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Abstract

Most of the assets on the balance sheet of a typical bank are illiquid. Therefore, liquidity risk is one of the key risks for banks. Since the risks of an asset affect its value, liquidity risk should be included in their valuation. Although models have been developed to include liquidity risk in the pricing of traded assets, these models do not easily extend to truly illiquid or non-traded assets. This paper develops a valuation framework for liquidity risk for these illiquid assets. Liquidity risk for illiquid assets is identified as the risk of assets being liquidated at a discount in a liquidity stress event (LSE). Whether or not a bank decides to liquidate an asset depends on its liquidation strategy. The appropriate strategy for valuation purposes is shown to be a pro rata liquidation. The main result is that the discount rate used for valuation includes a liquidity spread that is composed of three factors: 1. the probability of an LSE, 2. the severity of an LSE, and 3. the liquidation value of the asset.

*Earlier versions of this paper were titled "Discounting Cashflows of Illiquid Assets on Bank Balance Sheets" .

1 Introduction

One of the main risks of a bank is liquidity risk. The importance of liquidity risk is reflected by, for instance, the inclusion of liquidity risk measures in the Basel 3 framework [BCBS(2010)]. Already before Basel 3 the BIS issued the paper "Principles for Sound Liquidity Risk Management and Supervision" [BIS(2008)], aimed at strengthening liquidity risk management in banks. The BIS-paper stresses the importance of liquidity risk as follows: "Liquidity is the ability of a bank to fund increases in assets and meet obligations as they come due, without incurring unacceptable losses. The fundamental role of banks in the maturity transformation of short-term deposits into long-term loans makes banks inherently vulnerable to liquidity risk, both of an institution-specific nature and those affecting markets as a whole."

Since liquidity risk may result in actual losses, this paper argues liquidity risk should be included in the valuation of balance sheet items. This paper assumes that the liabilities are liquid and as such are valued consistently with market prices. Therefore, the impact of liquidity risk on the valuation of assets is considered. The aim is to develop a valuation framework for liquidity risk that can be applied consistently to the different assets on a bank balance sheet. In particular the aim is to include derivatives, other traded assets, but also banking book assets. Many banking book assets are included in financial reporting on historical cost basis. Therefore, their valuation is not required for financial reporting. Nevertheless valuation is important to calculate sensitivities such as duration and PV01's. Valuation of banking book assets is also important to determine the profitability of assets. Therefore, although the valuation of banking book items is not relevant for accounting purposes, these are included in the valuation framework developed here.

In the literature, a number of approaches to include liquidity risk or the liquidity of an asset have been developed. Extensions of the CAPM model confirm that investors price in liquidity risk, see e.g. the paper by [Acharya & Pedersen(2005)] or the review article by [Amihud *et al.*(2005)]. It is useful to recall one of the basic results that result from these CAPM extensions (see e.g. [Amihud & Mendelson(1986), Amihud *et al.*(2005)]). The expected return on an asset in an economy where investors are risk-neutral and have an identical trading intensity μ is given by

$$R = r + \mu c\,, \tag{1.1}$$

where r denotes the risk-free rate and c the liquidity cost of trading the asset as a fraction of its price. The application of this basic result to illiquid assets requires a re-interpretation [Amihud *et al.*(2005)]. In that case μ may be interpreted as the probability of a liquidity shock. In a liquidity shock, an investor will need to liquidate the asset and encounters a cost c. In this paper, the event of a liquidity shock will be called a liquidity stress event (LSE) which includes both systemic, as well as idiosyncratic (firm-specific) events.

However, this result cannot be applied directly to the valuation of assets on bank balance sheets for three reasons. 1) For illiquid assets there does not need to be a market and, therefore, no equilibrium price. 2) A bank holds many different assets of different liquidity. In an LSE the bank typically does not need to sell off all its assets to meet the liquidity demand, the bank can decide which assets to liquidate. 3) The probability of an LSE and its impact will depend on specifics of the bank's balance sheet. E.g. a bank whose funding consists mainly of short-term wholesale funding has a much larger probability of an LSE (with a larger impact) than a bank with mostly long-term funding. These complications are addressed in the paper.

This paper focuses on the discounting of cash flows generated by the different assets to address these questions. It recognizes that the liquidity of an asset determines the discount rate of cash flows generated by the asset. In particular, the possibility that the bank has to liquidate (a fraction of) the asset in the event of liquidity stress determines the liquidity spread included in the discount rate. The liquidity spread is composed of the probability of a liquidity stress event, the severity of the liquidity stress event, and the liquidation value of the asset.

The outline of this paper is as follows: Firstly, section 2 develops a liquidity risk valuation framework and discusses some consequences. Section 3 extends the model to include credit risk and optionality. Section 4 considers the impact of the funding composition. In section 5 a paradox is discussed and as an example the value of the assets on Barclays and UBS balance sheet (per end of 2014) is calculated. Lastly, the conclusions are summarized.

2 Liquidity Risk Valuation Framework

2.1 First pass: Liquidity risk and valuation

In recent years, the impact of liquidity risk on pricing of assets has been studied. In particular, research has been done to extend the CAPM model to include liquidity risk, such as the work of [Acharya & Pedersen(2005)]. It is useful to recall these extensions to clarify the differences between these CAPM extensions and the approach in this paper.

Acharya and Pedersen define a stochastic illiquidity cost C_i for security i that follows a normal process in discrete time. The illiquidity cost is interpreted as the cost of selling the security. Furthermore, it is assumed that an investor who buys a security at time t will sell the security at time $t+1$. Liquidity risk in this model comes from the uncertainty of the cost of selling the security. With this set-up, Acharya and Pedersen derive a liquidity-adjusted CAPM with three additional betas.

Although the extension of CAPM including liquidity risk is useful to understand prices of traded assets, such as securities, it not easily extended to the valuation of most of the assets on a bank's balance sheet. One reason is that most of these assets are non-traded. Loans, mortgage, and other assets in the banking book are intended to be held to maturity. Hence, the assumption that the asset will be sold with a stochastic cost is not appropriate for these assets. Even assets in the trading book may not be traded. For instance OTC derivatives, whose market risks are hedged through trading hedge instruments, may well be held to maturity. Hence the CAPM approach, which assumes that an asset needs to be sold and model liquidity risk by stochastic liquidity costs, is not appropriate for most assets on a bank balance sheet.

The question is how these assets are sensitive to liquidity risk. Whatever the changes in liquidity cost, as long as these assets are held to maturity as intended, their payoff is not affected by liquidity risk. Therefore, it seems that these assets are not sensitive to liquidity risk, which would imply that liquid and illiquid assets with the same payoff should have the same value.

The resolution this paper proposes is that, although the assets may be intended to be held to maturity, in a liquidity stress event the bank may be forced to liquidate some of its assets at a discount. Therefore, the payoff generated by the asset may be lower than the contractual payoff when a bank is exposed to liquidity risk. The value of the asset should reflect this discount. It is clear that an illiquid asset, which has a larger discount in a forced liquidation than a liquid asset, will have a lower value (when they have the same contractual payoff).

These considerations lead to the following definition of liquidity risk:

Liquidity risk is the risk for an event to occur that forces a bank to liquidate some of its assets.

Such an event is termed a liquidity stress event (LSE). In the next section, a simple model for such events is proposed.

2.2 Liquidity Risk Model

In this paper, LSEs are modeled as random events. The model consists of three components:

- The probability that an LSE occurs: $PL(t_1, t_2)$ will denote the probability of such an event between t_1 and t_2.
- The severity of an LSE. The fraction of the assets that a bank needs to liquidate f determines the severity. By definition $0 \leq f \leq 1$. For simplicity the severity f is modeled as a fixed (non-random) number.
- The dependence structure of LSEs and other events. The model assumes that LSEs are independent of each other and other events such as credit risk or market risk events.

In particular, the model assumes that LSEs follow a Poisson process with a constant intensity $p \geq 0$, which implies for an infinitesimal time interval dt

$$PL(t, t + dt) = pdt. \tag{2.1}$$

This set-up simplifies the modeling of complicated dynamics of an LSE to the probability and severity of an LSE. Hence, the value of an asset depends on above effective parameters.

Of course, more insight in the liquidity risk of a bank is obtained by considering all potential contributors, such as retail deposits run-off, wholesale funding risk, collateral outflows, intraday risks, etc. However for the valuation of an asset it only matters if and when it gets liquidated, not if the liquidation is a result of retail deposits or wholesale funding withdrawal.

The interpretation of the above model is that the bank gets hit at random times by an LSE. In particular, the bank has at any time the same risk of being hit by an LSE, there is no notion of increased risk. A possible extension of the model could support multiple states, such as "high risk" and "low risk" states. These states would have different probabilities of an LSE and some probabilities to migrate from one state to the other. Such an extension might result in a more realistic model, but would also have many more parameters to calibrate. As discussed later, the lack of traded instruments to hedge liquidity risk make it difficult to calibrate the parameters using traded market instruments. Because of the inherent difficulties to calibrate parameters for liquidity risk, this paper chooses the above set-up with a minimum number of parameters.

2.3 Valuation with liquidity risk

In an LSE, a bank will liquidate some of its assets. These assets will be sold at a discount depending on the liquidity of the asset. The realization of this discount in case of an LSE may be recognized by defining an effective payoff.

$$\text{Effective pay-off} = \begin{cases} \text{contractual pay-off} & \text{if no LSE occurs} \\ \text{stressed value} & \text{if LSE occurs} \end{cases} \tag{2.2}$$

The contractual payoff includes all cash flows of the asset, for example, optionality, cash flows in case of default, contingent cash flows, etc.

The stressed value includes the discount for liquidating part of the position in the LSE. In case of a single LSE at time τ the stressed value may be expressed as

$$\text{stressed value} = f_A V(\tau) LV + (1 - f_A) V(\tau)\,, \tag{2.3}$$

where $V(\tau)$ is the fair value of the asset at time τ, f_A is the fraction of the asset that the bank will liquidate, and LV is the liquidation value denoted as a fraction of the fair value of the asset. Equation 2.3 assumes that assets are divisible, and any part of the assets can be liquidated.

The fraction f_A is determined by a liquidation strategy. In the next section, the liquidation strategy that should be used in valuation is derived.

Definition: The value of an asset under liquidity risk is defined as the present value of the effective pay-off

$$V = PV[\text{Effective pay-off}]\,. \tag{2.4}$$

Consider a cash flow of an illiquid asset at some future time T. In absence of default risk the value at time t of the cash flow is related to the value at time $t+dt$ through

$$V(t) = e^{-rdt}V(t+dt)(1-pdt)+e^{-rdt}[f_A V(t+dt)LV+(1-f_A)V(t+dt)]pdt \tag{2.5}$$

The first term on the r.h.s. is the contribution from the scenario that no LSE occurs between t and $t+dt$. The second term is based on (2.3) and is the contribution from the scenario that an LSE occurs. The contribution from multiple LSEs between t and $t+dt$ may be neglected as long as p is finite, since this contribution is of order $(pdt)^2$ and dt is an infinitesimal time period.

Equation (2.5) may be rewritten as

$$V(t) = e^{-rdt}V(t+dt)[1-p(1-LV)f_A dt]\,. \tag{2.6}$$

By introducing a liquidity spread

$$l = p(1-LV)f_A\,, \tag{2.7}$$

this becomes

$$V(t) = e^{-rdt}V(t+dt)(1-ldt). \tag{2.8}$$

The value of a cash flow at a future time T of notional 1 in absence of default risk is derived by iterating (2.8)

$$V = e^{-(r+l)T}, \tag{2.9}$$

since $\lim_{dt\downarrow 0}(1-ldt)^{T/dt} = e^{-lT}$.

The liquidity spread (2.7) used in discounting depends on the fraction of the asset f_A that a bank liquidates. This fraction is determined in the next section.

2.4 Liquidation strategy

Consider a balance sheet with a set of assets A_i with $i = 1, 2, ..., N$, where A_i denotes the market value, and each asset has a unique liquidation value LV_i. Without loss of generality, an ordering of the assets can be assumed: $LV_i > LV_j$ if $i < j$.

Definition: A liquidation strategy for a set of assets A_i is a set of fractions s_i of assets to sell such that

$$\sum_{i=1}^{N} s_i A_i = f \sum_{i=1}^{N} A_i. \tag{2.10}$$

with $0 \leq s_i \leq 1$ and the sum over i covers all assets on the balance sheet. Here A_i denote the market values of the assets.

Such a strategy could be, for instance, to sell the most liquid assets until sufficient assets have been liquidated to reach $f \sum_i A_i$. Note that the strategy is allowed to depend on the order of the assets, but not on the liquidation values LV_i. A bank's liquidation strategy will be of the type to liquidate assets based on their relative liquidity (e.g. most liquid assets first) instead of on their exact liquidation values.

Definition: An admissible liquidation strategy is a strategy s_i^* such that the liquidity spreads implied by the strategy

$$l_i = p(1 - LV_i)s_i^*\,, \tag{2.11}$$

satisfy the condition that for any set LV_i

$$LV_i < LV_j \Rightarrow l_i > l_j\,. \tag{2.12}$$

Definition: An optimal admissible liquidation strategy is an admissible liquidation strategy with the lowest loss in an LSE. This loss is defined as

$$\text{loss} = \sum_i s_i A_i (1 - LV_i)\,. \tag{2.13}$$

To demonstrate that the optimal admissible liquidation strategy is given by $s_i^* = s_j^*$ for all i, j, it first needs to be noted that a strategy with $s_i > s_j$ for $i < j$ is not an admissible strategy. Consider e.g. $s_1 > s_2$. Then the choice $LV_1 = LV_2 + \frac{s_1 - s_2}{2s_1}(1 - LV_2)$ implies $l_1 > l_2$. (It can be checked that this expression for LV_1 is a valid choice in the sense that $LV_1 > LV_2$ and $LV_1 < 1$.) Therefore $s_1 > s_2$ violates the requirement (2.12). Note that the same reasoning can be applied to any i, j with $i < j$, and that it is sufficient to have one choice of LV's that violates (2.12), since definition (2.12) should hold for any set LV's.

It can be concluded that the set of admissible liquidation strategies may be characterized by: $s_1 \leq s_2 \leq s_3 \leq ... \leq s_N$, where N denotes the last asset. Within this set, the optimal choice is $s_1 = s_2 = s_3 = ... = s_N$ since it will lead to the lowest loss for the bank in an LSE. The conclusion is that the optimal admissible strategy is specified by $s_1 = s_2 = s_3 = ... = s_N = f$.

The final step in the completion of the valuation framework is the determination what fraction of an asset f in (2.7) a bank will liquidate in an LSE. The optimal admissible liquidation strategy has been defined to determine this fraction. This strategy is the natural choice for valuation out of possible liquidation strategies. Since it preserves the relation between liquidation values and liquidity spreads (2.12) and minimizes the loss of the liquidation of assets within this admissible set.

2.5 Summary of the model

Putting the above liquidity risk model, valuation approach and optimal admissible liquidation strategy together the result is the following.

The discount factor of a cash flow at time T of an asset A_i without default risk is

$$DF = e^{-(r+l_i)T}, \tag{2.14}$$

where the liquidity spread is given by

$$l_i = p(1 - LV_i)f. \tag{2.15}$$

Note that the discount factor of the cash flow depends on the liquidity of the asset that generates the cash flow through LV_i. The other two factors, the probability of an LSE p and the severity of an LSE f, are not asset specific but are determined by the balance sheet of the bank.

Note that the model is consistent with the basic CAPM result (1.1) mentioned in the introduction when the fraction $f = 1$, and the liquidity cost c is identified as the liquidity discount in an LSE: $c = 1 - LV$.

2.6 Some consequences of the model

Equation 2.15 implies a simple relation between liquidity spreads of different assets (on the same balance sheet) are related. Since in (2.15) the probability of an LSE and the fraction of assets that need to be liquidated are the same for all assets, it follows immediately that

$$\frac{l_i}{l_j} = \frac{1 - LV_i}{1 - LV_j}. \tag{2.16}$$

The liquidity spread of asset i and asset j are related through their liquidation values.

A nice feature of the model is that it allows to explain a different discount rate for a bond and a loan. Consider, for example, a zero-coupon bond and a loan with the same issuer/obligor, same maturity, notional, and seniority. The zero-coupon bond and loan, therefore, have exactly the same payoff (even in case of default). Nevertheless, if the zero-coupon bond is liquidly traded, a difference in valuation is expected. The model developed here, can provide an explanation for this difference. The above relation (2.16) shows that the liquidity spreads of the zero-coupon bond and the loan. For example, consider a balance sheet where the probability of an LSE for a bank is estimated at 5% per year, and the severity of the event at 20%. Furthermore, the liquidation value for the ZC-bond is estimated at 80% and for the loan at 0% (since the loan cannot be sold or securitized quickly enough). Then the liquidity spreads for the bond and loan are:

$$l_{\text{bond}} = 20\text{bp}, \tag{2.17}$$

$$l_{\text{loan}} = 100\text{bp}. \tag{2.18}$$

These spreads are based on above example, and may differ significantly between banks. Nevertheless, they clarify that it is natural in this framework that a different discount rate is used for loans and bonds.

In this framework also the position size will affect the discount rate. Empirical studies find a linear relation between the size of the sale and the price impact, see e.g. [Obizhaeva(2008), Cont *et al.*(2012)]. In the context of this paper, this translates into a linear relation between the position size and the liquidation value:

$$LV_i = 1 - cx_i \tag{2.19}$$

where x_i is the size of the position in asset i, e.g. the number of bonds, and c a constant. Consider a different position x_j in the same asset. From (2.16) it immediately follows that

$$\frac{l_i}{l_j} = \frac{x_i}{x_j}. \tag{2.20}$$

Given a linear relation between the size of a sale and the price impact, the framework derived here implies a linear relation between liquidity spread and position size.

2.7 Replication and Parameter Estimation

One of the important concepts in finance is the valuation of derivatives through the price of a (dynamic) replication strategy. Unfortunately, liquidity risk is a risk that cannot be replicated or hedged. In principle, it is conceivable that products will be developed that guarantee a certain price for a large sale. E.g. for a certain period the buyer of the guarantee can sell N shares for a value $N \times S$, where S denotes the value of a single share. Such products would help in determining market implied liquidation values, but it is difficult to imagine that such products will be developed that apply to large parts of the balance sheet.

In any case, currently liquidity risk cannot be hedged. Nevertheless, the risk should be valued. Therefore, it seems appropriate to use the physical probability of an LSE and liquidation value to determine the liquidity spread in (2.15) as opposed to an imaginary risk neutral probability and liquidation value. Clearly, if it would be possible to hedge this risk, then the risk neutral values implied by market prices should be used.

The physical probability of LSEs and the severity of the events are required to estimate the liquidity spread, see (2.15). These may be difficult to estimate. Perhaps more importantly, in the absence of hedge instruments and associated implied parameters, estimates may be less objective than desired.

On the other hand, a bank should already have a good insight in the liquidity risk exposure. E.g. through stress testing a bank has insight into the impact of different liquidity stress events. The BIS paper "Principles for Sound Liquidity Risk Management and Supervision" [BIS(2008)] gives guidance to banks how to perform stress tests. Such stress tests should provide insight in bank-specific risks, which in combination with the market perception of liquidity risk through e.g. liquidity spreads on traded instruments should provide estimates for p and f.

3 Extensions of the model

3.1 Including Credit Risk

This section adds credit risk to the framework. Recall (2.6) with (2.7). The inclusion of default risk is straightforward under the assumption that default events are independent of LSEs. The result is

$$V(t) = e^{-rdt} V(t+dt)[1 - ldt - pd \times \text{LGD} \times dt]\,, \tag{3.1}$$

where pd is the instantaneous probability of default and LGD the Loss Given Default. By introducing a credit spread

$$s_{\text{credit}} = pd \times LGD \tag{3.2}$$

and solving (3.1) in a similar way as (2.6) gives the following value of a cashflow of nominal 1

$$V = e^{-(r+l+s_{\text{credit}})T}. \tag{3.3}$$

The discount rate consists of a risk-free rate, a liquidity spread, and a credit spread.

3.2 Liquidity Risk for Derivatives

Liquidity risk also affects the value of derivatives. In a Black-Scholes framework liquidity risk results in an extra term in the PDE, see [Nauta(2015)].

A brief derivation starts from a delta-hedged derivative's position. Demanding that the value of riskless portfolio of derivative's position and delta hedge grows at the risk-free rate gives

$$dV - \Delta dS = r(V - \Delta S)dt\,, \tag{3.4}$$

where V denotes not the value of the derivative, but the value of the derivative's position, as indicated above. The Delta has the usual definition: $\Delta = \partial_S V$ and S denotes the underlying that follows a geometric Brownian motion. Including liquidity risk gives

$$dV = \partial_t V dt + \partial_S V dS + \frac{1}{2}\sigma^2 S^2 \partial_S^2 V - f(1 - LV_V)\max(V, 0)dN, \tag{3.5}$$

The last term on the r.h.s. is the extra term coming from liquidity risk, here N follows a Poisson process with intensity p. LV_V denotes the liquidation value of the derivative. The max function reflects that the value of the derivative can be both positive and negative (depending on the type of derivative) and that only positions with a positive value will be liquidated in an LSE.

Taking the expectation of the Poisson process dN, under the assumption of independence with dS gives

$$\partial_t V + rS\partial_S V + \frac{1}{2}\sigma^2 S^2 \partial_S^2 V = rV + l_V \max(V, 0)\,. \tag{3.6}$$

Here V denotes the value of the derivative's position, S the underlying stock, σ the volatility, and l_V the liquidity spread of the derivative's position. The last term on the r.h.s. is the extra term coming from liquidity risk and is, in fact, equivalent to the last term on the r.h.s. of (2.8). Note the derivation of (3.6) assumes that the underlying is perfectly liquid (in the sense that its liquidation value $LV = 1$).

In [Nauta(2015)] also extensions of (3.6) are discussed that include credit risk.

A remarkable feature of (3.6) is that it is similar to models that some authors have proposed for the inclusion of funding costs in the valuation of derivatives. In particular the extra term $l_V \max(V, 0)$ has the same form as the term for inclusion of funding costs derived by e.g. [Burgard & Kjaer(2011)]. However, with the funding spread replaced by the liquidity spread. The model (3.6 is more complex than the model including funding costs since the liquidity spread is position-dependent.

4 Funding costs and liquidity risk

The funding composition largely determines the probability and severity of an LSE. In the previous sections, we have treated the funding of a bank simply as a given. The resulting liquidity risk is included in the valuation of assets. In this section, the funding is considered more explicitly, through two examples:

1. adding an asset to the balance sheet that is term funded,

2. considering a special balance sheet where the income from the liquidity spreads compensates exactly the funding spread costs.

4.1 Adding an asset that is term funded

Consider the following simple balance sheet

A_i	L_j
	E

where all assets A_i have the same maturity T, without optionality or coupon payments. These assets can be thought of as a combination of zero coupon bonds and bullet loans. The liabilities have varying maturities and may include, for instance, non-maturity demand deposits.

Define the impact of liquidity risk on the total value of the assets as the Liquidity Risk Adjustment (LRA)

$$LRA = \sum_i A_i^0 - \sum_i A_i \,, \tag{4.1}$$

where A_i^0 is the value of asset i without liquidity risk

$$A_i^0 = A_i(l_i = 0) = A_i e^{l_i T} \,. \tag{4.2}$$

Now consider adding an asset A_{new} with the same maturity T that is term funded. The question is what is the impact on the LRA. The new LRA is

$$LRA_{\text{new}} = \sum_i A_i^0 - \sum_i A_i^{\text{new}} + A_{\text{new}}^0 - A_{\text{new}} \,, \tag{4.3}$$

where A_i^{new} is the value of asset i with the new liquidity spread after adding the new asset and its term funding. A_{new} is the value of the new asset with liquidity risk and A_{new}^0 the value without liquidity risk in a similar fashion as in (4.2).

The first step to estimate the impact on the LRA is to determine the new liquidity spread. Clearly the liquidation values LV_i of the assets do not change. Also, the probability of an LSE does not change, since the funding composition has not changed except for adding a liability with the same maturity as the assets. The only change is in the fraction of assets that need to be liquidated. Since the funding withdrawn in an LSE is the same before or after adding the asset when the asset is term-funded, the following relation holds:

$$[\sum_i A_i + A_{\text{new}}] f^{\text{new}} = [\sum_i A_i] f^{\text{old}} \,, \tag{4.4}$$

Hence the new fraction is

$$f^{\text{new}} = \frac{\sum_i A_i}{\sum_i A_i + A_{\text{new}}} f^{\text{old}} \,. \tag{4.5}$$

The old and new liquidity spreads are given by

$$l_i^{\text{old}} = p(1 - LV_i) f^{\text{old}} \,, \tag{4.6}$$

$$l_i^{\text{new}} = p(1 - LV_i) f^{\text{new}} \,. \tag{4.7}$$

The impact of adding the term funded asset on the LRA is

$$\begin{aligned} LRA_{\text{new}} - LRA &= \sum_i (A_i - A_i^{\text{new}}) + A_{\text{new}}^0 - A_{\text{new}} &(4.8)\\ &= \sum_i (A_i - A_i e^{-(l_i^{\text{new}} - l_i^{\text{old}})T}) + A_{\text{new}} e^{l_{\text{new}} T} - A_{\text{new}}\,, &(4.9) \end{aligned}$$

where the relations $A_i^{\text{new}} = A_i^0 e^{-l_i^{\text{new}} T}$, $A_i = A_i^0 e^{-l_i^{\text{old}} T}$, and $A_{\text{new}} = A_{\text{new}}^0 e^{-l_{\text{new}} T}$ were used. Expanding this expression to first order in $A_{\text{new}}/(\sum_i A_i)$ gives

$$LRA_{\text{new}} - LRA = A_{\text{new}}(l^{\text{new}} - l_{\text{av}}^{\text{old}})T\,, \tag{4.10}$$

where $l_{\text{av}}^{\text{old}} = (\sum_i l_i^{\text{old}} A_i)/(\sum_i A_i)$. Hence, even though the new asset is term-funded, the liquidity risk adjustment does change. The reason is that the new asset and its term funding are not isolated from the rest of the balance sheet. In an LSE, the new asset may also (partly) be liquidated. And indeed, in the liquidation strategy derived in section 2.4 for valuation, it will be pro rata liquidated.

Equation (4.10) shows that the LRA decreases when the new asset is more liquid than the other assets on average.

4.2 A special balance sheet that balances funding costs and liquidity spread income

Up to now only the valuation of assets has been considered. However, a bank also manages the income generated from these assets. From an income perspective, a bank would want that the liquidity spread it earns on its assets is (at least) equal to the funding spreads it pays on its liabilities and equity:

$$\sum_i l_i A_i = \sum_j s_j^F L_j + s^E E \tag{4.11}$$

where s_j^F is defined as the spread paid on liability L_j relative to the risk-free rate r and s^E the spread paid on equity.

Define the average funding spread as

$$s_F = \frac{\sum_j s_j^F L_j + s^E E}{\sum_j L_j + E} \tag{4.12}$$

Then it is clear that (4.11) implies that the average liquidity spread equals the average funding spread

$$s_F = l_{\text{av}} \tag{4.13}$$

Hence, the liquidity spread for asset A_i in this special case is related to the average funding spread by

$$l_i = \frac{(1 - LV_i)}{(1 - LV_{\text{av}})} r_F \tag{4.14}$$

where $LV_{\text{av}} = \sum_i LV_i A_i / \sum_i A_i$.

This result suggests that a bank can charge for liquidity risk through its funding costs when it corrects for the liquidity of the asset in this special case. In particular

- In the FTP framework of such a bank, funding costs would differentiate between liquid and illiquid assets through the factor $\frac{(1-LV_i)}{(1-LV_{\text{av}})}$. E.g. the FTP for a mortgage portfolio would decrease when a bank has securitized these (but have kept them on the balance sheet), since liquidation value LV of securitized mortgages is higher.
- The liquidity risk adjustment is similar to the Funding Valuation Adjustment that some authors have proposed. The LRA would, however, distinguish between liquid and less liquid derivatives, such as an OTC and exchange-traded option that are otherwise the same. An example is given in [Nauta(2015)].

Remains the question how "special" this special case is. Many banks would recognize (4.11) as something they apply ignoring the commercial margins on both sides of the balance sheet. However, most banks base their liquidity spreads on their funding costs, although (4.11) may be satisfied, the liquidity spreads do not accurately price the liquidity risk of the bank. Nevertheless, adjusting for the liquidity of an asset according to (4.14) may improve pricing to account for the liquidity of the asset.

An extension of the above model is developed in [Nauta(2013)], which includes both funding costs and liquidation losses in an LSE. A disadvantage of that model is that it requires more parameters to calibrate; the advantage is that it allows to determine the optimal funding term for an asset.

5 A paradox and an example

5.1 A paradox

As discussed in section 2 the liquidity spread is determined by the loss from a forced sale of part of the assets in a liquidity stress event. The applied sell strategy is to sell the same fraction of each asset. In practice however one would sell the most liquid assets as this results in a smaller loss. Since the valuation accounts for a larger loss, it seems that a risk-free profit can be obtained by holding an appropriate amount of liquid assets or cash as a buffer for a liquidity stress event.

To analyze the paradox, consider a bank with a simple balance sheet, as shown below

$A = 80$	$L = 80$
$C = 20$	$E = 20$

This bank has 80 illiquid assets, 20 cash, and its funding consists of 80 liabilities and 20 equity. It is exposed to an LSE where 20% of the funding is instantaneously removed.

If the stress event occurs, the resulting balance sheet used in the valuation is

$$\begin{array}{r|l} \hline A = 64 & L = 60 \\ C = 16 & E = 20 \end{array}$$

The sale of the assets will result in a loss $= (1 - LV_A)16$. This loss is borne by the equity holders, who in this setup, provide the amount $(1 - LV_A)16$. This amount combined with the result from the sale of the assets $LV_A 16$ and a cash amount of 4 covers the withdrawal of funding. Note that this can be viewed as a two-step approach whereby the cash covers the withdrawal and is immediately supplemented by the sale of the assets and the cash provided by the equity holders.

In practice, a bank will use its cash buffer to compensate the loss of funding. In contrast to the strategy of the pro-rata sale of assets used for valuation, this strategy will not lead to a loss. The resulting balance sheet is

$$\begin{array}{r|l} \hline A = 80 & L = 60 \\ C = 0 & E = 20 \end{array}$$

The paradox is that the value of the assets includes the possibility of a loss (through the liquidity spread), whereas this loss seems to be avoided in reality by using the cash as a buffer.

However, the bank is now vulnerable to a next LSE, whereby 20% of its funding is withdrawn. To be able to withstand such an event a cash buffer of 16 is required. This buffer should be realized immediately to avoid any liquidity risk, which can be achieved by the same sale of assets as in the strategy for valuation, resulting in the same loss. Therefore, to avoid any liquidity risk the same loss is borne by the equity holders, which resolves the paradox.

In practice the assets may be sold over a larger period, thereby the bank chooses to accept some liquidity risk to avoid the full loss by an immediate sale. The optimal strategy in practice is the result of risk-reward considerations.

5.2 Example for Barclays and UBS

In this section, the model is applied to the balance sheets of Barclays and UBS[1]. The financial data used in this section is based on the (publicly available) 2014 full year results: [Barclays FY Results(2014)] and [UBS financial results(2014)]. This data is not very detailed, and it is clear that the analysis can be improved when details of the balance sheet are known. The purpose of this section is to illustrate the application of the methodology and to show the approximate impact of liquidity risk on valuation.

In table 1 the assets on the Barclays balance sheet are shown as per 31 dec 2014.

[1] The author has no connections with either Barclays or UBS. All analysis is based solely on publicly available data.

Assets	mGBP
Cash and balances at central banks	39,695
Items in the course of collection from other banks	1,210
Trading portfolio assets	114,717
Financial assets designated at fair value	38,300
Derivative financial instruments	439,909
Available for sale financial investments	86,066
Loans and advances to banks	42,111
Loans and advances to customers	427,767
Reverse repurchase agreements and other similar secured lending	131,753
Current and deferred tax assets	4,464
Prepayments, accrued income and other assets	19,181
Investments in associates and joint ventures	711
Goodwill	4,887
Intangible assets	3,293
Property, plant and equipment	3,786
Retirement benefit assets	56
Total assets	1,357,906

Table 1: Barclays balance sheet per 31 dec 2014 [Barclays FY Results(2014)].

As an LSE, the 30-day event considered in the LCR is used. This event is described as a significant stress scenario and in this example a probability of 1 in 25 years is assigned to this scenario

$$p = 4\% \,. \tag{5.1}$$

According to the Q3 2014 results Barclays has a liquidity pool 149b GBP and LCR$= 124\%$. This suggests that a stress event, as considered in the LCR, results in a $149\text{b}/124\% = 120\text{b}$ net cash outflow in the 30-day stress period. This outflow results in a stress severity of

$$f = 120\text{b}/\sum_i A_i LV_i = 16\% \,. \tag{5.2}$$

For the various assets on the balance sheet, a liquidation value is estimated based on the general description of the asset type. Note that more detailed information could increase the accuracy of the estimates. The estimates for LV's are based on regulatory factors as in [Nauta(2015)]. Note that for derivatives the use of regulatory factor would imply $LV = 0\%$. However in the examples here $LV = 50\%$ is used assuming an approximately equal part of the position consisting of liquid derivatives and illiquid derivatives. The estimated LV's and the resulting liquidity spreads are summarized in table 2.

Assets	LV	l
Cash and balances at central banks	100%	0.00%
Items in the course of collection from other banks	100%	0.00%
Trading portfolio assets	100%	0.00%
Financial assets designated at fair value	80%	0.13%
Derivative financial instruments	50%	0.32%
Available for sale financial investments	80%	0.13%
Loans and advances to banks	100%	0.00%
Loans and advances to customers	25%	0.48%
Reverse repurchase agreements and other similar secured lending	95%	0.03%
Current and deferred tax assets	0%	0.64%
Prepayments, accrued income and other assets	0%	0.64%
Investments in associates and joint ventures	0%	0.64%
Goodwill	0%	0.64%
Intangible assets	0%	0.64%
Property, plant and equipment	0%	0.64%
Retirement benefit assets	0%	0.64%
Total assets		0.29%

Table 2: Liquidity spreads for the assets on Barclays balance sheet.

From table 2, it is seen that the liquidity spread ranges from 0bp (for e.g. cash) to 64bp for illiquid assets. The average spread $l_{\text{av}} = 0.29\%$ times the total assets gives 3.9b that is the total compensation required for liquidity risk per annum. This amount was a significant part (approx. 17%) of the net operating income of 24b in 2014.

Not to single out Barclays the results for UBS are included as well based on full year 2014 reports [UBS financial results(2014)]. The results may be found in table 3.

From table 3, it is seen that the liquidity spread ranges from 0bp (for e.g. cash) to 98bp for illiquid assets. This variation is somewhat larger than for Barclays. The reason is that the estimated severity of the LSE is larger with 25%. The average spread $l_{\text{av}} = 0.41\%$ times the total assets gives 4.4b CHF, which is the total compensation required for liquidity risk per annum. This amount was a significant part (approx. 16%) of the net operating income of 28b in 2014.

The main observations from this exercise are that liquidity spreads of different assets on the same balance sheet differ significantly, liquidity spreads between similar assets on different balance sheets may differ due to different sensitivity to liquidity risk, and liquidity risk is significant.

Assets	mCHF	LV	l
Cash and balances with central banks	104,073	100%	0.00%
Due from banks	13,334	100%	0.00%
Cash collateral on securities borrowed	24,063	100%	0.00%
Reverse repurchase agreements	68,414	95%	0.05%
Trading portfolio assets	138,156	90%	0.10%
Positive replacement values	256,978	50%	0.49%
Cash collateral receivables on derivative instruments	30,979	100%	0.00%
Financial assets designated at fair value	4,951	80%	0.20%
Loans	315,757	25%	0.74%
Financial investments available-for-sale	57,159	80%	0.20%
Investments in associates	927	0%	0.98%
Property and equipment	6,854	0%	0.98%
Goodwill and intangible assets	6,785	0%	0.98%
Deferred tax assets	11,060	0%	0.98%
Other assets	22,988	0%	0.98%
Total assets	1,062,478		0.41%

Table 3: Liquidity spreads for the assets on UBS balance sheet.

6 Summary

This paper develops a liquidity risk valuation framework. The framework implies that liquidity risk of a bank affects the economic value of its assets. The starting observation is that the bank needs to liquidate some of its assets in an LSE, which means these will be sold at a discount. To develop the valuation framework, a liquidation strategy of the bank needs to be determined. It is shown that the optimal liquidation strategy suitable for valuation is a strategy where of each asset the same fraction is liquidated. The result is that cash flows are discounted including a liquidity spread. This liquidity spread consists of three factors: the probability of an LSE, the severity of an LSE, and the asset-specific discount in case of a liquidation in an LSE.

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