# Law of one price and optimal consumption-leisure choice under price dispersion

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**Abstract**

If the demand under price dispersion is formed by consumers with zero search costs and consumers with positive search costs, the law of one price holds at the equilibrium price level, where the lowest willingness to pay between consumers with zero search costs meets the willingness to accept or to sell of consumers with positive search costs. The equilibrium price level is provided by the individual equality of marginal losses in labor income during the search with marginal savings on purchase. Suboptimal decisions of consumers with positive search costs create an opportunity of arbitrage with willingness to pay at the zero costs search level that results in a new equilibrium price and in optimal consumption-leisure choices of all consumers.

**Key words:** equilibrium price, consumption-leisure choice, cost of search, price dispersion, willingness to pay, willingness to accept
**JEL Classification**: D11, D83.

## Introduction

During last decades the problem of price dispersion has become one of the most intriguing issues of modern economics (Adams (1997), Burdett and Judd (1983), Carlson and McAfee (1983), Diamond (1971, 1987), Fishman (1992), Janssen and Moraga-González (2004), Janssen, Moraga-González, and Wildenbeest (2005), Lach (2002), Manning (1997), Pratt, Wise, and Zeckhauser (1979), Reinsdorf (1994), Rosenthal (1980), Rothschild (1974), Salop and Stiglitz (1977,1982), Stahl (1989), Stigler (1961), Stiglitz (1979), Varian (1980)). In 1994 J.McMillan and M.Rothschild summarized the growing interest to the question of price dispersion in the "Handbook of Game Theory". In 2006 M.R.Baye, J.Morgan and P.Scholten presented the comprehensive overview of that problem for "Economics and Information Systems" where they introduced that phenomenon with the proposition that empirical studies had revealed the fact that price dispersion was the rule rather than the exception in many homogeneous product markets. In addition, the authors addressed to the other famous introduction: "Economists have belatedly come to recognize that the "law of one price" is no law at all" (Varian 1980,p.651).

Consumers' heterogeneity is one of the most obvious reasons for the persistence of price dispersion. Usually the analytical approaches to consumers' heterogeneity envisage two groups of consumers. There are consumers that do not search, i.e., price-takers, and there are consumers that search for low price. The model of the optimal consumption-leisure choice under price dispersion (Malakhov 2013,2014a,2014b,2014c,2015) also uses this dual approach: "Some consumers have zero search costs, while all others have a positive search cost" (Stahl 1989,p.700). Examining *shoppers* with zero search costs and *searchers* with positive search costs, the model proposes some additional reasonings to the question whether Walras' law holds or not under price dispersion in homogeneous product markets.

The model of the optimal consumption-leisure choice under price dispersion argues that in everyday economic activity a consumer makes satisficing buying decisions that equalize marginal costs of search with its marginal benefits. The model describes the analytical framework that demonstrates why an explicit satisficing decision becomes optimal. Observing consumers that differ in their willingness to pay for the single unit they are trying to buy, this paper specifies the role of optimization of search costs in the establishment of the equilibrium price level.

**Willingness to pay, equilibrium price, and willingness to accept**

The optimal consumer choice under price dispersion represents the result of the trade-off between consumption and leisure with respect to two constraints – the wage rate $w$ and marginal savings on purchase, i.e., the price reduction with regard to the time of search at the moment of purchase $\partial P/\partial S<0$. The trade-off between consumption $Q$ and leisure $H$ is provided by the propensity to search $\partial L/\partial S<0$, i.e., the propensity to substitute labor for search as for another source of income. When the problem of the maximization of consumption-leisure utility $U(Q,H)$ is constrained by the equality of marginal values of search $w\partial L/\partial S=Q\partial P/\partial S$, where the left side of the equation represents the value of marginal loss in labor income during the search and the right side represents the value of marginal benefit of search, the marginal rate of substitution of leisure for consumption takes the following form (Fig.1):

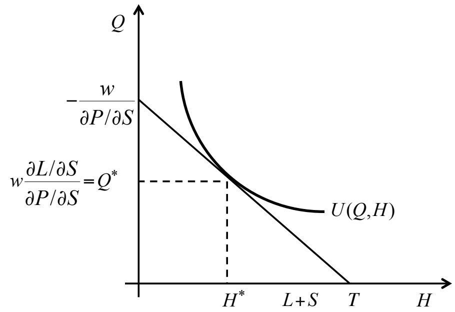

**Fig.1.Optimal consumption-leisure choice**

$$\frac{\partial U/\partial H}{\partial U/\partial Q} = -\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H \quad (1)$$

The analysis of the propensity to search discovers the time-based structure of this apparently complex psychological variable with respect to the time horizon $T=L+S+H$ of the consumption-leisure choice (Malakhov 2013,2015):

$$w\frac{\partial L}{\partial S} = -w\frac{L+S}{T} \quad (2)$$

And the derivative of the propensity to search with respect to leisure time simplifies the presentation of the *MRS(H for Q)*:

$$-\frac{L+S}{T} = \frac{H-T}{T} \Rightarrow \partial^2 L/\partial S\partial H = \frac{1}{T} \quad (3)$$

$$\frac{\partial U/\partial H}{\partial U/\partial Q} = -\frac{w}{T\partial P/\partial S} \quad (4)$$

And we see that this consideration gives us another form of the constraint for the utility maximization problem:

$$w\frac{\partial L}{\partial S} = Q\frac{\partial P}{\partial S} \Rightarrow w(L+S) = -T\partial P/\partial S \quad (5)$$

For the moment we get some value for the potential labor income $w(L+S)$ that is equal to a -$T\partial P/\partial S$ value. But the choice of the sequential optics for the search process explains the nature of Equation (5). Indeed, the static maximization problem simply requires the equality of marginal values of search $w\partial L/\partial S=Q\partial P/\partial S$. It tells us that any optimal choice should respect this equality. However, the choice can be the result of the sequential search for the predetermined quantity along the dispersion of prices that produce different marginal savings on purchase. Thus, the final decision, for example, the choice of the first quote below the reservation level of labor income $wL<wL_0$, that could be spent and restored after the purchase, determines the price of purchase $P_P=wL$ for the given quantity $Q$ and the corresponding marginal savings $\partial P/\partial S$. If this satisficing decision is optimal we get the following picture (Fig.2):

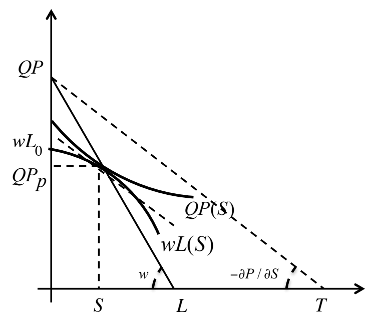


**Fig.2. Satisficing optimal decision**

We see that the equation $w(L+S)=-QT\partial P/\partial S$ generates some price $P$ at the zero-search-costs level. This price is greater than the willingness to pay $WTP=wL_0$. From one side, this price is equal to the potential labor income and from the other side it is equal to the rate of its reduction or "depreciation" during the lifecycle or utilization of an item until the following purchase of the same item.

To understand better the value of potential labor income let us take home production, say, preparing a meal, as a form of search. In this case the purchase price is equal to the price of inputs for home production or meal's ingredients. This assumption gives us an understanding that the value of potential labor income is equal to the willingness to accept or to sell the prepared meal. The same happens when a consumer decides at what price he should sell the bought item. If a consumer decides to sell an item he should recover not only labor costs $wL$ but also search costs $wS$.

The right side of Equation (5) represents the fall in price that happens when the search diminishes the time of consumption and therefore the attractiveness of an item to be bought. The fall in price, if we follow the assumption of the diminishing efficiency of search, or $\partial^2P/\partial S^2>0$, results in the fall of the absolute value of price reduction $|\partial P/\partial S|$ in a way that every price has its corresponding value of price reduction. For example, if search comes to correctly determined expiration date, the price of item should be equal to zero. Hence, we can suppose that at the zero-search-costs level this price exhibits the full attractiveness of an item. All other prices below this level need search.

Who can pay this price? Obviously, there are consumers that are not interesting in search. However, zero search costs don't eliminate the propensity to search. Equation (2) simply takes the following form:

$$w\frac{\partial L}{\partial S} = -w\frac{L}{T} \quad (6)$$

The search is not interesting for *shoppers* because at this price level, let's take for illustrative simplicity a single unit purchase, the search produces marginal savings that are not greater then marginal losses in labor income, or:

$$\left|-w\frac{L}{T}\right| \geq \left|\frac{\partial P}{\partial S}\right| \quad (7)$$

It means that the zero-search-costs level collects all buyers with willingness to pay higher or equal to the price that represents the full attractiveness of an item. However, the inequality of marginal values of search is not stable. *Shoppers* with very high wage rate need less time to restore their cash balances and they reduce the expected time-horizon. The cut in the expected time horizon by saving in labor time decreases the absolute value of propensity to search (Fig.3):

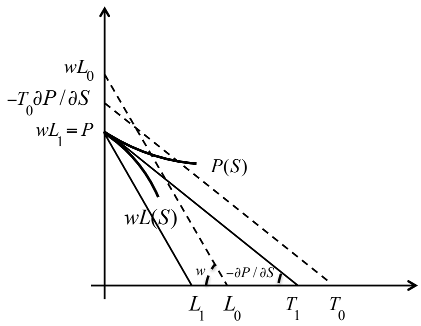


**Fig.3.Adjustment of high WTP to equilibrium price level**

$$wL_0 >> P; \left|w\frac{L_0}{T_0}\right| > \left|\frac{\partial P}{\partial S}\right| \Rightarrow wL_0 > -T_0\frac{\partial P}{\partial S};$$

$$L_1 < L_0; T_1 = T_0 - dL;$$

$$\left|w\frac{L_1}{T_1}\right| = \left|\frac{\partial P}{\partial S}\right| \Rightarrow wL_1 = P = -T_1\frac{\partial P}{\partial S} \quad (8)$$

The process of adjustment of time horizon of consumers with different high willingness to pay eliminates the inequality of marginal values of search in Equation (7) and all consumers with high willingness to pay equalize their marginal losses in labor income with marginal savings at this price level:

$$w_a L_a = w_b L_b = ... = w_n L_n = -T\partial P / \partial S = P_e \quad (9)$$

We see that at this price level market adjusts different perceptions of time horizons and makes itself really homogenous with the unique time horizon for a product.

This is the level of equilibrium price. If we do not take into account for the moment the existence of upper price niche where consumers with high willingness to pay, suffering from the "snob effect" at the equilibrium price level, can search and make ambitious purchases, we can say that the equilibrium price level is equal to the lowest willingness to pay between high-income consumers with zero search costs.

For the moment, these considerations follow the assumption that "the price in the high-price stores is the reservation price of shoppers with high willingness to pay, not their maximum willingness to pay for the good" (Diamond 1987, p.434). However, the possibility to adjust time horizon attracts to this price level or to the high-price store also some low-income consumers. Impatient low-income consumers can compensate at this price level the low wage rate by high propensity to search that results in earlier and more intensive consumption. Here, the decrease in the time horizon happens when consumer reduces the time of leisure (Malakhov 2014a). The reduction in the time horizon transforms the initial inequality of marginal values of search that should encourage the search into the optimal equation, which eliminates the need to look or to wait for low price:

$$\left| w\frac{\partial L}{\partial S} \right| < \left| \frac{\partial P}{\partial S} \right|; -w\frac{L}{T_0} < \frac{\partial P}{\partial S}; T_1 < T_0; -w\frac{L}{T_1} = \frac{\partial P}{\partial S} \quad (10)$$

In fact, the famous example of tourists, looking for a restaurant (Salop and Stiglitz 1977), can be revised under this assumption of impatience. Even low-income tourists don't want to waste time, they choose the restaurant for a lunch on their way, and in the evening they become hungry earlier and they are ready to take a dinner.

However, easy-going low-income consumers can take an advantage of their low wage rates and low propensity to search with respect to great marginal savings produced by the equilibrium price level. *Searchers* begin to look for low prices with regard to their willingness to pay. Some of them search in out-of-town commercial centers and some of them at factories' outlets. However, wherever they make purchases their willingness to accept comes to the equilibrium price level (Fig.4), or:

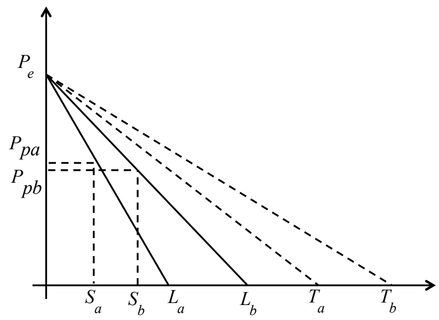


**Fig.4. Equilibrium price and WTA of heterogeneous searchers**

$$w_a(L_a + S_a) = -T_a \partial P_a / \partial S_a = w_b(L_b + S_b) = -T_b \partial P_b / \partial S_b = \ldots = w_n(L_n + S_n) = -T_n \partial P_n / \partial S_n = P_e \quad (11)$$

What happens when price dispersion is distorted and some prices don't result in corresponding marginal savings? This is the same thing that takes place when satisficing decision seems to be suboptimal. Generally, *searchers* begin to look for low prices when the search is interesting, or the marginal loss in labor income is less than the marginal saving:

$$\left| w\frac{\partial L}{\partial S} \right| < \left| \frac{\partial P}{\partial S} \right| \quad (12)$$

Let us suppose that the satisficing choice of the first quote below the reservation level ($\partial^2 L/\partial S^2 < 0$) stays suboptimal in accordance with Equation (12). However, if it is suboptimal, the *searchers*' willingness to accept or to sell stops below the equilibrium price level, more definitely, below the lowest zero-search-costs willingness to pay (Fig.5), or:

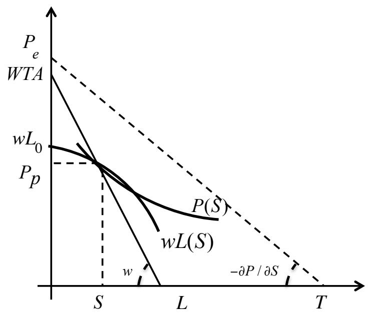


**Fig.5. Resale of suboptimal purchase**

$$\left| w\frac{\partial L}{\partial S} \right| < \left| \frac{\partial P}{\partial S} \right|; \left| -w\frac{L+S}{T} \right| < \left| \frac{\partial P}{\partial S} \right| \Rightarrow w(L+S) < -T\frac{\partial P}{\partial S} = P_e \quad (13)$$

If it happens, *searchers* will sell their purchases to *shoppers*. This extra supply drops the equilibrium zero-search-costs price level to the level where arbitrage becomes unprofitable for *searchers*, i.e., to the level where they equalize marginal values of search, and to its turn this equality matches their willingness to accept with a new equilibrium price. In addition, this new equilibrium price level reinforces the team of *shoppers* by newcomers from lower income bracket of *searchers* that makes the high-price store noisy and the equilibrium price level really becomes intolerable for snobs among *shoppers*.

The same effect takes place when a *searcher* finds an unexpected great discount, which results in unexpected low price. And, facing Equation (12), either the *searcher* adjusts the time horizon of his choice according to Equation (10), for example, due to shorten shelf-life of a product (Malakhov 2014a), or he makes an arbitrage. Adjustments of time horizon, i.e., decision to cut or to extend products' lifecycles at the moment of purchase reduce possibilities of arbitrage.[1] It

[1] The analysis of the increase in the time horizon with the increase in quantity to be purchased when the quote is dissatisfying is presented in Malakhov (2014b).

means that we can expect resale to be a common economic phenomenon in markets with fixed time horizons where adjustments are not possible. And it really takes place in markets of tickets for events (Courty 2003).

Hence, arbitrage and adjustment of time horizon transform suboptimal decisions of *searchers* into satisficing optimal choices. Finally, the equilibrium price level collects different willingness to accept of *searchers* with different wage rates and different propensities to search.

Thus, the equilibrium price is equal to the willingness to accept of *searchers*, which is equal to the lowest willingness to pay of *shoppers* with zero search costs and where *all individuals* equalize their marginal losses in labor income with their marginal savings on purchases.

This assumption takes us back to the classical optimal consumption-leisure choice:

$$\frac{\partial U/\partial H}{\partial U/\partial Q} = -\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H = -\frac{w}{T\partial P/\partial S} = \frac{w}{w(L+S)} = \frac{w}{P_e} \quad (14)$$

We see, that slight modifications in George Stigler's revolutionary equation do not change the general economic sense of exchange. The detailed answer to the question, whether sellers agree with that conclusion or not, stays beyond the scope of this paper but it might be framed by some comments.

In the model presented here sellers meet very different willingness to pay. They try to discriminate *shoppers* and to propose additional services to consumers with very high willingness to pay that could suffer from the "snob effect" at the equilibrium price level in order to separate them. The discrimination might be explicit when sales are made on high streets where consumers get a positive externality of prestige purchases, or implicit, like it happens in web 'clearinghouses' where a set of different prices does not exhibit the total sellers' heterogeneity. However, *searchers* can adjust their propensity to search to different quotes when they reconsider time horizons of their purchases with regard to seller's reputation, post-purchase services, etc. When this uncertainty begins to worry *shoppers* they become *searchers*. They either begin to investigate seller's reputation, or they look for a new market without problem. The appearance of stable upper price niche, i.e., the organization of a new market, does not change the logic of consumers' decision-making. The search model presented here slightly decorates a Walrasian market. The stable upper price niche can be considered as a new market if there a new group of zero-search-costs consumers arises. If such a group appears, other consumers with high willingness to pay become *searchers* and they either make satisficing optimal purchases when they search for prestigious items, or their purchases are suboptimal and these new *searchers* immediately find some *shopper* who can buy at zero search costs an item that has been already searched and bought. And numerous web sites for resale of luxuries demonstrate that it is possible. If such a group does not appear, all consumers with high willingness to pay can make resale only at the original equilibrium price level and a consumer who has overpaid for an item can sell it only with a loss at this level to other zero-search-costs consumers.[2]

The sellers' tactics in front of *searchers* is definitely artless – they redistribute transaction costs in order to charge consumers' costs of search, especially when search entails travel costs. The idea to sell for *searchers* at the equilibrium price level is not fruitful even if sellers have a power to reinforce consumers or they close points of sale for low prices; they incorporate all transaction costs and sell at the zero-search-costs level in the high-price store. If *searchers* should buy at the equilibrium price, they will bring to labor market all time of search. This extra labor supply decreases wage rates and makes the equilibrium price level unattainable.

However, the opportunity to redistribute transaction costs creates the other group of consumers that choose living near factory outlets. These marginal consumers get advantage of the neighboring lowest price level; they have zero search costs and the opportunity to make, legally or illegally, the resale. Really, Saturday markets in many countries exhibit both ways. To understand better the shopping behavior of these marginal consumers it is not sufficient to take into account only direct arbitrage costs. Quarters around a factory outlet can create negative externalities for *searchers* from upper income brackets, for example, at unsecured parking, and coming back from the shop, they could discover deep scratches on the car.

**Conclusion**

The analysis of propensity to search that optimizes satisficing purchasing decisions shows that the "law of one price' holds in an imperfect homogenous market if there are consumers with zero search costs. These consumers have different willingness to pay but they make purchases at the level of the lowest zero-search-costs willingness to pay. Consumers with positive search costs are also heterogeneous but they have the same willingness to accept or to sell that matches the lowest willingness to pay of consumers with zero search costs at the equilibrium price level. Arbitrage adjusts not only the price level but also the propensity to search that equalizes marginal losses in labor income during the search with marginal savings on purchase on a new equilibrium price level.

The equilibrium price level does not eliminate price dispersion. Consumers have different willingness to pay that meet heterogeneous sellers. Sellers try to discriminate consumers and, if they find a zero-search-costs demand, the new market is organized.

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[2] The satisficing purchases are made within the "common model" of behavior even on markets of luxuries. This is not true for the "leisure model" of behavior that produces Veblen effect (Malakhov 2015)

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