The Whole Economy Approach of the Input-Output Model

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Abstract

In macroeconomy research that uses technological table, most studies have only considered a special aspect. This paper develops an input-output model that extends to the whole economy by adding finance into transaction table. This forms a integrated capital flow cycling system for whole economy. Economic growth is used to reflect economic dynamic characteristics. Whole economy is divided into five subsystems. On the basis of subsystem's balance sheet, we set up the simultaneous equation of whole economy. As part of the disposal income, the new loan of the subsystems are affected by money supply badly and the income ratio is not stable. In this paper, expenditure ratio of subsystems is used to solve the simultaneous equation. The subsystem's balance sheets and the simultaneous equation can be applied to study fundamental economic issues effectively.

1 Introduction

In mathematical logic, Leontief built up the legitimate relationship among macro economic variables. It is a system of simultaneous equation of economic variables expressed with technological matrix [Leo86]. Input-output model (I-O in short) is very well known among economists and policy analysts, and it is widely used to study economic impacts of one sector on an economy in the national and the regional level [Mou00].

Leontief set up the relationship of environment pollution with consumption and production [Leo70]. M. Peneder illustrated the effect of industrial structure on aggregate income and growth [Pen03]. Swenson and Moore presented a method to determine full economic impacts of tax law [SM87]. Santos identified regional perturbations pursuant to disaster scenarios [JR12].

The application of technological table is too long to list. They have a common feature that every application is specialized in a particular aspect due to short of finance system. With finance in transaction table, it forms a integrated capital flow cycling system for the whole economy. Together with production, consumption and investment, taking finance into consideration is essential to build technological matrix, with which we can depict macroeconomy phenomena and explain its mechanism reasonably as a whole.

This paper attempts to fill the gap. On the closed economy, we divided whole economy into five subsystems: household, government, enterprise, investment and finance. As a result, the technological matrix is simplified to each subsystem's receipts and expenditure table. For every sectors on the technological table, input equals output and for every subsystem's on transaction table, disposal income equals payment. Both of them are in accordance with Walrasian general equilibrium theory [Wal54].

To illustrate economic dynamic characteristic, economic growth is introduced into transaction table. Then we get simultaneous equation of whole economy on the basis of transaction table and balance sheets. To solve the equation, we use output (expenditure) ratio instead of input (income) ratio. As part of disposal income, new loan amount is affected by money supply very badly, and as part of expenditure, the amount of loan to be repaid is decided by total loan of previous period. This suggests that expenditure ratio is more stable than income ratio. With the help of expenditure ratio, the dependent variables are replaced by independent ones. After plugging replaced variables into these equations, they become solvable. They can be used to illustrate

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economic mechanism, to perform economic simulating experiments, and to explain economic phenomena and fundamental issues such as money supply, calculation of inflation rate and economy growth, reasonable interest rate and exchange rate, and the cause of stagnation.

2 Transaction Table

To analyze macroeconomic situation as a whole, we divide closed economy system into 5 subsystems: household, government, enterprise, investment and finance. By tracing capital flows among subsystems, we get closed macroeconomy's technological table similar to Leontief's I-O table [Leo86]. Because subsystem's receipt is its input and payment is the output, the macroeconomy's technological table is also transaction table. We use x_{ij} to represent the capital flows among subsystems. x_{ij} indicates subsystem i paid x amount of money to subsystem j. The row sum $\sum_{j=1}^{n} x_{ij}$ represents subsystem's total payments and column sum $\sum_{i=1}^{n} x_{ij}$ total receipts.

	H	G	E	I	F
H	＼	x_{12}	x_{13}	x_{14}	x_{15}
G	x_{21}	＼	x_{23}	x_{24}	x_{25}
E	x_{31}	x_{32}	＼	x_{34}	x_{35}
I	x_{41}	x_{42}	x_{43}	＼	x_{45}
F	x_{51}	x_{52}	x_{53}	x_{54}	＼

Table 1: Transaction Table. H: Household; G: Government; E: Enterprise; I: Investment; F: Finance.

For Leontief's I-O table, it is evident that every sector's total input equals total output and for the transaction table above, receipts equal payments. Both of them are in accordance with the general equilibrium presented by Walras [Wal54]. The equilibrium of subsystems capital flows are exactly the balance sheets used in accounting. For the purpose of easy to understand and convenient to discuss in depth, the components of income and payment of every subsystem will be listed and their relationship will be studied.

3 Balance Sheets

The balance sheet discussed here is in a broad sense. The income is all the money a subsystem can get (including new loan), and payment consists of all the money a subsystem can use. To complete balance sheets, the variables and their meanings are given in Table 2.

Take household as an example to demonstrate the establishment of subsystem's balance sheets. Household incomes and payments are listed in Table 3.

Writing in symbols, household balance sheet becomes:

$$L_{ce} + B_e + P_{rh} + S_L + B_g + L_{cg} + L_{ci} + L_h + D_{ih} = C_h + C_{ih} + S + B_{en} + B_{gn} + L_{ah} + D_h + L_{ih} \,.$$

Inspecting the balance sheet, we find that the loan is still part of disposal income. Actually in the whole inspecting period, the loan was gradually used and used up at the end of period. The new loan added to entire economy caused the price goes up and quantity rises. In turn, comparing with last period, it enhanced every sub-system's actual income. In short, every sub-system's loan melt down in the entire economy by the work of economical mechanism and increased sub-systems' actual incomes by feedback. The problem appeared above is caused by analyzing dynamic economic issues with static analysis method. Therefore the balance sheet has to be reformatted to approach actual economic condition.

For simplicity, we assume constant increment to scale. That is if nominal growth increase by 5 percent, all the transaction variables in inspecting period will also increase by 5 percent, except for the variables related

Variables	Meaning
A_i	Asset bought from government for investment (mainly land).
AR	Available money to be loaned and deposit reserve.
B	Bond. B_e: bonds becoming due by enterprise, B_{en}: bonds newly issued by enterprise B_g: bonds becoming due by enterprise, B_{gn} bonds newly issued by government.
C	Consumption. C_g: Government consumption, C_h: Household consumption, C_i: Investment consumption, C_{ig}: Capital Construction consumption, C_{ih}: Housed investing consumption.
C_r	Unit material cost rate
D	Deposit. D_e: enterprise deposit, D_g: government deposit D_h: household deposit, D_i: investment deposit
D_i	Deposit interest. D_{ie}: Interest income from enterprise deposit, D_{ig}: Interest income from government deposit D_{ih}: Interest income from household deposit, D_{ii}: Interest income from investment deposit
I	Investment. I_{gi}: government investment to state owned companies.
L	Loan. L_e: Enterprise loan L_g: Government loan L_h: Household loan L_i: Investment loan
L_a	The amount of the loan to be repaid. L_{ae}: loan amount needed to be paid to finance by enterprise, L_{ag}: . . . government, L_{ah}: . . . household, L_{ai}: . . . investment.
L_c	Labor cost. L_{ce}: enterprise labor costs, L_{cg}: government labor costs, L_{ci}: investment labor costs.
L_i	Loan interest L_{ie}: loan interest paid to finance by enterprise, L_{ig}: . . . by government, L_{ih}: . . . by household, L_{ii}: . . . by investment.
M	Money. Newly issued money.
P	Price.
P_r	Profit. P_{rg}: profits paid to government by enterprise, P_{rh}: profits paid to household by enterprise, P_{ri}: profits used for new investment by enterprise.
Q	Quantity. The amount of goods and service.
S	Stock.
SL	Salary. Government employee salary.
T	Tax.

Table 2: List of variables and their meanings.

Income	Comes from	Payment	Goes to
Labour cost L_{ce}	E	Consumption C_h	E
Matured Bond B_e	E	Investment Consumption C_{ih}	E
Profit P_{rh}	E		
Salary SL	G	New bond B_{gn}	G
Matured Bond B_g	G		
Labour cost of capital consumption L_{cg}	G		
Labour cost L_{ci}	I	New bond B_{en}, New stock S	I
New loan L_h	F	Due loan L_{ah}	F
Deposit interest D_i	F	Loan interest L_{ih}	F
		Deposit D_h	F

Table 3: Household incomes and payments. Pensions are included in salary.

to finance x_{5j} and $x_{i5}(i,j=1,\dots,5)$. The reason that finance related variables do not move together with growth is that new loan is decided by new issued money and repaid loan is affected by total loan.

Take actual income from last period new plus loan and interest of deposit from this period as this period's disposal income, and take the sum of actual payment of last period and its nominal increment from this period, plus payback to finance from this period as this period's payments, the reformatted household balance sheet is shown below:

$$L^o_{ce}+B^o_e+P^o_{rh}+S^o_L+B^o_g+L^o_{cg}+L^o_{ci}+L'_h+D'_{ih}=(1+Q')(C^o_h+C^o_{ih}+S^o+B^o_{en}+B^o_{gn})+L'_{ah}+D'_h+L'_{ih}\,,$$

where superscripts $\cdot^o$ indicates Last period (year), and $\cdot'$ indicates the inspecting period (this year).

Likewise Government balance sheet:

$$T^o+P^o_{rg}+B^o_{gn}+A^o_i+L'_g+D'_{ig}=(1+Q')(C^o_g+C^o_{ig}+I^o_{gi}+SL^o+B^o_g+L^o_{cg})+L'_{ag}+D'_g+L'_{ig}\,,$$

Enterprise balance sheet

$$P^oQ^o+L'_e+D'_{ie}=(1+Q')(C^o_rQ^o+L^o_{ce}+B^o_e+P^o_{rh}+P^o_{ri}+P^o_{rg}+T^o)+L'_{ae}+D'_e+L'_{ie}\,.$$

Note that total sales PQ, intermediate goods C_rQ and final goods satisfy:

$$PQ-C_rQ=C_h+C_{ih}+C_g+C_{ig}+C_i\,.$$

If the ratio of intermediate goods to total sales λ is a constant. The intermediate goods could be replaced by λ and the final goods:

$$PQ=\frac{1}{1-\lambda}(C_h+C_{ih}+C_g+C_{ig}+C_i),\quad PQ=\frac{C_rQ}{\lambda}\,.$$

This gives

$$C_rQ=\frac{\lambda}{1-\lambda}(C_h+C_{ih}+C_g+C_{ig}+C_i),\quad Q'C_rQ=\frac{\lambda Q'}{1-\lambda}(C_h+C_{ih}+C_g+C_{ig}+C_i)\,.$$

Hence the transformed enterprise balance sheet will be:

$$(1-\frac{\lambda Q'}{1-\lambda})(C^o_h+C^o_{ih}+C^o_g+C^o_{ig}+C^o_i)+L'_e+D'_{ie}=(1+Q')(L^o_{ce}+B^o_e+P^o_{rh}+P^o_{ri}+P^o_{rg}+T^o)+L'_{ae}+D'_e+L'_{ie}\,.$$

Investment balance sheet

$$P_{ri}^o + S^o + B_{en}^o + I_{gi}^o + L_i' + D_{ii}' = (1+Q')(C_i^o + A_i^o + L_{ci}^o) + L_{ai}' + D_i' + L_{ii}' \,,$$

and the Finance balance sheet

$$\begin{aligned} & M' + L_{ah}' + D_h' + L_{ih}' + L_{ag}' + D_g' + L_{ig}' + L_{ae}' + D_e' + L_{ie}' + L_{ai}' + D_i' + L_{ii}' \\ = & \Delta AR' + L_h' + D_{ih}' + L_g' + D_{ig}' + L_e' + D_{ie}' + L_i' + D_{ii}' \,. \end{aligned}$$

4 Macro Economy Mechanism

Summing up all the money one subsystem can get from others forms capital flow variables x_{ij} used in transaction table. See Table 4.

x_{21}	x_{12}	x_{31}	x_{13}	x_{41}
$S_L + B_g + L_{cg}$	B_{gn}	$L_{ce} + P_{rh} + B_e$	$C_h + C_{ih}$	L_{ci}
x_{14}	x_{24}	x_{42}	x_{23}	x_{32}
$B_{en} + S$	I_{gi}	A_i	$C_g + C_{ig}$	$P_{rg} + T$
x_{34}	x_{43}	x_{51}	x_{15}	x_{52}
P_{ri}	C_i	$L_h + D_{ih}$	$L_{ah} + L_{ih} + D_h$	$L_g + D_{ig}$
x_{25}	x_{54}	x_{45}	x_{53}	x_{35}
$L_{ag} + L_{ig} + D_g$	$L_i + D_{ii}$	$L_{ai} + L_{ii} + D_i$	$L_e + D_{ie}$	$L_{ae} + L_{ie} + D_e$
$M_1 = M - \Delta AR$				

Table 4: Components of x_{ij}.

On the basis of sub-system's balance sheets and summed up capital flow variables, the working mechanism chart of entire-dynamic macro economy can be drawn as Figure 1.

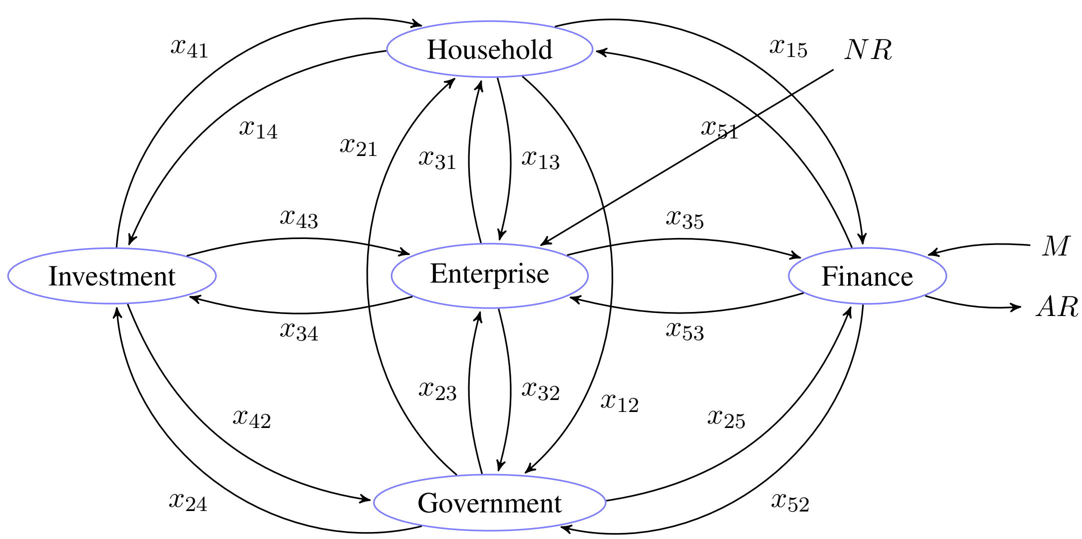

Figure 1: Illustration of the entire dynamic macro economy. NR: Natural Resource; M: Money Supply; AR: Available money to be loaned and deposit reserve.

Figure 1 depicts the working mechanism of entire dynamic macro economy and how it is running orderly and effectively. On the market, profits drive economic units, resource provides basic needs and monetary policy ensures the market moving smoothly. The capital flows formed in market transaction makes all the economic activities a whole macro economy. The changes of amount and direction of capital flows determine the state of macro economy.

5 Dynamic Transaction Table

Among sub-systems of macro economy, there are capital inter-flows. It is the capital inter-flows of sub-systems, which connect one sub-system to another inseparably, forming a whole organic macro economy. In the following discussion, the simultaneous equations will be set-up according to the quantities of capital flow among sub-systems, which contains the logical relationship of all the macro-economic variables.

Putting summed up variable x_{ij} into balance sheets, we get simultaneous equations composed of 5 sub-system balance sheets:

$$
\begin{aligned}
x^o_{21} + x^o_{31} + x^o_{41} + x'_{51} &= (1+Q')(x^o_{12} + x^o_{13} + x^o_{14}) + x'_{15} \\
x^o_{12} + x^o_{42} + x^o_{32} + x'_{52} &= (1+Q')(x^o_{21} + x^o_{24} + x^o_{23}) + x'_{25} \\
(1 - \frac{\lambda Q'}{1-\lambda})(x^o_{13} + x^o_{23} + x^o_{43}) + x'_{53} &= (1+Q')(x^o_{31} + x^o_{32} + x^o_{34}) + x'_{35} \\
x^o_{14} + x^o_{24} + x^o_{34} + x'_{54} &= (1+Q')(x^o_{41} + x^o_{42} + x^o_{43}) + x'_{45} \\
x'_{15} + x'_{25} + x'_{45} + x'_{35} + M'_1 &= x'_{51} + x'_{52} + x'_{54} + x'_{53}
\end{aligned}
$$

Table 1 is a static and settled accounts transaction matrix. We can establish a dynamic one according to the equations above. See Table 5.

	H	G	E	I	F
H		$(1+\theta')x^o_{12}$ x^o_{12}	$(1+\theta')x^o_{13}$ $(1-\frac{\lambda\theta'}{1-\lambda})x^o_{13}$	$(1+\theta')x^o_{14}$ x^o_{14}	x'_{15}
G	$(1+\theta')x^o_{21}$ x^o_{21}		$(1+\theta')x^o_{23}$ $(1-\frac{\lambda\theta'}{1-\lambda})x^o_{23}$	$(1+\theta')x^o_{24}$ x^o_{24}	x'_{25}
E	$(1+\theta')x^o_{31}$ x^o_{31}	$(1+\theta')x^o_{32}$ x^o_{32}		$(1+\theta')x^o_{34}$ x^o_{34}	x'_{35}
I	$(1+\theta')x^o_{41}$ x^o_{41}	$(1+\theta')x^o_{42}$ x^o_{42}	$(1+\theta')x^o_{43}$ $(1-\frac{\lambda\theta'}{1-\lambda})x^o_{43}$		x'_{45}
F	x'_{51}	x'_{52}	x'_{53}	x'_{54}	M'_1

Table 5: Dynamic transaction Table. The variables on the top of each cell are used to calculate payments and those on the bottom are used to calculate receipts.

6 Simultaneous Equation

Even taking M'_1 as known, the simultaneous equation has 20 unknowns and only 5 linear equations. To solve it, we need the second assumption. Similar to single process production function assumption used in I-O model [Leo86], we assume that there is a fixed expenditure pattern for every subsystem. The constant payment to scale is that subsystems single payment will increase with its total income by the same precent. The payment proportion to total income of subsystems is subject to economic structure. They are relatively stable, so we take it as constant. For example, during the 11 years from 2001 to 2012 in United Kingdom, the living cost and food expenditure remains steady, 489 pound per week (inflation adjusted). It is almost a level line [NS113]. The statistical analysis of Americans household expenditure proportion got similar conclusion [WSJ12].

Emphasizing the stability of expenditure ratio does not deny its little change. The payment ratio is only an instrument used to solve the equation here. In the process of applying them, we need to adjust them according to actual statistic results. The payment ratio can be used to reform the simultaneous equations. Payment ratio of sub-systems is in Table 6.

Household	Variable	x_{12}	x_{13}	x_{14}	x_{15}	
	Ratio	α_1	α_2	α_3	α_4	$\sum \alpha_i = 1$
Government	Variable	x_{21}	x_{24}	x_{23}	x_{25}	
	Ratio	β_1	β_2	β_3	β_4	$\sum \beta_i = 1$
Enterprise	Variable	x_{31}	x_{32}	x_{34}	x_{35}	
	Ratio	γ_1	γ_2	γ_3	γ_4	$\sum \gamma_i = 1$
Investment	Variable	x_{41}	x_{42}	x_{43}	x_{45}	
	Ratio	ζ_1	ζ_2	ζ_3	ζ_4	$\sum \zeta_i = 1$
Finance	Variable	x_{51}	x_{52}	x_{54}	x_{53}	
	Ratio	ε_1	ε_2	ε_3	ε_4	$\sum \varepsilon_i = 1$

Table 6: Payment ratio of subsystems.

The calculation of payment ratio is shown in Table 7.

α_1	α_2	α_3	α_4
$\frac{(1+Q')x^o_{12}}{x^o_{21}+x^o_{31}+x^o_{41}+x'_{51}}$	$\frac{(1+Q')x^o_{13}}{x^o_{21}+x^o_{31}+x^o_{41}+x'_{51}}$	$\frac{(1+Q')x^o_{14}}{x^o_{21}+x^o_{31}+x^o_{41}+x'_{51}}$	$\frac{x'_{15}}{x^o_{21}+x^o_{31}+x^o_{41}+x'_{51}}$
β_1	β_2	β_3	β_4
$\frac{(1+Q')x^o_{21}}{x^o_{12}+x^o_{42}+x^o_{32}+x'_{52}}$	$\frac{(1+Q')x^o_{24}}{x^o_{12}+x^o_{42}+x^o_{32}+x'_{52}}$	$\frac{(1+Q')x^o_{23}}{x^o_{12}+x^o_{42}+x^o_{32}+x'_{52}}$	$\frac{x'_{25}}{x^o_{12}+x^o_{42}+x^o_{32}+x'_{52}}$
ζ_1	ζ_2	ζ_3	ζ_4
$\frac{(1+Q')x^o_{41}}{x^o_{14}+x^o_{24}+x^o_{34}+x'_{54}}$	$\frac{(1+Q')x^o_{42}}{x^o_{14}+x^o_{24}+x^o_{34}+x'_{54}}$	$\frac{(1+Q')x^o_{43}}{x^o_{14}+x^o_{24}+x^o_{34}+x'_{54}}$	$\frac{x'_{45}}{x^o_{14}+x^o_{24}+x^o_{34}+x'_{54}}$
γ_1	γ_2	γ_3	γ_4
$\frac{(1+Q')x^o_{31}}{(1-\frac{\lambda Q'}{1-\lambda})(x^o_{13}+x^o_{23}+x^o_{43})+x'_{53}}$	$\frac{(1+Q')x^o_{32}}{(1-\frac{\lambda Q'}{1-\lambda})x^o_{13}+x^o_{23}+x^o_{43})+x'_{53}}$	$\frac{(1+Q')x^o_{34}}{(1-\frac{\lambda Q'}{1-\lambda})x^o_{13}+x^o_{23}+x^o_{43})+x'_{53}}$	$\frac{x'_{35}}{(1-\frac{\lambda Q'}{1-\lambda})x^o_{13}+x^o_{23}+x^o_{43})+x'_{53}}$
ε_1	ε_2	ε_3	ε_4
$\frac{x'_{51}}{x'_{15}+x'_{25}+x'_{45}+x'_{35}+M'_1}$	$\frac{x'_{52}}{x'_{15}+x'_{25}+x'_{45}+x'_{35}+M'_1}$	$\frac{x'_{54}}{x'_{15}+x'_{25}+x'_{45}+x'_{35}+M'_1}$	$\frac{x'_{53}}{x'_{15}+x'_{25}+x'_{45}+x'_{35}+M'_1}$

Table 7: Payment ratio calculation.

On the basis of the second assumption, every subsystem's expenditure variables are all interrelated and dependent variables. There is only one independent expenditure variable in each subsystem. They can be replaced one by the other. Therefore, the simultaneous equation has 5 independent variables with 5 linear equations. It can be solved uniquely. Choosing one payment variable such as x^o_{21} and 4 income variables x^o_{12}, x^o_{42}, x^o_{32}, x'_{52} in one subsystem's balance sheet as basic variables, we build new simultaneous equations, shown below.

$$
\begin{aligned}
x_{21}^o &= \frac{\beta_1}{1+Q'}(x_{12}^o + x_{42}^o + x_{32}^o + x_{52}') \\
x_{12}^o &= \frac{\alpha_1}{1+Q'}(x_{21}^o + x_{31}^o + x_{41}^o + x_{51}') \\
x_{42}^o &= \frac{\zeta_2}{1+Q'}(x_{14}^o + x_{24}^o + x_{34}^o + x_{54}') \\
x_{32}^o &= \frac{\gamma_2}{1+Q'}\left[(1-\frac{\lambda Q'}{1-\lambda})(x_{13}^o + x_{23}^o + x_{43}^o) + x_{53}'\right] \\
x_{52}' &= \varepsilon_2(x_{15}' + x_{25}' + x_{45}' + x_{35}' + M_1')
\end{aligned}
$$

To get solvable equations, we need to replace all other variables by basic ones with the aid of variable replacement table. See Table 8.

Government	Variable	x_{21}^o	x_{24}^o	x_{23}^o	x_{25}'
	Replacement	$\frac{\beta_1}{\beta_1}x_{21}^o$	$\frac{\beta_2}{\beta_1}x_{21}^o$	$\frac{\beta_3}{\beta_1}x_{21}^o$	$\frac{\beta_4(1+Q')}{\beta_1}x_{21}^o$
Household	Variable	x_{12}^o	x_{13}^o	x_{14}^o	x_{15}'
	Replacement	$\frac{\alpha_1}{\alpha_1}x_{12}^o$	$\frac{\alpha_2}{\alpha_1}x_{12}^o$	$\frac{\alpha_3}{\alpha_1}x_{12}^o$	$\frac{\alpha_4(1+Q')}{\alpha_1}x_{12}^o$
Investment	Variable	x_{42}^o	x_{41}^o	x_{43}^o	x_{45}'
	Replacement	$\frac{\zeta_2}{\zeta_2}x_{42}^o$	$\frac{\zeta_1}{\zeta_2}x_{42}^o$	$\frac{\zeta_3}{\zeta_2}x_{42}^o$	$\frac{\zeta_4(1+Q')}{\zeta_2}x_{42}^o$
Enterprise	Variable	x_{32}^o	x_{31}^o	x_{34}^o	x_{35}'
	Replacement	$\frac{\gamma_2}{\gamma_2}x_{32}^o$	$\frac{\gamma_1}{\gamma_2}x_{32}^o$	$\frac{\gamma_3}{\gamma_2}x_{32}^o$	$\frac{\gamma_4(1+Q')}{\gamma_2}x_{32}^o$
Finance	Variable	x_{52}'	x_{51}'	x_{54}'	x_{53}'
	Replacement	$\frac{\varepsilon_2}{\varepsilon_2}x_{52}'$	$\frac{\varepsilon_1}{\varepsilon_2}x_{52}'$	$\frac{\varepsilon_3}{\varepsilon_2}x_{52}'$	$\frac{\varepsilon_4}{\varepsilon_2}x_{52}'$

Table 8: Variable Replacement Table

Putting replaced variables into new simultaneous equations, we have re-organized 5 elements linear function simultaneous equations. See below.

$$
\begin{aligned}
x_{21}^o &= \frac{\beta_1}{1+Q'}(x_{12}^o + x_{42}^o + x_{32}^o + x_{52}') \\
x_{12}^o &= \frac{\alpha_1}{1+Q'}(x_{21}^o + \frac{\zeta_1}{\zeta_2}x_{42}^o + \frac{\gamma_1}{\gamma_2}x_{32}^o + \frac{\varepsilon_1}{\varepsilon_2}x_{52}') \\
x_{42}^o &= \frac{\zeta_1}{1+Q'}(\frac{\beta_2}{\beta_1}x_{21}^o + \frac{\alpha_3}{\alpha_1}x_{12}^o + \frac{\gamma_3}{\gamma_2}x_{32}^o + \frac{\varepsilon_3}{\varepsilon_2}x_{52}') \\
x_{32}^o &= \frac{\gamma_2}{1+Q'}\left[(1-\frac{\lambda Q'}{1-\lambda})(\frac{\beta_3}{\beta_1}x_{21}^o + \frac{\alpha_2}{\alpha_1}x_{12}^o + \frac{\zeta_3}{\zeta_2}x_{42}^o) + \frac{\varepsilon_4}{\varepsilon_2}x_{52}'\right] \\
x_{52}' &= \varepsilon_2(1+Q')(\frac{\beta_4}{\beta_1}x_{21}^o + \frac{\alpha_4}{\alpha_1}x_{12}^o + \frac{\zeta_4}{\zeta_2}x_{42}^o + \frac{\gamma_4}{\gamma_2}x_{32}^o) + \varepsilon_2 M_1'
\end{aligned}
$$

Taking economic structure ratio $\alpha_i, \beta_i, \gamma_i, \zeta_i, \varepsilon_i$ and nominal growth rate Q' as known. x_{ij} can be solved, which are expressed by M_1'. The determinant solution of the simultaneous equation is given below:

$$\begin{pmatrix} 1 & -\frac{\beta_1}{1+Q'} & -\frac{\beta_1}{1+Q'} & -\frac{\beta_1}{1+Q'} & -\frac{\beta_1}{1+Q'} \\ -\frac{\alpha_1}{1+Q'} & 1 & -\frac{\zeta_1\alpha_1}{\zeta_2(1+Q')} & -\frac{\gamma_1\alpha_1}{\gamma_2(1+Q')} & -\frac{\varepsilon_1\alpha_1}{\varepsilon_2(1+Q')} \\ -\frac{\beta_2\zeta_2}{\beta_1(1+Q')} & -\frac{\alpha_3\zeta_2}{\alpha_1(1+Q')} & 1 & -\frac{\gamma_3\zeta_2}{\gamma_2(1+Q')} & -\frac{\varepsilon_3\zeta_2}{\varepsilon_2(1+Q')} \\ -\frac{\beta_3\gamma_2(1-\frac{\lambda Q'}{1-\lambda})}{\beta_1(1+Q')} & -\frac{\alpha_2\gamma_2(1-\frac{\lambda Q'}{1-\lambda})}{\alpha_1(1+Q')} & -\frac{\zeta_3\gamma_2(1-\frac{\lambda Q'}{1-\lambda})}{\zeta_2(1+Q')} & 1 & -\frac{\varepsilon_4\gamma_2}{\varepsilon_2(1+Q')} \\ -\frac{\beta_4\varepsilon_2(1+Q')}{\beta_1} & -\frac{\varepsilon_2\alpha_4(1+Q')}{\alpha_1} & -\frac{\zeta_4\varepsilon_2(1+Q')}{\zeta_2} & -\frac{\gamma_4\varepsilon_2(1+Q')}{\gamma_2} & 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \varepsilon_2 M_1' \end{pmatrix}$$

7 The Usage of Simultaneous Equation

To demonstrate the usage of simultaneous equation, a hypothetical payment ratio table is given in Table 9.

Series Ratio	1	2	3	4
α	.05	.4	.1	.45
β	.15	.1	.55	.2
γ	.35	.3	.2	.15
η	.2	.1	.55	.15
ϵ	.12	.3	.4	.18

Table 9: Hypothetical Payment Ratio

If the hypothetical nominal growth θ' and intermediate goods ratio λ are $.25$ and $.4$ respectively. The calculated determinant solution of the simultaneous equation is

$$\begin{pmatrix} 1 & -.12 & -.12 & -.12 & -.12 \\ -.04 & 1 & -.08 & -.0466 & -.016 \\ -.0533 & -.16 & 1 & -.0533 & -.1067 \\ -.7333 & -1.6 & -1.1 & 1 & -.144 \\ -.5 & -3.375 & -.5625 & -.1875 & 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ .3M_1' \end{pmatrix}$$

After solving the equation, we get x_{ij}^o and x_{ij}' in Table 10.

	x_{12}^o	x_{13}^o	x_{14}^o	x_{15}'
	.04077	.32616	.08154	.45866
x_{21}^o		x_{23}^o	x_{24}^o	x_{25}'
.13687		.50816	.09125	.22812
x_{31}^o	x_{32}^o		x_{34}^o	x_{35}'
.42864	.36741		.24494	.22963
x_{41}^o	x_{42}^o	x_{43}^o		x_{45}'
.20158	.10079	.55435		.18898
x_{51}'	x_{52}'	x_{53}'	x_{54}'	
.25264	.63160	.37896	.84213	

Table 10: x_{ij} solutions. We omit the M_1' factor in all entries.

Now the dynamic transaction table becomes:

	H	G	E	I	F
H		.05096	.40770	.10193	.45866
		.04077	.27180	.08154	
G	.17109		.62733	.11406	.22812
	.13687		.41821	.09125	
E	.53580	.45926		.30618	.22963
	.42864	.36741		.24494	
I	.25198	.12600	.69294		.18898
	.20158	.10079	.46196		
F	.25264	.63160	.37896	.84213	1

Table 11: Hypothetical dynamic transaction table.

8 Applications

The result discussed above can be used widely in macroeconomy. We give a few examples demonstrating applications of balance sheets and the simultationeous equation.

8.1 Applications of Balance Sheets

Monetary Equation. The money supply can be captured by summing up sub-systems 5 balance sheets. After added up and reorganized, we have the money supply:

$$
\begin{aligned}
M' = \Delta AR \quad & \text{Available loan and reserve increment} \\
+ \Delta C_h^o + \Delta C_{ih}^o + \Delta C_g^o + \Delta C_{ig}^o + \Delta C_i^o + \Delta C_r^o \cdot Q^o \quad & \text{Total sales increment } \Delta(PQ) \\
+ \Delta L_{ce}^o + \Delta L_{ci}^o + \Delta L_{cg}^o + \Delta SL^o \quad & \text{Labor cost increment } \Delta L_c \\
+ \Delta P_{rh}^o + \Delta P_{rg}^o + \Delta P_{ri}^o \quad & \text{Profit increment } \Delta P_r \\
+ \Delta B_{en}^o + \Delta B_{gn}^o + \Delta B_e^o + \Delta B_g^o + \Delta S^o \quad & \text{security market volume increment } \Delta B \\
+ \Delta T^o \quad & \text{Tax increment} \\
+ \Delta A_i^o \quad & \text{Asset increment} \\
+ \Delta I_{gn}^o \quad & \text{State owned co investment increment by government}
\end{aligned}
$$

Let $Y = LC + P_r + B + T + A_i + I_{gi}$ denote the yield. The money supply is equal to total income increment plus available loan and reserve increment. Equation can be written in short:

$$M' = \Delta(PQ)^o + \Delta Y^o + \Delta AR' \quad \text{or} \quad M_1' = \Delta(PQ)^o + \Delta Y^o \,.$$

.

Compared with monetary exchange equation $MV = PQ$ by Irving Fisher in 1911 [Fis11], it is evident that its foundation is not enough. Except for total income increment of the year, last year's total income is supported by existing money in the system. Furthermore, the velocity of money has no direct connection with money supply, because money owner can only use their money once.

Inflation rate and economy growth calculation. By changing monetary equation into another form, the relationship among money supply, all sales, inflation rate and economic growth will surface. Dividing both sides of monetary equation with total sales P^oQ^o and reorganizing it we get inflation rate and the economy growth formula:

$$\frac{\Delta P}{P} + \frac{\Delta Q}{Q} = \frac{M' - \Delta AR' - \Delta Y^o}{P^oQ^o} \quad \text{or} \quad \frac{\Delta P}{P} + \frac{\Delta Q}{Q} = \frac{M_1' - \Delta Y^o}{P^oQ^o} \,.$$

The formula above tells that inflation rate plus growth equals new issued money minus available loan and reserve increment and yield increment divided by total sales of last year. Actually in the article “Relationship Between the Money Supply and Inflation” [FMI12] already got close conclusion with the formula above: The perception relationship between money supply and inflation should take economic growth into consideration.

Interest rate calculation in closed economy. By summing up the balance sheets of four subsystems (H, G, E, I), the relationship among economy growth, total loan and deposit of the term, accumulative total loan and deposit of last term, difference value between loan and deposit interest rate, time limit of loan and loan interest rate is uncovered. The reasonable loan interest rate in a closed enonomy setting can be calculated with the formula below:

$$i_\ell = \frac{TL' - TD'}{ATL^o - ATD^o} - \frac{TL^o/K + wTD^o}{ATL^o - ATD^o} - \frac{Q'((PQ)^o + Y^o)}{ATL^o - ATD^o} .$$

Here TL and TD are the total loan and deposit of this term respectively, ATL and ATD are accumulate total loan and deposit of last term resp., K is the time limit of the loan and w represents the difference value of loan and deposit interest rate.

In Taylor’s monetary and policy rule, interest rate responses to changes in inflation and output [Tay93]. The formula above provides a more close approach by taking new issued money into consideration.

Exchange rate calculation in open economy. Adding merchandise trading amount of export and import and capital flows in and out of the country to subsystem’s balance sheet of closed economy, we get subsystems’ balance sheets of open economy. By summing up five subsystems’ balance sheets in an open economy setting, the relationship among supply, economy growth, export of last term, import of this term, surplus of capital flow and exchange rate will appear. We can caculate the exchange rate from the formula below.

$$Re = \frac{M_1' - Q'(P^oQ^o + Y^o)}{(1 + Q')IM^o - EX^o - S_c'} ,$$

where Re means exchange rate, IM represents import, EX denotes export and S_c means capital flow surplus. OANDA only listed top 5 factors affecting exchange rate: interest rate, employment outlook, economic growth expectation, trade balance and central bank action [OA]. The formula also expounded the mathematical relationship among these factors.

8.2 Applications of Simultaneous Equation

The impacts of one variable to others. As the I-O model, simultaneous equation can be used to study the economic impacts of one variable change on others [Mou00]. Actually, if we take M_1' as a variable, the equation is a homogeneous system. Its solutions can be expressed with any one of six variables. So we can study the economic impacts on the other five variables caused by on variable change.

Economic simulating experiment. The simultaneous equation can be used to do economic simulating experiment. Input different set of macroeconomic control instruments like M_1', β_i and t (tax rate) with controlled level of inflation $\frac{\Delta p}{p}$ and expected target of growth $\frac{\Delta Q}{Q}$ together into simultaneous equation, we can get different set of x_{ij}. According to the desired economic state (x_{ij}), economy reality and feasibility of the instruments, policy maker can opt one set of macroeconomic control instrument (M_1', β_i, t) and put them into practice.

Reasons for stagnation. After the stagnation happened, the transaction variables x_{ij}^o in the period of economy crisis and x_{ij}' in the period of stagnation are all known. Rewrite simultaneous equation, the inflation rate $\Delta P/P$, subsystems’s actual growth $(\frac{\Delta Q_h}{Q_h}, \frac{\Delta Q_g}{Q_g}, \frac{\Delta Q_e}{Q_e}, \frac{\Delta Q_i}{Q_i})$ can be solved.

$$
\begin{aligned}
x^o_{21}+x^o_{31}+x^o_{41}+x^o_{51} =&(1+\frac{\Delta P}{P}+\frac{\Delta Q_h}{Q_h})(x^o_{12}+x^o_{13}+x^o_{14})+x'_{15}\\
x^o_{12}+x^o_{32}+x^o_{42}+x^o_{52} =&(1+\frac{\Delta P}{P}+\frac{\Delta Q_g}{Q_g})(x^o_{21}+x^o_{23}+x^o_{24})+x'_{25}\\
\left[1-\frac{\lambda}{1-\lambda}(\frac{\Delta P}{P}+\frac{\Delta Q_e}{Q_e})\right](x^o_{13}+x^o_{23}+x^o_{43})+x'_{53} =&(1+\frac{\Delta P}{P}+\frac{\Delta Q_e}{Q_e})(x^o_{31}+x^o_{32}+x^o_{34})+x'_{35}\\
x^o_{14}+x^o_{24}+x^o_{34}+x^o_{54} =&(1+\frac{\Delta P}{P}+\frac{\Delta Q_i}{Q_i})(x^o_{41}+x^o_{42}+x^o_{43})+x'_{45}
\end{aligned}
$$

To show how to use the equation above, we use the hypothetical x^o_{ij} and $x'_{i5}, i=1,\ldots,4$. Reallocate total loan with $\varepsilon_1(.1), \varepsilon_2(.4), \varepsilon_3(.35), \varepsilon_4(.15)$, and get

$$x'_{51}=.21584M'_1, x'_{52}=.86336M'_1, x'_{53}=.75544M'_1, x'_{54}=.32376M'_1\,.$$

Plug those known values (x^o_{ij}, x'_{ij}) into the equation we get:

$$\frac{\Delta P}{P}+\frac{\Delta Q_h}{Q_h}\approx .17, \frac{\Delta P}{P}+\frac{\Delta Q_g}{Q_g}\approx .57, \frac{\Delta P}{P}+\frac{\Delta Q_e}{Q_e}\approx .22, \frac{\Delta P}{P}+\frac{\Delta Q_i}{Q_i}\approx .15\,.$$

By the try and error method, we will obtain

Inflation rate	$\frac{\Delta P}{P}=28\%$
Household real consumption growth	$\frac{\Delta Q_h}{Q_h}=-11\%$
Government consumption growth	$\frac{\Delta Q_g}{Q_g}=29\%$
Enterprise real production growth	$\frac{\Delta Q_e}{Q_e}=-6\%$
Invest real growth	$\frac{\Delta Q_i}{Q_i}=-13\%$

Therefore on the basis of economic crisis, government's stimulating measures are the reason for stagnation. This conclusion provides a theoretical support to some existing deduction [CSM11].

9 Conclusions

We summarize below the main results and observations in this paper. Making use of balance sheets, we solve fundamental macro-economic issue.

- Monetary equation: $M' = \Delta(PQ)^o + \Delta Y^o + \Delta AR'$.
- Inflation rate and economy growth formula: $\frac{\Delta P}{P} + \frac{\Delta Q}{Q} = \frac{M' - \Delta AR' - \Delta Y^o}{P^o Q^o}$.
- Interest rate formula:

$$i_\ell = \frac{TL' - TD'}{ATL^o - ATD^o} - \frac{TL^o/K + wTD^o}{ATL^o - ATD^o} - \frac{Q'((PQ)^o + Y^o)}{ATL^o - ATD^o}.$$

- Exchange rate formula:

$$Re = \frac{M' - Q'(P^o Q^o + Y^o)}{(1 + Q')IM^o - EX^o - S'_c}.$$

Applying simultaneous equation, we can do economic simulating experiment on computer, to provide scientific basic for macro economy management. We also see that government's stimulating measures in economic crisis are the reason for stagnation.

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