The demand for euro banknotes in Germany: Structural modelling and forecasting

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Abstract

This paper explains and forecasts the demand for banknotes issued in Germany. For small and large denomination notes we estimate vector error correction models (VECM). The results suggest that the long-run demand for German small denomination notes is mainly driven by domestic transactions and demand from outside the euro area. The transaction motive in the rest of the euro area is part of the short-term dynamics. The long-run demand for German large denomination notes is mainly driven by foreign demand both from the rest of the euro area and outside the EMU. The global financial crisis led to a one-time increase in the (real) demand for these notes. Our results are in line with estimates according to which the level and dynamics of banknote demand are mainly determined by foreign demand. Additionally, we present RegARIMA models for the medium denominations as it was not possible to build a VECM for these denominations.

JEL Classification: C22, C32, E41

Keywords: banknotes, vector error correction, RegARIMA, forecasts

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1 Introduction[*]

Generally speaking, all euro-area national central banks issue euro banknotes. Following the introduction of euro cash at the start of 2002, the cumulated net issuance of euro notes by the Deutsche Bundesbank ("German" euro notes) increased from an initial €73 billion to €508 billion at the end of 2014. Figure 1 shows that the volume of these German euro banknotes outstanding has grown very much faster than could have been expected on the basis of earlier growth rates of D-Mark currency. For the first two years after the launch of euro cash, this strong growth could be explained by the need to replenish stocks of hoarded banknotes both inside and outside the euro area after the currency changeover. However, this should have ceased to have an effect at the end of 2003 when the volume of German banknotes outstanding returned to the hypothetical level that would have been reached had euro cash not been introduced. Nevertheless, the pace of growth in the volume of banknotes outstanding continued to be much more dynamic than in the D-Mark era in the 1990s. As shown in Bartzsch, Rösl and Seitz (2011a), this huge surge is due to foreign demand for euro banknotes. They find that, at the end of 2009, around 70% of the cumulated net issuance was held outside Germany. Of this, the lion's share (roughly 50%) was in non-euro-area countries, with the remainder in other euro-area countries. This also means that only a relatively small share – approximately 30% – was used for transaction purposes and hoarding in Germany.[1] In their opinion, 20% is a realistic figure for banknotes hoarded in Germany. Consequently, only around 10% were used for transaction purposes in Germany at the end of 2009. This was equivalent to around €430 *per capita* at the end of 2009.

While Bartzsch, Rösl and Seitz (2011a, b) have split up the cumulated net issuance of euro notes by the Deutsche Bundesbank into its components (transaction balance, hoarding and foreign demand), we want to further analyse the role of these underlying motives of banknote demand. These differ for the individual denominations. Therefore we estimate models of banknote demand for small, medium and large German euro banknotes. In these structural models, the demand for banknotes is explained by proxy

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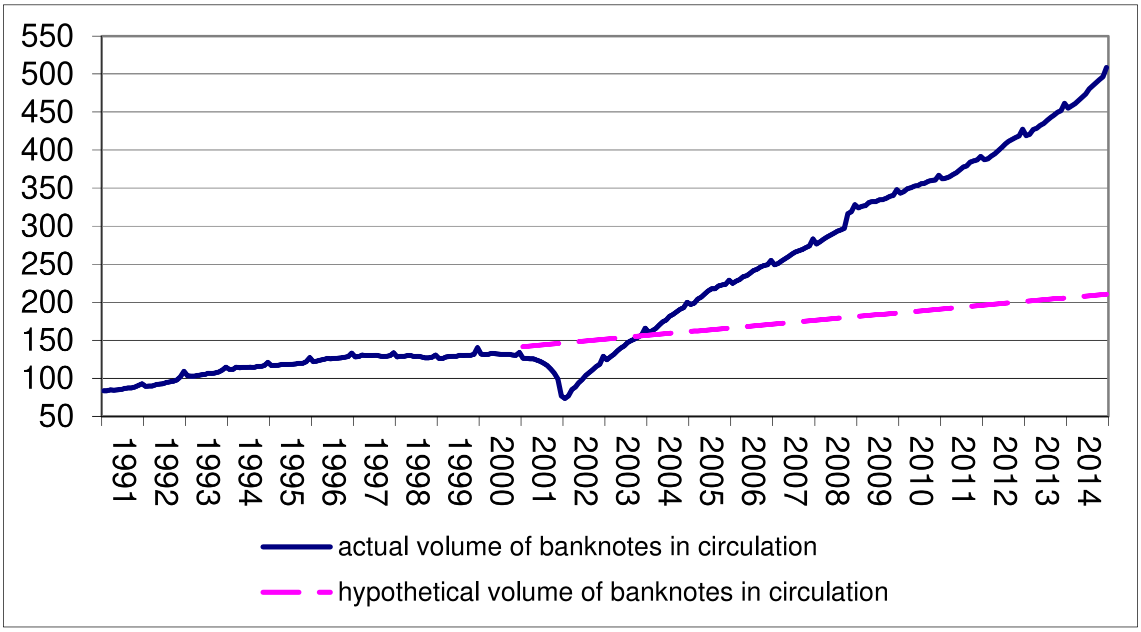
[1] Although the results of the indirect approaches are slightly higher than the figures obtained in the direct approaches (see Bartzsch, Rösl and Seitz, 2011b), the latter are largely confirmed. An update of the estimates in Bartzsch, Rösl and Seitz (2011b) is presented in chapter 3.

variables for the motives of holding banknotes. Amongst others, we estimate the interest (semi-)elasticities of banknote demand. This allows us to answer the question whether portfolio shifts from short-term bank deposits into cash are to be expected owing to the very low level of interest rates.

Figure 1: German banknotes in circulation (€ billion)



Note: The actual volume of banknotes in circulation in the period from January 1991 to December 2001 corresponds to the volume of D-Mark banknotes outstanding (converted to euros with the irrevocably fixed exchange rates of 1 January 1999) and, following the introduction of euro cash in January 2002, the volume of Bundesbank-issued euro banknotes outstanding. For the sake of simplicity, the volume of D-Mark banknotes outstanding in the period from January 1991 to December 2000, extrapolated using its linear trend, is taken as the hypothetical volume of banknotes in circulation as of January 2001.

Sources: Deutsche Bundesbank and the authors' own calculations.

This paper is closely related to Seitz and Setzer (2009) who also estimate structural models of the demand for German banknotes. With data available only up until the end of 2007 they use a mixed D-Mark/euro series from 1991 to 2007. By contrast, our models are estimated for the euro era only with data ranging from 2002 to 2011.

The paper is structured as follows. Chapter 2 contains a literature survey on banknote demand models. In chapter 3, some stylised facts are presented concerning the development and composition of banknotes in circulation in Germany and the rest of the euro area. The data we use to estimate banknote demand models are described in chapter 4. In chapter 5, we estimate structural models of banknote demand for small and large denominations. We have not been able to estimate a satisfying structural model for medium denominations. Instead, we present time series models which are used for forecasting. The results are summarised together with other conclusions in chapter 6.

2 Related literature

This literature survey focuses on more recent work since the beginning of the 2000s. For an overview of older papers on currency demand, see Boeschoten (1992, subsection 1.4.2).

Doyle (2000) estimates foreign demand for US, German and Swiss currency. He obtains higher shares than in previous studies. His results are based on currency demand equations within a cointegration framework. The cointegration space is made up of real currency balances, interest rates and retail sales. He uses retail sales as transaction variable owing to their monthly availability and because they are relevant to currency. Doyle obtains different results when he splits up currency into large and small-value denominations. The signs and significance of the coefficients are unchanged with just large denomination notes. By contrast, when only small-denomination notes are used, those same coefficients tend to be insignificant or have the wrong signs. These smaller notes are the ones that Doyle expects will more likely be used in the legitimate US economy. Therefore, he concludes that this result might indicate an invalidation of traditional explanations of currency demand or simply movements away from small to large notes (in real terms), or from cash to other instruments as incomes have risen.

Khamis and Leone (2001) find strong evidence that real currency demand in Mexico remained stable throughout and after the financial crisis in Mexico which started at the end of 1994. They find a strong cointegration relationship between currency balances, private consumption expenditures and the interest rate. The dynamic model also exhibits significant parameter constancy even after the financial crisis. The sample period from 1983 to 1997 includes the inflationary debt crisis period, the stabilisation period under the 1987 stabilisation plan, the ensuing financial crisis in December 1994 and the recovery period thereafter. The paper concludes that the significant reduction in currency demand in the course of the financial crisis can be appropriately explained by a change in the variables that historically explain cash demand in Mexico quite well.

Akinci (2003) models currency in circulation in Turkey using data between 1987 and 2003. Cointegration analysis reveals that there is a long-run relationship between currency issued, private consumption, interest rates and a bilateral exchange rate. The results reveal that economic agents are more sensitive to interest rate movements than to exchange rate movements in the long run. The exchange rate elasticity is more effective in the short run. This indicates that the exchange rate might be a powerful indicator in terms of capturing the dynamics of the demand for cash. Moreover, the high and significant exchange rate coefficient is consistent with the existence of currency substitution in Turkey. In the long run, however, real income and the interest rate variables appear to be the main determinants of the demand for cash balances.

Amromin and Chakravorti (2009) analyse cash demand for 13 advanced economies from 1988 to 2003 with panel regressions by separating cash into three denomination categories to disentangle its store of value and payment functions. They isolate the transactional role of cash by focusing on the small denomination class, which they define as banknotes (including coins) that are lower in value than those which are commonly dispensed by ATMs. Amromin and Chakravorti econometrically test a money demand equation where the currency-to-GDP ratio is a function of alternative payment infrastructure, cash infrastructure, proportion of small merchants, and the opportunity cost of cash. They also report results for the aggregate currency-to-GDP ratio. The substitution effects with respect to electronic payments are largely confined to the demand for small denominations. Moreover, they find that the demand for small denomination currency is not affected by changes in the interest rate. By contrast, the interest rate sensitivity of the demand for high denomination notes decreases as interest rates rise but is generally unaffected by changes in debit card usage. The interest rate sensitivity of demand for high-denomination notes is especially high in countries that do not have significant proportions of their currency stock circulating outside their borders. This suggests a persistent role for cash as a store of wealth.

Nachane, Chakraborty, Mitra and Bordoloi (2013) identify various factors influencing currency demand in India from 1989 to 2011 in a vector error correction framework for aggregate currency demand as well as for various currency sub-groups. They argue that the homogeneity postulate with respect to prices might be too restrictive. Hence, they model currency in nominal terms, using wholesale prices as price measure. The trends in currency in circulation at the individual denomination level show considerable fluctuations, in particular, which renders econometric modelling a complex task. However, there exists a cointegrating relationship between (total) currency circulation, real GDP, prices and deposit rates. The income elasticity of currency is found to be somewhat higher than is observed in similar studies for advanced countries.

Cusbert and Rohling (2013) analyse the strong increase in the demand for currency in Australia which began in mid-October 2008, around four weeks after the collapse of Lehman Brothers and concurrently with policy responses of the Reserve Bank of Australia and the Federal Government. They attempt to capture the effects of the global financial crisis on currency demand in three ways. First, they add dummy variables for the three quarters from December 2008 to June 2009 to their baseline model. Second, they introduce confidence, financial market and wealth variables to their model. They expect increases in the stock of currency to be associated with declines in confidence and wealth and rises in financial volatility. Finally, they examine whether these variables retain any explanatory power in the presence of dummy variables. In their baseline model, currency in circulation is modelled in a single equation error correction framework to exploit the possible cointegration between currency holdings, nominal GDP and interest rates. They also include ATMs, EFTPOS terminals, bank branches per capita and the ratio of self-employed to total employment in the long-run relationship. They estimate the model using data from 1993 to 2011 and find that only around 20% of the rise in Australian currency demand during the financial crisis can be attributed to the normal response of currency holdings to the lowering of interest rates and to the increase in income from the government stimulus. The remaining 80% may be due to an increase in precautionary holdings in response to financial market uncertainty, which is consistent with the larger increase in demand for high-denomination banknotes. In addition, Cusbert and Rohling estimate separate models of currency demand of the bank and non-bank sector for different denominations. The interest coefficients are broadly consistent with the idea that demand for larger denominations should be more sensitive to deposit rates. The insignificance of the financial crisis dummy variables in the low-denomination regression confirms that only larger denominations were behaving unusually in this period.

Besides these time series models there is one paper which estimates currency demand using micro data. *Briglevics and Schuh (2014)* investigate US consumer demand for cash using panel micro data for 2008–2010 with a special emphasis on the role of low interest rates and different kinds of credit cards. They find that cash demand by consumers using credit cards for convenience is much more interest elastic than those using credit cards to borrow. These findings may have implications for the welfare cost of inflation because consumers who revolve credit card debt are less likely to switch from cash to credit.

There are only two papers which concentrate on the euro area. *Fischer, Köhler and Seitz (2004)* analyse currency in circulation in the euro area since the beginning of the 1980s. They develop a theoretical model which extends traditional money demand models to also incorporate arguments for the informal economy and foreign demand for specific currencies. In the empirical part, they estimate the total demand for euro legacy currencies and for small and large denominations within a vector error correction framework. They find significant differences between the determinants of holdings of small and large denominations as well as overall currency demand. While the long-run demand for small-value banknotes is mainly driven by domestic transactions, the demand for large-value banknotes in the cointegrating relation depends on a short-term interest rate, the exchange rate of the euro as a proxy for foreign demand and inflation variability. Therefore, large value banknotes seem to be used to a large extent as a store of value both domestically and abroad.

An approach similar to that of *Fischer, Köhler and Seitz (2004)* is taken by *Seitz and Setzer (2009)*. They estimate the demand for small, medium and large denominations of German banknotes in a vector error correction framework for the period from the first quarter of 1991 to the fourth quarter of 2007. These comprise D-Mark banknotes and euro banknotes which were put into circulation by the Deutsche Bundesbank. They include the DM period, as the time series for the euro era alone was too short at that time. In the case of small and medium denominations, what stands out in the results is the obvious impact of the transaction volume. The large denominations, by contrast, appear to be unaffected by this. In their case, however, non-resident motives are important: first, via a long-term impact of the house prices in the euro area, whose dynamics are determined mainly by the real estate market outside Germany, and second, via private consumption in the euro area excluding Germany. The importance of German transactions in terms of the demand for large denominations is noticeable, at most, in the short-term impact of unemployment. Additionally, demand from non-euro-area countries is important for all denominations. Moreover, an influence of the shadow economy on banknote demand cannot be ruled out for any of the three banknote categories. Finally, opportunity costs in the form of interest rates seem to be of relevance only for the small denominations. Alternative means of payment (especially card payments) evidently influence only the small denominations, too. The error correction term indicates the fastest adjustment for the small denominations and the longest for the large denominations.

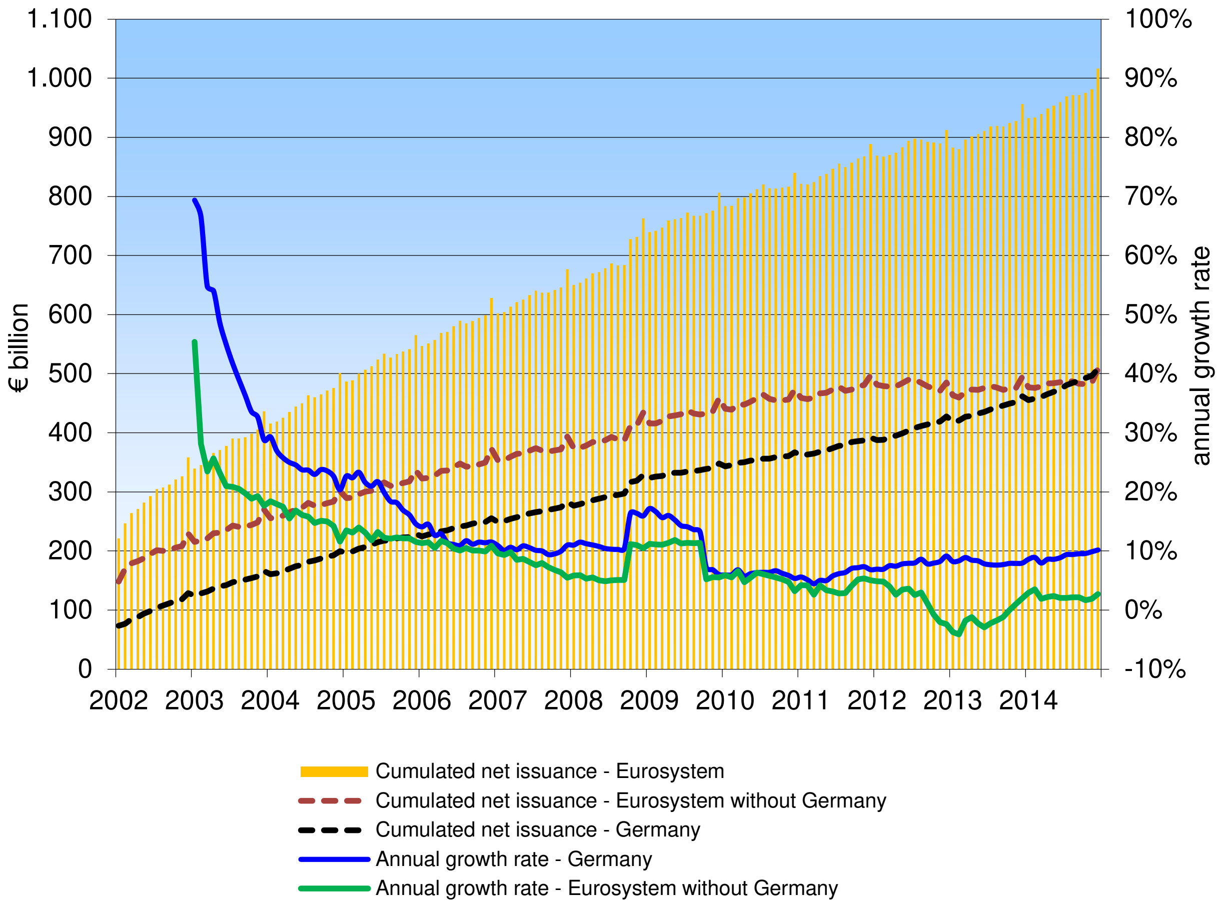
Our paper is the first of its kind to analyse cash demand using genuine euro data since 2002.

3 Stylised facts

In this chapter we present a number of stylised facts about the cumulated net issuance of euro banknotes by the Deutsche Bundesbank ("German" euro banknotes in circulation) and the Eurosystem. The development of euro banknotes in circulation is shown in figure 2. After the euro cash changeover at the beginning of 2002, the cumulated net issuance of German euro banknotes increased from an initial €73 billion to €508 billion (10.7 billion notes) at the end of 2014. This corresponds to an average annual growth rate of more than 16%. In the euro area, banknotes in circulation increased during the same period from an initial €221 billion to €1,017 billion (17.5 billion notes). Thus, the German share in the value of the cumulated net issuance of euro banknotes has increased from 33% to 50% since the euro cash changeover. This share is clearly above Germany's share in the ECB capital of about 25.7%, which is determined on the basis of the size of its population and its GDP.

The vast growth in German euro banknotes in circulation up until the end of 2003 can be attributed to the replenishment of stocks of hoarded banknotes both inside and outside the euro area (see figure 1). From 2004, the growth rates of banknotes in circulation began to decline steadily. In 2006, they stabilised at a level of about 10%. In the wake of the financial crisis, the US investment bank Lehman Brothers was forced to file for bankruptcy in September 2008. This represented a break with the existing "too big to fail" principle and greatly unsettled the financial markets. German households reacted to this in the fourth quarter of 2008 by making considerable shifts in their financial investment (see Deutsche Bundesbank 2009, p 52 f). This led to sharp inflows into liquid and (relatively) secure short-term types of investment, which also boosted the demand for cash. As a result, the German net issuance of banknotes rose by €16 billion in October 2008. In that month alone, the annual growth rate of the cumulated net issuance of German euro banknotes increased by six percentage points.

Figure 2: Euro banknotes in circulation



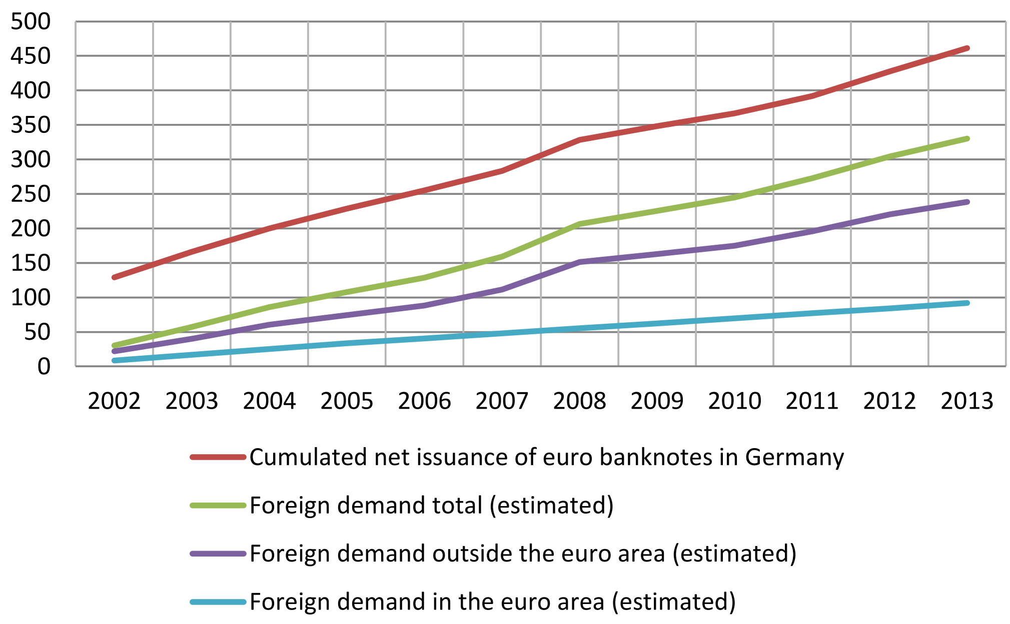
Note: Left scale: Cumulated net issuance; right scale: annual growth rate.

Sources: Deutsche Bundesbank and ECB.

As can be seen in figure 2, the hoards of "German" euro banknotes resulting from the crisis were partially reduced in the course of 2009 in the sense that the growth rates in Germany developed more in line with those in the rest of the euro area.

Since the beginning of 2012, the volume of "German" euro banknotes in circulation, which has seen annual growth rates of between 7% and 10%, has again been showing much stronger growth than the circulation of banknotes issued by other Eurosystem member states (see figure 2). Some euro-area countries have even recorded negative rates of increase. These differences in the development of euro banknotes in circulation can be explained by the large share of the Bundesbank's cumulated net issuance of euro banknotes in circulation outside Germany. A number of different approaches can be used to measure the foreign demand for "German" euro banknotes (see Bartzsch et al, 2011a, b). In this connection, the regional distribution of the cumulated net issuance is determined using the "net shipments and foreign travel" approach (Bartzsch et al, 2011b, section 3.1). The volume of "German" euro banknotes in circulation abroad is estimated using data collected as part of a household survey by the Bundesbank on foreign travel as well as available data on net shipments of euro banknotes by banks (international foreign currency traders) to countries outside the euro area. These net shipments correspond to the difference between the outpayments by the Bundesbank to international foreign currency traders and the inpayments by the international foreign currency traders at the Bundesbank. The estimated regional distribution of euro banknotes issued in Germany estimated using this approach can be seen in figures 3 and 4.

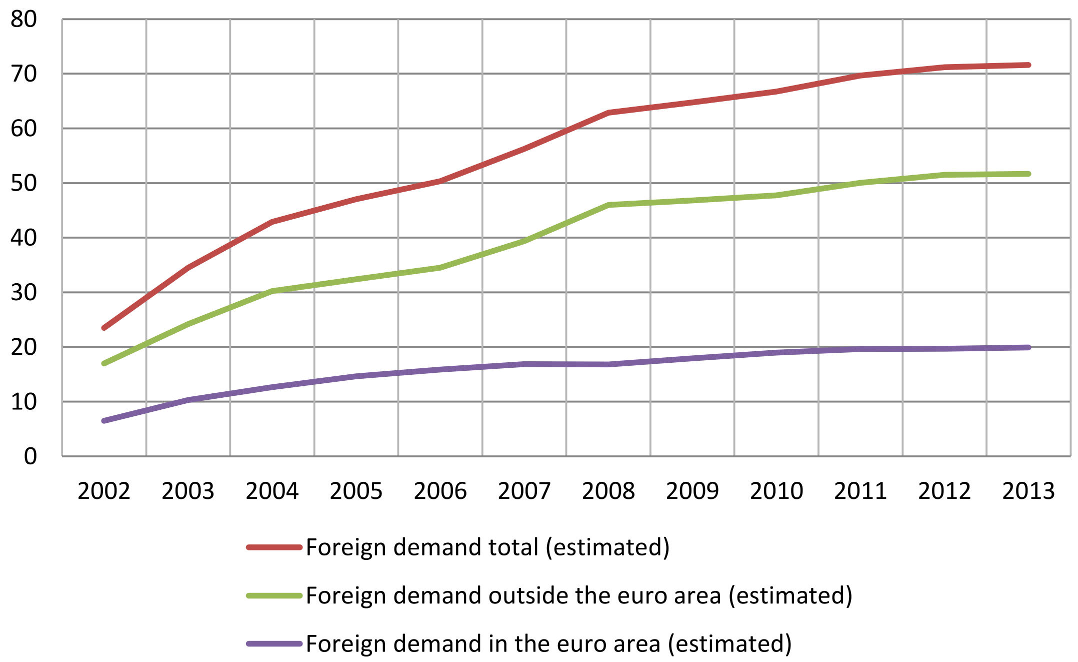
Figure 3: Regional distribution of euro banknotes issued in Germany (€ billion)



Sources: Deutsche Bundesbank and authors' own calculations.

At the end of 2013, the largest share of the Bundesbank's cumulated net issuance in the amount of just over €460 billion was accounted for by banknotes in circulation abroad (€330 billion or just over 70% of the cumulated net issuance), with the lion's share in circulation outside the euro area (just under €240 billion, or just over 50% of the cumulated net issuance). Germany was, however, also a major net exporter of euro banknotes - especially via foreign travel - to the rest of the euro area (just over €90 billion, or 20% of the cumulated net issuance). The banknotes in circulation outside the euro area can be attributed to foreign travel and to the net shipments, with the latter (cumulated) accounting for the greatest share (just over €140 billion, or just over 30%) of the cumulated German net issuance. In summary, the growth in the cumulated net issuance of banknotes in Germany can be primarily explained by the volume of German-issued euro banknotes in circulation abroad (foreign demand), whereas the domestic demand for banknotes (for transaction and hoarding purposes) remains largely constant and therefore does not make a notable contribution to the growth in the cumulated net issuance. The percentage share of the Bundesbank's cumulated net issuance of euro banknotes in circulation abroad has therefore increased significantly since the introduction of euro cash.

Figure 4: Regional distribution of euro banknotes issued in Germany

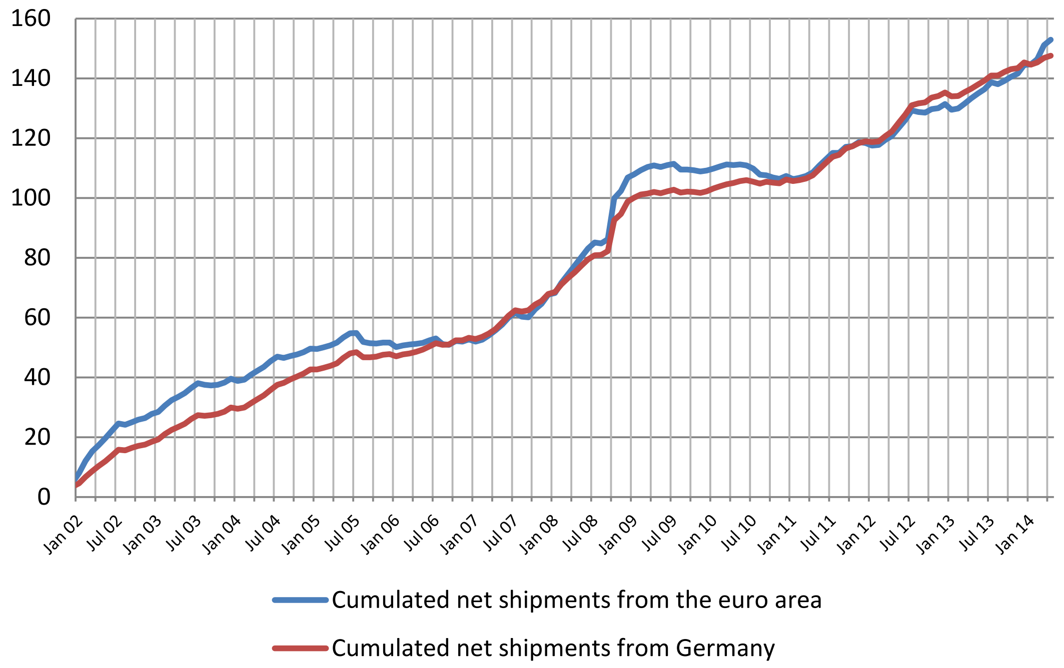


Note: Percentage shares in the cumulated net issuance.
Sources: Deutsche Bundesbank and authors' own calculations.

To complete the picture, figure 5 shows the cumulated net shipments from Germany and from the Eurosystem as a whole. Both time series lie close together over the entire time horizon and have been virtually congruent since the end of 2010. In other words, virtually all of the Eurosystem's total cumulated net issuance originates from Germany on balance. This can be explained, in part, by Germany's long-standing strong involvement since the D-Mark era in the international wholesale banknote market and its central geographical location and also its proximity to Frankfurt airport.

In addition to the cumulated net shipments, the (total) volume of euro banknotes in circulation outside the euro area is fed by other channels, such as foreign travel or cash sent home by foreign workers. The cumulated net shipments therefore only represent a lower limit. According to estimates by the ECB, around 25% of all euro banknotes issued by the Eurosystem are outside the euro area (ECB, 2014, p 23). At the end of 2013, this was equivalent to just under €240 billion. This corresponds exactly to the estimated value of the cumulated net issuance of "German" euro banknotes in circulation outside the euro area (see figure 3).

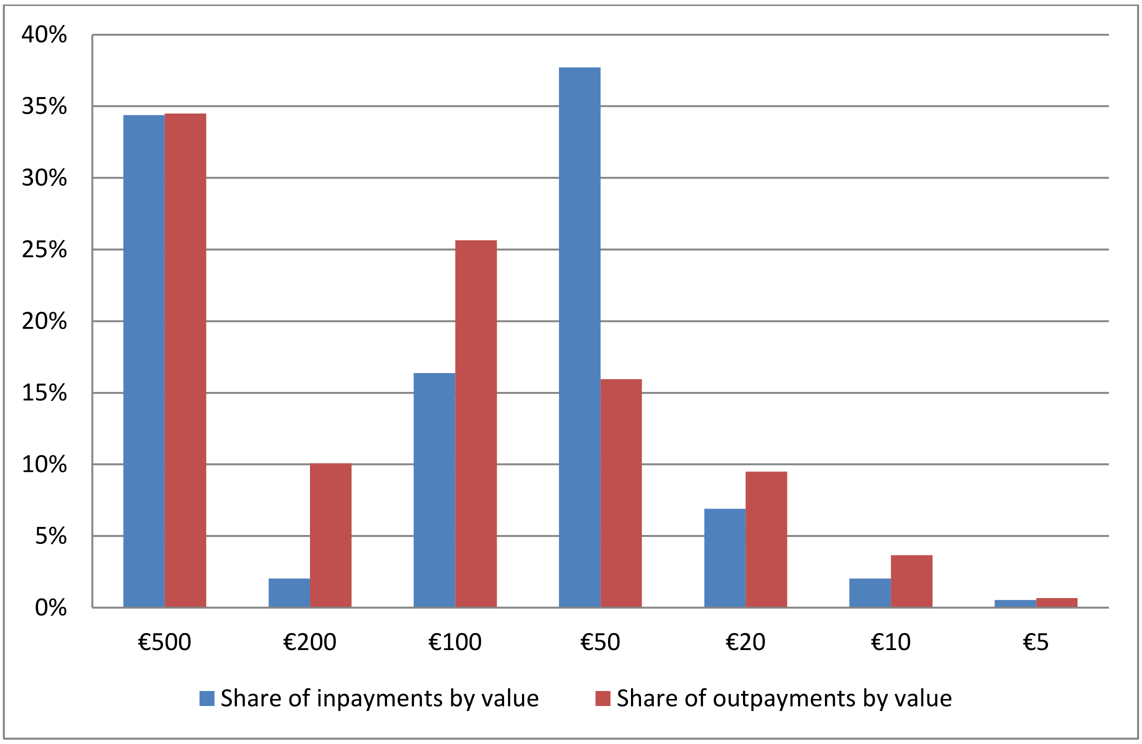
Figure 5: Cumulated net shipments of euro banknotes from Germany and the rest of the euro area (€ billion)



Sources: Deutsche Bundesbank and ECB.

In summary, the dynamic development of the cumulated net issuance of euro banknotes by the Bundesbank - unlike those issued by other Eurosystem member states - can be explained as follows. As shown in figure 5, practically the entire volume of euro banknotes in circulation abroad issued by the Eurosystem originates from Germany. Furthermore, Germany is a major (net) exporter of euro banknotes to other euro-area countries. Moreover, the growth in the cumulated net issuance in Germany is almost exclusively driven by foreign demand. In the meantime, foreign demand is presumably also the greatest driving force behind the development of banknotes in circulation for the Eurosystem as a whole.[2] In Germany's euro-area partner countries, the volumes of banknotes held for transaction purposes are likely to have grown less strongly owing to the subdued economic growth and the generally lesser significance of cash as a means of payment.

Figure 6: German shipments in 2013 by denomination



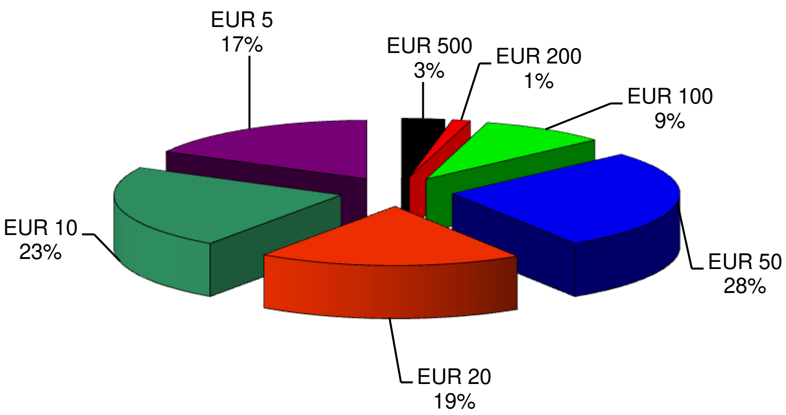
Source: Deutsche Bundesbank.

The lion's share of euro banknotes in circulation outside the euro area is presumably not used for transaction purposes, but is hoarded. Bartzsch et al (2011b, section 3.4)

[2] For further information on the stronger growth in the foreign demand for euro banknotes in recent years, see ECB (2014).

estimate that stocks of hoarded banknotes account for 70% of the total volume of "German" euro banknotes in circulation outside the euro area. This hypothesis is also supported by the breakdown (by denomination) of the shipments from Germany (by value). Figure 6 illustrates this by way of example for 2013, which is when inpayments at central banks, ie the purchases by wholesale currency banks, started being recorded by denomination.

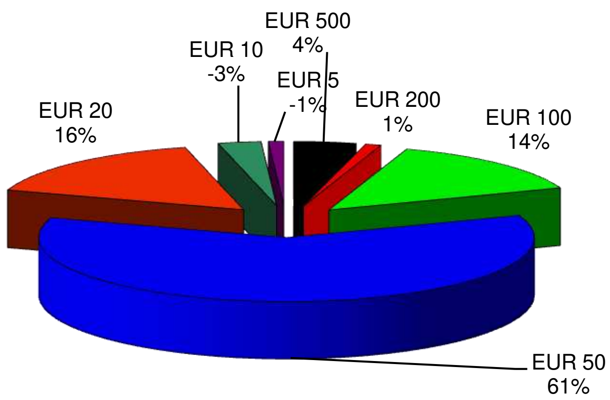
Figure 7: Denominational structure of the number of euro banknotes put into circulation by the Deutsche Bundesbank



Note: Percentage shares in the cumulated net issuance as at 15 June 2014.

Source: Deutsche Bundesbank.

Figure 8: Denominational structure of the number of euro banknotes put into circulation by the Eurosystem without Germany

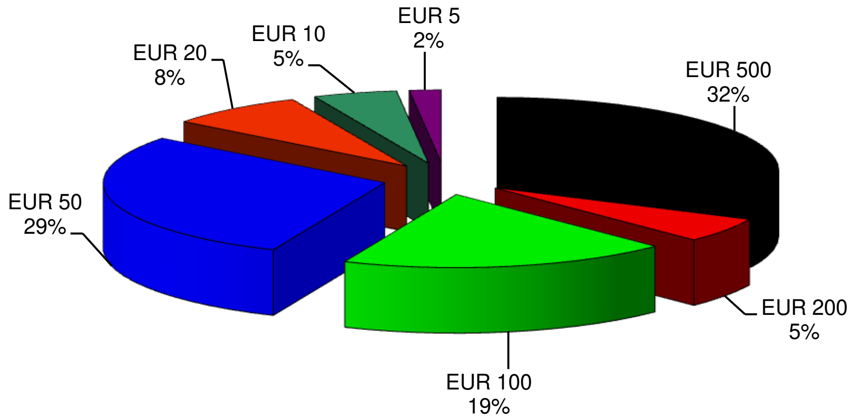


Note: Percentage shares in the cumulated net issuance as at 15 June 2014.

Source: ECB.

With regard to outpayments and inpayments, the bulk of these banknotes is accounted for by the €500, €100 and €50 denominations. The €500 and €100 banknotes are denominations that are typically used for hoarding, whereas the €50 note is used for both hoarding and transaction purposes.

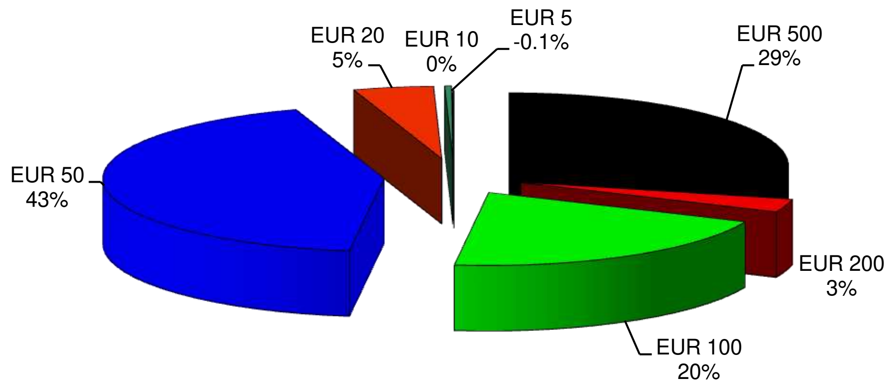
Figure 9: Denominational structure of the value of euro banknotes put into circulation by the Deutsche Bundesbank



Note: Percentage shares in the cumulated net issuance as at 15 June 2014.

Source: Deutsche Bundesbank.

Figure 10: Denominational structure of the value of euro banknotes put into circulation by the Eurosystem without Germany



Note: Percentage shares in the cumulated net issuance as at 15 June 2014.

Source: ECB.

The breakdown by denomination of the number of banknotes put into circulation is shown in figure 7 for the Bundesbank and in figure 8 for the Eurosystem without Germany. It is striking that the share of €5 notes and €10 notes for Germany (together 40%) is quite large, whereas it is negative for the rest of the euro area.[3] This means that the demand for these denominations is completely met by the Deutsche Bundesbank. Moreover, the share of €50 notes is much higher in the Eurosystem without Germany (61% compared with 28%). As can be seen in figures 9 and 10, these differences also hold for the composition of the value of banknotes put into circulation by denomination, albeit to a lesser degree.

4 Data and determinants of banknote holdings

The main objective of this section is to explain the data used in the econometric analysis and the determinants of the demand for German euro notes in circulation. The procedure is eclectic to the extent that we use a large set of variables which reflect the various motives for holding banknotes and which we test for statistical significance. This approach is followed by a look at limited data availability - a factor which is mainly related to the characteristic feature of banknotes, their anonymity.

In total, **we identify five different purposes of holding cash (or banknote) balances**. These are 1) transaction balances, 2) store of wealth (and, in this connection, opportunity cost) considerations, 3) the availability of alternative means of payment, 4) the size of the shadow economy and 5) demand by non-residents. In the following paragraphs, we describe the coding of these variables in the empirical analysis.[4]

In terms of the **transaction variable**, it would be optimal to include a variable capturing all cash transactions (Snellman and Vesala, 1999; Snellman, Vesala and Humphrey, 2000). Since no data are available on the number of cash transactions in Germany, one solution is to resort to total private consumption, retail sales or GDP as is the case in conventional money demand studies. This is, however, only a rough proxy given the large number of cashless transactions in an economy. Therefore, we additionally construct a variable based on those components of domestic private consumption which

[3] The percentage share is negative if the cumulated *net* issuance of the respective denomination is negative.

[4] Additional purposes and proxy variables can be found in Seitz and Setzer (2009).

are primarily carried out in cash (real “cash consumption”, *ccr*). These include 1) accommodation and hospitality services, 2) clothing and footwear, 3) leisure, entertainment and culture, 4) food and beverages, as well as 5) other purposes, such as body care and personal articles.

In addition to its function as a payment medium, cash also serves as a **store of value**. This is the case in particular for high-value, and to a certain extent, also for medium-value banknotes. Since cash bears no interest, interest rate levels can be used as an opportunity cost measure for holding cash.[5] Appropriate choices include the three-month money market rate and the ten-year government bond yield, both for Germany as well as for the global economy (measured by the GDP weighted average of the interest rates in the euro area, the US, the UK, Japan and Canada). Following Friedman (1977), we also include a measure of the whole term structure of interest rates as estimated, for instance, by the Nelson-Siegel-Svensson method, see Deutsche Bundesbank (1997) for details. The term structure of interest rates provides a precise measure of expectations in the money and bond markets. Its pattern can thus provide information about expected changes in interest rates or inflation - both variables which are directly related to the opportunity costs of cash holdings. Moreover, this procedure circumvents multicollinearity problems when taking more than one interest rate into account in empirical applications. Friedman argues that a demand-for-currency equation should include the key characteristics of the whole structure of yields: the “general” level, the “tilt” of the term structure to maturity and the difference between real and nominal yields. A steepening of the tilt of the term spread with an unchanged mean, for example, which implies higher long-term rates and lower short-term rates, will tend to reduce cash balances, and vice versa (Friedman, 1977, p 408).[6] The formula used for estimating the term structure specifies the interest rate as the sum of a constant and various exponential terms and reads as in Deutsche Bundesbank (1997, p 63f)

[5] In the Baumol-Tobin model (Baumol, 1952; Tobin, 1956), an inclusion of interest rates may also be rationalised by transaction demand, see Alvarez and Lippi (2007) for a modern version of this model.

[6] An empirical implementation of Friedman’s proposal within a money demand framework can be found in Friedman and Schwartz (1982) for the US and in Seitz (1998) for Germany.

$$i(T,\beta) = \beta_0 + \beta_1 \left(\frac{1-\exp(-T/\tau_1)}{(T/\tau_1)} \right) + \beta_2 \left(\frac{1-\exp(-T/\tau_1)}{(T/\tau_1)} - \exp(-T/\tau_1) \right)$$
$$+\beta_3 \left(\frac{1-\exp(-T/\tau_2)}{(T/\tau_2)} - \exp(-T/\tau_2) \right).$$

Here, *i(T, β)* denotes the interest rate for maturity *T* as a function of the parameter vector *β*. $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2$ are the parameters to be estimated. We include β_0 as a measure of the complete interest range into the analysis (*int*). It may be interpreted as a shift parameter to represent the generally prevailing interest rate level. An increase in this parameter means that the entire interest range shifts upwards.

Closely related to opportunity costs of holding banknotes are **alternative payment media**. In addition to the pressure from existing means of payment (eg, debit and credit cards), cash faces increasing competition from new payment instruments, such as contactless payment facilities in the retail trade, new payment procedures for internet purchases and the use of mobile phones.[7] While new payment opportunities may reduce the use of cash, their overall distribution is still negligible. Moreover, they will also compete with existing non-cash payment procedures. We therefore refrain from including a variable for these innovative means of payment.[8] Deutsche Bundesbank (2012b) investigates how payment behaviour in Germany has changed in recent years. This study is based on survey data. According to the study, individuals use cash for 53% of total expenditure (excluding regularly recurring payments such as rent). Although its share has fallen by almost 5 percentage points since 2008, it is still the most-used payment instrument in Germany. Debit cards are still the most commonly used cashless payment instrument. Their share in terms of turnover has increased by almost 3 percentage points to 28%. While cash is still the preferred method of payment for small purchases, cashless payments are mainly used to pay for high-value items. In principle, one would expect a negative impact on currency demand from card payments given that bank and credit cards provide a substitute for cash payments. For example, Amromin and Chakravorti (2009) found evidence that the demand for low denomination notes decreased with increasing debit card usage in the OECD countries during the period from 1988 to 2003. However, bank cards are also used to withdraw money from ATMs

[7] An overview of innovative payment instruments can be found in Deutsche Bundesbank (2012a).
[8] Generally, a time trend could be used as a crude proxy for the process of financial innovation.

and could thus increase currency in circulation. As a result, the effect of technological innovation on currency demand is ambiguous. In order to capture the increasing preference for cashless payments, we use the number of card payments. However, this measure is only available on an annual basis. For our analysis we converted it to a quarterly frequency using the quadratic (match sum) method.

Shadow economy transactions are often undertaken in the form of cash payments due to their anonymity (Schneider 2002). A rise in the size of the shadow economy should therefore increase the demand for currency. We proxy this influence by taking into account the share of the shadow economy in GDP.[9] However, this variable is not directly observable and can only be estimated with considerable uncertainty. Therefore, we also use a variable that may cause hidden economy transactions. The unemployment rate is expected to have a positive impact on the hidden economy (and thus on currency demand) since a high unemployment rate encourages people to work "underground".

The cumulated net issuance of euro banknotes put into circulation by the Bundesbank differs considerably from the domestic holdings of banknotes. Due to large inflows and outflows between countries, net issuance may systematically differ from the demand for cash within the economy. The foreign demand for banknotes issued in Germany can be divided in two groups: first, there is **the demand for German banknotes resulting from residents of other euro-area countries**. This is because German banknotes are perfect substitutes for other euro-area national issuances. In other words, the demand for cash in one euro-area country may be satisfied in part by inflows of cash coming from another member state. The transaction related part of this foreign demand is taken into account via house prices (*house*) and real private consumption (*diff_pc*) in the euro area without Germany in each case. The former are likely to be a good proxy for the preference for cash payments, as real property purchases are often made in cash. The ECB house price indicator for the euro area excluding Germany is chosen as the variable for capturing this effect. **The second category of foreign demand is demand from outside the euro area**. As shown in section 3, a significant portion of the demand for German euro banknotes stems from outside the currency union. In the absence of a

[9] We thank Friedrich Schneider for the provision of this time series on shadow economic activities in Germany. Since this time series has a yearly frequency, we converted it to a quarterly frequency using the quadratic (match average) method.

variable which directly indicates this demand from many different foreign countries, we proxy it with the euro exchange rate (see also Fischer et al, 2004; Seitz, 1995). An appreciating euro should be associated with a higher attractiveness and thus a higher euro demand from non-euro-area residents. As mentioned in section 3, those euro banknotes in circulation outside the euro area are presumably held, first and foremost, for hoarding purposes, ie utilised as a store of value. We use the real effective external value of the euro vis-à-vis the 12 as well as vis-à-vis the 20 most important trading partners (*er12* and *er20*).

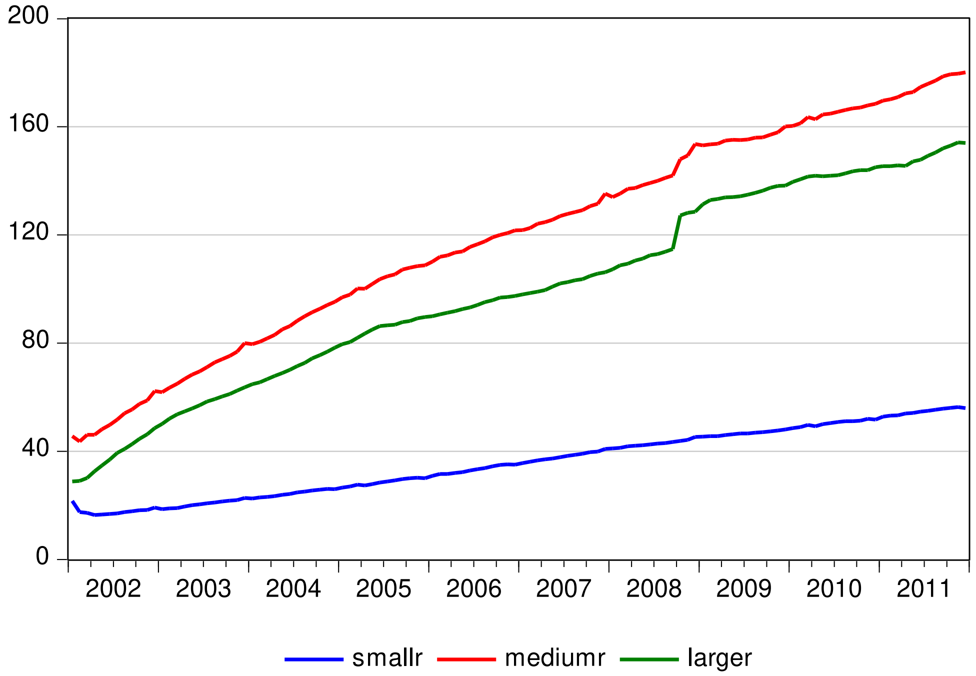
It is implausible to assume that the coefficients of the variables determining the demand for banknotes are the same for all denominations. For example, the transaction motive should be more important for small and medium-value banknotes. By contrast, store of wealth considerations may dominate with respect to high-value banknotes. At the same time, substitution effects may exist between banknotes of similar value. Therefore, we estimate three separate relations, one for small (*small*), one for medium (*medium*) and one for large (*large*) denomination German euro notes in circulation. Our preferred classification is €5 - €20 for "small" notes, €50 - €100 for "medium" notes and €200 - €500 for "large" notes. This classification is chosen because the large notes are not distributed by ATMs, which primarily serve to "top up" transaction balances.[10] Moreover, the €50 banknote should be the smallest denomination that is used (amongst other things) for hoarding purposes. We also include DM notes that are still in circulation as these can still be exchanged (on an unlimited basis) for euro notes at the Deutsche Bundesbank.[11] To do this in a meaningful way, the DM banknotes are converted into euro with the official conversion rate of €1 = DM1.95583. The application of the above classification to DM notes is facilitated by the fact that they have a similar denominational structure as euro notes. Therefore, *small* includes DM5 - DM50 notes, *medium* comprises DM100 and DM200 notes and *large* includes DM500 and DM1,000 notes. We estimate specifications in real terms (*r*). For the *small* and

[10] For a similar classification scheme to isolate transactional and store of wealth roles of currency in a multi-country study, see Amromin and Chakravorti (2009). They select the medium-note category by determining which denomination is prevalently distributed by ATMs. Denominations above this threshold are categorised as "large" while those below this threshold (including coins) are categorised as "small".

[11] The circulation of small, medium and large DM notes amounted to around €3 billion to €9 billion at the end of January 2002. It declined to around €1 billion to €2 billion at the end of January 2003. This is negligible when compared with the total circulation of German notes (DM and euro) (see figure 11).

medium categories we choose the price index of domestic cash consumption of households as price deflator and for the *large* category we use the price index of domestic consumption expenditures of households.[12] This means that we assume long-run price homogeneity to hold. The data are quarterly and (if necessary) seasonally adjusted (*sa*). Our sample covers the period from the first quarter of 2002 to the fourth quarter of 2011. In our view, the inclusion of data from 2012 and 2013 would not have a substantial impact on the results. Therefore, we have not updated our dataset. When using interest rates, we work with a semi-log specification. All other variables are in logarithms. The difference operator "d(…)" refers to the first (quarterly) difference. The three cash variables are shown in figure 11.

Figure 11: Small, medium and large value denominations, in real terms (€ billion)



Source: Deutsche Bundesbank.

5 Estimating the demand for banknotes

Our empirical approach relies on a vector error correction model. We use two kinds of unit root tests: the augmented Dickey-Fuller test (ADF) and the Zivot-Andrews test. In both of these tests the null hypothesis is that the series has a unit root, ie is I(1) in levels

[12] Taking the price index of domestic cash consumption of households as a deflator for the large category does not change the results.

and I(0) in first differences. We employ the Zivot-Andrews test when we presume structural breaks. For example, owing to the financial crisis there is a break in the intercept at the end of 2008 in *mediumr* and *larger* (see figure 11). Table 1 shows the results of the unit root test for the variables that we employ in our final specifications.

Table 1: Unit root tests

	ADF test statistic	Test specification[a)]	Zivot-Andrews test statistic	Test specification[b)]	Conclusion
smallr_sa	-2.26	C, T, 5			I(1)
d(*smallr_sa*)	-7.11***	C, 2			
mediumr_sa			-6.50***	C, 2	I(1) instead of trend-stationary
larger_sa			-8.53***	C, 1	I(1) instead of trend-stationary
ccr_sa	-2.66	C, T, 0			I(1)
d(*ccr_sa*)	-6.97***	C, 0			
int	-1.55	C, T, 6			I(1)
d(*int*)	-2.90*	C, 6			
er20	-2.87*	C, 0			I(1)
d(*er20*)	-4.61***	0			
er12	-2.72*	C, 0			I(1)
d(*er12*)	-4.35***	0			
house			-4.17	C and T, 1	I(1) instead of I(2)[c)]
d(*house*)			-3.22	C, 2	
d^2(*house*)			-8.55***	C, 2	
diff_pcr_sa			-3.57	C and T, 1	I(1) instead of I(2)[c)]
d(*diff_pcr_sa*)			-3.72	C, 0	
d^2(*diff_pcr_sa*)			-7.73***	C, 0	

Notes: ***/ **/ *: significant at the 1, 5 and 10 per cent level, respectively. The ADF test refers to the critical values in MacKinnon (1996). The Zivot-Andrews test refers to the critical values in Andrews and Zivot (1992). Sample: 2002 Q1 to 2011 Q4.
a) C: intercept, T: linear trend; 0, 1, 2, 3, …, 6: number of lags
b) C: break in intercept, T: break in linear trend, C and T: break in intercept and linear trend; 0, 1, 2: number of lags
c) Test is biased owing to the financial crisis ("bump" in 2008).

Source: Authors' own calculations.

As expected, the value of small denomination banknotes in circulation (*smallr_sa*), the real effective exchange rates (*er20*, *er12*) and cash consumption (*ccr_sa*) are clearly I(1). The Zivot-Andrews test indicates that the medium and large denominations (*mediumr_sa, larger_sa*) are trend-stationary. However, we assume that they are difference-stationary. Firstly, this is in line with the usual empirical specification. Secondly, the reliability of our unit root tests is impaired owing to the short sample. Moreover, we consider the house price indicator (*house*) and private consumption in the rest of the euro area (*diff_pcr_sa*) to be I(1) instead of I(2) as the tests, which indicate that these series are I(2), are biased owing to the financial crisis.[13]

Owing to the non-stationarity of the time series, the demand for the different denominations is estimated within a vector error correction model (VECM) based on the procedure developed by Johansen (1995, 2000). This approach seems to be particularly suitable for verifying the long-run equilibrium (cointegration) relationships on which the theoretical considerations are based.[14] The empirical analysis starts with an unrestricted VECM, which takes the following form:

(1) $$\mathrm{d}y_t = \mu + \Pi y_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \mathrm{d}y_{t-i} + \mathrm{B}x_t + \varepsilon_t,\ t = 1,\ldots,T\ ,$$

where $\{y_t\}$ represents the vector of the non-stationary endogenous variables. $\{\varepsilon_t\}$ denotes the vector of the independently and identically distributed residuals, B is the coefficient matrix of strictly exogenous (non-modelled) variables $\{x_t\}$, Γ is the coefficient matrix of the lagged endogenous variables and μ the vector of constants. The number of cointegration relationships corresponds to the rank of the matrix Π. Granger's representation theorem asserts that if the coefficient matrix Π has reduced rank $r < n$, then there exist (nxr) matrices α (the loading coefficients or speed-of-adjustment parameters) and β (the cointegrating vectors) each with rank r (number of cointegration relations) such that $\Pi = \alpha\beta'$ and $\beta' y_t$ is I(0). The cointegration vectors represent the long-term equilibrium relationships of the system. The loading coefficients denote the importance of these cointegration relationships in the individual equations and the speed of adjustment following deviations from long-term equilibrium.

[13] There is a "bump" in 2008 owing to the financial crisis, see the house price variable in figure 15.
[14] Rao (2007) compares our chosen econometric method with others to distinguish between short-term and long-term relationships. He finds that there are often only minor differences in the estimates.

Given the short sample with only 40 (quarterly) observations, the lag order (*k*) of the system is determined by the minimal lag order that is sufficient to eliminate autocorrelation of the residuals in the VECM. In any case, the chosen cointegration specification assumes an intercept both in the cointegrating equations and in the VAR. In other words, we assume that the level data $\{y_t\}$ have linear trends but the cointegrating equations have only intercepts.

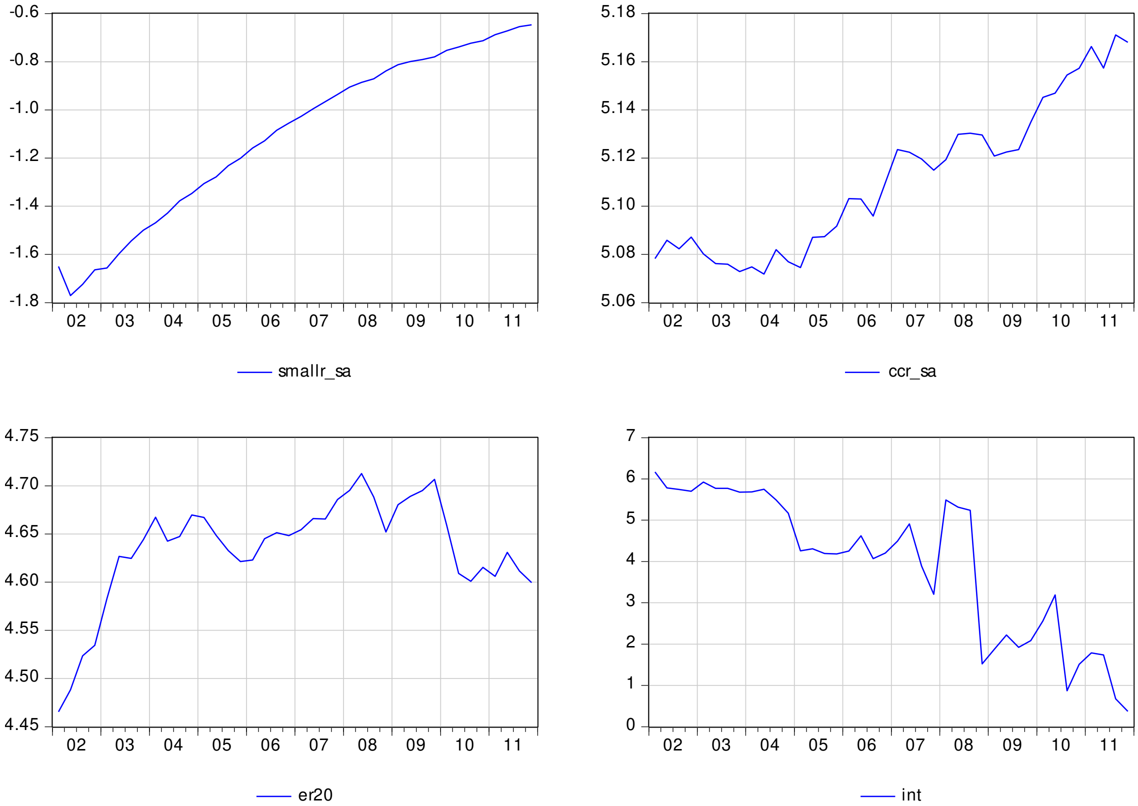
5.1 Structural model for the demand for small denomination notes

After pretesting, we select small denomination notes (*smallr_sa*), cash consumption (*ccr_sa*), the effective external value of the euro vis-à-vis the 20 most important trading partners (*er20*) and the shift parameter of the estimated term structure (*int*) to enter the cointegration space. These endogenous variables are shown in figure 12. As mentioned above, all of these variables have a stochastic trend, which is a necessary condition for the existence of cointegration relations. Furthermore, we add (the first difference of) private consumption in the rest of the euro area (*diff_pcr_sa*) as a strictly exogenous variable to the system of equations. Other potential variables are insignificant. In the case of card payments this might be attributed to the poor quality of our proxy variable.[15]

The chosen lag order to ensure white noise residuals is two in the VAR in levels, ie one in the corresponding VECM. This lag order is between that selected by different lag length information criteria (available upon request).

[15] We choose the number of card payments with cards issued in Germany (excluding transactions with e-money cards) as published in the payments statistics in Germany by the Deutsche Bundesbank. Due to redefinitions, this series exhibits a counterintuitive and unexplained downward shift in 2007. In Seitz and Setzer (2009), who use a similar econometric framework, this variable exerts a significant influence.

Figure 12: Endogenous variables of the VECM for small denomination banknotes



Sources: Deutsche Bundesbank and authors' own calculations.

The number of cointegration vectors is verified by determining the cointegration rank with the trace test and the maximum eigenvalue test (see table 2). These test statistics are subject to a small sample bias, which tends to reject the null of no cointegration too often. Therefore, we corrected them by the factor *(n-mk)/n*, where *n* is the number of observations, *m* the number of variables entering the cointegration space and *k* the number of lags, as suggested by Reimers (1992).[16] The tests yield conflicting results. While the trace test suggests one cointegration relationship, the maximum eigenvalue test indicates no cointegrating equations. Seitz and Setzer (2009) identify a (similar) cointegrating relationship for small German euro banknotes, cash consumption, an exchange rate argument and a proxy variable for opportunity costs in which all of these variables were defined as in our model. Therefore, it seems reasonable to restrict the *VECM* to one cointegration relationship.

[16] An alternative would be to adjust the critical values, see Cheung and Lai (1993). As they use an analogous correction to that of Reimers (1992), the results are in any case qualitatively equal.

Table 2: Cointegration rank tests

Number of cointegrating relationships	0		1		2		3	
	ts	**cv**	**ts**	**cv**	**ts**	**cv**	**ts**	**cv**
Trace test	51.57*	47.86	25.49	29.80	11.05	15.49	3.23	3.84
Max-eigenvalue test	26.08	27.58	14.44	21.13	7.82	14.26	3.23	3.84

* Denotes rejection of the hypothesis at the 0.05 level. ts: small sample adjusted test statistic according to Reimers (1992), cv: 0.05 level critical value.

Source: Authors' own calculations.

Table 3: Estimates and diagnostic test results of the VECM for small denominations

	Cointegrating equation
smallr_sa(-1)	1.00
ccr_sa(-1)	-6.55 (-8.7)
er20(-1)	-3.15 (-14.6)
int(-1)	0.04 (3.0)
constant	49.0
error correction term	-0.19 (-4.49)
constant	0.04 (8.6)
d(*diff_pcr_sa*)	0.004 (4.80)
adj. R^2	0.76
s.e.	0.019
F-statistic	21.48
AIC	-4.97
SC	-4.68
LM (1) [p-value]	22.76 [0.12]
LM (4) [p-value]	16.28 [0.43]
JB [p-value]	35.87 [0.00]

Notes: t-statistics in ( ); JB: Jarque-Bera VEC residual joint normality test; LM (.): VEC residual serial correlation LM Tests of lag (.); s.e.: standard error of equation; AIC (SC): Akaike (Schwarz) information criterion.

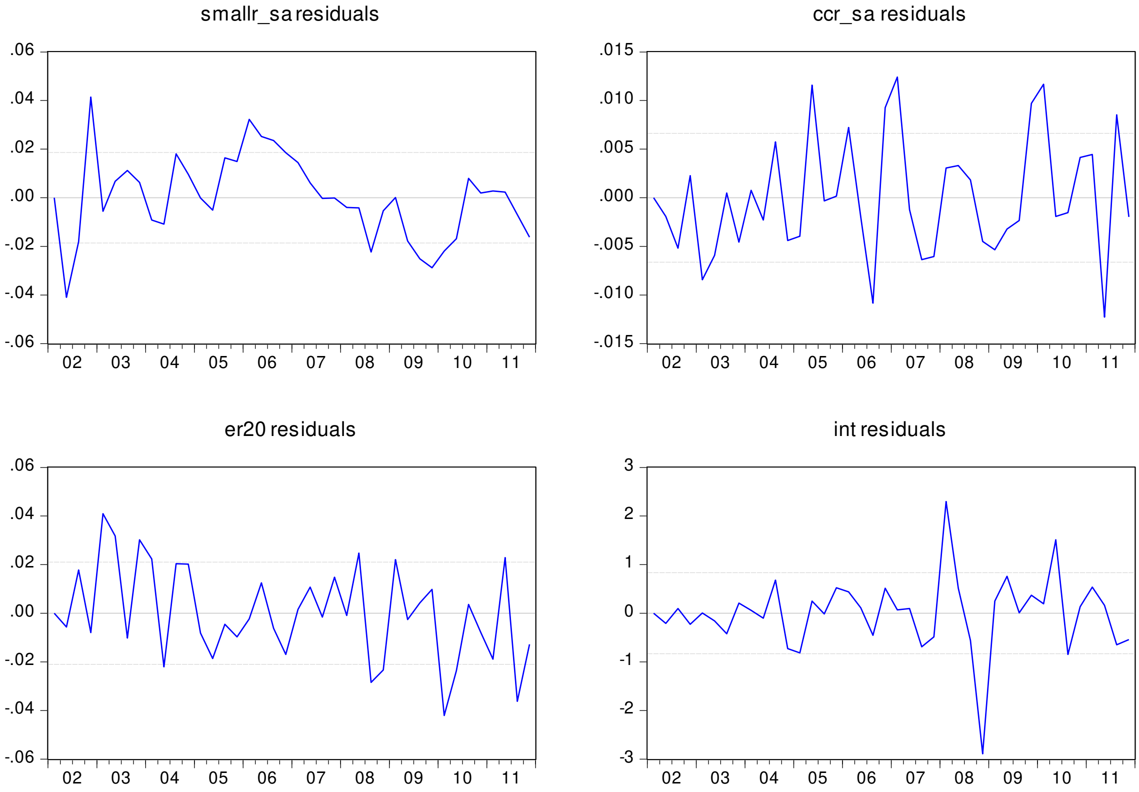
Source: Authors' own calculations.

Table 3 displays the estimation results of the VECM. The long-run determinants and the short-run coefficients of the exogenous variables are displayed together with the error correction term. We do not show the equations for the other endogenous variables (real cash consumption, real effective exchange rate and the proxy variable for interest rates) and the short-run coefficients of the lagged endogenous variables. The signs in the cointegrating equation are as expected: the demand for small banknotes rises when cash consumption and the exchange rate increase and declines when interest rates increase. It seems that the small denominations are mainly driven by domestic transactions and foreign demand in the long run. The interest rate semi-elasticity is fairly low. If the whole spectrum of yields rises by one percentage point, the growth rate of small banknotes declines by only 0.04 percentage point. The relatively high coefficient of cash consumption indicates that it was not possible to adequately model certain determinants. The adjustment to the long-term equilibrium is reflected in the error correction term: *ect* (*smallr_sa* - 6.55*ccr_sa* - 3.15*er20* + 0.04*int* + 49.0). The speed-of-adjustment parameter (*ect*) states how much of an existing disequilibrium is reduced within one quarter. Here, about 19% of the imbalance is corrected in one quarter. While cash consumption and interest rates are weakly exogenous, the speed-of-adjustment parameter is also significant in the equation for the real effective exchange rate. Therefore, in purely statistical terms, the cointegration equation in table 3 can neither be interpreted as a banknote demand function nor be estimated in a single equation framework. However, the adjustment of the exchange rate to deviations from the cointegrating relation lacks a convincing economic explanation. In our context it seems quite natural that only banknotes adjust to this deviation and not the other variables. Moreover, the adjustment of the exchange rate hardly affects the equation for banknotes in the system. Therefore, we interpret the latter as a banknote demand equation. While the cointegrating relation catches demand for small German banknotes in Germany and outside the euro area, the transaction motive in the rest of the euro area is part of the short-run dynamics. This motive is proxied by the (stationary transformed) seasonally adjusted real private consumption in the rest of the euro area. Its (positive) coefficient is highly significant with a t-value of 4.8. This is in line with the considerable issuance of €5 notes and €10 notes (typical transaction denominations) by the Deutsche Bundesbank. As shown in figures 7 to 10, for both of these denominations the cumulated net issuance of the Bundesbank clearly exceeds that of the Eurosystem without Germany. This is evidence that there are significant (net) exports of small denomination banknotes from Germany to the rest of the euro area.

The statistical fit of the system of equations is satisfactory with an adjusted R^2 of 76%. The Jarque Bera test statistic indicates non-normality of the residuals in the VECM. As cointegration theory is asymptotically valid under the assumption of independently and identically distributed residuals, this result should not be too serious a problem.

According to the LM test, the residuals are uncorrelated.[17] Figure 13 depicts the short-run error sequences, ie the estimated $\{\varepsilon_t\}$ series that equals the residuals in equation (1). By and large, they approximate a white noise process. Figure 14 shows deviations of *smallr_sa* from the long-run relationship. This long-run error series also conforms to its theoretical desired property in that the residuals from the long-run equilibrium appear to be stationary.

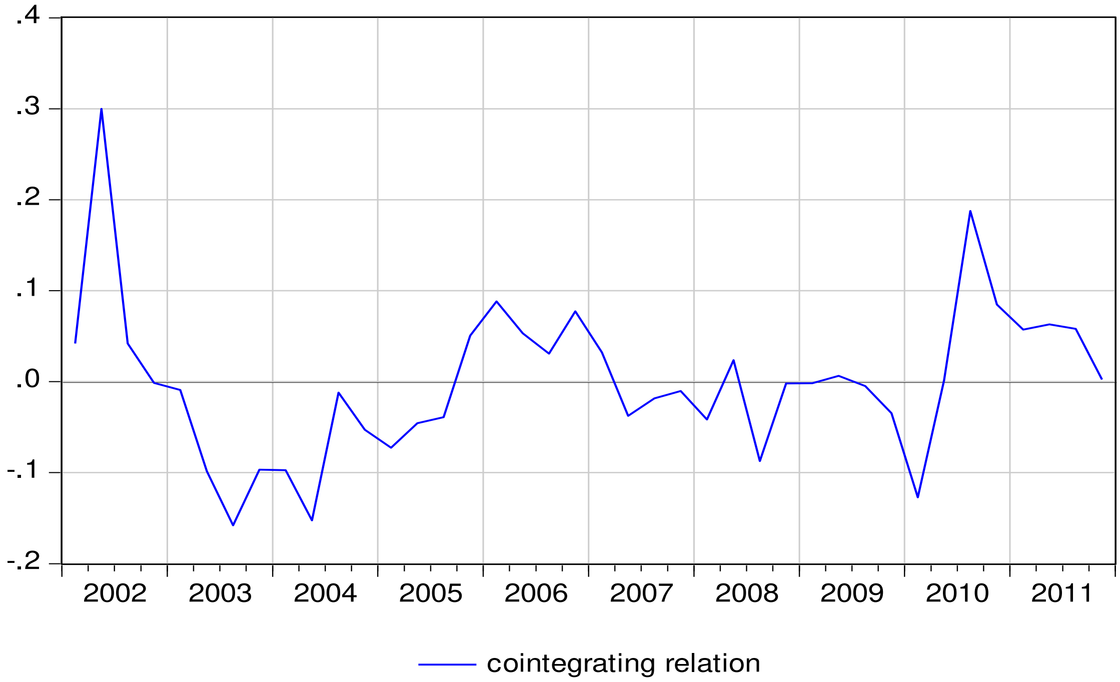
Figure 13: Short-run error sequences



Source: Authors' own calculations.

[17] The only hint at autocorrelation is at lag 3 where the p-value amounts to only 0.053.

Figure14: Long-run error series



Source: Authors' own calculations.

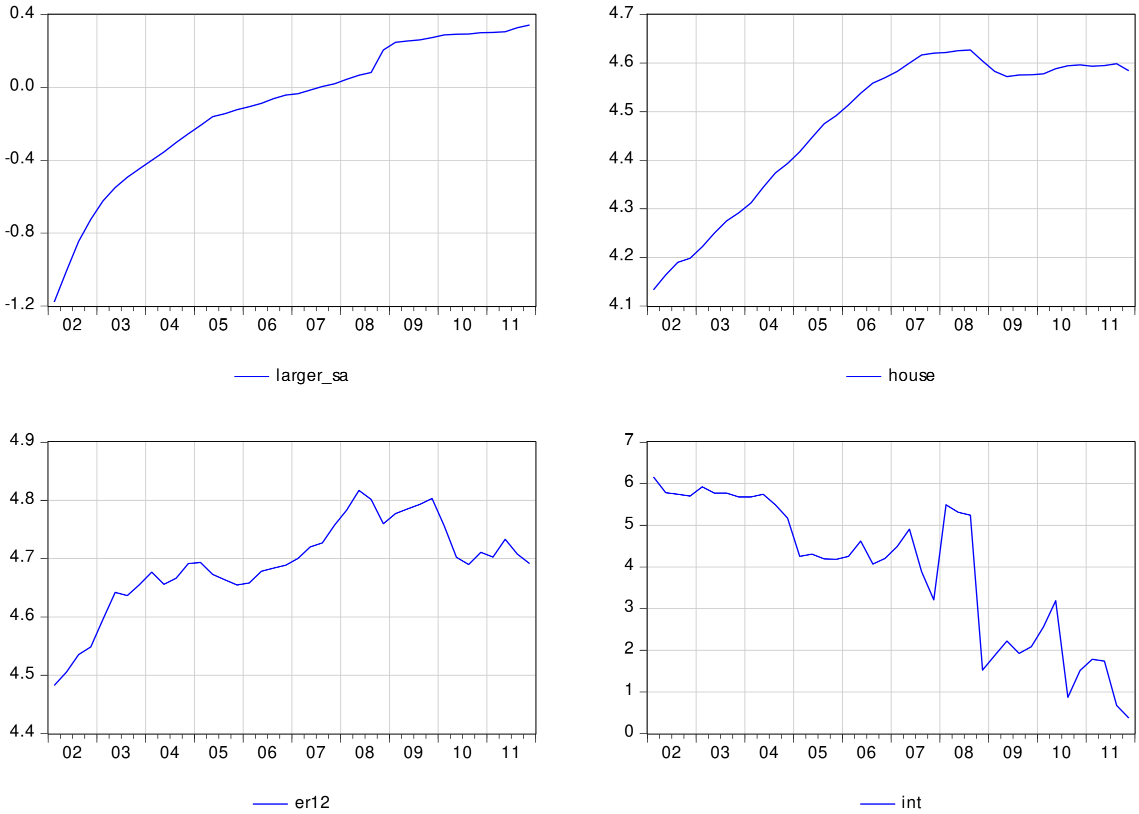
In view of the short sample of only 40 quarterly observations, we cannot employ valid tests of parameter stability. However, we have estimated the VECM for alternative samples ending in different quarters of 2011. Visual inspection suggests no significant changes of the cointegrating equation and the speed-of-adjustment coefficient.

5.2 Structural model for the demand for large denomination notes

The following variables enter the cointegrating space: large denominations (*larger_sa*), euro-area house prices outside Germany (*house*), the real effective external value of the euro vis-à-vis the 12 most important trading partners (*er12*) and again the term structure parameter (*int*). These endogenous variables are shown in figure 15. Furthermore, we add the following strictly exogenous variables to the system of equations: a dummy variable for the onset of the financial crises in the fourth quarter of 2008, *d2008q4*, and a dummy variable for the public debt crisis in the euro area that began in the first quarter of 2010, *d_debt2010q1*. The latter variable should capture the crisis-related increase in the demand for large denominations. Other potential exogenous variables are insignificant.

The lag order (k) of the system is again determined by the minimal lag order that is sufficient to eliminate autocorrelation of the residuals in the VECM. The chosen lag order is two for the VAR in levels, ie one in the corresponding VECM. This is also the lag order suggested by the Hannan-Quinn information criterion (result available upon request).

Figure 15: Endogenous variables of the VECM for large denomination banknotes



Source: Deutsche Bundesbank and authors' own calculations.

The results of the trace and maximum eigenvalue test on the number of cointegration vectors are shown in table 4. Both tests suggest one cointegration relationship. The critical values assume no deterministic exogenous series and this assumption is violated in our case. However, given the unambiguousness of the test results, we should be on the safe side in restricting the *VECM* to one cointegration relationship.[18]

[18] The sensitivity of the critical values in cointegration tests with respect to the deterministic specification (trend assumption) might be regarded as a benchmark here. See table 1 on page 276 in MacKinnon (1991).

Table 4: Cointegration rank tests

Number of cointegrating relationships	0		1		2		3	
	ts	**cv**	**ts**	**cv**	**ts**	**cv**	**ts**	**cv**
Trace test	68.72*	47.86	23.30	29.80	10.46	15.49	0.35	3.84
Max-eigenvalue test	45.42*	27.58	12.84	21.13	10.10	14.26	0.34	3.84

* Denotes rejection of the hypothesis at the 0.05 level. ts: small sample adjusted test statistic according to Reimers (1992), cv: 0.05 level critical value.

Source: Authors' own calculations.

Table 5: Estimates and diagnostic test results of the VECM for large denominations

	Cointegrating Equation
larger_sa(-1)	1.000
house(-1)	-0.82 (-7.9)
er12(-1)	-2.03 (-10.7)
int(-1)	0.09 (8.8)
constant	12.9
error correction term	-0.48 (-5.0)
constant	-0.02 (-1.4)
d2008q4	0.10 (2.5)
d_debt2010q1	0.03 (1.6)
adj. R^2	0.61
s.e.	0.039
F-statistic	9.79
AIC	-3.47
SC	-3.13
LM (1) [p-value]	21.38 [0.16]
LM (4) [p-value]	20.26 [0.21]
JB [p-value]	28.38 [0.00]

Notes: t-statistics in ( ); JB: Jarque-Bera VEC residual joint normality test; LM (.): VEC residual serial correlation LM Tests of lag (.); s.e.: standard error of equation; AIC (SC): Akaike (Schwarz) information criterion.

Source: Authors' own calculations.

Table 5 displays the estimation results of the VECM. Again we only show the equation for banknotes without the short-run coefficients of the lagged endogenous variables. The signs in the cointegrating equation are as expected: the demand for large denomination banknotes rises when house prices in the rest of the euro area and the exchange rate increase and it declines when interest rates increase. It seems that the large denomination notes in circulation are mainly driven by foreign demand in the long run. The semi-elasticity of interest rates is once again fairly low. If the whole spectrum of yields rises by one percentage point, the growth rate of large denomination banknotes declines by only 0.09 percentage points. The adjustment to the long-run equilibrium is reflected in the error correction term: *ect* (*larger_sa* - 0.82*house* - 2.03*er12* + 0.09*int* + 12.9). In the case of large denominations, about 50% of the imbalance is corrected in one quarter. While the real effective exchange rate is weakly exogenous, the speed-of-adjustment parameter in the equation for the house price indicator is highly significant and in the equation for interest rates it is marginally significant (p-value of 0.051). However, the adjustments of house prices and interest rates to deviations from the cointegrating relation again lack a convincing economic explanation and they hardly affect the equation for big banknote denominations in the system.[19] Therefore, as with small denominations, we interpret the latter as a banknote demand equation within a system.

We have also included two crises variables in the VECM as strictly exogenous variables. The escalation of the global financial crisis after the bankruptcy of Lehman Brothers in September 2008 resulted in a sharp increase in the issuance of German large denomination notes (Deutsche Bundesbank, 2009, pp 52f). This is modelled by the dummy variable *d2008q4*. It is an impulse variable that takes the value one in the fourth quarter of 2008 and zero otherwise. In other words, the financial crisis is assumed to have resulted in a *one-time* increase in the *real* demand for large denominations. Economic crises generally go hand in hand with an increase in demand for large banknote denominations. Therefore, we also tried to model the repercussions of the European public debt crisis which started at the beginning of 2010. The corresponding dummy variable *d_debt2010q1* is a shift variable. It is equal to one from the first quarter of 2010 to the end of the sample and zero in all other quarters. This corresponds to a continuously increasing *level* of (real) banknote demand. While having the right positive sign, the coefficient of *d_debt2010q1* is only marginally significant. However, the estimated coefficients of the VECM are robust with regard to the inclusion or omission of *d_debt2010q1*.

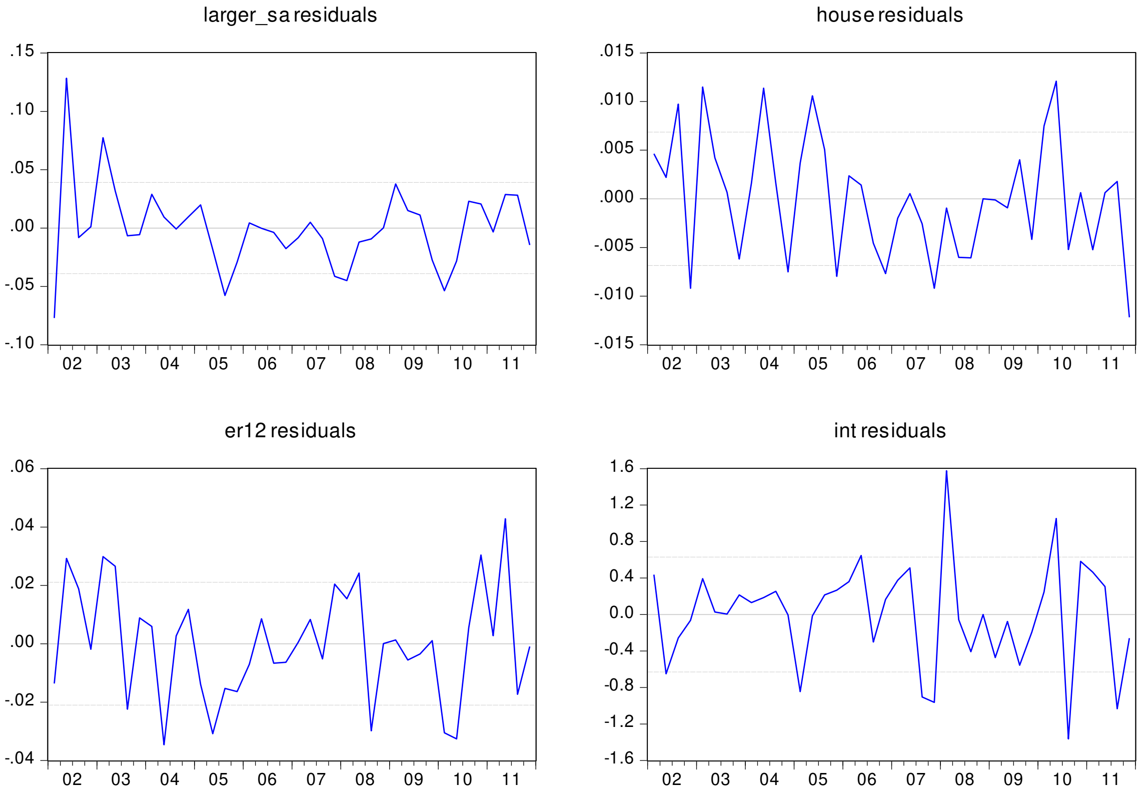
The statistical fit of the system of equations is rather good with an adjusted R^2 of 61%. The other test statistics reveal a similar performance to that of small denominations. Figure 16 depicts the short-run error sequences, ie the estimated $\{\varepsilon_t\}$ series (residuals)

[19] In our context, it seems quite natural that only banknotes adjust to this deviation and not the other variables.

in equation (1). By and large, they approximate a white noise process. Figure 17 shows deviations of actual banknote developments from the long-run relationship. This long-run error series also conforms to its theoretical desired property in that the residuals from the long-run equilibrium appear to be stationary.

In view of the short sample of only 40 quarterly observations, we cannot employ valid stability tests. However, we have estimated the VECM for alternative samples ending in different quarters of 2011. Visual inspection suggests no significant changes of the cointegrating equation and the speed-of-adjustment coefficient.

Figure 16: Short-run error sequences



Source: Authors' own calculations.

Figure 17: Long-run error series

Source: Authors' own calculations.

5.3 Time series models for forecasting medium notes in circulation

We did not succeed in modelling the medium denominations within a VECM. Unfortunately, the short sample period does not allow to establish a stable cointegration relation, which seems to be due, in particular, to the early part of the sample.[20] We presume that in order to estimate a suitable vector error correction model for medium denomination notes, we need several years of additional (quarterly) observations.

Since we cannot build structural models, we present times series models for these denominations (€50, €100). These models are used by the Deutsche Bundesbank within the scope of the annual banknote production planning in the Eurosystem. Each summer, the *increase* in the number of German banknotes in circulation (by denomination) is forecast *for the following year*. The (final) forecasts of the Deutsche Bundesbank result from a reconciliation of the statistical forecasts with expert forecasts. The statistical

[20] We also tried to estimate a VECM for the total of German euro notes in circulation. To do this, we have to model all of the motives of banknote demand (transactions, hoarding, foreign demand). Again, we were not able to find a sufficiently stable cointegration equation which may be interpreted as a cash demand function.

forecasting models are RegARIMA models which are estimated using monthly data.[21] In the forecasts made in July 2014, the sample ranged from January 2002 to June 2014. Both denominations display a stochastic trend. The estimation is therefore performed in first differences. The models are selected with regard to the following criteria: significant and stable coefficients, goodness of fit and uncorrelated residuals. In July 2014, the following RegARIMA model for the *number* of German €50 notes in circulation, *nn050*, evolved:

$$\text{(2)} \qquad \mathrm{d}(nn050_t) = c + a_1 easter + a_2 seas(1) + a_3 seas(12) + a_4 dum0810 + a_5 iv0810\_050 + \rho_{12} u_{t-12} + \varepsilon_t + \theta_1 \varepsilon_{t-1}, \; t = 1,\ldots,T.$$

The ARMA part of the model consists of a moving average term at lag 1, ma(1) (ε_{t-1}), and an autoregressive term at lag 12, ar(12) (u_{t-12}). The term *seas*(i) represents a seasonal dummy variable for month *i*. For instance, the rise in the demand for banknotes in December owing to the Christmas season is denoted by *seas*(12). The subsequent decline in January is captured by *seas*(1). The variable *easter* represents the increased demand for notes over Easter. Since the Easter holidays do not fall in the same month every year, they cannot be represented by a seasonal dummy variable. The impulse dummy *dum0810* is equal to one in October 2008 and zero otherwise. It is used to model the increase in German €50 notes in circulation (in the form of hoarding) as a result of the escalation of the financial crisis in October 2008. The intervention variable *iv0810_050* captures the subsequent complete back-formation of this increase. It is estimated by means of an auxiliary regression where *iv0810_050* in equation (2) was replaced with dummy variables for the months from November 2008 onwards (*dum0811*, *dum0812*, *dum0901*, …). While *dum0811* and *dum0812* are not significant, *dum0901*, *dum0902* and *dum0903* are and their estimated negative coefficients add up to about *minus* 40. A comparison with the estimated coefficient of *dum0810* of about *plus* 40 (see equation (2')) is interpreted as evidence that the crisis-induced increase in the number of German €50 notes in circulation was completely reversed by the end of the first quarter of 2009. Accordingly, *iv0810_050* is equal to the estimated coefficient

[21] ARIMA models are autoregressive integrated moving average models. RegARIMA or ARIMAX models are ARIMA models with exogenous input or rather right-hand side explanatory variables. The input can either be deterministic (eg dummy variables) or stochastic and exogenous. Similar RegARIMA models are also used to forecast German euro coins in circulation within the scope of coin requirement planning in Germany, see the box "Forecast model" on p 35 in Deutsche Bundesbank (2013).

of *dum0901* in January 2009, to the estimated coefficient of *dum0902* in February 2009 and to the estimated coefficient of *dum0903* in March 2009. It is zero otherwise. This is a simple way to gauge the effects of the global financial crisis on the number of German €50 notes in circulation. However, it involves additional uncertainty as *iv0810_050* is a generated variable. Pagan (1984) has shown that the standard error of the coefficient estimate is unbiased if the null hypothesis to be tested is a zero coefficient. This is exactly the case in our exercise. To alleviate this problem, we have modelled the highly significant dummy variable *dum0810* (with a t-statistic of about 10) separately.

Equation (2) is estimated using data from January 2002 up to and including June 2014. The estimation results together with test statistics are shown in equation (2') where we paid particular attention to ensure uncorrelated residuals and stable coefficient estimates (t-statistics in brackets below the coefficients). With an adjusted R^2 of 88%, the statistical fit is fairly good.

$$(2')\qquad \mathrm{d}(nn050_t) = \underset{(10.3)}{15.10} + \underset{(3.9)}{19.32}\,easter - \underset{(-9.0)}{95.56}\,seas(1) + \underset{(5.1)}{56.12}\,seas(12) + \underset{(9.8)}{40.88}\,dum0810 + \underset{(6.4)}{0.77}\,iv0810\_050 + \underset{(4.3)}{0.42}\,u_{t-12} + \varepsilon_t - \underset{(-4.1)}{0.36}\,\varepsilon_{t-1}$$

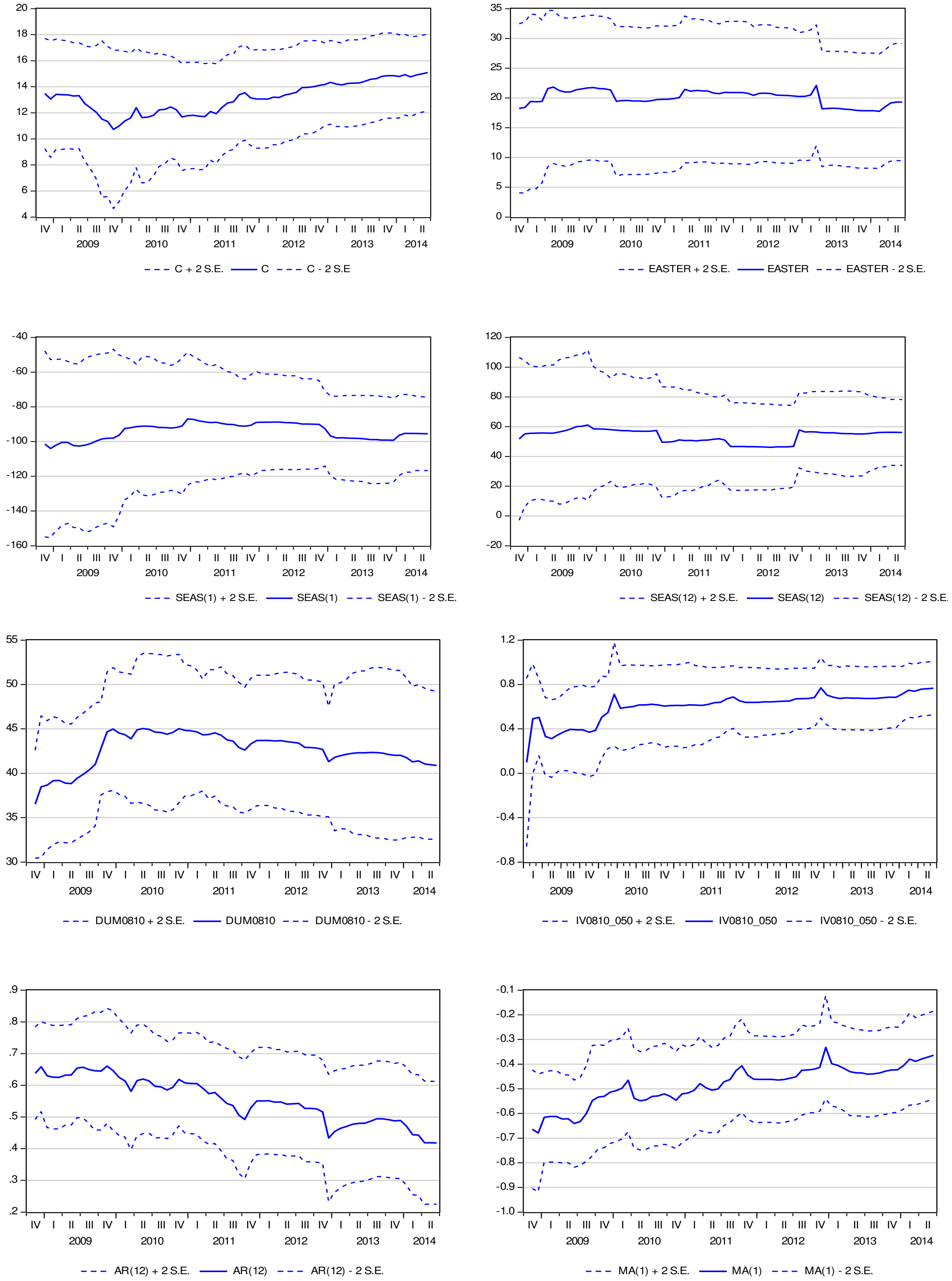
Notes: HAC standard errors and covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.000), adj. $R^2 = 0.88$, standard error of regression = 12.1, Wald F-statistic = 89.5.

The autocorrelations and partial autocorrelations roughly indicate serial correlation up to lag order 13, 14, 19 and 24 (results available upon request). However, according to the more precise serial correlation LM test, there is no autocorrelation in the residuals.[22] We test for parameter stability by recursive estimation of the coefficients. These are shown in figure 18 together with the corresponding ± 2 standard deviation bands marked by the dashed lines. All of the coefficients are sufficiently stable. However, the coefficients of the intercept, *c*, and of the autoregressive term at lag 12, *ar*(12), have only been stable since June 2011 and April 2011 respectively.[23]

[22] The Breusch-Godfrey LM-test statistic shows the following p-values: 0.60 for lag 13, 0.32 for lag 14, 0.13 for lag 19 and 0.16 for lag 24.

[23] A coefficient is classified as being stable at a certain period of time if within this period *each* of the (recursively estimated) coefficients is above the highest point of the - 2 standard deviation band and below the lowest point of the + 2 standard deviation band.

Figure 18: Recursive estimates of the coefficients of equation (2)



Source: Authors' own calculations.

In a next step, the forecast model is chosen. Besides the RegARIMA model in equation (2), we have considered three alternative time series models. Firstly, we use the so-called "constant absolute growth model" (CAG model). This model assumes that the change in the number of notes in circulation in each month is equal to the corresponding change in the same month of the previous year. Secondly, we employ the so-called "constant growth rate model" (CGR model). It presumes that the monthly growth rate in the number of notes in circulation in each month is equal to the monthly growth rate in the same month of the previous year. In addition, we use a Holt-Winters exponential smoothing model with a multiplicative seasonal component (HW model).[24] We select the model with the lowest root mean squared forecast error (RMSE). The total sample is from January 2002 to June 2014. The in-sample forecast evaluation period starts in April 2010 (t = 100) and ends in June 2014 (t = 150). Therefore, the out-of-sample forecast period starts in July 2014. In July 2014, the *increase* in €50 banknotes in the current and the following year has to be forecast. Finally, the *increase* to be forecast is the difference between the number of German €50 notes in circulation 18 months ahead (at the end of December 2015) and the number of German €50 notes in circulation six months ahead (at the end of December 2014). Therefore, we denote the corresponding root mean squared forecast error as RMSE_18_6. For the in-sample forecast period, it is defined as

$$(3) \quad \text{RMSE\_18\_6} = \sqrt{\frac{1}{T}\sum_{t=100}^{150}\left[(nn050_t - nn050_{t-12}) - (nn050f\_18_t - nn050f\_6_{t-12})\right]}.$$

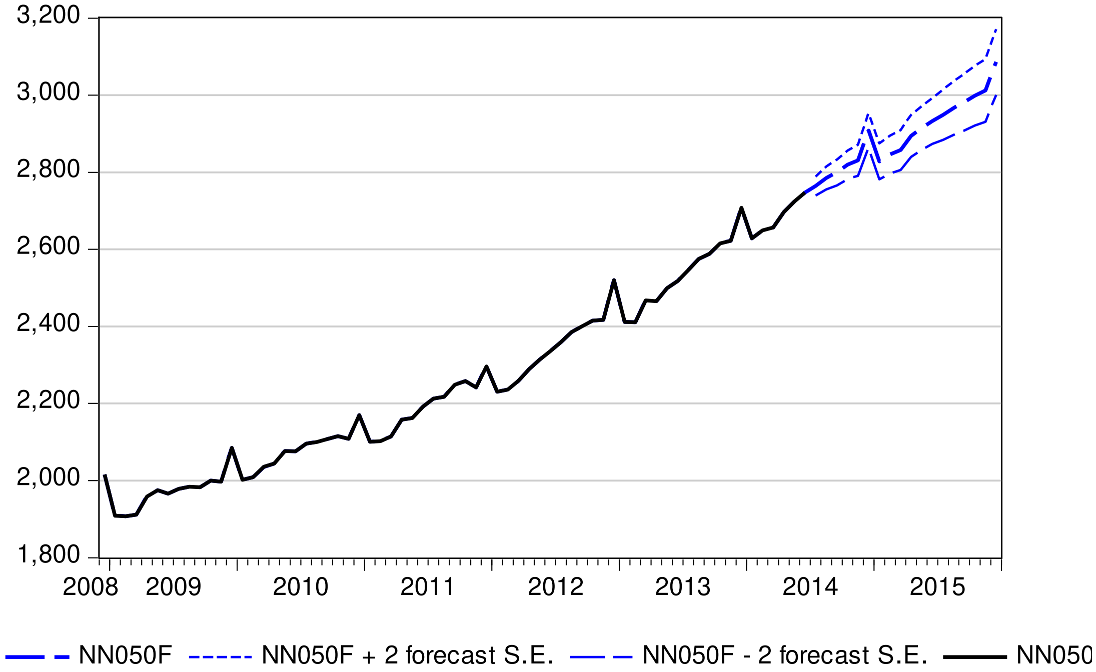
The term in square brackets is the (in-sample) forecast error. It conforms to the difference between the actual (yearly) increase in the number of German €50 notes ($nn050_t - nn050_{t-12}$) and the difference between the forecasted number of German €50 notes in circulation 18 months ahead ($nn050f\_18_t$) and the forecasted number of German €50 notes in circulation six month ahead ($nn050f\_6_{t-12}$).[25] Thus, the number of in-sample forecast errors is only 50. Therefore, the resulting root mean squared errors

[24] Unobserved component models as developed by Andrew Harvey generally offer a suitable alternative to the RegARIMA models. However, the number of observations is not yet sufficient to be able to use them. Unlike with RegARIMA models, the coefficients in these models are not fixed but vary over time. This means that the seasonal structure to be observed, which changes over time, may be better represented using unobserved component models.

[25] January 2002 corresponds to $t = 1$. For $t = 100$ in equation (3) the RegARIMA model without the intervention variable *iv0810_050* is estimated with data from January 2002 to October 2008. We discount the intervention variable *iv0810_050* because it is unknown at the time when the in-sample forecasts (or rather pseudo out-of-sample forecasts) are made. April 2010 is equal to October 2008 plus 18 months. For $t = 150$ in equation (3) the same RegARIMA model is estimated with data from January 2002 to December 2012. June 2014 corresponds to December 2012 plus 18 months.

should rather be considered as a reference point for the forecast evaluation. In this forecast exercise, the RegARIMA model performs best (RMSE_18_6 = 37.5), followed by the CGR model (RMSE_18_6 = 50.8), the CAG model (RMSE_18_6 = 51.6) and the HW model (RMSE_18_6 = 59.1). Consequently, we use the RegARIMA model in equation (2) to forecast the increase in German €50 notes in circulation in 2015 out-of-sample. Figure 19 displays this out-of-sample forecast made in July 2014, *nn050f*, together with standard error bands and the realised values of the time series until June 2014, *nn050*. The peaks in December are seasonal amplitudes. The two standard error bands provide an approximate 95% forecast interval.[26] The forecast horizon ranges from July 2014 to December 2015. The forecast increase in the number of German €50 notes in 2015 amounts to about 178 million pieces.

Figure 19: Out-of-sample forecast of the number of German €50 notes in circulation (in million pieces)



Source: Deutsche Bundesbank and authors’ own calculations.

Finally, we present the corresponding results for the number of German €100 banknotes in circulation, *nn100*. For this denomination, we chose the following RegARIMA model for the forecast in July 2014:

[26] If many forecasts are made, the actual value of the dependent variable will fall inside these bounds 95% of the time. The forecast standard errors account for both innovation and coefficient uncertainty.

$$(4)\qquad \mathrm{d}(nn100_t) = c + a_1 seas(1) + a_2 seas(12) + a_3 dum0508 + a_4 dum0810 + a_5 iv0810\_100 + a_6 iv1201\_100 + \rho_{12} u_{t-12} + \omega u_{t-1} - \rho_{12}\omega u_{t-13} + \varepsilon_t + \theta_1 \varepsilon_{t-1},\ t = 1,\ldots,T.$$

The ARMA part of the model consists of a moving average term at lag 1, *ma*(1) (ε_{t-1}), an autoregressive term at lag 12, *ar*(12) (u_{t-12}), as well as multiplicative autoregressive term at lag 1, *sar*(1) (u_{t-1}, u_{t-13}), where the latter introduces ω into the lag polynomial.[27] If ω is set equal to zero, the same ARMA structure as in the case of €50 notes results (see equation (2)). There is again a seasonal peak in December denoted by *seas*(12). The subsequent decline in January is captured by *seas*(1). In contrast to the RegARIMA model for €50 notes, the variable *easter* is not significant. This can be attributed to the fact that other than €50 notes, €100 notes are usually not used for transactions but rather as a store of value. The impulse dummy *dum0508* is equal to one in August 2005 and zero otherwise. It stands for the effect of the EU Directive on the taxation of savings income in the form of interest payments (Council Directive 2003/48/EC) which entered into force on 1 July 2005. This EU Directive aims at improving the exchange of information between EU member states in order to ensure that savings income paid to individuals in another Member State is taxed. As can be seen in equation (4’), the highly significant coefficient of *dum0508* is negative. This might be due to the liquidation of savings by Germans in other member states and the subsequent depositing of the corresponding cash (in the form of €100 notes) at banks in Germany. As before, the impulse dummy *dum0810* is equal to one in October 2008 and zero otherwise and captures the effect of the Lehman Brothers default. The intervention variable *iv0810_100* represents the subsequent further increase and the back-formation of the (cumulated) increase afterwards.[28] It is specified along the lines of *iv0810_050* in equation (2). It represents a further crisis-induced increase in the number of German €100 notes from November 2008 until January 2009. The cumulated increase was

[27] With monthly data, a multiplicative autoregressive term at lag 12 represents a *seasonal* multiplicative term. Here, the multiplicative autoregressive term *sar*(1) only serves the purpose of a parsimonious parametrization. Replacing it with *ar*(1) and *ar*(13) results in a similar, but unrestricted model. By contrast, the process in equation (4) contains non-linear restrictions on autoregressive coefficients.

[28] The global financial crisis had a significant influence only on the medium and large-value denominations. According to our regressions, the crisis-induced increase in the number of German €200 notes in circulation in October 2008 had completely reversed by the end of May 2009. The increase in the number of German €500 notes in circulation in October 2008 due to the financial crisis had largely been cancelled out by the end of 2010.

largely cancelled out by the end of 2010. *iv1201_100* is equal to 1 after and including January 2012 and zero otherwise. It allows for the strong growth in the number of German €100 notes since January 2012 which, to a certain extent, was caused by a substitution of €500 notes. Without this interaction term, the estimated coefficient of the intercept, *c*, would be unstable.

The exact estimation results are shown in equation (4') (t-statistics in brackets below the coefficients). With an adjusted R^2 of 88%, the statistical fit is fairly good.

$$\text{(4')} \quad d(nn100_t) = \underset{(7.5)}{4.78} - \underset{(-7.1)}{12.06}\, seas(1) + \underset{(6.7)}{8.84}\, seas(12) - \underset{(-4.0)}{3.46}\, dum0508$$
$$+ \underset{(24.0)}{30.65}\, dum0810 + \underset{(23.8)}{0.95}\, iv0810\_100 + \underset{(2.7)}{2.73}\, iv1201\_100$$
$$+ \underset{(8.7)}{0.55}\, u_{t-12} + \varepsilon_t - \underset{(-2.0)}{0.54}\, \varepsilon_{t-1} + \underset{(3.9)}{0.75}\, sar(1)$$

Notes: HAC standard errors and covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.000), adj. $R^2 = 0.88$, standard error of regression = 1.9, Wald F-statistic = 262.8.

The autocorrelations and partial autocorrelations roughly indicate serial correlation up to lag order 14, 15 and 20 (available upon request). However, as in the case of €50 notes, the serial correlation LM test indicates no autocorrelation in the residuals.[29]

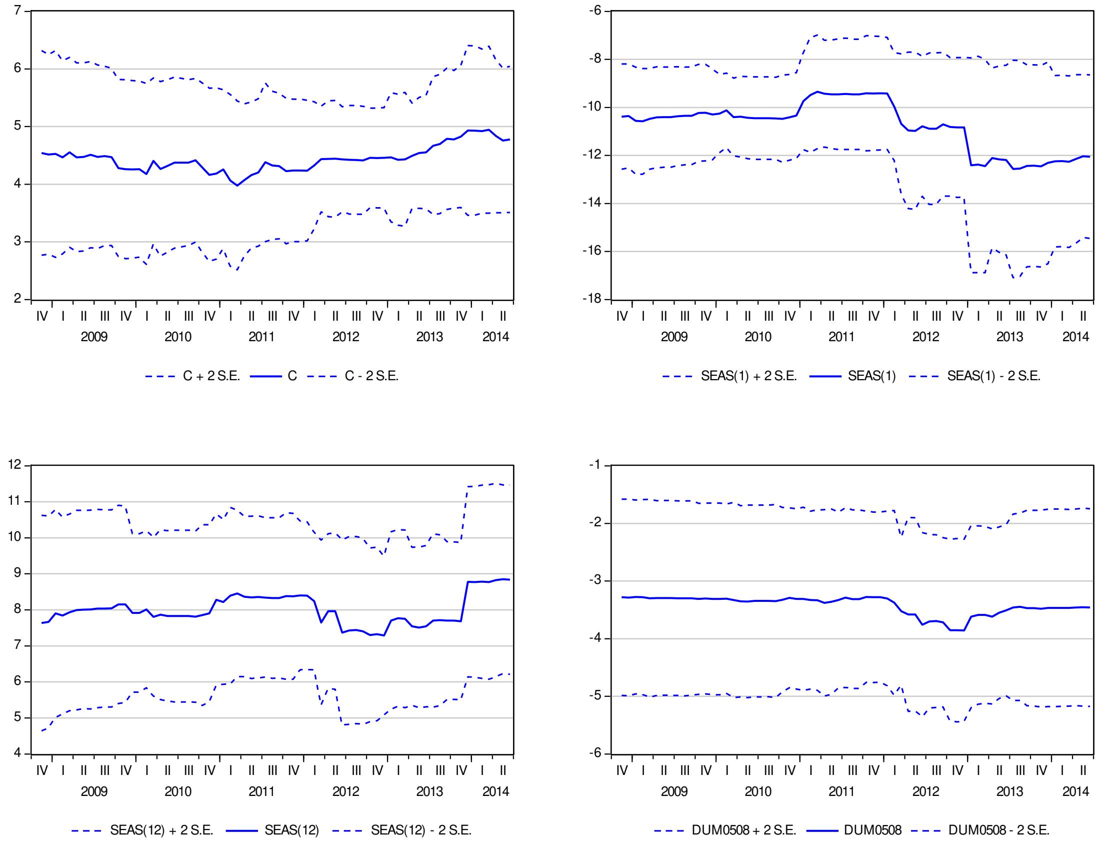
The recursive estimation results together with the corresponding ± 2 standard deviation bands are shown in figure 20. All coefficients except *sar*(1) and *ma*(1) are sufficiently stable. The latter have only been stable since July 2013. Notwithstanding that, we included them in the RegARIMA model in order to remove autocorrelation of the residuals. Moreover, the change in the estimated coefficients seems to be of a temporary nature. Since July 2013, they have been on a level comparable to the period up to February 2012. While a sustained change in a coefficient might indicate model misspecification, such a temporary change should not affect the quality of the out-of-sample forecast.

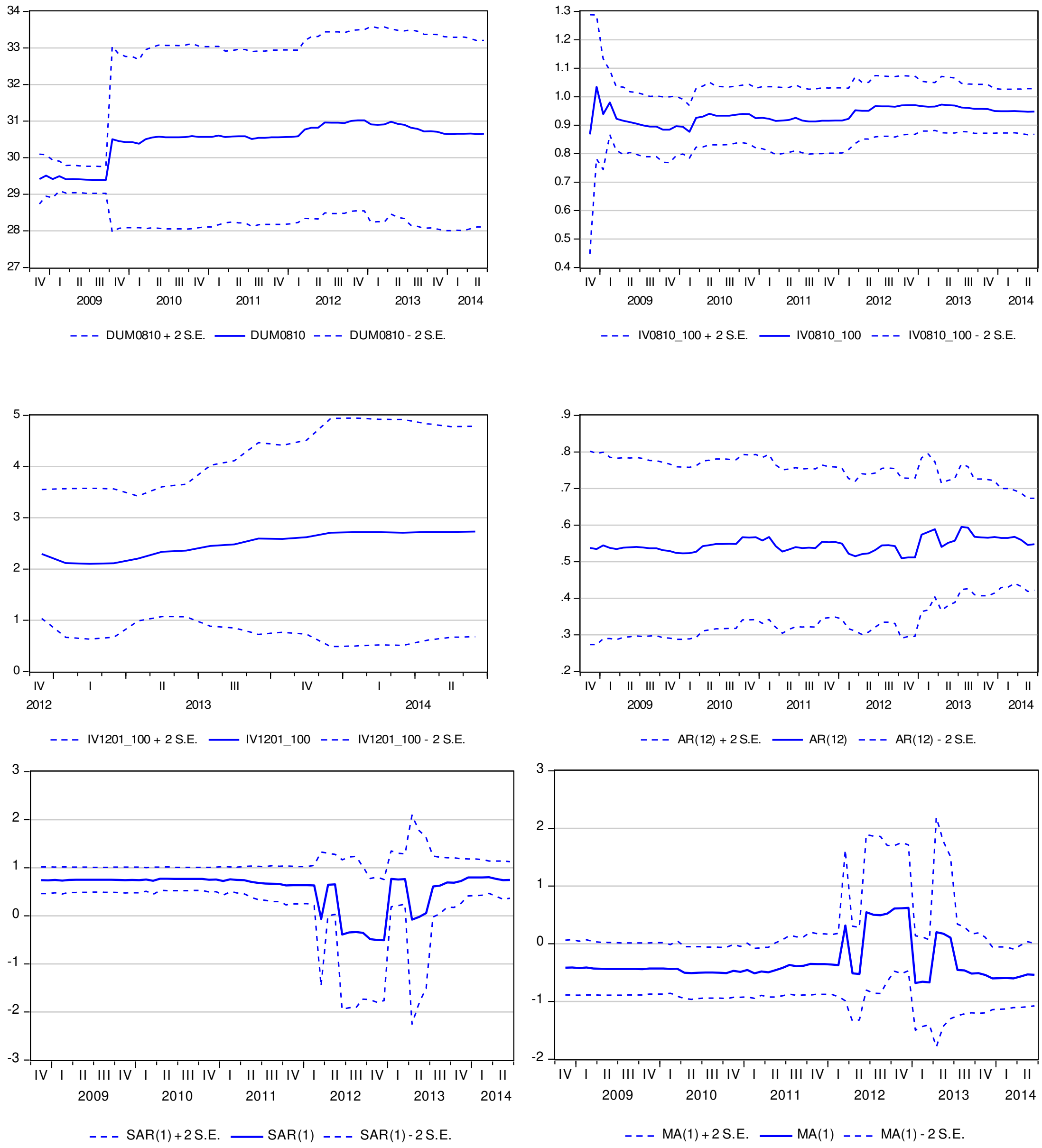
The forecast evaluation is performed in the same way as for German €50 notes. The in-sample forecast horizon runs from April 2010 ($t = 100$) to June 2014 ($t = 150$). The RMSE_18_6 of the RegARIMA model is once again the smallest one (29.0), followed by the HW model (34.6), the CAG model (35.2) and the CGR model (45.0). Therefore, the out-of-sample forecasts are made with the RegARIMA model in equation (4’).

[29] The Breusch-Godfrey LM-test statistic shows the following p-values: 0.29 for lag 14, 0.18 for lag 15 and 0.19 for lag 20.

Figure 21 displays these out-of-sample forecasts made in July 2014, *nn100f*, together with the (forecast) standard error bands and the realised values of the time series until June 2014, *nn100*. The seasonal amplitudes in December are much less pronounced than those for €50 notes as €100 notes are rarely used for transaction purposes. The forecast horizon ranges from July 2014 to December 2015. The forecast increase in the number of German €100 notes in 2015 amounts to about 94 million notes.

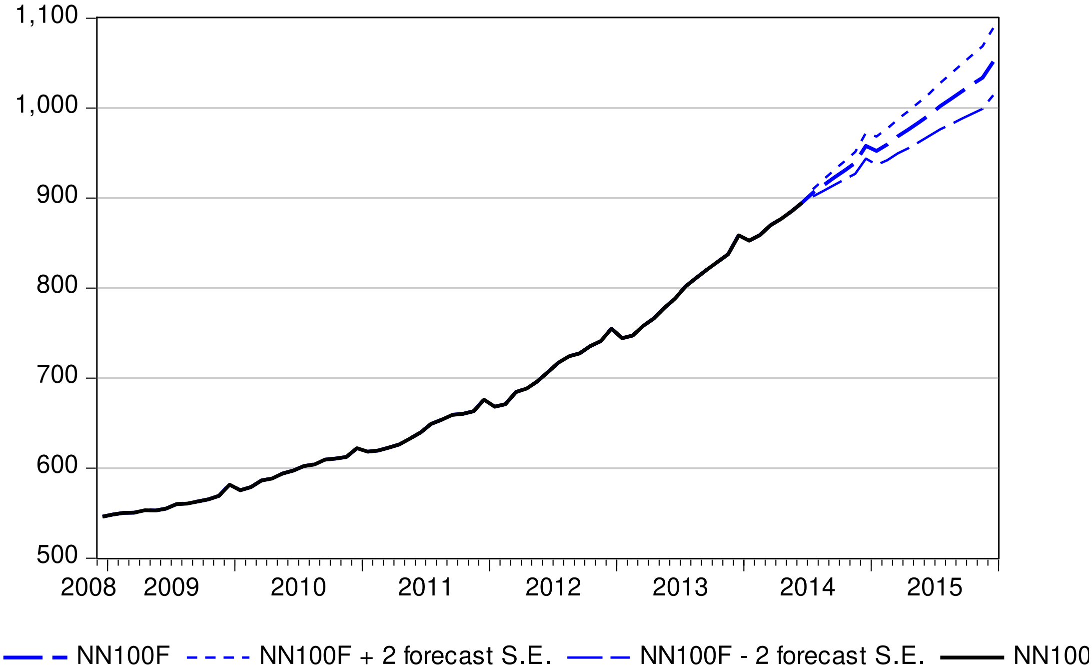
Figure 20: Recursive estimates of the coefficients of equation (4)





Source: Authors' own calculations.

Figure 21: Out-of-sample forecast of the number of German €100 notes (in millions)



Source: Deutsche Bundesbank and authors' own calculations.

6 Summary and conclusions

In this paper, we analysed the cumulated net issuance of euro banknotes by the Deutsche Bundesbank ("German" euro notes in circulation). The strong growth in German euro notes in contrast to the weak increase in other euro-area countries can be explained as follows. Firstly, the dynamics of euro notes in circulation are, to a large extent, driven by demand from outside the euro area and this demand is predominantly met by Germany. Secondly, Germany is also an important net exporter of euro notes to other euro-area countries. However, the lion's share of foreign holdings is in non-euro-area countries.

The importance of foreign demand is reflected in the vector error correction models, which we estimate using genuine euro data up to the end of 2011. It seems that the demand for small denominations is mainly driven by domestic transactions and foreign demand from outside the euro area in the long run. By contrast, interest rates seem to play only a minor role. The transaction motive in the rest of the euro area (without Germany) is part of the short-term dynamics. This is in line with the fact that the cumulated net issuance of €5 notes and €10 notes by the Deutsche Bundesbank exceeds that of the Eurosystem.

The cointegrating equation for the large denominations reveals that the demand for these denominations rises when house prices in the rest of the euro area and the real effective exchange rate increase and it declines when interest rates increase. Foreign demand from within and outside the euro area is the main driver in the long run. The effect of the escalation of the global financial crisis after the bankruptcy of the US investment bank Lehman Brothers in September 2008 also exerts a significant influence. By contrast, the European public debt crisis which started at the beginning of 2010 hardly influenced the demand for German euro notes.

With the reservation of the small sample period, the vector error correction models seem to be rather stable. In line with the low interest rate (semi-)elasticities, we do not expect significant portfolio shifts into cash owing to the currently very low level of interest rates. This is confirmed by financial accounts data on the acquisition of financial assets in Germany (until the end of 2014). By contrast, the declining value of the euro exchange rate since 2014 due to the unconventional monetary policy measures by the Eurosystem should exert a significant negative effect on the demand for banknotes.

We did not succeed in finding an appropriate structural model for German medium euro notes. We presume that several years of additional observations are necessary to fix this problem. Instead, we present RegARIMA models used by the Deutsche Bundesbank to forecast the number of German €50 notes and €100 notes within the scope of the annual banknote production planning in the Eurosystem. These models account for the global financial crisis and seasonal factors.

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