Could pension system make us happier?

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Abstract

We have analyzed the effect of a pension system on life expectancy and happiness level using a cross country data and an optimal dynamic problem of individuals who live in continuous and finite time. From the data and the optimization problem, we have found the following: 1) happiness can be almost explained by income per capita, 2) depending on the level of income per capita, the pension system can make lifespan longer or shorter and can raise or reduce the level of happiness, 3) the extension of lifespan without the income support may not always make our happiness higher, 4) under government budget constraint, even though pension system can make the lifespan longer, pension system cannot make the happiness level higher and can rather raise problems for aging population. The public pension system, which is a compulsory saving, can crowd out the private savings and can prevent individual’s utility maximization. This paper suggests that it is not always true that the pension system improves happiness level and that it may be necessary for us to reconsider about the reason for existence of the compulsory pension.

Keywords: pension system, optimized life expectancy, lifetime utility level, health investments
JEL Classification Codes: C61, H55, I31
1. Introduction

Believe it or not, according to an anecdote in Europe, as soon as a pension system was introduced, the number of people who jog in the park for their health increased. If we have a pension system, it looks like a good deal, if we live long enough to enjoy in pension. This research analyzes the effect of a pension system on life expectancy and happiness level. Especially, this research will offer answers to the following questions. Could pension system make us happier? Could pension system make our lifespan longer? Is it always true that longevity ensures happiness? Which one is better, we plan our own future by ourselves or we rely on government pension system? and so on.

There are many literatures on the effect of rising longevity on some economic variables as saving rate, growth rate, labor market, education, and so on. For example, Bloom et al. (2007) and Dushi et al. (2010), Lee et al. (2000) examine the effects of improvements in health or life expectancy on social security system and saving rate. Weil (2007), Acemoglu and Johnson (2007), Zhang and Zhang (2004), Zhang et al. (2001), and so on, analyze the effects of improvements in health or life expectancy on economic growth. Zhang et al. (2003) shows that rising longevity encourages both savings and earlier retirement. Gorski et al. (2007) studies the effects of a pension reform on the educational level of the economy. Pecchenino and Utendorf (1999), de la Croix and Licandro (1999), Cipriiani (2000), Boucekkine, et al. (2002, 2003), Pecchenino and Pollard (1997, 2002), and so on, analyze the effect of longer lifespan on economic growth through the level of schooling and human capital accumulation. Lorentzen et al. (2008) and Chakraborty et al. (2010), and so on, analyze the effect of mortality and disease on economic growth and growth trap. Lorentzen et al. (2008) mentions that higher adult mortality has bad influences on economic growth and could be the source of a poverty trap through increased levels of risky behavior, higher fertility, and lower investment in physical and human capital. Recently, Pestieau and Ponthiere (2012) surveys the various contributions to the impact of changes in longevity on various public policies. However, there is little research on the opposite direction, that is, how economic variables, except income per capita, affect life expectancy or health. This research is different from previous researches in the fact that a pension system is the cause, and the life expectancy and the happiness level are its effects.

A vast amount of empirical and theoretical researches about the economic welfare of a pension system has been accumulated. The main results of some previous studies on pension system and economic welfare can be summarized as follows: under a fully funded system, the economic welfare is not affected, however, under a pay-as-you-go pension system, depending on the economic situations and generations, the economic welfare might be both improved or worsened. The public pension system as a risk-hedging device can increase welfare by providing a certainty in the imperfect market, e.g., Shiller (1999), Krueger and Kuhler (2002), Sánchez-Marcos and Sánchez-Martin (2006), Bohn (2009), etc. The compulsory pension system which is one of the forced saving policies can lead to high saving rates, meanwhile, the public pension system crowds out the private savings. The pension system can have a negative effect on the capital accumulation and can retard growth, e.g., Cutler and Gruber (1996), Feldstein and Liebman (2002), Zhang and Zhang (2004), etc. The overall welfare impact depends on the balance between the insurance effect and the crowding-out effect. In this research, we show that the pension system has both positive and negative effect on welfare using a cross country data. More specifically, the pension system has a positive effect on welfare when income per capita is low, but the pension system has a negative effect on welfare when income per capita is high.

Many previous researches analyze economic welfare using overlapping generation models. We use an optimal dynamic problem of individuals who live in continuous time, instead of discrete time which has been used in the overlapping generation models, e.g., Chakraborty (2004), Sánchez-Marcos and Sánchez-Martin (2006), Ponthiere (2009), etc. This is one of the differences of this research model from the previous models. To tell more specifics, in many previous overlapping generation models, the maximum lifespan has been given (e.g., two-period or three-period) and the survival probability has been introduced and the life expectancy has been calculated by the average of the longevity of the people who live to the maximum lifespan and the people who die before the maximum lifespan depending on the survival probability. Actually, in two-period model, only two kinds of ages (i.e., one-period-old and two-period-old) exist and nobody survives more than the given period even though the life expectancy has variations. However, in this research model, the individual decides about the time when he/she will die, at the terminal point of the continuous time model, to maximize his/her happiness level. We consider that the individual’s longevity is based from the result of the individual’s utility maximization problem. Individuals could choose to live a short and intensely
happy life, or a longer and less intensely happy life, or a so-so long and so-so happy life. For example, there are many people who still smoke, even though they know all of the health risks and there are so many warnings and pictures showing the consequences on the cigarette packets. We can interpret their behavior by saying that they prefer some present pleasure from smoking, even if smoking plays havoc with their health. It can be an example of the first case, that is, they choose to live a short and intensely happy life. If the length of life is chosen optimally, the relationship of the length of life and the level of happiness could not be proportional.

We consider a lifetime utility maximization problem between the length of life and the level of happiness under individual’s budget constraint. This means that the individual’s longevity is an endogenous variable, not exogenous variable. An individual distributes his/her budget to his/her basic needs and to his/her health investments to maximize his/her lifetime utility. Like lifetime uncertainty models (e.g., Pecchenino and Pollard (1997), Chakraborty (2004), Momota et al. (2005), etc.), we assume that it is possible to extend lifespan by the effort of an individual through health investments. For example, eating good food, taking some nutritional supplements, getting in shape by going to the gym, investing in the development of medical technology, and so on. The longevity will arise due to the implementation of the previously mentioned examples of the health investments. In reality, it is well known that coronary heart disease (CHD) mortality is highly influenced by the major risk factors, e.g., serum cholesterol, systolic blood pressure, diabetes, smoking habits, high alcohol consumption, lack of exercise and stress, etc. Lifestyle changes through individual’s efforts (e.g., healthier diet, physical exercise, cessation of smoking and drinking, etc.) and medications have been shown to be effective in reducing coronary disease. If we can eliminate the risk factors, the life expectancy will undoubtedly grow.

We have investigated how the optimized lifespan and the lifetime utility level have been changed by the existence or non-existence of the pension system. We have compared the lifetime utility level under the restriction of the pension system as a compulsory saving with the lifetime utility level without the restriction. We have shown the following using the cross country data: 1) Income per capita can almost explain happiness. The positive correlation between life expectancy and happiness is a spurious relationship in which two variables have no direct causal connection. In reality, the income per capita has caused both. And the lifespan does not have much influence on happiness. 2) Depending on the level of income per capita, the pension system can make lifespan longer or shorter. 3) Depending on the level of income per capita, the pension system can raise or reduce the level of happiness. 4) Life expectancy itself is not always proportional to the happiness level. Moreover, we have shown following four important results using this research model: i) Life expectancy is not always proportional to lifetime utility level. ii) Pension system can make the lifespan longer or shorter. The lifespan depends on the type of pension system. iii) Under government budget constraint, even though pension system can make the lifespan longer, pension system cannot make the happiness level higher. The pension system can rather raise problems for aging population which affect the country’s productivity and growth rate negatively through the decline in the fraction of working-age population.1 iv) If the prediction of lifespan does not turn out to be completely wrong under lifetime uncertainty, it is not always true that the pension system improves the lifetime utility level.

This paper is organized as follows: Section 2 confirms the relationship among income per capita, existence or non-existence of pension system, happiness and life expectancy using a cross country data. Section 3 presents a benchmark model and drives the benchmark outcomes. Section 4 introduces a pension system to the benchmark model and solves the models numerically and analyzes the results. Section 5 introduces an uncertainty in the benchmark model and analyzes the results. Section 6 offers conclusions on this research. Finally, more information on each country and the detailed calculation can be found in Appendix.

2. Empirical Facts

2.1. Data

We have used data of GDP per capita, life expectancy and the existence or non-existence of a pension system as the determinants of happiness. Each of the variables is thought to be the variable that looks at an economic side, a biological side and a social systematic side to decide about the level of happiness, respectively. We will clarify how a pension system influences happiness using the data set. The data used in this research can be easily downloaded on the internet. The

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1 The public pension system which is a compulsory saving can raise the national saving rate and the growth rate if it does not crowd out the private savings.
happiness index and the life expectancy are available at World Database of Happiness and the GDP per capita is also available at Penn World Table. The data of the existence or non-existence of a pension system are found in Table 1 of Bloom et al. (2007). World Database of Happiness had released the averages of the happiness index from 2000 to 2009 and the averages of life expectancy from 2000 to 2009 for 10 years. The range of the happiness index is from 0 to 10. The GDP per capita has used the variable “rgdpcch” in Penn World Table 7.1. According to the Penn World Table 7.1, the variable “rgdpcch” is GDP per capita (chain series) converted using Purchasing Power Parity (PPP), at 2005 constant prices. We have calculated the average of GDP per capita from 2000 to 2009 to meet the happiness index and the life expectancy in World Database of Happiness. We used the logarithm scale for GDP per capita, in the following analysis. The pension data, which are dummy variable for the existence or non-existence of a pension system, show the situation in 2002. The value of dummy variable is one when the country has any pension system and the value of dummy variable is zero when the country does not have any pension system. Regarding pension data, we have used the figures under the name “Universal coverage” from the Table 1 of Bloom et al. (2007). According to Bloom et al. (2007), the dummy variable of “Universal coverage” indicates whether the system covers all workers or not.

We have reported the detailed data source in Table 1. World Database of Happiness, Penn World Table 7.1 and the pension data in 2002 of Bloom et al. (2007) listed 149, 190 and 61 countries, respectively. We focus on the 61 countries which have all the three data sets. Table 5 in the Appendix contains the basic information of the 61 countries.

Fig. 1 plots the relationship between income per capita and happiness, between life expectancy and happiness, and between the existence or non-existence of a pension system and happiness, visually. In Fig. 1, each vertical axis shows the happiness level. os represent the countries that have pension system while on the other side xs represent the countries that do not have any pension system. It appears that all of the three cases have positive relationships, between income per capita and happiness, between life expectancy and happiness, and between the existence or non-existence of a pension system and happiness, respectively. Each of the coefficients of correlation is \( r \) (income per capita, happiness) = 0.833, \( r \) (life expectancy, happiness) = 0.788, and \( r \) (pension system, happiness) = 0.390. We have reported the coefficients of correlation in Table 2.

2.2. Regression Analysis

2.2.1. Happiness

We have investigated the relationships between happiness and income per capita, between happiness and life expectancy, and between happiness and pension system, statistically. The happiness is treated as the dependent variable. We denote the happiness level, the income per capita, the life expectancy and the existence or non-existence of a pension system as \( H_i \), \( y \), \( L \) and \( P \), respectively.

We use linear regression models as Eq. (1) to Eq. (4). Eq. (1) has a single regressor, the income per capita \( y \). Eq. (2) has two regressors, the income per capita \( y \) and the life expectancy \( L \). And, Eq. (3) and Eq. (4) also have two regressors, the income per capita \( y \) and the existence or non-existence of a pension system \( P \).

We have estimated the variables using maximum likelihood estimation (MLE). The maximum likelihood estimation is a method to maximize the likelihood function to estimate the variables. The subscripts, \( i \) represent country \( i \). We have assumed that each of the errors, \( \epsilon_i \), is identically distributed, independent random variable with \( \epsilon_i \sim N(0, \sigma^2) \). Table 3 has reported the estimation results of Eq. (1) to (4), respectively.

\[
H_i = \beta_0 + \beta_1 y_i + \epsilon_i \quad (1)
\]
\[
H_i = \beta_0 + \beta_1 y_i + \beta_2 L_i + \epsilon_i \quad (2)
\]
\[
H_i = \beta_0 + \beta_1 y_i + \beta_2 P_i + \epsilon_i \quad (3)
\]
\[
H_i = (1 - P_i)\beta_0 + \beta_1 y_i + P_i(y_0 + \gamma_1 y_i) + \epsilon_i \quad (4)
\]

All of the \( \beta_i \)s for the income per capita \( y \) in Eq. (1), (2) and (3) are positive and significant, but the \( \beta_2 \) in Eq. (2) for the life expectancy \( L \) and the \( \beta_2 \) in Eq. (3) for the existence or non-existence of a pension system \( P \) are not significant, respectively. We know that the happiness can be almost explained by the income per capita, and additionally that, life expectancy and pension system, do not affect the level of happiness, significantly. Furthermore, we also know, from the result of Eq. (4), that depending on the existence or

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2 There may be both superior and inferior pension systems depending on the countries. However, in this research, we have checked weather there is a pension system or not. We will not consider about the quality of pension system.

3 There are fewer countries without pension system. Only 13 countries out of 61 (21.5%) do not have their pension system which means 48 countries (78.7%) have it.
Table 1: Data sources

<table>
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<tr>
<th>Indicators</th>
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<th>Sources</th>
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</tr>
<tr>
<td>Life expectancy</td>
<td>World Database of Happiness</td>
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<tr>
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<td>World Bank</td>
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</tr>
<tr>
<td>Per capita GDP</td>
<td>Penn World Table</td>
<td><a href="https://pwt.sas.upenn.edu/">https://pwt.sas.upenn.edu/</a></td>
</tr>
<tr>
<td>Pension</td>
<td>Bloom et al. (2007)</td>
<td>Table 1, Social security systems: retirement and pension provisions, page 105-108</td>
</tr>
</tbody>
</table>

*) The happy life years are calculated by the product of the satisfaction with life and the life expectancy.

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(1) Income and happiness

Income per capita vs Happiness

(2) Life expectancy and happiness

Life expectancy vs Happiness

(3) Pension system and happiness

The existence or non-existence of a pension system

Happiness vs The existence or non-existence of a pension system

Figure 1: Income, life expectancy, pension system and happiness

We visualize the regression results from Eq. (1) to Eq. (4) in Fig. 2. Fig. 2 (1) shows regression lines based on Eq. (1) and Eq. (4). By comparing the red line ($P = 0$) and the blue line ($P = 1$), the slope of the red line is steeper than that of the blue line. As the non-existence of a pension system, the estimated values of the intercept and the slope are quite different. The intercept ($\gamma_0$), where pension system exists, is bigger than the intercept ($\beta_0$) where pension system does not exist. $\beta_0 < \gamma_0$. The slope ($\gamma_1$), where pension system exists, is smaller than the slope ($\beta_1$) where pension system does not exist. $\beta_1 > \gamma_1$. It means that the effect of pension system on the happiness is different depending on the income level.
Table 2: Coefficients of correlation

<table>
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<tr>
<th>variables</th>
<th>happiness</th>
<th>GDP per capita</th>
<th>life expectancy</th>
<th>pension</th>
<th>happy life years</th>
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<td>happiness</td>
<td>1.000</td>
<td>0.833</td>
<td>0.788</td>
<td>0.390</td>
<td>0.961</td>
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<tr>
<td>GDP per capita</td>
<td>0.833</td>
<td>1.000</td>
<td>0.882</td>
<td>0.529</td>
<td>0.909</td>
</tr>
<tr>
<td>life expectancy</td>
<td>0.788</td>
<td>0.882</td>
<td>1.000</td>
<td>0.527</td>
<td>0.919</td>
</tr>
<tr>
<td>pension</td>
<td>0.390</td>
<td>0.529</td>
<td>0.527</td>
<td>1.000</td>
<td>0.467</td>
</tr>
<tr>
<td>happy life years</td>
<td>0.961</td>
<td>0.909</td>
<td>0.919</td>
<td>0.467</td>
<td>1.000</td>
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Table 3: Estimation results

<table>
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<tr>
<th>Parameters</th>
<th>Eq. (1)</th>
<th>Eq. (2)</th>
<th>Eq. (3)</th>
<th>Eq. (4)</th>
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<tr>
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<td>-1.422</td>
<td>-1.870</td>
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<td>0.587</td>
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<tr>
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<td>$\gamma_1$</td>
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<td>$t$ value</td>
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<td>-</td>
<td>8.922</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma$</td>
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<td>0.691</td>
<td>0.702</td>
<td>0.692</td>
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<td>11.045</td>
<td>11.045</td>
<td>11.045</td>
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<tr>
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<tr>
<td>reverse point</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.206</td>
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</table>

The upper and lower show the estimated values and $t$ values, respectively.

Income level increases, both lines cross each other and the reverse of their positions occurs. We can find that when the income per capita is low, the blue line is upper than the red line, that is, the happiness is higher with the pension system comparing to the one without the pension system, but when the income per capita is high, the opposite occurs, which means that the red line is upper than the blue line, that is, the happiness is higher without the pension system comparing to the one with the pension system. The reverse point happens around income per capita $1,347 = \exp(7.206)$ in 2005 value which is small. It can be inferred that when income level is low, the pension system makes people happier, on the other hand, when income level is high, the pension systems make people less happy. The pension system, which is a kind of the forced saving policies, can prevent individuals from maximizing their utility which is a measurement of their happiness. In other words, even though we face uncertainty in our life so we do not know when we will die exactly, the results suggest that above certain level of income, we can be happier by planning our own future and by optimizing our utility by ourselves without any compulsory saving like pension system.

In Fig. 2 (1), we have also drawn a black line which is the regression line based on Eq. (1). And then, for each country, we have measured the gap between the country’s actual level of happiness and the level predicted by the regression line. Fig. 2 (2) shows the relationship between the residual part of happiness and the life expectancy. Because the happiness is almost explained by the income per capita, there is not high relationship between the residuals and the life expectancy. The correlation coefficient is only $r=0.096$. Fig. 2 (2) means that the $\beta_2$ of Eq. (2) is not significant as we have seen.

Fig. 2 (3) shows the relationship between the residuals of life expectancy regressed on the income per
capita and the residuals of happiness regressed on the income per capita. When the influence of income on the life expectancy and the happiness is removed, the relationship between both variables, the controlled life expectancy and the controlled happiness, is not so strong, only \( r = 0.205 \).

From the results of Fig. 2 (2) and (3), we know that the positive correlation between the life expectancy and the happiness in Fig. 1 (2) is a spurious relationship. Even if we can see positive relationship between longer life expectancy and happiness, we now know that the two variables have no direct causal connection and the income per capita actually works behind the two variables, the life expectancy and the happiness. The increasing lifespan without increasing income does not necessarily increase happiness. Even though we may not like to admit it, it unfortunately suggests that the survival itself is not always making our happiness and utility high and the two of those combined make us happy.4

2.2.2. Life expectancy and happy life years

First, we have investigated the relationships between the life expectancy and the income per capita, and between the life expectancy and the existence or non-existence of a pension system. The life expectancy is treated as the dependent variable. We have done regression analysis using Eq. (5) to Eq. (7). Eq. (5) has a single regressor, income per capita \( y \). Eq. (6) and Eq. (7) have two regressors, income per capita \( y \) and the existence or non-existence of a pension system \( P \). We have estimated the variables by the maximum likelihood estimation like as what we have done in the Section 2.2.1.

Second, we also have investigated the relationships between the happy life years \( HL \) and the income per capita \( y \), and between the happy life years \( HL \) and the pension system \( P \). The happy life year is treated as the dependent variable. The happy life years are calculated by the product of the level of happiness and the life expectancy. We have done regression analysis using Eq. (8) to Eq. (10). \( HL_i \) represents the happy life year in country \( i \).

4Some researches have a different opinion about it, for example, Becker et al. (2005) shows that life expectancy gains have been an important component of improvements in welfare which is also affected by quantity of life as represented by longevity. That may be the case in the situation where their financial problems can be resolved.
bigger than the intercept (does not exist. The intercept and the slope of the regression line are quite different depending on the existence or non-existence of a pension system, the happy life years. We also know from Eq. (7) and Eq. (10), that depending on the existence or non-existence of a pension system, the intercept and the slope of the regression line are quite different.

The intercept ($\gamma_0$), where pension system exists, is bigger than the intercept ($\beta_0$) where pension system does not exist, $\beta_0 < \gamma_0$. The slope ($\gamma_1$), where pension system exists, is smaller than the slope ($\beta_1$) where pension system does not exist, $\beta_1 > \gamma_1$. It means that the effects of pension system on the life expectancy and the happy life years are different depending on the income level.

We visualize the regression results in Fig. 3 which shows regression lines. Fig. 3 (1) shows the relationship between the income per capita and the life expectancy. Fig. 3 (2) shows the relationship between the income per capita and the happy life expectancy. Fig. 3 shows that the red regression lines without the pension system ($P = 0$) are steeper than the blue regression lines with the pension system ($P = 1$). As the income level increases, both lines cross each other and the reverse of their positions occurs as we have seen at Fig. 2 (1).

We can find that when the income per capita is low, the life expectancy and the happy life year are longer with the pension system comparing to the one without the pension system, otherwise, when the income per capita is high, the life expectancy and the happy life year are longer without the pension system comparing to the one with the pension system. Each of the reverse points of Eq. (7) and Eq. (10) happens around income per capita 5,410 = exp(8.596) and 2,390 = exp(7.779), respectively. Anyhow, the values are low. In the countries with low income levels, both the life expectancy and the happy life year are increased by the pension system, on the other hand, in the countries with high income levels, both the life expectancy and the happy life years are decreased by the pension system. It suggests that in the countries above certain level of income, the individual’s utility optimizations can be hampered by the savings forced by the pension system.

### Table 4: Estimation results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Eq. (5)</th>
<th>Eq. (6)</th>
<th>Eq. (7)</th>
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<td>$t$ value</td>
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<td>-</td>
<td>10.131</td>
<td>-</td>
<td>-</td>
<td>12.653</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>5.156</td>
<td>5.097</td>
<td>4.762</td>
<td>5.845</td>
<td>5.841</td>
<td>5.726</td>
</tr>
<tr>
<td>$t$ value</td>
<td>11.045</td>
<td>11.045</td>
<td>11.045</td>
<td>11.045</td>
<td>11.045</td>
<td>11.045</td>
</tr>
<tr>
<td>log likelihood value</td>
<td>-186.60</td>
<td>-185.91</td>
<td>-181.76</td>
<td>-194.26</td>
<td>-194.21</td>
<td>-193.00</td>
</tr>
<tr>
<td>reverse point $\frac{\gamma_0 - \beta_0}{\beta_1 - \gamma_1}$</td>
<td>-</td>
<td>-</td>
<td>8.596</td>
<td>-</td>
<td>-</td>
<td>7.779</td>
</tr>
</tbody>
</table>

The upper and lower show the estimated values and $t$ values, respectively.
2.2.3. Nonparametric estimation

To loosen the assumption of linear relationship between the variables which we have assumed in the Section 2.2.1 and the Section 2.2.2, we have estimated the relationship by a nonparametric estimation which estimates the regression function without assuming any specific form. \( f(\cdot) \)s in Eq. (11) to Eq. (17) are the unspecified regression functions.

Each of the equations from Eq. (11) to Eq. (13) has one regressor, the income per capita \( y \) and each of the equations from Eq. (14) to Eq. (17) has two regressors, the income per capita \( y \) and the pension system \( P \) or the income per capita \( y \) and the life expectancy \( L \). The dependent variables in Eq. (11) to Eq. (13) are the happiness \( H \), the life expectancy \( L \) and the happy life years \( HL \), respectively. The dependent variables in Eq. (14) to Eq. (17) are the happiness \( H \), the happiness \( H \), the life expectancy \( L \) and the happy life years \( HL \), respectively.

\[
H_i = f(y_i, P_i) + \epsilon_i \\
L_i = f(y_i, L_i) + \epsilon_i \\
HL_i = f(y_i, P_i) + \epsilon_i
\]

Eq. (11) to Eq. (13) are the regression results of Eq. (11) to Eq. (13), respectively. The estimation results in Fig. 4 (1) to (3) are not much different to the results obtained by assuming the linear relationship as before. Fig. 4 (1) corresponds to Fig. 2 (1), and Fig. 4 (2) corresponds to Fig. 3 (1), and Fig. 4 (3) corresponds to Fig. 3 (2). The red lines \( (P = 0) \) without the pension system are steeper than the blue lines \( (P = 1) \) with the pension system.

Because the equations from Eq. (14) to Eq. (17) have two regressors and one dependent variable, we cannot depict the regression results on two dimensional space, so we depict the results using contour lines in Fig. 5.\(^5\) The contour line shows the same level of dependent variable in the different combinations of two regressors. For example, the contour lines in Fig. 5 (1) show the same levels of happiness in the different combinations of two regressors which are the income per capita and the pension system.

Fig. 5 (1) shows that when the income per capita

\(^5\)I have used the command “Tps” in R to estimate the nonparametric regression models.
is low, the contour lines are vertically-tilted. It means that regardless the involvement of the pension system, the level of happiness is almost the same. However, as the income per capita increases, the contour lines are no longer vertically-tilted. The contour lines begin to turn clockwise. For example, point A where income per capita which is 8.5 without the pension system and point B where income per capita which is 10.0 with the pension system have the same level of happiness, that is, the two points, A and B, are connected with a contour line where the same level of happiness is seven. In other words, if the income levels are the same, the level of happiness is higher when there is no pension system.

Fig. 5 (2) shows the happiness level in the different combinations of the life expectancy and the income per capita. We found that when the income per capita is low (left-below side), the contour lines are downward-slop, however, when the income per capita is high (right-upper side), there is no a specific pattern. By comparing the point C and the point D, which are densely-packed with the data, both levels of the happiness are the same, but the combinations of the income per capita and the life expectancy are different. The point C is a combination of high income per capita and long life expectancy, on the other hand, the point D is a combination of low income per capita and short life expectancy. The short life expectancy with less income is equivalent to the long life expectancy with more income which means the life expectancy itself is not always proportional to the happiness.

Fig. 5 (3) shows the life expectancy in the different combinations of the existence or non-existence of a pension system and the income per capita. We have found that when the income per capita is low (left-below side), the contour lines are downward-slop, however, as the income per capita increases, the contour lines change almost vertically-tilted, slightly in a clockwise direction. When the income per capita is low, the pension system makes the life expectancy longer, however, when the income per capita is high, the pension system is uninfluential or negative to the extension of the life expectancy. This result is consistent with the regression results of Eq. (6) and Eq. (7).

Fig. 5 (4) shows the happy life years in the different combinations of the existence or non-existence of a pension system and the income per capita. We also found that almost of the contour lines are vertically-tilted. This means that the existence or non-existence of a pension system does not affect to the happy life years. This result is consistent with the regression results of Eq. (9).

2.3. Summary

We have summarized some empirical factors which are obtained by the usage of the regression analysis.

1. The level of happiness can be almost explained by the income per capita. The relationships between the happiness and the other variables, which are
the life expectancy and the pension system, are spurious relationships. In reality, the income per capita have caused both.

2. The pension system can make the life expectancy longer or shorter depending on the level of income per capita. However, in many cases, the pension system may give bad effects on the extension of life expectancy.

3. When the level of income per capita is low, the pension system makes the happiness higher, on the contrary, when the level of income per capita is high, the level of happiness may be higher when there is no pension system.

4. Life expectancy itself is not always proportional to the happiness. The extension of lifespan without the income support may not always make our happiness higher. The extension of lifespan only accompanied by the income support may make our happiness higher.

3. The benchmark model

In this section, we have created a utility maximization model to analyze the relationship between the life
expectancy and the level of happiness, between the income per capita and the level of happiness, and between the income per capita and the life expectancy which we have seen in the previous section. We use the lifetime utility to measure the level of happiness which is an abstract variable. Happiness is not exactly the same with utility, but both happiness and utility have a positive relationship. According to Kimball and Willis (2006), Jeremy Bentham’s (1781) first definition of “utility” made the equation of utility and happiness explicit. We assume that both happiness and utility have a close relationship and the higher the utility level is, the higher the happiness level is.

3.1. Setting

We consider an individual’s utility maximization problem under the finite period. He/She can live up to $T$ years old and die at the age of $T$. There is no uncertainty in the model and individuals have perfect foresight. An individual maximizes his/her lifetime utility which is affected by consumption. The instantaneous utility function ($u(c)$) is specified in log form as follows:

$$u(c) = \ln c$$  \hfill (18)

where $c$ is consumption. We think that it is possible to extend the lifespan by the efforts of the individual. We assume that there is a linear relationship between health investment and the lifespan as follows

$$T = a + bz, \quad (a > 0, \quad b > 0)$$  \hfill (19)

where $T$ and $z$ are the lifespan and the health investment, respectively. And $a$ and $b$ are positive constants. When $z$ is decided, $T$ is automatically decided, on the contrary, when $T$ is decided, $z$ is automatically decided, which means if we invest amount of $z$, we can live until $T$ and if we want to live until $T$, we should invest amount of $z$. If an individual invests for his/her health more, as the result, his/her lifespan will be longer. We assume that the health investments do not affect the utility directly.\(^6\)

We also assume that the interest earning is the only source of income of the individual. And to simplify, a small open country is assumed, then the domestic interest rate is always constant. We denote the individual’s asset as $x$, then his/her budget constraint is written as follows:

$$\dot{x} = rx - c - z$$  \hfill (20)

where $r$ is interest rate. We put the initial asset which the individual has as $x(0) = x_0$.

The individual’s utility maximization problem can be written as follows:

$$\max_{c(t), z} \int_0^T e^{-\rho t} \ln c(t) \, dt, \quad (0 < \rho < 1)$$

s.t. \hspace{1cm} \dot{x}(t) = rx(t) - c(t) - z,  \hfill (21)

$$T = a + bz,$$

$$x(0) = x_0$$

where $\rho$ is discount rate. We assume the $\rho$ is constant, that is, this model is a exponential discounting model, not a hyperbolic discounting model which is treated in behavioral economics. We assume $r \geq \rho$.\(^7\) For simplification, we assume that $z$ has a constant value from the initial period until $T$ period, $z(0) = \cdots = z(T) = z$, and that $z$ is decided at the initial period. Life expectancy is the number of years a person can expect to live in given social environments when he/she is born. We assume that when an individual is born, he/she decides how much he/she invests for his/her health and how long he/she lives in the social environments surrounding him/her which are $x_0, a, b, r, \rho$, etc.

3.2. Solving the Model

The maximization problem is solved in two stages. At the first stage, we consider that $T$ and $z$ in Eq. (19) are given values, not control variables. At the second

---

\(^6\)For example, utility is a reflection of people’s choices and happiness is a reflection of people’s feelings.

\(^7\)Kimball and Willis (2006) mentioned that in the existing literature attempting to link utility and happiness, the dominant explicit or implicit hypothesis is that current felt happiness is equal to flow utility. Kahneman (1999), Gruber and Mullainathan (2002), Frey and Stutzer (2004), and Layard (2005) are some of the most explicit in equating happiness and flow utility.

\(^8\)We can divide consumption $c$ into two categories which are the general consumption $c^G$ and the consumption for health improvement $c^H$. It is unclear whether the direct effect of the latter $c^H$ on the utility of individual is positive or negative or neutral. For examples, there might be a person who takes wheatgrass powder for his/her health maintenance even though it is unpalatable, while on the other side, there might be a person who takes it with the thinking that it is delicious. Also, there might be a person who commutes to the gym for his/her health maintenance though it is painful, while on the other side, there might be a person who goes happily to the gym. Nutritional supplements are beneficial for health but are not delicious or tasteless. Therefore, we can assume that the consumption for health improvement $c^H$ is neutral to the individual’s utility and only the general consumption $c^G$ affects the individual’s utility. This means $\frac{\partial u}{\partial c^G} > 0$ and $\frac{\partial u}{\partial c^H} = 0$, so $u(c^G, c^H) = u(c^G)$.

\(^9\)If $r = \rho$, there is no transitional path, because the jump from the initial state up to the terminal state occurs. If $r < \rho$, there is an overshooting, the amount of his/her asset accumulation turns back to the terminal state and has a negative growth rate. We do not consider the negative growth in this research.
stage, we consider the $T$ and $z$ as control variables. First, we maximize over $c$ and $x$ for any given $T$ and $z$, and then the objective function which has been maximized with respect to $c$ and $x$ could be described as a function of $T$ and $z$. Second, we maximize over $T$ and $z$ instead of $c$ and $x$, because $c$ and $x$ have been maximized in the first stage, that means $c$ and $x$ are functions of time $t$, i.e., $c(t|T,z)$ and $x(t|T,z)$.

3.2.1. The First Stage

We use the Hamiltonian method to solve the maximization problem. The Hamiltonian is written as follows:

$$H = \ln c + \lambda (rx - c - z)$$  \hspace{1cm} (22)

By differentiating Eq. (22) with respect to $c$ and $x$, we can get Eq. (23) and Eq. (24).

$$\frac{\partial H}{\partial c} = \frac{1}{c} - \lambda = 0 \quad \Rightarrow \quad c = \lambda^{-1},$$  \hspace{1cm} (23)

$$\frac{\partial H}{\partial x} = \rho \lambda - \lambda r = \frac{\lambda}{\lambda} = \rho - r.$$  \hspace{1cm} (24)

We integrate Eq. (24) to time $t$, then we get

$$\ln \lambda = (\rho - r)t + k$$  \hspace{1cm} (25)

where $k$ is a constant of integration. Taking exponential both sides of Eq. (25), then we can get

$$\lambda = C_1 e^{(\rho - r)t}$$  \hspace{1cm} (26)

where $C_1 = e^k$. Substituting Eqs. (23) and (26) into Eq. (20), we obtain the following

$$x = \frac{1}{C_1} \left( e^{\rho t} - 1 \right) e^{rt} + C_2 e^{rt} + \frac{z}{r}$$  \hspace{1cm} (27)

This differential equation is solved as follows,

$$x = \frac{1}{C_1} \left( e^{\rho t} - 1 \right) e^{rt} + C_2 e^{rt} + \frac{z}{r}$$  \hspace{1cm} (28)

where $C_2$ is a constant. See Appendix A.1 for the detailed calculation. $C_1$ and $C_2$ can be obtained from substituting the initial condition and transversality condition. Because of $x(0) = x_0$, we get $C_2$ as follows,

$$C_2 = x_0 - \frac{z}{r}.$$  \hspace{1cm} (29)

To maximize his/her utility, when dying, he/she uses up all his/her assets and leaves nothing. In other words, $x(T) = 0$. We get $C_1$ as follows,

$$C_1 = \frac{1}{\rho x_0 - (1 - e^{\rho T})\frac{z}{r}}.$$  \hspace{1cm} (30)

Substituting Eqs. (29) and (30) into Eq. (28), we obtain the following

$$x(t) = \frac{x_0 - (1 - e^{-\rho T})\frac{z}{r}}{1 - e^{-\rho t}} e^{-\rho t} + \frac{z}{r}.$$  \hspace{1cm} (31)

Substituting Eq. (26) into Eq. (23), we can get

$$c(t) = \rho x_0 - (1 - e^{-\rho T})\frac{z}{r} e^{(\rho - r)t}.$$  \hspace{1cm} (32)

Eqs. (31) and (32) are the optimal paths of $x$ and $c$, respectively, in the situation where the variables $T$ and $z$ are fixed.

3.2.2. The Second Stage

In the second stage, to maximize his/her lifetime utility, the individual considers Eq. (19) by choosing his/her optimal $T$. We can rewrite the utility maximization problem as follows:

$$\max_T \int_0^T e^{-\rho t} \ln \left( \frac{x_0 - (1 - e^{-\rho t})\frac{z}{r}}{1 - e^{-\rho t}} e^{(\rho - r)t} \right) dt$$  \hspace{1cm} \text{s.t.} \quad T = a + bz$$  \hspace{1cm} (33)

We solve the integral in Eq. (33), then we can induce Eq. (34)

$$\int_0^T e^{-\rho t} \ln \left( \frac{x_0 - (1 - e^{-\rho T})\frac{z}{r}}{1 - e^{-\rho t}} e^{(\rho - r)t} \right) dt$$

$$= -\ln \left( \frac{x_0 - (1 - e^{-\rho T})\frac{z}{r}}{1 - e^{-\rho t}} e^{(\rho - r)t} - 1 \right)$$  \hspace{1cm} (34)

See Appendix A.2 for the detailed calculation. Substituting Eq. (19) into Eq. (34). The maximization problem can be rewritten as Eq. (35) which has no integral and has only one control variable $T$. Eq. (35) is just a static maximization problem, not a dynamic problem.

$$\max_T \ln \left( \frac{x_0 - (1 - e^{-\rho T})\frac{T - a}{b}}{1 - e^{-\rho t}} \right) \left( 1 - e^{-\rho T} \right)$$

$$+ (r - \rho) \left( \frac{1}{(\rho + 1) e^{-\rho T}} - 1 \right)$$  \hspace{1cm} (35)

We take the derivative of Eq. (35) with respect to $T$ and set the first derivative to zero as follows,

$$e^{-\rho T} \ln \left( \frac{x_0 - (1 - e^{-\rho T})\frac{T - a}{b}}{1 - e^{-\rho t}} \right)$$

$$- 1 - e^{-\rho T} e^{-\rho T} (T - a) + (1 - e^{-\rho T})\frac{z}{r}$$

$$- e^{-\rho T} + (r - \rho)T e^{-\rho T} = 0$$  \hspace{1cm} (36)
Eq. (36) is an implicit function as \( f(x_0, T|a, b, r, \rho) = 0 \) which is highly non-linear, so it is difficult to solve it analytically.

### 4. Pension system in case of lifetime certainty

One of the purposes of this research is to analyze the effect of the pension system on the maximized utility. We compare the maximized utility with the constraint by compulsory saving such as a pension system and the maximized utility without the constraint. We know that if other things are constant (ceteris paribus), less constrained individual is generally happier than more constrained individual. In the following sections, 4 and 5, we will deal with two pension models that include, lifetime certainty model and lifetime uncertainty model, respectively. In section 4, due to certainty, pension does not have a function of insurance which is a main function of pension. In section 5, we will introduce an uncertainty to add the function of insurance.

#### 4.1. Lifetime certainty model

Individuals will live until the time of death. In section 4, due to certainty, pension does not have the function of insurance which is a main function of pension. In the following sections, 4 and 5, we will deal with two pension models that include, lifetime certainty model and lifetime uncertainty model, respectively. In section 4, due to certainty, pension does not have a function of insurance which is a main function of pension. In section 5, we will introduce an uncertainty to add the function of insurance.

#### 4.1.1. Setup

We introduce a pension system into the benchmark model additionally. He/She pays a pension \( p \) from 0 to \( s \) period and gets a pension \( q \) after \( s \) period to death. The government decides about \( p, q \) and \( s \) which are constants as given to individuals. This pension system performs as a compulsory saving for individuals. The time from 0 to \( s \) is named as young period, while the time after \( s \) is named as old period. His/Her budget constraint in the benchmark model Eq. (20) is changed to Eq. (37).

\[
\dot{x} = \begin{cases} 
rx - c - z + p, & \text{if } 0 \leq t \leq s \text{ (young period)} \\
rx - c - z + q, & \text{if } s < t \leq T \text{ (old period)}.
\end{cases}
\]  

(37)

The way to solve the model with this pension system is similar to that of the benchmark model even though we have to divide it into young period and old period. Eq. (28) is changed as follows:

\[
x = \begin{cases} 
\frac{1}{C_1} \left( e^{\rho t} - 1 \right) e^{\rho t} + C_2^y e^{\rho t} + \frac{z}{\rho}, & \text{if } 0 \leq t \leq s \\
\frac{1}{C_1} \left( e^{\rho t} - 1 \right) e^{\rho t} + C_2^o e^{\rho t} + \frac{q}{\rho}, & \text{if } s < t \leq T.
\end{cases}
\]  

(38)

where, \( C_1^y, C_2^y, C_1^o \) and \( C_2^o \) are constants of integration which are as follows:

\[
C_1^y = \frac{1}{\rho} e^{\rho s} x_0 - \frac{1}{(1 - e^{\rho t})^{1/\rho}} - x(s)e^{\rho t}
\]

\[
C_2^y = x_0 - \frac{z + p}{\rho} \tag{40}
\]

\[
C_1^o = \frac{1}{\rho} \sum_{t=s}^T (1 - e^{\rho(t-T)}) - x(s) \tag{41}
\]

\[
C_2^o = \frac{1}{\rho} \sum_{t=s}^T (1 - e^{\rho(t-T)}) - x(s) - \frac{z-q}{\rho} e^{-\rho T} \tag{42}
\]

where, \( x(s) \) is interpreted as both the terminal value of young period and initial value of old period at the same time. By the same way as the previous, Eq. (32) is changed as follows

\[
c(t) = \begin{cases} 
\frac{1}{C_1} e^{\rho(s-t)} & \text{if } 0 \leq t \leq s \\
\frac{1}{C_1} e^{\rho(s-t)} & \text{if } s < t \leq T.
\end{cases}
\]  

(43)

Substituting Eq. (43) into the utility function, we have obtained the following

\[
\int_0^T e^{-\rho t} \ln \left( \frac{1}{C_1} e^{\rho(s-t)} \right) dt + \int_s^T e^{-\rho t} \ln \left( \frac{1}{C_1} e^{\rho(s-t)} \right) dt
\]  

(44)

We integrate Eq. (44) to time \( t \), then we have gotten

\[
\ln \left( \frac{1}{C_1} \right) \frac{1 - e^{\rho s}}{\rho} + \ln \left( \frac{1}{C_1} \right) e^{\rho s} e^{-\rho T} - \frac{(r - \rho)(\rho T + 1) e^{-\rho T} - 1}{\rho^2} \tag{45}
\]

There are \( s \) in \( C_1^y, C_2^y, C_1^o \) and \( C_2^o \). If we substitute \( z = \frac{T-s}{T} \) into \( C_1^y, C_2^y, C_1^o \) and \( C_2^o \), then, the original dynamic optimization problem with the pension system becomes static optimization problem with respect to \( T \) and \( x(s) \) as seen in Eq. (46). In other words, all he/she has to do is to decide his/her own life expectancy and the initial asset at the old period.

\[
\max_{T,s(x)} U(T, x(s)) = \ln \left( \frac{1}{C_1(T, x(s))} \right) \frac{1 - e^{\rho s}}{\rho} + \ln \left( \frac{1}{C_1(T, x(s))} \right) e^{\rho s} - e^{-\rho T} \tag{46}
\]

\[
- (r - \rho) \left( \frac{\rho T + 1) e^{-\rho T} - 1}{\rho^2} \right)
\]
4.1.2. Population structure and government budget

We assume that a certain number of people with endowment \( x_0 \) is born in every period. Even though time will go, in the economy, the number of population will be the same and the population structure will not change, because we have assumed \( a, b, r, \rho \) are constants, that is, the social environments surrounding individuals do not change over time.\(^{10}\) To simplify, we assume that there is no production division. Because the individuals are born with same endowment, the optimized \( T \)s, which the individuals choose, are also same. At a \( t \) period, the distribution of population structure is a uniform distribution from 0 year old to \( T \) year old as seen in Fig. 6.\(^{11}\) To be precise, it is difficult to divide the period into \( t \) and \( t+1 \), because we consider a continuous time.

The period-by-period budget constraints of government are given as follows,

\[
sp = (T - s)q. \tag{47}
\]

The government collects \( p \) from each individual between the age of 0 and \( s \) and gives \( q \) to each individual between the age of \( s \) and \( T \). It can be a pay-as-you-go pension system. Because the population structure does not change, the government’s budget constraint holds Eq. (47) in every period.

4.2. The unbalanced budget in case of lifetime certainty

First, we consider about unbalanced budget, in other words, the government can be free to make decisions about \( p, q \) and \( s \) without any constraint. It is an unrealistic assumption. Only few countries, which are rich in natural resources and are carefree about their government resources, for example, oil product countries, may do it. In the following section, we consider about the balanced budget, i.e., government budget constraint, that can be found in most of the countries in the world.

Taking the derivative of Eq. (46) with respect to \( T \) and \( x(s) \), and setting each first derivatives to zero, and solving the system of equations, we could obtain the optimal \( T^* \) and \( x(s)^* \). Since the profit function of Eq. (46) is highly non-linear and nested structure, it is very difficult to get an exact analytical solution for this problem. The alternative option is to provide the solutions numerically. The suitable parameter values are used for the calculation, though they are arbitrary. The parameter values that we use to calculate are the following: \( a = 20, b = 10, x_0 = 100, \rho = 0.01, r = 0.02 \). In order to investigate the effects of only the pension system, not including the effect of income, we put the initial income as the constant value. And we have controlled the parameters for pension system i.e., \( p, q \) and \( s \) which are the amount of payment for pension, the amount of pension gratuity and the period of payment for pension, respectively. To show the effects of \( p, q \) and \( s \) on the life expectancy and the lifetime utility, \( p \) and \( q \) are controlled from 0.0 to 2.0, respectively, and \( s \) is controlled from 0.0 to 20.0.

Because we deal with the three variables, \( p, q \) and \( s \), it is difficult to visualize the three variables at one plane at the same time. We put one variable fixed and change the remaining two variables to analyze the effect of the two variables on the life expectancy, the lifetime utility, and so on. In Section 4.2.1, we fix \( s \) and control \( p \) and \( q \). In Section 4.2.2, we fix \( q \) and control \( p \) and \( s \). In Section 4.2.3, we fix \( p \) and control \( q \) and \( s \).

4.2.1. The case where \( s \) is fixed

Each panel in Fig. 7 shows the results as the contour lines. Fig. 7 (1) and Fig. 7 (2) show the results of the life expectancy and the lifetime utility level, respectively, when \( s \) is fixed at 10.0 while \( p \) and \( q \) are changed. The red lines show the results of the benchmark model which has no pension system. In Fig. 7 (1) and Fig. 7 (2), the values on the left-upper side are high and the values on the right-lower side are low. Under fixed \( s \), when \( p \) is small and \( q \) is big, the life expectancy is longer and the lifetime utility level is higher. By a combination of \( p \) and \( q \), especially, when \( p \) is small and \( q \) is big, higher life expectancy and higher lifetime utility level can be realized compared to the results of benchmark model.

We have gotten a natural result in Fig. 7 (2) that if the period of payment for pension is fixed, when individuals pay smaller amount of money and get bigger amount of money from his/her pension, his/her lifetime utility level will be higher (on the left-upper side), otherwise, when individuals pay bigger amount of money and get smaller amount of money from his/her pension, his/her lifetime utility level will be lower comparing to the case of non-existing the pension system (on the right-lower side). The result accords with intuition.

\(^{10}\)As the population ages and fewer babies are born, pension system might cause inequality problem between young generation and old generation.

\(^{11}\)We only consider the unchanged period in population structure, which means the population structure is in the steady state, not in transitional path.
In Fig. 7 (3), we present the joint graphs of both Fig 7 (1) and Fig. 7 (2). Fig. 7 (3) shows that Fig. 7 (1) overlaid Fig. 7 (2). Comparing point “A” and point “B”, the life expectancy of point “A” is longer than that of point “B”, however, the lifetime utility level of point “B” is higher than that of point “A”. It is suggested that long life does not necessarily improve the lifetime utility level, that means, to live a short and intensely happy life can be better than to live a longer but less intensely happy life. We have calculated the difference of the present value at time 0 of benefit $q$ and the present value at time 0 of contribution $p$ using Eq. (48) and have shown the results in Fig. 7 (4). The contour lines shows upward curve in $p - q$ plane. We denote the difference of both present values as $PV$. Both the $PV$ contour lines and the lifetime utility contour lines are similar in the fact that both of them have positive slopes.

\[
PV = \int_{s}^{T} e^{-rt} q \, dt - \int_{0}^{s} e^{-rt} p \, dt = \frac{q}{p} (e^{-rs} - e^{-rT}) - \frac{p}{1 - e^{-rs}} \tag{48}
\]

4.2.2. The case where $q$ is fixed

The readings of Fig. 8 is the same with the readings of Fig. 7. Fig. 8 (1) and Fig. 8 (2) show the results of the life expectancy and the lifetime utility level, respectively, when $q$ is fixed at 1.0 while $p$ and $s$ are changed. The red lines show the results of the benchmark model which has no pension system. In Fig. 8 (1) and Fig. 8 (2), the values on the left-lower side are high and the values on the right-upper side are low. Under fixed $q$, when $p$ is small and $s$ is short, the life expectancy is longer and the lifetime utility level is higher. By a combination of $s$ and $p$, especially, when $s$ is short and $p$ is small, higher life expectancy and higher lifetime utility
level can be realized compared to the results of benchmark model.

We have gotten a natural result in Fig. 8 (2) that if the amount of pension gratuity is fixed, when individuals pay smaller amount of money for a short period of time, his/her lifetime utility level will be higher (on the left-lower side), otherwise, when individuals pay bigger amount of money for a long period of time, his/her lifetime utility level will be lower comparing to the case of non-existing the pension system (on the right-upper side). The result also accords with intuition.

In Fig. 8 (3), we present the joint graphs of both Fig 8 (1) and Fig. 8 (2). Fig. 8 (3) shows that Fig. 8 (1) overlaid Fig. 8 (2). Both contour lines draw quite parallel lines. The contour lines in Fig. 8 (4) show downward curve in $s - p$ plain. Both the PV contour lines and the lifetime utility contour lines are similar in the fact that both of them have negative slopes.

4.2.3. The case where $p$ is fixed

The readings of Fig. 9 is the same with the readings of Fig. 7 and Fig. 8. Fig. 9 (1) and Fig. 9 (2) show the results of the life expectancy and the lifetime utility level, respectively, when $p$ is fixed at 1.0 while $q$ and $s$ are changed. The red lines show the results of the benchmark model which has no pension system. In Fig. 9 (1) and Fig. 9 (2), the values on the right-lower side are high and the values on the left-upper side are low. Under fixed $p$, when $q$ is big and $s$ is short, the life expectancy is longer and the lifetime utility level is higher. By a combination of $q$ and $s$, especially, when $q$ is big and $s$ is short, higher life expectancy and higher lifetime utility level can be realized compared to the results...
of benchmark model.

We have gotten a natural result in Fig. 9 (2) that if the amount of payment for pension is fixed, when individuals get bigger amount of money from his/her pension and pay for a short period of time, his/her lifetime utility level will be higher (on the right-lower side), otherwise, when individuals get smaller amount of money from his/her pension and pay for a long period of time, his/her lifetime utility level will be lower comparing to the case of non-existing the pension system (on the left-upper side). The result also accords with intuition.

In Fig. 9 (3), we present the joint graphs of both Fig 9 (1) and Fig. 9 (2). Fig. 9 (3) shows that Fig. 9 (1) overlaid Fig. 9 (2). Comparing point “A” and point “B”, the life expectancy of point “A” is longer than that of point “B”, however, the lifetime utility level of point “B” is higher than that of point “A”. As it has been already suggested, to live a short and intensely happy life can be better than to live a longer and less intensely happy life, which means that long life does not necessarily improve the lifetime utility level. The contour lines in Fig. 9 (4) show downward curve in q – s plain. Both the PV contour lines and the lifetime utility contour lines are similar in the fact that both of them have positive slopes.

4.2.4. Grid search

We have gotten several natural results using the dynamic utility maximization problem. The results are that when p is small, when q is big, and when s is short, that is, when an individual pays a small amount of money for a short period of time and gets a big amount of money from his/her pension, the lifetime utility level
is higher. The increase in lifetime utility is hardly astonishing because the lifetime budget constraint of individuals increases when individuals receive more benefits from the pension system for which they do not have to pay so much. These results accord with intuition.

We will show the relationship among the life expectancy, the lifetime utility and the pension system through the combination of $p$, $q$ and $s$ and we will compare them with the results from the cross-country data in the Section 2.

Fig. 10 plots the relationship between the life expectancy and the lifetime utility level. By changing of the parameters for pension system, $p$, $q$ and $s$, which are the amount of payment for pension, the amount of pension gratuity and the period of payment for pension, respectively, we have gotten the pairs of the life expectancy and the lifetime utility. We have used the grid search to show the pairs. In using the grid search, we have to decide the range of three variables and the number of grids in advance. We choose an equispaced grid $\{p_1, \cdots, p_n\} = \{0.01, \cdots, 1.91\}$ for the amount of payment for pension $p$ with $n = 20$ nodes and $\{q_1, \cdots, q_n\} = \{0.05, \cdots, 1.95\}$ for the amount of pension gratuity $q$ with $n = 20$ nodes. For the period of payment for pension $s$, we also choose an equispaced grid $\{s_1, \cdots, s_n\} = \{0.01, \cdots, 19.01\}$ with $n = 20$ nodes.

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12The figures of the life expectancy tell nothing about the relative length of life expectancy. As the concept of the ordinal utility, the differences in the figures of the life expectancy are treated as meaningless. The figures do not mean the number of years.
The number of combinations of three variables $p$, $q$, and $s$ is 8,000. In Fig. 10, the horizontal line and the vertical line show the life expectancy and the lifetime utility level, respectively.

In Fig. 10, there are 8,000 dots which are the results of grid search and there is one dot which is point A $(LE, LU) = (24.556, 33.742)$ which shows the pair of the life expectancy and the lifetime utility level obtained from the benchmark model which has no pension system. All of these dots except the point A show the pairs when the pension system exists. We draw a vertical and horizontal line from the point A and divide the plain into 4 areas. In area I, the life expectancy is longer and the lifetime utility level is higher compared to the point A. In area II, the life expectancy is longer but the lifetime utility level is lower compared to the point A. In area III, the life expectancy is shorter and the lifetime utility level is lower compared to the point A. There is no pair in area IV.

The life expectancy is not always proportional to the lifetime utility level. When we compare any dot in the area II and the point A, even though the life expectancy of the dot in the area II is longer, its lifetime utility level is lower. Also, the pension system can make life expectancy longer or shorter and can make lifetime utility level higher or lower. If we get big amounts of pension in the future, the life expectancy can be extended and the lifetime utility can go up. It is the most preferable, however, in today’s reality, we cannot expect that the amount of the pension will increase due to the problem of financial resources.

The pension system could also lead to some kind of infirmity as follows: 1) Even though the life expectancy is extended, the lifetime utility level can go down. This is the case when he/she is forced to pay his/her pension, he/she chooses a dot in the area II, instead of the the
point A which is the best choice for individuals in case without pension system. Because of that an individual is forced to pay the pension during his/her young period, the pension system leads to less personal consumption in his/her young period. Even though he/she tries to prolong his/her life for a long time to get his/her money back which he/she paid mandatorily, his/her lifetime utility level can go down compared to the case without pension system. Even though rising longevity is incited by the pension system, the years they gain in life expectancy may not be healthy ones, so the increase in life expectancy requires more savings for health-care spending in his/her old age and less consumption through his/her whole life. It is also confirmed from the data in the Section 2 that the increase in life expectancy without an increase in income does not affect too much their happiness. 2) The life expectancy is decreased, moreover, the lifetime utility level can go down. This is the worst scenario. This is the case when he/she is forced to pay his/her pension, he/she chooses a dot in the area III, instead of the point A which is the best choice for individuals in case without pension system. He/She does not have enough money to invest for his/her health care because most of his/her money is paid for his/her pension. As an extreme example, we can take an individual who can choose a short life to refuse to pay the pension until such period s and to increase his/her consumption in his/her young period.13

4.3. The balanced budget in case of lifetime certainty

In the previous section, we have analyzed the relationship among the life expectancy, the lifetime utility and the pension system under the unbalanced budget of government which was an unrealistic assumption. From now on, we will analyze the relationship under the balanced budget of government. Under the budget constraint, the degree of freedom which the government can choose decreases by one. The government can only decide two variables among the three variables, p, q and s. We can explain it in the following order. In Section 4.3.1, the government decides about the two variables p and q while s is given by the budget constraint of government. In the Section 4.3.2, the government decides about the two variables s and p while q is given by the budget constraint of government. In Section 4.3.3, the government decides about the two variables q and s while p is given by the budget constraint of government. We will use the same values of parameter used in the Section 4.2, that is, p and q are controlled from 0.0 to 2.0, respectively, and s is controlled from 0.0 to 20.0.

4.3.1. The case where p and q are controlled

When p and q are controlled, s can be obtained from Eq. (49).

\[ s = \frac{Tq}{p+q} \]  

(49)

Fig. 11 (1) and Fig. 11 (2) show the results of the life expectancy and the lifetime utility level while p and q are changed under the government budget constraint, respectively. Fig. 11 (3) shows s when p and q are given. Fig. 11 (4) shows the PV, which is the difference of the present values of contribution p and the present values of benefit q, that is also calculated in the Section 4.2. Fig. 11 (1), (2), (3) and (4) show a blank portion on the left-upper area where the government budget constraint does not hold Eq. (49).14

From Fig. 11 (1), the life expectancy far from the point of origin (0,0) is longer comparing to the life expectancy near to the point of origin, that means the pension system makes the life expectancy extended, however, from Fig. 11 (2), the lifetime utility level at the point of origin is the highest, that is, it is impossible that the pension system makes the lifetime utility level higher under the government budget constraint. From Fig. 11 (1) and Fig. 11 (2), we know that when the life expectancy is short, the utility is higher and that even though the pension system makes the life expectancy longer, it does not make the lifetime utility level higher under the budget constraint. We can guess the reason why in order to make T longer, s also has to be longer to hold the budget constraint. From Fig. 11 (4), we know that by a combination of p and q under the budget constraint, the PV cannot be positive.

4.3.2. The case where s and p are controlled

When s and p are controlled, q can be obtained from Eq. (50).

\[ q = \frac{sp}{T-s} \]  

(50)

Fig. 12 (1) and Fig. 12 (2) show the results of the life expectancy and the lifetime utility level while s and

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13 There was an accident reported in South Korea in 2005, where a person, who was against the compulsory pension system and who was in arrears with his pension, took away his life. It is a case of the tail wagging the dog.

14 The government budget constraint does not always hold Eq. (49), because individuals decide T based on p, q and s. To describe Eq. (49) more accurately, \[ s = \frac{T_{PV_{c}}}{p+q} \].

21
$p$ are changed under the government budget constraint, respectively. Fig. 12 (3) shows $q$ when $s$ and $p$ are given. Fig. 12 (4) shows the PV. The upper-right areas are the areas where the government budget constraint does not hold Eq. (50).

We have gotten the same results, as in the case where $p$ and $q$ are controlled at the previous section, that from Fig. 12 (1), the further from the point of origin, the life expectancy is longer, however, from Fig. 12 (2), the further from the point of origin, the lower lifetime utility level is and the nearer to the point of origin, the higher lifetime utility level is. Even though the pension system makes the life expectancy longer, it does not make the lifetime utility level higher under the budget constraint. From Fig. 12 (4), we know that by a combination of $s$ and $p$ under the budget constraint, the PV cannot be positive.

4.3.3. The case where $q$ and $s$ are controlled

When $q$ and $s$ are controlled, $p$ can be obtained from Eq. (51).

$$p = q \left( \frac{T - s}{s} \right)$$  \hspace{1cm} (51)

Fig. 13 (1) and Fig. 13 (2) show the results of the life expectancy and the lifetime utility level while $s$ and $p$ are changed under the government budget constraint, respectively. Fig. 13 (3) shows $p$ when $q$ and $s$ are given. Fig. 13 (4) shows the PV. The lower-right areas are the areas where the government budget constraint does not hold Eq. (51).

We have gotten the same results as in the both previous cases, the shorter life expectancy, the higher lifetime
utility level is and that even though the pension system makes the life expectancy longer, it does not make lifetime utility level higher under the budget constraint. In Fig. 13 (2), if $s$ is same, when $q$ is small, the lifetime utility level is high. Although this may sound strange, because when $q$ is small, $p$ also should be small to hold the budget constraint. That can be confirmed from the PV in Fig. 13 (4).

4.3.4. Grid search

We have plotted the relationship between the life expectancy and the lifetime utility level under the unbalanced budget of government in Fig. 10 in the previous section. We have divided the dots into five areas, which are area i, area ii, area iii, area iv and area v, by the government budget constraint, the lifetime utility level and the life expectancy as follows,

\[
\begin{align*}
\text{area i,} & \quad \text{if } sp < (T - s)q \text{ and Lifetime utility} > LU \\
\text{area ii,} & \quad \text{if } sp < (T - s)q \text{ and Lifetime utility} < LU \\
\text{area iii,} & \quad \text{if } sp = (T - s)q \\
\text{area iv,} & \quad \text{if } sp > (T - s)q \text{ and Life expectancy} > LE \\
\text{area v,} & \quad \text{if } sp > (T - s)q \text{ and Life expectancy} < LE
\end{align*}
\]

Fig. 14 shows the results. In the area i and ii, the government revenue is less than the government expenditure for the pension. The area i and ii are unfeasible areas if there is no additional financial resources. The government should replenish the underfunded revenue by another way to meet the deficit budget e.g., raising tax or issuance of government bonds or sellout the natural resources, etc. The area iii, iv and v are feasible areas.
even if there is no additional financial resources. In the area iii, the government executes the balanced budget. In the area iv and v, the government expenditure for the pension is less than the government revenue. If pension system has any inefficiency which is liable to happen, the areas, iv and v, are possible.

From the area iii, we know that when the government holds the budget constraint, the lifetime utility level can not increase even though the life expectancy can prolong. In the area ii, when the government expenditures for pension are higher than the government revenues, even though the life expectancy increases, the lifetime utility goes down. On the contrary, in the area iv, when the government expenditures for pension are lower than the government revenues, even though the lifetime utility goes down, the life expectancy prolongs. Individuals know that if they live longer they will get more pension. They are motivated to live longer as it is the only way they could enjoy the pension they have been paying for a long time.

From these results, we can say that life expectancy can be longer when pension system exists, not only when the government has enough revenue but also when the government hardly has enough revenue, depending on how the government operates the pension system.

5. Pension system in case of lifetime uncertainty

Generally, we are prone to think that we have more need for a pension system in case that we do not know, due to uncertainty, when we will die exactly. In Section 5, we consider an uncertainty in lifetime.
5.1. Lifetime uncertainty model

We introduce an uncertainty in the \( T \) which individual chooses to maximize his/her lifetime utility. We add an uncertainty to Eq. (19).

\[
T = a + bz + \epsilon \quad (53)
\]

where, \( \epsilon \) is a random variable with \( E(\epsilon) = 0 \) and symmetric distribution with respect to the mean, \(-\phi \leq \epsilon \leq \phi\), where \( \phi \) is a positive constant. When \( \epsilon \) is positive, an individual lives longer than the planned period \( T \) and when \( \epsilon \) is negative, an individual dies earlier than the planned period \( T \). \( a + bz - \phi \leq T \leq a + bz + \phi \).

Because \( E(\epsilon) = 0 \), the expected longevity is \( E(T) = a + bz \). And the distributions of population by age look like Fig. 15 at any \( t \) period. Someone dies before \( T \)-year-old and someone continues to live more than \( T \). If there are lots of individuals in economy, the number of individuals who continue to live more than \( T \) will be equal to the number of individuals who die before \( T \), because the random variable \( \epsilon \) is symmetric with respect to zero. The amount of pension which is paid to the individual \( T \) years of age or older can be covered from the amount of pension which is collected from the individuals who die younger than \( T \). By law of large numbers, the budget constraint is no different from the one in the case of lifetime certainty in the previous section as Eq. (47).

We will compare Case I in which the individual saves an extra money by himself/herself for the extended period from \( T \) to \( T + \phi \) when he/she will be still alive unexpectedly and Case II in which the individual gets the benefit of the pension that has been analyzed in the previous section. If the individual is still alive and does not have money after period \( T \) which he/she has chosen optimally, he/she will have hard time. So, we assume that first, the individual chooses the \( T \) optimally.

---

\(^{15}\)There are many different ways to express the lifetime uncertainty problem, for example, we can model the uncertainty problem as follows, 
\[
\max E(U) = \int U(c(\epsilon)) f(\epsilon) \, d\epsilon,
\]
where \( f(\epsilon) \) means probability density function. However, this assumption makes the uncertainty problem very simple. We do not need to specify about the distribution of random variable \( \epsilon \), etc.
and then he/she maximizes his/her lifetime utility with respect to $T + \phi$ considering the case that he/she will be still alive until $T + \phi$. The individual’s utility maximization problem can be written as follows:

$$\max_{c(t)} \int_{0}^{T+\phi} e^{rt} \ln c(t) \, dt$$

where $T^*$ is the optimized $T$ in the Section 3.

5.2. Results

Fig. 16 shows that the green line, which shows the result of Case I, is added on Fig. 14 in the Section 4.3.4. The green line is located above the area iii which means the lifetime utility level of the Case I is higher than that of the Case II. Even though individual chooses $T^* + \phi$ as a second best instead of $T^*$ and saves his/her money for unexpected lifespan gain, his/her lifetime utility level is higher comparing to the case of existing the pension system. If the prediction of lifespan is completely wrong under lifetime uncertainty like point B which means $\phi$ is very big, the pension system will improve the lifetime utility level. The extended line “–?-?” from the area iii to the point B is drawn approximately but not exactly to show the area to hold the government budget constraint.

Even if the uncertainty has been introduced in this research model, the same conclusion could be reached, because the core of this problem is the existence or non-existence of the compulsory pension system which prevents the lifetime utility from maximizing. The existence of uncertainty is not of the essence in this research model.

6. Conclusion

This research has analyzed the effect of a pension system on life expectancy and happiness level using a cross country data and an optimal dynamic problem of individuals who live in continuous and finite time.
There are many studies which have analyzed the effect of rising longevity on pension system, but few studies have analyzed the opposite direction effect, which means how the pension system affects to the rising longevity. This is a contribution of this research.

We have gotten several interesting but radical results using the data and the optimization problem. Income per capita can almost explain happiness and the lifespan does not have much influence on happiness. Therefore, life expectancy itself is not always proportional to the happiness. The survival itself is not always making our happiness higher. The extension of lifespan only accompanied by the income support may make our happiness higher. Becker et al. (2005) mentions that life expectancy gains have been an important component of improvements in welfare, but that may be the case in the situation where their financial problems can be resolved.

Pension system can make lifespan longer or shorter and can raise or reduce the level of happiness, depending on income per capita and the type of pension system. Especially, under government budget constraint, even though pension system can make the lifespan longer, pension system cannot make the happiness level higher. If there is a pension system, we will try to live longer to get more pension, however, the public pension system, which is a compulsory saving, can crowd out the private savings and can prevent individual’s utility maximization. Even though there is an uncertainty in lifetime, unless the prediction of lifespan turns out to be completely wrong, there is a small possibility that the pension system will improve the happiness level. The pension system can rather raise problems for aging population which affect the country’s productivity and growth rate through the decline in the fraction of working-age population.

When the level of income per capita is low, the pension system can make the happiness higher, on the contrary, when the level of income per capita is high, the level of happiness may be higher when there is no pension system. It may be necessary to reconsider about the reason for existence of the compulsory pension system which has been a considerable economic and social burden on young generations.
Appendix

A.1 Derivation of Eq. (28)

Let us put $B = -C_1$. Multiplying both sides of Eq. (27) by $e^{-rt}$ and integrating to time $t$, we get the following

$$\int_0^t e^{-rt} \left( x - r x + z \right) e^{-rt} \, dt = B e^{-rt}$$

$$= B e^{-rt} + \int_0^t e^{-rt} z e^{-rt} + D_1 \, dt \quad \text{(A1)}$$

where $D_1$ and $D_2$ are constants of integration. Eq. (A1) can be arranged as follows

$$x e^{-rt} - \frac{z}{r} e^{-rt} = -B e^{-rt} + B \frac{1}{\rho} + C_2$$

$$\left( x - \frac{z}{r} \right) e^{-rt} = -B \left( \frac{e^{-rt} - 1}{\rho} \right) + C_2 \quad \text{(A2)}$$

where $C_2 = D_2 - D_1 - B \frac{1}{\rho}$. Multiplying both sides of Eq. (A2) by $e^{-rt}$ and substituting $B = -C_1$ into Eq. (A2), Eq. (A2) can be arranged as Eq. (28).

A.2 Derivation of Eq. (34)

Let us put $A = \ln \left( \frac{\mu - \left( 1 - e^{-rT} \right) \rho}{1 - e^{-rT}} \right)$. Then

$$\int_0^T e^{-rt} \ln \frac{\mu_0 - (1 - e^{-rT}) \rho}{1 - e^{-rT}} e^{(r-\rho)t} \, dt$$

$$= A \int_0^T e^{-rt} dt + (r-\rho) \int_0^T te^{-rt} \, dt \quad \text{(A3)}$$

$$= A \left[ \frac{e^{-rt}}{\rho} \right]_0^T + (r-\rho) \left[ \frac{(\rho t + 1)e^{-rt}}{\rho^2} \right]_0^T$$

$$= -A \left( \frac{e^{-rt} - 1}{\rho} \right) - (r-\rho) \left( \frac{(\rho T + 1)e^{-rT} - 1}{\rho^2} \right)$$

Substituting $A = \ln \left( \frac{\mu - \left( 1 - e^{-rT} \right) \rho}{1 - e^{-rT}} \right)$ into Eq. (A3), Eq. (A3) can be arranged as Eq. (34).

References


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