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Intra-Day Realized Volatility for European and USA Stock Indices

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Abstract

The paper constructs measures of intra-day realized volatility for 17 European and USA stock indices. We utilize a model-free de-noising method by assembling the realized volatility in sampling frequency selected according to the volatility signature plot which minimizes the micro-structure effects. Having verified the stylized facts of realized volatility, the dynamic behavior of correlation between realized volatilities is investigated. The correlation among realized volatilities is positive and extremely high, although for some periods it decreases dramatically. The correlation of volatilities within USA (or Europe) is much higher than the correlation of volatilities across USA and Europe. Moreover, we provide evidence that the inter-day adjusted realized volatility reduces significantly the underestimation of the true variability.

Keywords: correlation of volatilities, intra-day data, model-free de-noising, realized volatility, sampling frequency, ultra-high frequency, volatility signature plot.

JEL Classifications: C14; C32; C50; G11; G15.

1. Introduction

Empirical finance literature provides various methods of refining intra-day prices in order to measure the daily volatility. The objective of the paper is to fill the gap in the literature between theoretical aspects and empirical applications of constructing realized volatility. In particular, we extend the literature by examining a long length dataset for 17 European and USA stock indices. We consider a range of European and USA indices to be able to extend previous studies which mainly consider data from few markets[i]. The proposed method of constructing realized volatility offers useful findings for practitioners.

Following recent papers on realized volatility forecasting and market microstructure noise (e.g. Andersen et al., 2011), the paper advocates choosing a model-free[ii] de-noising approach for the construction of intra-day realized volatility measures. This method has the advantage of being accurate-to-estimate and, at the same time, straightforward to apply (simple in terms of numerical computations). The high frequency log-returns are constructed according to the previous tick method, in order to get volatility measures which do not converge in probability to zero. In addition, the realized volatility measures are assembled in sampling frequency selected according to the volatility signature plot by minimizing the noise accumulation due to market frictions.

The paper provides an extension of the earlier empirical investigations reported in Andersen et al. (2001a, 2001b, 2010, 2011), Hansen and Lunde (2005), Jungbacker and Koopman (2006), and Thomakos and Wang (2003). These papers provide evidence on the stylized facts of realized volatilities using traditional approaches under several assumptions. We extend these studies by explicitly accounting for the stylized facts of realized volatilities in a simple form; this is highly important for financial decision-makers who deal with high-frequency datasets.

In particular, we investigate the distributional properties of realized stock return volatilities of the major European and USA markets and verify the stylized facts noted in financial literature. The results from 17 European and USA markets support empirically the notion that inter-day adjusted realized standard deviation is highly leptokurtic and skewed to the right, while the daily realized volatility is

[i] This is important as the current debate on the 2008 financial crisis give emphasis to the *domino effect* of the major stock markets.

[ii] By the term *model-free* we note a de-noising method that does not assume a predefined model configuration for microstructure noise.

approximately log-normally distributed. The standardized log-returns with the realized standard deviation have a 94% lower standard deviation and 69% lower kurtosis than the raw log-returns.

Additionally, the paper provides evidence that the correlation between realized volatilities is not constant across time. Although, the correlation between realized volatilities is positive and high, for some periods it decreases dramatically. The correlation between USA (or European) volatilities is much higher than the correlation across USA and European volatilities. Pushing the analysis one step further, we confirm that the inter-day adjusted realized volatility reduces significantly the underestimation of the true variability (of the integrated variance).

In the section follows, the theoretical framework of *integrated variance* and the concept of the *realized volatility* are illustrated. Section 3 describes easy-to-implement adjustment procedures for the construction of realized volatility estimators; the linear interpolation and the previous tick methods for constructing the sequence of the calendar time sampling prices, as well as the volatility signature plot to identify the bias induced by microstructure frictions. Section 4 applies the proposed method to construct the realized volatility for 17 European and USA stock indices. Section 5 investigates the variance reduction of integrated volatility due to the inter-day adjustment. Section 6 investigates the time-varying correlation among the realized volatilities. Section 7 provides information for the distribution of *inter-day adjusted* realized standard deviations, whereas section 8 concludes and provides insights for future research.

2. Ultra-High-Frequency Realized Volatility

The time interval $[a,b]$ is partitioned in τ *equidistance points* (*sub-intervals*) in time $j=1,2,...,\tau$. At each point in time t_j, for $t_j \ni [a,b]$, the asset price is observed. The $\{P_{t_j}\}_{j=1}^{\tau}$ process is observed at *sampling frequency* $m=(b-a)/(\tau-1)$, for length of each sub-interval $m=t_j-t_{j-1}$.[iii] The $y_{t_j}=\log P_{t_j}-\log P_{t_{j-1}}$ denotes the *log-return* over the sub-interval $\lfloor t_j - t_{j-1} \rfloor$. The instantaneous prices $p(t)$ represent the latent

[iii] We denote the sampling frequency m, which lowers as the number of samples increases. On the contrary, the notion of *ultra-high frequency* defines the mean of high number of equidistance points in time τ. In the remaining of the manuscript, when the points in time, i.e. the size of the sample, increase we will note that the sampling frequency increases.

efficient prices generated by the true data generated mechanism, i.e. the diffusion process $d\log p(t) = \sigma(t)dW(t)$, where $\sigma(t)$ is the volatility of the instantaneous log-returns process and $W(t)$ is the standard Wiener process. The $\sigma^{2(IV)}_{[a,b]} = \int_a^b \sigma^2(t)dt$ denotes the *integrated variance* over the interval $[a,b]$, which is not observed.

Having considered that each sub-interval has length which tends to zero, $m \to 0$, we assume that $dt \approx t_j - t_{j-1}$. The realized volatility $RV^*_{[t_j,t_{j-1}]} = \left(\log p_{t_j} - \log p_{t_{j-1}}\right)^2$ is a consistent estimator for $\sigma^{2(IV)}_{[t_j,t_{j-1}]}$, for each sub-interval; consequently, $RV^*_{[a,b]}$ is a consistent estimator for $\sigma^{2(IV)}_{[a,b]}$: $RV^*_{[a,b]} = \sum_{j=1}^{\tau} \left(\log p_{t_j} - \log p_{t_{j-1}}\right)^2$. However, $RV^*_{[t_j,t_{j-1}]}$ is not estimable as the efficient prices are not observable. Therefore, we compute *realized volatility* based on the observed prices, for the time interval $[a,b]$ which is partitioned in τ equidistance points:

$$RV_{[a,b]} \equiv RV_t^{(\tau)} = \sum_{j=1}^{\tau} \left(\log P_{t_j} - \log P_{t_{j-1}}\right)^2 . \quad (1)$$

As $\tau \to \infty$, $\sqrt{\tau}\left(RV_{[a,b]} - \int_a^b \sigma^2(t)dt\right) \Big/ \sqrt{\int_a^b 2\sigma^4(t)dt} \xrightarrow{d} N(0,1)$. The asymptotic volatility of volatility is termed *integrated quarticity*: $\sigma^{2(IQ)}_{[a,b]} = \int_a^b 2\sigma^4(t)dt$. From Barndorff-Nielsen and Shephard (2004a, 2004b), the finite sample behaviour of the realized volatility is $\left(RV_{[a,b]} - \sigma^{2(IV)}_{[a,b]}\right) \Big/ \left(\sqrt{2\left(3^{-1}\right)\sum_{j=1}^{\tau}\left(\log P_{t_j} - \log P_{t_{j-1}}\right)^4}\right) \xrightarrow{d} N(0,1)$, for $RQ_{[a,b]} = \frac{\tau}{3}\sum_{j=1}^{\tau}\left(\log P_{t_j} - \log P_{t_{j-1}}\right)^4$ being the *realized quarticity*; a consistent estimator of $\sigma^{2(IQ)}_{[a,b]}$.

3. Measuring Realized Volatility

3.1. Equidistance Price Observations

Linear interpolated prices between preceding and immediately following quotes are computed weighting linearly their inverse relative distance to the desired point in time. Hansen and Lunde (2006a) noted that the linear interpolated realized volatility measure method converges in probability to zero as the number of sub-intervals tends to infinity: $RV_{[a,b]} \xrightarrow{p} 0$, as $\tau \to \infty$ or $m \to 0$.

Wasserfallen and Zimmermann (1985) proposed the previous tick method which is to always use the most recently published price:

$$P_{pre,t_j} = \frac{P_{bid,t_{i-1}} + P_{ask,t_{i-1}}}{2}. \quad (2)$$

The sequence of the one-minute sampling prices is constructing for this study according to the previous tick method, in order to get a volatility measure that does not converge in probability to zero, as the number of equidistance points in time tends to infinity.

3.2. Microstructure Frictions

Market frictions are anything that interferes with trade, such as transparency of transactions, discreteness of the data, transaction costs, taxes, regulatory costs, properties of the trading mechanism and bid-ask spreads. Market microstructure noise plays significant role in financial markets. Several methods have been proposed to de-noise the data in the context of volatility estimation. These methods have been constructed assuming various assumptions about the microstructure of the markets, see for example Andersen et al. (2010), Barndorff-Nielsen et al. (2008), Maheu and McCurdy (2002), Aït-Sahalia et al. (2005), Hansen and Lunde (2006a).

Based on previous studies (e.g. Andersen et al., 2011), we account for micro-structure noise without assuming a predefined model of estimating the noise. The realized volatility is assembled in sampling frequency selected according to the volatility signature plot which minimizes the micro-structure effects[iv].

Andersen et al. (2005) introduced a model-free adjustment procedure framework for the calculation of volatility loss functions based on practically feasible realized volatility benchmarks. Avoiding a specific framework for the market microstructure noise, the realized volatility will be subject to the measurement error; named *realized volatility error*: $U_{[a,b]} = RV_{[a,b]} - \sigma^{2(IV)}_{[a,b]}$. The variance of the realized volatility is $V\left(RV_{[a,b]}\right) = V\left(U_{[a,b]}\right) + V\left(\sigma^{2(IV)}_{[a,b]}\right) + 2Cov\left(U_{[a,b]}, \sigma^{2(IV)}_{[a,b]}\right)$. From Andersen et al. (2005) the covariance between realized volatility error and integrated variance is of

[iv] A number of papers propose accounting for jumps in intra-day log-prices; see Dumitru and Urga (2012) for a survey. For example, Fleming and Paye (2011) take into account the issue of jumps by using the bi-power variation (Barndorff-Nielsen and Shephard, 2006) measure of RV. A number of methods have been introduced; including the two-scale method (e.g. Aït-Sahalia et al., 2005), the kernel method (e.g. Barndorff-Nielsen et al., 2011), the multiple grid combination method (e.g. Zhang et al., 2005), the quasi-maximum likelihood method (e.g. Aït-Sahalia et al., 2010), non-parametric modelling of jumps in asset prices (e.g. Andersen et al., 2007, 2010 and Barndorff-Nielsen and Shephard, 2006), etc.

order less that τ^{-1}. Hence (based on Section 2), we get that $V(U_{[a,b]}) = \frac{2}{\tau} E(RQ_{[a,b]}) + o(\tau^{-1})$. Thus:

$$V\left(\sigma_{[a,b]}^{2(IV)}\right) = V\left(RV_{[a,b]}\right) - \frac{2}{\tau} E\left(RQ_{[a,b]}\right) - o\left(\tau^{-1}\right). \tag{3}$$

3.3. Optimal Sampling Frequency

The accuracy improves as the number of sub-intervals increases, or $\tau \to \infty$. On the other hand, as $m \to 0$, the market frictions are a source of additional noise in the estimate of volatility, as fluctuations in transaction prices over very small time intervals (sampling at high sampling frequency) are mainly reflected noise and carry no information about underlying return volatility. Thus, the points in time τ should be as many as the market microstructure features do not induce bias to the volatility estimator.

The sampling frequency, m, should be selected based on a trade-off between accuracy and potential biases due to market microstructure frictions. Fang (1996) and Andersen et al. (2000c) proposed the construction of the volatility signature plot, which provides a graphical representation of the average realized volatility against the sampling frequency. As the number of samples increases, the bias induced by microstructure frictions increases too. Thus, in the signature plot one should look for the highest frequency where the average realized volatility appears to stabilize. With the appropriate manipulations, the inter-day variance can be decomposed into the intra-day variance, $RV_t^{(\tau)}$, and the intra-day autocovariances $y_{t_i} y_{t_{i-j}}$:

$$y_t^2 = RV_t^{(\tau)} + 2\sum_{j=1}^{\tau-1} \sum_{i=j+1}^{\tau} y_{t_i} y_{t_{i-j}} . \tag{4}$$

The intra-day autocovariances comprise measurement errors, whose expected values are equal to zero, $E\left(y_{t_i} y_{t_{i-j}}\right) = 0$, for $j \neq 0$. The optimal sampling frequency can be chosen as the highest frequency for which the autocovariance bias term disappears. Corsi et al. (2001) presented a theoretical model that replicates the autocorrelation of tick-by-tick returns and the observed anomalous scaling of the volatility captured by the volatility signature plot[v].

[v] Awartani et al. (2009) propose a test that statistically deals with the optimal selection of sampling frequency. In order to avoid market microstructure frictions without lessening the accuracy, the majority of the studies propose a sampling frequency of 5-minutes or 30-minutes, i.e. Andersen et al. (2000a), Andersen et al. (2001b).

3.4. Inter-day Adjustment

Hansen and Lunde (2005) combined intraday volatility during the open-to-closed period with the closed-to-open inter-day volatility:

$$RV_{t(HL^*)}^{(\tau)} = RV_{[a,b](HL^*)} = \omega_1 \left(\log P_{t_1} - \log P_{t-1_\tau}\right)^2 + \omega_2 \sum_{j=1}^{\tau} \left(\log P_{t_j} - \log P_{t_{j-1}}\right)^2 . \qquad (5)$$

The advantage of $RV_{t(HL^*)}^{(\tau)}$ measure is that minimizes the squared distance between realized volatility measure and integrated volatility devoid of necessitate to define a specific relation for efficient prices and market microstructure noise. The parameters ω_1 and ω_2 are estimated such as $\min_{(\omega_1,\omega_2)} E\left(RV_{[a,b](HL^*)}^{(\tau)} - \sigma_{[a,b]}^{2(IV)}\right)^2$. As $\sigma_{[a,b]}^{2(IV)}$ is unobservable, Hansen and Lunde (2005) proposed to solve $\min_{(\omega_1,\omega_2)} V\left(RV_{[a,b](HL^*)}^{(\tau)}\right)$.[vi] The analytical estimates of ω_1 and ω_2 are presented in Table 2.

4. Intra-Day Realized Volatility for 17 European and USA Stock Indices

The dataset considers major EU and US stock market indices with the longest continuous history. The selection is based on i) the indices' market capitalization and ii) the fact that they are the most publicly quoted stock market indices. In addition, most indices are considered as benchmark indices for stocks (e.g. Nasdaq, S&P500) traded internationally as they contain about 70-80% of the value of their individual stocks. Moreover, the list includes world's top stock exchanges by value shares traded as reported by World Federation of Exchanges Industry Association. Table 1 presents information for the one-minute intra-day data for the 17 indices: S&P500, S&P100, Dow Jones Industrial Average, Nasdaq100, Russell2000, S&P400 Midcap, FTSE100, AEX25, IBEX35, DJ Euro Stoxx 50, DAX30, CAC40, DJ Stoxx 50, FTSE Euro Top 300, FTSE MIB Index, Swiss Market Index and Euronext100.

[Insert Table 1 about here]

The proposed realized volatility measures assume that i) a very high sampling frequency is available for the data (section 2), ii) the sequence of the sampling prices is constructing according to the previous tick method (section 3.1), and iii) the intra-day autocovariance comprises the measurement errors due to market frictions (section 3.2). The optimal sampling frequency is chosen as the highest frequency for which the

[vi] Hansen and Lunde (2005) noted that, for Y denoting a real random variable and for X_ω, for $\omega \in \Omega$, being a class of real random variables, if $E(X_\omega \mid Y) = Y$, for $\forall \omega \in \Omega$, then $\arg\min_{(\omega)} E(X_\omega - Y)^2 = \arg\min_{(\omega)} V(X_\omega)$.

autocovariance bias term disappears. Therefore, the sampling frequency, m, is selected based on a trade-off between accuracy and potential biases due to market microstructure frictions. As Andersen et al. (2006) noted "*Recognizing that the bid-ask spread (and other frictions) generally bias the realized volatility measure, this suggests choosing the highest frequency possible for which the average realized volatility appears to have stabilized*".[vii]

[Insert Figure 1 About here]

To conserve space, in Figure 1, the volatility signature plot is presented for 4 indices, indicatively. All the Figures for each of the 17 stock indices are available from the authors upon request. The Figure provides a graphical representation of the average intra-day autocovariances, $2T^{-1}\sum_{t=1}^{T}\sum_{j=1}^{\tau-1}\sum_{i=j+1}^{\tau} y_{t_i} y_{t_{i-j}}$, against the sampling frequency, $m = 1,2,...,40$. The last column of Table 1 presents the highest sampling frequency for which the autocovariance bias term minimizes.

The interday adjustment proposed by Hansen and Lunde (2005) is taken into consideration.The ω_1 and ω_2 estimates are extremely sensitive to the outliers of both closed-to-open interday volatility, $\left(\log P_{t_1} - \log P_{t-1_\tau}\right)^2$, and intra-day volatility, $\sum_{j=1}^{\tau}\left(\log P_{t_j} - \log P_{t_{j-1}}\right)^2$. Therefore, we compute the ω_1 and ω_2 estimates, for 200 iterations, excluding at each iteration either the highest value of the closed-to-open interday volatility or the highest value of the intra-day volatility. Figure 2 plots for 4 indices, indicatively, the ω_1 and ω_2 estimates for 200 iterations, excluding at each iteration the most extreme value. We should note that the extreme outliers are excluding form the computation of the ω_1, ω_2, μ_1, μ_2, μ_0, η_1, η_2 and η_{12} estimates, but not from the construction of the realized volatility measure, i.e. the realized volatility is constructed for the trading days that the extreme outliers are observed. The number of extreme outliers varies across indices. We remove as many outliers as necessary in order to stabilize the ω_1 and ω_2 estimates. Table 2 presents the parameter estimates of the inter-day adjusted realized volatility, $RV_{t(HL^*)}^{(\tau)}$. The last column presents the required number of excluding trading days in order to compute ω_1 and ω_2 estimates which are not sensitive to the outliers. On average, 1.1% of the

[vii] The program codes that carry out the necessary computations for constructing the realized volatility measures are available from the authors upon request.

observations as outliers are required for the estimates of the scaling parameters to be stabilized. However, there are cases, i.e. AEX25 and FTSE MIB, that require less than 0.5% of outliers, and cases, i.e. FTSE Eurotop300 and S&P400 Midcap, that require around 2.5% of outliers.

[Insert Figure 2 About here]

[Insert Table 2 about here]

The ratio of closed-to-open interday volatility and open-to-close intra-day volatility is not stable across indices. On average, $\mu_1/\mu_0 = 14\%$ of the volatility occurs during the inactive period. The stock indices are grouped into two categories. The first group mainly contains leading European and USA stock indices such as Euro Stoxx 50, DJStoxx50, DowJones, FTSE100, FTSE EuroTop300, Russell2000, S&P100, S&P400 MidCap and S&P500, whereas the second group mainly comprises from secondary European and USA indices such as AEX25, CAC40, DAX30, Euronext100, IBEX35, MIB, Nasdaq100, SMI. In the case of the leading stock indices about 4% of daily volatility occurs during the inactive period. As concerns the second group, the 25%, on average, of daily volatility occurs during the inactive period. Hansen and Lunde (2005) have found, for the 30 equities of the Dow Jones Industrial Average (during the period January 2001 to December 2004), that about 20% of daily volatility occurs during the inactive period. Therefore, the proportion of volatility that occurs during the inactive period ranges i) from equities to indices, ii) across time, as well as iii) from market to market. Figure 3 illustrates the annualized inter-day adjusted realized standard deviation, $\sqrt{252 RV_{t(HL^*)}^{(\tau)}}$, for 4 indices, indicatively.

[Insert Figure 3 About here]

5. Variability of Integrated Variance

The inter-day adjusted realized volatility reduces significantly the underestimation of the true variability. True integrated volatility, $\sigma_{[a,b]}^{2(IV)}$, is known only in simulation experiments, but we are able to estimate from the actual data the variance of the realized volatility measure, $V\left(RV_{[a,b]}\right)$, as well as, the expected mean estimate of *realized quarticity* $E\left(RQ_{[a,b]}\right)$. Based on equation (3), the ratio

$V\left(RV_{[a,b]}\right)/\left(V\left(RV_{[a,b]}\right)-2\tau^{-1}E\left(RQ_{[a,b]}\right)\right)$ defines an index[viii] of the overestimation of the true variability of the *integrated variance* when the ex post $RV_{[a,b]}$ is used in place of the latent $\sigma_{[a,b]}^{2(IV)}$. Table 3 presents the index for the unadjusted realized volatility, $RV_t^{(\tau)}$, as well as for the *inter-day adjusted* realized volatility, $RV_{t(HL^*)}^{(\tau)}$. For example, for the AEX25 index, the $RV_t^{(\tau)}$ overestimates the true variability of the *integrated variance* by 1.212, whereas the index of overestimation for the $RV_{t(HL^*)}^{(\tau)}$ is reduced to 1.109. The overestimation of the true variability of the $\sigma_{[a,b]}^{2(IV)}$ when the $RV_{t(HL^*)}^{(\tau)}$ is used in place of the latent $\sigma_{[a,b]}^{2(IV)}$ is reduced in all the cases.

Various adjustments on the construction of the realized volatility estimator have been investigated. Kalman filtering to $\log RV_{t(HL^*)}^{(\tau)}$, similar to Hansen and Lunde's (2005) smoothing of realized volatility estimates, has been applied. However, the ratio $V\left(RV_{[a,b]}\right)/\left(V\left(RV_{[a,b]}\right)-2\tau^{-1}E\left(RQ_{[a,b]}\right)\right)$ is much higher for the Kalman filtered $RV_{t(HL^*)}^{(\tau)}$, mainly because of the reduction of realized volatility's variability due to the filtering.[ix]

[Insert Table 3 about here]

6. Realized Volatilities Time-Varying Correlations

An analysis of the time-varying correlation among the realized volatilities would provide useful insights about the relation of volatilities across European and USA markets. We estimate an asymmetric Diag-VECH model[x] for the vector $\mathbf{y}_t = \left(\log\sqrt{252RV_{t(HL^*),1}^{(\tau)}} \quad \ldots \quad \log\sqrt{252RV_{t(HL^*),17}^{(\tau)}}\right)'$. The $\mathbf{y}_t$ is regressed on a vector of constants[xi]. The innovation process, $\boldsymbol{\varepsilon}_t$, has a conditional covariance matrix $\mathbf{H}_t$, such

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[viii] Andersen et al. (2005) note that the (feasible) R^2 from the Mincer-Zarnowitz regression will underestimate the true predictability as measured by the (infeasible) R^2 from the regression of the latent integrated volatility on the same set of predetermined regressors by the multiplicative factor $V\left(RV_{[a,b]}\right)/\left(V\left(RV_{[a,b]}\right)-2\tau^{-1}E\left(RQ_{[a,b]}\right)\right)$.

[ix] The filtered inter-day adjusted realized volatility for the 17 stock indices are available from the authors upon request.

[x] For technical details about Bollerslev's et al. (1988) Diag-VECH model, see Xekalaki and Degiannakis (2010).

[xi] The $\mathbf{y}_t$ is highly predictable, suggesting to model the conditional mean as a Vector AutoRegressive process (VAR); i.e. $\mathbf{y}_t = \boldsymbol{\beta} + \sum_{i=1}^{k}\mathbf{B}_k\mathbf{y}_{t-k} + \boldsymbol{\varepsilon}_t$. However, according to Degiannakis et al. (2013), in the Diag-VECH model, the non-diagonal elements of $\mathbf{H}_t$ express the time-varying correlations of the de-mean log-returns. On the contrary, in the case of modelling the conditional mean as a VAR(k) process, the non-diagonal elements of $\mathbf{H}_t$ would express the time-varying correlation of the unexplained part of the realized volatilities.

as $\boldsymbol{\varepsilon}_t = \mathbf{H}_t^{1/2}\mathbf{z}_t$, or $\boldsymbol{\varepsilon}_t \mid I_{t-1} \sim N[0, \mathbf{H}_t]$. For $\circ$ denoting the Hadamard product, the $\mathbf{H}_t$ has the form $vech(\mathbf{H}_t) = vech(\tilde{\mathbf{A}}_0) + vech(\tilde{\mathbf{A}}_1) \circ vech(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}) + vech(\tilde{\boldsymbol{\Gamma}}_1) \circ vech(\tilde{\boldsymbol{\varepsilon}}_{t-1}\tilde{\boldsymbol{\varepsilon}}'_{t-1})$ $+ vech(\tilde{\mathbf{B}}_1) \circ vech(\mathbf{H}_{t-1})$. The non-diagonal elements of $\tilde{\mathbf{A}}_0, \tilde{\mathbf{A}}_1$, $\tilde{\boldsymbol{\Gamma}}_1$ and $\tilde{\mathbf{B}}_1$ are time varying. Such a specification has the flexibility to estimate time-varying covariances. Otherwise, in case of constant non-diagonal elements of $\tilde{\mathbf{A}}_0, \tilde{\mathbf{A}}_1$, $\tilde{\boldsymbol{\Gamma}}_1$ and $\tilde{\mathbf{B}}_1$, a time-varying correlation due to the time-varying standard deviations would lead to an increase (decrease) in correlations in less (more) volatile periods. The i^{th} diagonal element of $\mathbf{H}_t$ is $\sigma_{i,t}^2 = a_{i,i} + \tilde{a}_{i,i}\varepsilon_{i,t-1}^2 + \gamma_{i,i}\varepsilon_{i,t-1}^2 d_{i,t-1} + \tilde{b}_{i,i}\sigma_{i,t-1}^2$, whereas the i^{th}, j^{th} non-diagonal elements are $\sigma_{i,j,t} = a_{i,j} + \tilde{a}_{i,j}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \gamma_{i,j}\varepsilon_{i,t-1}d_{i,t-1}\varepsilon_{j,t-1}d_{j,t-1} + \tilde{b}_{i,j}\sigma_{i,j,t-1}$, where $d_{i,t-1} = 1$ if $\varepsilon_{i,t-1} < 0$, and $d_{i,t-1} = 0$ otherwise. The time-varying correlations between i^{th} and j^{th} markets are estimated as $\rho_{i,j,t} = \sigma_{i,j,t}\left(\sigma_{i,t}^2\sigma_{j,t}^2\right)^{-1/2}$. The appropriateness of the asymmetric Diag-VECH specification for modeling the conditional covariance matrix has been tested for residuals' serial correlation and for presence of ARCH effects in the residuals. Bollerslev and Wooldridge's (1992) robust quasi-maximum likelihood standard errors are also considered[xii].

Figure 4 plots, indicatively, the dynamic correlation among the realized volatility of the DAX30, FTSE100, Nasdaq100 and S&P500 indices. In general the correlation across realized volatilities is positive and high. There are periods, i.e. July, 2002 - May, 2003 and October, 2008 - June, 2009, that the correlation across realized volatilities is even higher than 85%. However, for some periods the correlation decreases dramatically - i.e. during December 2007 - January 2008, the correlation between DAX30 and S&P500 volatilities is almost equal to zero. The same case holds for the correlation between FTSE100 and Nasdaq100 for December 2003. Undoubtedly, the correlation between realized volatilities is not considered constant across time. The correlation between USA (or European) volatilities is much higher than the correlation across USA and European volatilities[xiii]. Interestingly, the

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[xii] A model-free analysis of dynamic correlation among realized volatilities was also considered, according to the J.P. Morgan's (1996) multivariate Riskmetrics framework, which provides qualitatively similar results. The multivariate Riskmetrics model does not require the estimation of $\mathbf{H}_t$, but the assumption of imposing the same dynamics on every component is difficult to justify. Thus, we rely our analysis on the parametric quasi-maximum-likelihood estimated model.

[xiii] The time varying correlations among the 17 realized volatilities are available from the authors upon request.

increase or decrease of correlation between volatilities is not related with a specific trend (either upward or downward) of equity markets. In other words, during the October, 2008 - June, 2009 period of extremely high correlation (financial crisis of 2008), the trend of DAX30 and S&P500 indices was descending for the first half, whereas it was ascending for the second half.

[Insert Figure 4 About here]

7. Distributional Properties of Realized Volatility

Figure 5 represents, for 4 indices, the distributions of *inter-day adjusted* realized standard deviations, based on Kernel bandwidths method. The distributions are highly leptokurtic and skewed to the right. Figure 6 interprets the estimated density of the *inter-day adjusted* realized daily logarithmic standard deviations. The distributions of logarithmic standard deviations are approximately Gaussian. Figure 7 plots the estimated density of the standardized log-returns; the standardization of the log-returns with the realized standard deviation reduces kurtosis and skewness significantly.

[Insert Figure 5 About here]
[Insert Table 4 about here]

According to Table 4, the average value of the annualized standard deviation for the 17 stock indices is 17.5%. However, there is considerable variation among stock markets. In particular, the maximum annualized volatility is observed for the S&P100 index to 206%, for the day next of Black Monday on October 19th, 1987. The median value of annualized volatility across the 17 stock markets, ranging from 10.7% for S&P500 to 20.9% for Nasdaq100.

Barndorff-Nielsen and Shephard (2004b) studied the finite sample behaviour of the logarithmic realized variance: $\left(\log\left(RV_{[a,b]}\right)-\log\left(\sigma^{2(IV)}_{[a,b]}\right)\right)\left(2\tau^{-1}RQ_{[a,b]}RV^{-2}_{[a,b]}\right)^{-1/2}\xrightarrow{d}N(0,1)$, and provided theoretical and simulated evidence in favour of the logarithmic realized variance. The asymptotic approximation to the distribution of $\log\left(RV_{[a,b]}\right)$ works well even for moderately small values of τ, while the finite sample behaviour of $RV_{[a,b]}$ requires much higher value of τ to be empirically reliable. Table 5 provides the descriptive statistics of the annualized inter-day adjusted realized daily logarithmic standard deviations. The sample skewness is positive in all the cases. On average, the skewness of log-standard deviations, across the 17 stock indices, is reduced to 0.33 compared to 3.06 for the realized standard deviations. As concerns the kurtosis, the average value of kurtosis for the log-volatilities, across the 17 stock indices, is 3.25 compared to 22.81 for the realized standard deviations; the assumption of normality is much closer in the logarithmic transformation case.

According to Tables 6 and 7, on average, the standardized log-returns have a 95% lower standard deviation than the raw log-returns. The sample kurtosis of the standardized log-returns is, on average, 77% lower than the kurtosis of the unstandardized log-returns. The sample kurtosis for all of the 17 cases does not exceed the normal value of three.

[Insert Figure 6 About here]
[Insert Figure 7 About here]

We, therefore, provide evidence from the 17 stock market indices that i) the daily realized volatility constructed from high-frequency data is approximately log-normally distributed, as well as ii) the log-return series standardized with the realized standard deviation is approximately unconditionally normally distributed. The findings are in line with the literature (i.e. Thomakos and Wang, 2003). Andersen et al. (2001b), for Deutschemark and Yen returns against the Dollar, found that the distributions of the realized daily variances, $RV_t^{(\tau)}$, and standard deviations , $\sqrt{RV_t^{(\tau)}}$, are skewed to the right and leptokurtic, but the distributions of the logarithmic daily standard deviations, $\log\sqrt{RV_t^{(\tau)}}$, are approximately normal. In general the empirical analyses on realized volatility conclude that i) the distribution of log-returns scaled by the realized standard deviation is approximately Gaussian and ii) the realized logarithmic standard deviation is also nearly Gaussian. The innovative studies of the exploration of the distributional properties of realized volatility include Andersen et al. (2000b, 2001a, 2001b, 2003).

[Insert Table 5 about here]
[Insert Table 6 about here]
[Insert Table 7 about here]

8. Conclusion and Suggestions for Further Research

The studies of Hansen and Lunde (2006b) and Patton (2011) showed that the use of a volatility proxy can lead to an evaluation appreciably differing from what would be obtained if the true volatility were used. The noisier the proxy is, the less precise evaluation would have been obtained. The realized volatility is computed in sampling frequency that minimizes the noise accumulation due to market frictions.

The presence of market microstructure noise makes it optimal to sample less often than would otherwise be the case in the absence of noise. The sample frequency that minimizes the micro-structure noise is estimated according to the volatility signature plot, avoiding of need to define a specific relation for efficient prices and market microstructure noise. The sequence of the one-minute sampling prices is constructing according to the previous tick method, which provides realized volatility measure that does not converge in probability to zero, as the number of sub-intervals tends to infinity. Moreover, we take into consideration the inter-day adjustment that minimizes the distance between realized volatility measure and integrated volatility.

The empirical findings obtained from the 17 stock market indices suggest that: i) The proportion of volatility that occurs when the markets are closed ranges from market to market; for the leading (secondary) stock indices about 4% (25%) of daily volatility occurs during the inactive period. ii) The distribution of inter-day adjusted realized standard deviation is highly leptokurtic and skewed to the right. iii) The daily realized volatility constructed from high-frequency data is approximately log-normally distributed. iv) There is considerable variation among stock markets; the median annualized volatility across the 17 stock markets, ranging from 10.7% for S&P500 to 20.9% for Nasdaq100. v) The log-return series standardized with the realized standard deviation is approximately unconditionally normally distributed. vi) The standardization of the log-returns with the realized standard deviation reduces kurtosis and skewness significantly. The standardized log-returns have a 94% lower standard deviation and 69% lower kurtosis than the raw log-returns. vii) The correlation among realized volatilities is positive and high, but for some periods it decreases dramatically. viii) The correlation of volatilities within USA or Europe is much higher than the correlation of volatilities across USA or Europe. Overall, our empirical results provide a guidance regarding the construction of realized volatility measures based on a model-free de-noising approach. The reported findings are useful in financial decision-making, and are recommended to financial analysts and modelers dealing with European and USA stock markets.

Future research may focus on the use of all the data along with the modeling of the noise; as in Ait-Sahalia et al. (2005). Additionally, the range-based[xiv] realized volatility (proposed in Christensen and Podolskij, 2007), Andersen's et al. (2010)

[xiv] Under the assumption that the sample path of the process is available, the range-based realized volatility is consistent and has five times greater precision than that of the realized volatility.

event-time or financial-time return series[xv] and Christensen's et al. (2010) quantile-based realized variance may be compared to the realized volatility measures, in order to exploit possible advantages and drawbacks of each method. It is an interesting topic for further investigation: whether or not, in real world data, the sampling at the highest frequency (which comes along with the increase of microstructure noise), provides more accurately volatility estimates. When we use all the available data (irregularly sampled tick-by-tick data), which alternative estimate (range-based realized volatility, quantile-based realized variance, etc.) is superior to the realized volatility? Future research may also include the examination of the predictability of models that capture the properties of the constructed realized volatility measures.

Acknowledgment

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[xv] The financial-time log-returns are constructed from jump-adjusted intra-day returns spanning the expected continuous volatility component time units.

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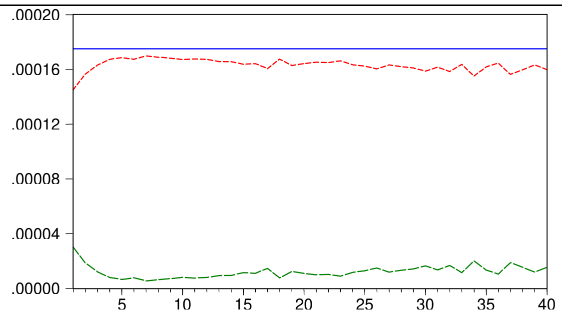
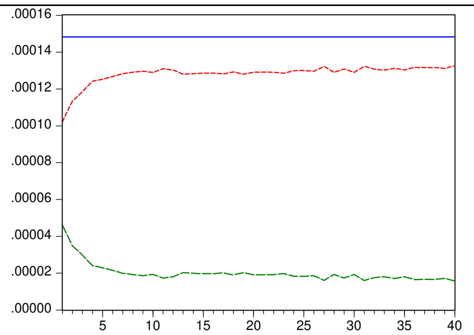
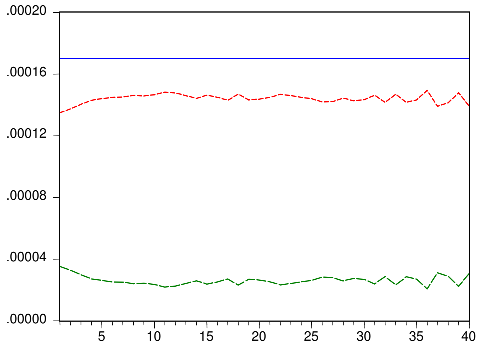
Figure 1. Average daily squared log-returns, $T^{-1}\sum_{t=1}^{T} y_t^2$, average daily realized volatility, $T^{-1}\sum_{t=1}^{T} RV_t^{(\tau)}$, and, average intra-day autocovariance, $2T^{-1}\sum_{t=1}^{T}\sum_{j=1}^{\tau-1}\sum_{i=j+1}^{\tau} y_{t_i} y_{t_{i-j}}$, for sampling frequency of $m = 1,2,\ldots,40$ minutes.

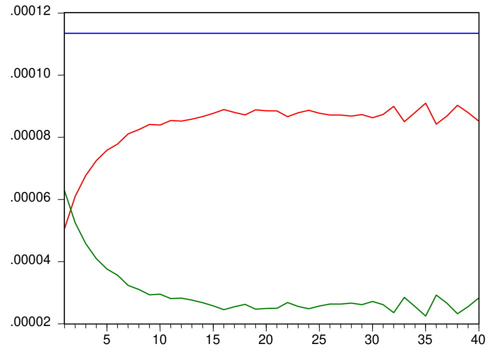
CAC 40	EURONEXT 100





IBEX35	S&P 500

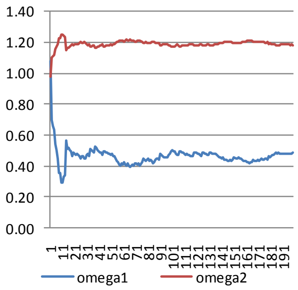


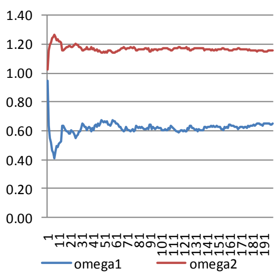


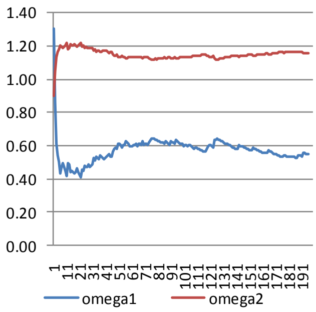
Average daily squared log-returns
average daily realized volatility
average intra-day autocovariance

Figure 2. The ω_1 and ω_2 estimates (for *inter-day adjustment* of realized volatility), for 200 iterations, excluding at each iteration either the highest value of the closed-to-open interday volatility or the highest value of the intra-day volatility.

CAC 40	EURONEXT 100	IBEX 35	S&P 500







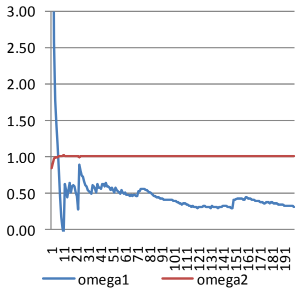
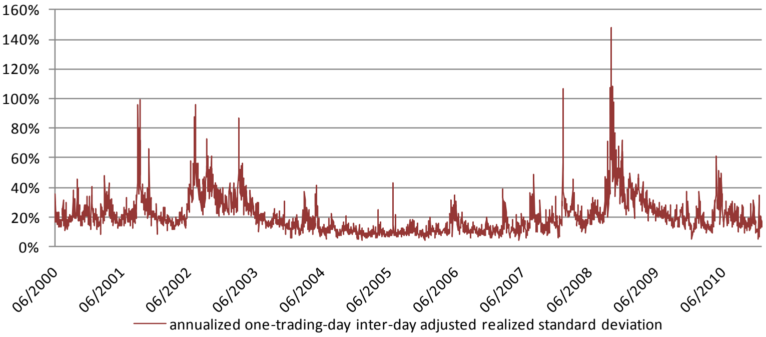
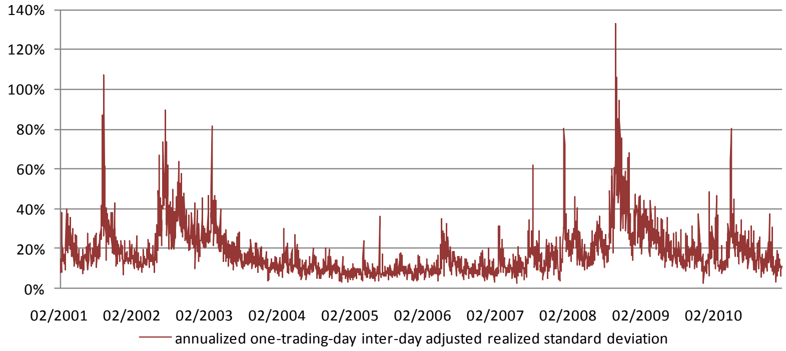


Figure 3. The annualized one-trading-day *inter-day adjusted* realized standard deviation, $\sqrt{252RV_{t(HL^*)}^{(\tau)}}$.

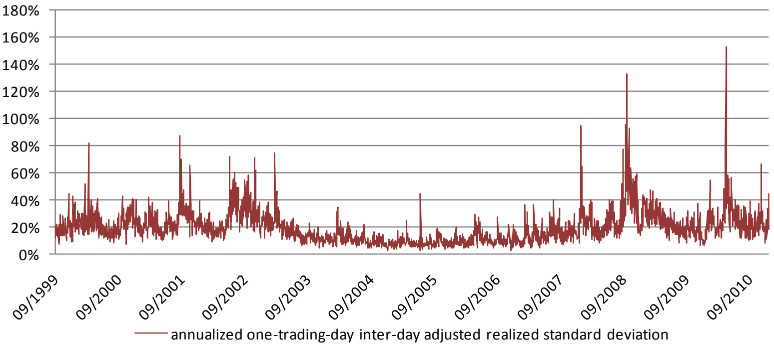
CAC 40 realized standard deviation, from 13th June 2000 to 12th January 2011.



EURONEXT 100 realized standard deviation, from 12th February 2001 to 19th January 2011.



IBEX35 realized standard deviation, from 21st September 1999 to 12th January 2011.



S&P 500 realized standard deviation, from 1st February 1983 to 12th January 2011.

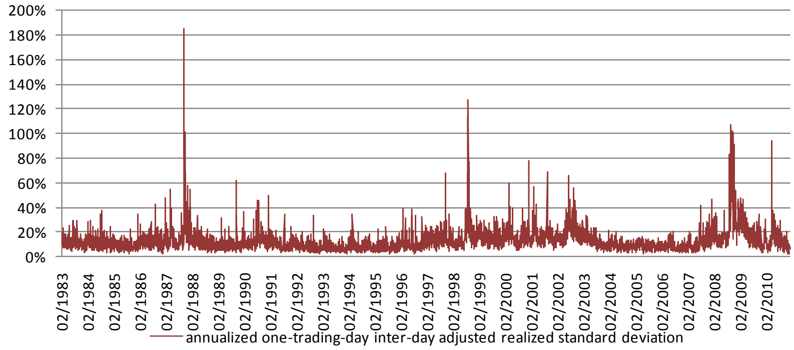


Figure 4. The dynamic correlation among the realized volatilities of the DAX30, FTSE100, NASDAQ100 and S&P500 indices.

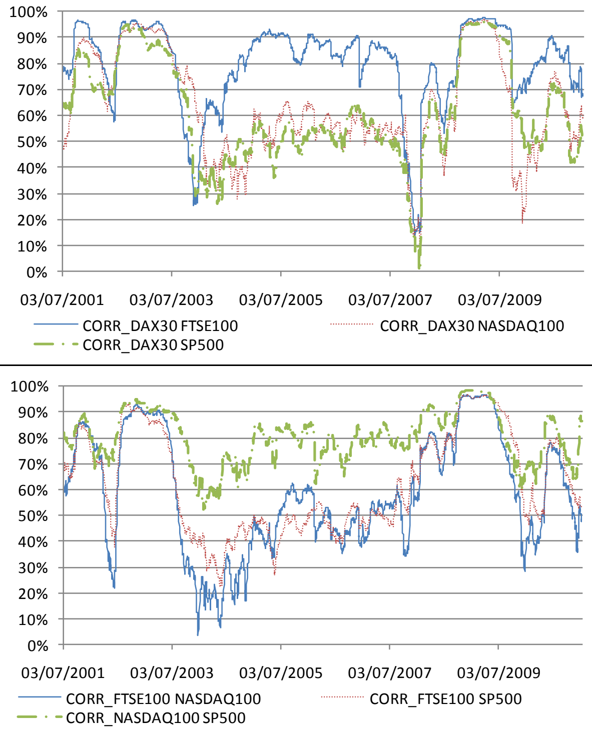
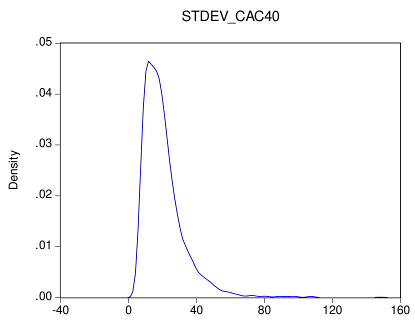
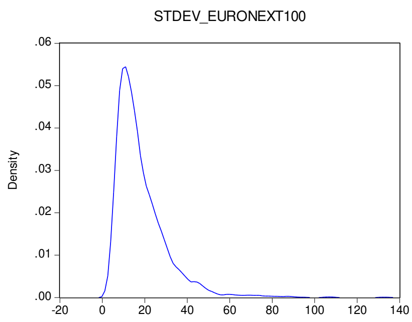


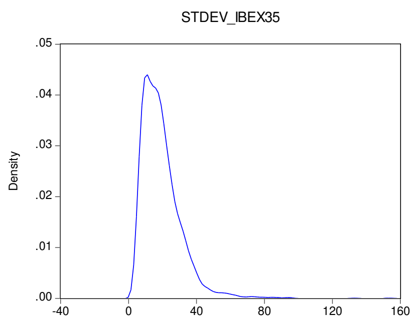
Figure 5. The estimated density of annualized one-trading-day *inter-day adjusted* realized daily standard deviations, $\sqrt{252RV_{t(HL^*)}^{(\tau)}}$.

CAC 40 index	EURONEXT index





IBEX35 index	S&P 500 index



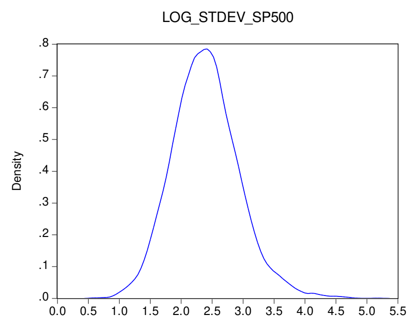
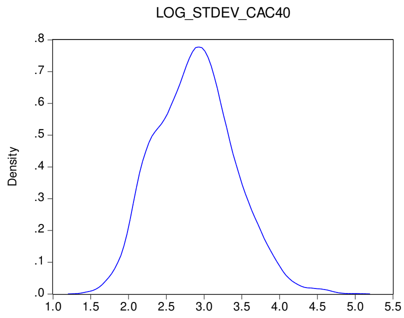
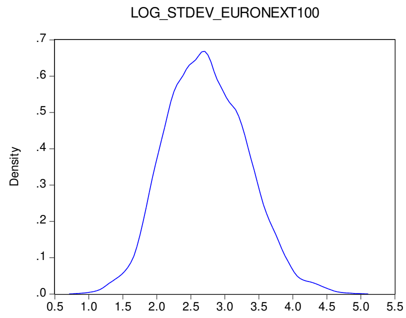


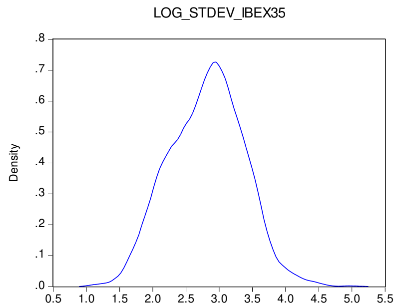
Figure 6. The estimated density of annualized *inter-day adjusted* realized daily logarithmic standard deviations, $\log \sqrt{252 RV_{t\left(HL^*\right)}^{(\tau)}}$.

CAC 40 index	EURONEXT index





IBEX35 index	S&P 500 index



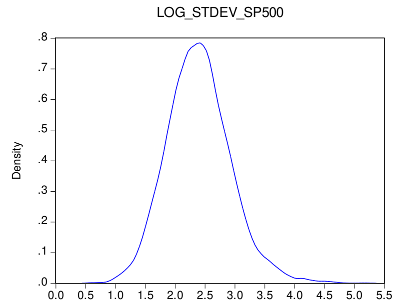
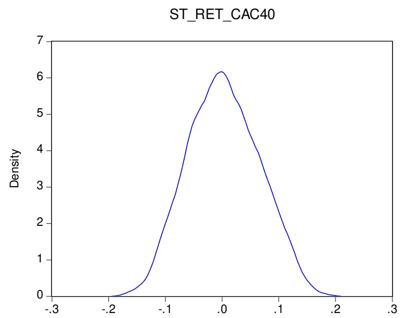
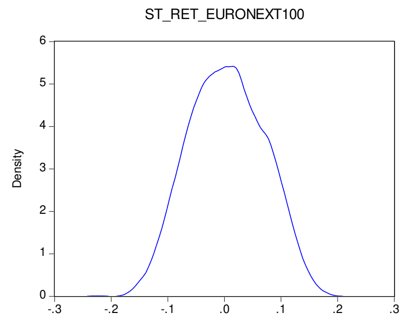


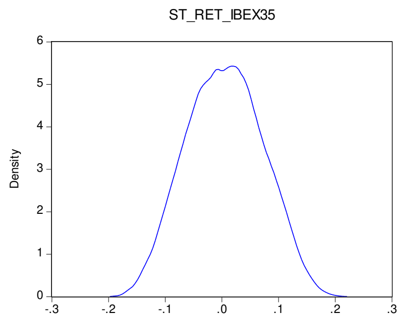
Figure 7. The estimated density of standardized log-return series, standardized with the annualized one-trading-day *inter-day adjusted* realized standard deviation, $y_t \Big/ \sqrt{252 RV_{t\left(HL^*\right)}^{(\tau)}}$.

CAC 40 index	EURONEXT index





IBEX35 index	S&P 500 index



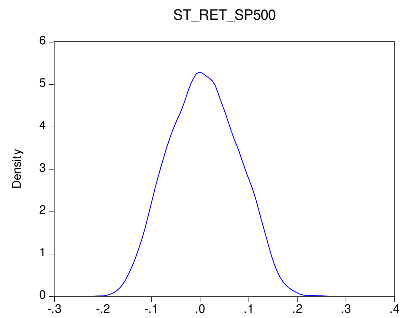


Table 1. Information for the intra-day data.

Index	Number of intra-day (1 minute) observations	Number of trading days	First day	Last day	*Optimal* sampling frequency
AEX 25	1,502,405	2,968	28th April 1999	12th January 2011	6 minutes
CAC 40	1,403,509	2,708	13th June 2000	12th January 2011	7 minutes
DAX 30	1,433,751	2,806	3rd January 2000	12th January 2011	13 minutes
DJ EURO STOXX 50	1,550,392	2,794	14th January 2000	12th January 2011	7 minutes
DJ STOXX 50	1,540,749	2,791	17th January 2000	12th January 2011	7 minutes
Dow Jones Ind	1,743,684	4,481	1st April 1993	12th January 2011	9 minutes
EURONEXT 100	1,376,389	2,532	12th February 2001	19th January 2011	7 minutes
FTSE100	1,576,347	3,128	20th August 1998	12th January 2011	7 minutes
FTSE EURO TOP 300	1,498,756	2,693	2nd August 2000	31st January 2011	7 minutes
IBEX35	1,456,892	2,854	21st September 1999	12th January 2011	5 minutes
FTSE MIB INDEX	1,314,690	2,646	8th August 2000	12th January 2011	6 minutes
NASDAQ 100	1,373,948	3,532	2nd January 1997	12th January 2011	9 minutes
RUSSELL 2000	1,377,229	3,531	2nd January 1997	12th January 2011	16 minutes
SWISS MARKET INDEX	1,312,897	2,638	24th July 2000	12th January 2011	3 minutes
S&P 100	2,363,072	6,060	2nd January 1987	12th January 2011	15 minutes
S&P400 MIDCAP	1,277,416	3,278	2nd January 1998	12th January 2011	16 minutes
S&P 500	2,754,688	7,045	1st February 1983	12th January 2011	16 minutes

Table 2. Estimation of the inter-day adjusted realized volatility, $RV_{t(HL^*)}^{(\tau)}$.

Index	ω_1	ω_2	$10^7\,\mu_1$	$10^7\,\mu_2$	$10^7\,\mu_0$	$10^{11}\,\eta_1$	$10^{11}\,\eta_2$	$10^{11}\,\eta_{12}$	# of outliers
AEX 25	0.571	1.169	515.1	1,304.7	1,819.8	1,676.6	3,549.5	721.8	11
CAC 40	0.467	1.186	507.4	1,451.3	1,958.7	1,180.7	2,770.6	584.6	19
DAX 30	0.559	1.072	275.8	1,679.2	1,955.0	710.3	4,620.5	425.0	22
DJ EURO STOXX 50	0.903	1.007	98.9	1,385.1	1,484.0	98.7	2,237.5	76.0	44
DJ STOXX 50	0.676	1.024	80.0	1,101.4	1,181.4	96.4	1,397.5	39.8	44
Dow Jones Ind	0.798	1.002	9.6	834.7	844.3	6.1	945.6	6.1	31
EURONEXT 100	0.620	1.166	500.3	1,148.4	1,648.7	1,187.1	2,439.8	562.1	16
FTSE100	0.522	1.014	27.5	934.0	961.5	31.1	825.0	8.4	55
FTSE EURO TOP 300	0.992	1.001	13.8	773.3	787.1	7.1	573.1	3.2	61
IBEX35	0.584	1.144	415.5	1,201.9	1,617.4	673.1	1,460.7	196.1	48
FTSE MIB INDEX	0.481	1.213	494.2	1,205.5	1,699.8	1,235.6	2,269.1	526.0	13
NASDAQ 100	0.379	1.162	643.7	2,467.7	3,111.4	2,530.2	8,940.6	1,647.4	25
RUSSELL 2000	0.426	1.036	74.0	1,186.2	1,260.1	58.7	2,338.3	124.8	20
SWISS MARKET INDEX	0.276	1.294	392.1	965.9	1,358.0	642.9	1,029.5	307.5	24
S&P 100	0.548	1.015	27.1	801.8	828.9	16.9	785.7	17.8	54
S&P400 MIDCAP	0.951	1.001	18.5	905.4	923.9	10.9	692.9	3.9	90
S&P 500	0.538	1.010	15.6	697.0	712.6	7.7	705.1	11.8	52

The estimates of the parameters have been computed as: $\omega_1 = \left(1 - \frac{\mu_2^2\eta_1 - \mu_1\mu_2\eta_{12}}{\mu_2^2\eta_1 + \mu_1^2\eta_2 - 2\mu_1\mu_2\eta_{12}}\right)\frac{\mu_0}{\mu_1}$, $\omega_2 = \frac{\mu_2^2\eta_1 - \mu_1\mu_2\eta_{12}}{\mu_2^2\eta_1 + \mu_1^2\eta_2 - 2\mu_1\mu_2\eta_{12}}\frac{\mu_0}{\mu_2}$, $\mu_1 = T^{-1}\sum_{t=1}^{T}\left(\log P_{t_1} - \log P_{t-1_\tau}\right)^2$, $\mu_2 = T^{-1}\sum_{t=1}^{T}\sum_{j=1}^{\tau}\left(\log P_{t_j} - \log P_{t_{j-1}}\right)^2$, $\mu_0 = \mu_1 + \mu_2$, $\eta_1 = T^{-1}\sum_{t=1}^{T}\left(\left(\log P_{t_1} - \log P_{t-1_\tau}\right)^2 - \mu_1\right)^2$, $\eta_2 = T^{-1}\sum_{t=1}^{T}\left(\sum_{j=1}^{\tau}\left(\log P_{t_j} - \log P_{t_{j-1}}\right)^2 - \mu_2\right)^2$, and $\eta_{12} = T^{-1}\sum_{t=1}^{T}\left(\left(\left(\log P_{t_1} - \log P_{t-1_\tau}\right)^2 - \mu_1\right)\left(\sum_{j=1}^{\tau}\left(\log P_{t_j} - \log P_{t_{j-1}}\right)^2 - \mu_2\right)\right)$.

Table 3. The $V\left(RV_{[a,b]}\right)/\left(V\left(RV_{[a,b]}\right) - 2\tau^{-1}E\left(RQ_{[a,b]}\right)\right)$ index of the overestimation of the true variability of the integrated variance when the ex post realized volatility is used in place of the latent $\sigma_{[a,b]}^{2(IV)}$.

Index	Index of overestimation of the true variability of the integrated volatility when the ex post $RV_t^{(\tau)}$ is used in place of the latent $\sigma_{[a,b]}^{2(IV)}$	Index of overestimation of the true variability of the integrated volatility when the ex post $RV_{t(HL^*)}^{(\tau)}$ is used in place of the latent $\sigma_{[a,b]}^{2(IV)}$
AEX 25	1.212	1.109
CAC 40	1.143	1.081
DAX 30	1.221	1.163
DJ EURO STOXX 50	1.246	1.213
DJ STOXX 50	1.281	1.229
Dow Jones Ind	1.132	1.129
EURONEXT 100	1.129	1.062
FTSE100	1.330	1.314
FTSE EURO TOP 300	1.356	1.350
IBEX35	1.100	1.035
FTSE MIB INDEX	1.157	1.082
NASDAQ 100	1.106	1.064
RUSSELL 2000	1.222	1.140
SWISS MARKET INDEX	1.117	1.065
S&P 100	1.168	1.144
S&P400 MIDCAP	1.222	1.176
S&P 500	1.169	1.101

Table 4. Descriptive statistics of annualized one-trading-day inter-day adjusted realized daily standard deviations, $\sqrt{252 RV_{t(HL^*)}^{(\tau)}}$.

Index	Mean[1]	Median[1]	Maximum[1]	Minimum[1]	Std.Dev[1]	Skewness	Kurtosis
AEX 25	19.6	16.3	167.1	3.1	13.0	2.8	17.3
CAC 40	20.6	17.8	148.1	4.1	12.5	2.5	14.6
DAX 30	20.4	17.0	136.2	3.3	13.3	2.6	14.3
DJ EURO STOXX 50	18.6	15.4	173.9	2.8	12.7	3.1	23.2
DJ STOXX 50	16.6	13.8	165.9	2.6	11.1	3.0	22.1
Dow Jones Ind	13.7	11.7	133.8	3.3	8.5	3.5	25.7
EURONEXT 100	18.3	14.9	132.7	2.6	12.3	2.4	12.9
FTSE100	15.4	13.3	166.9	2.9	10.2	3.5	29.7
FTSE EURO TOP 300	14.6	12.1	151.1	1.3	10.2	3.1	23.6
IBEX35	19.4	17.2	153.3	3.1	11.9	2.4	16.2
FTSE MIB INDEX	18.9	16.0	128.1	4.3	12.3	2.5	14.7
NASDAQ 100	25.5	20.9	199.9	4.5	16.0	2.4	14.6
RUSSELL 2000	16.3	13.7	117.4	1.6	10.9	2.8	17.0
SWISS MARKET INDEX	17.4	15.0	126.1	7.4	9.0	2.9	18.7
S&P 100	14.0	11.7	206.5	2.6	9.5	4.7	55.3
S&P400 MIDCAP	15.9	13.2	113.3	1.3	10.5	3.0	17.4
S&P 500	12.9	10.7	185.3	1.8	9.3	4.8	50.4

[1]The numbers are expresses in percentages.

Table 5. Descriptive statistics of annualized inter-day adjusted realized daily logarithmic standard deviations, $\log \sqrt{252 RV_{t(HL^*)}^{(\tau)}}$.

Index	Mean	Median	Maximum	Minimum	Std.Dev	Skewness	Kurtosis
AEX 25	2.82	2.79	5.12	1.12	0.55	0.35	3.22
CAC 40	2.88	2.88	5.00	1.40	0.52	0.27	3.01
DAX 30	2.85	2.83	4.91	1.18	0.55	0.34	3.16
DJ EURO STOXX 50	2.75	2.74	5.16	1.04	0.57	0.23	3.05
DJ STOXX 50	2.64	2.62	5.11	0.97	0.56	0.26	3.00
Dow Jones Ind	2.49	2.46	4.90	1.19	0.48	0.56	3.71
EURONEXT 100	2.73	2.70	4.89	0.94	0.58	0.22	2.93
FTSE100	2.58	2.59	5.12	1.05	0.54	0.29	3.18
FTSE EURO TOP 300	2.51	2.50	5.02	0.26	0.58	0.19	3.17
IBEX35	2.81	2.84	5.03	1.12	0.56	0.00	2.85
FTSE MIB INDEX	2.78	2.77	4.85	1.45	0.55	0.32	2.79
NASDAQ 100	3.09	3.04	5.30	1.51	0.54	0.32	2.93
RUSSELL 2000	2.62	2.62	4.77	0.44	0.58	0.02	3.37
SWISS MARKET INDEX	2.76	2.70	4.84	2.00	0.41	0.84	3.72
S&P 100	2.50	2.46	5.33	0.94	0.51	0.55	3.68
S&P400 MIDCAP	2.61	2.58	4.73	0.25	0.53	0.36	3.65
S&P 500	2.39	2.37	5.22	0.59	0.54	0.43	3.88

Table 6. Descriptive statistics of daily log-returns, y_t .

Index	Mean[1]	Median[1]	Maximum[1]	Minimum[1]	Std.Dev[1]	Skewness	Kurtosis
AEX 25	-0.016	0.044	9.714	-8.818	1.6	-0.1	8.8
CAC 40	-0.019	0.007	10.507	-8.405	1.5	0.1	8.1
DAX 30	0.002	0.073	11.868	-10.340	1.6	0.1	8.7
DJ EURO STOXX 50	-0.019	0.013	10.663	-8.615	1.6	0.0	7.7
DJ STOXX 50	-0.020	0.018	9.259	-8.835	1.4	0.1	8.5
Dow Jones Ind	0.027	0.052	10.561	-8.591	1.1	-0.1	11.0
EURONEXT 100	-0.012	0.049	10.308	-8.948	1.5	0.0	8.7
FTSE100	0.002	0.040	9.485	-8.926	1.3	-0.1	8.2
FTSE EURO TOP 300	-0.013	0.039	9.644	-8.072	1.4	-0.1	8.6
IBEX35	0.002	0.073	13.143	-9.609	1.5	0.1	8.5
FTSE MIB INDEX	-0.030	0.040	9.896	-8.131	1.4	0.0	8.8
NASDAQ 100	0.029	0.108	17.243	-10.435	2.1	0.1	7.3
RUSSELL 2000	0.023	0.088	8.706	-12.465	1.5	-0.3	7.7
SWISS MARKET INDEX	-0.007	0.042	8.911	-7.797	1.3	-0.1	8.2
S&P 100	0.026	0.056	10.696	-23.668	1.2	-1.3	32.7
S&P400 MIDCAP	0.031	0.088	9.688	-11.620	1.5	-0.3	8.6
S&P 500	0.031	0.059	10.714	-22.926	1.2	-1.3	33.1

[1]The numbers are expresses in percentages.

Table 7. Descriptive statistics of standardized log-returns, standardized with the annualized one-trading-day inter-day adjusted realized standard deviation, $y_t \Big/ \sqrt{252 RV_{t(HL^*)}^{(\tau)}}$.

Index	Mean[1]	Median[1]	Maximum[1]	Minimum[1]	Std.Dev[1]	Skewness	Kurtosis
AEX 25	0.004	0.003	0.204	-0.177	0.07	0.06	2.61
CAC 40	0.003	0.001	0.189	-0.177	0.06	0.08	2.56
DAX 30	0.006	0.005	0.227	-0.190	0.07	0.09	2.60
DJ EURO STOXX 50	0.004	0.001	0.222	-0.193	0.07	0.07	2.49
DJ STOXX 50	0.004	0.001	0.203	-0.246	0.07	0.08	2.53
Dow Jones Ind	0.007	0.005	0.254	-0.207	0.07	0.07	2.82
EURONEXT 100	0.004	0.004	0.186	-0.224	0.07	0.02	2.42
FTSE100	0.005	0.003	0.236	-0.265	0.07	0.08	2.75
FTSE EURO TOP 300	0.006	0.004	0.246	-0.250	0.08	0.10	2.63
IBEX35	0.005	0.005	0.200	-0.180	0.07	0.03	2.48
FTSE MIB INDEX	0.004	0.003	0.197	-0.179	0.06	0.04	2.64
NASDAQ 100	0.006	0.006	0.249	-0.195	0.07	0.14	2.79
RUSSELL 2000	0.009	0.008	0.229	-0.212	0.08	0.03	2.24
SWISS MARKET INDEX	0.003	0.003	0.194	-0.179	0.06	0.02	2.81
S&P 100	0.006	0.005	0.219	-0.210	0.07	0.06	2.71
S&P400 MIDCAP	0.008	0.008	0.323	-0.227	0.08	0.06	2.46
S&P 500	0.007	0.007	0.283	-0.215	0.07	0.05	2.54

[1]The numbers are expresses in percentages.