Revisiting the Conflicts between ‘Environmental Taxes vs Standard’ in the Context of International Trade: The Role of Waste Recycling

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ABSTRACT: The present paper throws light on the famous “tax versus standard” debate in the sphere of environmental economics by using general equilibrium framework and tries to examine which of the two, i.e., tax or standard is the better way to deal with pollution. The present paper has done so in the presence of a waste recycling sector which is the unique feature of it and has shown the impact of tax and standard separately on different polluting and non-polluting sectors of the economy. The paper has developed a unique as well as an interesting result that in the presence of a waste recycling sector in the economy, both pollution tax and environmental standard have the same impact.

Key words: Environmental Regulation, Green Capital, Waste Management and General Equilibrium.

JEL Classification: N50, F11, H23, D58

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1. Introduction

Over the last two and a half decades, perhaps the most relevant, worrying and most thought over issue worldwide has been global warming and the root cause of it, environmental pollution and how to deal with it for the sake of saving this planet. There may be no wrong in saying that an upsurge in the population growth and an urge for rapid development have contributed to the present scenario of environmental degradation. But the need for development cannot be denied either considering the fact that over two third of the population worldwide still belongs to developing economies. In order to become developed, it is necessary to fasten the production process and other economic activities and here lies the tragedy of the poor nations because such activities are contributing to the devastating environmental pollution. For sustainable development, it is therefore necessary to adopt proper measures that can continue the production process yet do not create pollution to an extreme level. The measures which are available to the controlling authority for dealing with pollution have given birth to the famous debate of environmental economics: “tax versus standard”, that is, the authority can either impose tax on the polluting industries or they may reduce the maximum allowable level of pollution for the sake of environment. Developed countries, with their good capital base, have started using those technologies that will not create pollution. This technology may be referred to as “green technology” or “green capital”. Developing nations, mostly, do not even possess such technology for being poor. Therefore, they either cannot use such sophisticated technology or even if they can, they use it in selected; few areas, not in all sectors of the economy because of lack of capital. Developing economies largely either imposes tax or restricts the emitting level and, if possible, uses “green technology” in very few areas.
Green technology generally enters in developing economies through the inflow of foreign capital or foreign investment and in the presence of globalization this trend is expected to grow where a developing nation would come across foreign entrepreneurs possessing pollution free technical knowhow. So the use of “green technology” in a developing nation initiates mainly through foreign investment. There are empirical evidences in support of this argument.

One more important aspect that has gained momentum and earned respect in the modern world is the recycling and reusing a product. During the production process there may be few things which become obsolete but there may be few things which may be re-used and may be given the shape of another product through the process of recycling. Recycling is the process of separating, collecting and remanufacturing or converting used or waste products into new materials. The recycling process involves a series of steps to produce new products. Recycling helps to extend the life and usefulness of something that has already served its initial purpose by producing something that is useable. Recycling has a lot of benefits and importance not only to us humans but especially to our planet. Recycling is very important as waste has a huge negative impact on the natural environment. A few of these impacts are:

1. Harmful chemicals and greenhouse gasses are released from rubbish in landfill sites. Recycling helps to reduce the pollution caused by waste.
2. Habitat destruction and global warming are some the affects caused by deforestation. Recycling reduces the need for raw materials so that the rainforests can be preserved.
3. Huge amounts of energy are used when making products from raw materials. Recycling requires much less energy and therefore helps to preserve natural resources.

Recycling is essential to cities around the world and to the people living in them because we have no space for waste. Our landfill sites are filling up fast. Recycling reduce financial expenditure in the economy. Making products from raw materials costs much more than if they were made from recycled products. It also preserves natural resources for future generations.

1 The term “globalization” is used in a broad sense in a developing nation where “greater integration of the world economy through heightened trade and investment flows and greater mobility of factors” are expected.
2 The investigating works of Mansfield (1961, 1968) may be mentioned.
Presently in India, about 960 million tonnes of solid waste is being generated annually as by-products during industrial, mining, municipal, agricultural and other processes. Of this, 350 million tonnes are organic wastes from agricultural sources; 290 million tonnes are inorganic waste of industrial and mining sectors and 4.5 million tonnes are hazardous in nature. There are many evidences that output or may be wastage of one sector is used by that sector or by a different sector to form a new product. So the dirty goods may contribute to the formation of pollution free goods. But the process of recycle and re-use needs the presence of sound, advanced technology which again generates the necessity of ‘green technology’.

The main motivation behind this study generates from the fact that there are only a few empirical works are there on the issue of pollution control through pollution tax or setting standard. From the theoretical angle also there are only few works to mention and even in this regard perhaps no work has been done by comprising the famous debate of “tax versus standard” in the presence of recycling sector in a general equilibrium framework. The only exception in this regard is the work by Gupta (2012). He has examined the trade off between tax and standard in a general equilibrium framework. The idea of the present study is generated from the work of Gupta (2012) in the presence of an additional recycling sector. Once recycling is introduced in a model which is similar to that of Gupta (2012), both the structure and results of the present study becomes widely different from those of Gupta (2012). This model helps us in understanding the inter-sectoral linkages and also to examine whether there is actually any trade off between tax and standard in the presence of recycling.

This study is based on a small open economy general equilibrium structure that has been used by several authors for dealing with environmental and trade related issues. The present study considers the impact of both pollution tax as well as setting environmental standard by regulating the level of emission in the presence of a recycling effect. The concept of green capital is also

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3 See Pappu, Saxena and Asolekar (2006) for details.

4 “Dirty goods are those goods that create pollution during the production process.

incorporated without categorising the different sectors of the economy into formal or informal sectors.

The paper is organised in the following sub sections. Section 2 considers the basic model. Comparative static analysis is done in section 3. Finally, the concluding remarks are made in section 4.

2. The Basic Model

We consider a small open economy with four sectors: a sector that requires no green capital and they need not recycle wastes (sector x), the sector which requires green capital to recycle wastes (sector z), the polluting-manufacturing sector (sector y) and the fourth sector is the waste-generating, non-traded intermediate good sector (sector v). This sector also uses green-capital. Its output is independent of the output of sector y. Wastes are generated by sector y during the process of producing its own output and by sector v from solid wastes of the community or from consumers. As sector v uses green capital, the wastes generated by this sector are all recycled by sector z. In other words, as sector v uses green capital, the wastes generated by this sector are not harmful for the economy as they can always be used for the purpose of recycling. Sector x and sector z together can be considered as “non-polluting” sectors within the economy. The assumption of small open economy implies that apart from the non-traded sector v, for all other sectors the product prices are fixed as the economy is a price-taker in the international market. Production function in each sector exhibits constant returns to scale (CRS), variable coefficient technology and diminishing marginal productivity to the variable factors. Sector x and Sector y use traditional capital, K, which is perfectly mobile between these two sectors. Sector z and sector v use much advanced, sector-specific “green capital” K_G which is perfectly mobile between these two sectors. The use of K_G by these two sectors implies the use of better, advanced, non-polluting technology by these sectors. Labour is assumed to be perfectly mobile among all the four sectors. Sector x and Sector z employ labour at a fixed wage rate \( w \). The remaining workers are used by the other two sectors, that is, sector y and sector v, at the 

\[ 6 \] Apart from sector V, all other sectors produce final commodities in this model.
competitive wage rate \( w \), which is assumed to be flexible and lower than \( \bar{w} \). The workers who fail to get a work in sectors \( x \) and sectors \( z \) are absorbed in sectors \( y \) and \( v \). So, there exists full employment in the labour market. The product of sector \( x \) is regarded as a numeraire and its price has been set equal to unity. A pollution tax \( \alpha \) is imposed on per unit of output of sector \( y \), which is the polluting-manufacturing sector.

The following notations are used in the formal presentation of the model:

\[
\begin{align*}
\alpha_{Li} &= \text{Labour-output ratio in the } i\text{-th sector, } i=x,y,z,v. \\
\alpha_{Ki} &= \text{Capital-output ratio in the } i\text{-th sector, } i=x,y,z,v. \\
P_i &= \text{Price of the product of the } i\text{-th sector, } i=x,y,z,v. \\
w &= \text{Competitive wage rate of sector } y \text{ and sector } v. \\
\bar{w} &= \text{Fixed wage rate of sector } x \text{ and sector } v. \\
X &= \text{Output of sector } x. \\
Y &= \text{Output of sector } y. \\
Z &= \text{Output of sector } z. \\
V &= \text{Output of sector } v. \\
r &= \text{Rate of return on traditional capital.} \\
R &= \text{Rate of return on green capital.} \\
L &= \text{Total endowment of labour in the economy.} \\
K &= \text{Total endowment of traditional capital.} \\
K_G &= \text{Total stock of green capital.} \\
\alpha &= \text{Tax rate on per unit of output of sector } y \\
\Omega &= \text{Maximum allowable pollution.} \\
\lambda_{ji} &= \text{Use of } j\text{-th factor by } i\text{-th sector with respect to total factor endowment (} j=L,K; i=x,y,z,v) \\
\theta_{ji} &= \text{distributive share of the } j\text{-th input in the } i\text{-th sector.}
\end{align*}
\]

2.1 Equational Structure of the Model
The competitive equilibrium conditions of the four sectors are given as
\[ w a_{ LX} + r a_{ KX} = 1 \]  \hspace{1cm} (1) \\
\[ w a_{ LY} + r a_{ KY} = P_y - \alpha = P_y' \]  \hspace{1cm} (2) \\
\[ w a_{ LZ} + R a_{ KZ} + P_v a_{ vz} = P_z \]  \hspace{1cm} (3) \\
\[ w a_{ LV} + R a_{ KV} = P_v \]  \hspace{1cm} (4)

The relationship between the waste-generating intermediate sector \( v \) and waste recycling sector \( z \) is given as:
\[ a_{ vz} Z = V \]  \hspace{1cm} (5)

Mobility of traditional capital between sectors \( x \) and sector \( y \) is given as
\[ a_{ KX} X + a_{ KY} Y = K \]  \hspace{1cm} (6)

Full employment condition of the labour market is given by
\[ a_{ LX} X + a_{ LY} Y + a_{ LZ} Z + a_{ LV} V = L \]  \hspace{1cm} (7)

The mobility of green capital between sector \( z \) and sector \( v \) is given by
\[ a_{ KZ} Z + a_{ KV} V = K_G(\Omega) \text{ where, } K'_G < 0 \]  \hspace{1cm} (8)

In equation (8) the reason behind \( K'_G < 0 \) is that when there is an improvement in the standard of environment, that is, when the maximum allowable pollution level is reduced, there is an increase in the use of green capital as the producers are compelled to adopt environment-friendly technology.

### 2.2 Working of the Model

The working of the model is simple. From equation (1) we can solve for \( r \). Using the value of \( r \) in equation (2) we can get \( w \). From equation (4) we can express \( R \) in terms of \( P_v \). As \( R \) can be expressed in terms of \( P_v \), by using this relation, from equation (3) we can get the value of \( P_v \), using which again in equation (4) one can get the value of \( R \). From equation (5) we can express \( V \) in terms of \( Z \), by using this relationship in equation (8), one can get the value of \( Z \) and again by using the value of \( Z \) in equation (5) we can get the value of \( V \). Thus, both \( Z \) and \( V \) becomes
known. Now we are left with two equations, that is, equation (6) and equation (7), and two unknowns – X and Y. So, we can solve for the values of X and Y from these two equations.

3. Comparative Statics

Here we would like to examine the impact of a strict environmental policy in the form of improvement in the environmental standard (by imposing reduction in maximum allowable pollution) and also in the form of an increase in the tax rate on output of the polluting sector. Both the measures can be interpreted as a drive towards an improvement in environmental quality of the economy.

First, we would like to examine the impact of a strict environmental policy, that is, a reduction in the maximum allowable level of emission in all the economy. A strict environmental policy implies a reduction in Ω. When we examine the change in Ω, then the system or the model becomes decomposable as the input-output co-efficients can be determined independent of the output system. From equation (5) we can see that V can be expressed in terms of Z. So equation (8) can be expressed in terms of Z only. In equation (8), as Ω falls because of a tough policy, we see that $K_G$ rises. To maintain the equilibrium, it implies that Z should also increase. As $\dot{Z} = V$, it also implies that V should rise. This is shown by the following two equations that express change in Z and V with respect to change in Ω.

\[
\frac{\dot{Z}}{\Omega} = \frac{\varepsilon}{(\lambda_{KZ} + \lambda_{KV})} \quad (9)
\]

and

\[
\frac{\dot{V}}{\Omega} = \frac{\varepsilon}{(\lambda_{KZ} + \lambda_{KV})} \quad (10)
\]

Both the two equations above show an inverse relationship between Ω and output of sector Z and sector V because $\varepsilon$ is negative (less than zero) in sign. So as Ω falls, the output levels of both sectors v and sector z increase, it implies that the availability of effective labour decreases in rest

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7 See appendix at the end of this chapter for detailed mathematical derivations.
of the two sectors, that is, in sector x and sector y. This creates a “Rybczynski effect” for which output of the capital intensive sector or sector x rises and that of the labour intensive sector or sector y falls. Mathematical expressions of these results are shown below. (see appendix for detailed derivations)

\[
\frac{\hat{Y}}{\Omega} = \left\{ -\frac{(\lambda_{KZ} + \lambda_{KV})\lambda_{KX}}{\lambda_{LX}\lambda_{KX} - \lambda_{KY} + \lambda_{LY}\lambda_{KX}} \frac{\varepsilon}{\lambda_{KZ} + \lambda_{KV}} \right\}
\]

(11)

and

\[
\frac{\hat{X}}{\Omega} = \lambda_{KY} \left\{ -\frac{(\lambda_{KZ} + \lambda_{KV})}{\lambda_{LX}\lambda_{KX} - \lambda_{KY} + \lambda_{LY}\lambda_{KX}} \frac{\varepsilon}{\lambda_{KZ} + \lambda_{KV}} \right\}
\]

(12)

Again as \(\varepsilon\) is negative in nature, the above two equations express a direct relation between Y and \(\Omega\) and an inverse relation between X and \(\Omega\), respectively.

These findings can be expressed with the help of following proposition.

**Proposition 1:** A strict environmental policy in the form of a reduction in the maximum allowable pollution increases the supply of green capital and also increases the production of wastes and output of waste recycling sector. It increases output of the non-polluting sector and reduces output of the polluting sector, given that the non-polluting sector is more capital-intensive than the polluting sector.

So far we have discussed the effect of change in environmental standard. Now, we would like to see the effect of change in tax rate (\(\alpha\)) on the output of all the sectors as well as on the input prices.\(^8\)

\(^8\) Here we have shown the impact of a change in tax rate \(\alpha\) in terms of change in the price of the polluting sector(sector Y) after tax being imposed, that is, in terms of \(\hat{P}'_Y\). As \(\alpha\) rises, \(P'_Y\) falls. So, \(\alpha\) and \(P'_Y\) have an opposite relation between them. This can be proved very easily. We know that \(P'_y - \alpha = P'_y\) (see equation 3.2). Differentiating this expression with respect to \(P'_Y\), we get,
From equation (13) we find that the between price of the waste-generating intermediate sector (sector v) and price of the price of the polluting sector (sector y) is given as:

\[
\frac{\hat{P}_v}{P'_y} = \frac{\theta_{kz}}{\theta_{lz}(\theta_{kz} + \theta_{vz}\theta_{kv})}
\]  

(13)

The above expression is positive, so, the relation between \(\hat{P}_v\) and \(P'_y\) is direct, that is, as \(P'_y\) falls, \(\hat{P}_v\) falls. The economic interpretation is simple. An increase in the tax rate on the output of the polluting sector reduces effective price of the product of the polluting sector, \(P'_y\), as \(r\) is already determined from equation (1), a fall in, \(P'_y\) causes a fall in \(w\), as we find from equation (2). For given \(P_v\), a fall in \(w\) causes a rise in \(R\) from equation (4). However, \(P_v\) is not given as sector v is the non-traded sector. This is evident from equation (3) where we find, for given \(P_z\), a rise in \(R\) implies a fall in \(P_v\). So we conclude a fall in \(P'_y\) causes a fall in \(P_v\). As \(R\) increases, it implies that \(a_{kz}\) and \(a_{kv}\) also falls, as a result of which \(Z\) increases and consequently \(V\) also increases, (as \(\hat{Z}=\hat{V}\)) it is shown by the following equation.

\[
\frac{d\alpha}{P'_y} = \frac{dP'_y}{P'_y}
\]

Or, \(-\frac{\alpha}{P'_y} \cdot \frac{d\alpha}{P'_y} = \hat{P}'_y\)

Or, \(-\frac{\alpha}{P_y - \alpha} = \hat{P}'_y\)

Or, \(-\gamma \hat{\alpha} = \hat{P}'_y\) where, \(\gamma = \frac{\alpha}{P_y - \alpha}\).

So the nature or sign (positive or negative) of any relationship which is expressed in terms of \(\hat{P}'_y\) will become exactly of the opposite nature or sign (positive or negative) when expressed in terms of \(\hat{\alpha}\). For example, if the impact of \(\hat{P}'_y\) on \(\hat{Z}\) is negative, it will become positive when expressed in terms of \(\hat{\alpha}\). This inverse relationship between \(\hat{P}'_y\) and \(\hat{\alpha}\) and its verbal explanation will be applicable on all those mathematical expression that shows a change in any variable with respect to a change in \(P'_y\) (\(\hat{P}'_y\)).

9 Equation (13) and equation (4.1) (in the appendix) are same, only for the sake of the simplicity, it is numbered differently, first, here and later in the appendix.
\[
\frac{\dot{Z}}{P'_y} = \left\{ -\frac{\lambda_{kz} A_2 + \lambda_{kv} A_4}{\lambda_{kz} + \lambda_{kv}} \right\} = \frac{\dot{V}}{P'_y}
\] (14)

As \( w \) falls (for given \( r \)), \( w/r \) falls or we can say that \( r/w \) increases. It also results in a fall in \( a_{kx} \) and \( a_{ky} \) for given \( X \) and \( Y \). So, there is excess supply of capital. From equation (7) we see that a fall in \( w \) results in an increase in \( a_{ly} \) and \( a_{lv} \). It results in a reduction in the availability of labour for sector \( x \) and sector \( y \) as \( \{ L - (a_{lz} + a_{lv} a_{vz}) Z \} \) falls. This is known as "Rybczynski-type effect". This will result in an increase in the output of sector \( x \) and a fall in that of sector \( y \), given that \( x \) is the capital-intensive sector than sector \( y \). We thus have:

\[
\frac{\dot{X}}{P'_y} = \frac{A_5 \lambda_{ky} - \lambda_{ly} A_6}{\lambda_{lx} \lambda_{ky} - \lambda_{kx} \lambda_{ly}}
\] (15)

and

\[
\frac{\dot{Y}}{P'_y} = \frac{\lambda_{lx} A_6 - \lambda_{ky} A_5}{\lambda_{lx} \lambda_{ky} - \lambda_{kx} \lambda_{ly}}
\] (16)

These results are presented by the following proposition.

**Proposition 2:** An increase in the tax rate on the output of the polluting sector increases the output of waste-generating intermediate sector as well as the output of waste recycling sector, it also increases the output of non-polluting capital-intensive sector but decreases the output of the labour-intensive polluting sector.

4. **Concluding Remarks**

In this paper we build up a model, using the H-O-S general equilibrium structure, assuming the presence of four sectors in the economy. Out of these four, two sectors are “non-polluting” and
other two are “polluting”. One sector from each category use green capital, that is, use advanced sophisticated, pollution free technology. A “non-polluting” sector takes the help of green technology to recycle wastes and a polluting sector does so to generate wastes for the purpose of recycling. In such a set up we have shown that the regulating authority possess the dual instrument to deal with pollution, that is, they may impose a pollution tax on the price of the product of the polluting industry and also they can reduce the maximum allowable level of emission which would compel industries to go for green technology. In this paper, we have shown the effects of both a reduction in the maximum allowable level of pollution as well as an increase in the tax rate on the output of the polluting sector.

In both the cases we have come across same results. In both the cases, we have seen that, either due to an increase in tax rate or because of a reduction in the maximum allowable emission level, the output levels of waste generating intermediate good sector and waste recycling sector have increased. We have also shown expansion of the non-polluting capital-intensive sector and contraction of the of the labour-intensive polluting sector. So, output levels of three of the four sectors go up out of which two are “non-polluting” in nature and that of the remaining sector goes down. One thing to be noticed here is that sectors which use green technology have expanded.

These results are significantly different to other findings in the field of “tax versus standard” studies. Most of the studies in this sphere have recommended either increase in pollution tax as superior to the reduction in the maximum emission level or vice-versa, depending upon their respective areas, under focus, which the impact of these two means are compared upon. But, our study reveals that both the measures would provide us the same result under some reasonable conditions.

**Appendix**

**Appendix 1  Mathematical Expressions of the Basic Model**

\[
\begin{align*}
\bar{a}_{LX} + r a_{KX} & = 1 \\
\bar{w} a_{LX} + r a_{KX} & = P_y - \alpha = P_y'
\end{align*}
\]
\[ \bar{w} a_{LZ} + Ra_{KZ} + P_v a_{VZ} = P_z \]  
\[ w a_{LV} + Ra_{KV} = P_v \]  
\[ a_{VZ} Z = V \]  
\[ a_{KX} X + a_{KY} Y = K \]  
\[ a_{LX} X + a_{LY} Y + a_{LZ} Z + a_{LV} V = L \]  
\[ a_{KZ} Z + a_{KV} V = K_G(\Omega), \quad K_G' < 0 \]  

**Appendix 1.A Detailed derivations of different expressions**

Derivations of equations considering change in the environmental standard, that is, a change in the maximum allowable level of emission (\( \Omega \)).

Differentiating equation (1) we get,
\[ a_{LX} \bar{w} + \bar{w} a_{LX} + a_{KX} dr + rd a_{KX} = 0 \]
As \( \bar{w} a_{LX} + rd a_{KX} = 0 \) (By Envelop Theorem) and \( d \bar{w} = 0 \), we get,
\[ \hat{r} \quad \theta_{KX} = 0 \]
Or, \( \hat{r} = 0 \) \hspace{1cm} (1.1)

Differentiating equation (2) we can get,
\[ a_{LY} dw + wd a_{LY} + a_{KV} dr + rd a_{KV} = d P'_Y \]
Again by using Envelop Theorem (\( wd a_{LY} + rd a_{KV} = 0 \)) from the above equation we get,
\[ \hat{w} \theta_{LY} = \hat{P}'_Y \]
Or, \( \hat{w} = \frac{\hat{P}'_Y}{\theta_{LY}} \) \hspace{1cm} (2.1)
Differentiating equation (3) and using Envelope theorem we get,

\[ \hat{R} \theta_{kz} + \hat{P}_v \theta_{vz} = 0 \]

Or, \[ \hat{R} = -\hat{P}_v \frac{\theta_{vz}}{\theta_{kz}} \] (3.1)

From equation (4) we get,

\[ wd a_{LV} + w b_{LV} dw + Rd a_{KV} + a_{KV} dR = 0 \]

By applying Envelope Theorem in the above expression we can get,

\[ \hat{w} \theta_{LV} + \hat{R} \theta_{KV} = \hat{P}_v \]

Using the value of \( \hat{R} \) from equation (3.1) in the above expression we get,

\[ \hat{w} \theta_{LV} - \hat{P}_v \frac{\theta_{vz}}{\theta_{kz}} \theta_{KV} = \hat{P}_v \]

Again, using the value of \( \hat{w} \) from equation (2.1) in the above expression we get,

\[ \frac{\hat{P}_v'}{\theta_{LV}} = \hat{P}_v (1 + \frac{\theta_{vz} \theta_{KV}}{\theta_{kz}}) \]

Or, \[ \frac{\hat{P}_v}{\hat{P}_v'} = \frac{\theta_{kz}}{\theta_{LV} (\theta_{kz} + \theta_{vz} \theta_{KV})} \] (4.1)

Using equation (4.1) in equation (3.1) we can get another expression of \( \hat{R} \), expressed as,

\[ \hat{R} = -\hat{P}_v' \frac{\theta_{vz}}{\theta_{LV} (\theta_{kz} + \theta_{vz} \theta_{KV})} \] (4.2)

From equation (5) we see that as \( a_{vz} \) is fixed, we can say,

\[ \hat{z} = \hat{v} \] (5.1)

From (6) we find that,
\[a_{kx} \, dx + x \, a_{kx} \, dy + y \, a_{ky} = 0\]

or, \[X \, \frac{a_{kx}}{k} \, \frac{dx}{x} + Xa_{kx} \, \frac{da_{kx}}{a_{kx}} + Ya_{ky} \, \frac{dy}{y} + Ya_{ky} \, \frac{da_{ky}}{a_{ky}} = 0\]

or, \[\lambda_{kx} \, \hat{x} + \lambda_{kx} \, \hat{a}_{kx} + \lambda_{ky} \, \hat{y} + \lambda_{ky} \, \hat{a}_{ky} = 0\]

or, \[\lambda_{kx} \, \hat{x} + \lambda_{kx} \, \hat{a}_{kx} = - (\lambda_{ky} \, \hat{y} + \lambda_{ky} \, \hat{a}_{ky})\]

From equation (1) and equation (2) we get-----

\[\hat{a}_{kx} \text{ and } \hat{a}_{ky} \text{ are fixed,}\]

So, \[\lambda_{kx} \, \hat{x} + \lambda_{ky} \, \hat{y} = 0\]

or, \[\hat{x} = - \frac{\lambda_{ky}}{\lambda_{kx}} \, \hat{y}\]  \hspace{1cm} (6.1)

From equation (7) we get,

\[a_{lx} \, dx + x \, a_{lx} \, dy + y \, a_{ly} \, dz + z \, a_{lz} \, dv + v \, a_{lv} = 0\]  \hspace{1cm} (7.1)

As \(\hat{a}_{lx}, \hat{a}_{ky}\) and \(\hat{a}_{lz}\) are fixed from equations (1), (2) and (3) respectively and by using elasticity of substitution from equation (4), we can express \(\hat{a}_{lv}\) as a function of \(\hat{p}_{y}'\). It is proved in equation (4.B.3), \textit{(proved later)}. But here we are dealing with change in maximum allowable emission level or \(\Omega\) and therefore the effect of pollution tax rate, \(\alpha\), is held constant. So, \(\hat{p}_{y}'\) which shows the price after tax \((P_y - \alpha\) ) can be considered as zero. So, we can say \(\hat{a}_{lv} = 0\).

By using \(\hat{a}_{lx}, \hat{a}_{ky}, \hat{a}_{lz}\) and \(\hat{a}_{lv} = 0\) in equation (7.1) we get,

Or, \[\lambda_{lx} \, \hat{x} + \lambda_{ly} \, \hat{y} + \lambda_{lz} \, \hat{z} + \lambda_{lv} \, \hat{v} = 0\]

Using the result of equation (5.1), in the above expression we can get,
\[ \lambda_{lx} \ddot{X} + \lambda_{ly} \ddot{Y} + (\lambda_{lz} + \lambda_{lv}) \ddot{Z} = 0 \]

Or, \[ \lambda_{lx} \ddot{X} + \lambda_{ly} \ddot{Y} = -(\lambda_{lz} + \lambda_{lv}) \ddot{Z} \]

Using the value of \( X \) from equation (6.1) in the above expression we get,

\[ \lambda_{lx} \left( -\frac{\lambda_{ky}}{\lambda_{kx}} \ddot{Y} \right) + \lambda_{ly} \ddot{Y} = -(\lambda_{lz} + \lambda_{lv}) \ddot{Z} \]

Or, \[ \ddot{Y} = -\frac{(\lambda_{lz} + \lambda_{kv})\lambda_{kx}}{(\lambda_{lx} \lambda_{kx} - \lambda_{ky} + \lambda_{ly} \lambda_{kx})} \ddot{Z} \] (7.2)

From equation (8) we get,

\[ a_{kz} dZ + Zd a_{kz} + a_{kv} dV + Vd a_{kv} = K_G' \]

Or, \[ \lambda_{kz} \dot{Z} + \lambda_{kz} \dot{a}_{kz} + \lambda_{kv} \dot{V} + \lambda_{kv} \dot{a}_{kv} = \epsilon \dot{\Omega} \]

As \( \dot{Z} = \dot{V} \) (from equation 5.1), the above equation can be written as,

\[ \dot{Z} (\lambda_{kz} + \lambda_{kv}) + \lambda_{kz} \dot{a}_{kz} + \lambda_{kv} \dot{a}_{kv} = \epsilon \dot{\Omega}, \text{ here} \]

Here also, like the previous case, we can get the value of \( \hat{a}_{kz} \) from equation (3) and the value of \( \hat{a}_{kv} \) from equation (4) in terms of \( \hat{P'}_y \). The fact that \( \hat{a}_{kz} \) and \( \hat{a}_{kv} \) can be expressed in terms of \( \hat{P'}_y \) is proved in equation (3.B) and in equation (4.B.2) respectively (proved later). Repeating the reasoning of the last case used after equation (7.1), we can say that the values of \( \hat{a}_{kz} \) and \( \hat{a}_{kv} \) are zero (0) in this case as we are dealing with change in \( \Omega \) and not in \( \alpha \). So, from the above equation we can get,

\[ \dot{Z} = \frac{\epsilon \dot{\Omega}}{(\lambda_{kz} + \lambda_{kv})} \] (8.1)
Again, by using the result obtained in equation (5.1), we get,

\[ \hat{V} = \frac{\hat{\omega}}{\lambda_{KZ} + \lambda_{KV}} \]  

(8.2)

Using equation (8.1) in equation (7.2) we get,

\[ \hat{Y} = \left\{ -\frac{(\lambda_{KZ} + \lambda_{KV})\hat{\lambda}_{KX}}{(\lambda_{Lx}\hat{\lambda}_{KX} - \hat{\lambda}_{KY} + \lambda_{LY}\lambda_{KK}\hat{\lambda}_{KX}) (\lambda_{KZ} + \lambda_{KV})} \right\} \]  

(7.3)

We use the above expression in the equation (6.1) and get,

\[ \hat{X} = -\frac{\lambda_{KY}}{\lambda_{KX}} \hat{Y} \]

Or, \[ \hat{X} = -\frac{\lambda_{KY}}{\lambda_{KX}} \left\{ -\frac{(\lambda_{KZ} + \lambda_{KV})\hat{\lambda}_{KX}}{(\lambda_{Lx}\hat{\lambda}_{KX} - \hat{\lambda}_{KY} + \lambda_{LY}\lambda_{KK}\hat{\lambda}_{KX}) (\lambda_{KZ} + \lambda_{KV})} \right\} \]

Or, \[ \hat{X} = \frac{\lambda_{KY}}{\lambda_{KX}} \left\{ \frac{(\lambda_{KZ} + \lambda_{KV})\hat{\lambda}_{KX}}{(\lambda_{Lx}\hat{\lambda}_{KX} - \hat{\lambda}_{KY} + \lambda_{LY}\lambda_{KK}\hat{\lambda}_{KX}) (\lambda_{KZ} + \lambda_{KV})} \right\} \]

Or, \[ \hat{X} = \lambda_{KY} \left\{ \frac{(\lambda_{KZ} + \lambda_{KV})\hat{\omega}}{(\lambda_{Lx}\hat{\lambda}_{KX} - \hat{\lambda}_{KY} + \lambda_{LY}\lambda_{KK}\hat{\lambda}_{KX}) (\lambda_{KZ} + \lambda_{KV})} \right\} \]  

(6.2)

**Appendix 1.B Detailed derivations of different expressions**

Derivations of equations considering change in the environmental pollution tax rate, that is, a change in \( \alpha \).

From equation (2) we get,
\[ w a_{LY} + r a_{KY} = P'_y \]
\[ wd a_{LY} + rd a_{KY} = 0 \]

Or, \[ \theta_{LY} \hat{a}_{LY} + \theta_{KY} \hat{a}_{KY} = 0 \]

Or, \[ \hat{a}_{LY} = -\frac{\theta_{KY}}{\theta_{LY}} \hat{a}_{KY} \] (2.B)

Again, \[ \sigma_y' = \frac{\hat{a}_{KY} - \hat{a}_{LY}}{\hat{w} - \hat{r}} \]

Or, \[ \sigma_y' \hat{w} = \frac{\hat{a}_{KY}}{\theta_{LY}} \] (as \( \hat{r} = 0 \))

Using the value of \( \hat{w} \) from equation (2.1) in the above expression we get,

\[ \hat{a}_{KY} = \sigma_y' \hat{P}_y' \]

Now from equation (2.B) we get the value of \( \hat{a}_{LY} \) as,

\[ \hat{a}_{LY} = -\frac{\theta_{KY}}{\theta_{LY}} \sigma_y' \hat{P}_y' \] (2.B.1)

From equation (3) we get,

\[ Rd a_{KZ} + P_v d a_{ vz} = 0 \]

Or, \[ \theta_{KZ} \hat{a}_{KZ} + \theta_{vz} \hat{a}_{vz} = 0 \]

Or, \[ \sigma_z = \frac{\hat{a}_{vz} - \hat{a}_{KZ}}{\hat{R} - \hat{P}_v} \]

As \( \hat{a}_{vz} = 0 \), we can say,

\[ \sigma_z = \frac{-\hat{a}_{KZ}}{\hat{R} - \hat{P}_v} \]

Or, \[ \sigma_z (\hat{R} - \hat{P}_v) = -\hat{a}_{KZ} \]

Or, \[ \hat{a}_{KZ} = \sigma_z (\hat{P}_v - \hat{R}) \]

Or, \[ \hat{a}_{KZ} = \sigma_z \left\{ \frac{\hat{P}_v' \theta_{KZ}}{\theta_{LY} (\theta_{KZ} + \theta_{vz} \theta_{KY})} + \frac{\hat{P}_y' \theta_{vz}}{\theta_{LY} (\theta_{KZ} + \theta_{vz} \theta_{KY})} \right\} \]
Or, \( \hat{a}_{KZ} = \hat{P}_y' \sigma_z \left\{ \frac{\theta_{KZ}}{\theta_{LY} (\theta_{KZ} + \theta_{VZ} \theta_{KV})} + \frac{\theta_{VZ}}{\theta_{LY} (\theta_{KZ} + \theta_{VZ} \theta_{KV})} \right\} \)

Or, \( \hat{a}_{KZ} = \hat{P}_y' A_1 \) \hspace{1cm} (3.B)

Where, \( A_1 = \sigma_z \left\{ \frac{\theta_{KZ}}{\theta_{LY} (\theta_{KZ} + \theta_{VZ} \theta_{KV})} + \frac{\theta_{VZ}}{\theta_{LY} (\theta_{KZ} + \theta_{VZ} \theta_{KV})} \right\} \), \( A_1 > 0 \).

Again from equation (4) we get,
\[ w_d a_{LV} + R_d a_{KV} = 0 \]

Or, \( \sigma_v = \frac{\hat{a}_{KV} - \hat{a}_{LV}}{\hat{w} - \hat{R}} \) \hspace{1cm} (4.B)

Again, \( w_d a_{LV} + R_d a_{KV} = 0 \) also implies
\[ \theta_{LV} \hat{a}_{LV} + \theta_{KV} \hat{a}_{KV} = 0 \]

Or, \( \hat{a}_{LV} = -\frac{\theta_{KV}}{\theta_{LV}} \hat{a}_{KV} \) \hspace{1cm} (4.B.1)

Using the above relation in the equation (4.B) we get,

\[ \hat{a}_{KV} = \theta_{LV} \sigma_v (\hat{w} - \hat{R}) \]

Or, \( \hat{a}_{KV} = \theta_{LV} \sigma_v \left\{ \frac{\hat{P}_y'}{\theta_{LY}} + \frac{\hat{P}_y' \theta_{VZ}}{\theta_{LY} (\theta_{KZ} + \theta_{VZ} \theta_{KV})} \right\} \)

Or, \( \hat{a}_{KV} = \theta_{LV} \sigma_v \hat{P}_y' \left\{ \frac{1}{\theta_{LY}} + \frac{\theta_{VZ}}{\theta_{LY} (\theta_{KZ} + \theta_{VZ} \theta_{KV})} \right\} \)

Or, \( \hat{a}_{KV} = A_2 \hat{P}_y' \) \hspace{1cm} (4.B.2)

Where, \( A_2 = \theta_{LV} \sigma_v \left\{ \frac{1}{\theta_{LY}} + \frac{\theta_{VZ}}{\theta_{LY} (\theta_{KZ} + \theta_{VZ} \theta_{KV})} \right\} > 0 \).

Using equation (4.B.2) in equation (4.B.1) we get,

\[ \hat{a}_{LV} = -\frac{\theta_{KV}}{\theta_{LV}} \hat{a}_{KV} \]

Or, \( \hat{a}_{LV} = -\frac{\theta_{KV}}{\theta_{LV}} A_2 \hat{P}_y' \)
Or, \( \hat{a}_{LV} = A_3 \hat{p}_y' \) \hspace{1cm} (4.B.3)

Where, \( A_3 = -\frac{\theta_{KV}}{\theta_{LV}} \). \( A_2 < 0 \).

Again equation (8) gives us,

\[
Zd a_{KZ} + a_{KZ} dZ + Vd a_{KV} + a_{KV} dV = 0
\]

Or, \( \lambda_{KZ} \hat{Z} + \lambda_{KZ} \hat{a}_{KZ} + \lambda_{KV} \hat{V} + \lambda_{KV} \hat{a}_{KV} = 0 \)

Using \( \hat{Z} = \hat{V} \) and the values of \( \hat{a}_{KZ} \) and \( \hat{a}_{KV} \) from equations (3.B) and (4.B.2) respectively in the above expression, we get,

\[
\hat{Z} = \left\{ -\frac{(\lambda_{KZ} A_2 + \lambda_{KV} A_1)}{(\lambda_{KZ} + \lambda_{KV})} \right\} \hat{p}_y'
\]

Or, \( \hat{Z} = A_4 \hat{p}_y' \) \hspace{1cm} (8.B)

Where, \( A_4 = \left\{ -\frac{(\lambda_{KZ} A_2 + \lambda_{KV} A_1)}{(\lambda_{KZ} + \lambda_{KV})} \right\} < 0 \). (As, \( A_1 > 0 \) and \( A_2 > 0 \)).

By differentiating equation (7) we get,

\[
a_{lx} dX + Xd a_{lx} + a_{ly} dY + Yd a_{ly} + a_{lz} dZ + Zd a_{lv} + a_{lv} dV + Vd a_{lv} = 0
\]

Or, \( \lambda_{lx} \hat{X} + \lambda_{lx} \hat{a}_{lx} + \lambda_{ly} \hat{Y} + \lambda_{ly} \hat{a}_{ly} + \lambda_{lz} \hat{Z} + \lambda_{lz} \hat{a}_{lz} + \lambda_{lv} \hat{V} + \lambda_{lv} \hat{a}_{lv} = 0 \)

Or, \( \lambda_{lx} \hat{X} + \lambda_{ly} \hat{Y} + \lambda_{ly} \hat{a}_{ly} + \lambda_{lz} \hat{Z} + \lambda_{lz} \hat{a}_{lz} + \lambda_{lv} \hat{V} + \lambda_{lv} \hat{a}_{lv} = 0 \) (as \( \hat{a}_{lx} = 0 \))

Or, \( \lambda_{lx} \hat{X} + \lambda_{ly} \hat{Y} + \lambda_{ly} \hat{a}_{ly} + \hat{Z} (\lambda_{lz} + \lambda_{lv}) + \lambda_{lv} \hat{a}_{lv} = 0 \) (as \( \hat{Z} = \hat{V} \))

Or, \( \lambda_{lx} \hat{X} + \lambda_{ly} \hat{Y} + \hat{Z} (\lambda_{lz} + \lambda_{lv}) + \lambda_{lv} \hat{a}_{lv} = 0 \)

Or, \( \lambda_{lx} \hat{X} + \lambda_{ly} \hat{Y} = -A_4 \hat{p}_y' (\lambda_{lz} + \lambda_{lv}) + \lambda_{ly} \frac{\theta_{KV}}{\theta_{LV}} \sigma_y \hat{p}_y' + \lambda_{lv} A_3 \hat{p}_y' = 0 \)

{by using the values of \( \hat{Z} , \hat{a}_{ly} , \hat{a}_{lv} \) from equations (8B), (2.B.1),and (4.B.3) respectively}

Or, \( \lambda_{lx} \hat{X} + \lambda_{ly} \hat{Y} = -A_4 \hat{p}_y' (\lambda_{lz} + \lambda_{lv}) + \lambda_{ly} \frac{\theta_{KV}}{\theta_{LV}} \sigma_y \hat{p}_y' - \lambda_{lv} A_3 \hat{p}_y' \)
Or, \( \dot{\lambda}_{LX} X + \dot{\lambda}_{LY} Y = \ddot{P}_Y \{ - A_4 (\dot{\lambda}_{LZ} + \dot{\lambda}_{LV}) + \dot{\lambda}_{LY} \frac{\theta_{KY}}{\theta_{LY}} \sigma'_Y - \lambda_{LV} A_3 \} \)

Or, \( \dot{\lambda}_{LX} X + \dot{\lambda}_{LY} Y = \ddot{P}_Y A_5 \) \hspace{1cm} (7.B)

Where, \( A_5 = \{ - A_4 (\dot{\lambda}_{LZ} + \dot{\lambda}_{LV}) + \dot{\lambda}_{LY} \frac{\theta_{KY}}{\theta_{LY}} \sigma'_Y - \lambda_{LV} A_3 \} > 0 \) (as \( A_4 < 0 \) and \( A_3 < 0 \))

Again, by differentiating equation (6) we get,
\[ a_{KX} dX + a_{KY} dY = 0 \]

Or, \( \dot{\lambda}_{KX} \dot{X} + \dot{\lambda}_{KX} \hat{a}_{KX} + \dot{\lambda}_{KY} \dot{Y} + \dot{\lambda}_{KY} \hat{a}_{KY} = 0 \)

Or, \( \dot{\lambda}_{KX} \dot{X} + \dot{\lambda}_{KY} \dot{Y} + \dot{\lambda}_{KY} \hat{a}_{KY} = 0 \) (as \( \hat{a}_{KX} \) is fixed)

Or, \( \dot{\lambda}_{KX} \dot{X} + \dot{\lambda}_{KY} \dot{Y} = - \dot{\lambda}_{KY} \hat{a}_{KY} \)

Or, \( \dot{\lambda}_{KX} \dot{X} + \dot{\lambda}_{KY} \dot{Y} = A_6 \ddot{P}_Y \) \hspace{1cm} (6.B)

Equations (6.B) and (7.B) can be written in the matrix form as:

\[
\begin{bmatrix}
\dot{\lambda}_{LX} & \dot{\lambda}_{LY} \\
\dot{\lambda}_{KX} & \dot{\lambda}_{KY}
\end{bmatrix}
\begin{bmatrix}
\dot{X} \\
\dot{Y}
\end{bmatrix} =
\begin{bmatrix}
\ddot{P}_Y A_5 \\
A_6 \ddot{P}_Y
\end{bmatrix}
\]

\[ \Delta = \lambda_{LX} \lambda_{KY} - \dot{\lambda}_{KX} \lambda_{LY} \]

\[ |\Delta| < 0 \] as \( X \) is the capital intensive sector and \( Y \) is the labour intensive sector.

By using Cramer’s rule we get the value of \( \dot{X} \) and \( \dot{Y} \) in terms of \( \dddot{P}_Y \) as,

\[ \dot{X} = \frac{\Delta_1}{\Delta} = \frac{A_5 \dot{\lambda}_{KY} - \dot{\lambda}_{LY} A_6}{\Delta} \dddot{P}_Y \] \hspace{1cm} (6.B.1)

and,

\[ \dot{Y} = \frac{\Delta_2}{\Delta} = \frac{\dot{\lambda}_{LX} A_6 - \dot{\lambda}_{KX} A_5}{\Delta} \dddot{P}_Y \] \hspace{1cm} (7.B.1)
References

