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December 2014

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Revised December 22, 2014

Abstract: We develop a new approach to discrete choice demand for differentiated products, using copulas to separate the marginal distribution of consumer values for product varieties from their dependence relationship, and apply it to the issue of how preference dependence affects market outcomes in symmetric multiproduct industries. We show that greater dependence lowers prices and profits under certain conditions, suggesting that preference dependence is a distinct indicator of product differentiation. We also find new sufficient conditions for the symmetric multiproduct monopoly and the symmetric single-product oligopoly prices to be above or below the single-product monopoly price.

Keywords: Product differentiation, discrete choice, copula, multiproduct industries.

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*Earlier versions, circulated under the titles "Preference and Equilibrium in Monopoly and Duopoly" and "Preferences, Prices, and Performance in Multiproduct Industries", were presented at the 2009 Summer Workshop in Industrial Organization (University of Auckland), 2009 Summer Workshop on Antitrust Economics and Competition Policy (SHUFE, Shanghai), 2009 IO Day Conference (NYU), Segundo Taller de Organización Industrial (Chile, 2009), 2010 Choice Symposium (Key Largo), and seminars at Ecole Polytechnique/CREST, Fudan University, Lingnan University, Mannheim University, National University of Singapore, University of Melbourne, and University of Rochester. The authors thank conference and seminar participants, especially discussants Simon Anderson and Barry Nalebuff, and several referees for useful comments.

I. INTRODUCTION

A key issue in the economics of differentiated product markets is how the relationship between consumer values for different product varieties matters for market outcomes. Typical discrete choice models of product differentiation assume either that consumer values for different product varieties are independent (e.g. Perloff and Salop, 1985) or follow a joint distribution of a specific form (e.g., Anderson, et al., 1992). Such special cases are insightful but lack the general structure needed for a more complete understanding of how the correlation of consumer values for alternative products affects market outcomes.

In this paper, we develop a new approach to discrete choice demand in differentiated product markets that is more general than the familiar approaches. The key feature of the new approach is to use copulas to separate the marginal distribution of consumer values for each variety from their dependence relationship. A copula is a multivariate uniform distribution that "couples" marginal distributions to form a joint distribution. Furthermore, by Sklar's Theorem, it is without loss of generality to represent a joint distribution of consumer values by its marginal distributions and a copula (Nelsen, 2006). The virtue of the copula approach is that all information about dependence (or correlation) of values is contained in the copula. Thus the copula representation of consumer preferences makes it straightforward to analyze how market outcomes are affected by the distribution of values for each variety holding the dependence relationship constant, or by the dependence properties among the values for arbitrary marginal distributions. In this way, the copula approach provides an elegant and useful representation of consumer preferences for differentiated products.

In Section 2, we present a model of consumer preferences over an arbitrary number of symmetric varieties of a good. Consumer values for the varieties are assumed to follow a smooth and symmetric joint probability distribution. We interpret the mean and variance of the marginal distribution as measures of preference strength and preference diversity respectively, and let the preference dependence properties of the copula capture the correlations of values. We define preference dependence using standard concepts of positive and negative dependence of random variables, and order copulas accordingly. We apply this approach to investigate two issues.

First, in Section 3, we study how prices and profits change with the degree of preference dependence in symmetric multiproduct markets. In discrete choice models of product differentiation that assume independence between values of different varieties, a higher variance of consumer values (i.e. preference diversity) raises price and profit under certain conditions, and thus can be interpreted as an indicator of product differentiation (e.g., Anderson et. al. al, 1992; Johnson and Myatt, 2006; Perloff and Salop, 1985). A natural question to ask is, when preferences for different varieties are not independent, how does preference dependence relate to product differentiation? Intuitively, greater dependence means that more consumers regard the varieties as closer substitutes, suggesting that product differentiation is less when preference dependence is greater. We find that price and profit decrease in preference dependence for a symmetric multiproduct monopoly or a symmetric single-product oligopoly under certain conditions. Therefore, preference depedenence can be interpreted as a distinct indicator of product differentiation, separate from preference diversity.

Second, in Section 4, we examine how prices differ across several market structures. This issue is relevant in various scenarios: (1) as a result of innovation, a single-product monopolist introduces new varieties to become a multiproduct monopolist; (2) as the result of lower entry barriers, such as expiration of a patent, competing single-product firms enter a previously monopolized market; (3) due to a change in merger policy, a single-product symmetric oligopoly merges into a multiproduct monopoly. Understanding the price effects in such scenarios is both theoretcally interesting and policy relevant. While Chen and Riordan (2008) analyzes the issue for the special case in which the marginal distribution of consumer values is exponential, the present paper provides general comparisons for arbitrary marginal distributions. Specifically, we find that the single-product monopoly price is higher than the symmetric oligopoly price if the hazard rate of the marginal distribution is non-decreasing and preferences are positively dependent, but lower if the hazard rate is non-increasing and preferences are negatively dependent.[1] Furthermore, the symmetric multiproduct monopoly price is higher than the single-product monopoly price when preferences are uniformly positively dependent or negatively dependent.

[1] As we can see from this result, the copula approach is more powerful than simply assuming a particular joint distribution of consumer values for alternative products. The bivariate normal distribution, for example, does have the virtue of neatly separating preference diversity (variance) and preference dependence (correlation), but is restrictive in part because the marginal normal distribution has a particular shape with an increasing hazard rate.

We make concluding remarks in Section 5, and gather proofs in the Appendix.

II. THE COPULA REPRESENTATION OF PREFERENCES

Consumers are assumed to purchase at most one of $n \geq 2$ possible varieties of a good. A consumer's value (or willingness to pay) for the i^{th} variety is w_i. To describe consumer preferences, the standard approach is to specify a joint distribution of $\mathbf{w} \equiv (w_1, ..., w_n)$. The analyses typically proceed by assuming independence or that the joint distribution has a particular function form, e.g. is a bivariate normal distribution. For tractability, it is often also assumed that the joint distribution function is symmetric. The copula approach to discrete choice demand provides a more general structure for modeling the correlation of consumer values for different varieties, while remaining tractable in the symmetric case.

The copula representation of preferences is based on Sklar's Theorem in statistics, which states that the probability distribution of a vector of random variables can be represented by a copula and marginal distributions. More specifically, a copula is a multivariate cumulative distribution function with uniform marginal distribution functions.[2] According to Sklar's Theorem, if $H(\mathbf{w})$ is a multivariate distribution function with marginal distribution functions $F_i(w_i)$, then there exists a copula $C(\mathbf{x})$ such that $H(\mathbf{w}) = C(F_1(w_1), ..., F_n(w_n))$. Conversely, if $F_i(w_i)$ for $i = 1, ..., n$ are univariate distribution functions and $C(\mathbf{x})$ is a copula, then the composition $H(\mathbf{w}) = C(F_1(w_1), ..., F_n(w_n))$ is a multivariate distribution function with marginal distribution functions $F_i(w_i)$.

Consider a population of consumers whose size is normalized to 1, and assume for simplicity that the joint distribution of $\mathbf{w}$ in the population is symmetric, and also that the marginal distribution function for each variety can be inverted to obtain a strictly-increasing continuous function $w_i = w(x_i)$. Then, by construction, x_i is dis-

[2] A copula $C(\mathbf{x})$ is an n-increasing function, defined for all $\mathbf{x} \equiv (x_1, ..., x_n) \in [0,1]^n$, satisfying

$$C(x, 1, ...1) = ... = C(1, ...1, x) = x$$

and

$$C(x_1, ...x_{n-1}, 0) = ... = C(0, x_2, ...x_n) = 0.$$

See Nelsen (2006).

tributed uniformly on the unit interval $I \equiv [0,1]$. Conversely, by Sklar's Theorem, the symmetric joint distribution of consumer values for the n varieties is fully described by a continuous strictly-increasing valuation function $w(x_i)$ and a symmetric copula $C(\mathbf{x})$. A consumer's type can be thought of as a point in the unit cube, $x \in I^n$, with the copula describing the population of types. Finally, assume for further simplicity that $w(x_i)$ and $C(\mathbf{x})$ are twice differentiable functions.

The copula approach to representing consumer preferences models the strength and dispersion of consumer tastes for individual varieties separately from the correlation of tastes for different varieties. Under the simplifying symmetry, monotonicity, and smoothness assumptions, the inverse of the valuation function defines the marginal distributions of consumer values, while the copula contains all of the information about the correlation of values. The approach enables a general treatment of how correlation (or dependence) matters, based on the properties of the symmetric copula, while still maintaining a tractable analysis, and without unduly restricting the smooth symmetric distribution functions under consideration.

The advantage of the copula representation of consumer preferences over the standard approach is to separate cleanly the dependence properties of the joint distribution of values from the properties of the marginal distributions. This enables us to investigate how the correlation of consumer values matters for market outcomes for a wider class of joint distributions, since different marginal distributions generate different joint distributions for a given copula. As noted above, the copula determines the statistical dependence of consumer values for the varieties. In particular, $C(\mathbf{x}) = \Pi_{i=1,\dots,n} x_i$ is the independence copula, and $C(\mathbf{x})$ is positively (negatively) orthant dependent if $C(\mathbf{x}) > (<)\ \Pi_{i=1,\dots,n} x_i$ for all $\mathbf{x} \in (0,1)^n$. Furthermore, $C_1(\mathbf{x}) \equiv \partial C(\mathbf{x}) / \partial x_1$ is the conditional distribution of $(x_2, \dots, x_n)$ given x_1, and $C_{11}(\mathbf{x}) \equiv \partial^2 C(\mathbf{x}) / \partial x_1^2 < (>)\ 0$ for all $\mathbf{x} \in (0,1)^n$ indicates positive (negative) stochastic dependence.[3] Because marginal distribution functions are monotonic, these properties of the copula translate directly into corresponding dependence properties of the joint distribution of consumer values. For example, positive orthant dependence means that the probability that a randomly drawn consumer's values for all varieties are high (or low) is greater than if the values were independent, and posi-

[3] By symmetry, these stochastic dependence properties can also be defined using $C_k(\cdot)$ and $C_{kk}(\cdot)$ for $k = 2, \dots, n$.

tive stochastic dependence means that a high realization of w_1 shifts the conditional distribution of $(w_2, ..., w_n)$ according to first-order stochastic dominance. In what follows, we shall say simply that consumer values are *positively dependent* or *negatively dependent* when both the appropriate orthant and stochastic dependence conditions are satisfied.

It is convenient for our purposes to consider an arbitrary family of copulas indexed by a parameter θ. The copula family is ordered by increasing orthant dependence if a higher θ indicates greater orthant dependence, i.e. $C_\theta(\mathbf{x}) \equiv \partial C(\mathbf{x};\theta)/\partial\theta > 0$ for interior $\mathbf{x}$. Similarly, the copula family is ordered by increasing stochastic dependence if $C_{11\theta}(\mathbf{x};\theta) \equiv \partial C_{11}(\mathbf{x};\theta)/\partial\theta < 0$ for interior $\mathbf{x}$. Roughly speaking, greater orthant dependence means that there is a lower probability that consumers have low values for some products and high values for the others, while greater stochastic dependence means that a higher value for one variety makes low values for the others more likely. We will refer to the orthant dependence and stochastic orders collectively as **increasing dependence**.[4]

We use the properties of the valuation function and the copula to measure consumer preferences along three dimensions: preference strength, preference diversity, and preference dependence. Preference strength refers to how much consumers on average value each variety, while preference diversity refers to the heterogeneity of those values. The mean and variance of consumer values for each variety are, respectively, $\mu \equiv \int_0^1 w(x)\,dx$ and $\sigma^2 \equiv \int_0^1 [w(x) - \mu]^2\,dx$. We interpret μ to measure preference strength and σ to measure preference diversity. Both are properties of the marginal distribution, and σ has been considered an indicator of the degree of product differentiation under the assumption that consumer values are independent (Perloff and Salop, 1985). Preference dependence refers to the correlation of consumer values for different varieties, and is measured by a parameter θ, indexing an ordered family of copulas. A higher value of θ indicates that the values for different varieties are more positively dependent or less negatively dependent. We argue below that θ can be interpreted as a distinct indicator of product differentiation.

Given the copula representation of preferences, it is straightforward to derive con-

[4]If $n = 2$, then $C_{11\theta}(\mathbf{x}) < 0$ implies $C_\theta(\mathbf{x}) > 0$ (Nelsen, 2006). Nelsen (2006) discusses various copula families and their dependence properties for the case of $n = 2$; see also Joe (1997) for related discussions.

sumer demand. Denote the price for good 1 by p, and the prices for the rest of the $n-1$ goods by r_i. It is convenient to normalize the consumer values and the prices by defining

$$u_i \equiv \frac{w_i - \mu}{\sigma}; \qquad \bar{\mu} \equiv \frac{\mu}{\sigma}; \qquad \bar{p} \equiv \frac{p-\mu}{\sigma}; \qquad \bar{r}_i \equiv \frac{r_i - \mu}{\sigma}.$$

Moreover, denoting the marginal distribution of $u_i = \frac{w(x_i)-\mu}{\sigma}$ by $F(u_i)$, by the Sklar's Theorem, the joint distribution of normalized values is $C(F(u_1), ..., F(u_n))$. A type $\mathbf{x}$ consumer will purchase good 1 under the following condtions:

$$\begin{aligned} x_1 &\geq F(\bar{p}); \\ F(u(x_1) - \bar{p} + \bar{r}_i) &\geq x_i, \ i = 2, ..., n. \end{aligned}$$

Therefore, defining $u(x_i) \equiv F^{-1}(x_i)$, the demand for good 1 is

$$Q(\bar{p}, \bar{r}_2, ..., \bar{r}_n) = \int_{F(\bar{p})}^{1} C_1(x_1, F(u(x) - \bar{p} + \bar{r}_2), ..., F(u(x_1) - \bar{p} + \bar{r}_n))\, dx_1. \quad (1)$$

The demand for other goods is derived similarly. It follows that any two goods are always substitutes because, for $j = 2, ..., n$,

$$\begin{aligned} &\frac{\partial Q(\bar{p}, \bar{r}_2, ..., \bar{r}_n)}{\partial \bar{r}_j} \\ = &\int_{F(\bar{p})}^{1} C_{1j}(x, F(u(x) - \bar{p} + \bar{r}_2), ..., F(u(x) - \bar{p} + \bar{r}_j))\, f(u(x) - \bar{p} + \bar{r}_j)\, dx > 0, \end{aligned}$$

where $f(u_i)$ is the density function. If only a single good is offered, then its demand is simply $Q(\bar{p}) = 1 - F(\bar{p})$.

We conclude this section by introducing the Farlie-Gumble-Morgenstern (FGM) copula family. Its general form for $n \geq 3$ is given in Nelsen (2006). When $n = 2$, it becomes

$$C(x_1, x_2) = x_1 x_2 + \theta x_1 x_2 (1 - x_1)(1 - x_2).$$

Our results in the next two sections can be illustrated with examples that combine an

FGM copula for $n = 2$ with the exponential marginal distribution $F(u_i) = 1 - e^{-u_i - 1}$. We shall refer to this as the FGM-exponential case. More details of these illustrative examples are contained in Chen and Riordan (2011).

III. IMPACT OF DEPENDENCE ON PRICE AND PROFIT

In this section, we consider how preference dependence affects price and profit. We maintain three additional simplifying assumptions for this and the next section. First, the average cost of production for each variety is constant, and without loss of generality normalized to zero. An appropriate interpretation of the normalization is that consumers reimburse the firm for the cost of producing the product in addition to paying a markup p. Consequently, μ can be interpreted as mean value minus constant average variable cost, and thus can be either positive or negative. Second, at least some consumers have positive values so that there are gains from trade, i.e. $w(1) > 0$. Third, equilibrium prices exist uniquely and are interior under all market structures, and they are symmetric under multiproduct monopoly or oligopoly when $n \geq 2$.[5]

As a benchmark, we first note that the single-product monopoly (gross) profit function is $\pi^m(\bar{p}) = \sigma(\bar{p} + \bar{\mu})[1 - F(\bar{p})]$. The profit-maximizing normalized price $(\bar{p}^m)$ satisfies the first-order condition

$$(\bar{p}^m + \bar{\mu})\lambda(\bar{p}^m) = 1 \tag{2}$$

and the second-order condition

$$(\bar{p}^m + \bar{\mu})\lambda'(\bar{p}^m) + \lambda(\bar{p}^m) > 0$$

at an interior solution, where $\lambda(u) \equiv \frac{f(u)}{1-F(u)}$ is the hazard rate determining the elasticity of demand. A standard regularity condition, for which an increasing hazard rate $(\lambda'(u) \geq 0)$ is sufficient but not necessary, guarantees a unique interior maximum:

[5] For convenience, we refer to optimal prices under monopoly as equilibrium prices. An interior price satisfies $p \in (w(0), w(1))$, so the market is neither shut down nor fully covered. Consequently, profit functions are differentiable at equilibrium prices. Given the symmetry of $C(\cdot)$, the symmetric price assumption is quite natural; it is satisfied, for example, in our FGM-uniform case.

$d\left[\left(u+\bar{\mu}\right)\lambda\left(u\right)\right]/du>0$.

Next, we consider a price-setting multiproduct monopoly producing $n\geq 2$ symmetric varieties of the good. Its profit function for a symmetric price is

$$\pi^{mm}\left(\overline{p}\right)=\sigma(\overline{p}+\overline{\mu})\left[1-C\left(F(\overline{p}),...F(\overline{p});\theta\right)\right]. \tag{3}$$

The profit-maximizing normalized price $\bar{p}^{mm}$ satisfies

$$\left(\bar{p}^{mm}+\bar{\mu}\right)\lambda^{C}(\bar{p}^{mm};\theta)=1 \tag{4}$$

and

$$\left(\bar{p}^{mm}+\bar{\mu}\right)\frac{d\lambda^{C}(\bar{p}^{mm};\theta)}{du}+\lambda^{C}(\bar{p}^{mm};\theta)>0 \tag{5}$$

where

$$\lambda^{C}(u;\theta)\equiv\frac{nC_{1}(F\left(u\right),...,F\left(u\right);\theta)}{1-C(F\left(u\right),...,F\left(u\right);\theta)}f\left(u\right) \tag{6}$$

is the hazard rate corresponding to the cumulative distribution function $F^{C}(u)\equiv C(F\left(u\right),...,F\left(u\right);\theta)$ on support $\left[u\left(0\right),u\left(1\right)\right]$. It is exactily as if the monopolist is selling to consumers a choice of varieties. An appropriate regularity condition, satisfied for example in our FGM-exponential case, plays the same role as for single-product monopoly:

$$\frac{d\left(u+\bar{\mu}\right)\lambda^{C}(u;\theta)}{du}>0.$$

A useful property of a copula family ordered by increasing orthant dependence is that the conditional copula $C_{1}\left(x,...,x;\theta\right)$ increases (decreases) in θ when x is small (large). This implies that greater positive dependence shifts up the hazard rate for the multiproduct monopolist when market coverage is high enough

Lemma 1 *Given increasing orthant dependence, there exists some* $u^{*}\in(u(0),u\left(1\right)]$ *such that* $\frac{\partial\lambda^{C}(\bar{p};\theta)}{\partial\theta}>0$ *if* $\bar{p}\leq u^{*}$.

Furthermore, it is straightforward that the market is fully covered, or nearly so, if demand is sufficiently great.[6] This consideration leads to the conclusion that

[6]Let $\bar{\mu}^{o}=\frac{1}{f(u(0))}-u(0)$. Then the market is fully covered for $\bar{\mu}\geq\bar{\mu}^{o}$ and almost fully covered for $\bar{\mu}=\bar{\mu}^{o}-\epsilon$ and ϵ a small positive number. Our maintained interiority assumption implicitly assumes $\bar{\mu}<\bar{\mu}^{o}$.

prices under multiproduct monopoly decrease with preference dependence if preference strength is high. The profit of the multiproduct monopolist, $\pi^{mm} \equiv \pi^{mm}(\bar{p}^{mm})$, however, always decreases with greater dependence, whether or not price increases, because of the resulting downward shift in demand. Formally:

Proposition 1 *Given increasing orthant dependence : (i) there exists* $\bar{\mu}^*$ *such that* $p^{mm} > u(0)$ *when* $\bar{\mu} = \bar{\mu}^*$ *and* $\frac{dp^{mm}}{d\theta} < 0$ *if* $\bar{\mu} \geq \bar{\mu}^*$*; and (ii)* $\frac{d\pi^{mm}}{d\theta} < 0$.

Therefore, a multiproduct monopolist would prefer that consumer values for its n products are less positively (more negatively) dependent. This is intuitive, since the more similar are product varieties the less valuable is choice. Thus a higher θ reduces quantity at any given price and hence reduces equilibrium profit, while the effect of θ on equilibrium price is more subtle. The lower quantity under a higher θ motivates the firm to lower price, but the slope of the demand curve also changes with θ, possibly having an opposing effect on price. Both effects work in the same direction if demand is sufficiently strong. It is possible, however, that p^{mm} increases with θ if demand is sufficiently weak. For example, in the FGM-exponential case, numerical analysis shows that p^{mm} increases in θ if $\bar{\mu}$ is below a critical value.

Now suppose that the n products are sold by n symmetric single-product oligopoly firms. Given that all other firms charge price r, the profit function of Firm 1 is

$$\pi^n(\overline{p}, \overline{r}) = \sigma(\overline{p} + \overline{\mu}) Q(\overline{p}, \overline{r}..., \overline{r}).$$

From (1),

$$\left|\frac{\partial Q}{\partial \bar{p}}\right._{\bar{p}=\bar{r}} = -C_1(F(\bar{p}), ..., F(\bar{p})) f(\bar{p}) + \int_{F(\bar{p})}^{1} (n-1) C_{12}(x, ..., x) f(u(x)) dx.$$

In equilibrium, $\overline{p} = \overline{r} = \overline{p}^n$, satisfying

$$(\bar{p}^n + \bar{\mu}) h(\bar{p}^n; \theta) = 1, \tag{7}$$

where we define the adjusted hazard rate for oligopoly competition

$$h(u;\theta) \equiv \lambda^C(u;\theta) + n(n-1) \frac{\int_{F(u)}^{1} C_{12}(x, ..., x; \theta) f(u(x)) dx}{1 - C(F(u), ..., F(u); \theta)}, \tag{8}$$

which is equal to the hazard rate under multiproduct monopoly plus an extra term. The extra term is the diversion ratio used in contemporary merger analysis (Shapiro 1996, Farrell and Shapiro, 2010), that is, the percentage demand increase from a price cut resulting from customers who change allegiance. A modified regularity condition, once again satisfied in the FGM-exponential case, guarantees a unique symmetric equilibrium:

$$\frac{d\left(u+\bar{\mu}\right)h(u;\theta)}{du}>0. \tag{9}$$

Assuming the regularity condition for multiproduct monopoly holds, the regularity condition for symmetric oligopoly additionally requires that the diversion ratio does not fall too quickly as price rises. Each firm's equilibrium profit is

$$\pi^n\equiv\sigma\bar{\pi}^n=\frac{1}{n}\sigma\left(\bar{p}^n+\bar{\mu}\right)\left[1-C\left(F\left(\bar{p}^n\right),...,F\left(\bar{p}^n\right);\theta\right)\right]. \tag{10}$$

It is intuitive to expect that oligopoly competition intensifies with more preference dependence, as more consumers regard any two varieties to be close substitutes. In general, however, the effect of preference dependence on prices and profits is ambiguous. As under multiproduct monopoly, the regularity condition is not enough to ensure that prices monotonically decrease with θ. For while a higher θ shifts demand downward, motivating a lower price (market share effect), it also may affect the slope of the residual demand curve, potentially providing an incentive to raise price (price sensitivity effect). Under oligopoly, a unilateral marginal reduction in price impacts a firm's residual demand on both an extensive margin (market expansion) and the intensive margin (business stealing). The ambiguity of the price sensitivity effect on the extensive margin explains why more substitutability between goods (e.g. $C_{12\theta}\left(x,...x;\theta\right)\geq 0$) may not be sufficient to conclude that p^n decreases with θ.

We next identify sufficient conditions under which p^n and π^n do decrease with θ. The lemma below provides technical conditions that are sufficient for $\frac{\partial h(\bar{p};\theta)}{\partial\theta}>0$

Lemma 2 *Given increasing dependence, $h(\bar{p};\theta)$ decreases in θ if*

$$h(u;\theta)+\frac{f'(u)}{f(u)}\geq 0 \tag{11}$$

and

$$\frac{d^2 \ln f(u)}{du^2} \geq \frac{nf^2(u(x))}{C_\theta(x,...,x;\theta)} C_{11\theta}(x,...,x;\theta). \tag{12}$$

Using the technical lemma, the next proposition identifies sufficient conditions on the copula and marginal distribution under which price and profits under symmetric oligopoly decrease in the degree of preference dependence: $\frac{dp^n}{d\theta} < 0$ and $\frac{d\pi^n}{d\theta} < 0$. Part (i) invokes positive stochastic dependence and limited log-curvature of the marginal density (e.g. when f is approximately uniform or exponential). Part (ii) invokes stronger log-curvature restrictions on the marginal density (e.g. when f is approximately uniform) without imposing restrictions on the copula.

Proposition 2 *If regularity condition (9) holds at p^n, then, given increasing dependence, p^n and π^n decrease in θ if either of the following conditions hold: (i) $C_{11} < 0$ and $\left|\frac{d^2 \ln f(u)}{du^2}\right|$ is sufficiently small; or (ii) $\frac{d \ln f(u)}{du}$ and $\frac{d^2 \ln f(u)}{du^2}$ both are not too negative.*

Propositions 1 and 2 suggest that preference dependence is a useful measure of product differentiation, disentangled from preference diversity. In fact, profits actually increase in preference diversity σ when μ is relatively small (Johnson and Myatt, 2006; Chen and Riordan, 2011), whereas profits always monotonically decrease in θ under multiproduct monopoly, and profits also monotonically decrease in θ under oligopoly for all μ when f is approximately uniform or when f is approximately exponential and C is positively dependent.

Thus the effects of preference dependence (θ) on prices and profits offer a new way to think about product differentiation. Both σ and θ can be interpreted as indicators of the degree of product differentiation: higher σ indicates more heterogeneity of consumer values for each product, while higher θ indicates greater similarity of these values *between* products for a randomly chosen consumer. They have rather different economic meanings and the copula approach to modeling consumer preferences disentangles their effects in a general way. Proposition 2 loosely suggests that two competing single-product firms have a mutual incentive to coordinate the design or promotion of their products so that consumer values are less positively dependent or more negatively dependent.

IV. MARKET STRUCTURE AND PRICE

The copula approach also enables us to derive new results on how prices differ across market structures, relating them to properties of the marginal distributions and the dependence relationship. This provides new insights on how market structure affects firm conduct. Specifically, we compare p^m, p^n, and p^{mm}, motivated by the scenarios in Section 1.

We start with comparing the equilibrium oligopoly price with the single-product monopoly price. While Chen and Riordan (2008) find sufficient conditions for $p^m \gtreqless p^n$ when $n = 2$ and the marginal distribution is exponential (i.e. $\lambda'(\cdot) = 0$), it has been an open question how the prices compare for arbitrary marginal distributions and for any $n \geq 2$. We can now answer with the following result:

Proposition 3 *If* $C_{11} < 0$ *(positive dependence) and* $\lambda'(p) \geq 0$, *then* $p^m > p^n$; *and if* $C_{11} > 0$ *(negative dependence) and* $\lambda'(p) \leq 0$, *then* $p^m < p^n$.

Thus positive dependence and a non-decreasing hazard rate for the marginal distribution ensures that competition from other products lowers prices; conversely, negative dependence and a non-increasing hazard rate ensures that oligopoly competition raises price.[7]

This result can be understood as follows. An oligopolist sells less output at the monopoly price, p^m, and thus a slight price reduction at p^m is less costly to the oligopolist since it applies to a smaller output. This "market share effect" is a standard reason why one expects more competition to lower price. However, as Chen and Riordan (2008) discuss in the context of a duopoly, there is a potentially offsetting "price sensitivity effect" when products are differentiated. Since an oligopolist sells on a different margin from a monopolist, the slope of an oligopolist's (residual) demand curve differs from the slope of the single-product monopolist's demand curve. Furthermore, greater negative dependence makes it more difficult for the oligopolist to win over marginal consumers who value its own product less but its rival's product more. Similarly, a non-increasing hazard rate tends to put less consumer density

[7] Chen and Riordan (2007) and Perloff, Suslow, and Sequin (1995) present more specific models of product differentiation in which entry can result in higher prices.

on the oligopolist's intensive margin, further reducing price sensitivity.[8] Together, negative dependence and a non-increasing hazard rate are sufficient for the price sensitivity effect to dominate the market share effect, resulting in a higher price under oligopoly competition.[9]

Although preference dependence and the number of firms are different economic concepts, our analysis suggests a common theme between their effects on equilibrium prices. Both greater preference dependence and more firms represent increased competition. Each has a market share effect—lower output— that favors lower prices, but each may also have a price sensitivity effect—potentially steepening the residual demand curve—that favors higher prices. Propositions 2 and 3 give the respective sufficient conditions for the net effect to lower prices.

Next, we compare the prices for the multiproduct monopoly with those under single-product monopoly and symmetric oligopoly.

Proposition 4 *If either $C_{11}(x,...,x) \geq 0$ for all $x \in (0,1)$ or $C_{11}(x,...,x) \leq 0$ for all $x \in (0,1)$, then $p^{mm} > p^n$ and $p^{mm} > p^m$.*

As one might expect, $p^{mm} > p^n$, or prices for n substitutes are higher under monopoly than under competition, extending the result for $n = 2$ in Chen and Riordan (2008). The familiar intuition is that a multiproduct monopolist internalizes the negative effects of reducing one product's price on profits from the other products. The comparison of prices under multiproduct monopoly (p^{mm}) and single-product monopoly (p^m) is more subtle. The multiproduct monopolist has higher total output at p^m than the single-product monopolist, which motivates it to raise its symmetric price above p^m. But, as with the oligopoly comparison, the marginal consumers of the multiproduct monopolist differ from those of the single-product monopolist, which can potentially make the slope of the multiproduct monopolist's demand curve steeper than that of the single-product monopolist. Interestingly, the market share

[8]When $n = 2$, the argument in the proof of Proposition 3 can be adapted to show more formally that, with $\lambda'(\cdot) \leq 0$, the (residual) demand curve of a duopolist is indeed steeper than that of the monopolist if $C(\cdot,\cdot)$ is negatively dependent, independent, or has sufficiently limited positive dependence.

[9]Note that due to more varieties under oligopoly, a higher price under oligopoly does not imply that consumer welfare is lower under oligopoly competition than under single-product monopoly.

effect unambiguously dominates, provided that $C(x, ...x)$ exhibits uniform positive or negative stochastic dependence.

Under general preference distributions, Propositions 3 and 4 largely settle the issue of how prices in symmetric multiproduct industries compare to the single-product monopoly price.

V. CONCLUSION

Using copulas to describe the distribution of consumer preferences is a convenient and intuitive approach to discrete choice demand. The approach enables us to identify preference dependence as a distinct indicator of product differentiation in multiproduct industries, disentangled from the effects of preference diversity, in the sense that greater correlation of consumer values for alternative products leads to lower prices and profits under certain conditions. The approach also leads to new results in price theory. The entry of symmetric differentiated competitors into an initial single-product monopoly lowers (raises) price if preferences are positively (negatively) dependent and the hazard rate of the marginal distribution is non-decreasing (non-increasing). Moreover, under a uniform dependence condition, price rises when a single-product monopolist adds symmetric differentiated varieties to its product line.

There are several directions for further research for which the copula approach is likely to be useful. One is to examine further the effects of entry into differentiated product markets. Whereas we have found that entry into an intitial monopoly raises or lowers price depending on preference dependence and the hazard rate, it is important additionally to determine the conditions under which the symmetric oligopoly price is decreasing or increasing in the number of firms. Similarly, it is important to understand the conditions under which a product line expansion by a multiproduct monopolist results in higher or lower prices. Also, relaxing symmetry is important, even though this is likely to challenge tractability. For example, a symmetric model seems inappropriate for understanding conditions under which generic entry results in higher or lower branded drug prices (Perloff, Suslow, and Seguin, 1995).

The copula representation of consumer preferences may be valuable for studying other applied microeconomics topics. Chen and Riordan (2013) applies the copula approach to study the profitability of product bundling, and further applications might shed more light on the positive and normative economics of bundling. Chen and Pearcy (2010) uses a specific class of copulas to model intertemporal dependence of consumer values. Other promising topic areas include the economics of search (e.g., Anderson and Renault 1999; Schultz and Stahl 1996; Bar Isaac, Caruana, and Cunat 2010), and the endogenous determination of market structure (e.g., Shaked and Sutton, 1990). Furthermore, the copula approach to discrete choice demand, and its potentially rich set of predictions about market structure, conduct, and performance, might open interesting new directions for empirical industrial organization research.[10]

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[10]See, for example, Chen and Savage (2011) for an empirical analysis of how preference dispersion affects prices and price differences between monopoly and duopoly markets for Internet services.

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APPENDIX: PROOFS

The appendix contains proofs for Lemma 1, Proposition 1, Lemma 2, Propositions 2, 3, and 4.

Proof of Lemma 1. Given increasing orthant dependence,

$$C\left(F(\bar{p}),...,F(\bar{p});\theta\right)=n\int_{0}^{F(\bar{p})}C_{1}\left(x,...,x;\theta\right)dx$$

increases in θ for any $\bar{p}>F^{-1}\left(0\right)$, which is possible only if $C_{1\theta}\left(x,...,x;\theta\right)>0$ for all θ when x is close to zero. Similarly, $C_{1\theta}\left(x,...,x;\theta\right)<0$ for all θ if x is sufficiently close to 1. Thus there must exist $x' > 0$ such that, for all θ, $C_{1\theta}(x', ..., x'; \theta) = 0$ and $C_{1\theta}(x, ..., x; \theta) > 0$ if $x < x'$.[11] Since

$$\begin{aligned} & \frac{\partial \lambda^C(\bar{p}; \theta)}{\partial \theta} \\ = \; & n \left[\frac{C_{1\theta}(F(\bar{p}), ..., F(\bar{p}); \theta)}{1 - C(F(\bar{p}), ..., F(\bar{p}); \theta)} + \frac{C_1(F(\bar{p}), ..., F(\bar{p}); \theta) C_\theta(F(\bar{p}), ..., F(\bar{p}); \theta)}{[1 - C(F(\bar{p}), ..., F(\bar{p}); \theta)]^2} \right] f(\bar{p}), \end{aligned}$$

there exists some $u^* \in [F^{-1}(x'),\ u(1)]$ such that $\partial \lambda^C(\bar{p}; \theta) / \partial \theta > 0$ for all θ if $\bar{p} \leq u^*$ ■

Proof of Proposition 1. (i) From (4), for any θ, let $\bar{\mu}^*$ be such that $[u^* + \bar{\mu}^*] \lambda^C(u^*; \theta) = 1$, where $u^* \geq F^{-1}(x_1') > u(0)$ is defined in Lemma 1. Then, $\bar{p}^{mm} = u^* > u(0)$ if $\bar{\mu} = \bar{\mu}^*$. If $\bar{\mu} \geq \bar{\mu}^*$, then $\bar{p}^{mm} \leq u^*$ and Lemma 1 implies $\frac{\partial \lambda^C(\bar{p}^{mm}; \theta)}{\partial \theta} > 0$. It follows from (4) and (5) that $\frac{d\bar{p}^{mm}}{d\theta} < 0$ and hence $\frac{dp^{mm}}{d\theta} < 0$. (ii) holds from application of the envelope theorem to (3) and $C_\theta > 0$. ■

Proof of Lemma 2. Notice that (suppressing the argument θ to simplify notation),

$$\frac{dC_1(x, ..., x)}{dx} = C_{11}(x, ..., x) + (n-1) C_{12}(x, ..., x),$$

or

$$(n-1) C_{12}(x, ..., x) = \frac{dC_1(x, ..., x)}{dx} - C_{11}(x, ..., x).$$

Thus,

$$h(\bar{p}) = \lambda^C(\bar{p}) + n(n-1) \frac{\int_{F(\bar{p})}^1 C_{12}(x, ..., x) f(u(x)) \, dx}{1 - C(F(\bar{p}), ...F(\bar{p}))}$$

$$= \frac{nC_1(F(\bar{p}), ...F(\bar{p})) f(\bar{p})}{1 - C(F(\bar{p}), ..., F(\bar{p}))} + n \frac{\int_{F(\bar{p})}^1 \left[\frac{dC_1(x, ..., x)}{dx} - C_{11}(x, ..., x) \right] f(u(x)) \, dx}{1 - C(F(\bar{p}), ..., F(\bar{p}))}$$

$$= \frac{nC_1(F(\bar{p}), ..., F(\bar{p})) f(\bar{p})}{1 - C(F(\bar{p}), ..., F(\bar{p}))} + n \frac{\int_{F(\bar{p})}^1 \frac{dC_1(x, ..., x)}{dx} f(u(x)) \, dx - \int_{F(\bar{p})}^1 C_{11}(x, ..., x) f(u(x)) \, dx}{1 - C(F(\bar{p}), ..., F(\bar{p}))}.$$

[11] Similarly, there exists $x'' \geq x'$ such that $C_{1\theta}(x, ..., x; \theta) < 0$ if $x > x'$. For the FGM family with $n = 2$, $x'' = x' = 1/2$.

Since

$$\begin{aligned}
&\int_{F(\bar{p})}^{1} \frac{dC_1(x,...,x)}{dx} f(u(x))\,dx \\
&= f(u(1)) - C_1(F(\bar{p}),...,F(\bar{p}))\,f(\bar{p}) - \int_{F(\bar{p})}^{1} C_1(x,...,x)\frac{f'(u(x))}{f(u(x))}dx,
\end{aligned}$$

we have

$$\begin{aligned}
h(\bar{p}) &= n\frac{f(u(1)) - \int_{F(\bar{p})}^{1} C_1(x,...,x)\frac{f'(u(x))}{f(u(x))}dx - \int_{F(\bar{p})}^{1} C_{11}(x,...,x) f(u(x))\,dx}{1 - C(F(\bar{p}),...F(\bar{p}))} \\
&= n\frac{f(u(1)) + \int_{F(\bar{p})}^{1} \frac{f'(u(x))}{f(u(x))}\frac{1}{n}\frac{d[1-C(x,...,x)]}{dx} - \int_{F(\bar{p})}^{1} C_{11}(x,...,x) f(u(x))\,dx}{1 - C(F(\bar{p}),...,F(\bar{p}))}
\end{aligned}$$

$$= -\frac{f'(\bar{p})}{f(\bar{p})} + n\frac{f(u(1)) - \frac{1}{n}\int_{F(\bar{p})}^{1} [1 - C(x,...,x)]\frac{d\frac{f'(u(x))}{f(u(x))}}{dx} - \int_{F(\bar{p})}^{1} C_{11}(x,...,x) f(u(x))\,dx}{1 - C(F(\bar{p}),...,F(\bar{p}))}.$$

Therefore, if (11) holds, then

$$\frac{\partial h(\bar{p})}{\partial\theta} = \frac{\int\limits_{F(\bar{p})}^{1}\left[\frac{C_\theta(x,...x)}{f(u(x))}\frac{d^2\ln f(u)}{du^2} - nC_{11\theta}(x,...,x) f(u(x))\right]dx + \left[h(\bar{p}) + \frac{f'(\bar{p})}{f(\bar{p})}\right]C_\theta(F(\bar{p}),...F(\bar{p}))}{1 - C(F(\bar{p}),...,F(\bar{p}))} > 0$$

because $C_\theta > 0$ and $C_{11\theta} < 0$ by increasing orthant dependence and increasing stochastic dependence respectively. ■

Proof of Proposition 2. First, observe that $\frac{dp^n}{d\theta}$ has the same sign as $\frac{d\bar{p}^n}{d\theta}$. Second, observe that π^n decreases in θ when $\frac{d\bar{p}^n}{d\theta} < 0$ because

$$\begin{aligned}
\frac{d\pi^n}{d\theta} &= \frac{\partial\pi^n}{\partial\theta} + \frac{\partial\pi^n}{\partial\bar{p}^n}\frac{d\bar{p}^n}{d\theta} \\
&= -\frac{1}{n}(\bar{p}^n\sigma + \mu)\,C_\theta\left(F\left(\bar{p}^d\right),...,F\left(\bar{p}^d\right);\theta\right) + \frac{\partial\pi^n}{\partial\bar{p}^n}\frac{d\bar{p}^n}{d\theta} < 0,
\end{aligned}$$

where $C_\theta\left(F\left(\bar{p}^d\right),...,F\left(\bar{p}^d\right);\theta\right) > 0$ from increasing orthant dependence, and $\frac{\partial\pi^n}{\partial\bar{p}^n} > 0$ by the envelope theorem and by the fact that a firm's demand increases in the other firm's price. Given these observations we focus on sufficient conditions for $\frac{d\bar{p}^d}{d\theta} < 0$.

Since $\frac{\partial h(\bar{p};\theta)}{\partial\theta} > 0$ and the regularity condition imply $\frac{d\bar{p}}{d\theta} < 0$, it is sufficient to verify the conditions for Lemma 2.

(i) If $\frac{d^2 \ln f(u)}{du^2} \to 0$ and $C_{11}(x...x;\theta) < 0$, then

$$h(\bar{p}) + \frac{f'(\bar{p})}{f(\bar{p})} \to n\frac{f(u(1)) - \int_{F(p)}^{1} C_{11}(x,...,x;\theta) f(u(x))\,dx}{1 - C(F(\bar{p}),...,F(\bar{p});\theta)} > 0$$

and $\frac{d^2 \ln f(u)}{du^2} > \frac{2f^2(u(x))}{C_\theta(x,...x)} C_{11\theta}(x,...,x;\theta)$ since $C_{11\theta}(x,...x;\theta) < 0$, thus satisfying Lemma 2.

(ii) If $\frac{d \ln f(u)}{du}$ and $\frac{d^2 \ln f(u)}{du^2}$ both are not too negative, then $h(u) + \frac{f'(u)}{f(u)} \geq 0$ and $\frac{d^2 \ln f(u)}{du^2} > \frac{2f^2(u(x))}{C_\theta(x,...,x)} C_{11\theta}(x,...,x)$ by increasing stochastic dependence, thus satisfying Lemma 2. ■

Proof of Proposition 3. It suffices to show that (i) $h(\bar{p}) > \lambda(\bar{p})$ if $C_{11} < 0$ and $\lambda'(\bar{p}) \geq 0$; and (ii) $h(\bar{p}) < \lambda(\bar{p})$ if $C_{11} > 0$ and $\lambda'(\bar{p}) \leq 0$.

(i) Suppose that $\lambda'(\bar{p}) \geq 0$. Then, since

$$\frac{dC_1(x_1,...,x_1)}{dx_1} - C_{11}(x_1,...,x_1) = (n-1)\,C_{12}(x_1,...,x_1) > 0,$$

$$\begin{aligned} h(\bar{p}) &= \frac{nC_1(F(\bar{p}),...,F(\bar{p}))\,f(\bar{p})}{1 - C(F(\bar{p}),...,F(\bar{p}))} + n\frac{\int_{F(\bar{p})}^{1} (1-x)(n-1)\,C_{12}(x,...,x)\,\frac{f(u(x))}{1-F(u(x))}dx}{1 - C(F(\bar{p}),...,F(\bar{p}))} \\ &= \frac{nC_1(F(\bar{p}),...,F(\bar{p}))\,f(\bar{p})}{1 - C(F(\bar{p}),...,F(\bar{p}))} + n\frac{\int_{F(\bar{p})}^{1} (1-x)\left[\frac{dC_1(x,...,x)}{dx} - C_{11}(x,...,x)\right]\frac{f(u(x))}{1-F(u(x))}dx}{1 - C(F(\bar{p}),...,F(\bar{p}))} \\ &\geq \frac{nC_1(F(\bar{p}),...,F(\bar{p}))\,f(\bar{p})}{1 - C(F(\bar{p}),...,F(\bar{p}))} + n\frac{f(\bar{p})}{1-F(\bar{p})}\frac{\int_{F(\bar{p})}^{1} (1-x)\left[\frac{dC_1(x,...,x)}{dx_1} - C_{11}(x,...,x)\right]dx}{1 - C(F(\bar{p}),...,F(\bar{p}))}. \end{aligned}$$

Substituting

$$\begin{aligned} &\int_{F(\bar{p})}^{1} (1-x)\left[\frac{dC_1(x,...,x)}{dx} - C_{11}(x,...,x)\right]dx \\ =\;& |(1-x)\,C_1(x,...,x)_{F(\bar{p})}^{1} + \int_{F(\bar{p})}^{1} C_1(x,...,x)\,dx - \int_{F(\bar{p})}^{1} (1-x)\,C_{11}(x,...,x)\,dx \end{aligned}$$

$$= -(1-F(\bar{p}))\,C_1(F(\bar{p}),...,F(\bar{p})) + \frac{1}{n}(1-C(F(\bar{p}),...,F(\bar{p}))) - \int_{F(\bar{p})}^{1}(1-x)\,C_{11}(x,...,x)\,dx,$$

and simplifying, we obtain

$$h(\bar{p}) \geq \lambda(\bar{p})\left[1 - n\frac{f(\bar{p})}{1-F(\bar{p})}\frac{\int_{F(\bar{p})}^{1}(1-x)\,C_{11}(x,...,x)\,dx}{1-C(F(\bar{p}),...,F(\bar{p}))}\right].$$

Hence $h(\bar{p}) > \lambda(\bar{p})$ if in addition $C_{11} < 0$.

(ii) Suppose that $\lambda' \leq 0$. By analogous derivations, we have

$$h(\bar{p}) < \lambda(\bar{p})\left[1 - n\frac{f(\bar{p})}{1-F(\bar{p})}\frac{\int_{F(\bar{p})}^{1}(1-x)\,C_{11}(x,...,x)\,dx}{1-C(F(\bar{p}),...,F(\bar{p}))}\right].$$

Hence $h(\bar{p}) < \lambda(\bar{p})$ if in addition $C_{11}(x,...,x) > 0$. ■

Proof of Proposition 4. (i) Since $h(\bar{p}) > \lambda^C(\bar{p})$ from (8), comparing (4) and (7) leads to $\bar{p}^n < \bar{p}^{mm}$.

(ii) It suffices to show that $\lambda^C(\bar{p}) < \lambda(\bar{p})$ for all $\bar{p} \in (u(0), u(1))$ if $C_{11}(x,...,x) \leq 0$, $C_{11}(x,...,x) = 0$, or $C_{11}(x,...x) \geq 0$ for all $x \in (0,1)$.

First,

$$\frac{\lambda^C(\bar{p})}{\lambda(\bar{p})} = \frac{\frac{nC_1(F(\bar{p}),...,F(\bar{p}))}{1-C(F(\bar{p}),...,F(\bar{p}))}f(\bar{p})}{\frac{f(\bar{p})}{1-F(\bar{p})}} = \frac{nC_1(F(\bar{p}),...,F(\bar{p}))}{1-C(F(\bar{p}),...,F(\bar{p}))}[1-F(\bar{p})].$$

If $C_{11} \geq 0$ or if $C_{11} = 0$, then, since

$$\int_{F(\bar{p})}^{1}(1-x)\,dC_1(x,...,x) = -(1-F(\bar{p}))\,C_1(F(\bar{p}),...,F(\bar{p})) + \frac{1}{n}[1-C(F(\bar{p}),...,F(\bar{p}))]$$

and $C_{1k}(x,...,x) = C_{12}(x,...,x) > 0$ for $x \in (0,1)$ and for all $k \neq 1$,

$$\begin{aligned}
\frac{\lambda^C(\bar{p})}{\lambda(\bar{p})} &= \frac{[1 - C(F(\bar{p}),...,F(\bar{p}))] - n\int_{F(\bar{p})}^{1}(1-x)\,dC_1(x,...,x)}{1 - C(F(\bar{p}),...,F(\bar{p}))} \\
&= \frac{[1 - C(F(\bar{p}),...,F(\bar{p}))] - n\int_{F(\bar{p})}^{1}(1-x)\,[C_{11}(x,...,x) + (n-1)C_{12}(x,...,x)]\,dx}{1 - C(F(\bar{p}),...,F(\bar{p}))} \\
&= 1 - n\frac{\int_{F(\bar{p})}^{1}(1-x)\,[C_{11}(x,...,x) + (n-1)C_{12}(x,...,x)]\,dx}{1 - C(F(\bar{p}),...,F(\bar{p}))} < 1.
\end{aligned}$$

Next, suppose $C_{11}(x,...,x) \leq 0$ for all $x \in (0,1)$, so that there is positive stochastic dependence. Then, $C_1(x,...,x) \leq \frac{C(x,...,x)}{x}$ for all $x \in (0,1)$ since

$$C(x,...,x) = \int_0^x C_1(t,x,...,x)\,dt \geq \int_0^x C_1(x,...,x)\,dt = C_1(x,...,x)\,x.$$

Hence, letting $x = F(\bar{p})$,

$$\frac{\lambda^C(u(x))}{\lambda(u(x))} = \frac{n(1-x)\,C_1(x,...,x)}{1 - C(x,...,x)} \leq \frac{n(1-x)\,C(x,...,x)}{x\,[1 - C(x,...,x)]}.$$

Now, suppose to the contrary that $\frac{\lambda^C(u(x))}{\lambda(u(x))} = \frac{n(1-x)C_1(x,...,x)}{1-C(x,...,x)} \geq 1$. Then $\frac{n(1-x)C(x,...,x)}{x[1-C(x,...,x)]} \geq 1$. We will show that this leads to a contradiction. First, notice that

$$\lim_{x\to 1}\frac{n(1-x)\,C(x,...,x)}{x\,[1 - C(x,...,x)]} = n\lim_{x\to 1}\frac{-C(x,...,x) + (1-x)\,nC_1(x,...,x)}{1 - C(x,...,x) - nxC_1(x,...,x)} = 1.$$

Next, letting $\mathbf{x} = (x,...x)$ to simplify notation, we have

$$\begin{aligned}
&\frac{d\left[\frac{(1-x)C(x,...,x)}{x[1-C(x,...,x)]}\right]}{dx} \\
= &\frac{[-C(\mathbf{x}) + n(1-x)\,C_1(\mathbf{x})]\,\{x\,[1 - C(\mathbf{x})]\} - (1-x)\,C(\mathbf{x})\,[1 - C(\mathbf{x}) - nxC_1(\mathbf{x})]}{\{x\,[1 - C(\mathbf{x})]\}^2} \\
= &\frac{n(1-x)\,C_1(\mathbf{x})\,x\,[1 - C(\mathbf{x})] - C(\mathbf{x})\,[1 - C(\mathbf{x})] + (1-x)\,C(\mathbf{x})\,nxC_1(\mathbf{x})}{\{x\,[1 - C(\mathbf{x})]\}^2}
\end{aligned}$$

$$= \frac{n(1-x)C_1(\mathbf{x})x - C(\mathbf{x})[1-C(\mathbf{x})]}{\{x[1-C(\mathbf{x})]\}^2} \geq \frac{[1-C(\mathbf{x})]x - C(\mathbf{x})[1-C(\mathbf{x})]}{\{x[1-C(\mathbf{x})]\}^2}$$
$$= \frac{[1-C(\mathbf{x})][x-C(\mathbf{x})]}{\{x[1-C(\mathbf{x})]\}^2} > 0 \text{ for } x \in (0,1),$$

where the first inequality is due to $\frac{n(1-x)C_1(x,...,x)}{1-C(x,...,x)} \geq 1$ by assumption, and the second inequality holds because $x > C(\mathbf{x})$. It follows that, for any interior x,

$$\frac{n[1-x]C(x,...x)}{x[1-C(x,...x)]} < 1,$$

which is a contradiction. Therefore $\frac{n[1-x]C_1(x,...,x)}{1-C(x,...,x)} < 1$ for any $x \in (0,1)$, or $\frac{\lambda^C(\bar{p})}{\lambda(\bar{p})} < 1$ for any $\bar{p} \in (u(0), u(1))$. ■