Investor Overconfidence and the Forward Discount Puzzle

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June 2005

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Current Version: December 2007

*We thank Bill Branch, Henry Cao, Mark Chen, Carol Osler, Nagpurnanand Prabhala, Stefan Krause, and participants at the Dice Finance seminar at Ohio State University, the finance seminar at the University of Maryland at College Park, the Washington Area Finance Association Research Conference, the China International Conference in Finance, and Shanghai University of Finance and Economics Conference, for helpful comments.

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ABSTRACT

This paper offers an explanation for the forward discount puzzle in foreign exchange markets based upon investor overconfidence. In our model, overconfident individuals overreact to their information about future inflation differential. The spot and the forward exchange rates differentially reflect such overreaction; as a result, the forward discount forecasts reversal in the spot rate. With plausible parameter values, the model explains the magnitude of the forward discount puzzle and stylized facts about how the forward discount bias varies with time horizon and time-series versus cross-sectional test method. Furthermore, the model generates new empirical predictions about the relation between the forward discount bias to foreign exchange trading volume, exchange rate volatility and predictability, as well as the degree of violation of the relative Purchasing Power Parity.

1 Introduction

Nominal interest rates reflect investor expectations about future inflation. If investors rationally foresee future inflation, then currencies in which bonds offer high nominal interest rates should on average depreciate relative to low-nominal-interest-rate currencies. Furthermore, when the interest rate differential is higher than usual, the rate of depreciation should be higher than usual.

A strong empirical finding, however, is that at times when short-term nominal interest rates are high in one currency relative to another, that currency subsequently *appreciates* on average (see, e.g., surveys of Hodrick 1987, Lewis 1995, and Engel 1996). An equivalent finding is that the forward discount (defined as the difference between the forward and spot exchange rates) *negatively* forecasts subsequent exchange rate changes, a pattern known as the forward discount puzzle.[1]

The most extensively explored explanation for the forward discount puzzle is that it reflects time-varying rational premia for systematic risk (e.g., Fama 1984). However, the survey of Hodrick (1987) concludes that "we do not yet have a model of expected returns that fits the data" in foreign exchange markets; Engel (1996) similarly concludes that equilibrium models do not explain the strong negative relation between the forward discount and the future exchange rate change for any degree of risk aversion, even when nonstandard utility functions are employed.[2] Since rational risk pricing has not explained the forward discount bias, Engle (1996) suggests that an approach based upon imperfect rationality can potentially offer new insights about the

[1] The average slope coefficient in regressing future change in the log spot exchange rate on the forward discount across some 75 published estimates surveyed by Froot and Thaler (1990) is -0.88.

[2] For example, Bekaert (1996) finds that his habit formation model would require unrealistically volatile exchange rates to deliver exchange rate risk premia that are variable enough to explain the forward discount puzzle. Verdelhan (2006) proposes a rational habit formation model that can generate negative covariance between exchange rate variation and interest rate differentials. Burnside et al. (2006) provide new evidence suggesting that time-varying exchange risk premia does not explain the forward discount bias.

puzzle.

In this paper we propose an explanation for the forward discount puzzle based upon investor overconfidence, a well-documented psychological bias. According to DeBondt and Thaler (1995), overconfidence is “perhaps the most robust finding in the psychology of judgement.” Our model of overconfidence is based on a large body of evidence from cognitive psychological experiments and surveys indicating that people, including those from various professional fields, overestimate the accuracy of their judgments in various setting. Froot and Frankel (1989) provide evidence of overreaction in currency traders' expectations about future exchange rate depreciations. Oberlechner and Osler (2004) provide direct survey evidence that currency market professionals tend to overestimate the precision of their information signals.

A growing analytical and empirical literature has argued that investor overconfidence explains puzzling patterns in stock markets of return predictability, return volatility, volume of trading, and individual trading losses (see Hirshleifer (2001) for a recent review). If a systematic bias such as overconfidence causes anomalies in stock markets, it should also leave footprints in bond and foreign exchange markets, and vice versa. A behavioral explanation for anomalies is more credible if it can explain a range of patterns across different kinds of markets, thereby obviating the need to tailor a different theory for each anomaly and type of market.

In our model, overconfident individuals think that the precision of their information signal about the future inflation differential is greater than it actually is. As a result, investor expectations overreact to the signal. This causes both the forward and spot exchange rates to overshoot in the same direction. However, the consumption price level and the spot exchange rate are influenced by a transactions demand for money, whereas forward rates are more heavily influenced by speculative considerations, i.e., the expected return from holding domestic or foreign bonds. Therefore, biased expectations cause the forward rate to overshoot more than the spot rate, which implies that the forward discount serves as a measure of investor overreaction (and is, in a sense we will make precise, a better measure of overreaction than the forward or spot rates alone). Later, the overreaction in the spot rate is on average reversed. The forward discount is a predictor of this correction, and hence on average is (under reasonable parameter values) a negative predictor of future exchange rate changes.

The sign of the slope coefficient in a regression of the future spot rate change on the forward discount depends on two opposing effects. The overreaction effect described above favors a negative coefficient. On the other hand, if the information investors receive is authentic, and there is no overreaction in the spot rate, then a higher forward discount positively predicts future spot rate changes—this is the conventional effect that makes the empirical findings a puzzle. We show that over short horizons the overreaction-correction effect dominates, but over long horizons the positive conventional effect eventually dominates. Intuitively, over time mispricing in the spot exchange rate attenuates, whereas the effects of foreseeable differences in expected inflation rates across countries accumulate. This model implication is consistent with evidence that the forward discount regression coefficients switch from negative to positive at long horizons (e.g., Gourinchas and Tornell 2004, Meredith and Chinn 2004).

Similarly, there is a tendency for countries with high average interest rates relative to the U.S. over long periods of time also to have high average depreciation relative to the dollar (e.g., Cochrane 1999). Our model predicts this contrasting pattern in cross-sectional versus time-series regressions. A substantial part of the long-run inflation differential across countries is foreseeable, fairly constant, and not a matter of subjective judgment—what we call the known component of the inflation differential. The long-run mean interest rate differentials between countries tend to reflect heavily the known component, and to average out the transitory effects of mispricing. In contrast, a time-series regression of exchange rate depreciation on the forward discount tends to eliminate (throw into the constant term) the known, predictable components of the inflation differentials and focus on judgment-sensitive fluctuations in expectations.

Our approach allows for, but does not require, violations of relative Purchasing Power Parity (PPP). In our model overreaction in investor expectations affects both price levels and spot exchange rates. In consequence, there is overshooting in both exchange rates and price levels even if the exchange rate and price levels are perfectly aligned (so that PPP holds). We show that deviations from PPP alone cannot generate the forward discount bias for reasonable parameter values. However, when investors are overconfident, PPP violations interact with overconfidence to affect the magnitude of the bias. Specifically, the greater the overshooting in the spot exchange rate relative to the inflation differential, the more negative will be the slope coefficient in the forward discount regression.

A few recent papers have provided insightful analyses of how investor irrationality can potentially explain the forward discount puzzle. An early application of irrationality to foreign exchange markets is provided by Frankel and Froot (1990a). Mark and Wu (1998) apply the noise trader model of DeLong et al. (1990), where the distortion in noise traders' belief is exogenously specified and only occurs in the first moment of exchange returns: noise traders overweigh the forward discount when predicting future changes in the exchange rate. Gourinchas and Tornell (2004) offer an explanation of the forward discount puzzle based upon a distortion in investors' beliefs about the dynamics of the forward discount: they overestimate the importance of transitory shocks relative to persistent shocks. The paper provides some empirical verification of such distortion, but is agnostic as to the source of the distorted beliefs.[3]

[3]Bacchetta and Wincoop (2006, 2007) propose a middle ground between behavioral and fully rational risk-

Some commentators (e.g., McCallum 1994) emphasize the need for behavioral approaches to provide an underlying motivation for their assumptions about the form of irrationality or noise trading. We agree that this serves as an important discipline, because in the absence of restrictions, some distribution of noise trading can always be found to fit any empirical fact about prices. Our paper differs from past behavioral explanations for the forward discount puzzle in possessing a combination of features: assumptions about belief formation based upon evidence from psychology, explicit modelling of the belief formation process, and explicit modelling of the equilibrium forward discount without making exogenous assumptions about its dynamics. Furthermore, our approach provides a distinctive additional set of predictions about the forward discount bias, and the psychological bias that we assume has been shown to have realistic implications for security markets in general, not just the foreign exchange market.

Specifically, we show that the average negative relationship between the forward discount and future exchange rate changes is a natural consequence of a well-documented cognitive bias—overconfidence. We derive price relationships from investor beliefs, rather than directly making assumptions about trading behavior. Furthermore, we do not assume that belief errors have a particular correlation with the forward discount, but rather derive this correlation from the psychological premise.

Our psychology-based explanation of the forward discount puzzle is not developed ex post specifically for the purpose of solving this puzzle. Investor overconfidence has been used to explain a range of other cross-sectional and time-series patterns of return predictability in securities markets as well as patterns in volume, volatility, and investor trading profits.[4] Thus, our ap-

premium explanations for the forward discount puzzle. In their approach, the forward discount bias can result from a combination of infrequent and partial information processing.

[4]Individual investors trade actively and on average lose money on their trades, which is consistent with overconfidence (e.g., DeBondt and Thaler, 1985, Barber and Odean, 2000). Investor overconfidence has been proposed as an explanation for several patterns in stock markets, such as aggressive trading and high return volatility (e.g., Odean 1998, Dumas, Kurshev, and Uppal 2006), price momentum, long-term reversals, and underreactions to

proach offers a parsimonious explanation for a range of anomalies in asset markets, which helps avoid possible concerns about overfitting the theoretical model to the anomaly being explained.

Our model also provides several distinctive new empirical predictions about the forward discount regressions. It predicts that the forward discount regression coefficient will be more negative in periods that are subject to greater investor overconfidence. Such periods can be identified by high trading volume, exchange rate volatility, or foreign exchange forecast dispersion. In addition, our model predicts a negative relationship between the magnitude of the forward discount bias and the exchange rate predictability, and a positive relationship between the bias and the sensitivity of exchange rate changes to the inflation differential.

In recent years currency carry trades have become very popular. This trading strategy involves borrowing money in a country with low interest rates and investing the money in another country with higher rates. As reckoned by some economists, as much as $1 trillion may be staked on the yen carry trade.[5] In our model currency carry trade emerges as a profitable strategy for rational investors when other investors are overconfident.

2 The Basic Idea

Existing models of overconfidence in securities markets imply that returns are predictable based on current market prices and fundamental measures. We review the intuition behind such predictability, and contrast it with the intuition developed here, which reflects the monetary aspect of the forward discount puzzle. We will show that the intuition underlying previous overconfidence models can explain why the forward discount regression yields slope coefficients less than one, but that the distinctive aspects of the foreign exchange setting explains why a

corporate events (e.g., Daniel, Hirshleifer and Subrahmanyam, 1998; 2001), return comovements (Peng and Xiong, 2006), and speculative price bubbles (Scheinkman and Xiong, 2003).

[5] "What keeps bankers awake at night?" *Economists*, February 1 2007.

negative regression coefficient (the forward discount puzzle) is possible.

Evidence of long-term stock market return reversals (e.g., DeBondt and Thaler, 1985), and that market-to-book and other price/fundamental ratios negatively predict future stock returns (e.g., Rosenberg, Reid and Lanstein, 1985) has often been interpreted as representing market overreaction. Expressed logarithmically, there is a negative slope coefficient in the regression

$$r_{t+1} = \Delta m_{t+1} = \alpha + \beta(m_t - b_t) + v_{t+1}, \tag{1}$$

where m and b are the log market value and book value respectively, and r_{t+1} is the subsequent stock return. Suppose for simplicity that the current book value b_t is an unbiased predictor of the firm's terminal cash flow based upon the existing information. The current market value reflects additional information about future payoffs, and any overreaction to such information. So the current market/book ratio $m_t - b_t$ contains information about overreaction. Overreaction, as proxied by $m_t - b_t$, and its eventual reversal can cause the slope coefficient to be negative in (1).[6] On the other hand, in the regression

$$\triangle b_{t+1} = \alpha + \beta(m_t - b_t) + w_{t+1}, \tag{2}$$

we expect to find $0 < \beta < 1$ so long as the market price reflects some meaningful additional information about the firm's terminal cash flow beyond that contained in the book value. The favorable information that $m_t > b_t$ predicts a positive change $\triangle b_{t+1}$. Because of overreaction, m_t tends to overpredict b_{t+1}, resulting in a slope coefficient less than 1. In the special case where overconfident investors react to pure noise signals, β should be zero.

[6] Low $m_t - b_t$ indicates that the market is too pessimistic about a stock's prospects, pushing the stock price too low. The low price tends to correct subsequently, causing a positive average return (see, e.g., Daniel, Hirshleifer, and Subrahmanyam, 2001).

In the foreign exchange setting, there is subjective judgment involved in forecasting future inflation, which creates scope for overconfidence.[7] Both the spot exchange rate s_t (like the book value b_t in (3)) and the forward exchange rate f_t (like the market value m_t) contain information about future fundamentals, here the inflation differential. Suppose for the moment that the spot exchange rate, in analogy to the current book value, has little or no average bias, whereas the forward exchange rate, like the current stock market value, is subject to substantial misreaction. Then the forward discount regression

$$s_{t+1} - s_t = \alpha + \beta(f_t - s_t) + u_{t+1} \tag{3}$$

is similar to the regression (2) in the stock market context. When the forward rate is high relative to the spot rate, the market expects a relatively high inflation differential and exchange rate depreciation.

The coefficient β in (3) can be less than one, because the overconfidence-induced overreaction in investors' expectations causes the forward rate to rise more than the increase in the rational expected future spot exchange rate. We illustrate this effect (which is much weaker than the forward discount bias) in Figure 1. The upper half of Figure 1 plots the path of movement for the spot and forward exchange rates from date 0 to date 2, conditional on a positive date-1 signal about date-2 inflation differential. For ease of presentation, we assume that at date 0, the expected future inflation differential is zero so that spot exchange rate s_0 coincides with the forward rate f_0. The expected movement of the spot rate if there is no overreaction is depicted in the line segment from s_0 to s_1^R, and then the segment to the point labelled $\mathrm{E}_1^R[s_2]$; the R

[7] The existence of an active industry selling macroeconomic forecasts is consistent with our assumption that at least some investors believe they can obtain superior information about future inflation. Previous studies share our assumption that individuals believe they possess private information about aggregate factors (e.g., Subrahmanyam, 1991). It is not crucial for our purposes whether investors are correct in thinking that they possess superior information about inflation, so long as they overreact.

superscript indicates values under rational belief. If there were no overreaction in the forward rate and if the investor is risk neutral, the forward rate would be $f_1^R \equiv \mathrm{E}_1^R[s_2]$. The overreaction in the forward rate is shown in the steep segment running from s_0 to $f_1 > \mathrm{E}_1^R[s_2]$. Under our temporary assumption of no overreaction in the spot rate, the forward discount is the vertical difference $f_1 - s_1^R$. The symmetrical case of a negative signal is in the lower half of the figure.

Comparing the upper and lower halves of the figure, it is evident that a positive forward discount is associated with a higher expected future spot rate than a negative forward discount– i.e., that the β coefficient in the forward discount regression is positive. It is also evident that the coefficient is less than one; the overreaction in the forward rate implies that the variation in the independent variable, $f_1 - s_1^R$, is larger than the average variation in the dependent variable, $\mathrm{E}_1^R[s_2] - s_1^R = f_1^R - s_1^R$ by the amount of overreaction $f_1 - f_1^R$.

The greater the importance of overreaction relative to genuine information in the forward rate movement, the lower the β coefficient. As overreaction becomes extreme, the date 1 forward rate swings wildly relative to the future spot rate. Thus, when there is no overreaction in the spot rate, the forward discount regression coefficient approaches zero, but does not become negative. The puzzle remains: why (in contrast with stock market models) is the coefficient negative?

Our answer relies on a difference between the foreign exchange setting and the stock market setting. Whereas book value of a stock is an historically-determined quantity, the spot exchange rate is a market price, subject to its own misreaction.[8] For example, suppose that investors receive a signal about an increase in the U.S. relative to German inflation. The forward rate (Dollar/DM) rises, incorporating the expected depreciation of U.S. dollar. The spot rate rises

[8]The asset market approach to exchange rate determination has long recognized that exchange rate movements are primarily driven by news that changes expectations (see, e.g., Obstfeld and Rogoff (1996, p. 529)).

too (and may also overshoot), because investors who expect higher future U.S. inflation are less willing to hold dollars today. In our model, the inflation signal endogenously has a stronger effect on the forward rate than on the current spot rate. Therefore, the forward discount is positively related to overconfidence-induced overreaction in the spot exchange rate and predicts its subsequent correction. This effect can result in a negative slope coefficient. Whether it does so depends upon the balance between the traditional effect (the fact that the forward rate reflects information about future inflation) and the overreaction/correction effect.

This intuition is also illustrated in Figure 1. After a positive private signal (the upper branch of the figure), owing to overreaction, the forward rate rises above the level of the new expected spot rate $\mathrm{E}_1^R[s_2]$; the spot rate rises less, because consumption good price levels and spot exchange rates are influenced by a transactions demand for money, not just speculative concerns about future inflation rates; so the forward discount $f_1 - s_1$ is positive. Symmetrically, on the lower branch, the forward rate declines more than the spot rate, so that the discount is negative. At date 2 the overreaction in the spot rate corrects. If the spot overreaction is strong enough, then in the upper branch of the figure in which the forward discount is positive, on average the spot rate declines to $\mathrm{E}_1^R[s_2]$; and in the lower branch in which the forward discount is negative, on average the spot rate increases to $\mathrm{E}_1^R[s_2]$. Thus, the forward discount negatively predicts the change in the spot rate.

A common challenge to psychology-based approaches to securities markets anomalies is to explain how irrational investors can have an important effect if there are smart arbitrageurs. As we show in Section 4.4, when some investors are overconfident there is an opportunity for rational investors to profit from the currency carry trade, a strategy that exploits the forward discount bias.

However, the risk inherent in carry trades limits the extent to which risk-averse investors will engage in arbitrage.[9] The recent experience of Goldman Sachs' Global Alpha fund is a good example. The fund's carry trade between Japanese yen and Australian dollar led to big losses in August 2007. Froot and Thaler (1990) and Burnside et al. (2006) show that portfolio strategies designed to take advantage of the forward discount anomaly do not represent unexploited profit opportunities when there are market frictions and other practical constraints.

The behavioral finance literature offers several reasons why irrational investors do not necessarily lose money competing with the rational ones, and why even if irrational investors are prone to losing money, imperfect rationality can still influence price.[10] For example, Dumas, Kurshev, and Uppal (2006) show that that rational investors have limited ability to offset the effect of overconfident investors. In the foreign exchange context, uncertainty about a country's inflation rate is a systematic risk, so that even if the market prices reflect incorrect expectations, rational investors are not presented with a risk-free arbitrage opportunity (on imperfect arbitrage of systematic misvaluation, see Daniel, Hirshleifer, and Subrahmanyam (2001)). Furthermore, even if less sophisticated currency users on average lose relative to a set of smart speculators, less sophisticated individuals will still need to hold money balances, so their money demands will still play a role in determining equilibrium price levels and therefore spot and forward exchange rates. Hence, we do not expect complete elimination of the forward discount bias.

[9] Predictability in excess currency returns implied by the forward discount puzzle is low (with R^2 typically less than 10%) and largely overshadowed by uncertainty about future exchange rate (Bacchetta and Wincoop (2006)). Trading on this excess return predictability involves high risk.

[10] Reasons that imperfectly rational investors may earn high expected profits and/or remain important include a possible greater willingness of overconfident investors to bear risk or to exploit information aggressively, limited investment horizons of the arbitrageurs, wealth reshuffling across generations, and the existence of market frictions (see Hirshleifer (2001) for a discussion of these issues).

3 The Model

The currencies of Countries A and B can be exchanged costlessly. Each country's products can only be purchased using its home currency.

3.1 Decisionmakers

There are two groups of decision makers. One consists of individuals who receive information signals about future money growth, and are overconfident in the sense that they overestimate the precision of their signal. The other group consists of individuals who do not receive signals and are thus rational (not overconfident). The inclusion of non-signal-receiving individuals allows us to interpret the signals received by informed individuals as 'private', and therefore overconfidence-inducing (Daniel, Hirshleifer and Subrahmanyam, 1998). For modelling convenience, we assume that the overconfident individuals are risk neutral, while the rational individuals are risk averse. This implies that prices are determined solely by the overconfident investors.[11]

In the rest of the paper, $\mathrm{E}^C[-]$ denotes expectations taken with respect to the beliefs of the overconfident investors. The information structure underlying the expectations will be detailed in Section 3.4.

[11] In a more general setting where all investors are risk averse, prices reflect a weighted average of the beliefs of different investors, and therefore still reflect overconfidence; see, e.g., Daniel, Hirshleifer and Subrahmanyam (2001). Similar results to those derived here would apply in such a setting. Similar results also would apply if there are some risk averse, fully rational "arbitrageurs" who received information signals. Only when the fraction of individuals who are fully rational approaches one would the effects described in this paper vanish. In the more general settings, risk is priced as well; such risk effects are not essential to our argument.

3.2 Money Markets

We assume a typical Cagan money demand function in logarithmic form,

$$\begin{aligned} m_t^d - p_t &= c - \alpha \mathrm{E}_t^C[\pi_{t+1}] \\ m_t^{d*} - p_t^* &= c^* - \alpha^* \mathrm{E}_t^C[\pi_{t+1}^*], \end{aligned}$$

where m_t^d and p_t are respectively the log money demand and price level in country A at date t, $\pi_{t+1} = p_{t+1} - p_t$ is the realized inflation from date t to $t+1$ in country A. An asterisk denotes a country B (foreign country) variable. The constants c and c^* represent the effect of the output and the real interest rate which are assumed to be fixed in the short run. Constant parameter $\alpha > 0$ measures the sensitivity of money demand to inflation expectations. For simplicity, we assume that $\alpha = \alpha^*$.

The log money supplies in both countries, m_t and m_t^*, are exogenously determined by the monetary authorities. We can view the exogenous money supplies as nominal wealth endowments in each period to the individuals. We define money growth rates in countries A and B from date $t-1$ to t, respectively as

$$\begin{aligned} \mu_t &= m_t - m_{t-1} \\ \mu_t^* &= m_t^* - m_{t-1}^*. \end{aligned}$$

The money growth differential between the two countries at date t, $\bar{\mu}_t \equiv \mu_t - \mu_t^*$, is the economic fundamental in our model. The money markets are continuously equilibrated. Money market equilibrium requires that $m_t^d = m_t$ and $m_t^{d*} = m_t^*$ for all t.

We define the *realized* inflation differential across the two countries on date t as

$$\bar{\pi}_t \equiv \pi_t - \pi_t^* = \triangle p_t - \triangle p_t^*.$$

The Cagan demand function implies that the date t realized inflation differential is related to the realized money growth differential as follows:

$$\bar{\pi}_t = \bar{\mu}_t + \alpha z_t, \tag{4}$$

where $z_t \equiv \mathrm{E}_t^C[\bar{\pi}_{t+1}] - \mathrm{E}_{t-1}^C[\bar{\pi}_t]$ is the *revision* of the expected inflation differential as new information arrives. Equation (4) implies that the realized inflation differential responds to changes in expectations of future inflation differential. If informed investors expect a higher inflation differential in the future, the inflation differential starts to widen today. How much price levels move today depends on the sensitivity of price indices to changes in inflation expectations, as reflected in α.

3.3 Spot Exchange Rates and the Forward Discount

The date t spot exchange rate depreciation is defined as $\triangle s_t \equiv s_t - s_{t-1}$, where s_t is the date t log spot exchange rate (the price of one unit of currency B in terms of currency A). In our base model we assume that the relative purchasing power parity (PPP) holds on average

$$\triangle s_t = \bar{\pi}_t + e_t, \tag{5}$$

where the error term e_t has mean zero, variance V_e and is uncorrelated with other variables in the model. We later extend the model to allow short-run deviations from the relative PPP and the long-run convergence to it.

Bonds denominated in currency A and currency B are traded. The log nominal returns on one-period bonds, i.e., the nominal interest rates, follow the Fisher equation:

$$\begin{aligned} i_t &= r_t + \mathrm{E}_t^C[\pi_{t+1}] \\ i_t^* &= r_t^* + \mathrm{E}_t^C[\pi_{t+1}^*], \end{aligned}$$

where r_t and r_t^* are log real rates of return on the bonds of countries A and B, respectively. For simplicity, and to focus on the market's ability to process information about future inflation, we assume that $r_t = r_t^*$. Thus, the determination of real rates is exogenous to the model.

Covered interest rate parity, a standard arbitrage condition in the forward exchange market, implies that the one-period forward discount is equal to the nominal interest rate differential, which is also the expected inflation differential in our model.

$$d_t \equiv f_t - s_t = i_t - i_t^* = \mathrm{E}_t^C[\bar{\pi}_{t+1}]. \tag{6}$$

3.4 Information Structure and Signals

Without loss of generality, we assume that on the initial date 0, $\triangle s_0 = \bar{\pi}_0 = \bar{\mu}_0$, where $\bar{\mu}_0$, a constant, represents the long-run unconditional mean money growth differential between countries A and B. Let η_t be the IID zero-mean innovation in the realized money growth differential on date t. Thus, on date 1 we have $\bar{\mu}_1 = \bar{\mu}_0 + \eta_1$. A persistent shock u in the money growth differential process arrives on date 2. Therefore,

$$\bar{\mu}_t = \bar{\mu}_0 + \eta_t + u, \quad t \geq 2. \tag{7}$$

We assume $u \sim N(0, V_u)$, and u is independent of η.[12]

On date 1, informed individuals receive a noisy signal about u that takes the form

$$\sigma = u + \epsilon, \tag{8}$$

where ϵ, the signal noise, is distributed as $N(0, V_\epsilon)$. Let the information precision be $\nu_\epsilon \equiv 1/V_\epsilon$ and $\nu_u \equiv 1/V_u$. We assume that the informed individuals overestimate the precision of their 'private' signals. In other words, they believe that the variance of the signal noise is lower than the true level: $V_\epsilon^C < V_\epsilon$, where a superscript C denotes an overconfident perception. This is equivalent to $\nu_\epsilon^C \equiv 1/V_\epsilon^C > \nu_\epsilon$, which implies that the overconfident investors take the noisy signal as more informative than it actually is, and overreact to it when they revise their expectations at date 1. On date 2, when the shock u is realized, overconfident investors correct their date 1 expectation errors.

4 The Forward Discount Bias

In this section, we derive the date 1 forward discount and the date 2 spot exchange rate change. In our model, overreaction in investor expectations affects country price levels (inflation) as well as spot and forward exchange rates. We show that overconfidence-induced overreaction to the money growth news and its subsequent correction can explain the negative relationship between the forward discount and the future exchange rate change.

[12] We focus on a persistent shock in the money growth differential because the inflation differentials and the forward discount, both driven by the money growth differential in our model, are known to be very persistent.

4.1 Expectations and Date 1 Spot and Forward Exchange Rates

After receiving the date 1 signal, informed investors update their expectations about the future money growth differential in a Bayesian fashion. Their expectations, however, are subject to the overconfidence bias. We define investor expectation sensitivities as follows.

$$\lambda^C \equiv \frac{\nu_\epsilon^C}{\nu_u + \nu_\epsilon^C}, \quad \lambda^R \equiv \frac{\nu_\epsilon}{\nu_u + \nu_\epsilon}, \quad \text{and} \quad \gamma \equiv \frac{\lambda^C - \lambda^R}{\lambda^C}, \tag{9}$$

where the superscript C and R denotes the overconfident and rational perceptions. Since $\nu_\epsilon^C > \nu_\epsilon$, it follows that $\lambda^C > \lambda^R$ and $0 < \gamma < 1$. In addition, γ increases monotonically with the degree of overconfidence $\nu_\epsilon^C/\nu_\epsilon$. Thus, the impact of overconfidence is captured by γ.

By (8), the overconfident individuals' date 1 expectation of next period's money growth differential conditional on the observed signal σ is

$$\mathrm{E}_1^C[\bar{\mu}_2|\sigma] = \bar{\mu}_0 + \lambda^C\sigma.$$

The fully rational conditional expectation is

$$\mathrm{E}_1^R[\bar{\mu}_2|\sigma] = \bar{\mu}_0 + \lambda^R\sigma = \bar{\mu}_0 + (1-\gamma)\lambda^C\sigma.$$

The difference between the two expectations, $\gamma\lambda^C\sigma = (\lambda^C - \lambda^R)\sigma$, is due to the overreaction in overconfident individuals' perceptions.

By applying the information structure in Section 3.4 and equations (4) to (6), we have

$$\bar{\pi}_1 = \bar{\mu}_1 + \alpha\lambda^C\sigma, \tag{10}$$

$$\triangle s_1 = \bar{\mu}_1 + \alpha\lambda^C\sigma + e_1, \tag{11}$$

$$d_1 = \mathrm{E}_1^C[\bar{\pi}_2] = \bar{\mu}_0 + \lambda^C\sigma. \tag{12}$$

We can see that following a positive signal about country A's future money growth rate ($\sigma > 0$), the date 1 inflation differential, the spot exchange rate, and the forward discount all move upward. All three variables contain an element of overconfidence-induced overreaction, as indicated by λ^C.

The intuition behind these equations is as follows. The signal about future money growth differential is informative about the relative value of currency A to currency B. In anticipation of an increase in inflation in country A, an informed individual will tend to hold less currency A, which leads to an immediate depreciation of currency A in the spot market. Meanwhile the expectation of future higher inflation in country A also makes currency A to be sold forward at a discount. Since the forward rate is more forward-looking and sensitive to expectations, the forward rate rises even more than the spot rate. Therefore, the spread between the forward and spot rates (i.e., the forward discount) contains the overconfidence-induced overreaction in the spot exchange rate.

4.2 Date 2 Exchange Rate Depreciation

A shock u to the money growth differential is realized on date 2 and persists to date 3. On date 2, the expected money growth differential over the next period is $\mathrm{E}_2^C[\bar{\mu}_3] = \bar{\mu}_0 + u$. Let

$$\delta \equiv \mathrm{E}_1^R[u] - u = \lambda^R \sigma - u \tag{13}$$

be the error in the rational expectations forecast of money growth differential. δ is orthogonal to the date 1 information set. Applying equation (4) on date 2 and substituting (13), we obtain the following proposition. Proof is provided in the appendix.

Proposition 1 *The date 2 spot exchange rate depreciation is a linear function of the long-run average money growth differential $\bar{\mu}_0$ and the forward discount d_1. Specifically,*

$$\triangle s_2 = \beta_0 \bar{\mu}_0 + \beta_1 d_1 + v_2, \tag{14}$$

where

$$\begin{aligned} \beta_0 &= (1+\alpha)\gamma, \\ \beta_1 &= 1 - (1+\alpha)\gamma, \\ v_2 &= \eta_2 - (1+\alpha)\delta + e_2, \end{aligned} \tag{15}$$

with $\mathrm{E}_1^R[v_2] = 0$ *and* $\mathrm{cov}(d_1, v_2) = 0$.

Proposition 1 is a key result of our paper. It shows that when change in the spot exchange rate is regressed on the lagged one-period forward discount, the slope coefficient is given by β_1, which can be decomposed into two terms. The first term of β_1 is unity, which represents the conventional effect (uncovered interest parity). The second term reflects investor overconfidence.

When there is no overconfidence (i.e., $\gamma = 0$), $\beta_1 = 1$, $\beta_0 = 0$. Uncovered interest rate parity holds. When there is overconfidence (i.e., $\gamma > 0$), β_1 is less than unity and becomes negative for sufficiently high level of investor overconfidence

$$\beta_1 < 0 \;\text{ if }\; \gamma > \frac{1}{1+\alpha}.$$

The more overconfident investors are, the more negative the relationship between the forward discount and the subsequent exchange rate depreciation.

There are two opposing forces influencing the relation between the forward discount and the subsequent exchange rate depreciation. On one hand, a higher inflation differential between countries A and B, when realized, depreciates currency A ($\triangle s > 0$). This is the conventional effect. On the other hand, the mispricing in the spot rate due to overconfidence eventually gets corrected, which promotes an appreciation of currency A ($\triangle s < 0$). This is the overreaction-correction effect, which favors a negative coefficient.

The sign of β_1 depends on which effect dominates. If the information signal is pure noise, the overreaction-correction effect must dominate, leading to the forward discount anomaly. Even for meaningful signals, greater overconfidence increases overreaction in the spot rate, thus strengthening the overreaction-correction effect. The slope coefficient becomes negative when the level of investor overconfidence is sufficiently high. Furthermore, by (15),

$$\frac{\partial \beta_1}{\partial \gamma} = -(1+\alpha) < 0. \tag{16}$$

We summarize the discussion above in the following proposition.

Proposition 2 *When investor overconfidence is sufficiently strong, the slope coefficient on the forward discount in a time series regression (equation (3)) is negative. The greater the degree of investor overconfidence, the more pronounced the forward discount bias.*

We end this section by illustrating the magnitude of the forward discount bias generated by our model under realistic model parameters. We pick $\alpha = 4$ based on many empirical estimates of Cagan's model (e.g., Cagan (1956), Barro (1970), Goodfriend (1982), Phylaktis and Taylor (1993)). For estimate of the overconfidence parameter γ, we rely on Friessen and Weller (2006). They use analyst earnings forecast data to estimate a model in which analysts are overconfident about the precision of their information, in the same manner as our model assumption. They find strong evidence that analysts are overconfident. The overconfidence parameter γ in our model equals $[a/(1+a)][\nu_u/(\nu_u+\nu_\epsilon)]$, where a is the overconfidence parameter defined in equation (17) in Friesen and Weller (2006). Estimates of a in various specifications of Friesen and Weller (2006) imply that $a/(1+a)$ is in the range of 0.4 to 0.6. Since $\nu_u/(\nu_u+\nu_\epsilon)$ is between 0 and 1, we consider values for γ between 0.3 and 0.5 to be reasonable. Under these parameter values, the theoretical slope coefficient β_1 in equation (15) ranges from -0.5 (when $\gamma = 0.3$) to -1.5 (when $\gamma = 0.5$). Thus, with plausible parameter values, our model can generate forward discount bias that closely matches the magnitude observed in the data.

4.3 Further Empirical Implications

A direct empirical implication of Proposition 2 is that the magnitude of the forward discount bias will change over time as the level of investor overconfidence shifts. The forward discount bias will be more pronounced in periods in which investors are more overconfident. Behavioral finance research on stock markets has found that overconfidence-induced overreaction is associated with excessive trading volume, high return volatility, and large cross-firm valuation dispersion.[13]

Prediction 1 *The forward discount bias is more pronounced in periods of abnormally high trading volume, excess exchange rate volatility (relative to the volatility of money growth rate or inflation rate), and high dispersion of exchange rate forecasts in foreign exchange markets.*

In our model, foreign exchange investors use their information signals to form expectations of future inflation differential and exchange rate movements. A larger λ^R, which is equivalent to a higher signal-to-noise ratio, corresponds to more predictability in inflation and exchange rates. By (16) and the definition of γ we have

$$\frac{\partial \beta_1}{\partial \lambda^R} = \frac{\partial \beta_1}{\partial \gamma} \frac{\partial \gamma}{\partial \lambda^R} > 0.$$

This leads to the following prediction.

Prediction 2 *The forward discount bias is less pronounced when exchange rate movements are more predictable.*

Bansal and Dahlquist (2000) find that the forward discount bias is weaker for currencies in high-inflation countries than for those in low-inflation countries. Using survey data of exchange rate forecasts, Chinn and Frankel (1994, 2002) find that exchange rate expectations appear much less biased for high-inflation currencies. They argue that the finding is intuitive, since it is relatively easy to guess the direction of changes in exchange rates when inflation is very high. Also, they find that forecasts for minor currencies exhibit less bias than those for major currencies (i.e., UK, DM, Yen, and Swiss Franc). If the exchange rate movements are more predictable for

[13]E.g., Odean (1998), Daniel, Hirshleifer, and Subrahmanyam (2001), Gervais and Odean (2001), Glaser and Weber (2007), Statman, Thorley and Vorkink (2005), and Jiang (2005).

high-inflation currencies, then Prediction 2 is consistent with Bansal and Dahlquist's finding.[14]

In our model, overconfidence causes investors' expectations to overreact. The forward discount reflects such overreaction and predicts its later correction. By Proposition 1, the exchange rate depreciation forecast error made by overconfident investors is

$$\mathrm{E}_1^C[\Delta s_2] - \Delta s_2 = -(1+\alpha)\gamma\bar{\mu}_0 + (1+\alpha)\gamma d_1 - v_2,$$

with $\mathrm{E}_1^R[v_2] = 0$ and $\mathrm{cov}(d_1, v_2) = 0$. Thus, when we regress overconfident investors' forecast error on the lagged forward discount, the slope coefficient is $(1+\alpha)\gamma > 0$.

Prediction 3 *When investors are overconfident, their prediction error is positively correlated with the forward discount.*

This implication is supported by the empirical findings in Froot and Frankel (1989) and Frankel and Chinn (1993). For example, Froot and Frankel (1989) examine the following regression

$$\Delta\hat{s}^e_{t+1} - \Delta s_{t+1} = \alpha_1 + \hat{\beta}_1(f_t - s_t) + v_{t+1},$$

where $\Delta\hat{s}^e_{t+1}$ is investors' expected change (based on survey data) in the spot exchange rate between date t and $t+1$. Thus the left-hand-side of the equation is the expectation error. The authors find that $\hat{\beta}_1$ is significantly greater than zero, and this finding is robust.

[14]A greater predictability in exchange rate movements in emerging economies is possible because in these economies a greater fraction of the variation in exchange rate movements may come from predictable money supply growth rather than unpredictable business cycle effects. Recessions are notoriously hard to predict. Money supply growth is predictable based upon both historical inflation patterns and observable information such as the extent of deficit spending. To the extent that emerging economies tend to have greater variation in expected inflation, the predictable component of exchange rate variation is increased relative to the unpredictable business cycle component.

Finally, in our model,

$$f_2 - f_1 = (f_2 - s_2) + (s_2 - s_1) - (f_1 - s_1) = d_2 - d_1 + \triangle s_2,$$

where $d_2 = \mathrm{E}_2^C[\bar{\pi}_3] = \bar{\mu}_0 + u$. It follows that the regression coefficient of $f_2 - f_1$ on d_1 is $\beta_1 - \gamma$, which is more negative than β_1 when investors are overconfident ($\gamma > 0$).

Prediction 4 *When investors are overconfident, the forward discount is more negatively related to the subsequent change in the forward exchange rate than to the subsequent change in the spot exchange rate.*

The intuition for this prediction is that the overconfidence-induced overreaction makes the forward rate overshoot more than the spot rate because the forward rate is more sensitive to expectations about future inflation differential. Thus, there is more subsequent correction in the forward rate than in the spot rate.

4.4 Currency Carry Trade

We have shown that the overconfidence-induced investor overreaction to shocks in money growth differential can cause mispricing in the currency markets (in spot as well as forward exchange rates). How does mispricing affect the risk-averse rational investors' optimal portfolio choice between the domestic bonds and the foreign bonds? Intuitively, the forward discount bias implies excess returns to a strategy that goes long on the high-nominal-interest-rate bond (e.g., domestic country A bond) and short on the low-nominal-interest-rate bond (e.g., foreign country B bond). This strategy is called currency carry trade. We will show that the risk-averse rational investors will engage in carry trade, while the overconfident investors will be indifferent between domestic bonds and foreign bonds.

Assume that the rational domestic investor has CARA utility with risk aversion coefficient ϕ. Let his date 1 wealth be normalized to one unit in country A's currency (which is without loss of generality because of the CARA utility). Let ω and $1-\omega$ be the fraction of his wealth invested in the one-period domestic bond and foreign bond (which would be converted to domestic currency at maturity). Thus, the domestic rational investor's date 2 wealth is

$$W_2 = \omega(1+i) + (1-\omega)(1+i^*)(1+\triangle s_2),$$

where i and i^* are the risk-free rates of return for the one-period domestic bond and foreign bond, respectively, and the exchange rate depreciation $\triangle s_2$ is given by (14) in Proposition 1. The optimal weight ω in the domestic bond for the rational investor maximizes $\mathrm{E}_1^R[W_2] - \left(\frac{\phi}{2}\right) Var(W_2)$. The solution is

$$\omega = 1 + \frac{(i-i^*) - \mathrm{E}_1^R[\triangle s_2]}{\phi(1+i^*)Var(\triangle s_2)}. \tag{17}$$

By Proposition 1, $\mathrm{E}_1^R[\triangle s_2] = \beta_0\bar{\mu}_0 + \beta_1 d_1$, with $\beta_0 = 1-\beta_1 = (1+\alpha)\gamma$. Substituting these and $i - i^* = d_1$ into (17), we obtain

$$\omega = 1 + \frac{(1+\alpha)\gamma\lambda^C\sigma}{\phi(1+i^*)Var(\triangle s_2)}. \tag{18}$$

Thus, when there is investor overconfidence ($\gamma > 0$) and when a signal $\sigma > 0$ arrives at date 1 indicting that the future money growth differential (thus the interest rate differential) between domestic and foreign countries is going to be higher, (18) suggests that the rational investor optimally chooses $\omega > 1$. That is, the rational investor will borrow money in the foreign bond market and leverages up his position in the domestic bond. This is exactly the carry trade.

In contrast, the overconfident investors are indifferent between domestic bonds and foreign bonds, because they perceive that the uncovered interest rate parity holds: $\mathrm{E}_1^C[\Delta s_2] = d_1$.[15] This is consistent with the empirical finding of Frankel and Froot (1990b) that when ex-ante measure of expected exchange rate change (based on survey data) instead of the ex post realizations is used as the dependent variable in regression (3), the coefficient on the forward discount β_1 is estimated to lie in the vicinity of +1.

4.5 Relative Predictive Power of Different Spot Rate Predictors

We have shown that investor overconfidence implies that the forward discount can negatively predict spot exchange rate changes, because the forward discount reflects overreaction in the spot rate and predicts its subsequent correction. However, similar reasoning implies that other variables that reflect mispricing, such as the inflation differential, the forward rate, the spot rate, the latest change in the spot or forward rate, also predict exchange rate changes. In this section, we show that when investors are overconfident, the forward discount is a stronger predictor of the subsequent spot rate change than the alternatives.

Proposition 3 *When investors are overconfident, the forward discount is the strongest predictor of subsequent exchange rate changes (highest R^2) among the possible alternatives suggested by our model (the forward discount, the inflation differential, the forward rate, the spot rate, the latest change in the forward rate, and the latest change in the spot rate).*

The proof of this proposition is provided in the appendix. Intuitively, the innovation in the money growth differential at date 1, η_1, is independent of the future money growth differential and unrelated to the private signal σ. Thus, for the purpose of predicting future spot exchange

[15] This follows from Proposition 1. The key is that $\mathrm{E}_1^C[\delta] = \lambda^R\sigma - \lambda^C\sigma = -\gamma\lambda^C\sigma$, and thus $\mathrm{E}_1^C[v_2] = -(1+\alpha)\mathrm{E}_1^C[\delta] = -(1+\alpha)\gamma\lambda^C\sigma$.

rates, η_1 is noise: it is unrelated to the overconfidence-induced mispricing in the spot rate. The money growth surprise η_1 is reflected in the realized inflation differential $\bar{\pi}_1$, spot rate s_1, forward rate f_1, and latest changes in these rates $\triangle s_1$ and $\triangle f_1$, but is differenced out from the forward discount $f_1 - s_1$. Thus, the forward discount, as a purer measure of the spot rate overreaction, has more predictive power in forecasting the future correction of such overreaction.

Prediction 5 *In univariate regressions of exchange rate changes on (1) the forward discount, (2) the inflation differential, (3) the forward rate, (4) the spot rate, (5) the latest change in the forward rate, and (6) the latest change in the spot rate, the forward discount regression will have the highest* R^2*.*

4.6 Long-Horizon Forward Discount Regressions

In Section 4.2, we show that there can be a negative relationship between the one-period forward discount and the subsequent one-period change in the spot exchange rate. We now examine the relation between the forward discount and the future spot rate change in a longer-horizon regression. Specifically, in a regression of the two-period change in the spot exchange rate $s_3 - s_1$ on the two-period forward discount $d_{1,3} \equiv f_{1,3} - s_1$, where $f_{1,3}$ is the forward exchange rate for a two-period forward contract, we examine whether the slope coefficient will be more or less negative than that in (14). Our result is summarized in the following proposition (proof in the appendix).

Proposition 4 *In the two-period forward discount regression,*

$$s_3 - s_1 = \beta_0' \overline{\mu}_0 + \beta_1' d_{1,3} + v_3,$$

where the slope coefficient is

$$\beta_1' = 1 - (1 + \alpha/2)\gamma.$$

The regression slope coefficient in the two-period regression is less negative than that in the one-period regression:

$$\beta_1 - \beta_1' = -\alpha\gamma/2 < 0.$$

Proposition 4 implies that when investors are overconfident, the two-period forward discount is still a biased predictor of the subsequent two-period exchange rate depreciation. However, the forward discount bias becomes less pronounced in the longer-horizon regression. This implication is consistent with the empirical findings of Gourinchas and Tornell (2004) (using 3-month, 6-month and 12-month forward discounts), and Meredith and Chinn (2004) (using 5 and 10 year forward discounts).

Intuitively, the sign of the slope coefficient in the forward discount regression depends on the relative strength of two opposing effects: the conventional effect (UIP) and the overreaction-correction effect. At short horizons, the overreaction-correction effect tends to dominate. In contrast, there are fairly objective and well understood differences in countries' expected money growth rates that can persist over very long periods of time. Therefore, at longer horizons the traditional effect tends to dominate.

The same intuition can be applied to the relationship between long-run average exchange rate depreciations and the long-run average forward discount. Substituting (12) into (14), and taking the unconditional expectations of both sides under the empirical probability measure, we have

$$\mathrm{E}[\triangle s_2] = (\beta_0 + \beta_1)\bar{\mu}_0 = \mathrm{E}[d_1], \tag{19}$$

Equation (19) implies that although the short-term forward discount negatively predicts the subsequent exchange rate depreciation, the long-run average forward discount correctly predicts the long-run average future exchange rate depreciation. In other words, our model implies that investors can earn excess returns by holding bonds from countries whose nominal interest rates are *temporarily higher than usual* relative to the interest rates of other countries, not by holding bonds from countries with high nominal interest rates. This is consistent with the empirical findings that countries with steadily higher interest rates (than that in the U.S.) have steady currency depreciations (against the U.S. dollar), as predicted by UIP (e.g., Cochrane 1999).

Prediction 6 *The forward discount bias becomes less pronounced (1) in a longer-horizon regression; and (2) in a regression of long-run average one-period exchange rate changes on long-run average one-period forward discount.*

4.7 Cross-Sectional Forward Discount Regressions

So far we have considered time-series forward discount regressions. In this section, we turn to the implications of our model for cross-sectional forward discount regressions. Consider the exchange rates between a fixed home country and N foreign countries, denoted by j, $j = 1, 2, ..., N$. As before, the exchange rates are defined as the prices of foreign currencies in units of the domestic currency. We make the same assumptions as in the basic model (including the same parameters) for all country pairs. We now add a superscript j to denote a given country pair. Then equation (14) applies to each country pair with the same β_0 and β_1 coefficients as given in Proposition 1:

$$\triangle s_2^j = \beta_0 \bar{\mu}_0^j + \beta_1 d_1^j + v_2^j, \quad i = 1, 2, ..., N. \tag{20}$$

There is, however, one difference between (20) and (14). $\bar{\mu}_0$ is a constant term in the time series regression (14), but is a random variable in (20), because different country pairs have different average money growth differentials. This implies that the cross-sectional variation in $\bar{\mu}_0^j$ will help explain part of the cross-sectional variation in future exchange rate depreciations across different country pairs. Furthermore, $\bar{\mu}_0^j$ is positively correlated with d_1^j in the cross-section ($d_1^j = \bar{\mu}_0^j + \lambda^C \sigma^j$), which implies that $\bar{\mu}_0^j$ would affect the slope coefficient b_1 in the cross-sectional regression

$$\triangle s_2^j = b_0 + b_1 d_1^j + v_2^j, \qquad j = 1, 2, ..., N, \tag{21}$$

where b_0 is a constant. In contrast, in the time-series regression (3) for a specific pair of countries, $\overline{\mu}_0$ is the same over time. It only affects the future exchange rate depreciation through the constant term in the regression, and has no effect on the slope coefficient.

To compute b_1 in (21), we first project $\bar{\mu}_0^i$ onto d_1^i. Assume that the $\bar{\mu}_0^i$'s are drawn from a normal distribution $\bar{\mu}_0^i \sim N(0, V_{\bar{\mu}})$. Then $d_1^i = \bar{\mu}_0^i + \lambda^C \sigma^i \sim N(0, V_{\bar{\mu}} + (\lambda^C)^2 V_u)$. Since $\bar{\mu}_0^i$ and σ^i are independent, we have

$$\mathrm{E}[\bar{\mu}_0^i | d_1^i] = \rho d_1^i, \tag{22}$$

where $\rho \equiv \frac{V_{\bar{\mu}}}{V_{\bar{\mu}} + (\lambda^C)^2 V_u}$, and the expectation is taken under the empirical probability measure. The parameter ρ is between 0 and 1, and decreases with investor overconfidence (measured by λ^C). By equations (20) and (22), the slope coefficient b_1 in the cross-sectional forward discount regression is

$$b_1 = \rho \beta_0 + \beta_1 = \rho + (1 - \rho)\beta_1.$$

Thus, b_1 is a weighted average of the time-series forward discount regression coefficient β_1 given in (15) and the regression coefficient implied by UIP (i.e., unity). Therefore,

$$\beta_1 < b_1 < 1.$$

The forward discount bias remains in the cross-sectional regression setting, but is weaker than in the time-series regression. But like β_1, b_1 is also decreasing in λ^C. We summarize these results in the following proposition.

Proposition 5 1. *In both time-series and cross-sectional forward discount regressions, the degree of forward discount bias increases in the level of investor overconfidence.*

2. *Given the level of overconfidence, the forward discount bias is less pronounced in a cross-sectional regression than it is in a time-series regression.*

The intuition for the relative weakening of the forward discount bias in the cross-sectional regression is related to the different roles of the innovation component (u) and the predictable long-run component ($\bar{\mu}_0$) of the money growth differential. The realization of u implies a change in the money growth differential process (and also in the inflation differential process in our model) from its past trend. The ex ante average level of the money growth differential $\bar{\mu}_0$ is publicly known and thus is not a matter for overconfident judgment. The overconfidence bias therefore implies greater overreaction to changes than to the more tangible long-run levels. For example, if the money growth differential across two countries has been steady around 3% for the past 10 years, then no one would overreact to this. Thus the innovation component tends to strengthen the overreaction/correction effect, while the long-run equilibrium component tends to support the conventional effect.

Prediction 7 *In univariate regressions of exchange rate changes on the forward discount, the* *coefficient on the forward discount will be more negative in a time-series test than in a cross-sectional test.*

4.8 PPP Deviations and Forward Discount Regressions

So far we have assumed that relative PPP holds. In this section we extend our model to allow for short-run violations of and long-run convergence to relative PPP. We show that deviations from relative PPP alone cannot generate the observed forward discount bias. But when there is investor overconfidence, PPP deviations can interact with investor overconfidence to affect the magnitude of the bias.

We assume that the dynamics of the spot rate depreciation is

$$\triangle s_t = \theta \bar{\pi}_t - \kappa(\triangle s_{t-1} - \bar{\pi}_{t-1}) + e_t, \tag{23}$$

This can be viewed as a reduced-form expression from some underlying equilibrium model (e.g., Sercu, Uppal, and Van Hulle 1995). It is motivated by several stylized facts in the international finance literature. First, when researchers regress $\triangle s$ on $\bar{\pi}$, the estimated slope coefficient often significantly deviates from unity (e.g., Krugman (1978), Frenkel (1981), Hakkio (1984)). Second, although PPP is violated in the short run, empirical research has documented mean-reversion in PPP deviations and long-run trends in nominal and real exchange rates that are consistent with PPP (e.g., Frankel (1986), Frankel and Rose (1995), Mark and Choi (1997)). Thus in (23), $\theta > 0$ measures the sensitivity of exchange rate to inflation differential and is allowed to be different from one, and κ is positive reflecting a gradual convergence towards PPP in the long run.

Proposition 6 *In a model with investor overconfidence and relative PPP deviations,*

$$\triangle s_2 = \beta_0^* \bar{\mu}_0 + \beta_1^* d_1 + v_2^*,$$

where

$$\begin{aligned}
\beta_0^* &= (1+\alpha)\gamma\theta + \kappa(\alpha - 1)(\theta - 1), \\
\beta_1^* &= 1 - (1+\alpha)\gamma\theta + (1 - \alpha\kappa)(\theta - 1), \\
v_2^* &= \theta\eta_2 - \kappa(\theta - 1)\eta_1 - (1+\alpha)\theta\delta - \kappa e_1 + e_2,
\end{aligned}$$

with $\mathrm{E}_1^R[v_2^*] = 0$ *and* $\mathrm{cov}(d_1, v_2^*) = 0$.

The proof of this proposition is provided in the appendix. Table 1 illustrates the slope coefficient β_1^* in the theoretical forward discount regression with both investor overconfidence and PPP deviations. The empirical literature has found that convergence of real exchange rates to relative PPP is very slow. The consensus view on the half-life of real exchange rate is between three and five years (Rogoff 1996). We choose $\kappa = 0.1733$ which gives a half life of four years. We continue to use $\alpha = 4$ and $0 \leq \gamma \leq 0.5$.

$\gamma \backslash\ \theta$	0.5	0.75	1.0	1.25	1.5
0.0	0.85	0.92	1.00	1.08	1.15
0.2	0.35	0.17	0.00	-0.17	-0.35
0.3	0.10	-0.20	-0.50	-0.80	-1.10
0.5	-0.40	-0.95	-1.50	-2.05	-2.60

Table 1: The Effects of Overconfidence and PPP Deviations on β_1^*

Table 1 suggests that PPP deviations alone cannot generate negative slope coefficient in the forward discount regression. The intuition is that without investor overconfidence, the forward discount contains no overreaction, and thus can not predict the subsequent reversal of the spot rate. To see this analytically, Proposition 6 implies that with γ being zero, $\beta_1^* = 1+(1-\alpha\kappa)(\theta-1)$, which is negative if and only if

$$\alpha\kappa > 1 \text{ and } \theta > 1 + \frac{1}{\alpha\kappa - 1}.$$

Reasonable estimates of α and κ based on empirical studies on Cagan's money demand function and the long-run validity of PPP typically give rise to values of $\alpha\kappa$ smaller than one. But even if $\alpha\kappa > 1$, e.g., $\alpha\kappa = 1.5$, we need $\theta > 3$ to have a negative β_1^*. Such high θ, however, is not observed in the empirical research.[16] Hence, our model implies that deviations from PPP alone do not explain the forward discount puzzle. This is consistent with the conclusion in Hollifield and Uppal (1997), which examines the effect of PPP deviations caused by international market segmentation on the forward discount bias. They find that even for extreme parameters the slope coefficient is not negative.

Although PPP deviations alone do not explain the forward discount puzzle, they interact with overconfidence to affect the magnitude of the forward discount bias. Table 1 shows that holding the level of investor overconfidence constant, the slope coefficient in the theoretical forward discount regression decreases with θ. To understand why, compare the cases of $\theta > 1$ and $\theta = 1$. When $\theta > 1$, the spot exchange rate overshoots relative to the inflation differential. So there are two drivers of spot rate overreaction: overconfidence about signals on inflation differential and deviations from PPP. Thus the date 1 spot rate overreaction and its subsequent correction are stronger.

[16] Cavallo et al. (2005) estimate real exchange rate overshooting and find the amount of overshooting to be below 50% except for two countries. The largest overshooting is about 150% in their sample.

Proposition 7 *When there are relative PPP deviations,*

1. *If investor overconfidence is sufficiently strong, the slope coefficient in the forward discount regression is negative.*

2. *The hurdle level of investor overconfidence needed to explain the forward discount bias decreases with the sensitivity of exchange rate movements to the inflation differential.*

3. *When investor overconfidence explains forward discount bias, the magnitude of the bias increases with the sensitivity of exchange rate to the inflation differential.*

Proposition 7 is proved in the appendix. It implies the following empirical prediction.

Prediction 8 *The greater the overshooting in the spot exchange rate relative to the inflation differential, the more negative will be the slope coefficient in the forward discount regression.*

Previous studies have documented that predictable deviations from PPP are highly correlated with the forward discount bias (see, e.g., the survey of Engle 1996). But the above prediction of our model has not been tested.

5 Conclusion

This paper investigates the role of investor overconfidence in explaining the forward discount puzzle and predictability in the foreign exchange market. In our model, investors overreact to macroeconomic news, which leads to overshooting of both forward and spot exchange rates, with a higher magnitude of overshooting in the forward rate than in the spot rate. Thus, the forward discount reflects the overreaction in the spot rate and predicts its subsequent correction. The forward discount bias results when this overreaction-correction effect dominates the conventional effect implied by uncovered interest rate parity.

In short-horizon time-series forward discount regressions, the overreaction-correction effect tends to dominate the conventional effect, resulting in the forward discount bias. In long-horizon time-series regressions, however, the conventional effect tends to gain importance relative to the overreaction-correction effect, because mispricing in the spot exchange rate attenuates over time, whereas the effect of foreseeable differences in the expected growth rates of different currencies (differences which are recognizable without much use of subjective judgment) accumulates. Thus, the forward discount bias weakens in long-horizon time-series regressions.

Similarly, our model implies that the forward discount bias is weaker in cross-sectional regression tests. The foreseeable part of the inflation differential plays a bigger role in cross-sectional regressions, strengthening the conventional effect. In time series regressions (especially short-horizon regressions), the innovation component of the inflation differential plays a larger role, increasing the importance of overreaction-correction effects.

Our analysis accommodates but does not require violations of relative purchasing power parity (PPP). We show that the existence of short-run violation of PPP by itself does not produce the forward discount bias under plausible parameter values. It can, however, amplify the overreaction-correction effect associated with investor overconfidence, thereby strengthening the forward discount bias.

Our model provides several distinctive new empirical predictions about forward discount regressions. The model predicts that the forward discount regression coefficient is more negative in periods that are subject to greater investor overconfidence. The model also links the magnitude of the forward discount bias to the predictability of exchange rate movements and the sensitivity of exchange rate changes to the inflation differential.

Our analysis suggests some broader directions for research. Violations of PPP in the form of overshooting of exchange rates relative to price levels is usually taken as exogenous in theoretical models. Such overshooting is not required for our main results, and therefore we do not explore its underpinnings. However, several considerations suggest that overconfidence can be a source of deviations from PPP. If there is a degree of market segmentation in which the prices of goods and services are influenced by the inflation expectations of participants in goods markets, whereas exchange rates reflect expectations of the currency traders, then the greater overconfidence among currency traders will tend to create greater overreaction in exchange rates than in the price levels for goods and services.

In foreign exchange markets, less than five percent of the transactions involves importers, exporters and other non-financial companies. Trading is dominated by institutional investors such as banks and hedge funds, who generally deal heavily with derivatives and speculate rather than hedge (Frankel and Rose 1995). Currency traders are in their business precisely because they believe they have superior talents at forecasting changes in exchange rates. In contrast, most participants in the real goods markets including consumers are not primarily in the business of forecasting exchange rates. Therefore, currency traders are likely to be more overconfident about forecasting exchange rate than participants in the goods markets are about forecasting inflation. Furthermore, since currency prices are highly volatile, investors of all sorts are likely to receive very noisy feedback about their abilities to forecast exchange rate movements; psychological evidence suggests that such noisy feedback tends to contribute to overconfidence.

Another interesting direction for extension of our approach is to the term structure of domestic interest rates. The bond pricing literature has provided findings that are in some ways analogous to the international forward discount puzzle (e.g., Fama and Bliss 1987, Campbell and Shiller 1991, Cochrane 1999, Bekaert and Hodrick 2001, Cochrane and Piazzesi 2005). A regression of the change in short term yields on the short-term forward-spot spread (difference between the forward interest rate and the short-term spot interest rate) gives a slope coefficient near zero and even negative, indicating the failure of the expectations hypothesis in the short horizons (the hypothesis predicts a slope coefficient of unity). The forward-spot spread also positively predicts holding period returns of long term bonds. Bekaert and Hodrick (2001) observe that researchers have had surprisingly little success explaining the empirical failure of the expectations hypothesis in terms of rational risk premia. It will be interesting to see whether overconfidence can offer an integrated explanation for these findings as well.

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Appendix

Proof of Proposition 1: By definition,

$$z_2 = \mathrm{E}_2^C[\bar{\mu}_3] - \mathrm{E}_1^C[\bar{\mu}_2] = u - \lambda^C \sigma.$$

Equation (13) can be rewritten as

$$u = \lambda^R \sigma - \delta = (1-\gamma)\lambda^C \sigma - \delta \tag{24}$$

Applying Equation (4) on date 2 and using the two equations above, we obtain

$$\bar{\pi}_2 = (\bar{\mu}_0 + u + \eta_2) - \alpha(\delta + \gamma\lambda^C \sigma). \tag{25}$$

The second term on the right hand side of (25) represents a correction of the date 1 error in overconfident individuals' expectation about inflation differential. The expectation error contains two elements: the surprise relative to the rational expectation (δ) and the correction of the overconfidence-induced date 1 overreaction ($\gamma\lambda^C = \lambda^C - \lambda^R$).

Substituting (24) and $\lambda^C \sigma = d_1 - \bar{\mu}_0$ into (25) and using $\triangle s_2 = \bar{\pi}_2 + e_2$, we obtain the results in Proposition 1.

Proof of Proposition 3: Since $\bar{\mu}_0$ and s_0 are constants, their values will not affect the correlations of the variables we consider here. Therefore, for simplicity we set $\bar{\mu}_0 = s_0 = 0$. On date 1, upon receiving the signal σ, the inflation differential ($\bar{\pi}_1$), the change in the spot rate ($\triangle s_1$), the spot rate (s_1), the forward rate (f_1), and the forward discount (d_1) are:

$$\begin{aligned}
\bar{\pi}_1 &= \eta_1 + \alpha\lambda^C \sigma, \\
\triangle s_1 &= \eta_1 + \alpha\lambda^C \sigma, \\
s_1 &= s_0 + \triangle s_1 = \eta_1 + \alpha\lambda^C \sigma + e_1, \\
f_1 &= s_1 + d_1 = \eta_1 + (1+\alpha)\lambda^C \sigma + e_1, \\
d_1 &= \lambda^C \sigma.
\end{aligned}$$

All the regressors, except d_1, contain η_1, the random realization of the money growth differential on date 1. $\bar{\pi}_1, \triangle s_1, s_1$ and f_1 can all be written in the same form of

$$\omega = b d_1 + c\eta_1 + h e_1,$$

for some non-negative constants b, c and h, but c and h are not both zero.

The R^2 of regressing $\triangle s_2$ onto a regressor of the form ω, denoted by R^2_ω, is

$$R^2_\omega = \frac{\mathrm{cov}(\triangle s_2, \omega)^2}{\mathrm{var}(\triangle s_2)\mathrm{var}(\omega)}.$$

By Proposition 1, $\triangle s_2 = \beta_1 d_1 + v_2$, where $v_2 = \eta_2 - (1+\alpha)\delta + e_2$. Note that η_1, η_2, e_1, e_2 and δ are uncorrelated with each other and with the date 1 signal σ. It follows that

$$\mathrm{cov}(\triangle s_2, \omega) = b\beta_1 \mathrm{var}(d_1),$$

and

$$\mathrm{var}(\omega) = b^2 \mathrm{var}(d_1) + c^2 \mathrm{var}(\eta_1) + h^2 V_e.$$

Thus,

$$R^2_\omega = \frac{[b\beta_1 \mathrm{var}(d_1)]^2}{\mathrm{var}(\triangle s_2)[b^2 \mathrm{var}(d_1) + c^2 \mathrm{var}(\eta_1) + h^2 V_e]}.$$

Similarly, the R^2 of regressing $\triangle s_2$ onto the forward discount d_1, denoted by $R^2_{d_1}$, is given by

$$R^2_{d_1} = \frac{\beta_1^2 \text{var}(d_1)}{\text{var}(\triangle s_2)}.$$

It follows that the difference in predictive power as measured by $R^2_{d_1} - R^2_\omega$ is

$$R^2_{d_1} - R^2_\omega = \frac{\beta_1^2 c^2 \text{var}(\eta_1)\text{var}(d_1) + \beta_1^2 h^2 \text{var}(\eta_1) V_e}{\text{var}(\triangle s_2)[b^2 \text{var}(d_1) + c^2 \text{var}(\eta_1) + h^2 V_e]} > 0.$$

Proof of Proposition 4: Let $f_{1,3}$ denote the two period forward exchange rate, and $d_{1,3} = f_{1,3} - s_1$ be the two period forward discount. By a standard arbitrage argument, we derive an equation similar to (6):

$$d_{1,3} = \mathrm{E}_1^C[\bar{\pi}_3 + \bar{\pi}_2].$$

By equations (4) and the law of iterated expectations

$$\begin{aligned} d_{1,3} &= \mathrm{E}_1^C[\bar{\mu}_2 + \bar{\mu}_3] + \alpha \mathrm{E}_1^C[\bar{\pi}_4 - \bar{\pi}_2] \\ &= 2(\bar{\mu}_0 + \lambda^C \sigma). \end{aligned} \tag{26}$$

In obtaining (26), we use (7) and the fact that

$$\begin{aligned} \mathrm{E}_1^C[\bar{\pi}_2] &= \bar{\mu}_0 + \lambda^C \sigma, \\ \mathrm{E}_3^C[\bar{\pi}_4] &= \bar{\mu}_0 + u, \end{aligned}$$

so that

$$\mathrm{E}_1^C[\bar{\pi}_4 - \bar{\pi}_2] = \mathrm{E}_1^C[u - \lambda^C \sigma] = 0,$$

By (12) and (26), $d_{1,3} = 2d_1$. By (5),

$$\triangle s_3 = \theta \bar{\pi}_3 = \bar{\mu}_0 + u + \eta_3 + e_3.$$

Combining this with $\triangle s_2$ given by Proposition 1, we have

$$\begin{aligned} s_3 - s_1 &= \triangle s_3 + \triangle s_2 \\ &= (\beta_0 + \gamma)\bar{\mu}_0 + (\beta_1 + 1 - \gamma) d_1 + v_3, \end{aligned}$$

where $v_3 = \eta_3 + e_3 + v_2 - \delta$, $\mathrm{E}_1^R[v_3] = 0$ and $\text{cov}(d_1, v_3) = 0$. Thus the slope coefficient of $s_3 - s_1$ on $d_{1,3}$ is

$$\beta_1' = (\beta_1 + 1 - \gamma)/2 = 1 - (1 + \alpha/2)\gamma,$$

as stated in the Proposition 4.

Proof of Proposition 6: By (10), (23), and (25), the exchange rate depreciation $\triangle s_2$ satisfies

$$\begin{aligned} \triangle s_2 &= \theta \bar{\pi}_2 - \kappa(\theta - 1)\bar{\pi}_1 - \kappa e_1 + e_2 \\ &= [\theta - \kappa(\theta - 1)]\bar{\mu}_0 + \theta(1 + \alpha)u - [\alpha\theta + \alpha\kappa(\theta - 1)]\lambda^C \sigma \\ &+ [\theta \eta_2 - \kappa(\theta - 1)\eta_1 - \kappa e_1 + e_2]. \end{aligned} \tag{27}$$

Proposition 6 follows immediately upon substituting (24) and $\lambda^C \sigma = d_1 - \bar{\mu}_0$ into (27).

Proof of Proposition 7: By Proposition 6, the slope coefficient β_1^* in the theoretical forward discount regression when there are relative PPP deviations is given by

$$\beta_1^* = 1 - (1 + \alpha)\gamma\theta + (1 - \alpha\kappa)(\theta - 1),$$

which implies that

$$\beta_1^* < 0 \ \text{ if } \ \gamma > \bar{\gamma},$$

where $\bar{\gamma}$ is

$$\bar{\gamma} \equiv \frac{1 + (1 - \alpha\kappa)(\theta - 1)}{(1 + \alpha)\theta} = \frac{1 - \alpha\kappa}{1 + \alpha} + \frac{\alpha\kappa}{(1 + \alpha)\theta}.$$

In other words, the slope coefficient in the theoretical forward discount regression is negative for sufficiently high level of overconfidence. Obviously, the hurdle level of overconfidence $\bar{\gamma}$ decreases with θ, the sensitivity of exchange rate movements to the inflation differential. In particular, overshooting of exchange rates relative to the inflation differential lowers the level of investor overconfidence required to generate a negative slope coefficient in the forward discount regression.

When the level of investor overconfidence is sufficiently strong ($\gamma > \bar{\gamma}$) so that $\beta_1^* < 0$, then it is automatically true that

$$\gamma > \frac{1 - \alpha\kappa}{1 + \alpha}, \tag{28}$$

because

$$\bar{\gamma} - \frac{1 - \alpha\kappa}{1 + \alpha} = \frac{\alpha\kappa}{(1 + \alpha)\theta} > 0.$$

Note that (28) implies that

$$\frac{\partial \beta_1^*}{\partial \theta} = (1 - \alpha\kappa) - (1 + \alpha)\gamma < 0.$$

Thus, when investor overconfident is sufficiently strong so that the slope coefficient in the forward discount regression is negative, the forward discount bias is more severe when the sensitivity of exchange rate movements to the inflation differential is higher.

Figure 1: **Overreaction and Correction of Exchange Rates**

This graph illustrates the expected path of movement for the spot and forward exchange rates from date 0 to date 2, conditional on a date-1 signal about date-2 money growth differential. The upper half of the figure depicts the case of a positive signal σ about money growth differential innovation. In response to a positive shock σ, the spot and the forward exchange rates increase to s_1 and f_1. They both overreact to σ: $s_1 > s_1^R; f_1 > f_1^R$, where the superscript R denotes the rational case without overconfidence-induced overreaction. The magnitude of overreaction is higher for the forward rate. After date-2 money growth differential is realized, the overreaction is on average corrected. The lower half of the figure depicts the case of a negative shock $-\sigma$.