31] Shimer, R. (2005), The Cyclical Behavior of Equilibrium Unemployment and Vacancies, *American Economic Review*, Vol. 95, No. 1, pp. 25-48.

[32] Silva, J. and Toledo, M. (2009), Labor Turnover Costs and the Cyclical Behavior of Vacancies and Unemployment, *Macroeconomic Dynamics*, Vol. 13, No. 1, pp. 76-96.

[33] Valletta, R. (2005), Help-Wanted Advertising and Job Vacancies, *FRBSF Economic Letter*, No. 2005-02.

[34] Zhang, M. (2008), Cyclical Behavior of Unemployment and Job Vacancies: A Comparison between Canada and United States, *B.E. Journal of Macroeconomics*, Vol. 8, No. 1, pp. 1-27.

Appendix 1: The ISAE indicator of labour scarcity (1992.q1-2005.q1)

As we stated in the introduction, In Italy, before 2004, there was no official data on vacancies and the only survey aimed at catching the unmatched recruiting efforts of firms was the one grounded on help-wanted job advertisements carried out by ISFOL.

However, at the beginning of the 90s, a number of scholars (e.g. Sestito 1991; Padoa-Schioppa 1991) retrieved estimations of the Beveridge curve by relying on a quarterly survey carried out on Italian manufacturing firms by the Institute for Economic Studies and Analysis (ISAE). This survey, among the other things, asked firms about their production restrictions due to recruiting difficulties. Drawing on this information, Sestito (1991) builds an indicator of labour scarcity that proxies the level of vacancies.[26] Aggregating regional figures, the ISAE survey allows to retrieve this additional measure of unfilled job openings over the period 1992-2005.[27]

The level and trend paths of vacancies and labour market tightness implied by the ISAE indicator of labour scarcity developed by Sestito (1991) are illustrated in figure A1 (all the series are seasonally adjusted; HP trends obtained by setting the smoothing parameter at $1,600$).

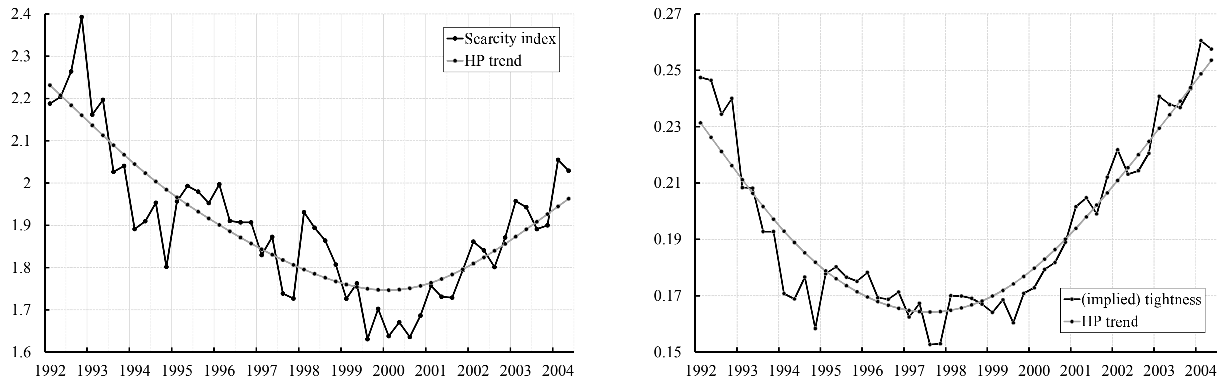

Figure A1: Vacancies and tightness implied by the ISAE indicator (1992.q1-2005.q1)

The LHS panel of figure A1 shows that the path level of vacancies implied by the ISAE indicator of labour scarcity is completely at odds with respect to the one tracked by the ISFOL HWTS. However, the path of labour market tightness on the RHS confirms the beneficial effects on employment prospects carried out by the Treu act after 1997 already stressed in section 2.

[26]Specifically, this indicator is a non-linear monotonic transformation of the percentage of firms constrained by the scarcity of labour. Additional details can be found in Sestito (1991) and, more recently, in Destefanis and Fonseca (2007).

[27]The overall national reference is retrieved by weighing the regional figures with the corresponding employment level.

A set of summary statistics concerning the implied trend log-deviation dynamics of unemployment, vacancies, labour market tightness and productivity all over the period covered by the ISAE indicator of labour scarcity can be found in table A1.

		$\ln(u)$	$\ln(v)$	$\ln(v/u)$	$\ln(a)$
Standard deviation		0.032	0.041	0.048	0.009
Quarterly autocorrelation		0.725	0.454	0.486	0.821
Correlation matrix	$\ln(u)$	1	0.131	-0.544	0.083
	$\ln(v)$	–	1	0.759	-0.594
	$\ln(v/u)$	–	–	1	-0.558
	$\ln(a)$	–	–	–	1

Table A1: Summary statistics, vacancies measured with the ISAE indicator (1992.q1-2005.q1)

The figures in table A1 replicate the positive relationship between vacancies and unemployment as well as the counter-cyclicality of productivity already disclosed by the ISFOL HWTS in table 1. Moreover, the vacancy and the tightness indicators display an unconventional correlation with productivity. However, in terms of standard errors, the ISAE indicator corroborates to some extent the empirical findings presented in section 2; indeed, even when labour scarcity index is taken as a proxy of vacancies, the volatility of tightness is higher than the volatility of productivity.

Appendix 2: Proof of proposition 1

Since for eq. (19) search effort s is a function of θ_b, eq.s (22) and (23) can be considered as implicit functions of only two endogenous variables, i.e., ϕ and θ_b. We denote the former $\mathbb{ZP}_b(\phi,\, \theta_b) = 0$ and the latter $\mathbb{ZP}_g(\phi,\, \theta_b) = 0$.

Consider first $\mathbb{ZP}_g(\phi,\, \theta_b) = 0$. It is easy to see that, as $\theta_b \to 0$, then $p_g \to z$ and, for eq. (3), this implies that ϕ tends to be a number strictly greater than zero. For the same eq. (3), if $\theta_b \to +\infty$, then $\phi \to 0$. Totally differentiating $\mathbb{ZP}_g(\phi,\, \theta_b) = 0$ and applying the implicit function theorem, we also get $d\,\phi/d\,\theta_b < 0$. As a consequence, this expression describes a decreasing relationship in the (ϕ, θ_b) space.

As far as $\mathbb{ZP}_b(\phi,\, \theta_b) = 0$ is concerned, if $\phi \to 0$, then for eq. (4), we have $p_b \to 1$. Under the assumption of $z < 1$, this implies that θ_b must be a number strictly greater than zero. Conversely, if $\phi \to 1$, then we have $p_b \to +\infty$ and $\theta_b \to +\infty$. Moreover, from eq. (19) we derive

$$\frac{ds}{d\theta_b} = \frac{s}{\theta_b\,(\sigma - 1)} \tag{B1}$$

The result in eq. (B1) leads to

$$\frac{d\,\mathbb{ZP}_b}{d\,\theta_b} = -\frac{\eta c_b\,(r+\delta)}{f\,(\theta_b)} - \beta c_b\,(1-s) - \frac{s c_b\,(\sigma-\beta)}{\sigma-1}\left(\frac{c_b}{c_g}\right)^{1-\eta} < 0 \tag{B2}$$

Using the implicit function theorem for $\mathbb{ZP}_b(\phi,\,\theta_b) = 0$, we have $d\,\phi/d\,\theta_b > 0$. This means that this expression describes an increasing relationship in the (ϕ, θ_b) space. As figure B1 makes clear, a unique equilibrium exists in ϕ and θ_b. In turn, the other endogenous components of the model can be easily determined. Specifically, from eqs. (18) – (21) one respectively obtain the equilibrium values of θ_g, s, w_g and w_b. Finally, the steady-state eq.s in (5) and (6) allow to derive the levels of employment and unemployment in each sector.

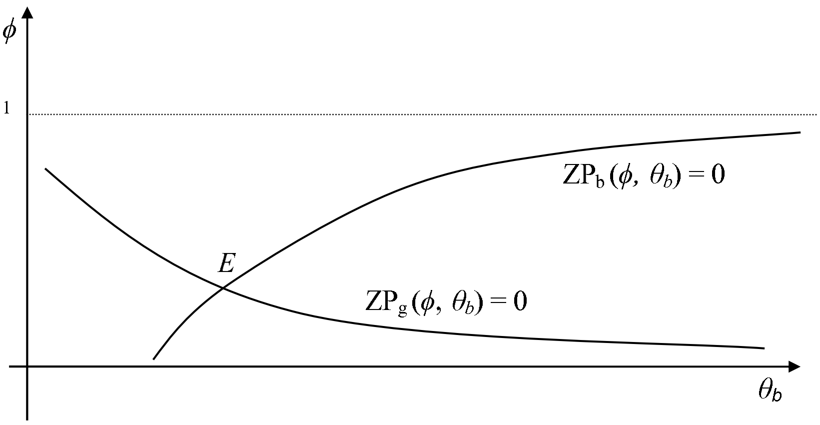

Figure B1: Existence and uniqueness of equilibrium