Subsidy or tax policy for new technology adoption in duopoly with quadratic and linear cost functions

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We present an analysis about subsidy (or tax) policy for adoption of new technology in a duopoly with a homogeneous good. Technology itself is free. However, firms must expend fixed set-up costs for adoption of new technology, for example, education costs of their staffs. We assume linear demand function, and consider two types of cost functions of firms. Quadratic cost functions and linear cost functions. There are various cases of optimal policies depending on the level of the set-up cost and the forms of cost functions. In particular, under linear cost functions there is the following case.

> The social welfare is maximized when one firm adopts new technology, however, both firms adopt new technology without subsidy nor tax. Then, the government should impose taxes on one firm or both firms.

Under quadratic cost functions there exists no taxation case. There are subsidization cases both under quadratic and linear cost functions.

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1 Introduction

We present an analysis about subsidy (or tax) policy for adoption of new technology in a duopoly with a homogeneous good. Technology itself is free. However, firms must expend fixed set-up costs for adoption of new technology, for example, education costs of their staffs. We assume linear demand function, and consider two types of cost functions of firms. Quadratic cost functions and linear cost functions. With quadratic cost functions marginal costs are increasing, and with linear cost functions marginal costs are constant.

There are several references about technology adoption or R& D in duopoly or oligopoly. Lots of researches focus on the relation between technology licensor and licensee. The difference of means of contracts which are royalties, up-front fees, the combinations of these two and auction are well discussed (Katz and Shapiro (1985), Kamien and Tauman (1986), Sen and Tauman (2007)). Kamien and Tauman (1986) shows that if the licensor does not have production capacity, fixed fee is better than royalty and it is also better for consumers. This topic under Stackelberg oligopoly is discussed in Kabiraj (2004) when the licensor does not have production capacity, and discussed in Wang and Yang (2004), Kabiraj (2005) and Filippini (2005) when the licensor has production capacity. A Cournot oligopoly with fixed fee under cost asymmetry is analyzed in La Manna (1993). He shows that if technologies can be replicated perfectly, a lower-cost firm has always the incentive to transfer its technology and hence a Cournot-Nash equilibrium cannot be fully asymmetric, but there exists no non-cooperative Nash equilibrium in pure strategies.

On the other hand, using cooperative game theory, Watanabe and Muto (2008) analyses bargaining between licensor with no production capacity and oligopolistic firms. In recent research, the relation between market structure and technology improvement is analyzed. Boone (2001) and Matsumura et. al. (2013), respectively, find a non-monotonic relation between intensity of competition and innovation. Also, Pal (2010) shows that if we consider technology adoption, Cournot competition makes more social welfare than Bertrand competition under differentiated goods market. Elberfeld and Nti (2004) examines the adoption of a new technology in oligopoly, where there is ex-ante uncertainty about variable costs of the new technology, and shows that if in equilibrium both old and new technologies are employed, more uncertainty about the new technology increases (decreases) the number of innovating firms and decreases (increases) the product price if the up-front investment is large (small). Zhang et. al. (2014) analyzes the effect of information spillovers when the outcome of R&D is uncertain in a two-stage Cournot oligopoly model where a subset of firms first make a choice between two alternative production technologies independently and then all firms compete in quantity.

This paper analyzes optimal subsidization or taxation policies about adoption of new technology by firms in a duopoly with a homogeneous good. We consider the following three-stage game.

(1) The first stage: The government determines the level of subsidies (or taxes) to the firms.

(2) The second stage: The firms decide whether they adopt new technology or not.

(3) The third stage: The firms determine their outputs.

The social welfare is defined to be consumers' surplus plus firms' profits, which is equal to consumers' utility minus productions costs including the set-up costs of new technology. Subsidies to the firms are financed by lump-sum taxes on the consumers, and revenues from taxes on the firms are transfered to the consumers in a lum-sum manner. These lump-sum taxes and transfers are not related to the good of this industry. Excluding income effects they do not affect the demand for the good, and they are canceled out in the social welfare.

There are various cases about optimal policies depending on the level of the set-up cost and the forms of cost functions. Under quadratic cost functions there are the following cases.

(1) The social welfare is maximized when both firms adopt new technology, but only one firm adopts new technology without subsidy. Then, the government should give subsidies to the firms. There are two subsidization schemes.

    a) The government gives a subsidy to one of the firms, and this firm adopts new technology. The other firm does not adopt new technology. It is a discriminatory policy.

    b) The government gives chances to receive subsidies to both firms, but actually only one firm receives a subsidy and adopts new technology. It is not a discriminatory policy.

In both schemes only one firm adopts new technology.

(2) The social welfare is maximized when both firms adopt new technology, and they adopt new technology without subsidy. Then, the government should do nothing.

Under linear cost functions there are the following cases.

(1) The social welfare is maximized when one firm adopts new technology, however, both firms adopt new technology without subsidy nor tax. Then, the government should impose taxes for new technology adoption on one firm, or both firms. There are the following two taxation schemes.

    a) The government imposes a tax on one firm to prevent adoption of new technology. It is a discriminatory policy.

    b) The government imposes taxes on both firms. At the equilibrium only one of the firms adopts new technology and this firm actually pays a tax. The other firm does not adopt. It is not a discriminatory policy because adoption of new technology is a choice of the firm.

In both schemes only one firm adopts new technology.

(2) The social welfare is maximized when one firm adopts new technology, and only one firm adopts new technology without subsidy nor tax. Then, the government should do nothing.

Under quadratic cost functions there exists no taxation case.

Note that our model is (at least mathematically) equivalent to a model of technology license with a fixed license fee.

2 The model

Two firms, Firm A and B, produce a homogeneous good, and consider adoption of new technology from a foreign country. Technology itself is free, but each firm must expend a fixed set-up cost, for example, education cost of its staff. Denote the outputs of Firm A and B by x_A and x_B, the price of the good by p. The utility function of consumers is

$$u = a(x_A + x_B) - \frac{1}{2}(x_A + x_B)^2,$$

where a is a positive constant. The inverse demand function is derived as follows.

$$p = a - x_A - x_B.$$

(1) Under quadratic cost functions the cost functions of the firms before adoption of new technology are cx_i^2, $i = A, B$, and the cost functions after adoption of new technology are $\frac{1}{2}cx_i^2$, $i = A.B$. A fixed set-up cost is e.

(2) Under linear cost functions the cost functions of the firms before adoption of new technology are cx_i, $i = A, B$, and the cost of each firm after adoption of new technology is zero. A fixed set-up cost is also e.

c in both cost functions and e are positive constants and common to both firms. There exists no fixed cost other than the set-up costs.

The social welfare W is defined to be the sum of consumers' surplus and firms' profits, which is equal to consumers' utility minus productions costs including the set-up costs of new technology, as follows;

$$W = a(x_A + x_B) - \frac{1}{2}(x_A + x_B)^2 - p(x_A + x_B) + [p(x_A + x_B) - c_A(x_A) - c_B(x_B)]$$

$$= a(x_A + x_B) - \frac{1}{2}(x_A + x_B)^2 - c_A(x_A) - c_B(x_B)$$

$c_A(x_A)$ and $c_B(x_B)$ generally denote the cost functions of firms. They may include set-up costs.

Subsidies to the firms are financed by lump-sum taxes on the consumers, and revenues from taxes on the firms are transfered to the consumers in a lum-sum manner. These lump-sum taxes and transfers are not related to the good of this industry. Excluding income effects they do not affect the demand for the good, and they are canceled out in the social welfare.

We analyze the optimal subsidization or taxation policies of the government for adoption of new technology by firms. If adoption of new technology and non-adoption are indifferent for a firm, then it adopts new technology.

3 Quadratic cost functions

In this section firms have quadratic cost functions. The profits of Firm A and B before adoption of new technology are

$$\pi_A = (a - x_A - x_B)x_A - cx_A^2, \ \pi_B = (a - x_A - x_B)x_B - cx_B^2.$$

After adoption of new technology they are

$$\pi_A = (a - x_A - x_B)x_A - \frac{1}{2}cx_A^2 - e, \ \pi_B = (a - x_A - x_B)x_B - \frac{1}{2}cx_B^2 - e.$$

We assume Cournot type behavior of firms. There are four cases.

(1) The conditions for profit maximization when no firm adopts new technology are

$$a - 2x_A - x_B - 2cx_A = 0, \ a - x_A - 2x_B - 2cx_B = 0.$$

We denote the equilibrium outputs as follows;

$$x_A^0 = x_B^0 = \frac{a}{2c + 3},$$

and the equilibrium profits by

$$\pi_A^0 = \pi_B^0 = \frac{(c + 1)a^2}{(2c + 3)^2}.$$

(2) The conditions for profit maximization when both firms adopt new technology are

$$a - 2x_A - x_B - cx_A = 0, \ a - x_A - 2x_B - cx_B = 0.$$

We denote the equilibrium outputs as follows;

$$\tilde{x}_A = \tilde{x}_B = \frac{a}{c + 3}.$$

and the equilibrium profits by

$$\tilde{\pi}_A = \tilde{\pi}_B = \frac{(c + 2)a^2}{2(c + 3)^2} - e.$$

(3) If only Firm A adopts new technology, the conditions for profit maximization are

$$a - 2x_A - x_B - cx_A = 0, \ a - x_A - 2x_B - 2cx_B = 0.$$

We denote the equilibrium outputs as follows;

$$x_A^A = \frac{(c + 2)a^2}{2c^2 + 6c + 3}, \ x_B^A = \frac{(c + 1)a}{2c^2 + 6c + 3}.$$

and the equilibrium profits by

$$\pi_A^A = \frac{(c + 2)(2c + 1)^2a^2}{2(2c^2 + 6c + 3)^2} - e, \pi_B^A = \frac{(c + 1)^3a^2}{(2c^2 + 6c + 3)^2}.$$

(4) If only Firm B adopts new technology, the equilibrium outputs are written as;

$$x_A^B = \frac{(c+1)a}{2c^2 + 6c + 3}, \; x_B^B = \frac{(c+2)a^2}{2c^2 + 6c + 3}.$$

and the equilibrium profits as

$$\pi_A^B = \frac{(c+1)^3 a^2}{(2c^2 + 6c + 3)^2}, \pi_B^B = \frac{(c+2)(2c+1)^2 a^2}{2(2c^2 + 6c + 3)^2} - e.$$

Let

$$e^1 = \tilde{\pi}_A + e - \pi_A^B = \tilde{\pi}_B + e - \pi_B^A = \frac{(2c^4 + 14c^3 + 36c^2 + 40c + 15)a^2 c}{2(2c+3)^2(2c^2 + 6c + 3)^2},$$

$$e^0 = \pi_A^A + e - \pi_A^0 = \pi_B^B + e - \pi_B^0 = \frac{(8c^4 + 40c^3 + 72c^2 + 56c + 15)a^2 c}{2(2c+3)^2(2c^2 + 6c + 3)^2}.$$

If $e \leq e^1$, the best response to adoption is adoption. If $e > e^1$, the best response to adoption is non-adoption. If $e \leq e^0$, the best response to non-adoption is adoption. If $e > e^0$, the best response to non-adoption is non-adoption.

We find

$$e^0 - e^1 = \frac{(c+2)(8c^3 + 38c^2 + 54c + 27)c^2 a^2}{2(c+3)^2(2c+3)^2(2c^2 + 6c + 3)^2} > 0.$$

The game after the second stage is depicted as follows.

		B	
		adoption of new technology	non-adoption
A	adoption of new technology	$\tilde{\pi}_B - e$ $\tilde{\pi}_A - e$	π_B^A $\pi_A^A - e$
	non-adoption	$\pi_B^B - e$ π_A^B	π_B^0 π_A^0

The sub-game perfect equilibria are as follows.

Lemma 1. (1) *If $e \leq e^1$, the sub-game perfect equilibrium is a state such that both firms adopt new technology. In this case $e \leq e^1$ and $e \leq e^0$, so adoption of new technology is a dominant strategy for each firm.*

(2) *If $e^1 < e \leq e^0$, the sub-game perfect equilibrium is a state such that one firm, Firm A or B, adopts new technology. In this case $e \leq e^0$ and $e > e^1$, so adoption of new technology is a best response to non-adoption, and non-adoption is a best response to adoption.*

(3) *If $e > e^0$, the sub-game perfect equilibrium is a state such that no firm adopts new technology. In this case $e > e^0$ and $e > e^1$, so non-adoption is a dominant strategy for both firms.*

Social welfare   Denote the social welfare when both firms adopt new technology by W^2, that when one firm adopts new technology by W^1 and that when no firm adopts new technology by W^0. Then, we have

$$W^2 = \frac{(c+4)a^2}{(c+3)^2} - 2e, \ W^1 = \frac{(6c^3 + 27c^2 + 27c + 8)a^2}{2(2c^2 + 6c + 3)^2} - e, \ W^0 = \frac{2(c+2)a^2}{(2c+3)^2}.$$

Let

$$e_a^0 = \frac{(8c^4 + 52c^3 + 102c^2 + 71c + 15)a^2 c}{2(2c+3)^2(2c^2 + 6c + 3)^2},$$

$$e_a^1 = \frac{(2c^4 + 17c^3 + 45c^2 + 43c + 15)}{2(c+3)^2(2c^2 + 6c + 3)^2}.$$

We have $W^0 = W^1$ when $e = e_a^0$, $W^1 = W^2$ when $e = e_a^1$, $W^0 > W^1$ (or $W^1 > W^0$) when $e > e_a^0$ (or $e < e_a^0$) and $W^1 > W^2$ (or $W^2 > W^1$) when $e > e_a^1$ (or $e < e_a^1$). We can show

$$e_a^0 - e_a^1 = \frac{(c^2 + 7c + 9)(4c^2 + 14c + 9)a^2 c^2}{(c+3)^2(2c+3)^2(2c^2 + 6c + 3)^2} > 0.$$

Thus, the following lemma is derived.

Lemma 2.   (1) *If $e \leq e_a^1$, W^2 is the maximum, and adoption of new technology by both firms is optimal.*

(2) *If $e_a^1 < e \leq e_a^0$, W^1 is the maximum, and adoption of new technology by one firm is optimal.*

(3) *If $e > e_a^0$, W^0 is the maximum, and non-adoption of new technology by any firm is optimal.*

Now we find

$$e_a^1 - e^0 = \frac{(4c^4 + 18c^3 + 17c^2 - 18c - 27)a^2 c^2}{2(c+3)^2(2c+3)^2(2c^2 + 6c + 3)^2}.$$

This is positive for reasonable value of c. For example, if $c > 1.07$, $e_a^1 - e^0 > 0$. Then, we obtain the following results.

Theorem 1. *Under quadratic cost functions the optimal policies should be as follows.*

(1) *If $e \leq e^1$, W^2 is optimal and both firms adopt new technology without subsidy. The government should do nothing.*

(2) *If $e^1 < e \leq e^0$, W^2 is optimal but one firm adopts new technology without subsidy. The government should give subsidies to both firms. The level of the subsidy must not be smaller than $e - e^1$. If the government gives a subsidy to only one firm, the other firm does not adopt new technology. Thus, in this case the government should give the subsidies to both firms.*

(3) *If $e^0 < e \leq e_a^1$, W^2 is optimal and no firm adopts new technology without subsidy. The government should give subsidies to both firms. The level of the subsidy must not be smaller than $e - e^1$.*

(4) *If $e_a^1 < e \leq e_a^0$, W^1 is optimal but no firm adopts new technology without subsidy. The government should give subsidies to the firms. There are two subsidization schemes.*

    a) *The government gives a subsidy to one of the firms, and this firm adopts new technology. The level of the subsidy must not be smaller than $e - e^0$. The other firm does not adopt. It is a discriminatory policy.*

    b) *The government gives subsidies to both firms. The level of the subsidy to each firm is between $e - e^0$ and $e - e^1$. Since at the equilibrium only one firm adopts new technology, the government actually gives the subsidy to one of the firms. It is not a discriminatory policy because both firms have chances to receive subsidies.*

    In both schemes only one firm adopts new technology.

(5) *If $e > e_a^0$, W^0 is optimal and no firm adopts new technology without subsidy. The government should do nothing.*

4 Linear cost functions

In this section we assume that firms have linear cost functions. Then, the marginal costs are constant. We use the same symbols as those in the previous section. The profits of Firm A and B before adoption of new technology are

$$\pi_A = (a - x_A - x_B)x_A - cx_A, \ \pi_B = (a - x_A - x_B)x_B - cx_B.$$

After adoption of new technology they are

$$\pi_A = (a - x_A - x_B)x_A - e, \ \pi_B = (a - x_A - x_B)x_B - e.$$

We assume Cournot type behavior of firms, and assume $a > 2c$. There are four cases.

(1) The conditions for profit maximization when no firm adopts new technology are

$$a - 2x_A - x_B - c = 0, \ a - x_A - 2x_B - c = 0.$$

The equilibrium outputs and profits are

$$x_A^0 = x_B^0 = \frac{a - c}{3}, \ \pi_A^0 = \pi_B^0 = \frac{(a - c)^2}{9}.$$

(2) The conditions for profit maximization when both firms adopt new technology are

$$a - 2x_A - x_B = 0, \ a - x_A - 2x_B = 0.$$

The equilibrium outputs and profits are

$$\tilde{x}_A = \tilde{x}_B = \frac{a}{3}, \ \tilde{\pi}_A = \tilde{\pi}_B = \frac{a^2}{9} - e.$$

(3) If only Firm A adopts new technology, the conditions for profit maximization are

$$a - 2x_A - x_B = 0, \ a - x_A - 2x_B - c = 0.$$

The equilibrium outputs and profits are

$$x_A^A = \frac{a+c}{3}, \ x_B^A = \frac{a-2c}{3}, \ \pi_A^A = \frac{(a+c)^2}{9} - e, \ \pi_B^A = \frac{(a-2c)^2}{9}.$$

(4) If only Firm B adopts new technology, the equilibrium outputs and profits are

$$x_A^B = \frac{a-2c}{3}, \ x_B^B = \frac{a+c}{3}, \ \pi_A^B = \frac{(a-2c)^2}{9}, \ \pi_B^B = \frac{(a+c)^2}{9} - e.$$

Let

$$e^1 = \tilde{\pi}_A + e - \pi_A^B = \tilde{\pi}_B + e - \pi_B^A = \frac{4(a-c)c}{9},$$

$$e^0 = \pi_A^A + e - \pi_A^0 = \pi_B^B + e - \pi_B^0 = \frac{4ac}{9}.$$

Clearly, $e^0 > e^1$. Thus, similarly to Lemma 1 the sub-game perfect equilibria of the game after the second stage are as follows.

Lemma 3. (1) *If $e \leq e^1$, the sub-game perfect equilibrium is a state such that both firms adopt new technology. In this case $e \leq e^1$ and $e \leq e^0$, so adoption of new technology is a dominant strategy for each firm.*

(2) *If $e^1 < e \leq e^0$, the sub-game perfect equilibrium is a state such that one firm, Firm A or B, adopts new technology. In this case $e \leq e^0$ and $e > e^1$, so adoption of new technology is a best response to non-adoption, and non-adoption is a best response to adoption.*

(3) *If $e > e^0$, the sub-game perfect equilibrium is a state such that no firm adopts new technology. In this case $e > e^0$ and $e > e^1$, so non-adoption is a dominant strategy for both firms.*

Social welfare The social welfare are obtained as follows;

$$W^2 = \frac{4a^2}{9} - 2e, \ W^1 = \frac{8a^2 - 8ac + 11c^2}{18} - e, \ W^0 = \frac{4(a-c)^2}{9}.$$

Let

$$e_a^0 = \frac{(8a+3c)c}{18}, \ e_a^1 = \frac{(8a-11c)c}{18}.$$

Then, $W^0 = W^1$ when $e = e_a^0$, $W^1 = W^2$ when $e = e_a^1$, $W^0 > W^1$ (or $W^1 > W^0$) when $e > e_a^0$ (or $e < e_a^0$) and $W^1 > W^2$ (or $W^2 > W^1$) when $e > e_a^1$ (or $e < e_a^1$). Clearly, $e_a^0 > e_a^1$. Similarly to Lemma 2 we get the following lemma.

Lemma 4. (1) *If $e \leq e_a^1$, W^2 is the maximum, and adoption of new technology by both firms is optimal.*

(2) *If $e_a^1 < e \leq e_a^0$, W^1 is the maximum, and adoption of new technology by one firm is optimal.*

(3) *If $e > e_a^0$, W^0 is the maximum, and non-adoption of new technology by any firm is optimal.*

Comparing e^0, e^1, e_a^0 and e_a^1, we find

$$e_a^1 < e^1 < e^0 < e_a^0.$$

This is different from the result in the previous section. Under quadratic cost functions we have $e^1 < e^0 < e_a^1 < e_a^0$.

We obtain the following results.

Theorem 2. *Under linear cost functions the optimal policies should be as follows.*

(1) *If $e \leq e_a^1$, W^2 is optimal and both firms adopt new technology without subsidy nor tax. The government should do nothing.*

(2) *If $e_a^1 < e \leq e^1$, W^1 is optimal but both firms adopt new technology without subsidy nor tax. The government should impose taxes for new technology adoption to one firm, or to both firms. There are the following two taxation schemes.*

    a) *The government imposes a tax on one firm, Firm A or B, to prevent adoption of new technology. The level of the tax must be larger than $e^1 - e$. It is a discriminatory policy. When one of the firms does not adopt, the other firm has an incentive to adopt.*

    b) *The government imposes taxes on both firms. The level of the tax on each firm is between $e^1 - e$ and $e^0 - e$. Then, at the equilibrium one of the firms adopts new technology and this firm actually pays a tax. The other firm does not adopt. It is not a discriminatory policy because adoption of new technology is a choice of the firm.*

    In both schemes only one firm adopts new technology.

(3) *If $e^1 < e \leq e^0$, W^1 is optimal and only one firm adopts new technology without subsidy nor tax. The government should do nothing.*

(4) *If $e^0 < e \leq e_a^0$, W^1 is optimal but no firm adopts new technology without subsidy nor tax. The government should give subsidies to both firms. The level of the subsidy must not be smaller than $e - e^0$. Since only one firm adopts new technology with this level of the subsidy, the government actually gives the subsidy to one of the firms which adopts new technology. However, it gives chances to receive subsidies to both firms. It is not a discriminatory policy.*

    Similarly to Theorem 1 there exists another discriminatory subsidization scheme such that the government gives a subsidy to only one firm.

(5) *If $e > e_a^0$, W^0 is optimal and no firm adopts new technology without subsidy nor tax. The government should do nothing.*

Remarkable results are (2) and (3) of this theorem. In (2) adoption of new technology by only one firm is optimal for the society, however both firms have incentives to adopt new technology. Thus, the government must impose taxes on one of the firms or to both firms so as to prevent adoption by one firm. On the other hand, in (2) of Theorem 1 adoption of new technology by both firms is optimal, but only one firm has an incentive to adopt. Thus, the government give subsidies to the firms. In (3) of this theorem adoption of new technology by one firm is optimal, and only one firm has an incentive to adopt. Thus, the government should do nothing. On the other hand, in (3) of Theorem 1 adoption of new technology by both firms is optimal, however no firm has an incentive to adopt. Therefore, the government give subsidies to the firms. (1), (4) and (5) are the same as those in Theorem 1.

In the future research we want to generalize the analyses in this paper to a case of general demand and cost functions.

References

Boone, J. (2001), "Intensity of competition and the incentive to innovate", *International Journal of Industrial Organization*, **19**, pp.705-726.

Elberfeld, W. and Nti, K. O. (2004), "Oligopolistic competition and new technology adoption under uncertainty", *Journal of Economics*, **82**, pp. 105-121.

Filippini, L. (2005), "Licensing contract in a Stackelberg model", *The Manchester School*, **73**, pp. 582-598.

Hattori, M. and Y. Tanaka (2014), "Incentive for adoption of new technology in duopoly under absolute and relative profit maximization", *Economics Bulletin*, **34**. pp. 2051-2059.

Kabiraj, T. (2004), "Patent licensing in a leadership structure", *The Manchester School*, **72**, pp.188-205.

Kabiraj, T. (2005), "Technology transfer in a Stackelberg structure: Licensing contracts and welfare", *The Manchester School*, **73**, pp.1-28.

Kamien, T. and Tauman, Y. (1986), "Fees versus royalties and the private value of a patent", *Quarterly Journal of Economics*, **101**, pp.471-492.

Katz, M. and Shapiro, C. (1985), "On the licensing of innovations", *Rand Journal of Economics*, **16**, pp. 504-520.

La Manna M. (1993), "Asymmetric Oligopoly and Technology Transfers", *Economic Journal*, **103**, pp.436-443.

Matsumura, T., Matsushima, N. and Cato, S. (2013), "Competitiveness and R&D competition revisited", *Economic modelling*, **31**, pp.541-547.

Pal, R. (2010), "Technology adoption in a differentiated duopoly: Cournot versus Bertrand", *Research in Economics*, **64**, pp. 128-136.

Sen, D. and Tauman, Y. (2007), "General licensing schemes for a cost-reducing Innovation", *Games and Economic Behavior*, **59**, pp.163-186.

Wang, X. H. and Yang, B. Z. (2004), "On technology licensing in a Stackelberg duopoly", *Australian Economic Papers*, **43**, pp. 448-458.

Watanabe N. and Muto, S. (2008), "Stable profit sharing in a patent licensing game: general bargaining outcomes", *International Journal of Game Theory*, **37**, pp.505-523.

Zhang, Y., Mei, S. and Zhong, W. (2014), "New technology adoption in a Cournot oligopoly with spillovers", *Journal of Economics*, **112**, pp. 115-136.